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Abstract

We study a contracting model of leverage and balance sheet size for financial intermediaries that fund their activities through collateralized borrowing. Leverage and balance sheet size increase together when measured risks decrease. When the loss distribution is exponential, the behavior of intermediaries conforms to the Value-at-Risk (VaR) rule, in which exposure is adjusted to maintain a constant probability of default. In a system context, increased risk reduces the debt capacity of the financial system as a whole, giving rise to amplified de-leveraging by institutions through the chain of repo transactions.

Key words: security brokers and dealers, contracting in financial institutions

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1. Introduction

Fluctuations in leverage have come to the fore in the debate on the propagation of financial distress. In particular, the phenomenon of "de-leveraging" in which financial intermediaries as a group attempt to contract their balance sheets simultaeneously has received attention as a key part in the propagation mechanism.[1]

When the financial system as a whole holds long-term, illiquid assets financed by short-term liabilities, a synchronized contraction of balance sheets will cause stresses that show up somewhere in the system. Even if some institutions can adjust down their balance sheets flexibly in response to the greater stress, not everyone can. This is because the system as a whole has a maturity mismatch. Something has to give. There will be pinch points in the system that will be exposed by the de-leveraging, and these are the institutions that suffer a liquidity crisis. Arguably, Bear Stearns was precisely such a pinch point in the de-leveraging episode in March 2008.

Fluctuations in leverage are therefore central to the proper understanding of financial market distress. Our task in this paper is a foundational one in which we model the determination of leverage and balance sheet size of financial intermediaries in their dealings with their creditors. Financial intermediaries borrow in order to lend. Hence, they are both debtors as well as creditors. As such, the bilateral contracting model can then be embedded in a system context to examine liquidity crises.

In a world where the Modigliani and Miller (MM) theorems hold, we can separate the decision on the size of the balance sheet (selection of the projects to take on) from the financing of the projects (debt versus equity). Even when

[1]See Brunnermeier (2008). Policy makers have also focused on this issue. See, for example, the minutes of the March 2008 meeting of the Federal Open Market Committee http://www.federalreserve.gov/monetarypolicy/files/fomcminutes20080318.pdf
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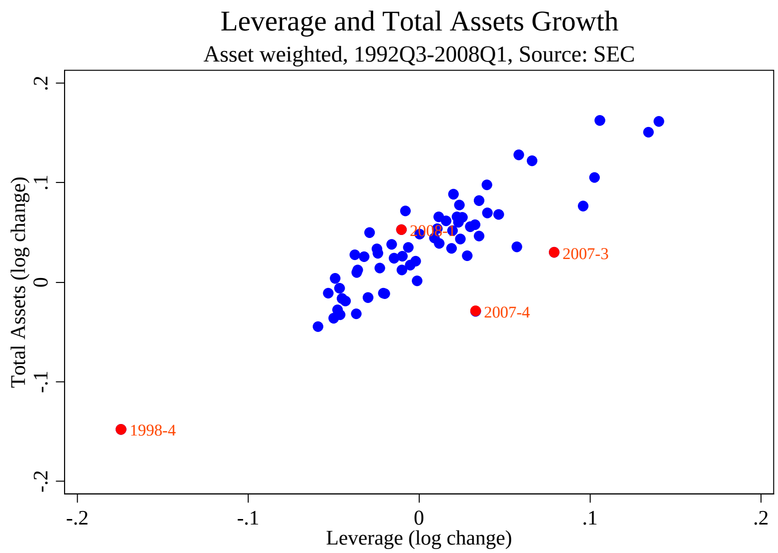

Figure 1.1: Leverage of US Investment Banks

the conditions for the MM theorems do not hold, it is often convenient to take the assets of the firm as given, in order to focus on the financing decision alone, as emphasized in corporate finance textbooks such as Brealey, Myers and Allen (2008) or Ross, Westerfield and Jaffe (2006). The literature on optimal capital structure in the tradition of the Merton (1974) approach also takes this route.[2]

However, the evidence on the capital structure of financial intermediaries points to a reversal of the usual order. Instead of assets being the driving variable, the evidence points to *equity* as being the driving variable. Total assets adjust once equity is given. The evidence is in figure 1.1, drawn from Adrian and Shin (2007). This figure plots quarterly changes in total assets against quarterly changes in leverage for US investment banks[3]. In figure 1.1, the horizontal axis

[2]See, for instance, Leland (1994), Leland and Toft (1996) and Anderson and Sundaresan (1996).

[3]Value weighted changes for the (then) five stand-alone US investment banks - namely, Bear

measures the quarterly percentage change in leverage, as measured by the change in log assets minus the change in log equity. The vertical axis measures the percentage change in assets as given by the change in log assets. Hence, the 45-degree line indicates the set of points where equity is unchanged. Above the 45-degree line equity is increasing, while below the 45-degree line, equity is decreasing. Any straight line with slope equal to 1 indicates constant growth of equity, with the intercept giving the growth rate of equity.

The key feature of figure 1.1 is that the slope of the fitted line in the scatter chart is close to 1, implying that equity is increasing at a constant rate on average. Thus, equity seems to play the role of the forcing variable, and all the adjustment in assets take place through shifts in leverage. The resulting flucutations in assets can be very substantial. For instance, between the third and fourth quarters of 1998 (during the LTCM crisis), the balance sheets of US investment banks contracted by 15%.[4]

One useful perspective on the matter is to consider the implicit maximum leverage that is permitted in collateralized borrowing transactions such as repurchase agreements (repos). Repos are the primary source of funding for market-based financial institutions, as well as being the marginal source of funding for traditional banks. In a repurchase agreement, the borrower sells a security today for a price below the current market price on the understanding that it will buy it back in the future at a pre-agreed price. The difference between the current market price of the security and the price at which it is sold is called the "haircut" in the repo, and fluctuates together with funding conditions in the market.

The fluctuations in the haircut largely determine the degree of funding available to a leveraged institution. The reason is that the haircut determines the

Stearns, Goldman Sachs,Lehman Brothers, Merrill Lynch and Morgan Stanley.

[4]See Adrian and Shin (2008a) and Greenlaw, et al. (2008) for why there has not been a similar contraction in balance sheets in the credit crisis of 2007/8.

maximum permissible leverage achieved by the borrower. If the haircut is 2%, the borrower can borrow 98 dollars for 100 dollars worth of securities pledged. Then, to hold 100 dollars worth of securities, the borrower must come up with 2 dollars of equity. Thus, if the repo haircut is 2%, the maximum permissible leverage (ratio of assets to equity) is 50.

Suppose that the borrower leverages up the maximum permitted level. Such an action would be consistent with the objective of maximizing the return on equity, since leverage magnifies return on equity. The borrower thus has a highly leveraged balance sheet with leverage of 50. If at this time, a shock to the financial system raises the market haircut, then the borrower faces a predicament. Suppose that the haircut rises to 4%. Then, the permitted leverage halves to 25, from 50. The borrower then faces a hard choice. Either it must raise new equity so that its equity doubles from its previous level, or it must sell half its assets, or some combination of both. However, asset disposals have spillover effects that exacerbate the distress for others. The "margin spiral" described by Brunnermeier and Pedersen (2007) models this type of phenomenon.

Considerations of repo haircuts suggest that measured risks will play a pivotal role in the determination of leverage. Our contracting model yields this outcome as a central prediction, and there is strong empirical backing for the prediction. We find that measures of Value-at-Risk (VaR) that are computed from the time series of daily equity returns explains shifts in total assets, leverage, and key components of the liabilities side of the balance sheet, such as the stock of repos.

In the benchmark case where losses are exponentially distributed, our contracting model yields the widely used Value-at-Risk (VaR) rule which stipulates that exposures be adjusted continuously so that equity exactly matches total Value-at-Risk. Among other things, the Value-at-Risk rule implies that exposures are adjusted continuously so that the probability of default is kept constant - at the level given by the VaR threshold. Given the ubiquitous use of the VaR rule both by private sector financial institutions and by the regulators of the Basel Committee, a microfoundations of the concept deserves serious attention. However, in spite of the widespread use of the concept, there has been comparatively little attention paid to the theoretical underpinnings of Value-at-Risk. One of our aims is to redress the balance.

To be sure, showing that the VaR rule is the outcome of a contracting model says little about the desirability of the widespread adoption of such practices from the point of view of economic efficiency. Indeed, we show that there are strong arguments to suggest that risk management tools such as Value-at-Risk causes spillover effects to other financial institutions that are detrimental to overall efficiency. For instance, the prudent reduction in exposures by the creditors to Bear Stearns is a run from the point of view of Bear Stearns itself. The spillover effects are a natural consequence of the contracts being bilateral arrangements. They do not take account of the spillover effects across more than one step in the financial network.

The plan of the paper is as follows. We begin in the next section by placing our approach in the context of the wider literature. We then describe the contracting environment. We proceed to solve for the optimal contract in our environment, and show the key comparative statics of the waxing and waning of leverage in response to shifts in underlying risk. Value-at-Risk is then shown to be an outcome in a special case of our framework where the loss distribution is exponential. We conclude with some implications of our analysis in a system context, where the financial system is modeled as a network of contracting relationships in which a creditor to a bank happens to be another bank, and draw some lessons for the credit crisis of 2007/8.

2. Related Literature

The classical approach to principal-agent contracting problems in the tradition of Mirrlees (1999), Holmstrom (1979) and Grossman and Hart (1983) have focused on eliciting the optimal level of effort given the tradeoff with risk-sharing. As such, a common approach has been to rely on the monotone likelihood ratio property (MLRP) where high effort is more likely to result in higher signals, and thereby better rewarded in the optimal contract. Innes (1990) has shown that debt is the optimal financial contract in these circumstances.

However, when risk is involved, the focus changes from first moments to second moments. Equity as a buffer takes center stage. Therefore, a more promising starting point are models where borrowers need to keep a stake in the project to align incentives. Holmstrom and Tirole (1997) is an example of such an approach. Chiesa (2001), Plantin and Rochet (2006) and Cerasi and Rochet (2007) show how regulatory capital in banking and in insurance can be rationalized in terms of such an approach. To the extent that the agent's stake can be interpreted as equity and the outside investor's stake is interpreted as debt, the Holmstrom and Tirole (1997) model provides a way to address how leverage is determined in the firm. However, as acknowledged by the authors themselves, their model is not well suited to answer questions concerning whether the outside financing is in the form of equity or debt since the outcome space in their model is binary - success or failure. Nor is the Holmstrom-Tirole model well suited to conduct comparative statics on shifts in risk.

Our approach is to build on Holmstrom and Tirole (1997) for the analysis of financial firms. The minimum stake held by the agent is the "haircut" in a repurchase agreement. We can solve for both the face value of debt (the promised repurchase price) and the market value of debt (the amount that can be raised by pledging assets). The key determinant of the haircut is the severity of the moral hazard induced by the underlying risks in the environment. When risks are heightened, moral hazard has a larger consequence. Haircuts must rise so as to restore incentives. In this way, we provide a theory of leverage as a function of the shifts in the risks inherent in the underlying environment.

In particular, we derive a set of special conditions on the distribution over asset outcomes that gives rise to the use of the Value-at-Risk (VaR) rule in which exposures are adjusted continuously to be matched with available capital, so as to leave the probability of default constant. Giving microfoundations for the use of Value-at-Risk has been an important unresolved issue in financial economics, especially given the ubiquitious use of Value-at-Risk at financial institutions as well as for the Basel capital regulations. Our modeling exercise addresses this need.

Our paper complements the recent literature on funding liquidity and asset pricing, and the role that leverage plays in the propagation of financial crises. Brunnermeier and Pedersen (2007) and Gromb and Vayanos (2002) have emphasized the importance of modeling the interaction of funding liquidity (the ease of borrowing) with market liquidity (the price impact of sales). Our approach provides potential avenues for giving further microfoundations for such effects.

More ambitiously, the goal would be to inform investigations into aggregate fluctuations and asset pricing. The work of Kiyotaki and Moore (2002, 2005) has focused on the implications for macroeconomic fluctuations. Asset pricing with liquidity effects are another important avenue of investigation. He and Krishnamurthy (2007) is a model of the asset pricing consequences of limited intermediary capital.

3. Model

There is one principal and one agent. Both the principal and agent are risk-neutral. The agent is a financial intermediary that finances its operation through collaterateralized borrowing. For ease of reference, we will simply refer to the agent as the "bank". The principal is the creditor to the bank. By modeling the bank as the agent in the principal agent relationship, we focus our attention on the dual nature of the bank, both as a lender but also as a borrower. It is the bank's status as the borrower that will be the key to our analysis. In this section, we treat the principal and agent in isolation from their other relationships. In a later section, the creditor is also a bank, but wears the hat as the lender to another bank.

There are two dates - date 0 and date 1. The bank invests in assets at date 0 and reaps any rewards and repays its creditors at date 1. The bank starts with fixed equity E, and chooses the size of its balance sheet. Denote by A the market value of assets of the bank, which we may interpret as the initial price of securities purchased by the bank at date 0. For instance, the assets could be risky fixed income securities, such as corporate bonds or mortgage-backed securities. The notional value of the securities is $(1 + \bar{r})\, A$, so that each dollar's worth of securities bought by the bank at date 0 is backed by a promise to repay $1+\bar{r}$ dollars at date 1.

The holding of securities worth A is funded partly by pledging the securities in a repurchase agreement. The bank sells the assets worth A for price D at date 0, and agrees to repurchase the assets at date 1 for price $\bar{D}$. Any shortfall $A - D$ is met by equity financing by the agent. Let E be the value of equity financing. The balance sheet in market values at date 0 is therefore

Assets	Liabilities
Assets A	Debt D
	Equity E

(3.1)

The notional value of the securities is $(1+\bar{r})\,A$, and the notional value of debt is the repurchase price $\bar{D}$. Thus, the balance sheet in notional values can be written as

Assets	Liabilities
Assets $A\,(1+\bar{r})$	Debt $\bar{D}$
	Equity $\bar{E}$

(3.2)

where $\bar{E}$ is the notional value of equity that sets the two sides of the balance sheet equal. The bank has the choice between two types of securities - good securities and bad securities. For each dollar invested at date 0, the bank can buy notional value of $1+\bar{r}$ of either security. However, for each dollar invested at date 0 the good security has expected payoff

$$1+r_H$$

with outcome density $f_H\,(.)$. The bad security has expected payoff $1+r_L$ with density $f_L\,(.)$. We assume that

$$r_L < 0 < r_H \tag{3.3}$$

so that investment in the bad security is inefficient. We assume that the bank's balance sheet is scalable in the sense that payoffs satisfy constant returns to scale. Assume for simplicity that the bank faces the binary choice of either holding good securities only or bad securities only.

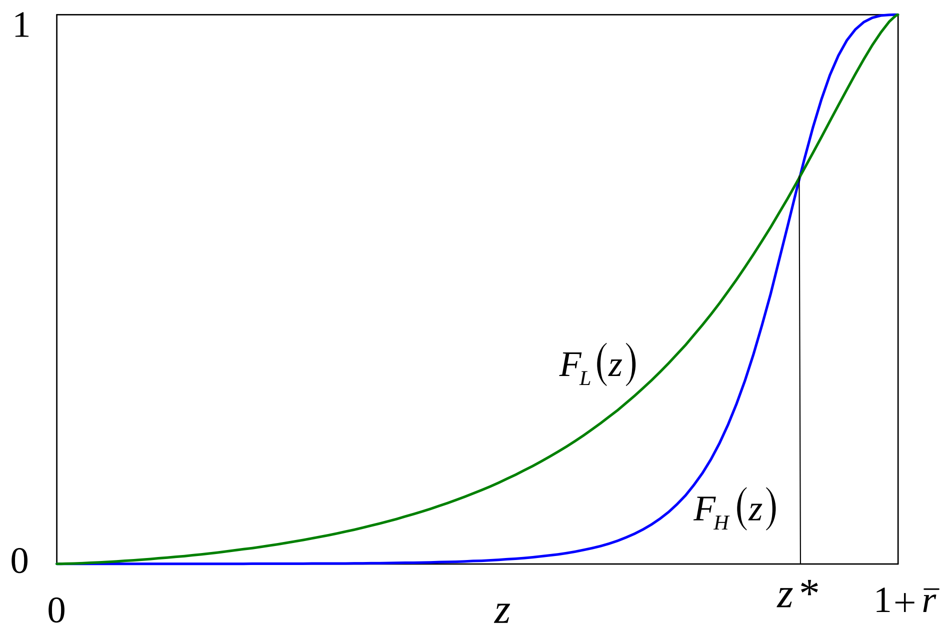

Figure 3.1: Cumulative Distribution Functions

Although the bad security has a lower expected return, it has higher upside risk relative to the good project in the following sense. Denote by $F_H(.)$ the cumulative distribution function associated with f_H and let $F_L(.)$ be the cumulative distribution function associated with f_L. The upside potential of the bad security comes from our assumed feature that F_H cuts F_L precisely once from below. That is, there is z^* such that $F_H(z^*) = F_L(z^*)$, and

$$(F_H(z) - F_L(z))(z - z^*) \geq 0 \tag{3.4}$$

for all z. Figure 3.1 illustrates this feature of our model.

4. Optimal contract

The bank's initial endownment is its equity E, which is given. The bank then decides on the total size of its balance sheet - its total assets A - by taking on debt as necessary. The debt financing decision involves both the face value of debt $\bar{D}$, as well as the market value of debt D. The repurchase price $\bar{D}$ in the repo contract can be interpreted as the face value of the debt owed by the bank. The face value $\bar{D}$ will be larger than the market value D, since the market value incorporates a discount for the possibility of default. The optimal contract maximes the bank's expected payoff by choice of A, D and $\bar{D}$ with E being the exogenous variable.

As noted by Merton (1974), the value of a defaultable debt claim with face value $\bar{D}$ is the price of a portfolio consisting of (i) cash of $\bar{D}$ and (ii) short position in a put option on the assets of the borrower with strike price $\bar{D}$. The net payoff of the creditor to the bank is illustrated in figure 4.1. The creditor loses her entire stake D if the realized asset value of the bank's assets is zero. However, if the realized asset value of the bank is $\bar{D}$ or higher, the creditor is fully repaid. We have $\bar{D} > D$, since the positive payoff when the bank does not default should compensate for the possibility that the creditor will lose because of default.

The equity holder is the residual claim holder, and his payoff is illustrated as the kinked convex function in figure 4.1. The equity holder can lose at most E, since this is the equity holder's initial stake. The sum of the equity holder's payoff and the creditor's payoff gives the net payoff from the total assets of the bank, given by the thin straight line passing through A in figure 4.1.

Creditor's Participation Constraint

Denote by $\pi_H\left(\bar{D}, A\right)$ the price of the put option with strike price $\bar{D}$ on the portfolio of good securities whose current value is A. We assume that the market for assets is competitive, so that the option price satisfies homogeneity of degree 1 -

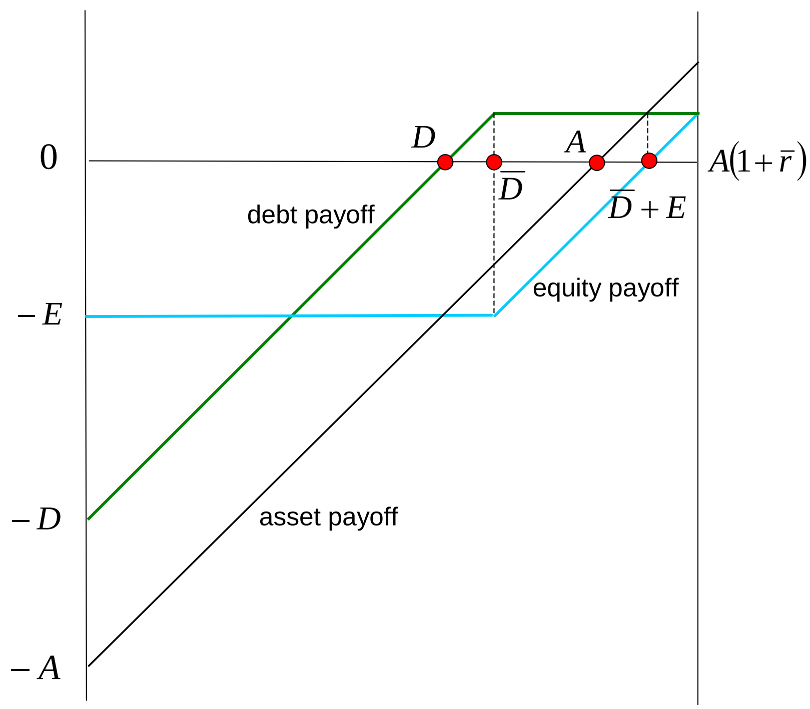

Figure 4.1: Net Payoffs

namely,

$$\pi_H\left(\bar{D}, A\right) = A\pi_H\left(\tfrac{\bar{D}}{A}, 1\right) \tag{4.1}$$

In other words, an option on A worth of securities with strike price $\bar{D}$ can be constructed by bundling together A options written on 1 dollar's worth of securities with strike price $\bar{D}/A$. Similarly, $\pi_L\left(\bar{D}, A\right) = A\pi_L\left(\frac{\bar{D}}{A}, 1\right)$, for portfolios consisting of bad securities.

Define $\bar{d}$ as the ratio of the promised repurchase price at date 1 to the market value of assets of the bank at date 0. In other words,

$$\bar{d} \equiv \frac{\bar{D}}{A} \tag{4.2}$$

Hence $\bar{d}$ is the ratio of the notional value of debt to the market value of assets. Define:

$$\pi_H\left(\bar{d}\right) \equiv \pi_H\left(\bar{d}, 1\right)$$

so that $\pi_H\left(\bar{d}\right)$ is the price of the put option on one dollar's worth of the bank's asset with strike price $\bar{d}$ when the bank's portfolio consists of good assets. $\pi_L\left(\bar{d}\right)$ is defined analogously for portfolio of bad securities.

The creditor's initial investment is D, while the expected value of the creditor's claim is the portfolio consisting of (i) cash of $\bar{D}$ and (ii) short position in put option on the assets of the bank with strike price $\bar{D}$. The (gross) expected payoff of the creditor when the bank's assets are good is therefore

$$\begin{aligned} & \bar{D} - A\pi_H\left(\bar{d}\right) \\ = \ & A\left(\bar{d} - \pi_H\left(\bar{d}\right)\right) \end{aligned}$$

Since the creditor's initial stake is D, her net expected payoff is

$$\begin{aligned} V &= \bar{D} - D - A\pi_H\left(\bar{d}\right) \\ &= A\left(\bar{d} - d - \pi_H\left(\bar{d}\right)\right) \end{aligned} \tag{4.3}$$

where $d \equiv D/A$ is the ratio of the market value of debt to the market value of assets. The participation constraint for the creditor requires that the expected payoff is large enough to recoup the initial investment D. That is,

$$\bar{d} - d - \pi_H\left(\bar{d}\right) \geq 0 \tag{IR}$$

Bank's Incentive Compatibility Constraint

The bank's equity holder is the residual claimant. The payoff of the equity holder is given by the difference between the net payoffs for the bank's assets as a whole and the creditor's net payoff, given by V in (4.3). Thus, the equity holder's payoff is

$$\begin{aligned} U(A) &= Ar - A\left(\bar{d} - d - \pi\left(\bar{d}\right)\right) \\ &= A\left(r - \bar{d} + d + \pi\left(\bar{d}\right)\right) \end{aligned}$$

Figure 4.1 plots the equity holder's payoff. The optimal contract maximizes U subject to the incentive compatibility constraint of the bank to hold good securities in his portfolio, and subject to the break-even constraint of the creditor. The equity holder's stake is a portfolio consisting of

- put option on the assets of the bank with strike price $\bar{D}$
- risky asset with expected payoff $A\left(r - \bar{d} + d\right)$

The expected return r and the value of the option $\pi\left(\bar{d}\right)$ depends on the bank's choice of assets. The expected payoff for the equity holder when the asset portfolio consists of the good asset is

$$A\left(r_H - \bar{d} + d + \pi_H\left(\bar{d}\right)\right) \tag{4.4}$$

while the expected payoff from holding bad assets is

$$A\left(r_L - \bar{d} + d + \pi_L\left(\bar{d}\right)\right) \tag{4.5}$$

where $\pi_L\left(\bar{d}\right)$ is the value of the put option on 1 dollar's worth of the bank's assets with strike price $\bar{d}$ when the bank holds bad assets. The incentive compatibility constraint is therefore

$$\begin{aligned} r_H - r_L &\geq \pi_L\left(\bar{d}\right) - \pi_H\left(\bar{d}\right) \\ &= \Delta\pi\left(\bar{d}\right) \end{aligned} \tag{IC}$$

where $\Delta\pi\left(\bar{d}\right)$ is defined as $\pi_L\left(\bar{d}\right) - \pi_H\left(\bar{d}\right)$. The term $\Delta\pi\left(\bar{d}\right)$ is analogous to the private benefit of exerting low effort in the moral hazard model of Holmstrom and Tirole (1997). The bank's equity holder trades off the greater option value of holding the riskier asset against the higher expected payoff from holding the good asset. The incentive compatibility constraint requires that the option value be small relative to the difference in expected returns.

We note that the IC constraint does not make reference to the market value of debt d, but only to the (normalized) face value of debt $\bar{d}$. This reflects the fact that the IC constraint is a condition on the strike price of the embedded option. In order to derive the market value of debt (and hence market leverage), we must make also use the IR constraint.

Given our assumptions on the densities governing the good and bad securities, we have the following feature of our model. Figure 4.2 illustrates.

Lemma 1. *$\Delta\pi\left(z\right)$ is a single-peaked function of z, and is maximized at the value of z where F_H cuts F_L from below.*

Proof. From the well-known result in option pricing due to Breeden and Litzenberger (1978), the price of the Arrow-Debreu contingent claim that pays 1 at z and

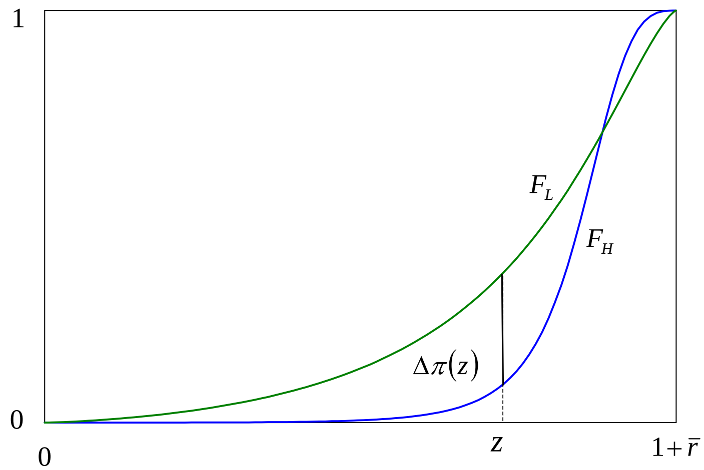

Figure 4.2: Difference in Option Values $\Delta\pi(z)$

zero otherwise is given by the second derivative of the option price with respect to the strike price evaluated at z (we give an intuition for this result below). Since both the principal and agent are risk-neutral, the state price is the probability. Thus, we have

$$\Delta\pi(z) = \int_0^z (F_L(s) - F_H(s))\, ds$$

Since F_H cuts F_L precisely once from below, $\Delta\pi(z)$ is increasing initially, is maximized at the point where $F_H = F_L$, and is then decreasing. ■

The intuition for Breeden and Litzenberger's result used in the proof of lemma 1 can be illustrated for a discrete outcome space in figure 4.3. Suppose for the purpose of this illustration only that the outcome space is $\{0, 1, 2, \cdots, Z\}$. Denoting by $p(s)$ the price of the Arrow-Debreu contingent claim at outcome s, the price of a put option with strike price z can be represented as the sum of the triangle of contingent claims as illustrated in figure 4.3. In other words, the price of the put option with strike price z is given by

$$p(z-1) + 2p(z-2) + 3p(z-3) + \cdots + zp(0)$$

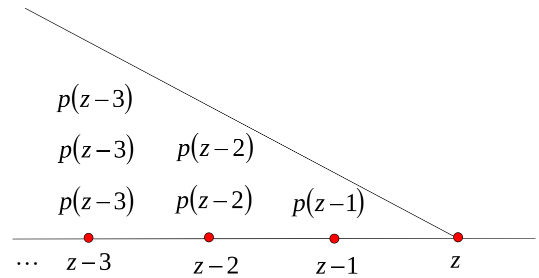

Figure 4.3: Put Option Value with Discrete Outcomes

Hence, the difference between the put option with strike price z and put option with strike price $z-1$ is given by

$$\begin{aligned} & p(z-1) + 2p(z-2) + 3p(z-3) + \cdots + zp(0) \\ & \qquad -p(z-2) - 2p(z-3) - \cdots - (z-1)p(0) \\ = \ & p(z-1) + p(z-2) + p(z-3) + \cdots + p(0) \end{aligned}$$

which is the cumulative distribution function evaluated at $z-1$. Thus, in the discrete outcome case, the state price density is the second difference in the option price with respect to the strike price. Breeden and Litzenberger (1978) showed that this intuition is valid when the outcome space is continuous. The state price is the second derivative of the option price with respect to the strike price.

Leverage Constraint

If the incentive compatibility constraint (IC) does not bind, then the contracting problem is trivial and the first best is attainable. We will focus on the case where the incentive compatibility constraint (IC) binds in the optimal contract. In what follows, therefore, we assume that (IC) binds. Since the value of the implicit put option held by the equity holder is increasing in the strike price $\bar{d}$, lemma 1 implies that there is an upper bound on the variable $\bar{d}$ for which the incentive constraint is satisfied. This upper bound is given by the smallest solution to the equation:

$$\Delta\pi\left(\bar{d}\right) = r_H - r_L \tag{4.6}$$

Denote this solution as $\bar{d}^*$. The quantity $\bar{d}^*$ is expressed in terms of the ratio of the repurchase price in the repo contract to the market value of assets, and so mixes notional and market values. However, we can solve for the pure debt ratio in market values by appealing to the participation constraint. The participation constraint binds in the optimal contract, so that we have:

$$d = \bar{d} - \pi_H\left(\bar{d}\right) \tag{4.7}$$

We can then solve for the debt to asset ratio d, which gives the ratio of the market value of debt to the market value of assets. Denoting by d^* the debt to asset ratio in the optimal contract, we have

$$d^* = \bar{d}^* - \pi_H\left(\bar{d}^*\right) \tag{4.8}$$

where $\bar{d}^*$ is the smallest solution to (4.6). The right hand side of (4.8) is the payoff of a creditor with a notional claim of $\bar{d}^*$. Hence, we can re-write (4.8) as

$$d^* = \int_0^{1+\bar{r}} \min\left\{\bar{d}^*, s\right\} f_H(s)\, ds \tag{4.9}$$

Clearly, d^* is increasing in $\bar{d}^*$, so that the debt ratio in market values is increasing in the notional debt ratio $\bar{d}^*$.

5. Balance Sheet Size

Having tied down the bank's leverage through (4.8), it remains to solve for the size of the bank's balance sheet. To do this, we note from (4.4) that the bank equity holder's expected payoff under the optimal contract is:

$$U(A) \equiv A\left(r_H - \bar{d}^* + d^* + \pi_H\left(\bar{d}^*\right)\right) \tag{5.1}$$

The expression inside the brackets is strictly positive, since the equity holder extracts the full surplus from a positive net present value relationship. Hence, the equity holder's payoff is strictly increasing in A. The equity holder maximizes the balance sheet size of the bank subject only to the leverage constraint (4.8). Let λ^* be the upper bound on leverage implied by d^*, defined as

$$\lambda^* \equiv \frac{1}{1 - d^*}$$

Then, the bank chooses total balance sheet size given by:

$$A = \lambda^* E \tag{5.2}$$

Note the contrast between this feature of our model and the textbook capital budgeting process as described in corporate finance textbooks. The usual approach is to deal first with the investment decisions by ranking investment projects according to their net present values, and undertaking only those projects that have positive net present value. Instead, in our model, the pivotal quantity is equity. For given equity E, total asset size A is determined as $\lambda^* \times E$, where λ^* is the maximum leverage permitted by the haircut in the repo contract. Thus, as λ^* fluctuates, so will the size of the bank's balance sheet.

Since the agent's payoff is increasing linearly in equity E, a very natural question is why the agent does not bring in more equity into the agency relationship, thereby magnifying the payoffs. The "pecking order" theories of corporate finance of Myers and Majluf (1984) and Jensen and Meckling (1976) shed some light on why equity may be so "sticky". In Myers and Majluf (1984), a firm that wishes to expand its balance sheet will first tap its internal funds, and then tap debt financing. Issuing equity is a last resort. The reasoning is that the firm has better information on the value of the growth opportunities of the firm and any attempt to raise new equity financing will encounter a lemons problem. In equilibrium, new equity is raised only by those firms that have low growth opportunities, and there is a fair discount that is applied to the new equity. Jensen and Meckling (1976) also predict a pecking order of corporate financing sources for the reason that agency costs associated with the actions of entrenched "inside" equity holders entail a discount when issuing new equity to "outside" equity holders. The stickiness of E is intimately tied to the phenomenon of "slow-moving capital" discussed by He and Krishnamurthy (2007).

6. Comparative Statics

Our main comparative statics result is with respect to increasing risk. Let the outcome densities be parameterized by σ, where higher σ indicates mean preserving spreads in the outcome density. Denote by

$$\pi_H(z, \sigma)$$

the value of the put option (parameterized by σ) on one dollar's worth of the bank's assets with strike price z when the bank's assets are good. $\pi_L(z, \sigma)$ is defined analogously when the assets are bad. Both π_H and π_L are increasing in σ, since payoffs densities undergo mean preserving spreads. We then have the following comparative statics result.

Proposition 1. *If $\Delta\pi(z, \sigma)$ is increasing in σ, then both $\bar{d}^*$ and d^* are decreasing in σ.*

We draw on three ingredients for the proof of this proposition. First, we draw on the the homogeneity of degree 1 of the option price as stated in equation (4.1). Second, we draw on the the IC constraint (IC). Finally, we draw on the supposition that $\Delta\pi(z, \sigma)$ is increasing in σ. From the IC constraint, and homogeneity of degree 1 of the option price, we have

$$\Delta\pi\left(\bar{d}^{*}(\sigma), \sigma\right) = r_H - r_L \tag{6.1}$$

where $\bar{d}^{*}(\sigma)$ is the value of $\bar{d}^{*}$ as a function of σ. The left hand side of (6.1) is increasing in both components, while the right hand side is constant. Hence, $\bar{d}^{*}(\sigma)$ is a decreasing function of σ.

To show that the market debt ratio d^{*} is decreasing in σ, we appeal to the participation constraint of the principal and the fact that the option value π_H is increasing in σ. From the participation constraint, we have

$$\begin{aligned} d^{*} &= \bar{d}^{*} - \pi_H\left(\bar{d}^{*}\right) \\ &= \int_0^{1+\bar{r}} \min\left\{\bar{d}^{*}, s\right\} f_H(s)\, ds \end{aligned} \tag{6.2}$$

Since $\bar{d}^{*}$ is decreasing in σ, so must d^{*} be decreasing in σ. This proves our result.

Proposition 1 is a sufficient condition for decline in leverage, but we may envisage other scenarios that also give rise to falls in leverage. For instance, a first-degree stochastic dominance shifts in the density of payoffs associated with a general deterioration in the prospects of the fundamental assets may also give rise to a fall in leverage. Suppose there is a deterioration in the fundamentals where the densities associated with the good asset and bad asset undergo a first degree stochastic dominance shift to the left but which leaves $r_H - r_L$ unchanged. Provided that the $\Delta\pi(z)$ function increases for small values of z but decreases for large values of z, we will have the intersection with the constant function $r_H - r_L$ shifting to the left. Hence, such a deterioration of fundamentals will also be associated with a decline in leverage.

7. Value-at-Risk

In practice, banks and other leveraged institutions follow Value-at-Risk (VaR) as a guide to their leverage decisions. Regulators have also encouraged this process. For a random variable W, the Value-at-Risk at confidence level c relative to some base level W_0 is defined as the smallest non-negative number V such that

$$\text{Prob}\left(W < W_0 - V\right) \leq 1 - c$$

In our context, W could be the realized asset value of the bank at date 1. Then the Value-at-Risk is the equity capital that the bank must hold in order to stay solvent with probability c. Financial intermediaries publish their Value-at-Risk numbers as part of their regulatory filings, and regularly disclose such numbers through their company reports. The term *economic capital* is also sometimes used to denote Value-at-Risk.

We consider a special case of our framework in which the leverage constraint can be expressed as a Value-at-Risk constraint in which a bank adjusts in balance sheet so that its equity capital is just sufficient to meet its Value-at-Risk. Consider the generalized extreme value distribution, which has the cumulative distribution function

$$G\left(z\right) = \exp\left\{-\left(1 + \xi\left(\frac{z-\theta}{\sigma}\right)\right)^{-1/\xi}\right\} \tag{7.1}$$

The parameter ξ can take any real number value, and the support depends on the sign of ξ. When ξ is negative, the support of the distribution is $(-\infty, \theta - \sigma/\xi)$. We consider a special case of (7.1) where $\xi = -1$ and $\sigma = 1$, and where we consider shifts in θ.

Consider the family of c.d.f.s $\{G_L, G_H\}_\theta$ parametrized by θ so that

$$G_L\left(z;\theta\right) = \exp\left\{z - \theta\right\} \quad \text{and} \quad G_H\left(z;\theta\right) = \exp\left\{z - k - \theta\right\} \tag{7.2}$$

where k is a positive constant. We will suppose that this family of distributions approximates the left tails of our outcome distributions, in line with the interpretation of the extreme value distribution as describing the frequency of rare, bad events. More formally, the extreme value limit theorem of Gnedenko (1948) states that if $z_1, z_2, ...$ is a sequence of independently and identically distributed random variables, $m_n = \min\{z_1, z_2, ..., z_n\}$, and sequences a_n and b_n exist such that $\lim \Pr\left(\left(m_n - b_n\right)/a_n\right) = G(z)$ as $n \to \infty$, then $G(z)$ is a generalized extreme value distribution. Consequently, we suppose that the following condition holds:

Condition 1. *There is $\hat{z}$ such that for all $z \in (0, \hat{z})$, we have*

$$F_L(z;\theta) = G_L(z;\theta) \text{ and } F_H(z;\theta) = G_H(z;\theta) \tag{7.3}$$

When $z = 0$, we have

$$F_L(0;\theta) = \int_{-\infty}^{0} G_L(s;\theta)\,ds \text{ and } F_H(0;\theta) = \int_{-\infty}^{0} G_H(s;\theta)\,ds \tag{7.4}$$

Let $\bar{d}^*(\theta)$ be the value of $\bar{d}^*$ in the contracting problem parameterized by θ. We then have the following result on Value-at-Risk.

Proposition 2. *For all $\theta \in \left[\underline{\theta}, \bar{\theta}\right]$ suppose that $\bar{d}^*(\theta) < \hat{z}$. Suppose also that $r_H - r_L$ stays constant to shifts in θ. Finally, suppose that condition 1 holds. Then the probability that the bank defaults is constant over all optimal contracts parameterized by $\theta \in \left[\underline{\theta}, \bar{\theta}\right]$.*

In other words, as θ varies over the interval $\left[\underline{\theta}, \bar{\theta}\right]$, the bank will adjust its leverage so that the equity is set equal to its Value-at-Risk at some given fixed confidence level c. In other words, leverage is fully determined by Value-at-Risk.

To prove proposition 2, note first from (7.2) that for all $z \in (0, \hat{z})$

$$\frac{F_L(z;\theta)}{F_H(z;\theta)} = \frac{G_L(z;\theta)}{G_H(z;\theta)} = e^k > 1$$

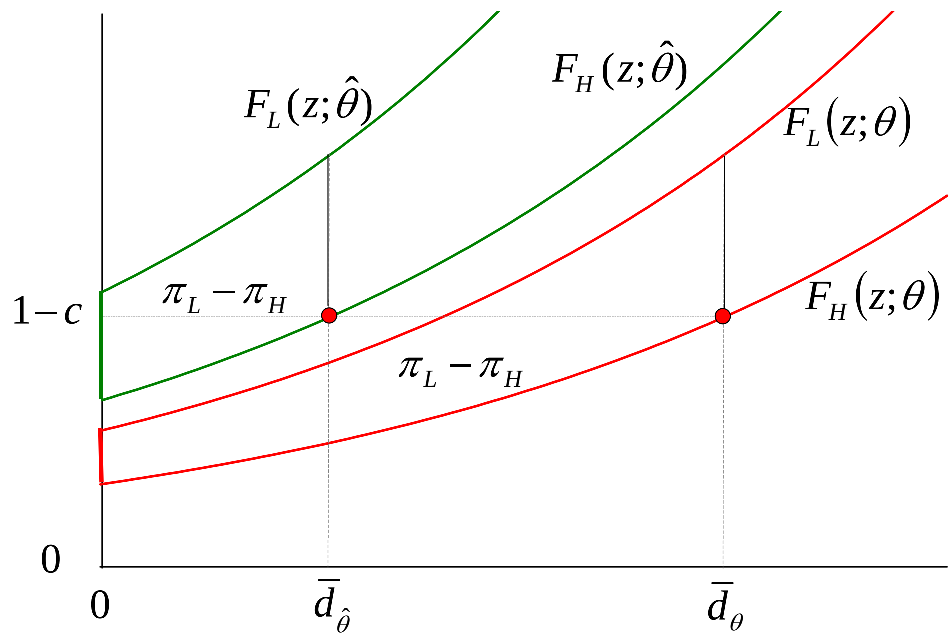

Figure 7.1: Value at Risk

Hence, from condition 1, we have

$$\begin{aligned}\Delta\pi(z;\theta) &= \int_0^z \left(F_L(s;\theta) - F_H(s;\theta)\right) ds \\ &= \int_{-\infty}^z \left(G_L(s;\theta) - G_H(s;\theta)\right) ds \qquad (7.5) \\ &= \left(e^k - 1\right)\int_{-\infty}^z G_H(s;\theta)\, ds \\ &= \left(e^k - 1\right) G_H(z;\theta) \qquad (7.6)\end{aligned}$$

From the IC constraint, we have $\Delta\pi\left(\bar{d}^*;\theta\right) = r_H - r_L$, so that for all $\theta \in \left[\underline{\theta}, \bar{\theta}\right]$, we have

$$\left(e^k - 1\right) G_H\left(\bar{d}^*;\theta\right) = r_H - r_L \qquad (7.7)$$

Therefore, from (7.6) and (7.7), we have that at every optimal contract $\bar{d}^*(\theta)$, the probability that the bank defaults is

$$G_H\left(\bar{d}^*;\theta\right) = \frac{r_H - r_L}{e^k - 1} \qquad (7.8)$$

which is constant. Therefore, as θ varies, the bank keeps just enough equity to meet its Value-at-Risk at a constant confidence level. Figure 7.1 illustrates the case of two values of θ, with $\hat{\theta} > \theta$ where the probability of default is kept at $1-c$. In our case, the right hand side of 7.8 is the probability of default. Hence the probability of default is

$$1 - c = \frac{r_H - r_L}{e^k - 1} \qquad (7.9)$$

8. Empirical Evidence

We now turn to the empirical evidence on the determination of leverage and balance sheet size. The key comparative statics result arising from our model is that both leverage and balance sheet size are driven by the the underlying riskiness of the environment, and that under special conditions on the tail density of outcomes the underlying riskiness can be captured by Value-at-Risk.

In order to put our model to test, we bring together balance sheet data of the (then) five stand-alone U.S. investment banks (Bear Stearns, Goldman Sachs, Lehman Brothers, Merrill Lynch and Morgan Stanley) from their regulatory 10-K and 10-Q filings with the Securities and Exchange Commission. Each quarter, intermediaries file extensive balance sheet information. The main variables that we investigate are total assets, collateralized borrowing (via repos and other secured borrowing transactions), and leverage defined by the ratio of total assets to equity. Because our main comparative static results are with respect to measured risks on the intermediary's balance sheet, we also compute the value-at-risk (VaR) of total assets. To estimate the asset VaR, we first compute the equity volatility from daily (market) returns of the respective investment bank. We then multiply the daily volatility by 2.33 and by the square root of 252 in order to obtain an annualized (market) equity VaR. We translate that number into a total asset VaR by multiplying by the market-to-book ratio, and dividing by leverage to express. All growth rates and VaRs are expressed in annualized percentage terms. We provide summary statistics for all variables in Table 8.1.

[Table 8.1]

As can be seen in table 8.1, total assets grew at an annualized rate of 16% with a standard deviation of 21%. There is some cross-sectional heterogeneity in these growth rates which partly reflects different strategies pursued by the banks, but also reflects the unbalanced nature of the panel arising from the different times at which these banks became public.

Interestingly, higher asset growth is not necessarily associated with higher asset growth volatility. Repo growth (on average) is somewhat slower than total asset growth, likely reflecting the increasing importance of derivatives on both the asset and liability side of the balance sheets. Repos are more volatile than total assets, reflecting the fact that repos are the primary margin of adjustment for asset growth. Leverage growth is positive on average across the banks, but this average masks the fact that some banks have negative average growth over the sample. For example, Goldman Sachs went public at a time when investment bank leverage was generally high, and thus experienced a decline in leverage over the 1999 - 2008 period. The volatility of leverage growth is of the same order of magnitude as the volatility of total asset growth. Equity growth is positive, but slightly lower than total asset growth. This reflects the fact that aggregate leverage growth is positive (as total asset growth is approximately equal to leverage growth plus equity growth).

[Table 8.2]

[Table 8.3]

Tables 8.2 and 8.3 test the main prediction from the model - namely, that leverage is decreasing in the bank's Value-at-Risk. We do indeed find a negative relationship between leverage and Value-at-Risk both for the panel regressions reported in Table 8.2, and the aggregate regressions in Table 8.3.

In Panel A of Table 8.2, the coefficient of -0.18 means that a 1% higher VaR corresponds to a leverage ratio that is 0.18% lower. We explain about 61% of the cross sectional variation, and 18% of the time series variation of leverage with VaR. When we include control variables, the time series R^2 increases to 64% for the investment bank aggregate), while the estimated elasticity of leverage with respect to Value-at-Risk does not change substantially.

[Table 8.4]

Intermediaries can expand balance sheets either when leverage constraints are looser (i.e. when intermediary VaR is lower), or when equity increases (either due to appreciating share prices, or by raising new capital). The results in Panels A and B of Table 8.4 indicate that relaxation of the Value-at-Risk constraints increase total assets. Also, it is notable that growth in equity is associated with larger assets. From the banks' VaR and equity growth, we explain 75% of the variation in total asset growth for the bank aggregate. The explanatory power increases to 80% when we add further control variables.

Consistent with the predictions from our theory, we also find that the growth of repos is constrained by variations in VaR. (Panel C of Table 8.4). Adrian and Shin (2007) find that repos are the primary channel through which investment bank balance sheets adjust, and so these findings further confirm the importance of repo financing. Since our model has been designed around the use of repos as the primary means of financing, the finding is also consistent with the key assumption in our model.

9. Debt Capacity of Financial System

We now apply our results to examine the debt capacity of the financial system as a whole. Suppose there are n banks, indexed by $i \in \{1, \cdots, n\}$. In addition, there is a non-leveraged sector which we label by $n+1$. The non-leveraged sector has equity and debt claims against the banks. In turn, the banks have claims against end-users of credit such as firms and households.

The banks in the financial system now have two roles - first as a borrower, but also simultaneously as a creditor. We will assume that there is an exogenous network structure that links the banks in the creditor-debtor relationship. We do not address how the network was formed.

The banks finance their assets by issuing securities against their own assets, and pledging these securities with other banks or outside funding sources. We have in mind the practice of securitization via special purpose vehicles (SPVs) - passive entities that issue liabilities collateralized by claims against other entities - and the issuing of securities such as collateralized debt obligations (CDOs). Although the special purpose vehicles are separate legal entities from the bank, they are supported by liquidity and credit enhancements by their sponsoring banks and can be considered as being part of the sponsoring bank from the standpoint of the bank's consolidated balance sheet. The credit crisis of 2007/8 has highlighted this particular feature of special purpose vehicles. Thus, in what follows, we will interpret the balance sheet of bank i has containing any sponsored SPVs in its consolidated balance sheet.

Moreover, we will assume that the individual rationality constraint binds for all banks in the financial system. In effect, the mark-to-market value of the balance sheets simply reflect the expected value of the claims and obligations on the balance sheet. In this way, any surplus in the contracting relationship is fully appropriated by the end-user borrowers in the financial system. Our focus, instead, will be on the debt capacity of the financial system as a whole, and on the comparative statics of such debt capacity.

For security brokers and dealers, the majority of debt is short-term debt in the form of repurchase agreements. Money market mutual funds lend cash against collateral to the bank, either overnight or for short terms (typically 30-90 days). The money market mutual fund determines the haircut, but the management of the haircut, pricing of the collateral, and monitoring of counter party credit risk is outsourced to clearing banks. Shareholders of financial institutions are primarily equity mutual funds and individuals.

Start with the balance sheet identity of bank i in market values:

$$y_i + \sum_j \pi_{ji} x_j = e_i + x_i$$

where y_i is the value of loans to end-users, x_i is the market of value of bank i's debt, e_i is the market value of bank i's equity and π_{ji} is the proportion of j's debt held by i. We can re-arrange the balance sheet identity in the following way.

$$x_i = \underbrace{y_i + \textstyle\sum_j \pi_{ji} x_j}_{\text{collateral value}} - \underbrace{e_i}_{\text{haircut}}$$

The left hand side is the debt capacity of bank i, which is given by the sum of three terms on the right hand side - namely, the collateral value of its direct claims on end-users, the collateral value of claims on other banks, minus the haircut that comes from the optimal contract. Gathering the debt capacities into a row vector, we can write

$$[x_1, \cdots, x_n] = [x_1, \cdots, x_n] \begin{bmatrix} & & \\ & \Pi & \\ & & \end{bmatrix} + [y_1, \cdots, y_n] - [e_1, \cdots, e_n] \tag{9.1}$$

or more succinctly,

$$x = x\Pi + y - e \tag{9.2}$$

Equation (9.2) shows the recursive nature of debt capacity in a financial system. Each bank's debt capacity is increasing in the debt capacity of other banks. The lower is the haircut, the greater is the asset value that can be pledged in the repo transaction. From (9.2) we can solve for y as

$$y = e\left(\Lambda\left(I - \Pi\right) + \Pi\right) \tag{9.3}$$

where Λ is the diagonal matrix whose ith element is λ_i, the leverage of bank i. Thus, total lending to end-users depends on (i) how much equity e there is in the banking system, (ii) how much leverage is permitted by the optimal contract (given by the diagonal matrix Λ), and (iii) the structure of the interbank market (given by Π). Total lending capacity is increasing in e and increasing in Λ.

9.1. Example of Single Chain

Consider the simple linear chain, where

$$\Lambda\left(I-\Pi\right)+\Pi=\begin{bmatrix} \lambda_1 & -\left(\lambda_1-1\right)h_1 & & & & \\ & \lambda_2 & -\left(\lambda_2-1\right)h_2 & & & \\ & & \lambda_3 & -\left(\lambda_3-1\right)h_3 & & \\ & & & \ddots & \ddots & \\ & & & & \lambda_{n-1} & -\left(\lambda_3-1\right)h_{n-1} \\ & & & & & \lambda_n \end{bmatrix}$$

and where $h_i = 1-\pi_{i,n+1}$ is the proportion of debt raised from within the banking system, rather than from outside creditors. Re-writing the equations for each individual bank, we have

$$\begin{aligned} y_1 &= \lambda_1 e_1 \\ y_2 &= \lambda_2 e_2 - \left(\lambda_1-1\right)h_1 e_1 \\ y_2 &= \lambda_3 e_3 - \left(\lambda_2-1\right)h_2 e_2 \\ &\vdots \\ y_n &= \lambda_n e_n - \left(\lambda_{n-1}-1\right)h_{n-1} e_{n-1} \end{aligned}$$

Consider now the special case where $y_i = 0$ for all $i > 1$ and $h_j = 1$ for all $j < n$. In other words, the outside asset y is held by bank 1 only, while all other banks hold assets that are the obligations of other banks higher up the chain. Figure 9.1 illustrates this chain of creditor relationships. Then we have

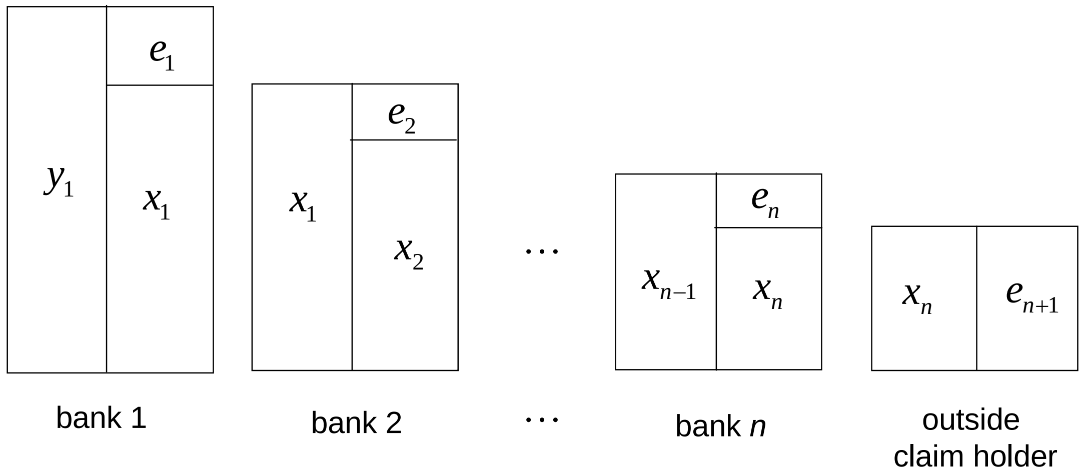

Figure 9.1: Chain of Credit Relationships

$$
\begin{aligned}
y_1 &= \lambda_1 e_1 \\
&= e_1 + \lambda_2 e_2 \\
&= e_1 + e_2 + \lambda_3 e_3 \\
&= e_1 + e_2 + \cdots + e_{n-1} + \lambda_n e_n
\end{aligned}
$$

For the financial system as a whole to support debt level of y_1, all of the following inequalities must hold.

$$
\begin{aligned}
y_1 &\leq \lambda_1 e_1 \\
y_1 &\leq e_1 + \lambda_2 e_2 \\
y_1 &\leq e_1 + e_2 + \lambda_3 e_3 \\
&\vdots \\
y_1 &\leq e_1 + e_2 + \cdots + e_{n-1} + \lambda_n e_n
\end{aligned}
$$

Thus, define y^* as follows.

$$y^* = \min_i \left\{ \sum_{j=1}^{i-1} e_j + \lambda_i e_i \right\}$$

Here y^* has the interpretation of the debt capacity of the financial system. Note that y^* is a function of all the equity levels $\{e_i\}$ and all the leverage ratios $\{\lambda_i\}$. The bank i for which $\sum_{j=1}^{i-1} e_j + \lambda_i e_i$ is the lowest among all banks could be seen as the "pinch point" in the financial system. The total lending capacity is restricted by this one bank.

If the debt capacity y^* falls below existing y_1, then the repo contract cannot be rolled over in the previous amounts. Then some of the loans y_1 will need to be sold. If the required sale is large enough, then bank 1 becomes insolvent. This will result in a Bear Stearns style run on bank 1. Even though bank 1 has the capacity to borrow, it is the bank further up the chain that prevents the rolling over of the debt. The system as a whole then runs out of lending capacity.

10. Concluding Remarks

In this paper, we have employed a contracting model for the determination of leverage and balance sheet size for financial intermediaries that finance their activities through collateralized borrowing.

The model gives rise to two features. First, as is observed in the data, leverage is procyclical in the sense that leverage is high when the balance sheet is large. Second, as also observed in the data, leverage and balance sheet size are both determined by the riskiness of the intermediary's assets.

In a system context, we have seen that a fall in the permitted leverage of the financial intermediaries as a group can lead to a funding crisis for a constituent of the system who is unable to reduce the size of its balance sheet accordingly as funding is withdrawn. In effect, a generalized fall in the permitted leverage in the financial system can lead to a "run" on a particular institution that has funded long-lived illiquid assets by borrowing short. Note that this mechanism is distinct from the more familiar "collateral squeeze" examined by Bernanke and Gertler (1989) and Kiyotaki and Moore (2002, 2005) that follows from a fall in the price of collateral assets.

The run on the US investment bank Bear Stearns in March 2008 has many of the features examined in this model. Similarly, we may regard the rescue of the UK bank Northern Rock in September 2007 in a similar light. Although the iconic TV images were of retail depositors queuing around the block outside branches of the bank, this depositor run was precipitated by the failure of Northern Rock to roll over its short term wholesale funding, and by its approach to the Bank of England. To the extent that Northern Rock's demise was due to a run by its wholesale creditors, our model is applicable to this case, too.

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Table 8.1: This Table reports summary statistics for balance sheet items of five large U.S. investment banks. The balance sheet items are from the quarterly 10-K and 10-Q filings with the Securities and Exchange Commission. Growth rates refer to annualized quarterly growth rates in percent. Assets refer to total assets. Repos refers to repurchase agreements and other collateralized borrowing. Leverage refers to the ratio of total assets to book equity, leverage growth refers to the annualized quarterly percentage growth rate of leverage. VaR refers to the total asset Value-at-Risk. The VaR is computed from the volatility of daily equity returns in each quarter and translated into total asset VaR by multiplying by the market-to-book ratio and dividing by the leverage ratio. The daily equity returns are from CRSP. BSC refers to Bear Stearns, GS to Goldman Sachs, LEH to Lehman Brothers, ML to Merrill Lynch, MS to Morgan Stanley, and Agg to the cross-sectional, total asset weighted average of the five banks. M is the sample mean of the respective variable, and Sd the sample standard deviation.

		(i)	(ii)	(iii)	(iv)	(v)	(vi)
		BSC	GS	LEH	ML	MS	Agg
Asset Growth	M	11.97	18.78	15.97	15.87	13.86	16.10
	Sd	23.83	15.95	26.33	25.55	23.29	21.82
Repo Growth	M	10.37	19.67	13.49	17.18	10.20	15.14
	Sd	42.76	87.34	46.63	46.41	40.94	38.02
Equity Growth	M	13.24	20.37	15.60	14.21	9.81	13.52
	Sd	19.60	23.05	16.59	18.76	16.34	11.58
Leverage	M	32.26	22.42	30.26	25.68	26.10	27.45
	Sd	3.95	4.28	4.19	5.80	3.75	3.85
Leverage Growth	M	-0.52	-0.50	0.82	2.13	4.25	2.95
	Sd	26.93	24.46	28.21	26.26	22.48	21.08
VaR	M	2.72	9.08	4.38	6.47	10.76	6.41
	Sd	0.81	5.76	2.53	3.81	8.27	4.40
VaR Change	M	0.03	-0.34	0.06	0.02	0.03	0.02
	Sd	0.70	3.14	1.92	1.87	4.52	2.13
Sample	from	1997Q1	1998Q4	1994Q4	1992Q1	1997Q2	1992Q1
	to	2008Q1	2008Q1	2008Q1	2008Q1	2008Q1	2008Q1

Table 8.2: This Table reports panel regressions of leverage on total asset Value-at-Risk (VaR). In Panel A, the leverage is regressed on VaR. In Panel B, (annualized) quarterly leverage growth is regressed on quarterly changes in VaR. The regressions with controls include the federal funds rate, the 10-year/3-months Treasury spread, the Moody's BAA/10-year Treasury credit spread, the Moody's AAA/10-year Treasury spread, and the 3-month Eurodollar/3-month Treasury spread. In Panel A, the level of controls is included in the regression, in Panel B, the changes of the controls are included. In the R-square column of Panel A, be refers to the between measure of fit, and wi to the within measure of fit. A moving average error term is estimated in all regressions, so standard errors are adjusted for autocorrelation. All regressions include a fixed effect.

Panel A: Leverage in Investment Bank Panel

	VaR	Constant	R^2	Obs	Specification
coef	-0.18	28.77	be: 61%	239	FE
t-stat	-3.68	233.03	wi: 18%		
coef	-0.18	30.75	be: 64%	239	FE, controls
t-stat	-3.64	136.40	wi: 26%		

Panel B: Leverage Growth in Investment Bank Panel

	VaR Change	Constant	R^2	Obs	Specification
coef	-2.31	1.30	be: 9%	239	FE
t-stat	-3.70	0.79	wi: 6%		
coef	-2.34	1.45	be: 9%	239	FE, controls
t-stat	-3.59	0.87	wi: 8%		

Table 8.3: This Table reports regressions of leverage on total asset Value-at-Risk (VaR) for the investment bank aggregate. In Panel A, the leverage is regressed on VaR. In Panel B, (annualized) quarterly leverage growth is regressed on quarterly changes in VaR. The regressions with controls include the federal funds rate, the 10-year/3-months Treasury spread, the Moody's BAA/10-year Treasury credit spread, the Moody's AAA/10-year Treasury spread, and the 3-month Eurodollar/3-month Treasury spread. In Panel A, the level of controls is included in the regression, in Panel B, the changes of the controls are included. In the R-square column of Panel A, be refers to the between measure of fit, and wi to the within measure of fit. In all regressions a moving average error term is estimated, so standard errors are adjusted for autocorrelation.

Panel A: Leverage in Investment Bank Aggregate

	VaR	Constant	R^2	Obs	Specification
coef	-1.59	13.12	8%	64	FE
t-stat	-3.14	3.61			
coef	-1.94	37.12	16%	64	FE, controls
t-stat	-2.06	1.26			

Panel B: Leverage Growth in Investment Bank Aggregate

	VaR Change	Constant	R^2	Obs	Specification
coef	-3.31	2.84	11%	64	FE
t-stat	-3.25	0.94			
coef	-2.49	3.52	24%	64	FE, controls
t-stat	-1.72	1.16			

Table 8.4: This Table reports the results from regressing asset growth on total asset VaR and intermediary equity growth (Panels A and B), and from regressing repo growth on intermediary VaR (Panel C). Panel regressions are reported in Panel A and C, and regressions for the investment bank aggregate are reported in Panels B. A moving average term is explicitly estimated in all specifications, and the panel regressions control for fixed effects. Specifications with controls include changes in the federal funds rate, the 10-year/3-months Treasury spread, Moody's BAA/10-year Treasury credit spread, Moody's AAA/10-year Treasury spread, and the 3-month Eurodollar/3-month Treasury spread. In the R-square column of Panel A, be refers to the between measure of fit, and wi to the within measure of fit.

Panel A: Asset Growth in Investment Bank Panel

	VaR Change	Equity Growth	Constant	R^2	Obs	Specification
coef	-2.13		13.74	be: 38%	239	FE
t-stat	-3.70		9.03	wi: 5%		
coef	-2.18	0.27	9.78	be: 75%	239	FE
t-stat	-3.89	3.35	5.31	wi: 10%		
coef	-1.85	0.23	10.62	be: 80%	239	FE, controls
t-stat	-3.21	2.82	5.69	wi: 15%		

Panel B: Asset Growth in Investment Bank Aggregate

	VaR Change	Equity Growth	Constant	R^2	Obs	Specification
coef	-0.21		33.26	8%	64	
t-stat	-2.77		4.75			
coef	-0.22	0.53	26.60	16%	64	
t-stat	-3.14	2.57	3.95			
coef	-0.23		35.98	22%	64	controls
t-stat	-2.49		4.16			

Panel C: Repo Growth in Investment Bank Panel

	VaR Change		Constant	R^2	Obs	Specification
coef	-4.82		9.87	be: 19%	64	FE
t-stat	-3.76		2.78	wi: 5%		
coef	-3.74		10.95	be: 79%	64	FE, controls
t-stat	-2.87		3.14	wi: 13%		