A Search and Learning Model of Export Dynamics

(Preliminary and Incomplete)

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December 2009

\textsuperscript{1}We gratefully acknowledge support from the National Science Foundation (Grant SES-0922358), the United States Census Bureau, and Banco de la República de Colombia. We also thank Monica Hernández for excellent research assistance, as well as Enrique Montes for expert data advice. This paper was written in part by Census Bureau staff. It has undergone a more limited review than official Census Bureau publications. All results were reviewed to ensure confidentiality. Any views, findings and opinions in the paper reflect the views of the authors and do not reflect the views of the U.S. Census Bureau.
1 Introduction

Economists have yet to develop models that reliably explain export dynamics at the micro level. Traditional gravity models focus on long run determinants of aggregate bilateral export flows, and are poorly-suited for the analysis of firm-level export fluctuations.\(^1\) Sunk-cost hysteresis models—which emphasize the start-up costs that new exporters face—do help us understand patterns of foreign market entry and exit by individual firms (Dixit, 1989; Baldwin and Krugman, 1989; Das, et al, 2007). But they provide little guidance as to why new exporters either exit or rapidly expand, while established exporters' sales are stable. Nor do they convincingly reconcile the substantial market entry costs that they posit with the fact that many firms export for short periods on a very small scale. Finally, while recent work by Arkolakis (2007, 2009) accounts for small-scale exporters and the age-dependence of export growth rates, it lacks the market frictions needed to explain the lags and irreversibilities observed in firms’ exporting behavior

This paper develops a model that explains small-scale exporting, age-dependent export growth, and lags and irreversibilities. It is based on the conjecture that firms’ exporting behavior reflects search and learning processes in foreign markets. That is, producers who are interested in a particular foreign market devote resources to identifying potential buyers there. When they find one, they learn something (receive a noisy signal) about the appeal of their products in this market. They also learn about foreign demand for their product from their experiences in their home markets. Taking stock of the available information,

\(^1\)Recent contributions to the gravity literature include Helpman et al (2008) and Anderson and van Wincoop (2003). Deardorff (1998) provides a survey of the earlier literature.
these firms update their beliefs concerning the scope for export profits, and they adjust the intensity of their search efforts accordingly, attempting to maximize their net expected profit streams. Export surges take place when home-market firms receive positive early signals about the scope for profits—both from their own experiences and from the experiences of rivals—and they intensify their search/marketing efforts, adding quickly to their foreign client base. Export collapses occur when firms allow their portfolio of buyers to shrink.

The motivation for this paper comes from descriptive analysis of a decade’s worth of individual merchandise shipments from Colombia to the United States. We begin by reviewing the stylized facts that come out of this analysis, including a number of finding that we have not reported in our earlier work (Eaton et al, 2008). Then we introduce our model, discuss its calibration, and demonstrate that, by adopting the assumptions mentioned in the previous paragraph, we are able to explain the basic features of the shipments data.

2 Firm-Level Trade: Transaction Level Evidence

The empirical motivation for our model comes from two sources. The first is a comprehensive data set that describes all shipments from Colombia to the United States (and elsewhere) that passed through Colombian customs during the period 1996-2005. Each customs record includes a date, the US dollar value of the product shipped, a 6-digit harmonized system product code (augmented by addition product information), a quantity index, a seller ID code, and the location of the buyer.\footnote{Because we use the same data that are used for official statistics, the merchandise exports in our data set aggregate to within one percent of total merchandise exports reported by the Colombian Bureau of Statistics (Departamento Administrativo Nacional de Estadística or DANE). The deviation is due to mistakes in the} The second data base provides analogous information
for the period 1992-2005. However it is based on U.S. Customs records, and it describes imports by buyers in the United States from Colombian exporters (as well as other sources). Critically, in addition to providing all of the information contained in the Colombian records, the U.S. customs data include ID codes for both sellers and buyers. It therefore allows us to identify the formation and dissolution of business relationships between individual buyers in the U.S. and sellers in Colombia, hereafter referred to as "matches."

### 2.1 Evidence from Colombian Customs Records

Following Brooks (2006) and Eaton et al. (2008), Table 1 provides various annual measures of Colombian exports to the United States for the years 1996-2007. Each column follows an exporting cohort—i.e., a group of firms that began exporting in a particular year, after at least one year of no exporting—from the year of its appearance through time. (Since we don’t know the history of firms before 1996, the 1996 “cohort” consists of all firms present that year regardless of when they began exporting.) The panels of the Table report number of exporters, total exports, and exports per firm, respectively.

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### Table 1a: Number of Exporting Firms, by Entry Cohort

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### Table 1b: Value of Exports, by Entry Cohort (millions of $US)

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<td>665</td>
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</tbody>
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### Table 1c: Exports per Firm, by Entry Cohort (thousands of $US)

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<tr>
<td>2002</td>
<td>4446</td>
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<tr>
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<td>1053</td>
<td>620</td>
<td>689</td>
<td>783</td>
<td>252</td>
<td>212</td>
<td>69</td>
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<td>0</td>
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<tr>
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<td>482</td>
<td>313</td>
<td>291</td>
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<td>0</td>
</tr>
<tr>
<td>2005</td>
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<td>1247</td>
<td>1444</td>
<td>1764</td>
<td>766</td>
<td>510</td>
<td>454</td>
<td>585</td>
<td>131</td>
</tr>
</tbody>
</table>
Consider panel 1a first. Naturally, each cohort’s membership falls as it matures. Note that there is very high attrition the first year, with at least half and up to three-fourths of firms dropping out. Conditional on making it to the second year, the survival probability is much higher, however, with an average attrition rate around 20 percent. Thus, in terms of numbers, the most recent cohort is always larger than any previous one (excepting the 1996 cohort, which is a special case). Note that firms that were exporting to the United States in 1996 account for only about one seventh of the firms exporting to the United States in 2005.

Panels 1b shows that, despite the rapid initial decline in its membership, the total sales of a cohort tends to rise over time, although quite unevenly. By the end of the period the 1996 cohort contributes about 76 percent of total sales, despite their relatively small number. The 2005 cohort contributes the second largest share.

The decline in number of firms per cohort along with their increasing contribution to total sales means, of course, that sales per firm are growing substantially (panel 1c). In fact, export sales for young survivors in each cohort tend to grow far more rapidly than total export sales, so that cohorts’ market shares tend to rise despite rapid attrition during their early years. Finally, note that cohort size and success (in terms of survival and sales) vary substantially across entry years. For example, the 2000 cohort appears very robust both in terms of number of exporters and exports per firm, with 1998 weak by comparison. This suggests that entry selection mechanisms vary over time in response to market-wide forces.
2.2 Evidence from U.S. Customs records

Individual buyers and sellers are identified in the transaction level data collected by the United States Census Bureau. Accordingly, this data set allows us to keep track of how many buyers each Colombian exporter is shipping to, and to see when buyers are dropped or added. We next use these data to characterize the buyer-seller matchings that took place during our sample period of 1992-2005.

2.2.1 Monogamous and polygamous matches

The number of Colombian exporters appearing in the sample grew from 3,742 in 1992 to 5,297 in 2005, a growth of 3.5 per annum, while the number of U.S. importing firms grew by 4.4 percent (Table 2). The number of Colombian exporter-U.S. importer pairs (representing at least one transaction between them in a year) grew at an annual rate of 3.3 percent. Roughly 80 percent of matches are monogamous in the sense that the buyer deals with only one Colombian exporter and the exporter ships to only one buyer in the United States. However, since the remainder of the matches are polygamous, the average Colombian exporter was involved in around 1.4 relationships with U.S. firms while the average U.S. buyer was involved in around 4 relationships with Colombian firms. Both figures declined slightly over the period.

<table>
<thead>
<tr>
<th></th>
<th>Colombian Exporters</th>
<th>U.S. Importers</th>
<th>Exporter-Importer Pairs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1992</td>
<td>3,742</td>
<td>1,265</td>
<td>5,297</td>
</tr>
<tr>
<td>2005</td>
<td>5,297</td>
<td>2,214</td>
<td>8,046</td>
</tr>
</tbody>
</table>
2.2.2 Transition Probabilities

Most matches are very short-lived. Of the buyer-seller matches that existed at the beginning of the period, 47 percent didn’t make it to 1993. But of those that survived into that year, almost 70 percent made it into the next year. Similarly, of the relationships that existed in 2005, 48 percent started that year, but of those that started the previous year, 65 percent had been around at least 3 years before. Of the 5,297 matches identified in 1992, only 85 endure (are present every year) throughout the period.

Table 3 reports the probability with which a Colombian firm participating in certain number of relationships with buyers transits into different number of relationships the following year. This table reports the annual average for 1992-1997 across all industries. Numbers for later periods are very similar. Thus, of firms not exporting to the United States in year $t$ but exporting in year $t + 1$, 92.5 percent sell to only one U.S. firm, etc. Of those that sell to one U.S. buyer in a year, 63 percent don’t export the next year, while only about 6 percent go on to establish a larger number of relationships. For firms with two relationships in a year, about 14 percent enter into a larger number of relationships, etc. Hence there is an enormous amount of churning at the lower end. Even for firms with a large number of relationships the most likely outcome is to have fewer the next year.
We can ask what this pattern of entry and growth implies about the ergodic distribution of relationships. If we assume that the number of entrants in a year replace exiters to the extent that the overall number of firms rises by 3.5 percent a year, the ergodic distribution implied by this transition matrix is given by:

For purposes of comparison, the year-specific average share of Colombian firms in each group is reported as well. Note that the ergodic distribution implied by the transition matrix is very close to the distribution in the data.

### 2.2.3 Match maturation

The survival probability of new matches increases with initial sales volume. Table 5 sorts observations on matches according to their size in their first year of existence and reports year-to-year separation rates. In addition to the very low survival rates, two patterns stand out. First, those matches that begin with sales in the top quartile among all new matches
are more likely to survive than matches that begin with smaller sales volumes. Second, survival probabilities improve after the initial year, especially for the surviving matches from the smallest quartile.

These patterns are suggestive of Rauch and Watson’s (2003) model, in which buyers place small trial orders with exporters of questionable quality. Many matches fail thereafter as the buyers examine the shipments and learn more about the sellers, but those that survive move on to larger shipments and are less likely to fail in future periods. Thus, if the trial stage is completed and more substantial orders are placed during the first period, survival probabilities will be correlated with first-period sales. This correlation is strengthened if some exporters are observably high quality, as in Rauch and Watson’s model, since their matches will immediately involve substantial orders and will be relatively likely to survive an additional period.

<table>
<thead>
<tr>
<th>Table 5: Separation Rates, Age of Match, and Initial Sales*</th>
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<td>Age of match (in years)</td>
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<tr>
<td></td>
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<tr>
<td>1st quartile</td>
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<td>2nd quartile</td>
</tr>
<tr>
<td>3rd quartile</td>
</tr>
<tr>
<td>4th quartile</td>
</tr>
</tbody>
</table>

*Figures exclude matches between affiliated parties, as well as those that involve trade in minerals, unprocessed agricultural products, and services.

Further evidence for the Rauch and Watson (2003) model comes from patterns of intra-match sales growth (Table 6). Matches that begin in the smallest quartile and survive to their second year grow an average of 66 percent, while surviving matches in the larger quartiles grow substantially less during their second year. Thereafter matches exhibit declining
sales on average, although there is considerable intra-quartile heterogeneity in growth rates. Accordingly, after the first year of a match, sales volumes tend to converge toward similar levels, and importantly for our purposes, firms wishing to maintain or expand their export sales must continually replenish their stock of foreign buyers.

<table>
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<th>Initial sales volume</th>
<th>Age of match (in years)</th>
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<th>4</th>
<th>5+</th>
</tr>
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<td>1st quartile</td>
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<td>-0.04</td>
<td>-0.07</td>
<td>-0.16</td>
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<td></td>
<td>(0.81)</td>
<td>(0.86)</td>
<td>(0.81)</td>
<td>(0.75)</td>
<td>(0.77)</td>
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<td>2nd quartile</td>
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<td>-0.09</td>
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<td></td>
<td></td>
<td>(0.88)</td>
<td>(0.84)</td>
<td>(0.71)</td>
<td>(0.73)</td>
<td>(0.63)</td>
</tr>
<tr>
<td>3rd quartile</td>
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<td>-0.20</td>
<td>-0.17</td>
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<tr>
<td></td>
<td></td>
<td>(0.89)</td>
<td>(0.75)</td>
<td>(0.64)</td>
<td>(0.65)</td>
<td>(0.72)</td>
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<td>4th quartile</td>
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<tr>
<td></td>
<td></td>
<td>(0.84)</td>
<td>(0.75)</td>
<td>(0.71)</td>
<td>(0.78)</td>
<td>(0.74)</td>
</tr>
</tbody>
</table>

*Variances are in parentheses. Figures exclude matches between affiliated parties, as well as those that involve trade in minerals, unprocessed agricultural products or services. Growth rates are conditioned on survival, and are calculated as \((X_{ijt} - X_{ijt-1})/\frac{1}{2}(X_{ijt} + X_{ijt-1})\).

3 A Model of Exporting at the Transactions Level

We propose a model that is consistent with the patterns documented in the previous section, and that provides new micro foundations for export booms. It explains firm-specific export adjustments on three margins: clients (buyers) per destination market, per-period sales per client, and duration of the buyer-seller relationship. The model is consistent with four key patterns documented above: (1) many new exporting firms appear each period; (2) most new exporters sell tiny amounts and disappear from export markets in the following period; (3) those exporters who survive expand their export volume very rapidly over the following
period, often accumulating additional buyers; and (4) firms that sell more initially are more likely to survive into the following period. It also explains (5) inter-temporal fluctuations in the size of the entering cohort, and (6) market-wide and relationship-specific fluctuations in per-period sales volumes.

The model builds on existing models of firm heterogeneity and exporting. As in Melitz (2003) and Bernard et al. (2003), firms are heterogeneous in terms of their underlying efficiency, with more efficient firms having greater incentive to overcome trade costs to sell in foreign markets. As in Das et al. (2007) and Irarrazabal and Opromolla (2007) firms experience shocks to their efficiency that lead them to switch into or out of exporting. As in Arkolakis (2008), by incurring a larger fixed cost a firm can increase the number of buyers it can reach. Finally, as in Rauch and Watson (2003), learning takes place after matches are made.

What we add to these models is a characterization of decision-making and learning by exporters. Before it enters an export market a firm is unsure of the appeal that its product has to buyers there. However, the firm can invest in activities that bring its product to the attention of individual buyers, such as advertising, participation in trade fairs, and maintenance of a foreign sales office. The more a firm spends on these activities, the more likely it will encounter a foreign buyer per unit of time. And when a match does occur, its sale not only generates a profit for the firm, it conveys information to the firm about its product’s appeal in that market. On the basis of this information the firm updates its beliefs about its product’s ultimate chances for success in that market. Good news means that future matches are likely to be more profitable, so it strengthens its efforts to encounter buyers, while bad
news discourages the firm from putting in so much effort.

3.1 Profits

To characterize the profit flow, consider firm $j$ with an efficiency $\varphi_{jt}$ (taking into account transport costs) at time $t$. This efficiency is known to the firm and evolves over time with idiosyncratic shocks. Given that it pays a wage (or more generally, unit input price) $w_t$ it can produce at cost $w_t/\varphi_{jt}$ in terms of local currency. If the exchange rate is $e_t$, its unit cost in the foreign market is $e_t w_t/\varphi_{jt}$. So assuming that all foreign buyers have Dixit-Stiglitz preferences with known demand elasticity $\eta$, seller $j$ offers price:

$$p_{jt} = \frac{\eta}{\eta - 1} \frac{e_t w_t}{\varphi_{jt}}$$

(1)

to any foreign buyer $i$ with whom it matches.$^4$

If potential buyer $i$ is confronted with an opportunity to purchase firm $j$’s product, that is, if $j$ matches with $i$ in period $\tau_{ij}$, its period $t$ sales to $i$ (conditioned on match survival) are:

$$X_{ijt} = \exp([z_j + \epsilon_{ij}] a_{t-\tau_{ij}} + \xi_{it}) \left(\frac{p_{jt}}{P_t}\right)^{1-\eta} B_t.$$  (2)

Here we introduce the market-wide spending levels among potential buyers, $B_t$, a price index for all competing products in the destination market, $P_t$, and several additional match-specific

$^4$For simplicity we assume that the firm makes a take-it-or-leave-it price offer. An alternative specification would introduce bilateral bargaining between buyer and seller, although the seller’s uncertainty about the buyer’s evaluation of the product renders this second approach substantially more complicated. Drozd and Nozal (2008) incorporate this type of bargaining in their model.
variables. First, $z_j + \epsilon_{ij}$ is a product appeal index with a component that is general to all buyers and a component that is idiosyncratic to buyer $i$. Second, $a_{t-\tau}$ is a known function of the period of time that the match has survived. This specification captures the pattern of sales growth documented in Table 6 if $a_{t-\tau}$ is positive but falls toward zero as $t-\tau$ increases. Finally, $\xi_{it}$ allows buyer $i$’s demand for $j$’s product to exhibit transitory fluctuations as idiosyncratic shocks occur in its (buyer $i$’s) product markets.

The flow profit in home currency implied by (1) and (2) is:

$$\pi(P_t, P_{ht}, \epsilon_t, z_j, a_{t-\tau}, \epsilon_{ij}, \xi_{it}, \varphi_{jt})$$

$$\begin{align*}
= \frac{1}{\eta} \frac{B_t}{\epsilon_t P_t^h} \exp([z_j + \epsilon_{ij}] a_{t-\tau} + \xi_{it}) \left( \frac{\epsilon_t w_t \eta / (\eta - 1)}{\varphi_{jt} P_t} \right)^{1-\eta},
\end{align*}$$

where $P_t^h$ is the price level in the home country. Or, combining all the aggregate variables and constants:

$$\pi(X_t, z_j, a_{t-\tau}, \epsilon_{ij}, \xi_{it}, \varphi_{jt}) = B_t \exp([z_j + \epsilon_{ij}] a_{t-\tau} + \xi_{it}) \varphi_{jt}^{\eta-1}$$

where $B_t = \frac{1}{\eta} \frac{\overline{P_t}}{\epsilon_t P_t^h} \left( \frac{\epsilon_t w_t \eta / (\eta - 1)}{P_t} \right)^{1-\eta}$ captures the market-wide forces that influence the payoff to all matches. We assume that $B_t$ and $\varphi_{jt}$ evolve over time according to a Markov process, so that given $(B_t, \varphi_{jt})$ in period $t$, the period $t+1$ values have a joint distribution $G(B', \varphi' | B_t, \varphi_{jt})$.

For purposes of the dynamic optimization problem to be introduced below, it will be convenient to define $\overline{\pi}_0(B_t, z_j, \varphi_{jt})$ as the expected present value of firm $j$’s entire profit stream associated with a new match as perceived at time $t$, conditional on $(B_t, z_j, \varphi_{jt})$. That is, $\overline{\pi}_0(B_t, z_j, \varphi_{jt})$. We treat idiosyncratic components of the price index as $P_t$ as reflected in $\epsilon_{ij} + \xi_{it}$. 

---

5Not all buyers necessarily face the same range of goods and hence the same aggregate price index $P_t$. We treat idiosyncratic components of the price index as $P_t$ as reflected in $\epsilon_{ij} + \xi_{it}$. \[\]
\( \pi_0(B_t, z_j, \varphi_{jt}) \) is the discounted expected value of the \( \pi(B_t, z_j, a_{t-\tau}, \epsilon_{ij}, \xi_{it}, \varphi_{jt}) \) trajectory from period \( t \) forward, with expectations taken over \( \epsilon_{ij}, \xi_{it}, \) and the future trajectory of \( (B_t, \varphi_{jt}) \).

In addition to its arguments, the value of \( \pi_0(B_t, z_j, \varphi_{jt}) \) depends on the firm’s discount rate \( r \), the rate at which matches terminate for exogenous reasons, \( \delta \), and the per-period fixed cost \( F \) that firms must pay to maintain each existing client relationship. More precisely, in period \( t \) the present value of a relationship that began in period \( \tau \leq t \) is:

\[
\pi_{t-\tau}(B_t, z_j, \varphi_{jt}) = B_t \exp \left[ \frac{\sigma_v^2}{2} a_{t-\tau} + \frac{\sigma_\xi^2}{2} \varphi_{jt}^{\eta-1} \right] + \frac{1 - \delta}{1 + r} \max \left\{ \int_{X'} \int_{\varphi'} \pi_{t+1-\tau}(B', z_j, \varphi') dG(B', \varphi'|B_t, \varphi_{jt}) - F, 0 \right\}.
\]

Thus \( \pi_0 \) can be recovered by evaluating (5) at \( \tau = t \).

This formulation of match pay-offs has several desirable features. First, once matches are formed, sales continue to fluctuate in response to market-wide shocks \( B_t \), idiosyncratic shocks to the buyer, \( \xi_{it} \), and shocks to the exporter’s efficiency \( \varphi_{jt} \). Second, as these fluctuations occur, matches dissolve endogenously if their continuation value falls below the fixed costs of maintaining them, \( F \). Finally, matches that generate relatively low sales volumes relative to \( F \) only survive a single period, so the model is capable of capturing both the association between initial sales and match survival and the rising survival rates documented in table 5.

At the same time that firms are matching with buyers in foreign markets, they are doing so at home. We assume that (5) characterizes the payoff to these home market matches as well, but we allow each firm’s product appeal in the home market \( (z^h_{ij}) \) to differ from its product appeal abroad. Similarly, we allow market-wide forces at home \( (B^h_t) \) to differ from foreign market-wide forces. Given that tastes are correlated across countries we expect that
cov(z_j, z^h_j) \neq 0. Also, since Colombian factor prices and the real exchange rate affect profits for all firms in both markets, cov(B_t, B^h_t) \neq 0.

3.2 Information about product appeal

In addition to generating profits, each match conveys information to an exporting firm about its product’s appeal to foreign consumers, and thereby affects its efforts to locate more buyers abroad. We assume that exporting firms are able to observe all market-wide variables and the transitory shocks that their buyers incur in product markets, \( \xi_{it} \). They also know the trajectory for \( a_{t-\tau} \), which is common to all matches. Hence, after making its first sale to buyer \( i \), firm \( j \) can use (2) to infer the associated demand shifter \( s_{ij} = z_j + \epsilon_{ij} \). This statistic serves as a noisy signal of its product appeal \( z_j \) in the foreign market, and thereby affects its search intensity. 6

More precisely, before it has met any foreign buyers, firm \( j \)’s beliefs concerning \( z_j \) are based solely on its home market product appeal index, \( z^h_j \), which we assume has been revealed to it through many matches with domestic buyers. Given that \( z_j \) and \( z^h_j \) are jointly normally distributed with zero means across the population of firms, these prior beliefs are distributed \( N(\alpha z^h_j, \sigma^2_u) \) where \( \alpha = \text{cov}(z_j, z^h_j)/\text{var}(z^h_j) \) and \( \sigma^2_u = \text{var}(z_j - \alpha z^h_j) \). However, each time a firm matches with a foreign buyer it learns something about its product’s appeal to foreign consumers. Let the buyer-specific component of foreign product appeal, \( \epsilon_{ij} \), be distributed \( N(0, \sigma^2) \) across the population of possible matches. Then after meeting \( n \) foreign buyers, firm

---

6 It would be possible to treat \( \xi \) as unobserved to the exporter. This would complicate the learning process, however, since exporters when then continue to extract information from all active matches. We feel the benefits of this extra complexity do not warrant the computational costs.
j’s posterior beliefs concerning $z_j$ are distributed $N(\tilde{z}_j^n, \sigma_n^2)$ where:

$$\tilde{z}_j^n = \alpha z_j^n \frac{\sigma_{\nu}^{-2}}{\sigma_{\nu}^{-2} + n\sigma_{\varepsilon}^{-2}} + \bar{s}_j^n \frac{n\sigma_{\varepsilon}^{-2}}{\sigma_{\nu}^{-2} + n\sigma_{\varepsilon}^{-2}},$$

$$\sigma_n = (\sigma_{\nu}^{-2} + n\sigma_{\varepsilon}^{-2})^{-1/2},$$

and $\bar{s}_j^n = n^{-1} \sum_{i=1}^n s_{ij}$.

### 3.3 Search intensity

It remains to characterize the optimal search policy. Let firm $j$ experience new matches with hazard $\lambda$ when it spends $c(\lambda)$ on search activities, where $c(\cdot)$ is increasing and convex.\(^7\) Then if the firm has received an average signal of $\bar{s}_n$ after $n$ encounters, the value of continued search

\(^7\)Following Arkolakis (2008), if we think that the market has $M$ potential buyers and sampling occurs without replacement we can generalize the hazard rate to be $\tilde{\lambda} = \lambda \cdot h(n)$ where $h(n)$ is decreasing in $n$, bounded on $[0,1]$, and $h(M) = 0$. For example, if the probability of a match is proportional to the pool of potential buyers who have not yet been visited, this function might take the form: $h(n) = \frac{M-n}{M}$. Working against this effect is the possibility that as matches accumulate, a firm’s reputation grows, and it becomes less costly to reach new customers. Hence a general expression for $h(n)$ that does not impose a sign on its derivative may be the most appropriate formulation. If this function is identified, it provides a test of Arkolakis (2008).
searching in the foreign market is \( V(\tilde{z}_j^n, n, X_t, \varphi_{jt}) \), where:

\[
V(\tilde{z}_j^n, n, B_t, \varphi_{jt}) = \max_{\lambda}
\left\{-c(\lambda) + \lambda \int_{\tilde{z}_n} \pi_0(B_t, z, \varphi_{jt}) dF(z | \tilde{z}_n, n) \right.
\]
\[
+ \frac{1 - \lambda}{1 + r} \int_{B_t'} \int_{\varphi'} V(\tilde{z}_j^n, n, B', \varphi') dG(B', \varphi' | B_t, \varphi_{jt})
\]
\[
+ \frac{\lambda}{1 + r} \int_{B_t'} \int_{\varphi'} \int_{\tilde{z}_j^n} V(\tilde{z}', n + 1, B', \varphi') d\Phi(\tilde{z}' | \tilde{z}_j^n) dG(B', \varphi' | B_t, \varphi_{jt}) \right\}
\]

Here \( r \) is the discount rate, \( F(z | \tilde{z}_n, n) \) is the posterior distribution for \( z \) after the \( n \)th match, and \( \Phi(\tilde{z}' | \tilde{z}_j^n) = N(\tilde{z}_j^n, \sigma_{n+1}) \) is the posterior distribution for \( z \) that the firm expects to prevail after the \( n + 1 \)st match, given \( \tilde{z}_n \).

A simplified version of (8) characterizes firms’ search behavior in their home market, since they have already learned their products’ appeal to domestic consumers:

\[
V^h(\tilde{z}_j^h, B_t^h, \varphi_{jt}) = \max_{\lambda^h}
\left\{-c(\lambda^h) + \lambda^h \pi_0(B_t^h, \tilde{z}_j^h, \varphi_{jt}) \right.
\]
\[
+ \frac{1 - \lambda^h}{1 + r} \int_{B_t'^h} \int_{\varphi'} V^h(\tilde{z}_j^h, B_t'^h, \varphi') dG(B_t'^h, \varphi' | B_t^h, \varphi_{jt}) \right\}
\]

Two margins of firm-level export response to idiosyncratic and market-wide shocks are characterized by these value functions: the present value of sales per buyer, and the number of buyers per firm (which is governed by \( \lambda \) and \( \lambda^h \)).

### 3.4 Stationary State

We consider an environment where firms are buffeted by shocks to their macroeconomic environment and to their own productivity. Each producer starts out ignorant of his product’s
appeal to foreign consumers, but learns about it over time. Hence some key variables in our model are highly nonstationary, and it is necessary to use numerical techniques to characterize its transition dynamics. Nevertheless it is useful to consider what happens in a stable environment in which all learning has taken place.

We thus ask what happens if \((B_t, \varphi_j) = (B', \varphi')\) and \(n \to \infty\) so that \(\bar{\pi}_n \to z\) and new matches convey no further information. Asymptotically, the distinction between \(V(\tilde{\pi}_n, n, B', \varphi')\) and \(V(\tilde{\pi}', n+1, B', \varphi')\) disappears, and the problem becomes \(rV(z) = \max \left\{ -c(\lambda) + \lambda \bar{\pi}_0(B, z, \varphi_j) \right\} \).

The solution is:

\[
V(z) = \frac{-c(\lambda^*) + \lambda^* \bar{\pi}_0(B, z, \varphi_j)}{r},
\]

where \(\lambda^*\) solves \(c'(\lambda^*) = \bar{\pi}_0(B, z, \varphi_j)\). So, not surprisingly, steady state search efforts and the present value of participating in foreign markets are monotonically increasing in the payoff to a successful match. As in Arkolakis (2008), more efficient firms (with higher \(\varphi_j\)) undertake more search effort and encounter more buyers. However, firms learn about their product appeal as they acquire buyers in our model, they adjust their search intensity accordingly, and they lose buyers over time as matches go sour. In a stationary equilibrium— with no macro or idiosyncratic shocks, and after all learning has taken place— firms settle into constant search intensities. If firm \(j\) chooses match hazard \(\lambda_j^*\) in this stationary equilibrium, it sells to an average number of buyers \(n(j)\) that satisfies \(\delta n(j) = \lambda_j^*\).

### 3.5 Specification for Numerical Solution

To solve our model numerically we parameterize the cost of matching as:
\[ c(\lambda) = b \left( \frac{\lambda}{1 - \lambda} \right) + f \cdot 1[\lambda > 0], \quad \lambda \in [0, 1) \] (10)

where \( f \) is the fixed cost of maintaining positive levels of search. Also, we treat shocks to efficiency and macroeconomic shocks as following independent first-order autoregressive processes, so that:

\[
\begin{align*}
\ln \varphi_{jt} &= \psi^\varphi \ln \varphi_{jt-1} + \nu^\varphi_t, \\
\ln B_t &= \psi^B \ln B_{t-1} + \nu^B_t, \\
\ln B^h_t &= \psi^{B^h} \ln B^h_{t-1} + \nu^{B^h}_t
\end{align*}
\] (11-13)

where:

\[
\nu^\varphi_t \sim \text{i.i.d. } N(0, \sigma^2_{\varphi})
\]

\[
\left( \nu^B_t, \nu^{B^h}_t \right) \sim \text{i.i.d. } N(0, \Sigma_B)
\]

To summarize, the model incorporate seven types of random shocks: cross-firm variation in foreign product appeal \( z \), cross-firm variation in home market product appeal, \( z^h \), noise around true product appeal associated with each match, \( \epsilon \), transitory shocks to buyers, \( \xi \), shocks to productivity, \( \nu^\varphi_t \), and the market-wide shocks, \( \nu^B_t \) and \( \nu^{B^C}_t \). It is fully described by the expression for profit (3), from which we can calculate the expected value of a relationship (5), the equation for updating beliefs about product appeal (6), the value function (8) and the cost function (10).
3.6 Fitting the model to data

3.6.1 Efficiency process

To implement our model we require values for a variety of parameters. First, we need to estimate the AR process that governs firm-level efficiency trajectories (11). The Colombian data are unusually well-suited to this task, since they allow us to construct firm-level quantity indices for both inputs and outputs (e.g., Eslava et al, 2004). Nonetheless, these indices are measured in different units at different firms, since each produces its own variety of output with its own input varieties. We therefore sweep out units of measurement by estimating (11) in growth terms:

$$\Delta \ln \varphi_{jt} = \psi \Delta \ln \varphi_{jt-1} + \Delta \nu^\varphi_{jt}. \quad (16)$$

Several econometric issues arise here. One is that the error in a differenced AR(1) model is correlated with the lagged dependent variable, since $\nu^\varphi_{jt-1}$ helps determine $\ln \varphi_{jt-1}$. We handle this problem by using Blundell and Bond’s (1998) GMM estimator, with $\ln \varphi_{jt-2}$, $\ln \varphi_{jt-3}$ and other twice-lagged plant characteristics (like output and capital stocks) serving as instruments. There is also a selection problem, since disproportionate exit occurs among low-productivity firms. This we handle with Mills ratios based on survival probabilities. Finally, to recover $\text{var}(\nu^\varphi)$, we note that by (14), $\text{var}(\nu^\varphi) = \frac{1}{2} \text{var}(\Delta \nu^\varphi)$.

Preliminary estimates are reported in Table 7 below. As is typically the case, differencing the data removes much of the persistence in measured efficiency, but lagged efficiency remains highly significant. In the present context we interpret this result to imply that permanent differences in product appeal and/or units of measurement account for the observed strong persistence in domestic sales. It should be noted that the reported specification fails a Wald
specification test for the exogeneity of the instrument (twice lagged productivity).

3.6.2 market-wide shocks

We also require estimates for the processes that generate $\ln B_t$ and $\ln B^h_t$. These we obtain (up to the intercept) using aggregate real consumption of manufactured goods in the United States and Colombia, respectively. Both are expressed in real pesos, so $\ln B_t$ incorporates the effects of exchange rate fluctuations. We denote by $\ln B_t$ the effect, which includes the exchange rate and domestic expenditure fluctuations.

When industries are pooled, we allow for industry-specific intercepts and we estimate a differenced form of (12) and (13), as with the efficiency process. When we focus on an individual industry, this is of course not necessary.

3.6.3 Remaining parameters

Four types of parameters remain, all of which we identify using the simulated method of moments. First, there are those that characterize the joint distribution of product appeal determinants: $z_j, z^h_j$ and $\epsilon_j$. These are all normalized to zero, so the variances of each ($\sigma_{z_j}^2, \sigma_{z^h_j}^2, \sigma_{\epsilon_j}^2$), plus the coefficient $\alpha$ are sufficient to identify their joint density. Since they govern the varation in sales across buyers for a given seller, the variation in exports across sellers, and covariance between foreign and domestic sales among exporters, key moments are: $\text{var}(\ln X^h_{jt})$, $\text{var}(\ln X_{jt})$, $\text{var}(\ln X_{ijt}/X_{jt}/n_{jt})$ $\text{cov}(\ln X^h_{jt}, \ln X_{jt})$, and $\text{cov}(\Delta \ln X^h_{jt}, \Delta \ln X_{jt})$ where $X^h_{jt}$ is firm $i$’s sales in its home market, and $X_{jt}/n_{jt}$ is its average sales per foreign client.

\[^8\text{If both the log exchange rate and the log of domestic expenditures on manufactured goods in the U.S. follow AR1 processes, then ln B_t is the sum of two AR1’s, which is generally an ARMA(2,1) process. To avoid introducing another state variable in our model we treat ln B_t as a simple AR1, implicitly assuming that both the log exchange rate and the log of domestic expenditures have the same root. This assumption could easily be relaxed at the expense of computational speed.\]
Second, there are parameters that govern matching and separation processes: $f, F, \delta$. The fixed costs of maintaining a relationship and the exogenous match destruction rate $\delta$ determine observed rates of match separations. Their effects are distinctive in that $\delta$ affects all firms equally, while the effect of $f$ declines as $\tilde{\pi}_0$ increases. Hence key moments for identifying these parameters are the level of failure rates and the covariance between failure rates and $\ln X_{jt}$. The fixed costs of searching, $F$, determine which firms abstain from exploring export markets altogether, so the fraction of firms that never export helps to identify this parameter.

Third, there parameters that govern match age effects: $a_{t-\tau}$. To limit the number of these, we impose a simple functional form:

$$a_{t-\tau} = \begin{cases} 
\mu_0 + \mu_1 (t - \tau) & t < t_{\text{max}} \\
\mu_0 + \mu_1 (t_{\text{max}} - \tau) & t \geq t_{\text{max}}
\end{cases}.
$$

The parameters $\mu_0$ and $\mu_1$ are then identified by the quantile and age specific average growth rates reported in Table 6.

Finally, there are several nuisance parameters: the rate of time preference, $r$, and the profit function scale parameter. Since rates of time preference are typically poorly identified in dynamic structural models, we follow convention and simply fix $r$ at a plausible value. Given other parameters, intercepts for the $\ln B$ and $\ln B^h$ processes are chosen to replicate observed industry-level sales volumes as closely as possible.

### 3.7 Parameterization

Estimation of the parameters is in progress. To give a preliminary sense for the behavior of our model, we solve it for the somewhat arbitrary parameter values reported in Table 7 below.
Table 7: Parameters for Simulations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>rate of time preference</td>
<td>$r$</td>
</tr>
<tr>
<td>rate of exogenous separation</td>
<td>$\delta$</td>
</tr>
<tr>
<td>profit function scale parameter, exports</td>
<td>$s^{us}$</td>
</tr>
<tr>
<td>profit function scale parameter, domestic</td>
<td>$s^{col}$</td>
</tr>
<tr>
<td>fixed cost of searching</td>
<td>$f$</td>
</tr>
<tr>
<td>fixed cost of sustaining match</td>
<td>$F'$</td>
</tr>
<tr>
<td>intercept, match age function</td>
<td>$\mu_0$</td>
</tr>
<tr>
<td>slope, match age function</td>
<td>$\mu_1$</td>
</tr>
<tr>
<td>time horizon for buyer learning</td>
<td>$\alpha_{\text{max}}$</td>
</tr>
<tr>
<td>standard deviation of noise in signal</td>
<td>$\sigma^2_{\epsilon}$</td>
</tr>
<tr>
<td>standard deviation of product appeal</td>
<td>$\sigma^2_{\beta}$</td>
</tr>
<tr>
<td>correlation, home and foreign mkt. appeal</td>
<td>$\alpha$</td>
</tr>
<tr>
<td>root of efficiency process</td>
<td>$\psi^{\varphi}$</td>
</tr>
<tr>
<td>root of foreign mkt. process</td>
<td>$\psi^B$</td>
</tr>
<tr>
<td>root of home mkt. process</td>
<td>$\psi^{B_h}$</td>
</tr>
<tr>
<td>standard deviation of efficiency innovation</td>
<td>$\sigma_{\psi^\varphi}$</td>
</tr>
<tr>
<td>standard deviation of foreign mkt. shock</td>
<td>$\sigma_{\psi^B}$</td>
</tr>
<tr>
<td>standard deviation of home mkt. shock</td>
<td>$\sigma_{\psi^{B_h}}$</td>
</tr>
</tbody>
</table>

3.8 Policy functions

The first panel of figure 1 above shows the value of access to foreign buyers that firms perceive after one signal, as a function of the signal they have received. Not surprisingly, there is a positive relationship, and firms that receive better signals choose to search more intensively. The second panel of this figure shows how values and search intensities have changed after 5 signals have accrued. (The horizontal axis is the posterior mean after 5 signals, $\tilde{z}_5^\beta$.) Note that the value of search has fallen relative to its value after one signal for those firms with low average signals because these signals become increasing precise as experience accumulates. (When five buyers tell you they don’t care for your product, there is a good chance that your product has poor market potential.) The last two panels of figure 1 translate search values into match probabilities, and tell the same qualitative story. Below some threshold signal, the
return to search is less than the associated fixed cost \( f \), and so no search takes place. If \( f \) were to increase, this cutoff would shift to the right (not pictured).

Figure 2 shows how the policy function characterized in figure 1 translates into behavior for a simulated set of 1,000 firms. Here the horizontal axis is true \( z \) value rather than signal. The first panel describes match hazards for a new cohort of firms, none of which has received any signals yet. Since all firms share the same priors at this point there is no relationship between \( z \) values and search intensity. However, some firms don’t search very intensively because their current productivity level is low. After 5 periods, a relationship between \( z \) and search intensity emerges, with many low-\( z \) firms dropping out of foreign markets. This replicates patterns seen in Tables 1 and 4. Note that considerable heterogeneity in behavior remains, given \( z \). This reflects both productivity differences and differences in the idiosyncratic features of the buyers (\( \epsilon \)’s and \( \xi \)’s) that the exporters have randomly matched with. It also reflects the magnitude of fixed search costs.

3.9 Match Separations

Figures 3 shows that even at made-up parameter values, the model replicates the patterns of match duration documented in Table 5. Most matches last only a single period, given their low expected payoff and the fixed costs of maintaining a relationship. Thereafter, there is a mild tendency for separation hazards to fall. Interestingly, most of the ongoing separation is endogenous to our model, since the average rate remains between 0.40 and 0.50 (as in table 4), while the exogenous rate of separation is chosen to be \( \delta = 0.1 \) (Table 7).

Our model also captures the association between first-year sales and match duration doc-
umented in section 2. Figure 4 shows that matches beginning with sales in the smallest size deciles almost always fail: their average duration is less than one and one-half years, with the smallest deciles almost always failing in a single period. On the other hand, matches that start in the top two sizes deciles last more than two years, on average.

It is nonetheless true that those initially-small matches that don’t fail grow more rapidly than the initially-large matches that survive, as Table 5 documents. This property of our model (not depicted in a figure) is ensured by the fact that our age function (17) has a slope of $\mu_1 = -0.3$ over the early years of the match, implying that demand grows for surviving matches with below-average (negative) $z$ values, while it falls for others. We chose a negative slope to capture the Rauch and Watson (2003) argument that exporters who shipped small samples to buyers during the first period and survived to the second period must have favorably resolved buyers’ uncertainty about their product.

3.10 Foreign and domestic sales

Figure 5 summarizes the behavior of 1000 simulated firms over a 75 year time horizon. All firms are assumed to begin with no matches at home or abroad, so the early years in figure 6 correspond to a transition period during which customer bases are being developed in both markets. Accordingly, while the first 5-10 years are of possible interest in analyzing the maturation of new export markets, they should not be interpreted to characterize the ergodic distributions of the variables depicted.

The first panel of figure 5 shows that, among exporting firms, 10 to 20 percent of sales revenue comes from foreign buyers, with considerable fluctuation over time. This matches up
well to plant-level survey data from Colombia, which imply average export shares fluctuated over a similar range during the period 1986-1996. It is remarkable that these export shares are not larger, given that the only difference between the expected pay-off to a home versus match is the profit function scaler, which is roughly twice as high at home ($s^{us} = 0.80$, $s^c = 1.50$). In a model that abstracts from market-specific matching processes, this would imply export shares of $0.80/(0.80+1.50) = 0.35$, on average, since sales revenues are proportional to profits. Our model generates a lower export share because search intensities move in sympathy with expected profits per match, and these are higher in domestic markets because of transport costs. That is, our model explains the border puzzle as partly due to lower search intensities in foreign markets.

Our model also replicates the well-known tendency for larger firms to be exporters, and the lack of association between export shares and size among firms with a foreign market presence. These features are documented in the second and third panel of figure 5, respectively. Both are due to the fact that cross-firm variation in behavior is induced by cross firm differences in productivity and product appeal. The former are equally important in both markets, and the latter are correlated across markets.

### 3.11 Export trajectories

The search frictions in our model lead to a new kind of inertia and hysteresis in export markets, especially for higher quality exporters, who tend to form more durable relationships. Unlike the earlier sunk cost hysteresis literature, which emphasized substantial market entry costs, our formulation accommodates the stylized fact that many small-scale exporters appear
and disappear each period. The other formulation which does this, Arkolakis (2009), does so in a frictionless environment and thus does not speak to response lags or irreversibilities in exporter behavior.

Are these features of our model important? Figure 6-7 aggregate the simulated firm-level export trajectories and numbers of foreign clients used to construct figure 6, thereby imputing economy-wide series for each. Both series are in logs and normalized to zero in the initial year. Figure 6 depicts the log of total exports and the log market-wide shifter \( (B_t) \) through time; the latter can be thought of as mainly reflecting movements in the real exchange rate. Clearly, exports are responsive to the exchange rate, and much more volatile. Partly this reflects the fact that the elasticity of demand is \( \eta = 5 \), causing small changes in export prices to trigger large changes in demand. But this does not explain why responses are larger in some periods than in others—something that would not occur in a frictionless model. Note in particular the large drop in exports that occurs around period 50, despite the mild appreciation of the exchange rate.

The reason for the relatively dramatic response here is clear in figure 7, which shows the log of the total number of clients, again with the log exchange rate. This series drops dramatically around period 50, reflecting the fact that many firms reduced their search efforts and allowed their existing matches to expire. Thus it appears that there are threshold values of expected profitability per match below which many producers dramatically curtail their foreign business relationships. And, unlike most trade models with heterogeneous firms, the fixed costs that create these threshold values influence both large and small-scale exports, since all have matches that are marginally profitable.
4 Conclusions (to come)
References


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Figure 1:

Signal, value, match hazard, and learning
Figure 2:

True product appeal and match hazard: initial and change after 5 signals
Figure 3:

Match age and average separation rates
Figure 4:

Average match duration and first-period sales
Figure 5:

Firm-level exports and domestic sales
Figure 6:

Total log exports and log foreign market shifter ($B$)
Figure 7:

Total log clients and log foreign market shifter ($B$)