Firm Operating and Financial Responses to a Financial Crisis: "Maximising Value" while "Managing for Cash" During a Credit Crunch*

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Abstract

I develop a model for how heterogeneous firms within an industry respond to a financial shock that temporarily raises the cost of external finance, relative to internally generated funds. The model incorporates three elements that lead to substantial variation across firms in policy response to the shock and affect the time path of aggregate outcomes: endogenous entry, heterogeneity in firm productivity, and gradual scaling up of firm capital stock. I find that these elements lead to firm policies that have off-setting effects on the initial aggregate impact of the crisis, such as the drop in output and investment, but reinforcing effects to slow down the subsequent recovery.

JEL Codes: L11, L25, O16

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1 Introduction

Financial crises have occurred in rich and poor countries, recently and in the past (Reinhart and Rogoff (2008, 2009)). Often the financial crisis is part of a broader crisis, for instance, twin-crises of banking and balance of payments (Kaminsky and Reinhart (1998, 1999)). Also, some crises have been more broad and severe, whereas other crises more narrowly a financial crisis. Considering the effect on firms of a crisis, the empirical patterns suggest substantially heterogeneous effects across firms, even within industries. The impact of the recent financial crisis on firm investment in R&D and capital, and employment differs sharply across firms depending on the extent to which these firms face constraints in accessing external finance (Campello, Graham and Harvey (2009), Almeida, Murillo, Laranjeira and Weisbenner (2009)). Looking further back in time, during the Great Depression within industries the change in labor hiring and investment differed substantially across firms, and there was ongoing entry and exit (Bresnahan and Raff (1991), Harber (1992)). Also, financial policies ranged from some firms accumulating large financial resources to other firms struggling to access external funding (Hunter (1982)). In addition, within the banking sector the effect of a recession is all but uniform across banks (Kashyap and Stein (2000) and Ivashina and Scharfstein (2008)). Consequently, underlying the aggregate statistics there is much within industry richness in firm response to an economic crisis, indicating that how firms adjust to a crisis is likely to vary substantially across firms, including firm operational and financial policies as well as exit and entry.

In this paper I develop a model that incorporates the heterogeneous response of firms within an industry to a financial crisis. As my focus is on how policies vary across firms and over time I model a simple financial shock that substantially increases the cost wedge to access external finance, with a gradual return back to pre-crisis levels. This approach to modeling financial frictions is of value, as in my case, when the focus is less on changes in capital structure or particular financial instruments, but more on the link between financial conditions and firm operating decisions (e.g., Froot, Scharfstein and Stein (1993), Almeida, Campello and Weisbach (2006)).[1] The simple approach of introducing a cost wedge to access external finance is convenient as this is sufficient for

[1] In contrast, introducing alternative sources of external finance requires specification of the financial frictions for each source, for instance for each of debt and equity (e.g., Albuquerque and Hopenhayn (2004), Clementi and Hopenhayn (2002), Cooley and Quadrini (2001)). Underlying each of these frictions are a range of issues highlighted by theoretical models with financial frictions, for instance, weak external investor rights (e.g., as in Hart and Moore (1994)) or information asymmetry (e.g., as in Stiglitz and Weiss (1981)). In addition, there may be additional variation depending on the institutional context considered, as, for instance, bankruptcy law, which varies substantially across countries (La Porta, Lopez-de-Silanes, Shleifer and Vishny (1998)).

Modigliani and Miller (1958) conditions not to hold. Consequently, in the model firms set policy to maximize the value of the firm taking account of the links between financial and operational policies, as policies that may be set independently with no financial frictions need to be considered jointly in the presence of financial frictions.

The model has three main characteristics: firm policies include operational and financial polices, as well as exit and entry; firms are within an industry, in competition with a heterogeneous set of firms; and the industry is embedded in a general equilibrium set-up.

Firm policy choices are over investment, labor hiring, use of external finance, and whether to exit. There is also ongoing potential entry. Consequently, in the model firm policy response may vary across firms not only in the extent of reaction (e.g., reduction of investment) but also in the mix of policies changed to adapt to the financial shock (e.g., reduction in investment versus labor hiring). Also, firm policy choice is inter-temporally linked as taken with a forward-looking perspective, and hence firm initial policy response to the crisis reflects the future evolution of the industry due to the shock. This set-up is in line with recent literature emphasizing the simultaneous consideration of external financing, liquidity, and investment policies (Almeida, Campello and Weisbach (2004)) building on earlier literature on the link between financial constraints and investment (Fazzari, Hubbard and Petersen (1988), Kaplan and Zingales (1997)). Also, the importance of considering the firm's polices in a dynamic setting have been highlighted, as firms policies reflect future expectations of investment opportunities and costs to access external finance (Titman and Tsyplakov (2007), and Almeida, Campello and Weisbach (2006)). This literature has generally focused on firms during 'normal' times, considering either a single firm or sets of firms that do not have competitive or general equilibrium linkages across firms. In contrast, as I consider periods of financial crisis, I do include competitive links across firms and also general equilibrium effects.

I develop a model of heterogeneous firms within an industry, in particular building on models such as of Hopenhayn (1992) and Melitz (2003). Hence, the response of firms to the crisis is also dependent on the response of other firms. Within the industry I focus on the effect of two aspects of firm heterogeneity. One aspect is the variation across firms in productivity, which is important as leading to selection effects. A second aspect is the speed at which firms may scale up capital stock, at least relative to the duration of the financial shock, which is important as determining how fast entrants grow to full scale. Each of these aspects affects separately and in combination the size distribution of firms, which is significant in a financial crisis as there is substantial theoretical and empirical literature emphasizing how firm's external financial dependence reduces with firm size and thus financial frictions most affect small firms. Theoretically, credit constraints are able to explain the within-industry patterns of how firm growth and exit vary with firm age and size, as well as the evolution of firm financial structure (e.g., Cooley and Quadrini (2001), Albuquerque and Hopenhayn (2004), Clementi and Hopenhayn (2002)). Limited access to external finance constrains the growth of firms that would prefer to grow at a rate requiring more financial resources than the cash flow generated by the firm's current activities. In the theoretical models firms that grow to a sufficient size typically cease to have dependence on external finance, as the funds generated from current internal activities are sufficient to fund future activities. Also, there is empirical evidence that is consistent with smaller, younger firms having their growth limited by poor access to external finance (Beck, Demirgüç-Kunt and Maksimovic (2005), Beck, Demirgüç-Kunt, Laeven and Levine (2008) and Cabral and Mata (2003)), and estimates suggest substantial costs to access external finance, with higher costs for smaller firms (Hennessy and Whited (2007)).

I embed the model of an industry with heterogeneous firms within a simple general equilibrium framework. This is important as financial crises are major economic events, with substantial feedback from micro firm level adjustments to macro level aggregate outcomes. Consequently, a major area of focus has been the link between financial crises and more general economic slowdowns, recessions and depressions. In particular, the role of the credit channel in exacerbating recessions (Bernanke and Gertler (1989), Bernanke and Gertler (1995)). The credit channel operates through the differential effect across firms of the restriction of credit by financial intermediaries due to a more general economic slowdown. Also, recessions have a marked effect on firm entry and exit (Caballero and Hammour (2005)), suggesting an important role for entry and exit in the adjustment to a financial crisis. Further, an empirical baseline pattern is that within industries there is typically ongoing entry and exit (Dunne, Roberts, and Samuelson (1988), Bartelsman, Scarpetta, and Schivardi (2003)).

The model captures the firm dynamics and, by aggregation across firms, the industry evolution over the course of the crisis: the immediate effect at the onset of the crisis, the subsequent periods during the crisis, and the gradual return to pre-crisis conditions. Thus the model includes both short-run responses and long-run transitions. Throughout the crisis firm policies are consistent with competing firm policies, and thus the aggregate industry equilibrium, both within a time period and inter-temporally. The main contribution of the model is the consideration of the equilibrium transition path in response to a financial shock, starting from a pre-shock initial stationary equilibrium. I focus on the transition through a financial shock with eventual return to pre-crisis conditions, though the model enables consideration along any arbitrary path of costs to access external finance. The transition dynamics are affected by firms factoring into current policy decisions the future evolution of cost of access to external finance and the current and future policy decisions of other firms. I rely on numerical methods to solve for these equilibria, developing a general computational algorithm that can be used to solve a wide set of related dynamic industry evolution models.[2] However, the algorithm is computationally intensive and the discussion of results is necessarily limited to comparing and contrasting selected scenarios.

In the numerical solutions I start with a model with very limited firm heterogeneity: productivity is the same for all firms, and firms may very rapidly scale up capital stock from entry to full scale. Consequently, the firm size distribution is almost entirely comprised of large firms. In response to the shock entry initially falls thus reducing competitive pressure on incumbents, in part alleviating the effect of the crisis. As the effects of the crisis subsist entry re-starts, with the new entrants rapidly achieving full scale. Overall this leads to a sharp initial drop in aggregate output with a rapid recovery that includes some overshooting. I then consider the effect of introducing gradual scaling up of firm size and heterogeneity in productivity separately and then jointly. With a more gradual scaling up of firm size the initial drop in output is less severe; however the recovery is slower as once entry resumes the entrants only gradually achieve full scale. With heterogeneity in productivity there is initially a sharper decline in output, as there is additional endogenous exit, and recovery is slower, as the productivity of each cohort of entrants improves over time due to selection effects.

Consequently, in combination, a gradual scaling up of firms and heterogeneity in productivity have off-setting effects on the initial decline in aggregate output and investment, but together slow the aggregate recovery in aggregate output and investment. Underlying these aggregate patterns, the firm policies vary over time, such as entry and exit, and across firms at a point in time. For instance,at the onset of the crisis, amongst firms with relatively low capital stock, the higher the productivity the less the cut-back in investment to grow capital stock and the more the cut-back in hiring of labor. In summary, the numerical solutions illustrate how consideration of firm

[2] These methods have also been concurrently used to study the effects of credit constraints on industrial evolution and the effect of trade opening on industrial evolution (Costantini (2006) and Costantini and Melitz (2007)). Similar methods applied to a continuous innovation decision in a general equilibrium setting have also recently been developed by Atkeson and Burstein (2006). The computational methods I use in the current paper apply to a monopolistically competitive sector with a large number of competing firms (where the mass of firms evolves endogenously). Hence, these methods are radically different from the seminal contribution to the computation of such equilibria with a small number of firms under oligopoly in Pakes and McGuire (1994), following the development of the theoretical version of the model in Erikson and Pakes (1995).

heterogeneity affects the characterization of firm policy response to a financial shock as well as the time path of aggregate outcomes.

In the rest of the paper I first describe the model set up and then the equilibrium conditions. Then I present the numerical solutions, first describing the calibration for the numerical solutions, next the properties of the initial stationary state, and then the transition in response to the financial shock. Finally, I have a concluding section, and an appendix with a detailed description of the numerical algorithm.

2 Model Setup

As highlighted above, I develop the model to analyze the evolution of an industry comprised of differentiated firms in response to a shock to the cost of external finance within a general equilibrium set up. As my main focus is on response within the firms and across firms, I keep the general equilibrium part relatively straightforward, basing the set up on Bernanke, Gertler and Gilchrist (1999) and Ghironi and Melitz (2005) . The differentiated firms are embedded in a general equilibrium setting that also comprises households, a final good sector, and a financial sector. Households consume a final good, provide labor to firms and save by providing funds to a financial sector. The final good sector aggregates the output of the differentiated goods firms. The financial sector takes the household savings and uses these funds to provide external financing to firms. For simplicity I do not include a government sector. An overview of the links across the households and sectors is in Figure 1.

I next describe in detail each part of the model, the equilibrium, and how I calibrate the model to generate the numerical solutions.

Households

I assume each of the L households provides a unit of labor inelastically in each period at the nominal wage p_t^w, that I normalize to unity. Households share identical preferences and maximize utility of consumption over an infinite horizon:

$$\sum_{k=0}^{\infty} \beta^k \ln(C_{t+k}^i) \tag{1}$$

where $\beta \in (0,1)$ is the subjective discount rate factor and C_t^i is consumption of household i at time t. As I do not have aggregate uncertainty there is no expectations operator. I keep to this logarithmic utility function, with unitary inter-temporal elasticity of substitution, as I aim to keep the general equilibrium set-up as straightforward as necessary.[3]

The individual household budget constraint is given by:

$$C_t^i = (p_t^w + R_t S_t^i - S_{t+1}^i)/P_t = (1 + R_t S_t^i - S_{t+1}^i)/P_t \tag{2}$$

as wages are normalized to unity, and where P_t is the price of the final good, S_t^i are the savings provided by the household to the financial sector at the start of the period, R_t is the gross return on these savings, and S_{t+1}^i are the savings for next period.

The household chooses C_t^i and S_{t+1}^i to maximize (1) subject to (2). Solving yields the first order Euler conditions:

$$\frac{1}{C_t^i} = \frac{\beta R_{t+1} P_t}{P_{t+1} C_{t+1}^i} \tag{3}$$

As all households are identical, aggregate consumption $\bar{C}_t = LC_t^i$ and aggregate saving $\bar{S}_t = LS_t^i$, with aggregate household budget constraint:

$$\bar{C}_t = (L + R_t \bar{S}_t - \bar{S}_{t+1})/P_t \tag{4}$$

Final goods sector

The firms in the sector purchase a C.E.S. aggregate of the differentiated goods as an input to produce the final good. I assume there is a continuum of varieties of differentiated goods $\omega_t \in \Omega_t$. The aggregate of the differentiated goods is $Q_t \equiv \left[\int_{\omega_t \in \Omega_t} q_t(\omega_t)^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)}$ at time t, where $q_t(\omega)$ is the quantity of each of the differentiated goods ω_t, and where $\sigma > 1$ is the elasticity of substitution across goods. The C.E.S. price index for the aggregate of the differentiated goods is $P_t = \left[\int_{\omega \in \Omega} p_t(\omega)^{1-\sigma}\right]^{1/(1-\sigma)}$ where $p_t(\omega)$ is the price of each differentiated good. Hence, total revenues of the differentiated goods producers are $D_t = Q_t P_t$.

For simplicity I assume the final goods sector does not require any other inputs other than the differentiated goods. I also assume the final goods sector is competitive. Consequently, the final good sector output $\bar{Y}_t = Q_t$ and the price of the final good is P_t. The final good may not be stored and is used by the households in consumption, $\bar{C}_t$, and, as explained below, by the differentiated good producers for investment $\bar{I}_t$ and by the financial sector for the activities related to the external

[3] The use of the logarithmin utility is in line with, for instance, Grossman and Helpman (1997), Bernanke, Gertler and Gilchrist (1999) and Akcigit (2009). The simple labor market set up is similar to that of Ghironi and Melitz (2005).

financial cost wedge $\bar{W}_t$, and thus:

$$\bar{Y}_t = \bar{C}_t + \bar{I}_t + \bar{W}_t \tag{5}$$

Differentiated goods sector

The firms in the differentiated goods sector each produce a distinct variety of good. The assumption is for a continuum of competing firms, with firms j at time t distinguished by its productivity v_t^j, capital stock z_t^j, and cash stock c_t^j. Hereon I do not show the superscript j to ease notation.

Within each period, the firm's policy choices are operational and financial. The firm's operational choices are the quantity of labor to hire for production and the amount of investment to increase capital stock. The firm's financial policies are the net flow of funds to or from the financial sector.

The timing of events within a time period are as follows, as illustrated in Figure 2. At the start of each period new entrants pay a sunk cost of entry and thereon are indistinguishable from incumbent firms surviving from the prior period with the same productivity, capital stock and cash. Each firm then decides whether to continue or exit. The value of continuation depends on the firm's policy choices. At the start of the period the firms decide the operational choices of labor to hire for production and investment to grow capital stock. The revenues from production are received at the end of the period. The firm's financial policies comprise choice of net flow of funds to the financial sector, with positive flows indicating an outflow from the firm and negative flows an inflow to the firm (and hence firms with negative flows are dependent on external finance). A net inflow of external finance results in a corresponding additional external finance cost wedge incurred by the firm. The choices of net financial flow, labor and investment determine the evolution of the firm's stock of cash. Finally, firm uncertainty is resolved, in terms of investment outcome and productivity outcome, and firms enter the next period.

There are two main sources of heterogeneity that I subsequently focus on. One is the variation in capital stock levels across firms. I assume there is no instantaneous scaling up of firm capital stock and so firms take several periods to grow capital stock. With ongoing entry of firms with limited initial capital stock, gradual growth in capital stock leads to firms with a range of capital stock at any given point in time. Subsequently, I consider the effect of changing the assumption of how fast firms may scale up capital stock. A second source of firm heterogeneity is due to firm productivity. I assume this depends on the productivity firms have upon entry and the subsequent stochastic evolution of firm productivity. Later I consider the effect of varying the range of firm productivity at entry and subsequent evolution.

Production

Firms produce with a Cobb-Douglas technology, with weights α_v on productivity, α_z on capital and $(1 - \alpha_v - \alpha_z)$ on labor, along with an overhead per-period fixed cost l^F (measured in labor units). At the start of each period, the firm takes its productivity and capital stock as given when optimizing the quantity of labor hired for production labor, $l_t(v_t, z_t, c_t)$.[4] The firm's cash does not directly affect production, as firm output is due to the combination of labor, capital and productivity.[5] However, cash may affect labor choice indirectly. Cash affects the firm's need for external finance. If the firm is externally financially dependent, the full marginal cost of labor includes the external finance cost wedge. Thus at the start of the period the firm incurs costs of $(l_t(v_t, z_t, c_t) + l^F)w_t = (l_t(v_t, z_t, c_t) + l^F)$, as wages are normalized to unity.

As firms face a residual demand curve with constant elasticity σ, firms set price $p_t(v_t, z_t, c_t)$ at a markup $\sigma/(\sigma - 1)$ over marginal cost. At the end of the period the firm receives revenues of: $r(v_t, z_t, c_t) = D_t P_t^{\sigma-1} p_t(v_t, z_t, c_t)^{1-\sigma}$. Consequently, the per-period gross profit from production, $\pi_t(v_t, z_t, c_t)$, is:

$$\pi_t = r_t - l_t - l^F \tag{6}$$

Evolution of Capital stock

One aspect of firm heterogeneity that I focus on is the distribution of firms over capital stock generated by the gradual growth of firm capital stock from entry to full scale. For simplicity I assume there is a maximum growth at which a firm may accumulate capital stock each period: in effect, a growth rate above which adjustment costs make investment prohibitive. Thus changes in the maximum growth rate alter how long it takes entrants to reach full scale and, consequently, due to ongoing entry and exit, the distribution of firms over capital stock. In essence, the assumption on maximal growth may be considered as the limit a firm may increase scale concurrently with the financial shock, within each time period. As discussed below, over several time periods firms may

[4] The relative timing of capital and labor decisions follows Olley and Pakes (1996). I do not include adjustment costs for labor for simplicity, whereas I do have adjustment costs for capital stock. This is in line with Bloom (2009) which, in the context of uncertainty shocks, highlights the importance of including capital adjustment costs more so than labor adjustment costs. In addition, the fixed labor cost does provide a persistent component to firm labor hiring, though common across all firms.

[5] This is a simplification as, for instance, assuming that wages are not influenced by the financial condition of the firm and thus by firm stock of cash. More generally, financial distress could affect the firm's transactions with suppliers and customers. For simplicity I do not include these effects.

invest repeatedly so as to grow capital stock beyond the limits imposed within a time period.

During a time period, the firm's capital stock z_t depreciates by a factor δ_z and increases due to firm investment. At the start of each period the firm decides how much to invest, $I_t(v_t, z_t, c_t)$ with a consequent increase in capital stock is $z_t^*(I_t)$, which is below the maximum growth rate of capital stock. Also, there is a stochastic multiplicative shock, ε_z, to reflect the uncertainty in firm investment outcomes.[6] Consequently, next period capital stock is given by:

$$z_{t+1} = (1 - \delta_z)z_t + z_t^*(I_t)(1 + \varepsilon_z) \tag{7}$$

Below the maximum growth rate, I assume that the costs per unit of investment increase with the amount invested in each time period, due to adjustment costs that increase with the scale of investment, such as time-compression diseconomies of scale, and that the costs per unit of investment increase with current level of capital stock, so as to introduce an aspect of diminishing returns to investment.[7] Specifically, I assume that the cost of investment is $P_t I_t$, with $I_t(v_t, z_t, c_t)$:

$$I_t = P_t \gamma_C (z_t^* - (1 - \delta_z)z_t) \exp\{\gamma_I (z_t^* - (1 - \delta_z)z_t) + \gamma_z z_t)\} \tag{8}$$

where γ_C is the number of consumption units per unit of capital stock, γ_I affects the cost based on level of investment, and γ_z affects the cost based on current stock level.

Evolution of Productivity

A second aspect of firm heterogeneity I consider is productivity. At any point in time there is a distribution of productivity across firms that depends on the distribution of entrant's initial productivity and the subsequent evolution of productivity for each entrant cohort. Consequently, to vary the extent of productivity differences across firms I change both the entrant productivity distribution and the ongoing evolution of productivity. For incumbents, I assume the firm's productivity evolves stochastically in each time period with a known martingale process, ε_v, so that $v_{t+1} = v_t(1 + \varepsilon_v)$.

[6] The capital stock of the firm is used in production. For a given amount of investment spending there is generally some uncertainty over the realized increase in effective capital stock (e.g. due to cost-over runs and unforseen events). In addition, introducing this stochastic process generates smoother firm distributions across capital stock in the numerical solutions.

[7] This is in line with the approach taken by Atkeson and Burstein (2006). Also, Bloom (2009) highlights the importance of non-convex adjustment costs for capital in response to uncertainty shocks. I have partially irreversible capital stock, in that depreciation is gradual and liquidation, at exit, only leads to partial recovery of capital stock.

Productivity differences if present at entry and/or from the stochastic evolution of productivity generate endogenous exit, as firms with sufficiently poor outcomes for productivity (and sufficiently low capital stock and cash) will choose to exit. In addition, firms exit due an exogenous death-inducing shock with probability δ, thus generating exogenous exit as irrespective of the firm's capital stock and cash holding.[8]

Evolution of Cash

The firm stock of cash varies during a period with the financial flows, and across periods depending on firm policy choices and stochastic outcomes. In terms of timing of cash flows, I assume that the firm, say at the end of period t, has cash c_t and receives revenues r_t due to period t production. This total of internal funds, $c_t + r_t$, is then used at the start of period $t+1$ to fund l_{t+1} and l^F labor costs and $P_{t+1}I_{t+1}$ investment cost. Also, at the start of the period the firm chooses the net external finance flow f_{t+1}. The net external finance flow, if negative, has an associated w_{t+1} external finance cost wedge, with the specification for the external finance cost wedge explained in a subsequent section. The equation linking the period t cash to the period $t+1$ cash is as follows:

$$c_t + r_t = c_{t+1} + l_{t+1} + l^F + P_{t+1}I_{t+1} + f_{t+1} + P_{t+1}w_{t+1} \tag{9}$$

The firm is constrained to have non-negative cash, $c_t \geq 0$. Negative cash is avoided through access to external finance. I assume firms do not earn any interest on cash balances. In particular, I do not allow firms to invest cash holdings so as to achieve the same returns as household savings, so as to avoid firms accumulating cash balances just for investment purposes. Consequently, cash stock is valuable to the firm only in so far as to reduce external financial dependence, and thus the the external finance cost wedge, in current and/or future periods.

Value Functions and Firm Policy Decisions

I next discuss how the firm policy decisions are embedded in the firm value functions. In addition to the operational and financial policies discussed above, firms decide whether to continue in the industry or exit. This is based on the maximization of firm value $V_t(v_t, z_t, c_t)$, comparing the value

[8] The death shock could be considered as one part of the stochastic process for the evolution of productivity: a probability of a sufficiently low productivity draw to induce exit. For clarity I keep this separate, as I maintain the death shock throughout while I do vary the entrant productivity distribution and the evolution of productivity.

of continuing, $V_t^C(v_t, z_t, c_t)$, to the value of exit $V_t^L(v_t, z_t, c_t)$:

$$V_t = \max\left[V_t^C, V_t^L\right]. \tag{10}$$

The value of exit is $V_t^L(v_t, z_t, c_t) = \alpha_L P_t z_t + c_{t-1} + r_{t-1}$, with α_L a parameter for the proportion of capital sold when the firm liquidates (with the remaining $1 - \alpha_L$ proportion of capital scrapped). The subscript $(t-1)$ on cash and revenues reflect that exit occurs at the start of the period, before the labor, investment and external finance policy choices are taken. At that point the cash holdings of the firm are prior period cash plus prior period revenues, from the left hand side of (9).

Continuing firms maximize their value by optimally choosing labor, investment and net external finance. Firms discount next period profits at the rate R_t. The firm policy choices must satisfy the Bellman equation:

$$\begin{aligned} V_t^C(v_t, z_t, c_t) &= \max_{l_t,\, I_t,\, f_t} \left\{ \begin{array}{c} +f_t(v_t, z_t, c_t) \\ +\frac{1}{R_t}\int_{v',z'} V_{t+1}(v', z', c_{t+1}) dG\left[v' \mid v_t\right] dG\left[z' \mid z_t, I_t\right] \end{array} \right\} \\ & \quad s.t. \\ c_{t-1} + r_{t-1} &= c_t + l_t + l^F + P_t I_t + f_t + P_t w_t \end{aligned} \tag{11}$$

With this set up, the firm policy choices are interdependent. The cost wedge for external finance incurred by the firm depends on whether the combined effect of the firm policy choices results in the need for external finance. Also, if the combined set of policies results in the firm being externally financially dependent, then the full marginal cost of labor or investment increases due to inclusion of the external financial cost wedge. In addition there is an inter-temporal link, as current policy choices affect cash for future periods, and hence external financial dependence for future periods. Thus in the presence of financial frictions the firm's operational and financial policies become intertwined, both within each time period and across time periods: hence the complexity of "managing for value" while "managing for cash". Even if a firm were not to have a choice over cash holdings, the choice of labor and investment would not be separable. Thus allowing for cash holdings provides firms an additional margin for adjustment in the face of shocks, in addition to adjustments in labor and investment choices and whether or not to exit. In contrast, if there is no cost wedge for external finance then the above set up simplifies substantially to a standard model with no financing frictions, as discussed below.

Simplification if no cost wedge for external finance

With no financing frictions $w_t(v_t, z_t, c_t, f_t) = 0 \ \forall \ \{t, v, z, c, f\}$, and there is no need for the firm to hold cash as internal and external funding has the same cost, including inter-temporally: hence, I assume zero cash holding. Thus financing policy is just the reflection of labour and investment choices: negative if the firm accesses external financing, positive otherwise. Specifically, if $w_t = 0$ $\forall \ \{t, v, z, c, f\}$ and the firm holds zero cash, (9) simplifies to:

$$r_t = l_{t+1} + l^F + P_{t+1} I_{t+1} + f_{t+1} \tag{12}$$

and consequently the choice of external finance is fully determined by the choice of labor and investment. Hence, (11) simplifies to:

$$\begin{aligned} V_t^C(v_t, z_t, c_t) &= \max_{l_t,\, I_t} \left\{ \begin{array}{c} r_{t-1} - l_t - l^F - P_t I_t \\ +\frac{1}{R_t} \int_{v', z'} V_{t+1}(v', z', c_{t+1}) dG\left[v' \mid v_t\right] dG\left[z' \mid z_t, I_t\right] \end{array} \right\} \qquad (13) \\ &= \max_{l_t} \left\{ r_{t-1} - l_t - l^F \right\} \\ &\quad + \max_{I_t} \left\{ -P_t I_t + \frac{1}{R_t} \int_{v', z'} V_{t+1}(v', z', c_{t+1}) dG\left[v' \mid v_t\right] dG\left[z' \mid z_t, I_t\right] \right\} \\ &= \max_{l_t} \{\pi_t\} + \max_{I_t} \left\{ -P_{t+1} I_{t+1} + \frac{1}{R_t} \int_{v', z'} V_{t+1}(v', z', c_{t+1}) dG\left[v' \mid v_t\right] dG\left[z' \mid z_t, I_t\right] \right\} \end{aligned}$$

Consequently with no external finance cost wedge the policy choices of labor and investment are separable. The choice of labor is conditional just on the firm's capital stock and productivity, with labor choice optimized by maximizing per-period profit π_t. The choice of investment I_t may be considered separately, as not dependent on current profits and hence not on choice of labor. Thus the firm's policy choice problem is much simplified, with two policy choices that may be considered independently, labor and investment, with these choices in turn determining the net external financial flow. Following I return to discussing the set up of the model with cost wedge for external finance.

Entrants

At the start of each period, new entrants can potentially enter the industry. An entrant pays a sunk cost of entry, l^S, in labor units, and then realizes its initial productivity draw from a known invariant distribution $G_E(v, z, c)$, and pays for the value of the initial capital stock and cash. Entry is not otherwise restricted. Thus entrants arrive into the industry with a range of initial productivity, capital stock and cash. Thereon, entrants are indistinguishable from incumbent firms with the same productivity, capital stock and cash. A prospective entrant faces a net value of entry of:

$$V_t^E = \int_{v,z,c} [V_t(v,z,c) - l^S - P_t z - c - P_t w_t^E] dG_E(v,z,c) \tag{14}$$

where w_t^E is the financing wedge, in consumption units, due to the initial funding of $\int_{v,z,c}[l^S + P_t z + c + P_t w_t^E] dG_E(v,z,c)$. When the value of entry is negative entry is unprofitable and there is zero entry; when the value of entry is positive there is entry and, in equilibrium, sufficient entry to drive down the value of entry to zero.

Financial sector

The financial sector each period collects savings from households and provides external finance to firms. The financial sector pays back to the households a gross return R_t at the end of a period. Thus the financial sector may considered from the perspective of households as a mutual fund that comprises a full range of savings instruments that result in an overall gross return of R_t. The benefit of this set up is that household response to a shock is evident in how consumption, savings and gross return vary over time.

From the perspective of the firms, the financial sector provides firms with all their external financing needs and so may be considered to use a mixture of debt, equity and any other financial instrument. I do not model the variety of potential financial instruments. I assume the external finance cost wedge w_t is associated with net external financial flows f_t. I take this approach as my focus is on the link between the firm internal policy choices, in particular the links between the operational and financial policies, in response to changes in the cost of external finance. My approach is in line with Almeida, Campello and Weisbach (2006) in which firms access external finance, without specification of the associated financial instrument, and for this incur a cost wedge to reflect the additional cost of using external versus internal funds. As Almeida, Campello and Weisbach (2006) discuss, this approach may incorporate alternative specific potential explanations for financial frictions.

An alternative approach is to specify a particular financial friction. A variety of reasons may lead to financial frictions, for instance, information asymmetries, as in Stiglitz and Weiss (1981), and/or limited rights of providers of external finance, as in Hart and Moore (1984). For example, a focus on capital structure requires specification of frictions for each source of external finance, such as in Cooley and Quadrini (2001) in which firms access external finance from debt and equity with a different type of financial friction for each of these sources of funds. As I do not focus on capital structure there is limited value in specifying particular financial instruments. Also, if I were to introduce just one financial instrument, say debt, I would nonetheless need to specify firm access (or lack of access) to other financial instruments, such as equity.

In general, the external cost wedge would be dependent on the amount of external finance demanded by a firm and the firm's characteristics. As the cost wedge represents a blend of financial instruments, I do not have a specific financial friction to motivate a functional form for the cost wedge $w_t(v_t, z_t, c_t, f_t)$. The cost wedge represents activities, such as monitoring, required to avoid the underlying financial frictions. Hence, the cost wedge is likely to be lower for firms with higher productivity, higher capital stock, higher cash. However, for simplicity, I assume a simple wedge, which is a constant factor of the inflows of external finance:

$$w_t = -\phi_t \min(f_t, 0) \tag{15}$$

Nonetheless, as the inflows of external finance vary across firms depending on firm characteristics, the effect of the shock is not uniform across firms. I vary ϕ_t over time to generate a shock to the cost of external finance which does not depend on firm characteristics. Aggregating across firms, the total external finance cost wedge is $\bar{W}_t$ in units of final good at a cost of $P_t\bar{W}_t$.

I assume the financial sector is competitive and thus generates no profits. Hence, firms pay for the cost wedge to the financial sector with no markup, as the input for this is just the final good. The cost wedge is only relevant to firms in the differentiated goods sector, as the firms in the final goods sector do not require external finance.

The financial sector is a conduit for the households to own all of the firms: I assume there are no agency or other problems with the financial sector acting on behalf of the households, the owners of the firms, in dealing with the firms. Hence, firms evaluate policy choices based on maximizing firm value using the gross return R_t as the appropriate discount factor, as this is what the households would do if considered as direct owners. For households, the return on the savings reflects the net financial flows received (at the start of the period) plus the change in value of the firms over the course of the period:

$$(R_t - 1)\bar{S}_t = R_t f_t + \bar{V}_{t+1} - \bar{V}_t \tag{16}$$

where $\bar{V}_t$ is the aggregate value of all the firms. The household aggregate savings $\bar{S}_t$ are used to purchase the firms, $\bar{S}_t = \bar{V}_t$, and consequently:

$$\bar{V}_t = f_t + (\frac{1}{R_t})\bar{V}_{t+1} \tag{17}$$

3 Equilibrium

Let $\mu_{v,z,c,t}$ represent the measure function for producing firms over states (v, z, c) in period t. This function summarizes all information on the distribution of producing firms across productivity levels, as well as the total mass of producing firms in state (v, z, c), $M_{v,z,c,t} = \mu_{v,z,c,t}(\Upsilon)$. A dynamic equilibrium is characterized by a time path for the price index $\{P_t\}$, the measure of firms in each state, $\{\mu_{v,z,c,t}\}$, the mass of entrants $\{M_{E,t}\}$, the time path of the gross interest rate $\{R_t\}$, and total output $\{Y_t\}$. Note that a choice of $\{P_t, R_t, Y_t\}$ uniquely determines the time path for $\{V_t^C(v, z, c)\}$ and thus determines all the optimal choices for any firm, given its productivity v, capital stock z and cash c. An equilibrium $\{P_t, R_t, Y_t, \mu_{v,z,c,t}, M_{E,t}\}$ must then satisfy the following conditions:

Firm Value Maximization All firms' choices for exit/continuation, and, if continuing, for labor, investment and net flow of external finance, conditional on v, z and c, must satisfy (10) and (11). In the aggregate, this means that $\mu_{v,z,c,t}$ is entirely determined by $\mu_{v,z,c,t-1}$ and the choices for $\{P_t\}$ and $\{M_{E,t}\}$. Starting with a mass and distribution of firms at time $t-1$, a share δ of firms receive the exogenous death shock. The remaining $(1-\delta)$ share of firms update capital stock and cash, based on choice of labor, investment and net flow of external finance. Firm productivity is updated based on the realization of the productivity shock. To these firms are added the $M_{E,t}$ new entrants, with a distribution determined by $G_E(v, z, c)$. All firms then make their endogenous exit decisions. The remaining firms result in a distribution and mass of firms for every state. In equilibrium this must match the chosen $\mu_{v,z,c,t}$.

Free Entry In equilibrium, the net value of entry V_t^E must be non-positive, since there is an unbounded pool of prospective entrants and entry is not limited beyond the sunk entry cost and cost of initial capital stock and cash. Furthermore, entry must be zero whenever V_t^E is negative.

Aggregate Industry Accounting The mass and distribution of firms over productivity levels (aggregating over states) implies a mass and distribution of prices (applying the profit maximizing markup rule to firm marginal cost). Aggregating these prices into the C.E.S. price index must yield the chosen P_t in every period.

Household Optimization Household choices of consumption and saving need to satisfy (3) and the household budget (4) hold.

Market Clearing Total output needs to match uses, and so satisfy (5). The aggregate investment $\bar{I}_t$ is the investment made by continuing firms plus the initial investment by entrants minus the capital stock remaining from firms that exit.

Stationary equilibrium

A time invariant level of external finance cost $\phi_t = \phi$ leads to a stationary equilibrium with a time invariant price index P, measure of firms $\mu_{v,z,c}$, mass of entrants M_E, gross return R_t, and total output Y_t. In such a stationary equilibrium, entry must be positive since there is always an exogenous component to exit. Thus V_t^E must be zero in this equilibrium. Although an equal mass of firms enter and exit, their distributions over productivity, capital stock and cash will not generally match in equilibrium. This is due to the productivity transition dynamics among incumbent firms and the labor, investment and financing choices of firms. Jointly, these productivity, capital stock and cash transitions, along with the distribution of entrants and exiting firms, lead to a stationary distribution of firms for every state. Also, as there is no aggregate uncertainty the consumption and savings path are constant over time and hence $R_t = \beta$.

Equilibrium during financial shock

The firms are initially in a stationary state in which firms expect the cost of external finance and other parameters to remain stable over time. The firms are then informed of the finance shock, which is a step-change in the cost of external finance with a subsequent gradual return to pre-shock levels. I refer to $t = 1$ as the last period of the stationary state, with firms informed of the shock at the end of period $t = 1$. Once the shock is known there is no aggregate uncertainty and so firms know the future time path of all aggregate variables so as to determine their policies. Although there is no aggregate uncertainty, the dynamic response of firms is complex. The firm's response to the shock reflects both near-term changes in conditions (e.g., increase in cost of external finance) as well as the future, eventual, convergence back towards the pre-shock stationary state. Also, the response of firms is heterogeneous, reflecting both the differences in firm characteristics and the competitive dynamics within the industry, for instance the choice of some firms to exit.

The equilibrium conditions hold throughout the response to the shock. The equilibrium path for the price index $\{P_t\}$, measure of firms $\{\mu_{v,z,v,t}\}$, entrants $\{M_{E,t}\}$, total output $\{Y_t\}$ and gross return $\{R_t\}$ will thus begin at their initial stationary levels until a change in costs of external finance is announced, then follow a transition path through the shock and with gradual return to the prior stationary state levels, and remain constant thereafter. Thus the model is of an industry that transitions through a disruptive shock, during which the dynamics present in the stationary equilibrium change substantially. For instance, during the transition, as opposed to the stationary states, the net value of entry may be negative resulting in periods of zero entry. Also, the gross return may differ from the subjective discount rate β, as the households trade-off consumption and savings choices during the shock. Finally, the time path of the shock and the effects of the shock may differ in timing, as the transition of the aggregate variables back towards the stationary equilibrium typically takes time. A main reason for this is due to the presence of sunk costs that lead to hysteresis effects.

4 Simulated Results

I search for the equilibrium path $\{P_t, R_t, Y_t, \mu_{v,z,c,t}, M_{E,t}\}$ using numerical methods. The appendix provides a description of the algorithm used. In essence: I first compute the values of P, R, Y, $\mu_{v,z,c}$ and M_E in the initial and final stationary equilibria. The algorithm then iterates over candidate equilibrium paths for $\{P_t, R_t, Y_t\}$ and $\{M_{E,t}\}$. The choice for $\{P_t, R_t, Y_t\}$ determines all of the policy choices for any incumbent firm: with $\{P_t, R_t, Y_t\}$ known each firm may optimize (l_t, I_t, f_t) conditional on its characteristics (v_t, z_t, c_t). This is the crucial benefit of abstracting from strategic interactions in the monopolistic competition equilibrium. Since $\mu_{v,z,c}$ in the initial stationary state is known, I can thus compute $\{\mu_{v,z,c,t}\}$ based on those policy choices, and the choice for the number of entrants. In turn, I can then compute a new price index $\{P_t\}$ based on the distribution and mass of firms (which implies a distribution of prices), new total output $\{Y_t\}$ based on consumption, investment and financing choices, and new gross return $\{R_t\}$ based on generated consumption and savings. I iterate until the new path for $\{P_t, R_t, Y_t\}$ matches the prior choices of paths $\{P_t, R_t, Y_t\}$.

The equilibrium, after the shock, gradually converges back towards the pre-crisis stationary state equilibrium. I consider a sufficiently long time path such that by the final period the industry has converged back, arbitrarily close, to the pre-crisis stationary equilibrium.

Calibration

I next describe how I set the parameters of the model to run the model simulations. The model is calibrated to reflect the typical patterns of firm dynamics within industries, firm access to external finance and aggregate effects of crises, in particular: Bartelsman, Scarpetta and Schivardi (2000); Cooley and Quadrini (2001); Olley and Pakes (1996); Hennessy and Whited (2007); Almeida, Campello, Laranjeira and Weisbenner (2009); Calvo, Izquierdo and Talvi (2006) and Kaminsky and Reinhart (1999). The key parameter choices are described in Table 1. Here I highlight selected aspects of the calibration. The grid over time periods, productivity levels, capital stock, and cash holding on which to run the model is set wide enough that grid size is exogenous to any firm decisions.[9] Also, I set the total number of time periods, correspond to one quarter, to 100 (i.e., 25 years) as this is long enough to ensure that by the final period the industry has converged close to the stationary equilibrium corresponding to the final set of parameters.

The cost wedge for external finance I set initially at $\phi_1 = 1.1$ in the stationary state. The shock involves a step-change increase to $\phi_1 = 1.7$, and then a gradual return back to initial levels, with the shock reduced by 1/3 each period: Figure 3. Hence, after one year (four periods) the shock is substantially reduced, to around $\phi_t = 1.2$, and after three years (12 periods) is almost eliminated. This is meant to represent a major financial crisis, though with no lasting effect on the cost of external finance. A severe crisis may include rationing of external finance and substantial disruptions to the liquidity of financial markets: I do not include such effects directly but do set the shock high enough that many firms will seek to substantially reduce or eliminate their external financial dependence. I also consider the resulting total drop in output generated in the model as compared to the fall in output during macroeconomic crises. I subsequently consider the effect of a shock that is smaller or decays faster.

Below I compare four scenarios that differ in the variation across firms in productivity and/or how fast firms may accumulate capital stock. I consider one case in which the productivity is the same for all firms and persistent, and another case in which firms enter with different productivity levels and thereafter firm productivity evolve stochastically. When all firms have the same productivity there is minimal endogenous exit, with exit primarily driven by the death shock (set at 5% per year). In contrast, with heterogeneous productivity there is endogenous exit by firms that

[9] I set the number of grid points to 10x100x12 = 12000; high enough that there are sufficient grid points to reduce any effects from the discreteness of the grid. For instance, a finer grid allows for the exit region to more smoothly adjust over time and for firms to more smoothly vary the amount invested.

enter with a poor productivity draw or that evolve to have productivity below the exit threshold.[10] Overall, in the simulations, entrants enter with an average productivity, capital stock and cash lower than that of incumbent firms. Thus, the simulations replicate the robust empirical findings that recent entrants are on average smaller, and exhibit higher exit rates than incumbent firms.

I also subsequently consider the effect of varying three main parameters to illustrate how the patterns discussed vary across industries. Initially these parameters are set as follows: the weights of the Cobb-Douglas production function on capital stock is $\alpha_z = 1/3$, with weights on productivity $\alpha_v = 1/3$ and labour $\alpha_l = 1/3$; the fixed overhead cost $F = 300$; and the liquidation value of capital stock is $\alpha_L = 50\%$.

Stationary state

In the model the capital stock and productivity potentially varies over the range of grid points: for ease of exposition I will discuss firms in terms of 'low' and 'high' capital stock, corresponding respectively to firms with capital stock typical at entry and at around full scale, the capital stock at which firms choose not to grow further. Also, I will refer to 'high' and 'low' productivity firms: in the scenarios with limited heteregeneity in productivity almost all firms have 'high' productivity.[11]

I first consider the numerical properties of the stationary state in which there are no differences in firm productivity and firms may rapidly increase capital stock. Firm policies differ primarily based on firm capital stock, as almost all firms have high productivity. Firms with low capital stock, such as entrants, set policies so as to increase rapidly capital stock, and receive an inflow of external finance (Table 2, panel (i)). In contrast, high capital stock firms invest to offset depreciation so as to maintain current capital stock levels. The firm size distribution resulting from these policies is almost completely composed of high stock firms (Figure 4). In the stationary equilibrium the entry and exit rates are equal at each point in time. As entrants grow rapidly, most firms at a point in time are high capital stock firms: in essence, this is a scenario with one main type of representative firm.

I next consider a scenario with gradual growth of firm capital stock and no variation in pro-

[10] The exogenous exit due to death shock is set to 5% per year, which is in line with firm level exit rates observed empirically (of around 3-7% per year). Endogenous exit is additional, hence exit rates may be above those observed empirically for firms. However, the firms in the model could be interpreted to be product lines or parts of larger firms operating within the industry considered for which exit rates would typically be higher than for firms.

[11] I set up the productivity parameters so that the average productivity is similar with and without heterogeneity in productivity. Hence, in the scenarios with heterogeneity in productivity, relative to the scenarios with no productivity differences, the average productivity of the 'high' productivity firms is higher as there are also substantial proportion of firms with 'low' productivity.

ductivity across firms or over time. The stationary state policies (Table 2, panel (ii)) and firm size distribution (Figure 5) are similar to that just described. The main difference in policies is that for low capital stock firms as growth is slower their external financial dependence lower (a less negative net flow of external finance). Also, as capital stock accumulation is gradual, the firm size distribution has a higher proportion of low stock firms.

Instead, the scenario with fast growth of firm capital stock and heterogeneity in firm productivity has firm policies differ primarily based on firm productivity (Table 2, panel (iii)). Entrants have a distribution of productivity levels and thus some choose to immediate exit: this endogenous exit results in higher overall exit rates as compared to the scenarios with no heterogeneity in productivity. Amongst firms with low capital stock, firms with higher productivity grow fast with heavy dependence on external finance and, as a consequence, have lower labor hiring.[12] These firms emphasize future profits from investment against current profits from labor hiring. In contrast, low productivity firms emphasize current profits, with higher labor hiring and more moderate investment. In part this reflects the very high exit rates for firms with low productivity and low capital stock, which leads to a greater emphasis on current profits. Over time firm productivity evolves and at higher levels of capital stock the exit boundary is at a lower level of productivity: hence the range of productivity for high capital stock firms is greater (Figure 6). Again, as there is fast growth of capital stock the equilibrium firm size distribution has most firms with high capital stock. Also, the optimal full scale capital stock increases with productivity. Consequently, amongst the high stock firms the higher the productivity the higher the growth. In addition, these more productive firms have greater outflows of external finance and consequently hire close to the optimal amount of labor.

Finally, in the scenario with heterogeneity in productivity and gradual growth in capital stock there is substantial variation across firms in policy choices (Table 2, panel (iv)). The main differences in policies across firms have been highlighted above in the discussion of the other scenarios. The distribution of firms over productivity highlights the selection effect at entry and how subsequent endogenous exit policy choices shift the distribution of firms over productivity (Figure 7, panel (b)). The effect of firm investment choices is evident in the distribution over capital stock of incumbent firms relative to entrants (Figure 7, panel (c)). In contrast to the other scenarios, there are significant proportions of all types of firms in equilibrium, as reflected in the firm size distri-

[12] Specifically, for the low capital stock firms the gap between the labor hired and the labor that would be hired were there no cost wedge to access external finance is larger than the equivalent gap for high capital stock firms.

bution (Figure 7, panel (a)). In particular, large firms have high productivity and capital stock, whereas firms on the margin of exit have low productivity and low capital stock. The intermediate small to mid-size firms comprise a wide mix of firms, from high productivity and low capital stock to the opposite: the policies across these small firms differs widely in the stationary state, and this will also be the case in response to the financial shock, as discussed below.

Consequently, firm policies and the initial firm size distribution change markedly depending on the variation across firms in productivity and the speed at which firms may achieve full scale. The next section highlights how these differences in aspects of firm heterogeneity also affect the response of firms to the financial shock and the evolution of aggregate variables.

Dynamics during financial shock

Next I discuss a sequence of scenarios that highlight the separate effects due to heterogeneity in productivity and gradual growth of capital stock. As in the prior section, I start by considering the scenario in which firms may grow fast and there is no variation in productivity. I then consider the separate effects of gradual growth in capital stock and of heterogeneity in productivity, and then the combined effects. The firm policies in response to the financial shock are in Tables 3 and 4, which shows the change in firm policies relative to policies in the initial stationary state. The policy change is shown for the first year and the average of the second and third years post shock, with a breakdown for firms with high and low productivity and capital stock. The aggregate time path of selected variables for each scenario is in Figures 8, 9, 10 and 11.

Scenario (i) with no heterogeneity in productivity and fast growth in capital stock

A main effect of the financial shock is to reduce entry, which falls to zero for several periods. The rise in external finance cost wedge directly raises the cost of entry, as the initial entry cost is funded by external finance (as in (14)), and hence the value of entry drops to below zero. With no entry and ongoing exit the number of firms in the industry declines and consequently the price index rises (Figure 8, panels (b) and (e)). The rise in price index, though in part offset by the drop in total output[13], and the reduction in the external finance cost wedge contribute to increasing the value of entry. As wages are normalized to unity, the rise in prices leads to a drop in real wages. Once the value of entry rises to be non-negative entry resumes (Figure 8, panels (f) and (g)). The

[13] The value of entry is forward looking and based on the value of continuation, equation (14), and thus dependent on many variables, including the price index, total output and the gross interest rate. One effect of the price index is on firm revenues, as firm revenues depend on $D_t P_t^{\sigma-1} = Y_t P_t^{\sigma}$.

drop in entry to zero is also a consistent feature across the scenarios discussed below, reflecting rise in entry costs due to the financial shock.

Consequently, the impact of the shock on incumbent firms is in part buffered by the decline in competitive pressure due to the drop in entry (Table 3, panel (i), for first year post crisis, focusing on the rows with high productivity as these are the bulk of firms). Nonetheless, the incumbents with low capital stock sharply reduce: investment to grow capital stock, labor hiring relative to optimal levels, and external financial dependence. In contrast, the response of firms with high capital stock, most of the firms, is moderate, as these firms are on average not dependent on external finance: there is a small drop in investment to grow capital stock and labor hiring, and a small increase in outflow of external finance.

Once entry resumes the smaller firms increase growth rates and labor hiring whereas the larger firms cut growth rates. The spike in entry increases competitive pressure and thus reduces the incentive for large firms to invest to grow capital stock (Table 4, panel (i), for second and third years post crisis). In aggregate, the entry spike results in a boom in output and investment, relative to the initial stationary state (and final long run equilibrium, as the same as the initial stationary state): Figure 8, panels (a) and (i).

In terms of aggregate financing, initially firms dependent on external finance sharply reduce inflows of external finance, and consequently the aggregate external finance cost wedge decreases (Figure 8, panels (l) and (p)). Firms that have net outflows of external finance increase their outflows and, overall, cash stock declines early on in response to the crisis (Figure 8, panels (k) and (o)). Labor hiring returns relatively fast to long-run levels, in part reflecting the lack of adjustment costs to labor and as per period labor hiring is largely driven by the concurrent cost of external finance (Figure 8, panel (j)).

Scenario (ii) no heterogeneity in productivity and with gradual growth in capital stock

In this scenario, similar to above, entry stops for several periods and the largest incumbents only moderately change policies. The main difference is with the low capital stock incumbents. At the onset of the crisis, these firms do not materially reduce investment to grow capital stock (Table 3, panel (ii), firms with high productivity and low capital stock). This is because with gradual growth there is a lower benefit to delaying investment to when the shock has subsided, as a smaller proportion of overall investment is earlier in time when the external finance costs wedge is highest. Hence, these firms primarily adjust to the financial shock by reducing labor hiring and reducing external financial dependence.

Scenario (iii) with heterogeneity in productivity and fast growth in capital stock and

In this scenario the heterogeneity in productivity leads to endogenous exit, as highlighted in the discussion of the initial stationary state. Thus the overall entry and exit rate is higher in the initial stationary state, as compared to with no heterogeneity in productivity. In particular, many marginal firms enter and exit each period. The financial shock reduces entry, as discussed above, and also exit rates (Table 3, panel (iii), and Figure 10, panels (c) and (g)).[14] The drop in exit rates is comprised of two off-setting effects. For the firms with low productivity and capital stock the exit rates increase. These firms have low profitability (due to the fixed costs) and heavily depend on external finance. Consequently, the financial shock induces many of these firms to exit. Counterbalancing this effect is the drop in entry, as the main source of weak firms are weak entrants. Hence with no entry the firm size distribution shifts towards a mix with high capital stock and productivity, leading to lower overall exit rates.

Despite the drop in exit rates, the overall exit rate remains higher than if there were no heterogeneity in productivity. This results in a steep drop in number of firms and rise in price index, more so than if there were no heterogeneity in productivity (Figure 10, panels (a), (b) and (e)). Consequently, entry restarts sooner and there is a larger entry spike. The exit and entry patterns affect aggregate productivity that first rises, as exit mostly affects low productivity firms, and then falls, as entrants on average have lower productivity than incumbent firms (Figure 10, panel (n)).

Scenario (iv) with heterogeneity in productivity and gradual growth in capital stock

The combined effect of heterogeneity in productivity and gradual accumulation of capital stock is that across firms there are substantial differences in policy response to the financial shock (Table 3, panel (iv)). Exit is concentrated in the low productivity, low capital stock firms. Continuing firms with low capital stock reduce external financial dependence and: if with higher productivity cut back on labor; whereas if with lower productivity cut back on investment. This difference in policy reflects the trade-off made between investing, which increases future profits, and hiring labor, which increases current profits, with these policy choices jointly set as these firms are externally

[14] The adjustment to the financial shock through an initial decline in both entry and exit, respectively creation and destruction of firms, is similar to the concurrent decline of entry and exit in a recession, as in Caballero and Hammour (1994) and (2005).

financially dependent. The benefit of investment is greater for higher productivity firms, whereas current profits are relatively more valuable for lower productivity firms close to the exit boundary.[15]

Continuing firms with high capital stock increase investment at the onset of the crisis (Table 3, panel (iv)). This is in response to the rise in the price index due to no entry and limited investment by other firms (Figure 11, panels (e) and (i)). Across the high capital stock firms, those with lower productivity cut back on labor whereas the more productive firms increase labor. The substantial rise in the price index increases firm profits. Firms with high capital stock barely change external financial flows, contributing to these firms accumulating substantial cash holdings. Once entry restarts, the high capital stock firms reverse their investment patterns, reducing capital stock, and increase the outflow of external finance, reducing cash balances (Table 4, panel (iv)) In aggregate the entry spike leads to a temporary boom in investment on the path to recovery and hence aggregate capital stock and productivity only gradually return to towards long-run levels (Figure 11, panels (i), (m) and (n)).

Comparing across the four scenarios (Table 5), in aggregate heterogeneity in productivity and the gradual growth in capital stock have off-setting effects on the initial drop in output and reinforcing effects on slowing the recovery of output long-run levels. The heterogeneity in productivity increases exit rates due to increased endogenous exit, accelerating the decline in output. The gradual growth in capital stock reduces the value to firms of waiting to invest once the shock subsides, and for the largest firms the decline in competitive pressure is sufficient to induce initially an increase in investment, softening the decline in output. In contrast, the heterogeneity in productivity and gradual growth in capital stock both slow recovery. The spike in entry at the start of the recovery has a lower average productivity than incumbent firms, as there is ongoing selection based on productivity as entrants grow, and hence average productivity only rises as the wave of entrants reaches full scale. Also, the wave of entrants at the start of the recovery take time to grow to full scale, so there is a small boom in output around the time of the spike in entry but this is not sustained.

Change in size and duration of shock Within this scenario (iv), I next vary the size and duration of the shock to assess the effect on the response of firms: the basic patterns are similar, though the timing and size of peaks and troughs changes. Not surprisingly, a smaller shock substan-

[15] The policy choices of the low productivity low capital stock firms to pursue the relatively less risky policy of hiring of labor (versus investing to grow capital stock, which is partly sunk and riskier due to potential exit), is consistent with Ameida et. al. (2009), in which firms under financial pressure do not risk shift their investments.

tially reduces the peak drop in output and investment (Figure 13, comparing panels (a) and (b)). However, the duration of the initial drop does not materially change, as this is primarily driven by when the external finance costs wedge drops sufficiently to enable entry to resume. Though the initial shock is only half the size, the half-life of the shock is sufficiently high that a given level of cost wedge is reached only one to two periods earlier than with the larger shock. Once entry resumes, the recovery of output and investment are almost immediate as there is a smaller gap to close, although aggregate capital stock recovers more slowly. In contrast, a shock with a shorter duration leads to a much milder initial effect on output and investment and a faster re-start of entry (Figure 13, panels (a) and (c)). As firms set policies in anticipation, knowing that the financial shock is of shorter duration makes this a less material shock for a larger proportion of firms. In this case, the entry spike results in an output boom relative to the long run equilibrium.

Change in selected parameters Next I consider the effect of changes in selected parameters, relative to scenario (iv). As compared to variations in the shock, which only affects the response of the firms to the transition, the effect of a parameter change is more complex as this also leads to a change in the initial stationary state.

A decrease in the weight to capital in the production function to $\alpha_v = 17\%$ (from 33% in scenario (iv)) leads to an initial stationary state with a distribution of firms that is more compressed along the dimension of capital stock, as the optimal capital stock decreases. Consequently, the effect of gradual accumulation of capital stock on the response to the shock is diminished. Also, investment is a less significant part of firm expenditures. Nonetheless, entry costs are largely labor and hence still affected by the increase in external finance cost wedge. Hence, the initial fall in output is similar to scenario (iv) but the recovery after the initial fall in output is faster and includes a small boom (Figure 14, panel (i)).

Next I consider a lower labor fixed costs of $F = 200$ (versus $F = 300$). In the stationary state firm size distribution this leads to more firms at lower levels of productivity, as profitability is less sensitive to scale and thus productivity. The main difference in the response to the financial shock is that investment and output starts to recover sooner, with the peak of the entry occurring sooner, as lower fixed costs facilitate survival of entrants which are on average smaller than incumbents (Figure 14, panel (ii)).

Finally, I consider a lower value of recovery of capital stock at exit, with $\alpha_L = 25\%$ (versus 50%). This has very limited effects on the initial stationary state and the response to the crisis

(Figure 14, panel (iii)). Two reasons for this are that even a major change in value of capital stock at exit has a moderate effect on the value of exit, as at exit the firm also recovers the full value of cash balances: hence, the average recovery on all assets changes by a more modest amount. Also, as there are adjustment costs to investment the replacement cost, which is linked to the value of continuation, is substantially higher than the value realized at liquidation. Consequently, even a major change in liquidation value is modest relative to the replacement value and thus has a small effect on firm policies.

5 Conclusion

I develop a model for how an industry with heterogeneous firms responds to a financial shock that temporarily raises the cost of external finance. The model enables characterization of the variation in policy choices across firms in the industry and, with aggregation within a general equilibrium set up, the time path of aggregate variables. The non-linear dynamics require numerical methods to solve for the equilibria, and I develop a general computational algorithm that can be used to solve a wide set of related dynamic industry evolution models. However, the algorithm is computationally intensive and the discussion of results is necessarily limited to comparing and contrasting selected scenarios. Even though I solve the model numerically, a number of substantive simplifications are necessary, in part to enable the effect of key aspects of the model to be highlighted as well as to ease computational burden. For instance, the treatment of the financial shock as leading to a uniform shock to the cost of external finance is clearly a major simplification. A straightforward extension would be to exogenously vary the shock across firms based on firm characteristics. However, for the most part this would reinforce the patterns discussed. Typically the impact of the financial shock would be considered higher for entrants and smaller firms, which in the current set up are the firms most externally financially dependent and thus already most directly affected. The main reason I set up a simple form for the financial shock is so as to allow for greater potential range of firm operating policies: in future work, greater emphasis could be placed on the external finance cost wedge with a more simple set of firm operating policy choices. On a similar vein, at present the general equilibrium set up is just sufficient to provide general equilibrium feedback and this could be enriched, for instance, the labor market set up. Also, I do not consider policy interventions though the heterogeneity across firms suggests that the response to particular policy interventions could well be highly varied across firms and thus the aggregate effects could depend on the particular design of the policies, such as the scope of the firms covered and the timing the policy intervention.

Within the current set up, I highlight how the effects of endogenous entry, gradual growth of firm capital stock and heterogeneity in firm productivity affects the variation in firm policies in response to the financial shock, across firms and over time. I find that introducing these elements leads to firm policies that have off-setting effects on the initial aggregate impact of the crisis, such as the drop in aggregate output and investment, but reinforcing effects to slow down the subsequent recovery.

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Table 1: Calibration: Model Timing and Productivity Grid

Variable		Empirical evidence*	Explanation of model calibration
Timing			
Duration of time periods	t	Most empirical data is quarterly or annual	Time period corresponds to one quarter. Total number of time periods so that by final period industry has converged close to final stationary state
Discount rate	β		5% per year, thus 1.3% per quarter (i.e., per time period)
State space grid			
Productivity	v	Relative size of largest to smallest firms often over 100x [B et al]	Set v to [0.01, 1] to allow a sufficiently broad range of firm sizes, with 10 grid points
Capital stock	z	Relative size of largest to smallest firms often over 100x [B et al]; Size of 75%ile firm about 30x size of 25%ile firm by assets [ACLW]	Set z to [20, 260] to allow a sufficiently broad range of firm sizes, with 100 grid points
Cash	c		Set c to [200, 2500] to allow sufficiently broad range, with 12 grid points
Normalization			
Wages			Normalize monthly wage to one
Scaling			
Number of workers	L		Choice of L scales size of market: Set L=100,000.
Finance costs			
Cost wedge for external finance	φ	Marginal cost of equity 5%-11% of capital for large versus small publich firms [H and W]; New share premium of 30% used by [C and Q]; Libor jumped from 0.53% to 1.31% from July to August 2007 [ACLW]; Declines in growth of output -4 to -8% typical in a crisis [CIT] [KR]	Set baseline for all firms to φ_f=1.1 pre-crisis. External finance in model includes any instruments, such as debt and equity, and thus premium less than just for equity. Set policy shock to start with no pre-announcement or lag: cost wedge of external finance rising to 1.7 at start of crisis (t=2). Thereafter, costs wedge falls by 1/3 each period back to pre-crisis levels, so after 3 years (12 periods) substantiavely back at pre-crisis levels. Output drop at trough in -5% to -12% range.
Demand			
Elasticity of substitution	σ		Set to 4
Production			
Weights	α_z α_l	Weights around 1/3 for capital, 2/3 labour for telecom equipment industry [O and P]	Set weights in Cobb-Douglas production function as α_v=1/3, α_z=1/3, and α_l=1/3
Labor hiring	l_t		Three choices of quantity of labor to hire, relative to 100% representing labor hired with no financial constraints: 100%, 80%, and 60%
Fixed costs	F		F=300 so that on average fixed labor cost is around 2/3 of total labor cost
Productivity transitions			
Transition for firm productivity			Scenario without heterogeneity has persistent shocks. Scenario with heterogeneity has productivity evolve according to truncated lognormal evolution with mean log(v) and 0.2 standard deviation (hence, mean zero change in productivity), and with truncation of increase/decrease to future productivity to within [0.5x,2x] of current.
Capital stock transition			
Depreciation of capital stock	δ_z	7% per year used by [C and Q], and 4% for buildings and 12% for equipment per year used by [O and P]	Set δ_z at 10% per year, so around 2.6% per quarter
Investment outcome: Growth of capital stock		Variation in investment/PPE for public firms 3% to 10% for 25%ile to 75%ile firm [ACLW]	Firm choice of investment is over a mean change in capital stock. With fast firm growth choice over {-δ,0,2δ,4δ,8δ} per period (i.e., maximum growth about 20% per quarter, 130% per year). With gradual firm growth choice over {-δ,0,δ,2δ,3δ} per period (i.e., maximum growth about 8% per quarter, 35% per year). Realization of increase in capital stock stochastic around mean based on truncated lognormal with 0.1 standard deviation with increase/decrease of capital stock to within [0.9x,1.1x] of mean
Investment cost			The cost of investment factors are γ_c=1 , γl=0.03 and γ_z=0.006
Exit			
Death shock	δ	Exit rate ~3-7% per year [B et al]	Set to 5% per year, about 1% per quarter (i.e., per time period), with additional exit from firm productivity dropping below exit productivity cutoff
Liquidation value of capital	α_L	Bankruptcy costs 8-15% of capital for large versus small publich firms [H and W]	Set α_L=50%, for scrap value of capital stock, with cash on hand recovered 100%: hence, total costs of liquidation below 50% of total firm capital
Entrant size		Entrants smaller than incumbents on average. Also, around 50% of entrants survive to 7 years, with 20% hazard in year 1 and around 10% hazard thereafter [B et al]	Set entrants as distributed with independent draws on productivity and capital stock. Productivity draw lognormal with mean and standard deviation respecitively: log(0.05) and 1 for more heterogeneity, log(0.15) and 0.01 for less heterogeneity. Capital stock draw lognormal, with mean log(25) and std dev=0.1. Cash stock draw lognormal, with mean log(200) and std dev=0.5. This results in entrants with, relative to incumbents, lower average productivity and capital stock, and higher exit rates.
Entry sunk cost	S		Set S=500, which correponds to a quarterly interest charge of 0.6 (i.e., around 2% of per period fixed costs).

*References: [B et al] Bartelsman (2003), [O and P] Olley and Pakes (1996), [C and Q] Cooley and Quadrini (2001), [H and W] Hennessy and Whited (2007), [ACLW] Almeida, Campello, Laranjeira and Weisbenner (2009), [CIT] Calvo, Izquierdo and Talvi (2006), and [KR] Kaminsky and Reinhart (1999)

Table 2: Initial stationary state firm policies

Firm characteristics*		Initial stationary state policies					
Produ-ctivity	Capital stock	% of firms	Entry, annual % rate	Exit, annual % rate	Growth in capital stock, %	Labour hiring**	Net flow of external finance
(i) Fast growth of capital stock and no heterogeneity in productivity							
All	All	100%	5%	5%	8%	99%	163
Low	Low	0%	360%	1%	106%	100%	(911)
High	Low	7%	38%	5%	89%	100%	(617)
Low	High	3%	0%	5%	31%	100%	(208)
High	High	90%	0%	5%	2%	99%	250
(ii) Gradual growth of capital stock and no heterogeneity in productivity							
All	All	100%	5%	5%	7%	99%	184
Low	Low	1%	139%	3%	23%	100%	(522)
High	Low	21%	13%	5%	23%	100%	(191)
Low	High	3%	0%	5%	23%	97%	(68)
High	High	75%	0%	5%	2%	99%	316
(iii) Fast growth of capital stock and with heterogeneity in productivity							
All	All	100%	15%	15%	17%	98%	426
Low	Low	7%	99%	117%	48%	98%	(422)
High	Low	9%	58%	4%	87%	96%	(464)
Low	High	32%	0%	5%	3%	98%	132
High	High	52%	1%	5%	9%	99%	927
(iv) Gradual growth of capital stock and with heterogeneity in productivity							
All	All	100%	15%	15%	11%	98%	443
Low	Low	16%	51%	62%	14%	97%	(208)
High	Low	20%	28%	5%	23%	97%	55
Low	High	23%	0%	5%	3%	99%	211
High	High	41%	1%	5%	9%	98%	1,070

* Firms split into four sets based on productivity and capital stock: each row has average for firms in a set

** Labor % is relative to 100% which is labor hiring if no financial contraints

Table 3: Change in firm policies, relative to initial stationary state, in response to financial shock for first four periods 2 to 5 (year 1)

Firm characteristics*			Policy difference versus initial stationary state				
Productivity	Capital stock	% of firms	Entry, annual % rate	Exit, annual % rate	Growth in capital stock, %	Labour hiring**	Net flow of external finance
(i) Fast growth of capital stock and no heterogeneity in productivity							
All	All	100%	-5.1%	0.0%	-3.7%	-1.0%	51
Low	Low	0%	-359.9%	-0.4%	-0.1%	-16.9%	182
High	Low	6%	-38.0%	-0.1%	-37.1%	-8.9%	156
Low	High	3%	0.0%	0.1%	-18.2%	0.0%	213
High	High	91%	0.0%	0.0%	-0.2%	-0.5%	18
(ii) Gradual growth of capital stock and no heterogeneity in productivity							
All	All	100%	-5.1%	0.0%	-0.8%	-1.3%	52
Low	Low	1%	-139.2%	0.0%	0.0%	-14.1%	194
High	Low	19%	-13.4%	-0.1%	0.0%	-4.1%	23
Low	High	4%	0.0%	0.1%	-1.7%	-7.9%	43
High	High	76%	0.0%	0.0%	-0.4%	-0.2%	39
(iii) Fast growth of capital stock and with heterogeneity in productivity							
All	All	100%	-14.9%	-2.0%	-7.8%	-1.9%	96
Low	Low	7%	-99.4%	2.3%	-43.3%	-1.4%	271
High	Low	7%	-58.1%	-0.4%	-25.5%	-11.6%	261
Low	High	32%	0.0%	-0.1%	-2.2%	-1.6%	19
High	High	54%	-0.9%	0.0%	-0.5%	-1.2%	(6)
(iv) Gradual growth of capital stock and with heterogeneity in productivity							
All	All	100%	-15.1%	-2.8%	-0.9%	-1.8%	70
Low	Low	15%	-50.5%	-9.7%	-5.2%	-3.9%	91
High	Low	19%	-27.6%	-0.1%	-0.1%	-4.9%	105
Low	High	24%	0.0%	0.0%	0.4%	-1.8%	8
High	High	42%	-1.2%	0.0%	0.9%	0.2%	(6)

* Firms split into four sets based on productivity and capital stock: each row has average for firms in a set

** Labor % is relative to 100% which is labor hiring if no financial contraints

Table 4: Change in firm policies, relative to initial stationary state, in response to financial shock for periods 6-13 (years 2 and 3)

Firm characteristics*			Policy difference versus initial stationary state				
Produ-ctivity	Capital stock	% of firms	Entry, annual % rate	Exit, annual % rate	Growth in capital stock, %	Labour hiring**	Net flow of external finance
(i) Fast growth of capital stock and no heterogeneity in productivity							
All	All	100%	4.6%	0.0%	-0.3%	-0.1%	54
Low	Low	1%	0.2%	0.0%	0.0%	-11.2%	41
High	Low	8%	25.5%	0.6%	-9.3%	-1.4%	(1)
Low	High	3%	0.0%	-0.3%	-7.3%	-0.9%	120
High	High	87%	0.0%	0.0%	-1.3%	0.2%	73
(ii) Gradual growth of capital stock and no heterogeneity in productivity							
All	All	100%	4.1%	0.0%	-0.7%	-0.4%	58
Low	Low	2%	43.5%	0.0%	0.0%	-1.9%	(56)
High	Low	19%	13.0%	0.0%	0.0%	-2.2%	12
Low	High	3%	0.0%	-0.1%	-0.4%	-7.6%	26
High	High	75%	0.0%	0.0%	-0.6%	0.4%	68
(iii) Fast growth of capital stock and with heterogeneity in productivity							
All	All	100%	15.2%	-0.7%	-2.5%	0.0%	157
Low	Low	13%	34.3%	-42.6%	-10.9%	-0.1%	28
High	Low	10%	60.4%	0.6%	-6.8%	-0.5%	(76)
Low	High	28%	0.0%	0.0%	-4.8%	0.5%	192
High	High	49%	1.0%	0.0%	-4.7%	-0.2%	297
(iv) Gradual growth of capital stock and with heterogeneity in productivity							
All	All	100%	11.6%	-2.1%	-3.2%	0.2%	197
Low	Low	17%	40.8%	-11.2%	-5.5%	0.6%	14
High	Low	16%	38.0%	0.0%	0.0%	-1.6%	15
Low	High	25%	0.0%	0.0%	-1.7%	-0.6%	151
High	High	42%	0.8%	0.0%	-2.3%	0.9%	288

* Firms split into four sets based on productivity and capital stock: each row has average for firms in a set

** Labor % is relative to 100% which is labor hiring if no financial contraints

Table 5: Summary for selected variables of time path of initial decline and subsequent recovery

Scenario		Decrease in response to financial shock		Subsequent recovery	
Productivity heterogeneity	Capital stock growth	Cumulative decline, relative to initial stationary state	Time period with maximum cumulative decline from initial stationary state	Time period when first return to initial stationary state level	Peak reached during recovery, relative to initial stationary state
(a) Output					
(i) No	Fast	-5%	8	14	2%
(ii) Yes	Fast	-3%	7	13	2%
(iii) No	Gradual	-12%	11	15	4%
(iv) Yes	Gradual	-11%	11	15	0%
(b) Investment					
(i) No	Fast	-25%	10	14	14%
(ii) Yes	Fast	-12%	9	15	5%
(iii) No	Gradual	-42%	11	15	22%
(iv) Yes	Gradual	-28%	12	16	1%
(c) Labor hiring*					
(i) No	Fast	-1.0%	5	15	0.3%
(ii) Yes	Fast	-1.3%	5	19	0.1%
(iii) No	Gradual	-1.9%	5	9	0.4%
(iv) Yes	Gradual	-1.7%	5	8	0.9%
(d) External finance flow into firms					
(i) No	Fast	-68%	8	11	88%
(ii) Yes	Fast	-43%	7	13	16%
(iii) No	Gradual	-79%	8	12	116%
(iv) Yes	Gradual	-63%	7	13	62%

* Labor % is relative to 100% which is labor hiring if no financial contraints

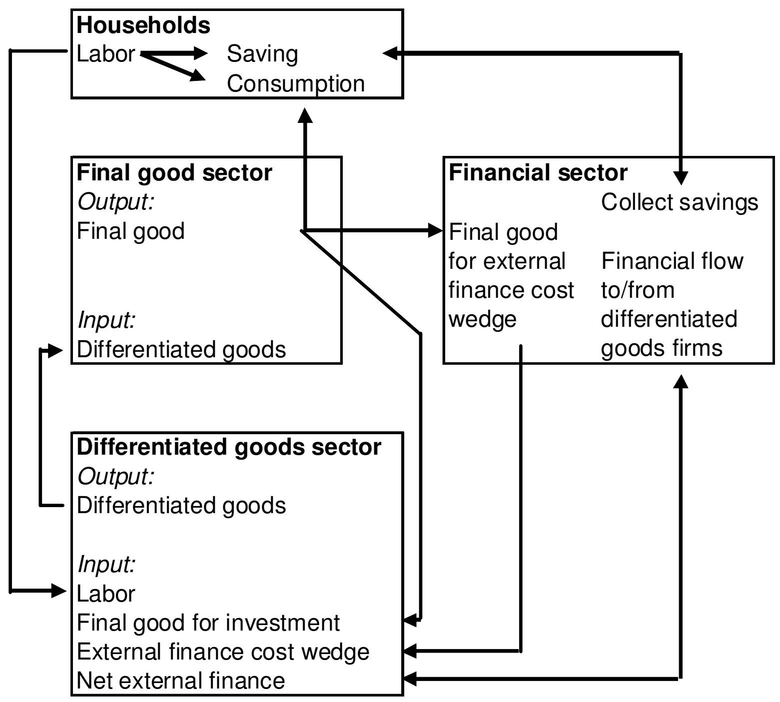

Figure 1: Overview of model set up with links across households and sectors

Event	Description
Entry	Entrants pay sunk cost of entry and then discover their initial productivity, capital stock and cash
Continuation / Exit decision	Entrants and incumbent firms decide if to continue in industry or exit
Operational policy choices	Labor to hire for production Investment to increase capital stock
Financial policy choice	Net external finance
Cash at start of period	Prior period cash plus revenues
Cash flows at start of period	Pay labor cost and investment; receive/pay out net external finance and any corresponding finance cost wedge
Cash flows at end of period	Receive revenues
Cash at end of period	Cash plus revenues
Transition to next period	Uncertainty resolved about transition to next period: death shock for all firms; productivity transition and capital stock transition

Figure 2: Timing and description of events for firms in differentiated goods sector

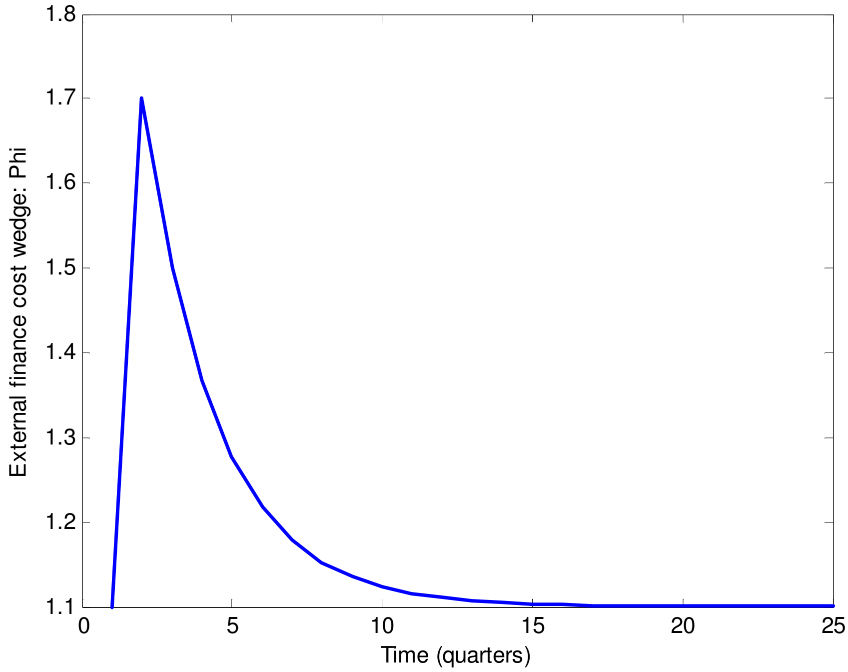

Figure 3: Financial shock: External finance cost wedge ϕ over time

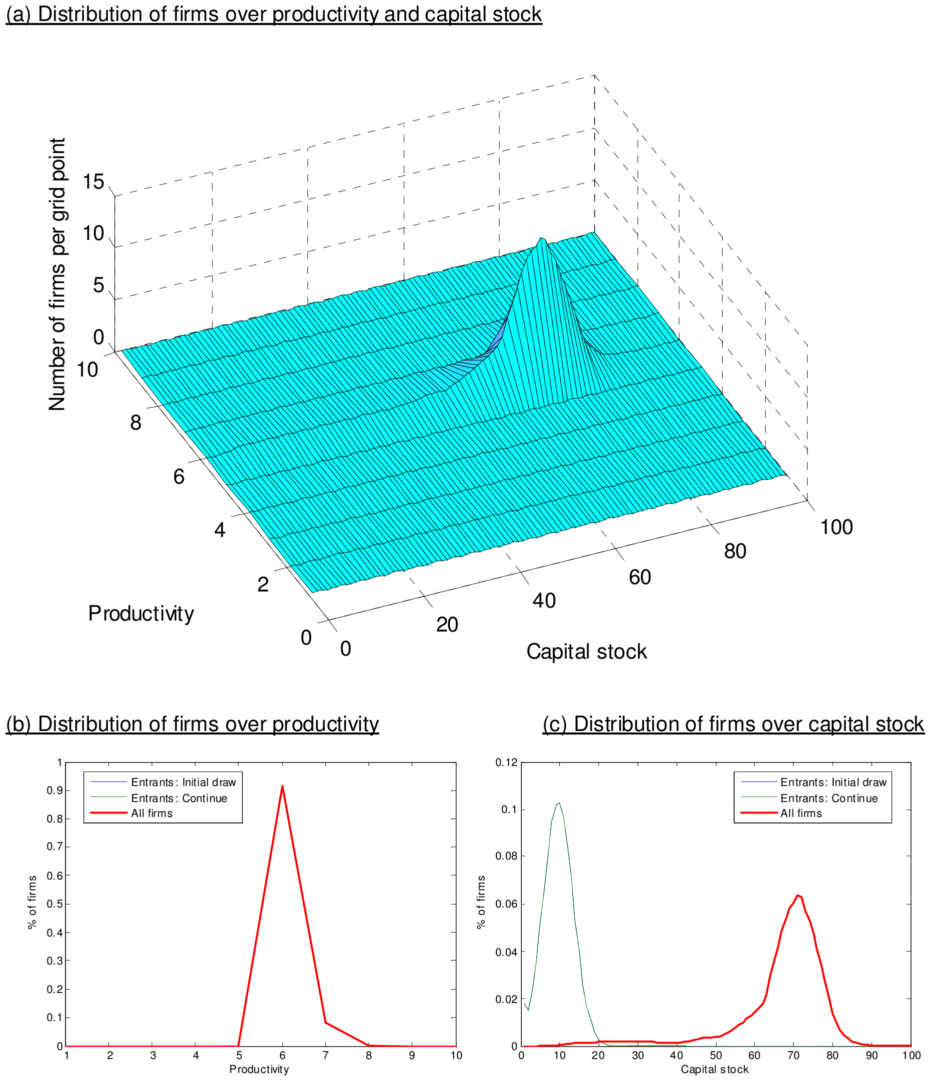

Figure 4: Firm size distribution over productivity and capital stock grid for scenario (i) without heterogeneity in productivity and with fast growth of capital stock

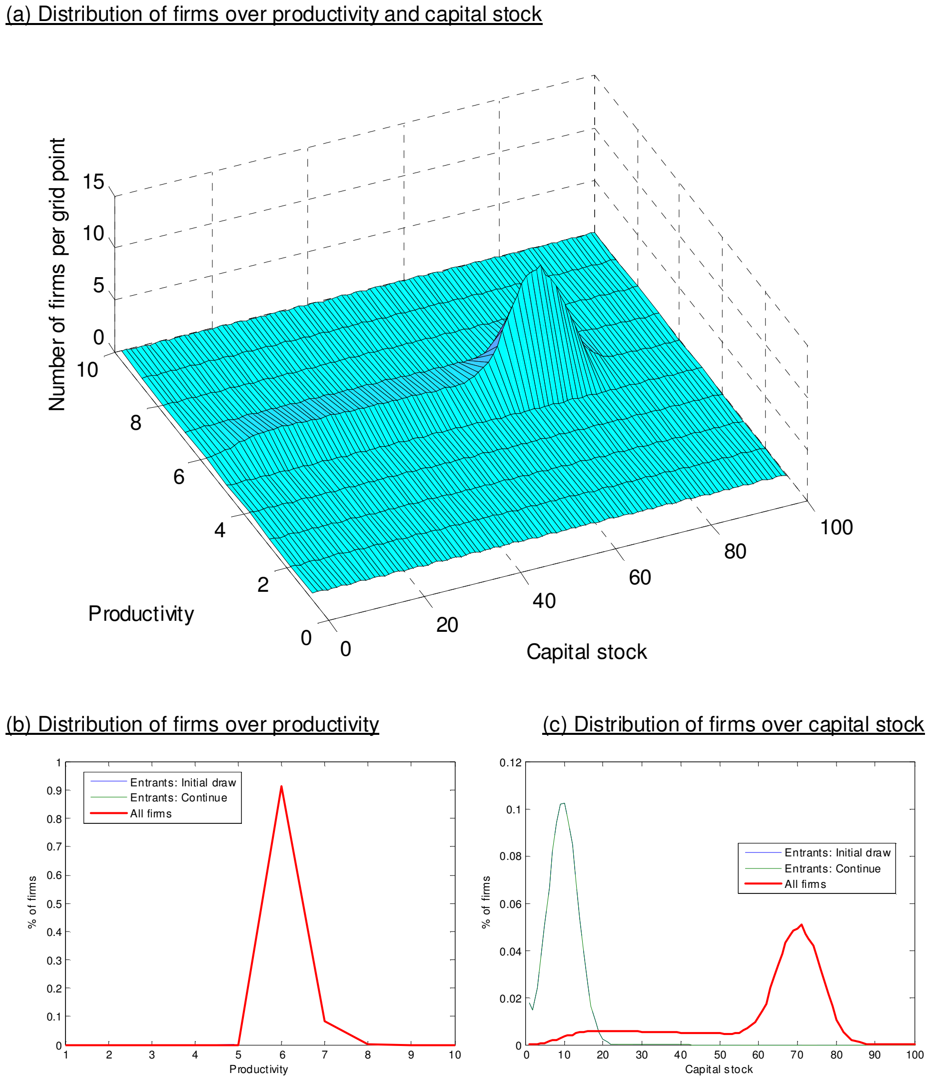

Figure 5: Firm size distribution over productivity and capital stock grid for scenario (ii) without heterogeneity in productivity and with gradual growth of capital stock

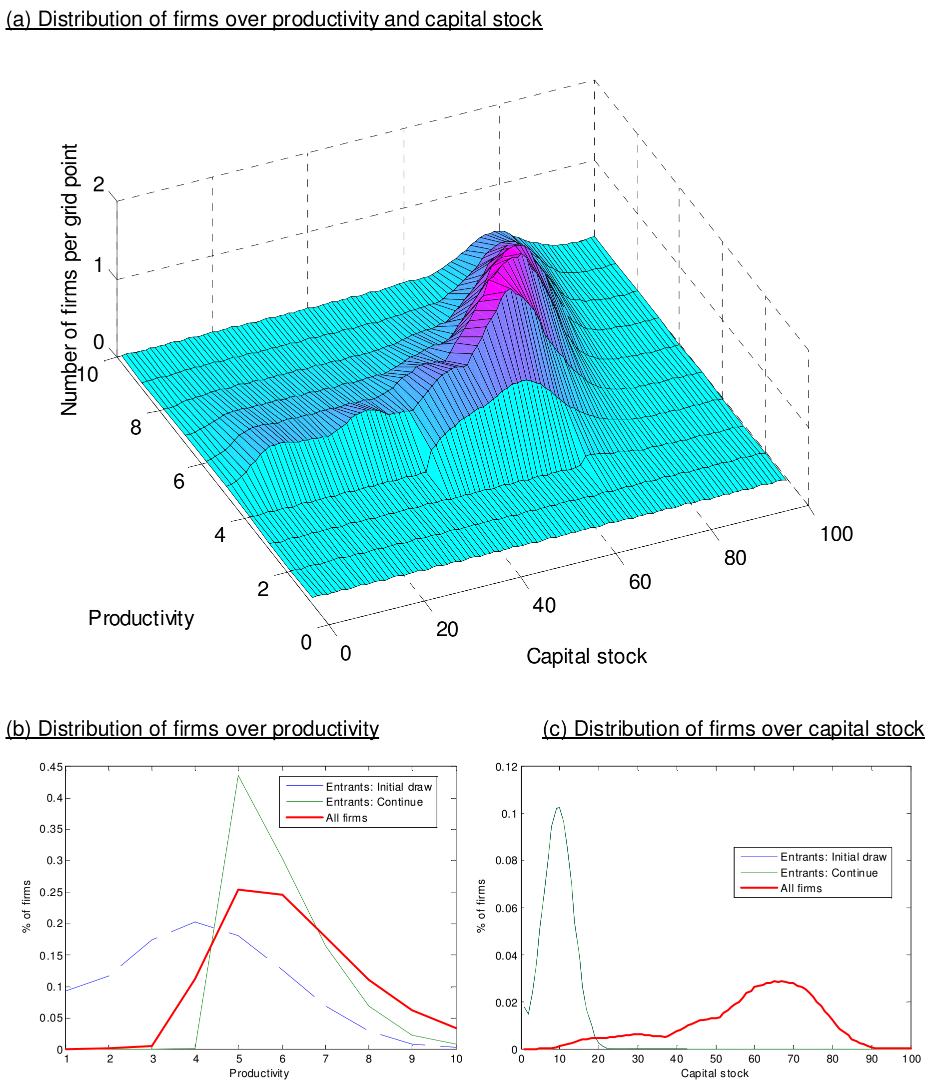

Figure 6: Firm size distribution over productivity and capital stock grid for scenario (iii) with heterogeneity in productivity and with fast growth of capital stock

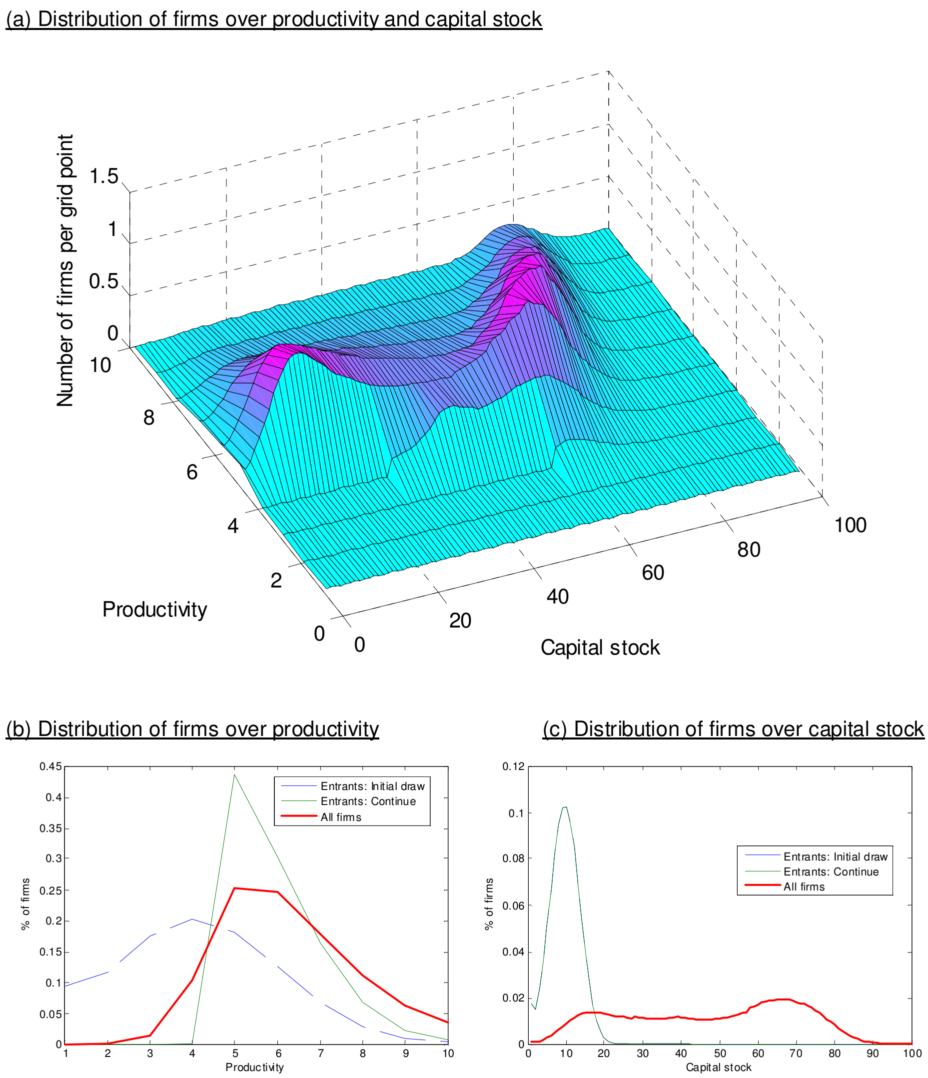

Figure 7: Firm size distribution over productivity and capital stock grid for scenario (iv) with heterogeneity in productivity and with gradual growth of capital stock

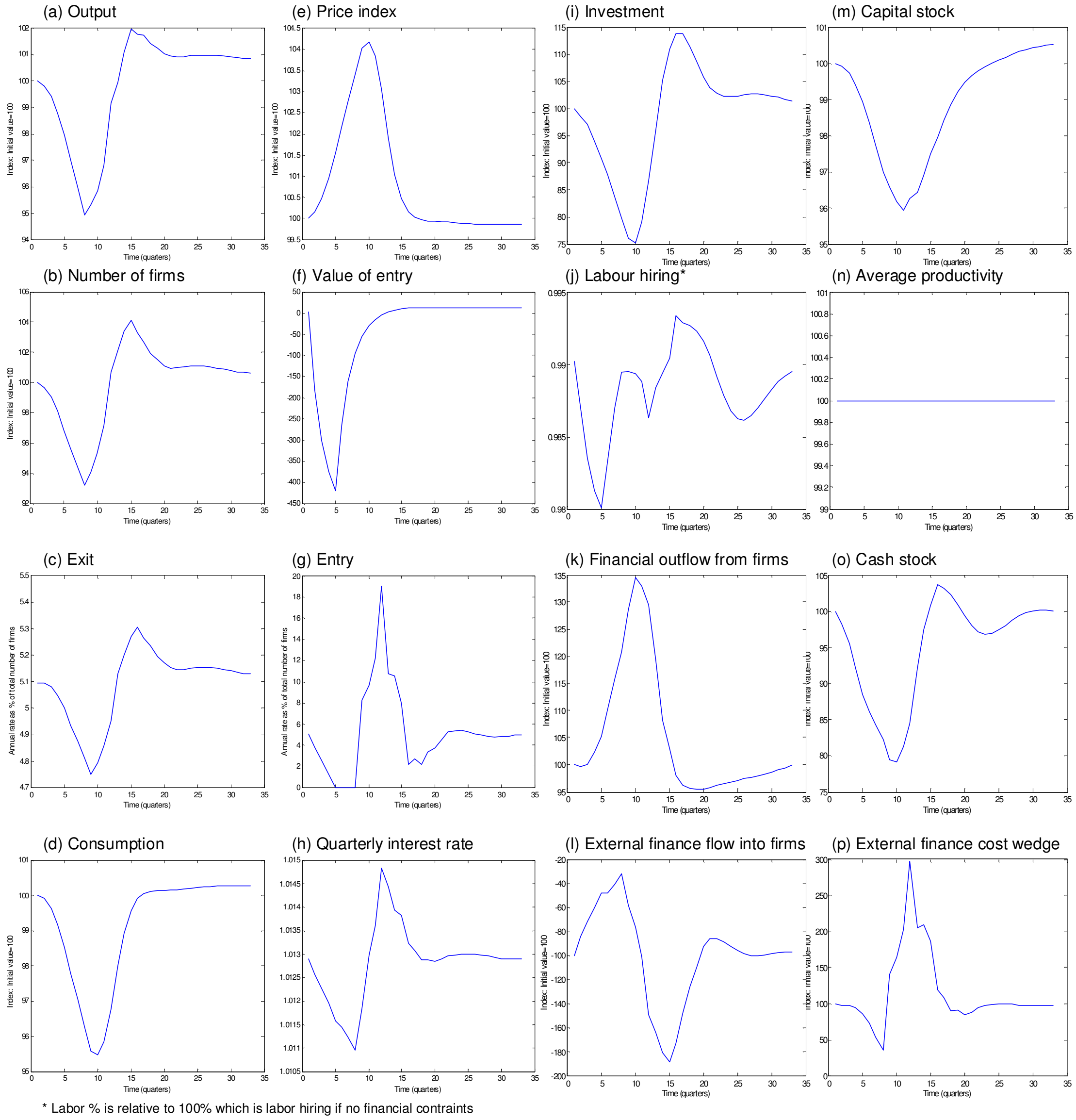

Figure 8: Aggregate response to financial shock for scenario (i) without heterogeneity in productivity and with fast growth of capital stock. Plot are moving annual averages on a quarterly basis. Period 1 corresponds to last period of initial stationary state. Periods 2 to 35 are for response to financial shock (remaining time periods in solution not shown)

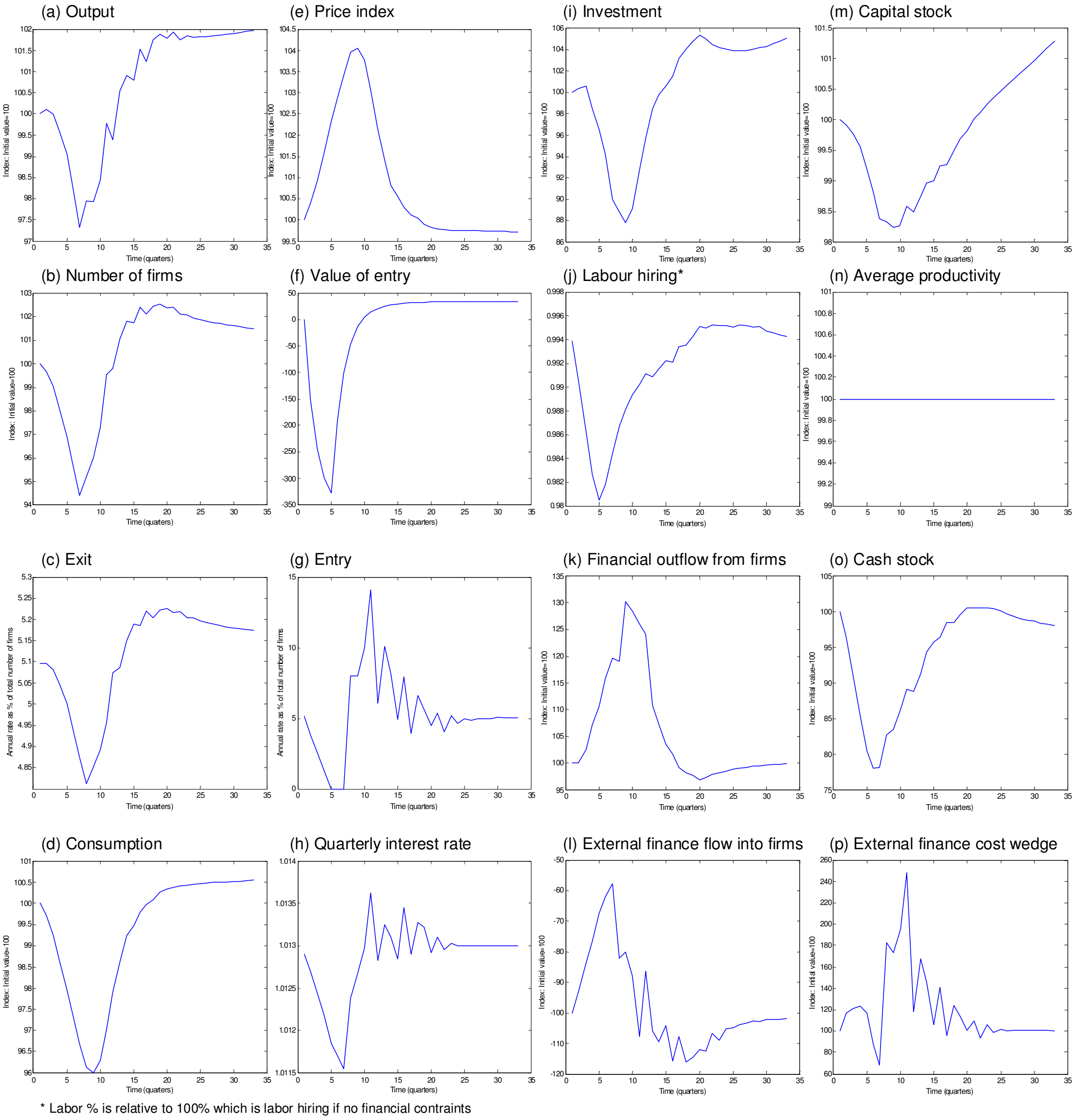

Figure 9: Aggregate response to financial shock for scenario (ii) without heterogeneity in productivity and with gradual growth of capital stock. Plot are moving annual averages on a quarterly basis. Period 1 corresponds to last period of initial stationary state. Periods 2 to 35 are for response to financial shock (remaining time periods in solution not shown)

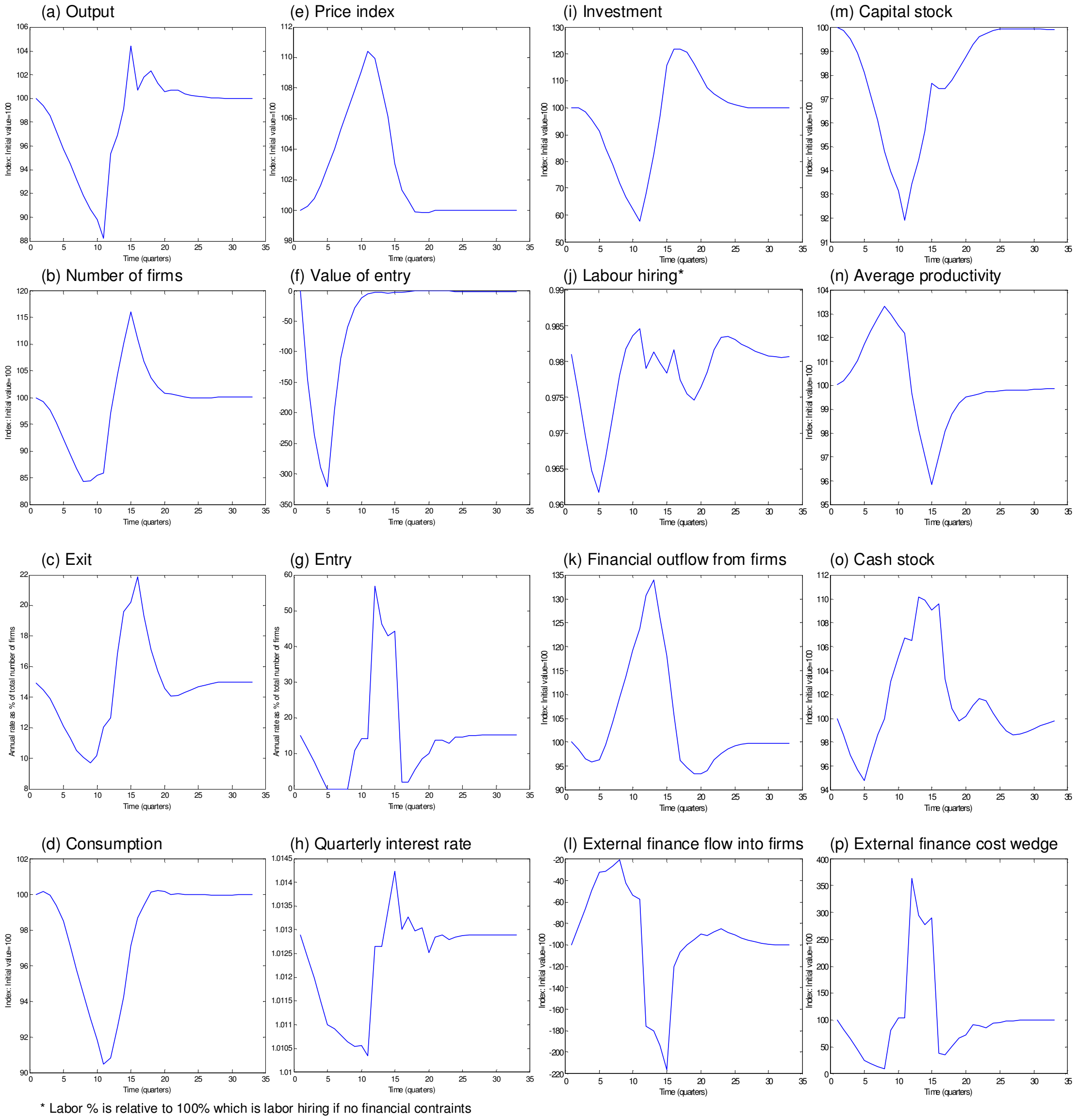

Figure 10: Aggregate response to financial shock for scenario (iii) with heterogeneity in productivity and with fast growth of capital stock. Plot are moving annual averages on a quarterly basis. Period 1 corresponds to last period of initial stationary state. Periods 2 to 35 are for response to financial shock (remaining time periods in solution not shown)

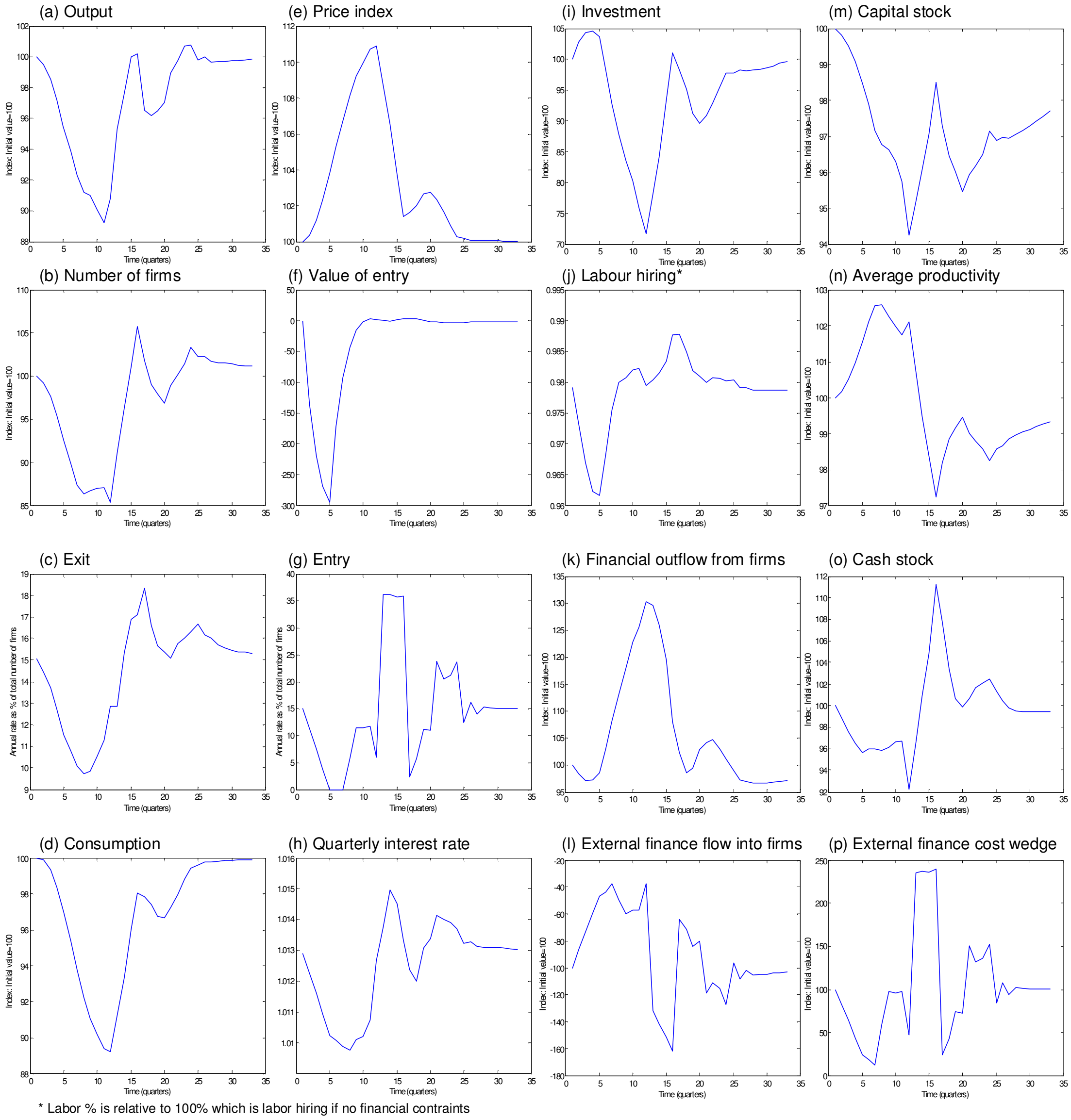

Figure 11: Aggregate response to financial shock for scenario (iv) with heterogeneity in productivity and with gradual growth of capital stock. Plot are moving annual averages on a quarterly basis. Period 1 corresponds to last period of initial stationary state. Periods 2 to 35 are for response to financial shock (remaining time periods in solution not shown).

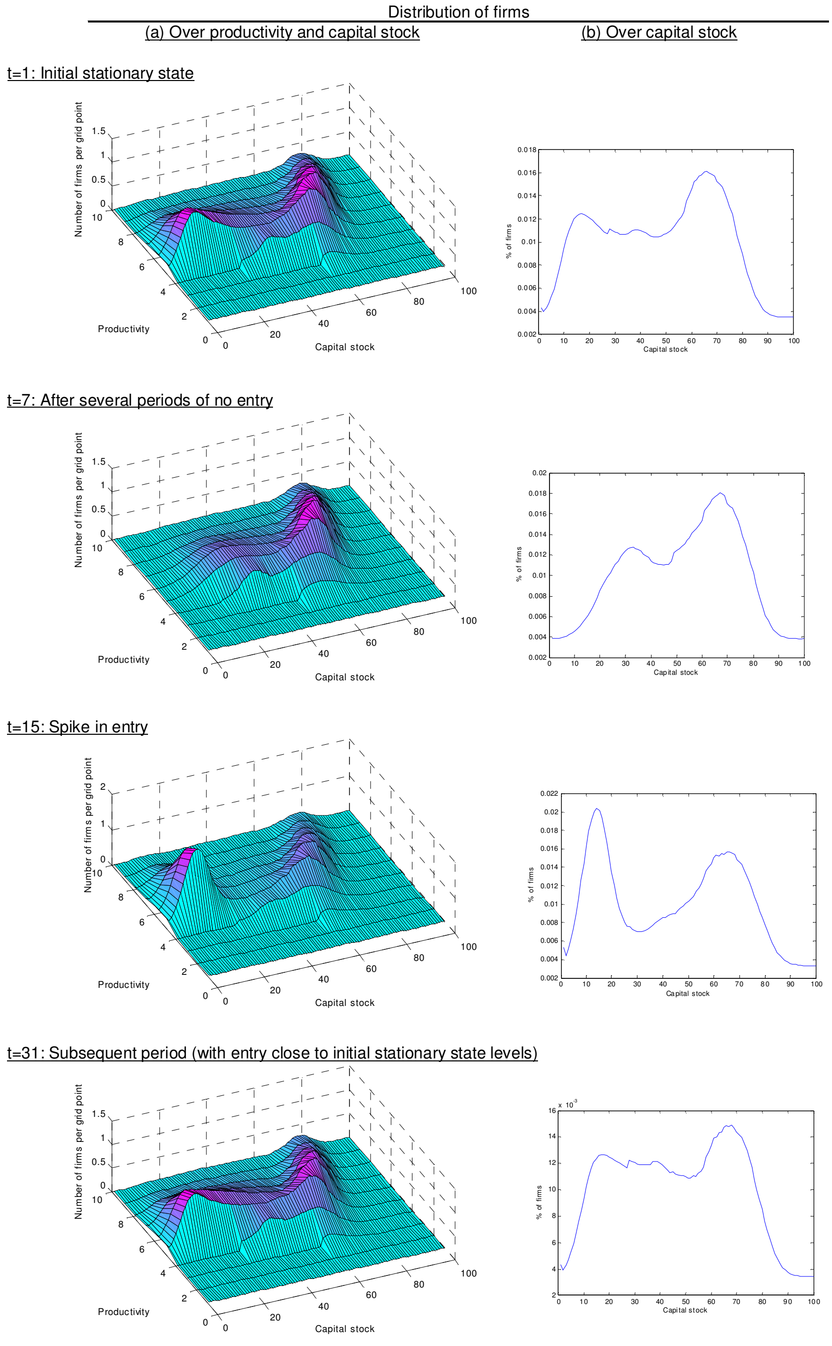

Figure 12: Distribution of firms for selected time periods in response to financial shock for scenario (iv) with heterogeneity in productivity and with gradual growth of capital stock.

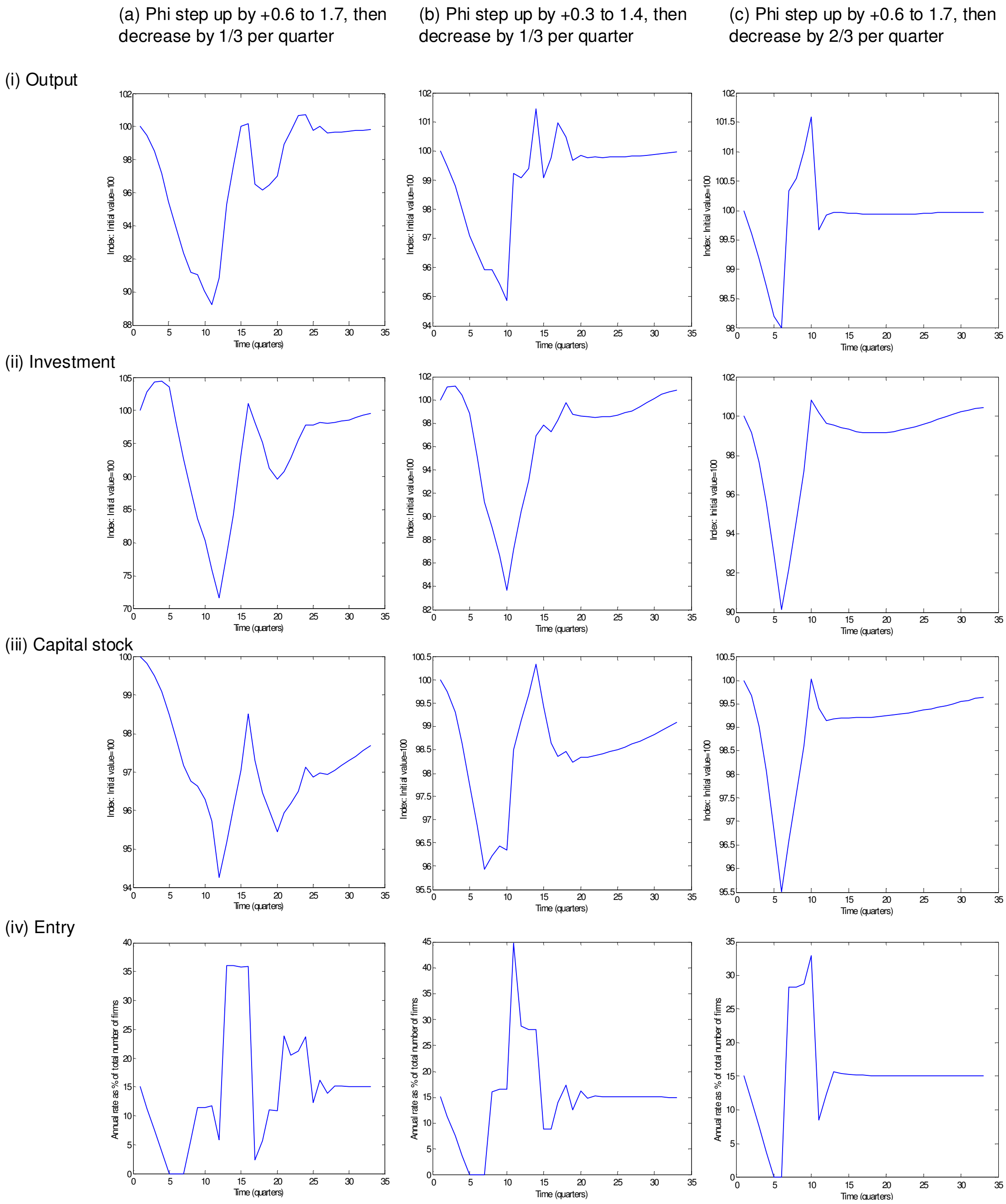

Figure 13: Comparison of response to financial shocks differing in magnitude and persistence

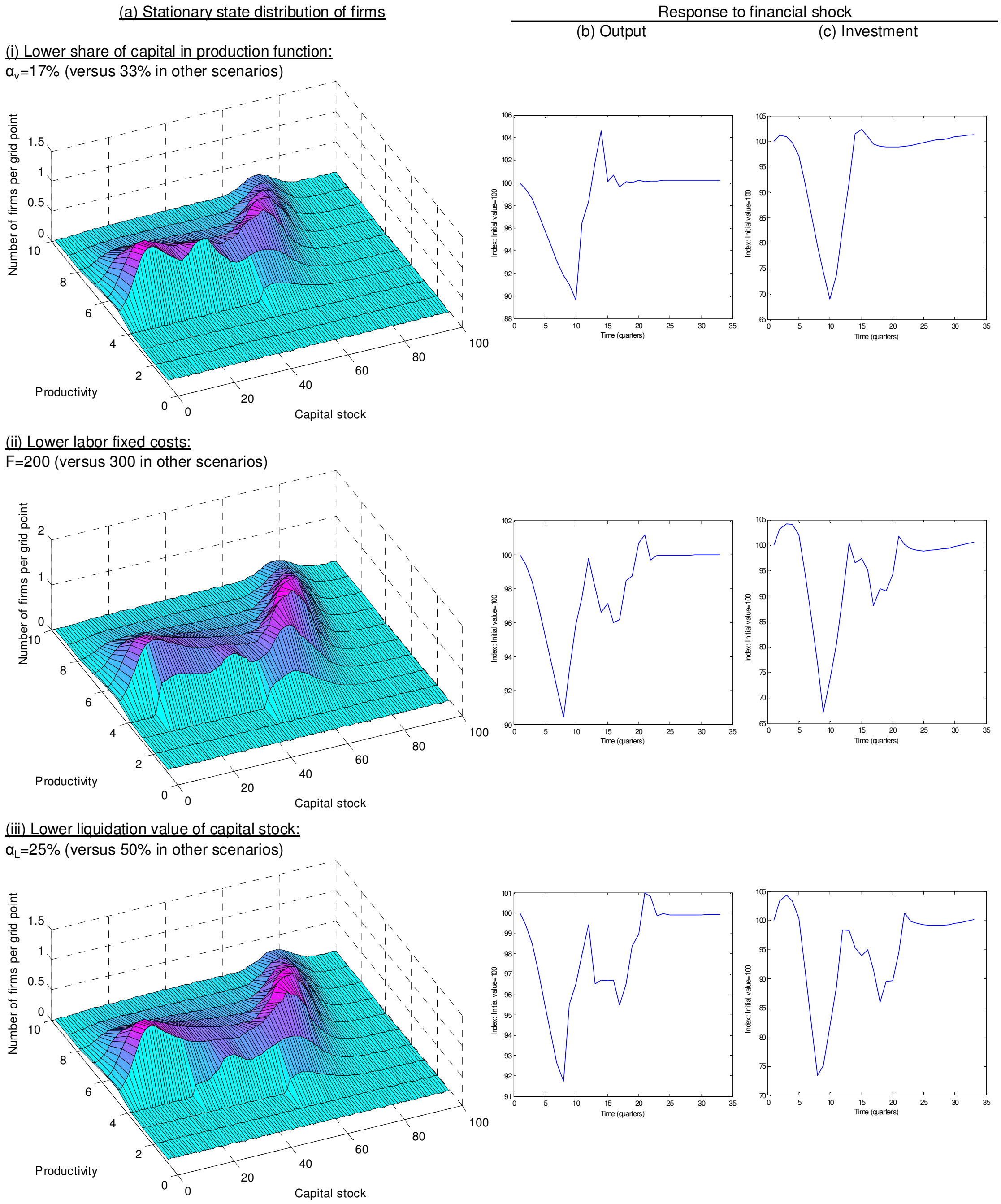

Figure 14: Response to financial shock at different parameter values for scenario (iv) with heterogeneity in productivity and gradual growth of capital stock

Appendix

A Model Algorithm

Following I describe the algorithm for numerically solving the model, focusing on the equilibrium conditions required and the sequence of calculations performed. The demand structure leads to monopolistic competition. In particular, this means that each firm in each time period t need only know industry aggregate outcomes for industry price P, discount rate R, and total output Y from time t onwards, $\{P_t, .., P_T; R_t, .., R_T; Y_t, .., Y_T\}$, to determine its specific policies conditional on its current productivity v, capital stock z, and cash c. Firm policy choices are whether to $\{Continue, Exit\}$, and, if continuing, labor to hire, investment, and dividends.

The algorithm comprise three steps. Step 1 is to set parameters. Step 2 is to compute the firm policies and firm-size distribution $\mu_{v,z,c,1}$ corresponding to the initial parameter values, the initial stationary state equilibrium at $t = 1$. Within Step 2, there is an iteration over the aggregate price for the stationary state P_1 and total output Y_1, as the discount rate in the stationary equilibrium is β. Step 3 computes the firm policies and firm-size distribution for the evolution from the initial stationary state through to period T. Within the Step 3, there is an iteration over the price path $\{P_2, .., P_T; R_2, .., R_T; Y_2, .., Y_T\}$.

1) Set initial parameters, including for industry characteristics and grid structure.

2) (P_1, Y_1) iteration:

- Choose candidate value for (P_1, Y_1).
- Firm Value and Policy Iteration:
  - Compute profit $\pi(v, z; l)$ at each productivity v and capital stock z and potential choice of labor to hire, based on the specific demand system and production function chosen.
  - Pick a candidate value function $V_1(v, z, c)$.
  - Determine $\{Continuation/Exit\}$ and choice of policy for labor, investment and dividends at each $\{v, z, c\}$.
  - Iterate the value function: The set of firm policies over continuation and choice of policy state imply a next iteration value for the value function, $V_1'(v, z, c)$, based on computing the value of continuing and comparing to the value of exit.
  - Check whether new $V_1'(v, z, c)$ is sufficiently close to $V_1(v, z, c)$. If not, continue iteration with $V_1'(v, z, c)$. If close enough, return to (P_1, Y_1) iteration.

- Check the equilibrium conditions: value of entry, and final good production and usage. As seek equilibria with positive entry the condition should be close to zero. Compute firm-size distribution $\mu_{v,z,c,1}$.
  - If value of entry close enough to zero, P_1 iteration is complete. If not, then adjust candidate P_1 accordingly: if condition is positive lower P_1, if negative raise P_1.
  - If production Y_1 is sufficiently close to total uses of final output, then Y_1 iteration is complete. If not, adjust Y_1 accordingly.

3) $\{P_t, .., P_T; R_t, .., R_T; Y_t, .., Y_T\}$ iteration:

- Choose candidate value for $\{P_t, .., P_T; R_t, .., R_T; Y_t, .., Y_T\}$.
  - Compute price corresponding to stationary state at final parameter values.
  - Set initial guess for $\{P_t, .., P_T; R_t, .., R_T; Y_t, .., Y_T\}$ based on prices corresponding to initial and final parameter values.
- Firm value and policy iteration
  - Firm Value and Policy Iteration for $t = T$:
    * Compute profit $\pi(v, z; l)$ at each productivity v and capital stock z and potential choice of labor to hire, based on the specific demand system and production function chosen.
    * Pick a candidate value function $V_T(v, z, c)$.
    * Determine $\{Continuation/Exit\}$ and choice of policy for labor, investment and net external finance at each $\{v, z, c\}$.
    * Iterate the value function: The set of firm policies over continuation and choice of policy state imply a next iteration value for the value function, $V_T'(v, z, c)$, based on computing the value of continuing and comparing to the value of exit.
    * Check whether new $V_T'(v, z, c)$ is sufficiently close to $V_T(v, z, c)$. If close enough, return to $\{P_t, .., P_T; R_t, .., R_T; Y_t, .., Y_T\}$ iteration. If not, continue iteration with $V_T'(v, z, c)$.
  - Firm Value and Policy Iteration for $t = \{2, ..., T-1\}$:
    * Compute profit $\pi(v, z; l)$ at each productivity v and capital stock z and potential choice of labor to hire, based on the specific demand system and production function chosen.

    * Iterate back to compute $V_{T-1}(v,z,c)$ based on $\pi_t(v,z)$ and $V_T'(v,z,c)$, and period T policies, based on computing the value of continuing and comparing to the value of exit. Hence, determine period $T-1$ policies $\{Continuation/Exit\}$ and choice of policy for labor, investment and dividends at each $\{v,z,c\}$.
    * Iterate back to period $t=2$.

- Compute value of entry in each time period $t=\{2,...,T-1\}$.
- Compute the size-distribution of firms $\mu_{v,z,c}=\{\mu_{v,z,c,2},...,\mu_{v,z,c,T}\}$ consistent with the computed firm policies.
  - Compute $\mu_{v,z,c,2}$ based on $\mu_{v,z,c,1}$ and firm policies computed for $t=2$.
  - Determine number of entrants:
    * If value of entry negative for $t=2$, set entry to zero.
    * If value of entry is non-negative, set entry such that:
      · Case 1: If the distribution of incumbents implies a price below P_2 then entry is zero, as adding entrants would further distance the firm distribution from the current value of price path
      · Case 2: If the distribution of incumbents implies a price above P_2, then add entrants until the firm distribution (including entrants) implies a price equal to P_2
  - Iterate forward to compute $\mu_{v,z,c}=\{\mu_{v,z,c,3},...,\mu_{v,z,c,T}\}$.
- Check whether path $\{P_t,..,P_T;R_t,..,R_T;Y_t,..,Y_T\}$ is close enough to an equilibrium:
  - Objective function for price path comprised of two parts:
    * The first part measures the distance between the price path and firm distribution: $(P^{max}-P)$, where P^{max} is the price implied by the distribution of firms.
    * The second part measures an equivalent gap based on the value of entry: $(P^{fe}-P)$.
      · This is zero if value of entry is negative (to capture instances when this value is close to zero but negative, I consider this to be zero if value of entry/sunk cost of entry is larger than -10^{-4}).
      · This is negative if the value of entry is positive. I calculate P^{fe} as what the price in the time period in question would need to change to in order to close part of the gap in value

of free entry. Hence, if value of entry is positive the price change is negative so as to lower profitability and thus lower the value of entry. The adjustment is moderated by the extent to which price adjustments for future periods (which have been determined as the algorithm work backs through time periods) are for increases or decreases in prices.

  * The objective function is then the Euclidian distance of these two measures: $((P^{max} - P)^2 + (P^{fe} - P)^2)^{\frac{1}{2}}$

- If objective function not sufficiently small, construct new candidate price path. The suggested price adjustment is the average of $(P^{max} - P)$ and $(P^{fe} - P)$. The actual price adjustment is only part of the suggested price adjustment, to reduce the risk of cycling over successive iterations of the price path.

- The objective function for $\{R_t, .., R_T; Y_t, .., Y_T\}$ is the Euclidean distance between the production $\{Y_t, .., Y_T\}$ and the uses of final output, and between the discount rate $\{R_t, .., R_T\}$ and the discount rate implied by the household savings (16).