Imperfect Risk Sharing, Output-Inflation Tradeoffs and Business Cycles

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Abstract

How do frictions in asset markets affect business cycle dynamics? This paper argues that imperfect risk-sharing among heterogeneous households, due to frictions in asset markets, amplifies price stickiness endogenously and consequently increases the persistence and volatility of business cycles. The main economic mechanism is an idiosyncratic wealth effect on individual household's labor supply. This result provides a way to understand two conflicting observations based on macro and micro data: (1) The model is able to generate persistent *aggregate* dynamics, being consistent with *micro* evidence of frequent price adjustments. (2) The model is consistent with large elasticity of *aggregate* labor supply as well as small elasticity of *individual* labor supply.

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1 Introduction

How do frictions in asset markets affect the aggregate dynamics of inflation and output? The main theme of this paper is that even a very small friction in risk-sharing, if combined by a relatively small nominal rigidity, can have quantitatively non-trivial effects on macroeconomic dynamics. More specifically the paper shows that imperfect risk-sharing due to frictions in asset markets amplifies price stickiness endogenously and increases the persistence and volatility of the business cycles. The main economic mechanism is an idiosyncratic wealth effect on labor supply that makes each individual household's labor supply inelastic to a change in real wage, which in turn makes each firm's marginal cost more elastic to a change in the firm's price and output.

Based on the result above, this paper studies and documents four important implications for business cycle research. The first two are the implications for discrepancies between micro and macro observations. The imperfect risk-sharing appears to reconcile the tension between those two. I then discuss a couple of policy implications.

First, since imperfect risk-sharing model amplifies price stickiness endogenously (or, equivalently, it generates real rigidity), the model can fit key aggregate time series data with a more moderate degree of nominal rigidity relative to perfect risk-sharing models. Consequently the model can be consistent with micro evidence of the frequency of price adjustment, which has turned out to be higher than many macroeconomists had thought (Bills and Klenow 2004). The model in this paper generates persistent aggregate dynamics with only 1.78 quarter of price stickiness when we take the possibility of imperfect risk-sharing into account. As a consequence, imperfect risk sharing can explain why monetary policy shocks can have large and persistent real effects even when many firms update their prices frequently.

Second, as stated in the first paragraph, the reason for real rigidity is a smaller individual labor supply elasticity due to idiosyncratic wealth effects. The idiosyncratic wealth effects, however, cancel each other out through aggregation across households. Therefore aggregate labor supply elasticity needs not be identical to individual labor supply elasticity. I indeed show that the macro elasticity is always larger than or equal to the micro elasticity, with the equality being realized only when the risk-sharing is perfect. The estimated general equilibrium model gives 0.35 for the micro elasticity and 3.18 for the macro elasticity, which appears to be consistent with evidence at both micro and macro level.

Third, the main result provides a new channel through which improving financial markets leads to higher welfare. It is generally agreed that imperfect asset markets can have adverse effects on the welfare of our society in two ways. First, people would be perfectly insured against idiosyncratic income risks if asset markets were ideal. Second, a better asset market would promote long term economic growth more efficiently. However the channel presented in this paper is new because in principle we can think of the previous two channels separately from business cycles. I show that asset market frictions affect welfare exactly because they amplify business cycle fluctuations: for a given degree of nominal rigidity, output, once either nominal or real shocks hit the economy, deviates from its efficient level by a larger amount and for a longer period of time due to the *real rigidity* generated by imperfect risk-sharing. This new channel therefore generates an additional source of inefficiency, which is separate from those discussed in previous studies. The estimated model predicts that, if there were no frictions in asset markets, the volatility of the U.S. business cycles would be less than half of the volatility we currently observe.

Finally the output-inflation tradeoff has been an important subject of study because it has implications for optimal monetary policy. I show that the real rigidity created by imperfect risk-sharing is reflected in the slope of Phillips curve. The Phillips curve gets flatter as frictions in risk-sharing get bigger. As a consequence, policy makers have to deal with less favorable inflation-output tradeoffs, and potentially larger sacrifice ratios, when financial frictions are more stringent.

I investigate the questions above by introducing an asset market friction into an otherwise simple prototype New Keynesian (NK) model. There are several reasons that the NK model is used as a starting point in this paper. First, the model and its extensions have been the workhorse for the analysis of business cycles and monetary policy. Second, the model is still simple enough for me to show the main result of this paper analytically. Third, few papers have investigated the aggregate implications of incomplete asset markets and heterogeneous households in the sticky-price framework.

The key features of the standard NK model include nominal price rigidity and monopolistic competition with differentiated products, each requiring a different labor skill to be produced. Since labor markets are segmented for different types of products and the households can be employed in different labor markets (perhaps because each household has a different skill set), the households are heterogenous in terms of their labor incomes. However, with complete asset markets, which is another standard assumption of the NK model, in equilibrium the households become homogeneous in asset holdings and consumption. Hence the model economy becomes identical to the economy with a representative household who supplies every type of labor.

In contrast, when risk-sharing is imperfect due to some frictions in asset markets, each household's consumption depends positively on its labor income and thus on the wage rate and output of the firm in which the household is employed. This feature of the model makes the wage elasticity of labor supply smaller due to the *wealth effect*: for instance, when the wage rate is high, household's consumption level is also high, and consequently the household has less incentive to supply labor.

To see how the wealth effect on labor supply influences a firm's pricing decision, consider a firm hit by a shock that reduces the firm's marginal cost. The firm then has an incentive to lower its price, which would induce more demand for its product. To produce more, the firm would demand more labor hours, which would shift the labor demand curve to the right. This would raise the equilibrium wage rate and consequently the firm's marginal cost, which might offset the initial decrease in the marginal cost. However, the later increase in the marginal cost must be larger when there is a friction in asset markets because the labor supply curve is steeper due to the wealth effect. The firm therefore decides to reduce its price by a smaller amount than it would do if the asset markets were perfect. To summarize, imperfect risk-sharing can explain inertial aggregate inflation and persistent business cycles, even if firms change prices relatively frequently, because when firms change prices they do so by a smaller amount.

I introduce a friction in asset markets following the same approach taken by Schulhofer-Wohl (2007). I include a convex transaction cost in the households' budget constraints, which makes transferring resources between the households costly. With the transaction cost, the households transfer their resources to insure the income risks by a smaller amount than they otherwise would. Consequently, consumption insurance is less than perfect: a typical household relative consumption fluctuates in the same direction as its relative income. This is true even when there exists a complete set of state contingent financial securities. Although one could view that the transaction cost is a shortcut to generate imperfect risk-sharing, I nevertheless take that approach for three reasons. First, it provides me with the simplest and cleanest way to derive my results. Second, alternative ways of modeling asset market frictions would not change the main insight of this paper as long as a household's relative income fluctuations result in relative consumption fluctuations as a result of the imperfect asset market institutions.[1] Finally in my view, the other approaches are not obviously more realistic.

In the real world, asset markets are not ideal in one way or another, and we often observe that people with bad income shocks also suffer from reduced level of consumption. So the asset market imperfection itself is less controversial. It is quantitative importance that is more controversial. The quantitative importance of the imperfect asset markets and of the less ideal risk-sharing on aggregate dynamics are still being debated and has been one of the main research areas. A number of papers have focused on how relaxing the assumption of complete asset markets and a representative household affects aggregate dynamics quantitatively. This research agenda is relatively young, but is growing rapidly. Important early contributions include Huggett (1993), Aiyagari (1994), and Krusell and Smith (1998). However up to my knowledge, few papers have analyzed the implications of imperfect risk sharing on inflation-output dynamics and the Phillips curve in a sticky-price framework.

This paper is also related to the literature on real rigidities. Early contributions include Ball and Romer (1990), Kimball (1995), Basu (1995), Bergin and Feenstra (2000), Altig et al. (2004), Eichenbaum and Fisher (2007), Woodford (2005) and Carvalho (2006). Chari

[1] Lee (2007) shows the same result assuming a different financial market institution. In that paper, the asset markets are incomplete and risk sharing is imperfect becuase the households can trade only one type of assets: riskless bonds.

et al. (2000) also emphasize the importance of endogenous stickiness. They argue that sticky-price dynamic stochastic general equilibrium (DSGE) models need to amplify the price stickiness endogenously to explain persistent aggregate dynamics with a reasonable degree of nominal price stickiness.

These two literatures, the literature on imperfect risk-sharing and on real rigidities have been important research ares in macroeconomics, but they have been developed rather separately. This paper argues that there could indeed be a connection between the imperfect risk-sharing among heterogeneous households and the real rigidity: the imperfect risk-sharing is a new source of real rigidity.

The literature on the labor supply elasticity at the micro and macro levels is another line of research related to this paper. This paper provides a bridge between the research on the elasticity and the literature on real rigidity, recognizing that a source of real rigidity can also provide an explanation for the apparent tension between the micro and macro labor elasticity. The closest to my work is Chang and Kim (2006) in that they also identify household heterogeneity and imperfect risk-sharing as a main driving force for the reconciliation. The economic mechanism in this paper, however, is different.[2]

The rest of the paper is organized as follows. In the next section, I develop a baseline NK model with asset market frictions and present the main *theoretical* results. In the following section, I take the model to the U.S. data and document some *quantitative* implications of imperfect risk-sharing using the estimated NK model developed in the previous section. Section 4 summarizes the findings and concludes.

2 Model

I describe the model economy in this section. The model is similar to the standard NK model in Woodford (2003, chapter 3). The only difference is the existence of a cost of transferring resources among the households as in Schulhofer-Wohl (2007). As a result, the households are not able to insure their income risks perfectly.

2.1 Households

There is a continuum of industries indexed by $i \in [0, 1]$, each of which produces a different type of product. Each industry i is represented by one firm called type-i firm. Different labor skills are required to produce different types of product, that is labor skill is industry-specific. In each industry i, there is a representative household called type-i household. The type-i household possesses a labor skill specialized exclusively for industry i, and thus supplies labor service to type-i firm.

Type-i household maximizes the following discounted expected utility function:

[2] A more detailed literature review will be added here.

$$E_0\left(\sum_{t=0}^{\infty}\beta^t\Gamma_t\left[\frac{C_t(i)^{1-\sigma}-1}{1-\sigma}-\Xi_t\frac{H_t(i)^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}}\right]\right),$$

where $C_t(i)$ denotes type-i household's consumption, $H_t(i)$ denotes the hours of labor services supplied to industry i, Γ_t and Ξ_t are preference shocks common to all households, β is the discount factor, ψ is the elasticity of labor supply and σ is the coefficient of relative risk aversion. The preference parameters ψ and σ are non-negative and $\beta \in (0,1)$.

The household's dynamic budget constraint is given by

$$P_tC_t(i)+E_t\left[Q_{t,t+1}B_{t+1}(i)\right]+P_t\underbrace{\phi\Phi\left(C_t(i),X_t(i)\right)}_{\text{real transaction cost}}=B_t(i)+W_t(i)H_t(i)+\Pi_t-P_tT_t,$$

where P_t denotes aggregate price level, $W_t(i)$ is competitive wage rate in industry i, T_t is lump-sum tax and Π_t is the aggregate profit of the economy, $\Pi_t=\int_0^1\Pi_t(i)di$. A household's *total* income at time t is the sum of *labor* income $W_t(i)H_t(i)$ and *capital* income Π_t. I let $X_t(i)$ denote type-i household's *total* after-tax real income at time t:

$$X_t(i)\equiv\frac{W_t(i)H_t(i)+\Pi_t-P_tT_t}{P_t}.$$

Unlike labor income, the households' capital incomes and the taxes are not idiosyncratic. Every household holds the same mutual fund so that the economy's total profit is equally distributed among the households, and the government collects the same amount of tax from each household. Consequently, the income differential between any two households is entirely due to a difference in labor income. The equal capital ownerships and taxes are simplifying assumptions that do not affect the main results of this paper.[3]

Households can trade nominal securities with arbitrary patterns of state-contingent payoffs. In budget constraint, $B(i)$ denotes type-i household's holding of one period state contingent nominal securities and $Q_{t,t+1}$ is a stochastic discount factor. At time t when the household makes its portfolio decision, $B_{t+1}(i)$ is a random variable that can have different values depending on the state realized at time $t+1$. Households completely specify whole distribution of $B_{t+1}(i)$ at time t, taking the market prices for the state-contingent payoffs as given.

Making consumption different from its income is costly. If a household's consumption $C_t(i)$ is different from its income $X_t(i)$ then the amount $\phi\Phi\left(C(i),X(i)\right)$ of consumption good is taken away from the household's resource. I assume $\phi\geq 0$. An important special case is when $\phi=0$. The model presented here is then the same as the standard NK model. Following Schulhofer-Wohl (2007), I let the transaction cost function $\Phi(\cdot)$ have the following

[3] In the model, the steady state values of the profit and tax are zero. Consequently they do not have first order effects on a household's income anyway. Therefore, up to first order approximation, we can regard that $X_t(i)$ contains only idiosyncratic part of the household's income. Again, this assumption is not necessary for the result of this paper.

form:

$$\Phi\left(C,X\right)=\frac{C}{2}\left(\log\frac{C}{X}\right)^{2}.$$

However, the specific function given above is not necessary because any convex transaction cost would give the same result.

Type-i household's first order conditions are

$$\beta\frac{P_tC_t(i)^{\sigma}\left\{1+\phi\Phi_C(C_t(i),X_t(i))\right\}\Gamma_{t+1}}{P_{t+1}C_{t+1}(i)^{\sigma}\left\{1+\phi\Phi_C(C_{t+1}(i),X_{t+1}(i))\right\}\Gamma_t}=Q_{t.t+1},\tag{1}$$

$$H_t(i)^{\varphi}C_t(i)^{\sigma}\frac{\left\{1+\phi\Phi_C(C_t(i),X_t(i))\right\}}{\left\{1-\phi\Phi_X(C_t(i),X_t(i))\right\}}\Xi_t=\frac{W_t(i)}{P_t},\tag{2}$$

where Φ_C and Φ_X are the partial derivatives of $\Phi\left(\cdot\right)$ with respect to the level of consumption and income:

$$\Phi_C\equiv\frac{\partial\Phi}{\partial C}=\left(\log\frac{C_t(i)}{X_t(i)}\right)+\frac{1}{2}\left(\log\frac{C_t(i)}{X_t(i)}\right)^{2},$$

$$\Phi_X\equiv\frac{\partial\Phi}{\partial X}=-\frac{C_t(i)}{X_t(i)}\left(\log\frac{C_t(i)}{X_t(i)}\right).$$

The gross nominal interest rate R_t is determined by $R_t^{-1}=E_t\left[Q_{t,t+1}\right]$ as R_t^{-1} is the price of portfolio in which $B_{t+1}(i)=1$ for every state of the economy at time $t+1$. The equation $R_t^{-1}=E_t\left[Q_{t,t+1}\right]$ and (1) together yield a consumption Euler equation

$$R_t^{-1}=\beta E_t\left[\frac{P_tC_t(i)^{\sigma}\left\{1+\phi\Phi_C(C_t(i),X_t(i))\right\}\Gamma_{t+1}}{P_{t+1}C_{t+1}(i)^{\sigma}\left\{1+\phi\Phi_C(C_{t+1}(i),X_{t+1}(i))\right\}\Gamma_t}\right].\tag{3}$$

One can show that in the case of no transaction cost where $\phi=0$, the economy is characterized by perfect consumption insurance. Especially with a normalizing assumption on the distribution of the households' initial wealth, one necessarily obtains from (1) that

$$C_t\equiv C_t(i)=C_t(j)\quad\forall i,j\in\left[0,1\right],\tag{4}$$

which should hold for every time period t and also for every possible state of the economy. In this case, (2) should instead be

$$H_t(i)^{\frac{1}{\psi}}C_t^{\sigma}\Xi_t=\frac{W_t(i)}{P_t}.\tag{5}$$

However, (4) no longer holds when ϕ is positive. A household relative consumption is higher when its relative income is higher, and consequently cross-household consumption distribution departs from the efficient distribution that would be realized when $\phi=0$.

Comparing (2) and (5) provides an idea of how the asset market friction leads to a greater degree of real rigidity. In the case of imperfect risk-sharing, type-i household's consumption $C_t(i)$ depends positively on labor income, thus real wage $\frac{W_t(i)}{P_t}$ as well as labor hour $H_t(i)$, which makes the wage elasticity of labor supply smaller due to the *wealth effect*.[4] As mentioned in the introduction, the less elastic labor supply makes the firm's marginal cost depends more negatively on the direction of price change, and as a result the firms would not change the prices as much. On the other hand, there is no such idiosyncratic wealth effect in the case of perfect risk sharing since each industry is so small relative to the whole economy that a change in the industry wage rate $\frac{W_t(i)}{P_t}$ or labor hour $H_t(i)$ do not affect directly the aggregate consumption C_t.

The equilibrium conditions can be log-linearized around the symmetric non-stochastic steady state. The log-linear approximations of (1), (2) and (3) take the form

$$c_t(i) = c_{t+1}(i) + \frac{1}{\sigma+\phi}\left(q_{t,t+1} + \pi_{t+1} + \gamma_t - \gamma_{t+1}\right) + \frac{\phi}{\sigma+\phi}\left(x_t(i) - x_{t+1}(i)\right), \tag{6}$$

$$w_t(i) - p_t = \psi^{-1}h_t(i) + \sigma c_t(i) + \xi_t, \tag{7}$$

$$c_t(i) = E_t c_{t+1}(i) - \frac{1}{\sigma+\phi}\left(r_t - E_t\pi_{t+1} + E_t\left(\gamma_t - \gamma_{t+1}\right)\right) + \frac{\phi}{\sigma+\phi}E_t\left(x_t(i) - x_{t+1}(i)\right), \tag{8}$$

where I use the lowercase letters to denote percentage deviations from the steady state.[5]

From (6), one can derive an analytical expression for a household's consumption as a function of its idiosyncratic income and aggregate output.

Proposition 1 *Up to the first order approximation, a typical household's consumption function can be expressed as a weighted sum of the idiosyncratic and aggregate incomes,*

$$c_t(i) = \frac{\omega}{1+\omega}x_t(i) + \left(1 - \frac{\omega}{1+\omega}\right)y_t, \tag{9}$$

where y_t is aggregate output (,which is equal to the aggregate consumption c_t in equilibrium), and the parameter, ω is the ratio of transaction cost to risk aversion,

$$\omega \equiv \phi/\sigma.$$

The proof of this (and all other results) is in the appendix. The result is intuitive. A household's consumption, $c_t(i)$ responds more to $x_t(i)$ relative to y_t as the transaction cost, ϕ increases. The equation (9) gives the elasticity of consumption to idiosyncratic income

[4] Note that the marginal rate of substitution,

$$MRS_t(i) = H_t(i)^{\frac{1}{\psi}}C_t(i)^{\sigma}\frac{\{1+\phi\Phi_C(C_t(i),X_t(i))\}}{\{1-\phi\Phi_X(C_t(i),X_t(i))\}}\Xi_t$$

is an increasing function of $C_t(i)$. And $C_t(i)$ is an increasing function of $X_t(i)$ and thus $H_t(i)$ and $W_t(i)/P_t$. Since $MRS_t(i) = W_t(i)/P_t$ in equilibrium, it can be easily shown that the supply of labor hours $H_t(i)$ is less sensitive to the real wage $W_t(i)/P_t$.

[5] For instance,

$$c_t(i) \equiv \log C_t(i) - \log C,$$

where C is the common steady state level of consumption of households.

and thus provides an idea of how large a given ϕ is, in the context of consumption function.

An alternative way of writing the equation (9) is

$$c_t^R(i) = \frac{\omega}{1+\omega} x_t^R(i). \tag{10}$$

The variables with superscript R, $c_t^R(i)$ and $x_t^R(i)$, denote type-i household's consumption and income relative to the aggregate consumption and income. The equation (10) indicates that a household's relative consumption moves in the same direction with its relative income as long as ϕ is positive and $c_t^R(i) = 0$ when $\phi = 0$.

Schulhofer-Wohl (2007) estimates the ratio of transaction cost parameter to risk aversion, ω and shows that a reasonable value of ω should be in the range of 0.117 to 0.205 if one accounts for the heterogeneous risk and time preferences among the households. If one does not account for the heterogeneous preferences as in this paper, the estimated ω is much larger and it can be greater than 0.5. As noted by Schulhofer-Wohl, his estimate of transaction cost parameter is considered one of the smallest. In this paper I assume small values for ω prior to the estimation, to be conservative.[6]

To have an idea of the magnitude of ω, let us consider a couple of examples. If $\omega = 0.2$, the equation (10) implies that a household's consumption would be higher by only 16.7% than the economy's average consumption, if its real income were larger by 100% than the economy's average income. Put it differently, the relative standard deviation of relative consumption to relative income, $\frac{std\left(c_t^R(i)\right)}{std\left(x_t^R(i)\right)}$ is only 0.167. Another way to put it is that the correlation between a typical household's consumption and aggregate consumption (and income) is 0.833 while the correlation between a household's consumption and its income is only 0.167 (see equation (9)). Therefore, according to this estimate, the transaction cost is indeed very small and people seem to share risks quite well. Even a larger value, such as $\omega = 0.5$, does not appear to lead to an unreasonably large transaction cost since $\frac{std(\left(c_t^R(i)\right)}{std(\left(x_t^R(i)\right)}$ is only about 0.33.

As mentioned above, the wage elasticity of labor supply is an important factor that determines the degree of real rigidity that a model generate. Combining the consumption equation, (9) and a household optimality condition (7), we can derive a household's labor supply schedule.

$$h_t(i) = \tilde{\psi}\left[w_t(i) - p_t\right] - \chi_1 y_t - \chi_2 \xi_t, \tag{11}$$

where

$$\tilde{\psi} \equiv \frac{1 - \frac{\sigma\omega}{1+\omega}}{\psi^{-1} + \frac{\sigma\omega}{1+\omega}}, \qquad \chi_1 \equiv \frac{\frac{\sigma}{1+\omega}}{\psi^{-1} + \frac{\sigma\omega}{1+\omega}}, \qquad \chi_2 \equiv \frac{1}{\psi^{-1} + \frac{\sigma\omega}{1+\omega}}.$$

The wage elasticity $\tilde{\psi}$ is decreasing in ω (and ϕ). Therefore the firms in industry i would face a steeper labor supply curve if there were a financial friction, and consequently the

[6] For instance, I consider 0.14 as the most probable value for ω before I actually estimate it, taking the model to aggregate time series data.

firms' marginal cost would be more sensitive to a change in price and output of the firms.

Note that even when the Frish elasticity of labor supply ψ is very large, the model can endogenously generate a quite small elasticity, $\tilde{\psi}$. This fact might reconcile the discrepancy between the elasticity of labor supply at micro level and at macro level. By aggregating (11) over the unit interval and then using the equilibrium condition, $h_t + w_t - p_t = y_t$, we can obtain the following aggregate labor supply curve,[7]

$$h_t = \psi(w_t - p_t) - \psi\sigma y_t - \psi\xi_t. \tag{12}$$

Therefore, for a given Frish elasticity of labor supply, ψ, the elasticity of labor supply at micro-level is always smaller than the macro elasticity as long as there is some financial friction.

I summarize the observations that we just made in the following proposition.

Proposition 2 *Up to the first order approximation, a typical household's labor supply schedule is given by (11) and the aggregate labor supply schedule is given by (12). The wage elasticity of individual labor supply is decreasing in risk-sharing friction, ω whereas that of aggregate labor supply does not depend on the friction. Moreover, the former is smaller than or equal to the later, that is,*

$$\tilde{\psi} \leq \psi$$

holds, with the equality being true when ω (and ϕ) is equal to zero.

2.2 Firms

This subsection describes the supply side of the economy. As mentioned above, there is a continuum of industries indexed by $i \in [0, 1]$, each of which produces a distinguished type of product $Y_t(i)$. In each industry i, there is a representative firm called type-i firm.

The differentiated products $\{Y_t(i)\}_{i\in[0,1]}$ are used to produce final consumption good Y_t, through a CES technology given by

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\theta_t - 1}{\theta_t}} di\right)^{\frac{\theta_t}{\theta_t - 1}},$$

[7] The aggregate hour and wage are defined as

$$\begin{aligned} H_t &= \int_0^1 H_t(i)di, \\ W_t &= \int_0^1 W_t(i)di. \end{aligned}$$

The aggregate labor supply curve also can be obtained by simply integrating (7) over the unit interval.

We will see that the total profit is equal to zero in the steady state. Consequently, up to first order approximation, the aggregate labor income must be equal to aggregate output, i.e. $h_t + w_t - p_t = y_t$.

where θ_t denotes stochastic elasticity of substitution between different types of goods. I assume $\theta_t \geq 1$. The corresponding price index P_t for the final consumption good is

$$P_t = \left(\int_0^1 P_t(i)^{1-\theta_t} di \right)^{\frac{1}{1-\theta_t}},$$

where $P_t(i)$ is the price of type-i product. The optimal demand for each type of good is then given by

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\theta_t} Y_t. \tag{13}$$

Type-i firm's production function is given by

$$Y_t(i) = A_t H_t(i), \tag{14}$$

where A_t denotes the level of economy-wide technology.

As a benchmark case, I first consider the economy in which all prices are flexible. Type-i firm chooses its price, $P_t(i)$ that maximizes its profit,

$$\Pi_t(i) = (1-\tau) P_t(i) Y_t(i) - W_t(i) H_t(i)$$

with the given technology (14) and the demand function for its product (13). The government collects a sales tax, with a fixed rate of $\tau = \frac{1}{\theta}$, where θ is the steady state value of θ_t. The presence of sales tax makes the equilibrium output in steady state to be efficient. I assume that the tax revenues are wasted.[8]

If the prices were flexible, the firms would choose a common optimal price and the households choose a common level of consumption. The natural output, the level of aggregate output that would be realized with flexible prices, is then given by

$$Y_t^N = S_t^{-1} A_t^{\frac{1+\psi^{-1}}{\sigma+\psi^{-1}}} \Xi_t^{-\frac{1}{\sigma+\psi^{-1}}},$$

where $S_t \equiv \frac{\theta_t}{(\theta_t - 1)(1-\tau)}$ denotes the stochastic mark-up. Note that if the markets were competitive the mark-up would not show up in the equation and the output produced would always be equal to the efficient level of output, $Y_t^E \equiv A_t^{\frac{1+\psi^{-1}}{\sigma+\psi^{-1}}} \Xi_t^{-\frac{1}{\sigma+\psi^{-1}}}$. The natural output thus can be expressed as

$$Y_t^N = S_t^{-1} Y_t^E$$

[8] These assumptions, that the government collects sales tax and that the tax revenues are wasted, are not necessary for the results of this paper. But they are often convenient to have, especially when one takes the model to the welfare analysis. In this paper, the presence of the tax makes algebra a little easier and leads to a cleaner expression of the reduced-form equilibrium conditions. But the results would be essentially unchanged without the tax.

Taking log gives

$$\begin{aligned} y_t^N &= \left(\frac{1+\psi^{-1}}{\sigma+\psi^{-1}}\right) a_t - \left(\frac{1}{\sigma+\psi^{-1}}\right) \xi_t - s_t \\ &= y_t^E - s_t. \end{aligned}$$

The equation above shows that the natural level of output can be different from the efficient level of output in the presence of the stochastic mark-up.

As emphasized by Lee (2007), when there is no idiosyncratic shock and the prices are flexible, the asset market imperfection by itself does not make any difference, not only for aggregate dynamics but also for cross-sectional distributions. In the absence of idiosyncratic shocks, every firm charges the same prices and produces the same amount of outputs. Consequently, the households' incomes are symmetric and there is no need for the asset markets at all. Then the aggregate dynamics would not be effected: the natural output Y_t^N does not depend on ϕ.

However the result above does not hold under sticky prices. When the prices are not flexible, even the aggregate shocks act like idiosyncratic shocks as the timing of price adjustments are not synchronized among the firms and consequently the labor incomes across the households are different. Therefore the equilibrium outcome in principle should be different where there is a friction in asset markets.

Following Calvo (1983) and Yun (1996) I assume the firms adjust their prices with probability $1-\alpha$ each period. A firm that re-optimizes at time t choose the optimal price $P_t^*(i)$ that solves the following profit maximization problem:

$$\max_{P_t^*(i)} E_t \sum_{k=0}^{\infty} \alpha^k Q_{t,t+k} \tilde{\Pi}_{t+k}(i),$$

where

$$\begin{aligned} \tilde{\Pi}_{t+k}(i) &= (1-\tau) P_t^*(i) Y_{t+k}(i) - W_{t+k}(i) H_{t+k}(i) \\ &= (1-\tau) P_t^*(i) \left(\frac{P_t^*(i)}{P_{t+k}}\right)^{-\theta_{t+k}} Y_{t+k} - \frac{W_{t+k}(i)}{A_{t+k}} \left(\frac{P_t^*(i)}{P_{t+k}}\right)^{-\theta_{t+k}} Y_{t+k}. \end{aligned}$$

The first order condition is

$$E_t \sum_{k=0}^{\infty} \alpha^k Q_{t,t+k} \left(\frac{P_t^*(i)}{P_{t+k}}\right)^{-\theta_{t+k}} Y_{t+k} \left\{ P_t^*(i) - S_{t+k} \frac{W_{t+k}(i)}{A_{t+k}} \right\} = 0. \tag{15}$$

By loglinearizing the first order condition above, I can obtain the generalized NK Phillips curve which nests the case of perfect consumption insurance as a special case.

Proposition 3 *Consider the heterogeneous household model with imperfect risk-sharing described in this paper. The aggregate output and inflation must satisfy a Phillips curve (or* *an aggregate supply relation) of the form*

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \left(y_t - y_t^E \right) + \mu_t,$$

where

$$\begin{aligned} \kappa &\equiv \left\{ \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \right\} \left\{ \frac{\sigma + \psi^{-1}}{1 + \theta \left(\psi^{-1} + \sigma\gamma \right)} \right\}, \\ \gamma &\equiv \frac{\omega \left(1 + \psi^{-1} \right)}{1 + \omega - \sigma\omega}. \end{aligned}$$

The reduced-form of the Phillips curve therefore remains the same as the baseline NK model. However the slope of the Phillips curve, κ gets smaller, for a given degree of nominal rigidity, α, as the degree of the transaction cost ϕ gets larger as long as $\phi \leq \frac{\sigma}{\sigma - 1}$.

The proof is outlined in the appendix. The residual term $\mu_t \equiv \kappa \left(\sigma + \psi^{-1} \right)^{-1} s_t$ is proportional to the stochastic mark-up, and is often called a "cost-push shock" in the literature. The inequality $\phi \leq \frac{\sigma}{\sigma-1}$ is a condition that makes γ positive and thus makes the slope, κ smaller. If the inequality does not hold, the Phillips curve gets steeper. Intuitively when the transaction cost is too large, the wealth effect is so large that not only the labor supply curve becomes less elastic but also the sign of slope switches to be negative: the households supply less hours when the real wage is higher. Since I view this is rather an unusual case, I will focus only on the case in which the transaction cost is non-negative but not too large; i.e. $0 \leq \phi \leq \frac{\sigma}{\sigma-1}$.

Figure B plots the slope of the Phillips curve for $\phi \in [0, 1.5]$ and $\sigma = 3$. In the special case in which $\phi = 0$, the slope of the Phillips curve would be exactly same as the one in the benchmark perfect risk-sharing model. On the other hand, if ϕ is 0.6 so that $\omega = 0.2$, then the slope is only about one third of the slope under perfect risk-sharing.[9] If a higher value of either σ or ϕ were used, then the slope would get even smaller. This suggests that the real rigidity due to the asset market friction can be substantial.

2.3 Government

Assuming the government does not issue the state-contingent assets, the government budget constraint should be

$$R_t^{-1} D_{t+1} + P_t G_t = D_t + P_t T_t + \tau P_t Y_t,$$

where D_t is government's risk-less bond supply, G_t is government expenditure, and $P_t T_t$ and $\tau P_t Y_t$ are the tax revenues from the households and the firms respectively. For simplicity I assume

$$G_t = \tau Y_t, \quad D_t = 0, \quad T_t = 0$$

[9] Figure B uses the benchmark parameter values: $\alpha = 0.5$, $\beta = 0.99$, $\sigma = 3$, $\varphi = 3$ and $\theta = 6$. These numbers are also used as the prior means in a later section.

Monetary policy is characterized by a Taylor rule:

$$R_t = \beta^{-1} R_{t-1}^{\rho_r} \left[\left(\frac{P_t}{P_{t-1}} \right)^{\phi_\pi} \left(\frac{Y_t}{Y} \right)^{\phi_y} \right]^{(1-\rho_r)} \exp(m_t).$$

2.4 Equilibrium

Equilibrium is characterized by allocation of the resources and prices that satisfy the households' optimality conditions and budget constraint, the firms' optimality conditions, the monetary policy rule and finally the market clearing conditions:

$$\int_0^1 \{C_t(i)+\phi k\left(C_t(i), X_t(i)\right)\} di + G_t = Y_t, \qquad \int_0^1 B_t(i) di = 0$$

for every time t and every state of the economy.

In the log-linear approximation, however, if we were only interested in the dynamics of the aggregate variables $\{y_t, \pi_t, r_t\}$, then the three familiar equations

$$y_t = E_t y_{t+1} - \frac{1}{\sigma}\left(r_t - E_t \pi_{t+1} + \gamma_t - E_t \gamma_{t+1}\right), \tag{16}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa\left(y_t - y_t^E\right) + \mu_t, \tag{17}$$

$$r_t = \rho_r r_{t-1} + (1-\rho_r)\left\{\phi_\pi \pi_t + \phi_y y_t\right\} + m_t, \tag{18}$$

would completely determine the dynamics of those variables. The first equation is the aggregate demand equation (or the IS equation) and it can be derived by integrating the households' Euler equations (8) over unit interval. The second equation is the generalized Phillips curve and the third equation is a log-linearized Taylor rule. The system of the equilibrium conditions looks exactly same as the standard NK model except the slope of the Phillips curve κ is a different function of structural parameters. This suggests that once we correctly adjust the slope, the imperfect risk-sharing does not play a role as long as one's focus is only on the aggregate dynamics up to the first order approximation. Especially one does not have to keep track of the cross-sectional distribution of the households' consumptions and asset holdings.

Finally one can also study dynamics of two labor market variables, hours and real wage, by adding two more equilibrium conditions,

$$w_t - p_t = \psi^{-1} h_t + \sigma y_t + \xi_t, \tag{19}$$

$$y_t = a_t + h_t. \tag{20}$$

The first equation can be obtained by aggregating the households' intra-temporal first order conditions and aggregating the firms production functions yields the second equation. To summarize, the equations (16), (17), (18), (19) and (20) determine the equilibrium time paths of $\{y_t, \pi_t, r_t, h_t, w_t - p_t\}$.

3 Applying the model to the U.S. business cycles

The previous section presented the simple NK general equilibrium model and discussed the main theoretical result. In this section, I take the model to the U.S. data, estimate the model and use the estimated model to answer several questions of interest. How large is the asset market friction implied by the model and the data? Is the estimated friction implausibly large or small? How does the presence of the asset market friction affect the estimate of nominal rigidity implied by the model? Is the estimated elasticity of labor supply consistent with both micro and macro evidence? How much does the friction matter in term of persistence and volatility of business cycles? Does it matter at all? Put it differently, how much could we moderate the business cycles if we improve the financial market institutions so that people can share the risk better, while other things being equal?

3.1 Estimation

The questions of interest above are quantitative in nature. It therefore is necessary to choose a set of model parameters to make the model useful for answering those questions and performing a counterfactual analysis.

3.1.1 Exogenous Process Specification

First of all, I specify the stochastic exogenous processes to complete the model. I assume that the two preferences shocks and the technology shock follow independent AR(1) processes whereas the cost-push shock and monetary policy shock follow independent IID processes:

$$\begin{pmatrix} \gamma_t \\ \xi_t \\ a_t \\ \mu_t \\ m_t \end{pmatrix} = \begin{pmatrix} \rho_\gamma & 0 & 0 & 0 & 0 \\ 0 & \rho_\xi & 0 & 0 & 0 \\ 0 & 0 & \rho_a & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \gamma_{t-1} \\ \xi_{t-1} \\ a_{t-1} \\ \mu_{t-1} \\ m_{t-1} \end{pmatrix} + \begin{pmatrix} \sigma_\gamma & 0 & 0 & 0 & 0 \\ 0 & \sigma_\xi & 0 & 0 & 0 \\ 0 & 0 & \sigma_a & 0 & 0 \\ 0 & 0 & 0 & \sigma_\mu & 0 \\ 0 & 0 & 0 & 0 & \sigma_m \end{pmatrix} \begin{pmatrix} \varepsilon_{\gamma,t} \\ \varepsilon_{\xi,t} \\ \varepsilon_{a,t} \\ \varepsilon_{\mu,t} \\ \varepsilon_{m,t} \end{pmatrix}.$$

This is a conventional specification in the DSGE literature.

3.1.2 The Data

Being consistent with the equilibrium conditions (16), (17), (18), (19) and (20), I take the model to fit the five quarterly aggregate time series data of the United States: real output, price inflation, interest rates, labor hours and real wages. I use the nonfarm business sector output as a measure of output and its price deflator as a measure of price levels. The total hours and real compensation per hour from the nonfarm business sector are used as a measure of hours and real wages respectively. The effective federal funds rate measures the nominal interest rate. The sample period is from 1986:Q3 to 2008:Q4. Since the size of the model economy has been normalized to one, I divide the real output and the hours by the total civilian non-institutional population over age of 16. I detrend the real variables using a linear trend and demean the nominal variables.

3.1.3 The Likelihood Function and Bayes Theorem

The model has seventeen parameters. I let $\boldsymbol{\lambda}$ denote the vector of parameters

$$\boldsymbol{\lambda} \equiv \left(\beta, \sigma, \psi, \theta, \alpha, \omega, \phi_\pi, \phi_y, \rho_r, \rho_\gamma, \rho_\xi, \rho_a, \sigma_\mu, \sigma_\gamma, \sigma_\xi, \sigma_a, \sigma_s\right)'$$

I estimate the model taking the Bayesian full-information approach that exploits restrictions imposed by general equilibrium of the model economy. With the Bayesian approach, we can incorporate our prior belief about the structural parameters $\boldsymbol{\lambda}$ with estimation by specifying a prior distribution $f(\boldsymbol{\lambda})$. Given the data set $\mathbf{X}^T$, the model gives the likelihood function $f(\mathbf{X}^T|\boldsymbol{\lambda})$. Then the posterior distribution of $\boldsymbol{\lambda}$ is determined by Bayes theorem:

$$f(\boldsymbol{\lambda}|\mathbf{X}^T) = \frac{f(\mathbf{X}^T|\boldsymbol{\lambda})f(\boldsymbol{\lambda})}{f(\mathbf{X}^T)} = \frac{f(\mathbf{X}^T|\boldsymbol{\lambda})f(\boldsymbol{\lambda})}{\int f(\mathbf{X}^T|\boldsymbol{\lambda})f(\boldsymbol{\lambda})d\boldsymbol{\lambda}}.$$

Since it is impossible to obtain the analytical solution for the posterior distribution $f(\boldsymbol{\lambda}|\mathbf{X}^T)$, I simulate the posterior distribution by Markov Chain Monte Carlo method.

3.1.4 The Priors

For most parameters, the prior distribution follows the convention in the literature of Bayesian DSGE estimation. The prior distributions are summarized in the first four columns in Table 2. I impose dogmatic prior on two parameters θ and β. I fix the discount factor β to be 0.99 throughout this paper. I also set θ to be 6 so that a firm's mark-up, in the absence of the sales tax, is 20 percent in the steady state.

Even with a fixed (θ, β), there is an identification problem between the measure of nominal rigidity α and the measure of financial market friction ω because only the slope of the Phillips curve contains the two parameters. But unlike the previous two parameters (θ, β) , I do not fix them because they are the key parameters of interests in this paper. Instead, to overcome the identification problem, I specify informative priors for α and ω based on the information external to the data used to estimate this model. The in-frequency of price change parameter, α, follows a beta distribution with support of [0,1]. Both the prior mean and mode are 0.5 and standard deviation is 0.15, which implies a 95% region of [0.2, 0.8]. This prior choice is based on the recent empirical studies of frequency of price change. The mean ω is set to 0.2, with standard deviation of 0.1. The prior mode ω of is only 0.14. I assume a small friction as a priori to be conservative but I do not exclude completely the possibility of larger or smaller ω.

The preference parameters have relatively diffuse priors, which reflects the wide variety of estimates for these parameters. Both the elasticity of labor supply, ψ and the coefficient of relative risk aversion, σ have prior mean of 3 and standard deviation of 1. A high value of the elasticity of labor supply is often assumed in business cycle models. For instance, the early RBC models often assume $\psi = \infty$. But some models use a much smaller number based on the estimates at the micro-level data. The prior mean of ψ is large enough, being consistent with the business cycle literature, but its relatively large standard deviation allows the case of either much larger or smaller ψ. For the risk aversion parameter σ, people often use relatively small values such as 1, 2, or 3. But much larger values are also used in the literature. Also the estimated σ is often quite large. Rabanal and Rubio-Ramirez (2005) estimate a similar model and report that the estimate of σ is in the range of 4.5$\sim$8.3. This observation leads me to specify a somewhat diffuse prior.

Finally, the priors for the exogenous shocks and Taylor rule parameters are quite standard.

3.1.5 The Posteriors

Next to the summary of priors, the last two columns of Table 2 show some key moments of the posterior distributions. Figure 2 shows the prior and posterior densities. In most cases, the estimates are in line with the previous studies that estimate the baseline NK models, and the data appears to be informative for the parameters.

Both the coefficient of relative risk aversion and the elasticity of labor supply are relatively large but they are in reasonable ranges. The exogenous variables are quite persistent, which indicates that the model still does not have sufficient internal propagation mechanism. This is expected because the model is still highly stylized even after the new source of propagation, the imperfect risk-sharing, has been added to the model. I deliberately have the model abstract from such features like habit persistence in household expenditure and past inflation indexation in firms price setting, often included in the medium-scale NK DSGE models, in the effort of making the model as simple and transparent as possible.

3.2 Implication for Nominal Price Rigidity

The two most important parameters of interest in this paper are the nominal price stickiness α and the measure of asset market friction ω. The posterior mean of α is 0.57 which is smaller than the typical estimates in the Bayesian DSGE literature. It implies that the average duration of price contracts is 1.78 quarters, which is consistent with the recent evidence by Bills and Klenow (2004). They report that the median duration of prices is about between 4 and 6 month. On the other hand, another recent study by Nakamura and Steinsson (2008) reports that the duration should be around between 8 to 11 month.

The difference between the two studies comes from the fact that Nakamura and Steinsson exclude the price changes associated with temporary sales whereas Bills and Klenow do not. There is not yet wide agreement regarding how one should deal with temporary sales in macroeconomic studies. In my view it is reasonable to regard Bills and Klenow's estimate as a conservative evidence and Nakamura and Steinsson's estimate as a liberal evidence on the nominal price rigidity.

The small estimate of α is due to the presence of asset market friction. Since imperfect risk-sharing due to the friction in asset market generates real rigidity, the model does not need an implausibly large degree of nominal rigidity to explain the persistent aggregate dynamics. The estimated α is much larger when I impose a restriction $\phi = 0$ (i.e. $\omega = 0$). In that case the posterior mean of α is 0.78 which implies the average duration of price contract is more than one year. Figure 3 shows the the posterior distribution of α in those two cases. The figure also shows the posterior distribution of the price duration implied by α. We can see that in the case of imperfect risk-sharing, the support of duration is roughly given by the interval (0.5Q, 5Q). On the other hand the support of duration is roughly (2Q, 8Q) in the case of ideal risk-sharing. For the asset market friction ω, I obtain estimate of 0.22 with standard deviation of 0.09. I view this is a quite small number and is consistent with one of the most conservative estimate based on micro-data (Schulhofer-Wohl, 2007).

In conclusion, the model, even in its basic form, can fit the major U.S. aggregate data while being consistent with the micro-level evidence of nominal price rigidity and risk-sharing friction.

3.3 Implication for Elasticity of Labor Supply

In the case of perfect risk-sharing, where we can think of a fictional representative household, the macro-elasticity and micro-elasticity of labor supply are identical. However, as noted in a previous section, the micro-elasticity should be smaller than the macro-elasticity in the imperfect risk-sharing model.

At the posterior mean, the elasticities are given as

$$\begin{aligned} \tilde{\psi} &= 0.35 \quad \text{(the wage elasticity of individual labor supply)} \\ \psi &= 3.18 \quad \text{(the wage elasticity of aggregate labor supply).} \end{aligned}$$

Therefore the model can reconcile the discrepancy between the estimate of elasticity from micro and macro studies.

3.4 Impulse Responses

In this and the following subsections, I document the quantitative importance of the small friction in risk-sharing found in the previous section. To do so, I assume the model is a true data generating process (DGP) and compare the DGP with a counterfactual model, called "ideal risk-sharing model", in which the households enjoy perfect consumption insurance. The ideal risk-sharing model are exactly same as the DGP, including the driving forces (the exogenous shocks), the characterization of monetary and fiscal policies, and the structural parameters, except there is no transaction cost, $\phi = 0$.

Figure 4 shows the impulse responses of the output gap, inflation, nominal interest rates, hours and real wage to one-standard deviation shocks to the five exogenous variables at the posterior mean. Not surprisingly, every shock has larger and more persistent effects on the output gap whereas inflation is less responsive to the shocks when there is friction in risk-sharing.

3.5 Implication for Business Cycles

The impulse responses shown above suggest that the friction in risk-sharing might have a big implication for the business cycles. The relevant measures for the severity of the business cycles include the volatility and persistence. I use the standard deviation and autocorrelation of output gap as a measure of volatility and persistence of the business cycles.

Figure 5 shows the autocorrelation functions of output gap at the posterior mean, and Table 3 reports the corresponding numerical values. They suggest that the business cycle would be much less persistent if people were insured against the idiosyncratic income shocks perfectly. Table 4 reports the standard deviations. Just like the persistence case, the business cycles would be much less volatile under ideal asset markets. In the ideal world, the volatility would be only 47% of that of the less-ideal world.

This result has a policy implication. Improving financial market institutions in the direction of a more efficient risk-sharing have additional benefit of moderating the business cycles on top of those benefits that have already been established in the existing studies, for example a more efficient long run growth.

4 Conclusion

In this paper, I have shown that an apparently small friction in risk-sharing can amplify the persistence and volatility of business cycles in a nontrivial way. The main mechanism is the idiosyncratic wealth effect on labor supply that is induced when firms change their prices.

An immediate consequence in the equilibrium equations is that the Phillips curve becomes flatter, which in turn have implications on the model-implied duration of price contract. Another consequence in the equilibrium equations is the de-coupling of micro and macro elasticity of labor supply. These two consequences enable the model to fit the aggregate data while being consistent with micro evidence of nominal price rigidity and labor supply elasticity.

I have shown that the asset market friction is having a quantitatively large effect on the business cycles of the United States. If the asset markets were perfect, both the volatility and persistence of the business cycles would be much smaller. This result suggests that improving the financial market institutions may be important in moderating the business cycles. This is a new advantage of better financial markets in *addition* to more efficient economic growth and better insurance against idiosyncratic income risks.

The model has a testable prediction: *other things being equal*, the business cycles of a country with larger financial frictions should be more persistent and volatile. A careful cross-country empirical analysis would be an interesting future research.

Appendix

A Proof

A.1 Proof of Proposition 1

From (6) one can derive the following equation:

$$c_t^R(i) - \frac{\phi}{\sigma+\phi} x_t^R(i) = c_{t+1}^R(i) - \frac{\phi}{\sigma+\phi} x_{t+1}^R(i), \tag{21}$$

which must hold for every time period t and for every state of economy. The equation (21) implies that $c_t^R(i) - \frac{\phi}{\sigma+\phi} x_t^R(i)$ should be some constant. Let

$$c_t^R(i) - \frac{\phi}{\sigma+\phi} x_t^R(i) = z, \tag{22}$$

for some constant z. Then it is necessary that $z = 0$ because $\int_0^1 c_t^R(i)di = \int_0^1 x_t^R(i)di = 0$. Using the relation, $x_t = c_t = y_t$, we can obtain

$$c_t(i) = \frac{\omega}{1+\omega} x_t(i) + \left(1 - \frac{\omega}{1+\omega}\right) y_t, \tag{23}$$

from (22).

A.2 Proof of Proposition 2

Start from the household's intra-temporal first order condition,

$$w_t(i) - p_t = \psi^{-1} h_t(i) + \sigma c_t(i) + \xi_t.$$

Substituting (23) into the equation above gives

$$w_t(i) - p_t = \psi^{-1} h_t(i) + \sigma \left[\frac{\omega}{1+\omega} x_t(i) + \left(1 - \frac{\omega}{1+\omega}\right) y_t \right] + \xi_t$$

Note $x_t(i) = w_t(i) - p_t + h_t(i)$. Plugging this into the equation above and solving for $h_t(i)$ gives the labor supply curve.

A.3 Proof of Proposition 3

Loglinearizing a firm's first order condition (15) gives

$$E_t \sum_{k=0}^{\infty} (\alpha\beta)^k \left[p_t^*(i) - p_{t+k}\right] = E_t \sum_{k=0}^{\infty} (\alpha\beta)^k \left[w_{t+k}(i) - a_{t+k} + s_{t+k} - p_{t+k}\right]$$

$$= E_t \sum_{k=0}^{\infty} (\alpha\beta)^k \left[\psi^{-1} h_{t+k}(i) + \sigma c_{t+k}(i) - a_{t+k} + \xi_{t+k} + s_{t+k}\right]$$

$$= E_t \sum_{k=0}^{\infty} (\alpha\beta)^k \left[\psi^{-1} y_{t+k}(i) + \sigma c_{t+k}(i) - (1+\psi^{-1}) a_{t+k} + \xi_{t+k} + s_{t+k}\right]$$

$$= E_t \sum_{k=0}^{\infty} (\alpha\beta)^k \left[\psi^{-1} y^R_{t+k}(i) + \sigma c^R_{t+k}(i) + (\sigma + \psi^{-1}) y_{t+k} - (1+\psi^{-1}) a_{t+k} + \xi_{t+k} + s_{t+k}\right] \tag{24}$$

The household's intra-temporal first order condition,

$$w_t(i) - p_t = \psi^{-1} h_t(i) + \sigma c_t(i) + \xi_t,$$

can be written as

$$w^R_t(i) = \psi^{-1} h^R_t(i) + \sigma c^R_t(i).$$

Add $h^R_t(i) \left(= y^R_t(i)\right)$ to both sides,

$$\underbrace{w^R_t(i) + h^R_t(i)}_{=x^R_t(i)} = \left(1 + \psi^{-1}\right) y^R_t(i) + \sigma c^R_t(i).$$

From the proposition 1, we have $c^R_t(i) = \frac{\omega}{1+\omega} x^R_t(i)$. Replace $x^R_t(i)$ with $w^R_t(i) + h^R_t(i)$:

$$\begin{aligned} c^R_t(i) &= \frac{\omega}{1+\omega}\left[w^R_t(i) + h^R_t(i)\right] \\ &= \frac{\omega}{1+\omega}\left[\left(1+\psi^{-1}\right) y^R_t(i) + \sigma c^R_t(i)\right] \end{aligned}$$

Solve for $c^R_t(i)$.

$$c^R_t(i) = \gamma y^R_t(i), \tag{25}$$

where

$$\gamma \equiv \frac{\omega}{1+\omega} \frac{1+\psi^{-1}}{1-\sigma\frac{\omega}{1+\omega}}$$

By substituting (25) into (24), I can rewrite the linearized first order condition as

$$\begin{aligned} & E_t \sum_{k=0}^{\infty} (\alpha\beta)^k \left[p^*_t(i) - p_{t+k}\right] \qquad (26) \\ = \quad & E_t \sum_{k=0}^{\infty} (\alpha\beta)^k \left[\left(\psi^{-1} + \sigma\gamma\right) y^R_{t+k}(i) + (\sigma + \psi^{-1}) y_{t+k} - (1+\psi^{-1}) a_{t+k} + \xi_{t+k} + s_{t+k}\right], \end{aligned}$$

Note that linearizing the demand function gives

$$y^R_{t+k}(i) = -\theta\left[p^*_t(i) - p_{t+k}\right] \tag{27}$$

Then substituting (27) into (26) gives

$$\begin{aligned}
&\left(1+\theta\left(\psi^{-1}+\sigma\gamma\right)\right)E_t\sum_{k=0}^{\infty}(\alpha\beta)^k\left[p_t^*(i)-p_{t+k}\right] \qquad (28)\\
=\quad&(\sigma+\psi^{-1})E_t\sum_{k=0}^{\infty}(\alpha\beta)^k\left[\underbrace{y_{t+k}-\left(\frac{1+\psi^{-1}}{\sigma+\psi^{-1}}\right)a_{t+k}+\left(\frac{1}{\sigma+\psi^{-1}}\right)\xi_{t+k}}_{=y_{t+k}-y_{t+k}^E}+\left(\frac{1}{\sigma+\psi^{-1}}\right)s_{t+k}\right].
\end{aligned}$$

It is now a standard procedure to obtain the following Phillips curve from (28):

$$\pi_t=\beta E_t\pi_{t+1}+\kappa\left(y_t-y_t^E\right)+\mu_t,$$

where

$$\kappa=\left\{\frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\right\}\left\{\frac{\sigma+\psi^{-1}}{1+\theta\left(\psi^{-1}+\sigma\gamma\right)}\right\}$$

$$\mu_t=\kappa\left(\sigma+\psi^{-1}\right)^{-1}s_t.$$

B Tables and Figures

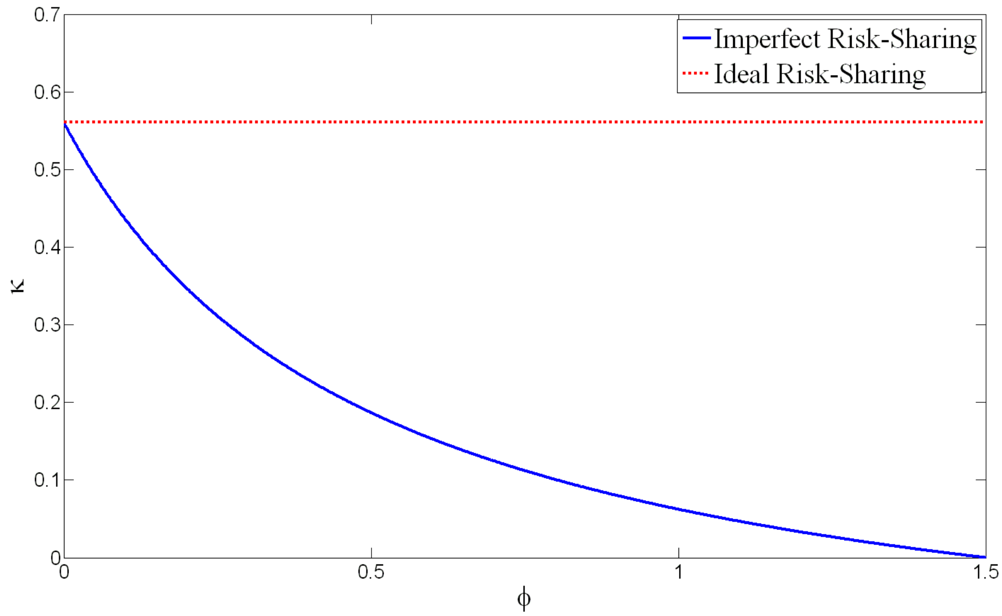

Figure 1: The Slope of Phillips Curve

Table 1. The Slope of Phillips Curve

ϕ	the slope, κ	$\frac{\kappa_{\text{imperfect risk-sharing}}}{\kappa_{\text{ideal risk-sharing}}} \times 100$ (%)
0	0.5611	100%
0.3	0.2806	50%
0.6	0.1530	27.3%
0.9	0.0802	14.3%
1.2	0.0330	5.9%
1.5	0	0%

Table 2. Priors and Posteriors

	Prior	Prior Mode	Prior Mean (Std)	Posterior Mean (Std)	95% HPD
σ	G	2.67	3 (1)	3.67 (0.77)	[2.75, 4.70]
ψ	G	2.67	3 (1)	3.18 (0.96)	[2.04, 4.47]
ρ_r	B	0.67	0.8 (0.2)	0.82 (0.02)	[0.80, 0.85]
ρ_γ	B	0.67	0.8 (0.2)	0.94 (0.015)	[0.92, 0.96]
ρ_ξ	B	0.67	0.8 (0.2)	0.996 (0.002)	[0.994, 0.998]
ρ_a	B	0.67	0.8 (0.2)	0.92 (0.02)	[0.89, 0.95]
σ_γ	IG	1	2 (2)	2.19 (0.56)	[1.60, 2.89]
σ_ξ	IG	1	2 (2)	3.15 (0.60)	[2.44, 3.96]
σ_a	IG	1	2 (2)	0.65 (0.05)	[0.58, 0.71]
σ_m	IG	0.125	0.25 (0.25)	0.14 (0.012)	[0.13, 0.16]
σ_μ	IG	1	2 (2)	0.32 (0.03)	[0.28, 0.36]
ϕ_y	G	0.045	0.125 (0.1)	0.06 (0.02)	[0.0315, 0.0889]
ϕ_π	G	1.47	1.5 (0.2)	2.03 (0.19)	[1.80, 2.28]
α	B	0.5	0.5 (0.15)	0.57 (0.11)	[0.41, 0.70]
ω	IG	0.14	0.2 (0.1)	0..22 (0.09)	[0.12, 0.34]

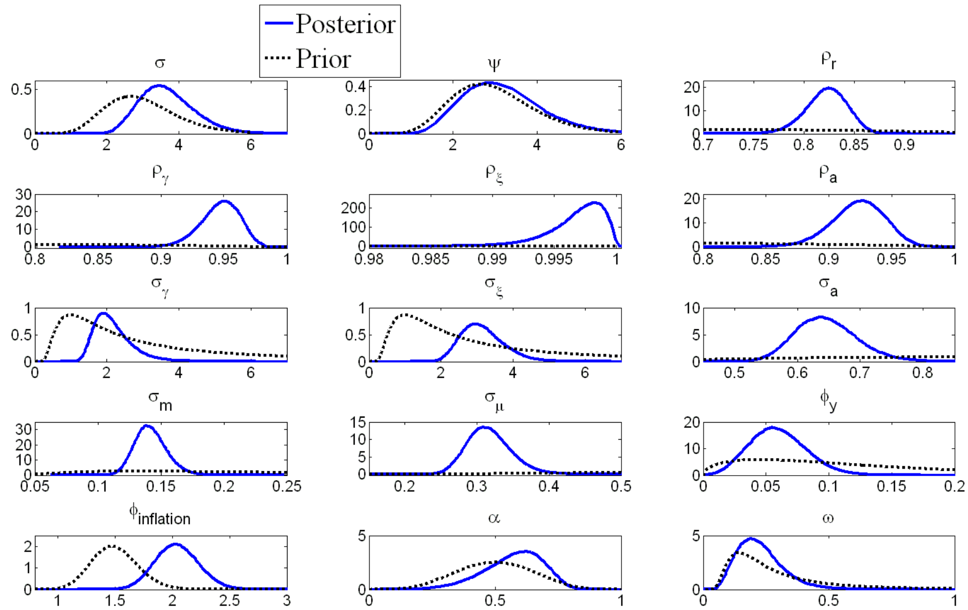

Figure 2: The Prior and Posterior Distributions

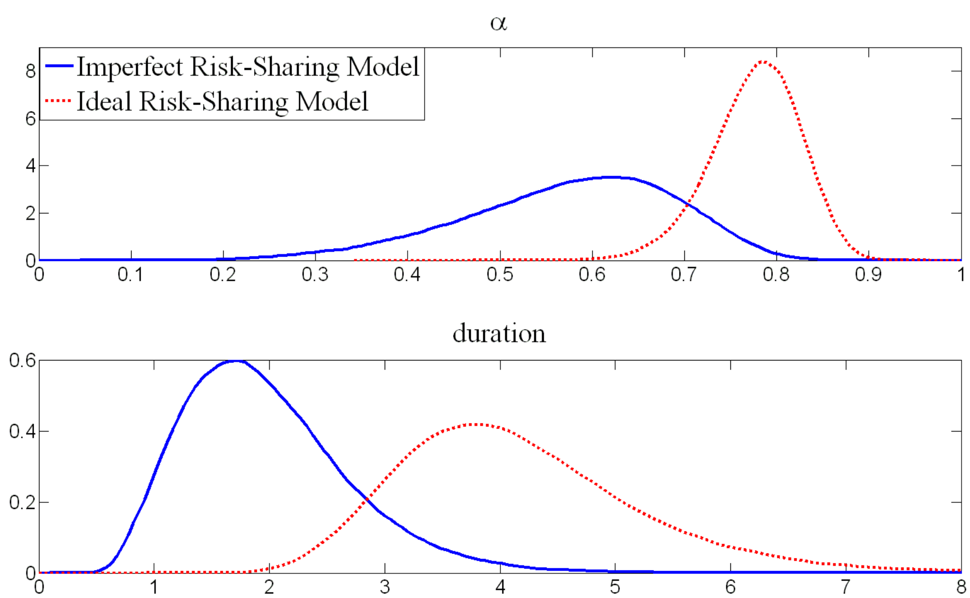

Figure 3: In-frequency and Duration

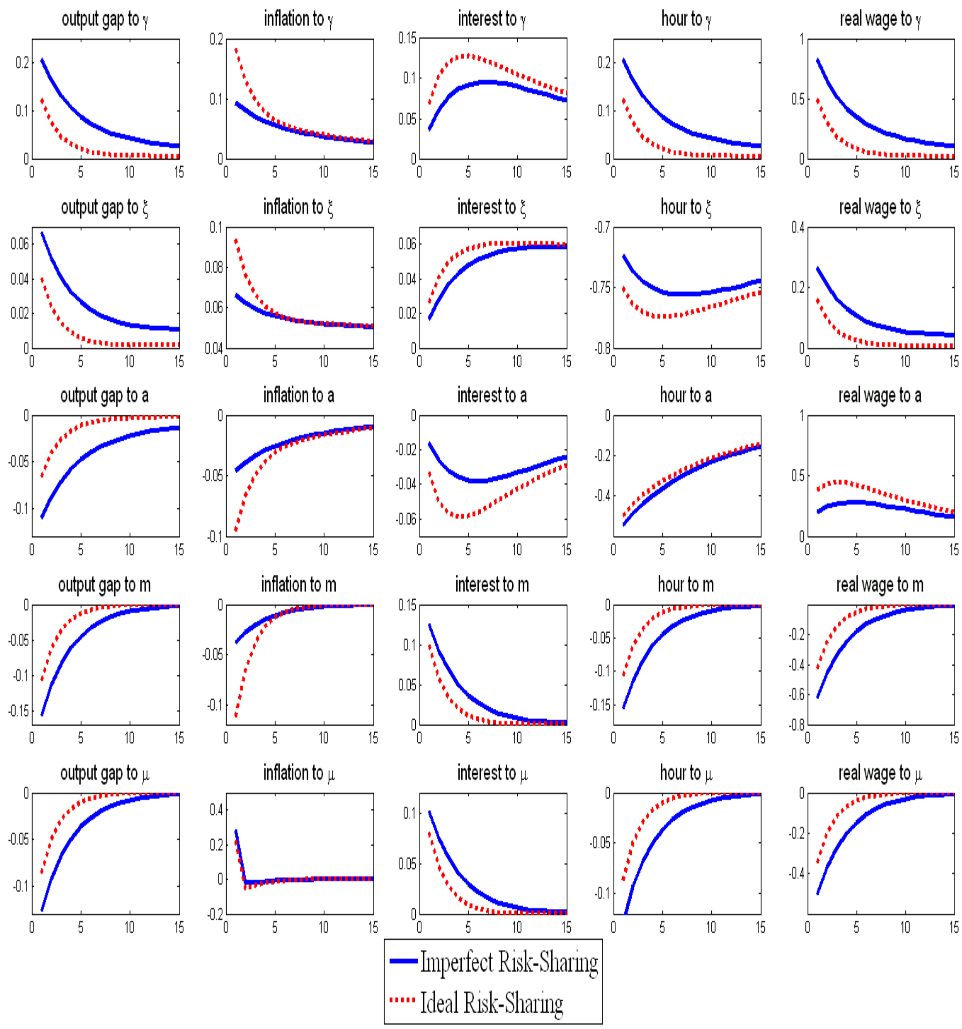

Figure 4: Impulse Response Functions at Posterior Means

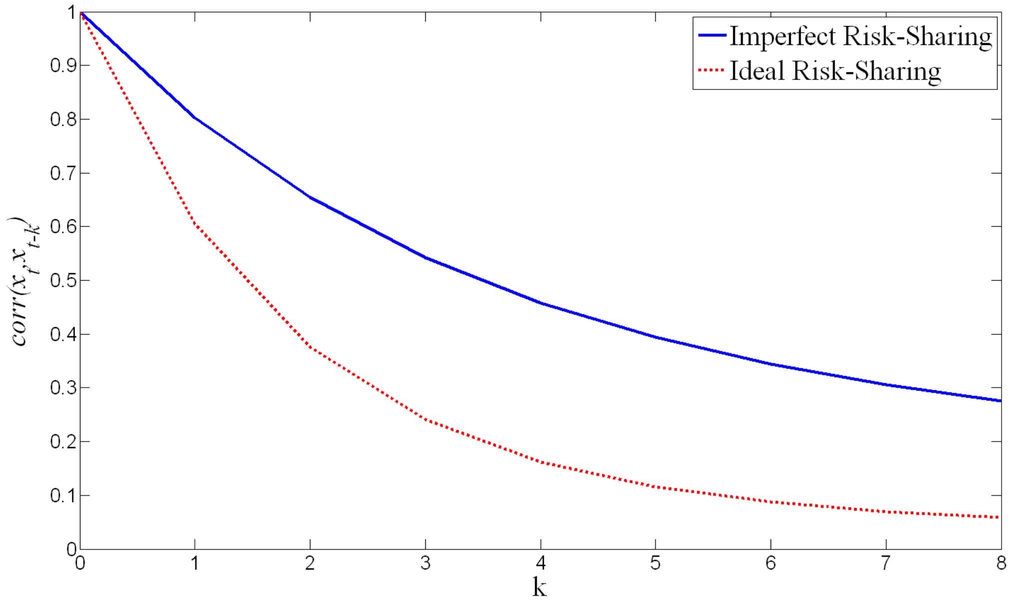

Figure 5: Persistence of Output Gap: Autocorrelation Functions

Table 3. Persistence

	Imperfect Risk-Sharing	Ideal Risk-Sharing
$\text{corr}(\tilde{y}_t, \tilde{y}_{t-1})$	0.8021	0.6056
$\text{corr}(\tilde{y}_t, \tilde{y}_{t-2})$	0.6450	0.3758
$\text{corr}(\tilde{y}_t, \tilde{y}_{t-3})$	0.5426	0.2415
$\text{corr}(\tilde{y}_t, \tilde{y}_{t-4})$	0.4582	0.1624
$\text{corr}(\tilde{y}_t, \tilde{y}_{t-5})$	0.3938	0.1155
$\text{corr}(\tilde{y}_t, \tilde{y}_{t-6})$	0.3442	0.0873
$\text{corr}(\tilde{y}_t, \tilde{y}_{t-7})$	0.3056	0.0700
$\text{corr}(\tilde{y}_t, \tilde{y}_{t-8})$	0.2752	0.0589

note: $\tilde{y}_t$ denotes the output gap, the deviation of output from its efficient level, $\tilde{y}_t \equiv y_t - y_t^E$.

Table 4. Volatility

Imperfect Risk-Sharing	Ideal Risk-Sharing	The Ratio
$\text{std}(\tilde{y}_t) = 0.5336$	$\text{std}(\tilde{y}_t) = 0.2515$	$\frac{0.2515}{0.5336} \times 100 = 47.13\%$

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