The Seeds of a Crisis: A Theory of Bank Liquidity and Risk-Taking over the Business Cycle

Viral Acharya∗ and Hassan Naqvi†‡

December 19, 2009

Abstract

We examine how the banking sector may ignite the formation of asset price bubbles when there is access to abundant liquidity. Inside banks, given limited liability and lack of observability of effort, loan officers (or risk takers) are compensated based on the volume of loans. Outside banks, when there is heightened macroeconomic risk, investors reduce direct investment and hold more bank deposits. This ‘flight to quality’ leaves banks flush with liquidity, lowering the sensitivity of managerial payoffs to the downside risk of loans and inducing excessive credit growth and asset price bubbles. The seeds of a crisis are thus sown. A Central Bank, that can detect the macroeconomic risk or the

∗New York University Stern School of Business, NBER, CEPR and ECGI. e-mail: vacharya@stern.nyu.edu
†National University of Singapore and NUS Risk Management Institute. e-mail: naqvi@nus.edu.sg
‡We would like to acknowledge Hanh Le and Michelle Zemel for research assistance. Comments by seminar participants at National University of Singapore, CEMFI, and Lancaster University Management School are also appreciated.
flight to quality effect, can curb the risk-taking incentives at banks with a contractionary monetary policy that draws out excess bank liquidity. Conversely, an expansionary monetary policy in such times only enhances the liquidity insurance enjoyed by banks, aggravating their risk-taking incentives.

“For too long, the debate has got sidetracked. Into whether we can rely on monetary policy ‘mopping up’ after bubbles burst. Or into whether monetary policy could be used to control asset prices as well as doing its orthodox job of steering nominal trends in the economy...” - Paul Tucker, Executive Director for Markets and Monetary Policy Committee (MPC) member at the Bank of England.1

1 Introduction

It is clear that in the period leading up to the global financial crisis of 2007-2009, credit and asset prices were growing at a ferocious pace.2 In the United States, for example, in the five-year period from 2002 to 2007, the ratio of debt to national income went up from 3.75 to one, to 4.75 to one. It had taken the prior full decade to accomplish this feat, and fifteen years prior to that. During this same period, house prices grew at an unprecedented rate of 11% per year while there was no evidence of appreciating borrower quality over this period. The credit growth was across board, in mortgages, especially sub-prime ones, in financing of leveraged buy-out transactions (LBOs), and through increasing issuance of low-rated bonds. This rapid rise in asset volume and prices met with a precipitous fall. For instance, the

---

2The series of facts to follow are borrowed from Acharya and Richardson (2009a).
Figure 1: House Price to Rent Ratio. The Figure graphs the demeaned value of the ratio of the Office of Federal Housing Enterprise Oversight (OFHEO) repeat-sale house price index to the Bureau of Labor Statistics (BLS) shelter index (i.e., gross rent plus utilities components of the CPI). Because of demeaning, the average value of this ratio is zero.

The median house price divided by rent in the United States\(^3\) over the 1975 to 2003 period varied within a relatively tight band around its long-run mean. Yet starting in late 2003, this ratio increased at an alarming rate. In mid 2006, however, the ratio flattened and has been falling sharply ever since albeit with much more room to fall. (See Figure 1.)

What caused this tremendous asset growth and the subsequent puncture is likely to intrigue economists for years. Some have argued that the global

\(^3\)In particular, this is the ratio of the Office of Federal Housing Enterprise Oversight (OFHEO) repeat-sale house price index to the Bureau of Labor Statistics (BLS) shelter index (i.e., gross rent plus utilities components of the CPI)
economy was in a relatively benign low-volatility environment in the decade leading up to the ongoing crisis (the so-called “Great Moderation”, see Stock and Watson, 2002). In departure from this view, we argue that it is likely not a coincidence that the phase of remarkable asset growth described above started at the turn of the global recession of 2001–2002. In fact, in response to the unprecedented rate of corporate defaults and heightened macroeconomic risk during the crisis, the Federal Reserve lowered interest rates to 1%, the lowest level since 1958. A period of abundant availability of liquidity to the financial sector ensued, bank balance-sheets grew two-fold within four years, and as the “bubble burst”, a number of agency problems within banks in those years came to the fore. Such problems were primarily concentrated in centers that were in charge of undertaking large risks and took the form of risk-takers being paid out huge bonuses based on the volume of assets they created rather than on (long-term) profits they generated.4

In this paper, we present a theoretical model that combines these ingredients and explains why access to abundant liquidity aggravates the risk-taking moral hazard at banks, giving rise to asset price bubbles. We argue that these bubbles can be counteracted by Central Banks with a contractionary monetary policy, and conversely are exacerbated by expansionary monetary policy. Somewhat perversely, the seeds of crisis are sown when the macroeconomic risk is high and investors in the economy switch from investments to savings in the form of bank deposits. Expansionary monetary policy is tempting in such times, but this may flush banks with liquidity, which can lead to credit booms and asset price bubbles.

We first develop a benchmark model in Section 2.1 wherein the representative bank collects deposits from investors and then allocates a fraction of these deposits to investment projects. The bank faces random deposit withdrawals and in case of liquidity shortfalls suffers a penalty cost. The penalty cost could be interpreted as the cost of fire sales or alternatively the cost of raising finance from markets. In order to avoid such costs the bank has an incentive to set aside some reserves. The rest of the deposits are invested in projects depending on the demand for loans. The problem of the bank is to choose the optimal lending rate that maximizes its expected profits subject to the depositors’ participation constraint. We show in this benchmark model that the bank lending rate appropriately reflects the underlying risk of the project.

In Section 2.2 we enrich the model to study how agency problems within the bank affect the pricing of loans. In practice, bank managers often have incentives to give out excessive loans since their payoffs are proportional to the amount of loans that are advanced.\(^5\) We show that such incentives can arise as part of an optimal contracting outcome of a principal-agent problem when managerial action or effort is unobservable. Further, under the optimal contract, bank managers may have an incentive to underprice the risk of the investments. However, we show that such mispricing of risk only occurs when the bank is awash with liquidity (deposits). This is because in presence of excessive liquidity the manager attaches too little weight to the scenario where the bank might later face liquidity shortfalls. In other cases.

words, excessive liquidity encourages managers to overinvest and underprice the underlying risks.

We then show in Section 2.2.3 that such behavior ultimately has an impact on asset prices. We assume that the demand for loans arises from investments by the corporate sector in underlying assets of the economy. To show how asset price “bubbles” are formed we first define the “fundamental” asset prices as those that arise in the absence of any agency frictions within banks. We construct the optimal demand function for assets by bank borrowers and then solve for the underlying asset price given the market clearing condition that the aggregate demand for assets should equal their finite supply. If the bank lending rate underprices risks, then there is an increase in aggregate borrowing from banks. This in turn fuels an excessive demand for assets which leads to prices rising above their fundamental values. We interpret this asset price inflation as a “bubble”. Importantly, such bubbles are formed only when bank liquidity is high enough as only then do loan officers underprice risk. We would like to clarify at this point that our paper is not concerned with the discussion of the welfare implications of bubbles but the focus is on explaining how bank-level agency problems can induce bubbles in asset prices.

In Section 3 we study when asset price bubbles are most likely to be formed. We show that this is the case when the macroeconomic risk is high. When macroeconomic risk increases, depositors avoid direct entrepreneurial investments and prefer to save their money in bank deposits which are perceived to be safer. Gatev and Strahan (2006) offers direct empirical evidence consistent with this. This “flight to quality” results in excessive bank liquidity and induces bubble formation in line with our earlier results.

Finally, we study the implications of this result for the monetary policy.
We show that if the Central Bank adopts a contractionary monetary policy in times of increasing macroeconomic risk, then it can counter the flight to quality by drawing out the increases in bank liquidity and avoiding the emergence of bubbles. On the contrary, if the central bank adopts a loose monetary policy then this accentuates the formation of bubbles. Intuitively, an increase in the money supply only serves to increase bank liquidity further when there is already a flight to quality of deposits. Our model can thus explain how lax monetary policy by the Scandinavian Central Banks in 1980's, Bank of Japan during 1986-1987, and the Federal Reserve during most of the Greenspan era culminated in housing and real estate bubbles in these countries.

Proponents of the ‘Greenspan camp’ argue that monetary policy should not be geared towards avoiding the emergence of bubbles and should focus on targeting the natural interest rate and the natural rate of employment as has traditionally been the case. This is justified on the basis that central banks cannot pinpoint an asset price bubble. Since the asset price bubble is intuitively tied to macroeconomic risk and flight to quality in deposits, we believe that the central banks’ task in identifying times for employing a contractionary policy is not as onerous as is often suggested. Indeed, we show that even in the absence of perfect information regarding macroeconomic fundamentals, a monetary policy based on some ‘leaning against the wind’ approach can be justified.  

\[\text{Indeed, a number of economists, including those who traditionally believed that monetary policy should not react to asset price bubbles, have revised their priors on its conduct. Some examples include: (i) “Given the events of the last eight months, it would be foolish not to reconsider the Greenspan doctrine,” by Kenneth Rogoff, Financial Times, 16 May 2008; (ii) “I think I am still with the orthodoxy but I have to admit that recent events}\]
While Jensen and Meckling (1976) showed that leverage induces equityholders to prefer excessive risk, our point is concerned with risk-taking incentives as a function of liquidity. On this front, our result is similar to Myers and Rajan (1998) wherein access to liquidity allows firms to switch to riskier assets, and in anticipation of such behavior, renders them illiquid ex ante. The channel in our model is different though: When banks are flush with liquidity, managers are hedged from the downside of risks they undertake, and this induces risk-taking incentives.

Allen and Gale (2000) show in a model of risk-shifting that uncertainty in monetary policy acts to exacerbate the risk-taking incentives ex ante and fosters an asset price bubble. Diamond and Rajan (2008) show that lowering interest rates ex post may be desirable for a central bank to avoid bank runs and fire sales, but that this can induce moral hazard and incentivize banks to hold more illiquid assets. It may thus be desirable for the Central Bank to commit to raising interest rates when they are low. In our model, the focus is similarly on the ex-ante effects of central bank intervention. An expansionary monetary policy serves to hedge banks against liquidity shocks and this makes risk-taking more attractive ex ante. The important difference in our model arises from its embedding of the principal-agent problem between dispersed shareholders of banks and the loan officers or the risk takers. The severity of the agency problem is affected by the availability of liquidity, and thus by central bank interventions. Importantly, in the absence of bank-level agency problems, there is no excess in credit growth

---

and central bank interventions do not distort credit outcomes.

2 Benchmark model

2.1 Base case

We consider a three-period model of a bank wherein the bank at \( t = 0 \) receives deposits \( D \) from risk neutral investors. Each investor deposits 1 unit of his endowment in the bank. The reservation utility of depositors is given by \( \bar{u} \). Hence in order to secure deposits the bank needs to set the rate of return on deposits, \( r_D \), such that the depositors earn an expected payoff of at least \( \bar{u} \).

The bank subsequently makes investments in projects while holding a fraction of the deposits as liquid reserves, \( R \). The bank-funded projects either succeed or fail at \( t = 2 \). The probability of success of bank projects is given by \( \theta \) and in the event the project is successful it pays off at \( t = 2 \). The project is illiquid in the sense that if it were to be liquidated prematurely at \( t = 1 \) the bank faces a penalty cost. The bank observes \( \theta \) and sets \( r_L \) which is the (gross) rate of return on loans. When choosing the lending rate, the bank takes into account the demand function for loans (say, by the corporate sector) which is given by \( L (r_L) \) where \( L' (r_L) < 0 \). Bank reserves are given by:

\[
R = D - L (r_L)
\]

The bank experiences withdrawals at \( t = 1 \) and for simplicity we assume that the fraction of depositors who experience a liquidity shock is a random variable given by \( \tilde{x} \), where \( x \in [0, 1] \). The cumulative distribution function

\[
F(x) = \Phi \left( \frac{x - \mu}{\sigma} \right)
\]

As in Allen and Gale (1998) and Naqvi (2007) we could have assumed that \( \tilde{x} \) is
of $\tilde{x}$ is given by $F(x)$ while the probability distribution function is denoted by $f(x)$. Each depositor who withdraws early receives 1 unit of his endowment back at $t = 1$. Thus the total amount of withdrawals at $t = 1$ is given by $\tilde{x}D$. If the realization of $\tilde{x}D$ is greater than $R$, then the bank faces a liquidity shortage, and it incurs a penalty cost, given by $r_p(\tilde{x}D - R)$, which is proportional to the liquidity shortage, where $r_p > r_L > 1$.

The penalty cost can be justified in a number of ways. The bank may be forced to cover the shortfall in a costly manner by selling some of its assets prematurely at fire-sale prices. This is particularly likely when firms in other industries are also facing difficulties. Alternatively the bank can raise external financing via capital markets. However, this is also costly because raising equity leads to dilution of existing shareholders due to the debt overhang problem (Myers, 1977). Furthermore, raising external finance correlated with asset quality news in the sense that depositors receive a noisy signal of $\theta$ on which they base their decision on whether or not to run. While this is more realistic, it does not affect our qualitative result and highly complicates the analysis. Hence similar to Diamond and Dybvig (1983) and Prisman, Slovin and Sushka (1986) we assume that $\tilde{x}$ is random.

Shleifer and Vishny (1992) argue that the price that distressed firms receive for their assets is based on industry conditions. In particular, the distressed firm is forced to sell assets for less than full value to industry outsiders when other industry firms are also experiencing difficulties. There is strong empirical support for this idea in the corporate-finance literature, as shown, for example, by Pulvino (1998) for the airline industry, and by Acharya, Bharath, and Srinivasan (2006) for the entire universe of defaulted firms in the US over the period 1981 to 1999 (see also Berger, Ofek, and Swary, 1996, and Stromberg, 2000). In the evidence of such specificity for banks and financial institutions, James (1991) shows that the liquidation value of a bank is typically lower than its market value as a going concern. In particular, his empirical analysis of the determinants of the losses from bank failures reveals a significant difference in the value of assets that are liquidated and similar assets that are assumed by acquiring banks.
entails a price impact due to the adverse selection problem a la Myers and Majluf (1984). Capital-raising can also entail costs related to monitoring that the new financier must undertake. Finally, if the bank attempts to cover the shortfall by emergency borrowing from the central bank, this can also be costly as the central bank may charge a penalty rate. Finally, apart from pecuniary costs, the bank may also suffer non-pecuniary costs such as a reputational cost, e.g., the stigma associated with borrowing from the central bank’s emergency facilities.

If the projects financed by bank borrowings are successful, then the bank is solvent and is able to repay the patient depositors the promised rate of return of $r_D$ at $t = 2$, whilst the equityholders consume the residual returns from bank investments after these depositors have been paid off. However, in case of the failure of bank-funded projects, the surplus reserves, $R - xD$, if any, are divided amongst the patient depositors whilst the equityholders consume zero. The sequence of events is summarized in the timeline depicted in Figure 2.
Given this setup the risk neutral bank’s problem is as follows:

$$\max_{r_L, r_D} \Pi = \pi - r_D E [\max (\tilde{x}D - R, 0)]$$

subject to

$$E (\tilde{x}) + (1 - E (\tilde{x})) \left[ \theta r_D + (1 - \theta) \frac{E [\max (R - \tilde{x}D, 0)]}{(1 - E (\tilde{x})) D} \right] \geq \bar{u}$$

where $\pi$ is given by:

$$\pi = \theta \{ r_L L (r_L) - r_D D (1 - E (\tilde{x})) + E [\max (R - \tilde{x}D, 0)] \}.$$
Note that for simplicity we have considered a setup with a given penalty cost. In the appendix we consider a setup wherein the penalty costs are explicitly calculated in an environment where the bank finances the shortfall by selling its assets at fire-sale prices. We show that in this three-period environment, the objective function of the bank is analogous to equation (1) and is given by $\pi$ minus a cost term which is proportional to the bank’s liquidity shortfall. Hence our qualitative results are unchanged and thus we use the simpler setup given its parsimony and tractability.

We next solve the bank’s optimization problem and derive the first best lending and deposit rates. Subsequently we study how the riskiness of bank projects as well as bank liquidity affects the loans rate offered by the bank. The results are summarized in Proposition 1.

**Proposition 1**  
1. The optimal gross lending rate is given by

$$r^*_L = \frac{1 + (r_p - 1) \Pr (\tilde{x}D \geq R)}{\theta \left(1 - \frac{1}{\eta_L}\right)} \tag{4}$$

where $\eta_L = -r_L L' (r_L)/L > 0$ is the elasticity of the demand for loans.

The optimal gross deposit rate is given by

$$r^*_D = \frac{(\bar{u} - E(\bar{x})) D - (1 - \theta) \Pr (\tilde{x}D < R) (R - E(\tilde{x}D|\tilde{x}D < R))}{\theta (1 - E(\tilde{x})) D} \tag{5}$$

while the optimal level of reserves is given by:

$$R = D - L (r^*_L)$$

2. (Risk effect) $\frac{\partial r^*_L}{\partial \theta} < 0$, i.e., an increase in risk $(1 - \theta)$, ceteris paribus, increases the equilibrium lending rate.
3. **(Liquidity effect)** \( \frac{\partial r^*}{\partial D} < 0 \), i.e., an increase in bank liquidity, *ceteris paribus*, decreases the equilibrium lending rate.

**Proof.** See Appendix. ■

It is interesting to note that as the elasticity of demand for loans decreases, the lending rate increases and hence the spread between the loan rate and deposit rate increases. This result is consistent with the Monti-Klein (Klein, 1971 and Monti, 1972) model. As expected, the greater the market power of the bank, the higher will be the lending rate that it can charge and vice versa.

The second and third parts of the proposition are also intuitive. The lending rate prices both risk and liquidity. An increase in liquidity lowers the expected cost of liquidity shortage and the bank passes some of this benefit to the borrowers via a lower loan rate.

### 2.2 Agency problem, over-lending and asset price bubbles

#### 2.2.1 Setting of the problem

So far we have abstracted from any agency issues between the bank equity-holders and the bank manager. Let us now consider a setting where such issues are relevant.

A study by OCC (1988) found that “Management-driven weaknesses played a significant role in the decline of 90 percent of the failed and problem banks the OCC evaluated...directors’ or managements’ overly aggressive behavior resulted in imprudent lending practices and excessive loan growth." They also found that 73% of the failed banks had indulged in over-lending. This indicates that principal-agent problems within banks have been one of the key reasons for bank failures and that bank managers tend to engage
in ‘overly aggressive risk-taking behavior’.\textsuperscript{9} Perhaps even more striking evidence is presented by the financial crisis of 2007-2009 which has revealed that in the period preceding the crisis, traders and large profit/risk centers at a large number of financial institutions were paying themselves bonuses based on the size of their risky positions rather than their long-run profitability. Moreover, in many cases, it was a conscious choice of senior management to silence the risk management groups that had spotted weaknesses in the portfolio of building risks.\textsuperscript{10}

To study how managerial problems can have an effect on bank policies, we model the agency problem within banks explicitly. Let $e$ denote the unobservable effort level of the manager, such that $e \in \{e_L, e_H\}$, where $e_H > e_L$. We assume that although the loans are affected by effort, they are not fully determined by it. The stochastic relationship is necessary to ensure that effort level remains unobservable. We assume that the distribution of $L(r_L)$ conditional on $e_H$ first-order stochastically dominates the distribution conditional on $e_L$. In other words, for a given level of lending rate, the manager on average makes a higher volume of loans when he exerts high effort relative to the case where he exerts lower effort, i.e. $E[L(r_L)|e_H] > E[L(r_L)|e_L]$. In addition to $L(r_L)$, both the principal and agent can observe the realization of bank’s net profit, $\Pi$.

The manager is risk averse and his utility is increasing in wages, $w$, but

\textsuperscript{9}The OCC’s study is based on an analysis of banks that failed, became problems and recovered, or remained healthy during the period 1979-1987. The study analysed 171 failed banks to identify characteristics and conditions present when the banks deteriorated.

\textsuperscript{10}See Chapter 8 of Acharya and Richardson (2009b), which contains a detailed account of governance and management failures at a large number of financial institutions. The most detailed evidence is for UBS based on its “Shareholder Report on UBS’s Write Downs” prepared for the Swiss Federal Banking Commission.
Figure 3: Timeline of events.

decreasing in effort. Furthermore, we suppose that $E[\Pi |e_H] - E[\Pi |e_L] > E[w |e_H] - E[w |e_L]$. This means that the incremental increase in the expected profit from implementing a high effort is greater than the increase in the expected wage costs from implementing the high effort. In other words the principal has an incentive to implement the high effort level since the gains from doing so are greater than the associated costs.

We assume that bank liquidity is non-verifiable. This is plausible given that liquidity is not even well-defined as it can take several forms. Furthermore, bank liquidity may be lent out to other banks via the interbank market or conversely it may be the excess liquidity of other banks that makes it way to the bank in question. It is also particularly difficult to verify off-balance sheet liquidity which may take the form of unused loan commitments.

The timeline is as follows: At $t = 0$ the bank raises deposits, the principal offers a contract to the manager (such that $e_H$ is chosen) and the manager chooses $r_L$. At $t = 0.5$, for a given level of $r_L$ the volume of loans will be realized. As before, at $t = 1$ there may be early withdrawals and finally at $t = 2$ the payoffs are realized and divided between the parties given the contractual terms. The timeline is depicted in figure 3.
In the presence of symmetric information, there is no agency problem and the bank’s problem is analogous to that of section 2.1 with the bank maximizing

\[ \Pi = \pi - r_p E[\max(\hat{x}D - R, 0)|e = e_H] \]

subject to the following participation constraint

\[ E(\hat{x}) + (1 - E(\hat{x})) \left[ \theta r_D + (1 - \theta) \frac{E[\max(R - \hat{x}D, 0)|e = e_H]}{(1 - E(\hat{x})) D} \right] \geq \bar{u} \]

where \( \pi \) is given by

\[ \pi = \theta \{ r_L E[L(r_L)|e_H] - r_D D (1 - E(\hat{x})) + E[\max(R - \hat{x}D, 0)|e = e_H] \} \]

The first-best lending rate analogous to equation (4) is given by

\[ r_L^f = \frac{1 + (r_p - 1) \Pr[(\hat{x}D \geq R)|e = e_H]}{\theta \left( 1 - \frac{1}{\bar{\eta}_L} \right)} \]

where \( \bar{\eta}_L = -r_L \left. \frac{\partial E[L(r_L)|e_H]}{E[L(r_L)|e_H]} \right|_{r_L} > 0 \).

Next, we study the case of asymmetric information. We assume that the manager can observe \( \theta \) and \( F(x) \) but this information is not observable to the principal and hence the principal cannot infer the first-best loan rate. Consequently, the principal will be unable to enforce the first-best lending rate and hence there will be a deviation from the first-best even though the principal is able to implement \( e_H \) via the incentive compatibility condition. However, the risk premium required on deposits (defined as \( \kappa^* \equiv r_D^* - \bar{u} \)) is

---

\(^{11}\)If \( \theta \) and \( F(x) \) are observable then the principal would be able to ascertain \( r_L^f \) given (9) and hence enforce the first-best rate, for instance by penalizing the manager if the actual rate diverges from the first best. Thus this assumption ensures that there is asymmetry of information regarding the optimal loan rate.
public information and hence the participation constraint (7) still needs to be satisfied.\textsuperscript{12}

Given this setup with asymmetric information, we prove the following proposition in the appendix.\textsuperscript{13}

**Proposition 2** The optimal contract between the principal and the manager entails that the manager’s payoffs be monotonically increasing in the amount of loans as well as bank profits net of penalty costs.

**Proof.** See Appendix. \( \blacksquare \)

Such contracts are ubiquitous in practice whereby the bonuses of managers are increasing in the volume of loans and profits. For simplicity we limit the analysis to linear contracts such that the manager’s payoff is a proportion \( \beta \) of loans and a proportion \( \delta \) of profits net of penalty costs.\textsuperscript{14}

Hence the bank manager maximizes the following program:

\[
\max_{r^*_L, r^*_D} \Pi_m = \beta E[L(r_L)|e_H] + \delta (\pi - r_pE[\max(\tilde{x}D - R, 0)|e = e_H]) \quad (10)
\]

\textsuperscript{12}As before, the participation constraint (7) will hold with equality and the optimal deposit rate can be written in generic form as \( r^*_D = \bar{u} + \kappa^* \) where \( \kappa^* = \kappa(\theta, F(x), E[L|e_H], D) \). The assumption that \( \kappa^* \) is public information implies that the participation constraint needs to be satisfied and at the same time we remain in an environment with asymmetric information.

\textsuperscript{13}In order to ensure monotonocity we use the monotone likelihood ratio property as is usual in contract theory literature. MLRP ensures that both \( \beta \) and \( \delta \) in expression (10) are positive.

\textsuperscript{14}The assumption of linear contracts is for simplicity and has no bearing on our results. Furthermore, Holmstrom and Milgrom (1987) argue that real-world compensation schemes involve linear contracts as more complex contracts are subject to costly “gaming” by agents. Holmstrom and Milgrom show that in a dynamic environment with CARA preferences and (binomial) i.i.d. increments in output, optimal linear contracts can be obtained.
subject to the participation constraint (7) and where $\pi$ is given by (8). We can now prove the following proposition.

**Proposition 3** *In the presence of an agency problem between the bank manager and equityholders, the loan rate is lower (compared to the case where there is no agency problem) and consequently the volume of credit in the economy is higher.*

**Proof.** See Appendix. ■

The intuition behind the above proposition is straightforward. The bank manager has an additional incentive to lower lending rates as this allows him to make more loans and his payoff is increasing in the amount of lending.

### 2.2.2 Liquidity-induced agency problem

The intuition as to why the optimal contract in the preceding section depends on both the volume of loans and net profit is as follows: Managerial compensation solely dependent on profits would be suboptimal from a risk-sharing standpoint since profits are affected not just by loans but also by random withdrawals by depositors. In other words, net profit is not a sufficient statistic for loans with respect to effort. On the other hand, if managerial compensation only depends on the volume of loans, then the manager will be incentivized to lower lending rates as much as possible because he will not be penalized when low lending rates adversely impact net profits. In this case, the volume of loans will be excessive, reserves will be too low and hence liquidity shortages very likely. Hence the optimal contract is a function of *both* loans and net profits which are observable to the principal.

Ideally the principal would want the manager to exert high effort as well as set the first-best lending rate. But given asymmetric information
the manager cannot be explicitly penalized for setting a lending rate lower than the first-best. Now let us suppose the principal can conduct a costly audit which verifies whether or not the manager acted overly aggressively. More specifically the contracting problem involves the following stages: The principal offers a contract to the manager. The manager optimizes his payoff under this contract by choosing the effort level and setting the lending rate. The principal conducts an audit following a liquidity shortfall and if it is inferred that the manager had acted overly aggressively by setting a lending rate below the first best then the manager is penalized a proportion $\gamma$ of the penalty costs, where $\gamma \in (0, 1]$.\footnote{This is a realistic setup given that in practice audits are carried out following liquidity shortages to ascertain the cause of the banking crisis. In the event that it is inferred that bank managers were responsible due to excessive risk-taking, then the managers are likely to suffer reputational costs, job losses, congressional hearing costs, etc.} It is assumed that the manager has limited liability and hence there is an upper bound on the punishment that can be imposed. Given that $\gamma \leq 1$, this implies that at most the manager incurs the entire penalty costs.

Optimal risk sharing now dictates that the manager’s payoff only be a function of loan volume as long as there is no liquidity shortfall. In the event of a liquidity shortfall, however, the manager is penalized if it is inferred that he had set a lending rate below the first best. The manager now faces a trade-off. If he behaves overly aggressively, his utility is higher as long as there are no liquidity shortfalls and subsequently no penalties. However, if a liquidity shortfall is realized then the manager is penalized and would be worse off. We can then prove the following proposition.

**Proposition 4** The manager will engage in overly-aggressive behavior if and only if the expected penalty cost (i.e. the expected cost of liquidity short-
falls) is low enough. In terms of the parameter $\gamma$ this will be the case if the following condition is satisfied:

$$
\gamma < \frac{\beta \Delta}{r_p E [\max (\bar{x}D - R, 0) | e = e_H]}.
$$

where $\Delta$ represents the increase in loan volume if the manager acts over-aggressively conditional on high effort. Since the expected penalty costs are decreasing in $D$ the above condition is satisfied if bank liquidity, $D$, is sufficiently high.

**Proof.** See Appendix. ■

The above proposition says that for high enough bank liquidity the manager has an incentive to engage in overly-aggressive behavior by mispricing the underlying risk of loans. More specifically, the loan rate misprices the underlying risk. It should be noted that the agency problem is only actuated when bank liquidity is high enough. This is because even though the manager bears a proportion of the penalty costs, in the presence of excessive liquidity, the probability of experiencing a liquidity shortage is low and this encourages the manager to indulge in excessive lending. On the other hand, for low enough liquidity the agency problem is immaterial since in this case the manager does not sanction excessive loans for the fear of incurring a penalty in the event of a liquidity shortfall.

### 2.2.3 Asset pricing

Next we consider the asset pricing implications of our model. We define the fundamental asset price as the price that holds in the absence of any distortions created by agency problems. We then compare the fundamental asset price with the actual asset price which may or may not be distorted.
depending on whether or not agency problems have been actuated within the banking system. To facilitate this comparison we first derive the asset demand by bank borrowers. We assume that there exists a continuum, \( n \), of risk neutral borrowers who have no wealth and hence need to borrow in order to finance investments. Since there is a continuum of borrowers and loans cannot be conditioned on their size, they can borrow as much as they like at the going rate of interest.

Although there is a continuum of borrowers, we shall analyze the behavior of a representative borrower. This implies that the equilibrium is symmetric and that all borrowers choose the same portfolio. The fact that all borrowers are identical ex post means that the bank cannot discriminate between borrowers by conditioning the terms of the loan on the amount borrowed or any other characteristic.

Let \( X_d \) denote the number of units of the asset demanded by the representative borrower and \( X_s \) denote the total supply of the risky asset. We assume for simplicity that the supply of the asset is finite and fixed. The asset returns a cash flow of \( C \) per unit with a probability of \( \theta \). Let \( P \) denote the price of one unit of the asset.

As in Allen and Gale (2000) we assume there is a non-pecuniary cost of investing in the risky asset \( b(X_d) \) such that it satisfies the usual neoclassical properties: \( b(0) = b'(0) \), \( b'(X_d) > 0 \) and \( b''(X_d) > 0 \) for all \( X_d > 0 \). The purpose of the investment cost is to restrict the size of the individual portfolios and to ensure the concavity of the borrower’s objective function.

The optimization problem faced by the representative borrower is to choose the amount of borrowing so as to maximize expected profits. That
is
\[ \max_{X_d} \theta [CX_d - r_L PX_d] - b(X_d). \]  \hspace{1cm} (11)

Note that the borrower has to pay an interest of \( r_L \) on its borrowing. The market-clearing condition for the asset is:
\[ nX_d = X_s. \]  \hspace{1cm} (12)

The first order condition of the above problem is as follows:
\[ \theta [C - r_L P] - b' (X_d) = 0 \]

Setting \( X_d = 1 \) in the first order condition and letting \( \tau (X_d) = b' (X_d) \) denote the marginal investment cost, the per unit asset price can be written as:
\[ P = \frac{\theta C - \tau (1)}{\theta r_L}. \]  \hspace{1cm} (13)

As expected, the asset price is the discounted value of the expected cash flows net of the investment cost. It is clear that there is a one-to-one mapping from the (gross) lending rate, \( r_L \), to the asset price, \( P \). Let \( r^f_L \) denote the fundamental (gross) lending rate which is the rate obtained in the absence of any distortions such as agency problems. Recall that \( r^f_L \) is the rate which maximizes the equity value of the bank. Since depositors always earn \( \bar{\mu} \) on average, \( r^f_L \) is also the rate which maximizes the value of the bank. Hence \( r^f_L \) is given by expression (9). Then the fundamental asset price is given by:
\[ P^f = \frac{\theta C - \tau (1)}{\theta r^f_L}. \]  \hspace{1cm} (14)

Since the total amount of loans for a given \( r_L \) is \( L(r_L) \) and the amount borrowed by an individual borrower is \( PX_d \), it has to be the case that
\( nPX_d = L(r_L) \). Hence, using the market-clearing condition (12) the asset price can also be written as:

\[
P = \frac{L(r_L)}{X_s}.
\]

(15)

Using the above formulation it is again clear that there is a one-to-one mapping from the lending rate to the asset price.\(^{16}\) Thus the fundamental asset price can also be rewritten as:

\[
P^f = \frac{L(r^f_L)}{X_s}.
\]

(16)

Having derived the fundamental asset price we can next define an asset price bubble. An asset price bubble is formed whenever \( P > P^f \). This is because whenever \( P > P^f \) the asset is overpriced and we interpret such asset price inflation as a “bubble”. Note that \( P > P^f \) as long as \( r_L < r^f_L \). Intuitively, a lending rate lower than the fundamental rate creates a high demand for the asset which in turn leads to an increase in asset prices over and above the fundamental values.

From Proposition 3 we know that the loan rate in the presence of an agency problem is lower than the fundamental rate. Furthermore, from Proposition 4 we know that this agency problem is actuated for high enough bank liquidity. Given that the asset price is given by expression (13) and the fundamental asset price is defined by (14) we immediately have the following corollary to Propositions 3 and 4.

**Corollary 1** In the presence of an agency problem between the bank manager and the equityholders, an asset price bubble is formed for high enough bank liquidity.

\(^{16}\) Note that \( \thetaC_{-1}(1) = \frac{L(r_L)}{X_s} \) and hence \( r_L = \frac{(\thetaC_{-1}(1))X_s}{\theta L (r_L)} \). Since \( nPX_d = L(r_L) \) and \( nX_d = X_s \) we again get \( P = \frac{\thetaC_{-1}(1)}{\theta r_L} \).
To better understand the mechanics behind the formation of a bubble, the four-quadrant diagram in figure 4 is useful. Quadrant I in the figure depicts the relationship between the risk of project failure, $1 - \theta$, and the loan rate, $r_L$, charged by the bank. In general the higher the riskiness the higher would be the equilibrium lending rate as is captured by the line $AA$. The loan rate in turn determines the demand for loans and the volume of credit in the economy. For any given lending rate, the amount of bank loans is given by $L(r_L)$. Since $L'(r_L) < 0$ we know that the lower the loan rate the higher is the amount of investment in the economy as is captured by the line $BB$ in quadrant II. From the expression derived in equation (15) we know that the market clearing condition is $PX_s = L(r_L)$. In other words, the price of the assets multiplied by the supply of the asset has to be equal to the total demand for the asset. Hence the demand for the asset determines the asset price and this relationship is captured by the line $YY$ in quadrant III. Notice that the higher the demand for the asset, the higher is its price. Finally quadrant IV derives the relationship between the asset price and risk. In general, the higher is the underlying risk the lower will be the asset price as is depicted by the line $ZZ$. The equilibrium relationship between asset price and risk is derived by tracing the effect of risk on the loan rate, which in turn has an effect on the amount of investment which subsequently determines the asset price.

Let the line $AA$ represent the fundamental relationship between risk and the bank loan rate, i.e. the relationship that would be obtained in the absence of agency issues. Then for any given level of risk, the fundamental asset price would be represented by the line $ZZ$. However, as we showed in Proposition 4 an agency problem is actuated for sufficiently high bank liquidity levels where the bank loan rate is lower for any given level of risk.
Figure 4: The mechanics of the formation of asset price bubbles.
This in turn shifts the $AA$ line to $A^1A^1$. From quadrant II we know that the volume of credit in the economy increases following lower loan rates. Consequently asset prices increase as a result of market-clearing as is shown in quadrant III. The final relationship between asset prices and risk is shown in quadrant IV and it is obvious that the actuation of the principal-agent problem shifts the $ZZ$ line to $Z^1Z^1$. In the end we see that the asset price is higher for the same level of risk leading to the formation of a bubble.

The intuition behind the formation of an asset price bubble is as follows: For the same level of risk and the same underlying cash flows, asset prices are higher if the bank misprices the risk given the distortions that arise from agency issues.

It is also interesting to note that our model implies that the size of the bubble is monotonic in the leverage of bank borrowers. This is because bank borrowers in the model borrow more the lower the lending rates offered by the banks. The greater the severity of the agency problems, the lower are the lending rates, which encourages excessive bank borrowing. This increase in leverage in turn increases asset demand which pushes up asset prices.

Our model helps explain how agency problems in the banking sector can induce the formation of asset price bubbles. In terms of the four-quadrant diagram we would be reducing our attention to quadrant IV alone if we fail to consider the role of the banking sector; embedding of the banking sector in a pricing framework gives us a fuller picture of how the banking sector contributes to the formation of asset price bubbles.
3 When are bubbles likely to be formed?

3.1 High macroeconomic risk

We have established so far that asset price bubbles are formed when bank liquidity is substantially high. So the question that arises is when are banks most likely to be flushed with liquidity. In an empirical study, Gatev and Strahan (2006) find that as spreads in the commercial paper market increase, bank deposits increase while bank asset (loan) growth also increases. The spreads on commercial paper are a measure of the investors’ perception of risk in the real economy. Intuitively, when investors are apprehensive of the risk in the entrepreneurial sector they are more likely to deposit their investments in banks rather than make direct investments.\textsuperscript{17}

To formalize the above intuition we develop a simple model of the entrepreneurial sector and show that bank deposits will increase at a time when the underlying risk increases. Consider an economy where entrepreneurs have access to projects that yield a terminal cash flow $C$ if it succeeds and 0 otherwise. The probability of success depends partly on the realization of the state variable, $\tilde{\theta}$, and partly on the entrepreneurs’ effort decision, $e$, which identifies whether the entrepreneur is diligent ($e = 1$) or shirks

\textsuperscript{17}The flight of depositors to banks may be due to banks having greater expertise in screening borrowers during stress times, inducing a natural negative correlation between the usage of lines of credit and deposit withdrawals as argued by Kashyap, Rajan and Stein (2002). Alternatively, the flight may simply be due to the fact that bank deposits are insured (up to a threshold) by the Federal Deposit Insurance Corporation (FDIC) whereas commercial paper and money market funds are uninsured, at least until the extraordinary actions taken by the Federal Reserve during 2008 and 2009. Pennacchi (2006) finds evidence supportive of this latter hypothesis by examining lending behavior of banks during crises prior to the creation of the FDIC.
\((e = 0)\) in which case, entrepreneurs extract a private benefit of \(B\). If the entrepreneur is diligent, the probability of success is \(\theta\) but in the presence of shirking the probability of success is \(\phi\theta\), where \(\phi \in (0, 1)\). The realization of the state variable \(\theta\) is observable to the entrepreneurs.

Entrepreneurs promise to pay the risk neutral investors who invest directly in their projects a face value of \(d\). Thus the expected payoff of the risk neutral entrepreneurs whose projects are financed by direct investment is given by:

\[
E(p) = \begin{cases} 
\theta (C - d) & \text{if } e = 1 \\
\phi\theta (C - d) + B & \text{if } e = 0
\end{cases}
\]

(17)

One can then prove the following proposition.

**Proposition 5** Entrepreneurs will shirk (i.e. \(e = 0\)) if and only if \(\theta < \theta^*\) where \(\theta^* = \frac{B}{(1 - \phi)(C - d)}\).

**Proof.** See Appendix.

The above proposition says that entrepreneurs are more likely to shirk when macroeconomic risk is high. Intuitively, an increase in macroeconomic risk lowers the success probability of entrepreneurial projects which encourages entrepreneurs to shirk so as to consume their private benefits.

We know that investors on average earn \(\bar{u}\) from bank investment. We assume that the expected utility of investors from direct investment will be higher than that from bank investment as long as \(\bar{\theta} \geq \theta^*\) and conversely the expected utility from bank investment will be higher than that from direct investment as long as \(\bar{\theta} < \theta^*\). The implicit assumption is that in bad times banks are better monitors of entrepreneurial activity and thus entrepreneurs cannot shirk if financed by banks.\(^{18}\) Let \(E(u_e|\theta)\) denote the expected utility

\(^{18}\)This view is consistent with the evidence that covenants in private financing arrange-
of the investor from entrepreneurial investment. This assumption can be written more precisely as:

\[ E(u_e|\theta < \theta^*) < \bar{u} < E(u_e|\theta \geq \theta^*) . \]

If investors perfectly observe \( \theta \) before deciding whether to invest directly in projects or in banks, all investments will either be channeled directly into projects or into banks depending on the realization of \( \theta \). Hence in order to make a more realistic distribution of investments we assume that depositors do not observe \( \theta \) but receive an imperfect signal, \( s \), on the basis of which they decide whether to allocate their endowments to direct entrepreneurial investment or bank deposits. A signal \( s_j = g \) received by investor \( j \) is a good signal which implies that \( \theta \geq \theta^* \) but a signal \( s_j = b \) is a bad signal which would be an indication to the investor that \( \theta < \theta^* \). Thus an investor allocates his capital to entrepreneurial projects only if he receives a good signal.\(^{19}\)

The probability distribution of the signals is a simple one: \( \Pr (s = g) = \nu \theta \) and \( \Pr (s = b) = 1 - \nu \theta \), where \( \nu \in (0, 1) \). Investors only observe their own signals and are not aware of the probability distribution of the signals. The simple formulation of the probability distribution implies that a proportion \( \nu \tilde{\theta} \) of the investors will allocate their endowments to entrepreneurial projects.

\(^{19}\) This will have implications for the participation constraint (2). In the constraint, \( \theta_i \) would need to be replaced by a Bayesian expectation of \( \theta \) which will slightly alter the equations in Proposition 1. However since all qualitative results are unchanged we can assume for simplicity that when offered a banking contract depositors are able to ascertain whether \( r_D \) is high enough to satisfy their reservation utility. This will be the case if the required risk premium on deposits, defined as \( \kappa^* \equiv r_D^* - \bar{u} \), is public information.
while a proportion $1 - \nu \hat{\theta}$ will allocate their endowments to bank deposits. This formulation is quite intuitive and it implies that the higher the $\theta$, the higher would be the amount of direct investments. However a low realization of $\theta$ would imply that a larger amount of endowments will be tunnelled to bank deposits. Hence, bank deposits are given by $D = (1 - \nu \theta) I$ assuming that there are a total of $I$ risk neutral investors.

It is clear from the formulation of $D$ that bank liquidity increases when the underlying macroeconomic risk is high. This is consistent with the findings of Gatev and Strahan (2006). However, we hypothesize that for extremely high macroeconomic risk (very low $\theta$) bank liquidity may be adversely affected. This is because casual observation of crises suggests that once macroeconomic risk crosses a certain threshold the bubble bursts subsequently, adversely affecting banks’ liquidity. Nevertheless since this paper focuses on the formation of bubbles rather than the burst of bubbles we do not model this phenomenon but simply capture this by assuming that $D = (1 - \nu \theta) I \ \forall \theta \in [\underline{\theta}, 1]$ where $\underline{\theta}$ is the threshold below which bank liquidity decreases as $\theta$ decreases.

We can then prove the following proposition.

**Proposition 6** A bubble is formed in the economy when the macroeconomic risk is high enough. More formally, there exists a threshold $\theta^c$ such that $P > P^f$ if $\theta < \theta^c$ where $\theta^c \in [\underline{\theta}, 1]$.

**Proof.** See Appendix.  

The intuition behind this result is as follows. As macroeconomic risk increases, there is a flight to quality whereby investors prefer to invest in bank deposits rather than engage in direct lending. Subsequently, banks find themselves flushed with liquidity during times when spreads in the
commercial paper market are high. Excessive liquidity in turn encourages bank managers to increase the volume of credit in the economy via the mispricing of underlying risk. The mispricing of risk in turn fuels a bubble in asset prices.

3.2 Loose monetary policy

Before we turn to the implications for central bank’s monetary policy, we briefly discuss how monetary policy has a direct effect on bank’s liquidity. When embarking on an expansionary monetary policy via open market operations, central banks buy government securities from primary dealers who have accounts with depositary institutions. The way this transaction works in practice is that the central bank directly credits the reserves which commercial banks have with the central bank, hence effectively increasing the deposit base of the bank. On the other hand, in order to implement a contractionary monetary policy, the central banks sell government securities to primary dealers and at the same time debit their accounts which effectively reduces the deposit base of banks. Hence bank deposits are a function of both macroeconomic risk ($\theta$) as well as monetary policy ($M$):

$$D = D(\theta, M)$$

(18)

The above relationship is depicted in Figure 5. As discussed in the previous section, as macroeconomic risk increases there is a flight to quality whereby bank deposits increase and this continues until risk crosses the threshold $1 - \theta$ after which more and more investors prefer to just consume their reservation utility. In the absence of an active monetary policy, the relationship between bank liquidity and risk is given by $DD$. However, following an expansionary monetary policy, bank liquidity increases for the same level of
risk and the $DD$ line shifts upwards to $D^+D^+$. Conversely, subsequent to a contractionary monetary policy, bank liquidity decreases for the same level of risk and consequently the $DD$ line moves downwards to $D^-D^-$. Since monetary policy has a direct impact on banks’ liquidity, our setup enables us to study how central banks can use monetary policy to avoid the emergence of bubbles. Assuming for now that the central bank has perfect information regarding macroeconomic and bank fundamentals, we show in the following proposition how the central bank can use monetary policy to avoid the formation of a bubble.

**Proposition 7** A central bank can avoid the formation of a bubble by adopting a contractionary monetary policy. More specifically, if $\theta$ falls below the threshold $\theta^c$ such that it is in the interval $[0, \theta^c]$, a perfectly informed central bank can avoid the emergence of a bubble by instantaneously selling government securities worth at least $(\theta^c - \theta)\nu I$ to the primary dealers and thus effectively reducing the money supply.

**Proof.** See Appendix. ■

We know from Proposition 6 that when macroeconomic risk increases above $1 - \theta^c$, asset prices are conducive to bubble formation since there is an influx of liquidity flow to the banking system. The above proposition says that the formation of a bubble can be avoided if the central bank acts swiftly and sells enough government bonds to offset the influx of liquidity in the banking system. This is demonstrated in Figure 5. In the figure, $D^* = (1 - \nu \theta^c) I$ is the liquidity threshold above which asset price bubbles are formed. When macroeconomic risk increases above $1 - \theta^c$ to say $1 - \theta^1$, bank liquidity crosses the threshold $D^*$ to $D^1$ leading to the formation of a bubble. However the central bank can offset this effect via a contractionary
Figure 5: The interplay between bank liquidity, macroeconomic risk and monetary policy.

monetary policy which will shift the $DD$ line downwards. The magnitude of
the contractionary monetary policy should be such that the $DD$ line moves
downwards to at least $D^-D^-$. As can be seen from the figure this is the
minimum shift that is required to ensure that for the new level of risk $1-\theta^1$,
bank liquidity is at or below $D^*$.20

The above analysis indicates that the central banks can prevent the
emergence of bubbles by adopting a tight monetary policy at times when
macroeconomic risk is increasing in order to offset the flight to quality in
the banking sector. However, if the central bank acts too late and tightens
monetary policy after a bubble has already been formed (as was the case
in the current subprime crisis) this would simply ‘prick the bubble’ and

\[ \theta_c - \theta^1 \nu I \]

20 As is proved in the appendix the proposition implies that the minimum reduction in
money supply that is required on the part of the central bank is given by $(\theta^c - \theta^1) \nu I$. 

34
would be much more costly compared to our suggested policy of tightening monetary policy before the formation of a bubble.

We next relax the assumption of a perfectly informed central bank and show that even in this environment a ‘leaning against the wind’ policy can be rationalized. Suppose that the central bank’s objective function is given by:

\[ E(W) = \frac{1}{2} a E \left[ \left( P(\theta, M) - P^f \right) \ell \right]^2 + \frac{1}{2} b \Gamma(\theta, M)^2 \]  

(19)

where \( E(W) \) denotes expected welfare, \( \Gamma \) is a measure for the expected growth or aggregate investment of the economy and

\[ \ell = \begin{cases} 
1 & \text{if } P > P^f \\
0 & \text{otherwise} 
\end{cases} \]

The first term in (19) implies that asset price bubbles are costly. Our model shows that asset price bubbles can be caused by agency problems within banks while at the same time these agency problems reduce the expected profits of the banks. Also note that \( P = P(\theta, M) \) since as shown earlier (and illustrated in Figure 5) monetary policy has an impact on asset prices. More specifically \( \partial P/\partial M > 0 \) since an expansionary monetary policy increases the aggregate liquidity in the economy and we have shown that an increase in liquidity can induce agency problems which in turn inflames asset prices.

The second term captures the welfare gains from an increase in the aggregate investment level. \( \partial \Gamma/\partial M > 0 \) since an expansionary monetary policy increases bank liquidity and as illustrated in Figure 4, this in turn reduces loan rates and hence increases the aggregate investment level in the economy.\(^{21}\)

\(^{21}\)In our model depositors ex ante recieve their reservation utility; bank profits are
Thus the objective function captures the trade-off faced by the policymaker: An expansionary monetary policy can induce growth and investment but at the same time can be conducive to asset price bubbles. Taking the first order condition of (19) with respect to $M$ we get:

$$b \frac{\partial \Gamma}{\partial M} = a \left[ \Pr (P > P_f) \frac{\partial P}{\partial M} + (P - P_f) \frac{\partial \Pr (P > P_f)}{\partial M} \right].$$ (20)

The LHS of the above expression measures the marginal benefits ($MB$) of monetary expansion in terms of higher investment growth while the RHS represents the marginal cost ($MC$) of monetary expansion in terms of a higher likelihood of an asset price bubble. Hence the above condition has a natural interpretation: optimal monetary policy dictates monetary expansion up to the point where the marginal benefits of expansion just equal the marginal costs.\(^{22}\)

Recall that the liquidity of the banking system is likely to increase when the macroeconomic risk is high. We next show that if the central bank had initially set monetary policy optimally such that it satisfied condition (20), then a subsequent increase in bank liquidity, say due to an increase in macroeconomic risk, reduces the optimal level of money supply. To see this, note that an increase in bank liquidity increases the probability of a bubble adversely affected by agency problems which cause bubbles; and finally entrepreneurs gain from higher investments via low loan rates. Hence our objective function succinctly captures the expected welfare of agents in the model. Nevertheless a general equilibrium model will be better able to measure the expected welfare of all the agents in the economy and at the same time can also incorporate the effects of monetary policy on price inflation.\(^ {22}\)

In order to ensure that the maximand (19) is quasi-concave we assume that the second order condition is satisfied, i.e the marginal cost of monetary expansion increases at a faster rate w.r.t. $M$ relative to the increase in marginal benefits. More formally: $b \frac{\partial^2 \Gamma}{\partial M^2} < a \left[ \Pr (P > P_f) \frac{\partial^2 P}{\partial M^2} + (P - P_f) \frac{\partial^2 \Pr (P > P_f)}{\partial M^2} + 2 \frac{\partial P}{\partial M} \frac{\partial \Pr (P > P_f)}{\partial M} \right].$
and hence increases the marginal cost of monetary expansion. Higher bank liquidity also increases investment. Nevertheless the increase in the marginal cost with respect to bank liquidity outweighs any increase in the associated marginal benefits. This follows directly from the second order condition.\footnote{We know from footnote 22 that the SOC is satisfied when $\frac{\partial MC}{\partial M} > \frac{\partial MB}{\partial M}$. Since $M$ affects both the probability of a bubble (and hence $MC$) and investment growth (and thus $MB$) via its affect on bank liquidity it follows that the SOC will be satisfied if and only if $\frac{\partial MC}{\partial M} > \frac{\partial MB}{\partial M}$.} Thus the optimal response of the central bank following excessive liquidity in the banking sector is to implement a contractionary monetary policy. Hence our framework allows us to justify a ‘leaning against the wind’ approach. We summarize this in the following proposition.

**Proposition 8** The optimal monetary policy, $M^*$, satisfies eqn. (20) and hence under optimality the marginal benefits of monetary expansion just equal the associated marginal costs, i.e. $MB(M^*) = MC(M^*)$. This implies a ‘leaning against the wind’ approach, i.e. tightening monetary policy in times of excessive bank liquidity.

We know from section 3.1 that in an environment of heightened macroeconomic risk bank liquidity increases as investors shift their endowments from direct investments to bank deposits. The increase in bank liquidity can actuate agency problems within banks which in turn increases the probability of asset price bubbles. Thus for the same level of money supply the marginal cost of monetary expansion increases and given the SOC this outweighs any increase in marginal benefits. Thus the FOC (20) will be obtained only at a lower level of $M$, which explains the intuition behind Proposition 8.
3.2.1 Discussion

As also argued by Blanchard (2000), we suggest that the traditional Taylor rule should be modified to include a third term relating to the difference between actual and fundamental asset prices rather than just focusing on inflation and output alone. In other words, monetary policy should target not just interest rates and employment but also asset prices.

It is interesting to note from Figure 5 that even if the macroeconomic risk level is below \( 1 - \theta \), central banks can fuel asset price bubbles by adopting loose monetary policies thereby shifting the \( DD \) line upwards such that the liquidity level crosses the threshold \( D^* \). Indeed Kindleberger (2005) in his study on the history of financial crises notes that: “Speculative manias gather speed through expansion of money and credit.”

Furthermore, Allen and Gale in their book “Understanding financial crises” document the following: “In Finland an expansionary budget in 1987 resulted in massive credit expansion. The ratio of bank loans to nominal GDP increased from 55 percent in 1984 to 90 percent in 1990. Housing prices rose by a total of 68 percent in 1987 and 1988... In Sweden a steady credit expansion through the late 1980's led to a property boom.” These observations are perfectly in line with our model. Loose monetary policies can potentially lead to excessive liquidity in the banking system which in turn encourages bank mangers to underprice the underlying risk and thereby increase the volume of credit in the economy. This in turn creates an asset price bubble.

Our model can thus explain how lax monetary policy in Japan during the mid 1980s led to asset price inflation. Bank of Japan (BOJ) reduced the official discount rate five times between January, 1986 and February, 1987,
leaving it finally at 2.5 percent. It is widely accepted that the easy credit policies adopted by BOJ created excess liquidity in the Japanese economy, as also acknowledged by Goyal and Yamada (2004). The sequence of events started with the Plaza Accord (1985), in which the G5 countries agreed on a stronger yen so as to lower the U.S. trade deficit. However, BOJ’s intervention in foreign exchange markets appreciated the yen rapidly. Responding to the strengthening yen and seeking to avert deflationary effects in the domestic economy, Bank of Japan lowered interest rates and consequently increased liquidity in the economy. In the subsequent years a large real estate bubble was formed.

One of the causes of the current subprime crisis has been suggested to be the loose monetary policy adopted by the Federal Reserve in the United States. In 2003, the Fed lowered the federal funds rate to 1% - a level that was last seen only in 1958. Subsequently banks mispriced risk and indulged in over-lending which finally culminated in the subprime crisis. It would be incorrect to call the present crises a ‘liquidity crisis’. In fact the world was awash with liquidity prior to the crisis. We would thus argue that the excess of liquidity rather than the shortage of liquidity in the period leading up to the crisis is what contributed to the crisis.

Finally, Naqvi (2007) shows that the central bank’s lender of last resort operations need to be complemented ex ante by an efficient supervisory framework so as to avoid the moral hazard repercussions of bail-outs. What we learn from our paper is that such supervision is even more essential during times when the banking system is flushed with liquidity. This is because during such times bank managers are more likely to under-price risk and hence over-invest. Furthermore, if the central bank does resort to a loose monetary policy, for instance either to counter deflation or to stimulate
the economy, such monetary policy needs to be accompanied by adequate supervision of the banking system in order to curtail the risk-taking appetites of banks.

4 Conclusion

We develop a theory of banking explaining how the seeds of a crisis may be sown when banks are flush with liquidity. The main empirical implication of our model is that excessive liquidity induces risk-taking behavior on the part of bank managers. In summary, we obtain the following results: (a) bank managers will behave in an overly-aggressive manner by mispricing risk when bank liquidity is sufficiently high; (b) asset price bubbles are formed for high enough bank liquidity; (c) bubbles are more likely to be formed when the underlying macroeconomic risk is high inducing investors to save with banks rather than make direct entrepreneurial investments; and, finally (d) bubbles are more likely to be formed following loose monetary policies adopted by the central bank.

We also show that the central bank can avoid the emergence of bubbles by adopting a contractionary monetary policy at times when banks are awash with liquidity by drawing out their reserves. Some, most notably Alan Greenspan, have argued that “we are never certain where we are in the cycle”\textsuperscript{24} and hence monetary policy should not be used to target asset prices. Nevertheless our model showed that even in the absence of precise knowledge of macroeconomic fundamentals a ‘leaning against the wind policy’ can be rationalized. Thus we argue that monetary policy should target not just interest rates and employment but also asset prices.

\textsuperscript{24}Alan Greenspan, Financial Times, 27 May 08.
Appendix

Extension: Bank’s objective function in the presence of fire-sale prices for assets liquidated prematurely. In this appendix we will derive the bank’s objective function in a setup where instead of taking the penalty cost term as exogenous we derive the term in an environment where the bank can finance the liquidity shortfall by liquidating assets at fire-sale prices. We show that the objective function in this setup has the same generic form as that in the main body of the paper and hence the qualitative results are unaffected.

The timing of the game is as follows. At time $t = 0$ the bank receives deposits and then invests a fraction of these deposits in investment projects via loans to entrepreneurs. A proportion of the total loans, $\mathcal{L}$, are of very good quality such that they repay early with probability 1 at $t = 1$. These loans are indexed by $L^e$ and the rate of return on these loans is $r^e_L$. However such investments are scarce so the bank cannot make its entire investments in such assets. The rest of the investments are made in assets which repay at $t = 2$ with a probability of $\theta$. These investments are indexed by $L$ and the rate of return on these investments is given by $r^\theta_L$. Thus the total investment is given by $\mathcal{L} = L + L^e$. The rest of the deposits are retained as reserves, $R$.

At time $t = 1$ the bank distributes a fraction of its ‘early returns’, $r^e_L L^e$, to its equityholders in the form of dividends where the dividend payout ratio is given by $d$. For simplicity we normalize $d = 1$, which implies that the early returns are accrued by the equityholders. Furthermore, at $t = 1$ the bank experiences a random liquidity shock such that there are withdrawals of $\tilde{x}D$. If the bank reserves at $t = 1$ are insufficient to service these withdrawals then the bank will have to inefficiently liquidate a frac-
tion, \( \xi \), of its portfolio. The value of the bank’s portfolio at \( t = 1 \) is given by \( r^e_L L + r_L \theta L \). However, the fire-sale value of the portfolio is \( \lambda (r^e_L L + r_L \theta L) \), where \( \lambda \in (0, 1) \). Thus the fraction of premature liquidation is given by \( \xi = (xD - R) / [\lambda (r^e_L L + r_L \theta L)] \). We assume for simplicity that the value of the bank’s portfolio is high enough so that it can service its withdrawals without going bankrupt. This is just for simplicity and allowing for bankruptcy does not change the results.

Finally at time \( t = 2 \) the returns from bank investments, if any, are divided amongst the depositors and the bank equityholders. With probability \( \theta \), the return from the ‘late’ projects will be realized and the equityholders will consume \( r_L L - r_D D (1 - \tilde{x}) + (R - \tilde{x} D)_{+} \) where \( (R - \tilde{x} D)_{+} = R - \tilde{x} D \) if \( \tilde{x} D < R \) and 0 otherwise. Also note that the equityholders need to make the promised payment \( r_D D \) to the patient depositors. With probability \( 1 - \theta \) the late projects fail and the value of reserves is divided amongst the depositors. In this case the equityholders only consume their dividends at \( t = 1 \).

Given this setup the expected profits of the bank’s equityholders is given by:

\[
\Pi_1 = \text{Pr}(\tilde{x} D < R) \left[ \theta \{r_L L - r_D D (1 - E(\tilde{x})) + E(R - \tilde{x} D)\} + r^e_L L \right] \\
+ \text{Pr}(\tilde{x} D \geq R) \left[ \theta \{r_L (1 - \xi) L - r_D D (1 - E(\tilde{x}))\} + r^e_L (1 - \xi) L \right].
\]

Since \( \text{Pr}(\tilde{x} D < R) = 1 - \text{Pr}(\tilde{x} D \geq R) \) and defining

\[
\pi_{NF} = \theta \{r_L L - r_D D (1 - E(\tilde{x})) + E(R - \tilde{x} D)\} \quad \text{and} \\
\pi_F = \theta \{r_L (1 - \xi) L - r_D D (1 - E(\tilde{x}))\}
\]

we can rewrite the above as follows:

\[
\Pi_1 = (\pi_{NF} + r^e_L L) - \text{Pr}(\tilde{x} D \geq R) \left[ (\pi_{NF} - \pi_F) + \xi r^e_L L \right]
\]

where \( (\pi_{NF} - \pi_F) \) denotes the difference in the profits from the late projects.
with and without fire sales while $\xi r_L^e L^e$ denotes the loss in value from the early projects due to fire sales. Thus the expected profit of the bank is given by the expected profit in the absence of any fire sales, $(\pi_{NF} + r_L^e L^e)$, minus the probability of a liquidity shortage times the loss in value due to fire sales.

To show the qualitative equivalence of the above dynamic setup with that of the static setup, we first note that $\pi_{NF} = \pi$ as defined in equation (3) given that $E(R - \tilde{x}D) = \Pr (\tilde{x}D < R) [R - E(\tilde{x}D|\tilde{x}D < R)]$. Thus,

$$\Pi_1 = (\pi + r_L^e L^e) - \Pr (\tilde{x} \geq R) \left[ (\pi_{NF} - \pi_F) + \xi r_L^e L^e \right]$$

Note that $r_L^e L^e$ is a constant and hence maximizing $\Pi_1$ w.r.t. $r_L$ and $r_D$ is equivalent to maximizing the following

$$\Pi_2 = \pi - \Pr (\tilde{x}D \geq R) \left[ (\pi_{NF} - \pi_F) + \xi r_L^e L^e \right]$$

(21)

Also note that $[(\pi_{NF} - \pi_F) + \xi r_L^e L^e]$ is increasing in $(\tilde{x}D - R)$ given that $\frac{\partial \pi_F}{\partial \xi} < 0$ and $\frac{\partial \xi}{\partial (\tilde{x}D - R)} > 0$. Thus $[(\pi_{NF} - \pi_F) + \xi r_L^e L^e]$ is increasing in $(\tilde{x}D - R)$. Note the stark similarity of maximizing $\Pi$ as in equation (1) with that of equation (21) above. In both cases, the expected profit of the bank is given by $\pi$ minus the probability of a liquidity shortfall times a cost term that is proportional to the shortfall. Thus in the main body of the paper, we use the simpler setup with an exogenous penalty cost given that the qualitative results are unaffected.

**Proof of Proposition 1.** The participation constraint of the bank will be binding because otherwise the bank can increase its expected profits by slightly reducing $r_D$. Thus, $r_D^\ast$ is given by the solution to the following:

$$E(\tilde{x}) + (1 - E(\tilde{x})) \left[ \theta r_D + (1 - \theta) \frac{E[\max (R - \tilde{x}D) \cdot \frac{0}{D}]}{D} \right] = \bar{u}$$

where $E[\max (R - \tilde{x}D) \cdot \frac{0}{D}] = \Pr (\tilde{x}D < R) [R - E(\tilde{x}D|\tilde{x}D < R)]$. Solving for $r_D^\ast$ we get (5).
We can then substitute $r^*_D$ in the bank’s objective function and hence $r^*_L$ will be the solution to the following unconstrained maximization problem:

$$\max_{r^*_L} \Pi = \theta \{r_LL(r_L) - r^*_D D (1 - E(\tilde{x})) + Pr(\tilde{x}D < R) [R - E(\tilde{x}|\tilde{x}D < R)]\}$$

$$-r_p E[\max(\tilde{x}D - R, 0)].$$

Assuming that $\Pi$ is quasi-concave in $r_L$ and noting that $R = D - L$, the maximum is characterized by the following first order condition:

$$\frac{\partial \Pi}{\partial r_L} = \theta L(r_L) - \theta Pr[\tilde{x}D < R] L'(r_L) + \theta r_LL'(r_L)$$

$$-r_p Pr[\tilde{x}D \geq R] L'(r_L) - \theta D (1 - E(\tilde{x})) \frac{\partial r^*_D}{\partial r_L} = 0. \tag{22}$$

Noting that $\frac{\partial r^*_D}{\partial r_L} = (1 - \theta) Pr[\tilde{x}D < R] L'(r_L) / \theta D$ and solving for $r_L$ after some simplification we get (4). Thus the optimal reserve level is given by $R = D - L(r^*_L)$ which proves the first part of the proposition.

From the FOC (22), if we solve for $r^*_L$ directly without exploiting the definition of $\eta_L$ we get the following expression for the return on loans:

$$r^*_L = \frac{1}{\theta} - \frac{L}{D} + \frac{(r_p - 1) Pr(\tilde{x}D \geq R)}{\theta} \tag{23}$$

Taking the partial derivative of the above expression w.r.t. $\theta$ we get:

$$\frac{\partial r^*_L}{\partial \theta} = - \frac{1 + (r_p - 1) Pr(\tilde{x}D \geq R)}{\theta^2} < 0 \tag{24}$$

since $r_p > r_L > 1$, which proves the second part of the proposition.

Next note that $\partial Pr(\tilde{x}D \geq R) / \partial D < 0$, i.e. an increase in bank liquidity (deposits) lowers the probability of liquidity shortfalls since $R = D - L$. Then taking the partial derivative of (23) w.r.t. $1 - F(R) = Pr(\tilde{x}D \geq R)$ we get:

$$\frac{\partial r^*_L}{\partial [1 - F(R)]} = \frac{r_p - 1}{\theta} > 0 \tag{25}$$
Hence \( \frac{\partial r^*_L}{\partial D} = \frac{\partial r^*_L}{\partial [1-F(R)]} \frac{\partial [1-F(R)]}{\partial D} < 0 \), which proves the third part of the proposition. Q.E.D.

**Proof of Proposition 2.** Both the principal and the agent can observe the realized amount of loans and the bank’s net profit. The joint density of loans and net profits conditional on \( e \) is given by \( g \left( L(r_L), \tilde{\Pi}|e \right) \) whilst the distribution function is given by \( G \left( L(r_L), \tilde{\Pi}|e \right) \), where \( \tilde{\Pi} \) is the realized value of net profits.

In our setup \( L \) is not a sufficient statistic for \( \tilde{\Pi} \) w.r.t. effort and vice versa. Hence given the results of Holmstrom (1979) the optimal compensation schedule should be a function of both \( L(r_L) \) and \( \tilde{\Pi} \).

The manager is an expected utility maximizer with a Bernoulli utility function \( u(w, e) \) over his payoffs or wage \( w \) and effort \( e \). The utility function satisfies \( u_w(w, e) > 0 \), \( u_{ww}(w, e) < 0 \) and \( u_e(w, e) < 0 \) (where the subscripts denote the partial derivatives). This implies that the manager prefers more wealth to less, he is risk averse and dislikes high effort. More specifically we assume that the utility function is given by \( u(w, e) = v(w) - e \), where \( v'(w) > 0 \) and \( v''(w) < 0 \).

We can now write down the optimal contract. Since bank profits are decreasing in managerial wages, the optimal compensation scheme solves the following:

\[
\text{Min}_{w(L, \pi)} \int_{L(r_L)} \int_{\tilde{\Pi}} w(L, \pi) \, dG \left( L(r_L), \tilde{\Pi}|e \right)
\]

subject to

\[
\int_{L(r_L)} \int_{\tilde{\Pi}} v(w(L, \pi)) \, dG \left( L(r_L), \tilde{\Pi}|e \right) - e \geq u^o \quad (26)
\]

\[
\int_{L(r_L)} \int_{\tilde{\Pi}} v(w(L, \pi)) \, dG \left( L(r_L), \tilde{\Pi}|e_H \right) - e_H \geq \int_{L(r_L)} \int_{\tilde{\Pi}} v(w(L, \pi)) \, dG \left( L(r_L), \tilde{\Pi}|e_L \right) - e_L \quad (27)
\]
where \( u^0 \) is the manager’s reservation utility. Hence, the second best contract minimizes the expected wages subject to the participation constraint of the manager and subject to the incentive compatibility constraint.

Letting \( \mu_1 \) and \( \mu_2 \) denote the multipliers for constraints (26) and (27) respectively, then optimization of the Lagrangian yields the following characterization for the optimal compensation schedule:

\[
\frac{1}{v'(w(L, \pi))} = \mu_1 + \mu_2 \left[ 1 - \frac{g \left( L(r_L), \tilde{\Pi}|e_L \right)}{g \left( L(r_L), \tilde{\Pi}|e_H \right)} \right]
\]

We know from standard contract theory that in any solution to the optimization problem with \( e = e_H \), both \( \mu_1 \) and \( \mu_1 \) must be strictly positive. (See for instance, Holmstrom (1979)). This implies that both constraints (26) and (27) are binding. Hence unlike the first best solution, the optimal compensation scheme is not a constant but varies with \( L \) and \( \tilde{\Pi} \).

As is common in the literature, we then invoke the monotone likelihood ratio property (MLRP), i.e. \( \left[ g \left( L(r_L), \tilde{\Pi}|e_L \right) / g \left( L(r_L), \tilde{\Pi}|e_H \right) \right] \) is decreasing in \( L \) and \( \tilde{\Pi} \). In words, this means that as bank loans and bank profits increase, the likelihood of getting a given level of loans and profits if effort is \( e_H \), relative to the likelihood if effort is \( e_L \) must increase. Given that \( v' > 0 \), this implies that the manager’s wages are monotonically increasing in \( L \) and \( \tilde{\Pi} \). Q.E.D.

**Proof of Proposition 3.** The proof is along the same lines as the proof to Proposition (1). Define \( \alpha = \beta / \delta \). Thus maximizing (10) is equivalent to maximizing the following objective function:

\[
\max_{r_L^*, r_D} \Pi_m = \alpha E \left[ L(r_L) | e_H \right] + (\pi - r_p E \left[ \max(\tilde{x}D - R, 0) | e = e_H \right])
\]

Let \( h(L(r_L) | e) \) denote the density function of loans conditional on effort.

Define \( \bar{L}_H = E[L(r_L) | e_H] \) and \( \bar{L}'_H = \partial E[L(r_L) | e_H] / \partial r_L = \int L'(r_L) h(L(r_L) | e_H) \, dL. \)
Then taking the partial derivative of $\Pi_m$ w.r.t. $r_L$ we get the following FOC:

$$
\alpha \bar{L}_H' + \theta \bar{L}_H - \theta \Pr[(\tilde{x}D < R) | e = e_H] \bar{L}_H' + \theta r_L \bar{L}_H' - r_p \Pr[(\tilde{x}D \geq R) | e = e_H] \bar{L}_H' - \theta D (1 - E(\tilde{x})) \frac{\partial r_D^*}{\partial r_L} = 0.
$$

The participation constraint is given by (7) and as before it holds with equality. Solving for $r_D^*$ and taking the partial derivative w.r.t. $r_L$ we get

$$
\frac{\partial r_D^*}{\partial r_L} = \frac{1 - \alpha + (r_p - 1) \Pr[(\tilde{x}D \geq R) | e = e_H]}{\theta (1 - \frac{1}{\bar{\eta}_L})}
$$

where $r_D^*$ denotes the lending rate in the presence of an agency problem and

$$
\bar{\eta}_L = -r_L \frac{\partial E[L(r_L) | e_H]}{E[L(r_L) | e_H]} = -r_L \frac{\bar{L}_H'}{\bar{L}_H}.
$$

Comparing the above expression with (9) it is clear that the loan rate is lower, i.e. $r_L^{na} < r_L^f$. Since $L'(r_L) < 0$, the volume of bank credit increases. Q.E.D.

**Proof of Proposition 4.** The manager’s expected payoff is now given as follows:

$$
\Pi_m = \beta E[L(r_L) | e_H] - \nu \gamma r_p E[\max(\tilde{x}D - R, 0)]
$$

where $\gamma \in (0, 1]$ and

$$
l = \begin{cases} 
1 & \text{if } r_L \neq r_L^f \\
0 & \text{otherwise}
\end{cases}
$$

In the absence of an agency problem the manager will choose the first best lending rate, $r_L^f$, so as to maximize the equityholders net profit and hence his expected payoff will be given by:

$$
\Pi_m^{na} = \beta E[L(r_L^f) | e_H].
$$
where $r_f^L$ is given by (9). However if the manager engages in overly-aggressive behavior, his expected payoff is given by:

$$\Pi_m = \beta E[L(r^a_L)|e_H] - \gamma r_p E[\max(\tilde{x}D - R, 0) | e = e_H]$$  \hspace{1cm} (30)$$

where $r_f^a$ is chosen so as to maximize (30). $\Pi_m^a > \Pi_m^a$ if and only if expression (29) is greater than expression (30). This will be the case if and only if

$$\beta \left[ E[L(r^a_L)|e_H] - E[L(r^f_L)|e_H] \right] - \gamma r_p E[\max(\tilde{x}D - R, 0) | e = e_H] > 0.$$  \hspace{1cm} (31)$$

where

$$E[\max(\tilde{x}D - R, 0) | e = e_H] = \Pr[(\tilde{x}D > R) | e = e_H] E[(\tilde{x}D - R) | e = e_H]$$

$$= \Pr[(\tilde{x}D > R) | e = e_H] \left[ E(\tilde{x})D - \bar{R} \right]$$

$$= \Pr[(D(1 - \tilde{x}) < L) | e = e_H] \left[ E[L(r^a_L)|e_H] - D (1 - E(\tilde{x})) \right]$$

where the third equality is derived by substituting $R = D - L$ and $\bar{R} = D - E[L(r^a_L)|e_H]$.

Since $r^a_L < r^f_L$, it is the case that $E[L(r^a_L)|e_H] > E[L(r^f_L)|e_H]$. Hence the above inequality will only hold if the expected penalty cost is low enough or if and only if

$$\gamma < \frac{\beta \Delta}{r_p E[\max(\tilde{x}D - R, 0) | e = e_H]},$$

where $\Delta = \left[ E[L(r^a_L)|e_H] - E[L(r^f_L)|e_H] \right]$.

Since $E[\max(\tilde{x}D - R, 0) | e = e_H]$ is decreasing in $D$, the expected penalty cost is decreasing in $D$ and hence the LHS of the above inequality is monotonically increasing in $D$. Even if $\gamma = 1$ the above inequality will hold for sufficiently large $D$. More generally for any given level of $\gamma$ the above inequality would hold if and only if

$$\gamma < \frac{\beta \Delta}{r_p E[\max(\tilde{x}D - R, 0) | e = e_H]}.$$
will hold for high enough \( D \) and hence in this case the manager will engage in overly-aggressive behavior. Conversely, \( \Pi_m^a < \Pi_m^{aa} \) for low enough \( D \). Q.E.D.

**Proof of Proposition 5.** We know from (17) that entrepreneurs will choose \( e = 0 \) if and only if \( \phi \theta (C - d) + B > \theta (C - d) \). This implies that entrepreneurs will shirk if and only if \( \theta < \frac{B}{(1 - \phi)(C - d)} \). Q.E.D.

**Proof of Proposition 6.** Comparing (13) with (14) we know that \( P > P_f \) if and only if \( r_L < r_f^L \). From the proof to Proposition 4 we know that \( r_L < r_f^L \) for sufficiently high \( D \). Let \( D^* \) denote the threshold below which \( r_L < r_f^L \) and assume the plausible that the number of investors \( I \) is big enough so that \( D^* \) exists. Hence all we need to show is that \( \frac{dD}{d\theta} < 0 \) \( \forall \theta \in [\theta, 1] \). Since \( D = (1 - \nu \theta) I \) \( \forall \theta \in [\theta, 1] \) it follows that \( \frac{dD}{d\theta} = -\nu I < 0 \) \( \forall \theta \in [\theta, 1] \). Since \( D \) is monotonically decreasing in \( \theta \) for all \( \theta \in [\theta, 1] \) it follows that there exists a threshold \( \theta^c \) below which \( D > D^* \) and hence \( P > P_f \), where \( \theta^c \) is such that it solves \( D^* = (1 - \nu \theta^c) I \). Q.E.D.

**Proof of Proposition 7.** We have established so far that:

\[
P = \begin{cases} 
  P_f & \text{if } D \leq D^* \\
  P_f > & \text{if } D > D^*
\end{cases}
\]

Furthermore from Proposition 6 we know that in the absence of monetary

25Note that if the penalty costs incurred by the manager were negligible then there will always exist an agency problem even if deposits are very low. In other words, expression (31) may always hold for negligible penalty costs which will make the problem not only uninteresting but also unrealistic. To rule out this possibility we assume that \( \gamma r_p > \frac{\Delta}{\max{\{2D - R, 0\}}[c = e_H]} \) where \( D \) denotes the lower bound on deposits that the bank receives. This condition ensures that penalty costs are high enough so that there may or may not exist an agency problem depending on the volume of bank deposits. In fact it can be shown that if \( \gamma \) were a choice variable chosen by the principal, its optimal value would be 1 since this minimizes the probability of agency problems.
policy:

\[
D \begin{cases} 
< D^* & \text{if } \theta > \theta^c \\
> D^* & \text{if } \theta \in [\theta, \theta^c]
\end{cases}
\]

Hence \( P = P^f \) as long \( D < D^* \) which is the case when \( \theta > \theta^c \). However for \( \theta \in [\theta, \theta^c] \), \( P > P^f \). Since \( D = D(\theta, M) \), this can be offset by contractionary monetary policy which ensures that \( D \leq D^* \). The increase in deposits above \( D^* \) when \( \theta \in [\theta, \theta^c] \) is given by \( \frac{dD}{d\theta}(\theta^c - \theta) \). Since \( D = (1 - \nu\theta)I \ \forall \theta \in [\theta, 1] \) in the absence of monetary policy, we have \( \frac{dD}{d\theta} = -\nu I \ \forall \theta \in [\theta, 1] \). Thus in order to ensure \( D \leq D^* \) and hence \( P = P^f \), the central bank will have to reduce liquidity by at least \( (\theta^c - \theta) \nu I \). This can be done via open market operations by selling government securities to primary dealers. If this is done instantaneously then a bubble formation is avoided. Q.E.D.
References


