Optimal Securitization with Moral Hazard *

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Abstract

This paper considers the optimal design of mortgage backed securities (MBS) in dynamic setting with moral hazard. A mortgage underwriter with limited liability can engage in costly effort to screen for low risk borrowers and can sell loans to a secondary market. Secondary market investors cannot observe the effort of the mortgage underwriter, but they can make their payments to the underwriter conditional on the mortgage defaults. We find the optimal contract between the underwriter and the investors involves a single payment to the underwriter after a waiting period. We also show how to implement the optimal contract using a credit default swap (CDS). The dynamic setting of our model admits three new findings. First, unlike static models that focus on underwriter retention as a means of providing incentives, our model show that the *timing* of payments to the underwriter is the key incentive mechanism. Specifically, we show the optimal contract delivers substantial gains in efficiency over other standard contracts which require the underwriter to retain a fraction of the pool of mortgages or a "first loss piece." Second, the maturity of the optimal contract can be short even though the mortgages are long-lived . Third, selling mortgages in a pool is more efficient that selling mortgages individually, because it allows investors to learn about underwriter effort more quickly (*information enhancement effect*).

1 Introduction

Mortgage underwriters face a dilemma: either to implement high underwriting standards and underwrite only high quality mortgages or relax underwriting standards in order to save on expenses. For example, an underwriter can collect as much information as possible about each mortgage applicant and fund only the most creditworthy borrowers. Alternatively, an underwriter could collect no information at all and simply make loans to every mortgage applicant. Clearly, the second approach, while less costly in terms of underwriting expenses, will result in higher default risks for the

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underwritten mortgages. Moreover, mortgage underwriters typically wish to sell their loans in a secondary market rather than hold loans in their portfolio. Investors do not observe the underwriter's effort and consequently do not observe the quality of the mortgages they are buying.[1] This paper examines how security design of mortgage backed securities (MBS) can address this agency conflict.

To address this issue we consider an optimal contracting problem between a mortgage underwriter and secondary-market investors. At the origination date, the underwriter can choose to undertake costly effort that results in low expected default rate of the underwritten mortgages. If the underwriter chooses to shirk, the mortgages will have a high expected default rate. Thus, the effort technology of our model results in mortgage performance that occurs through time, rather than on a single date as in previous model. In addition to costly hidden effort, we include a motivation to securitize by assuming that by selling mortgages, rather than holding them in her portfolio, the mortgage underwriter can exploit new investment opportunities, i.e., underwrite more mortgages. We model this feature by assuming that the underwriter is impatient, as in DeMarzo and Duffie (1999), so that the underwriter has a higher discount rate than the investors.[2] Investors do not observe the actions of the underwriter, however the timing of mortgage defaults is publicly observable and contractable.

We derive the optimal contract between the underwriter and the investors that implements costly effort and maximizes the expected payoff for the underwriter, provided the investors are making non-negative profits in expectation. We do not make restrictive assumptions on the form of the contract. Instead, we include all possible payment schedules between the investors and the underwriter in the space of admissible contracts, so long as they depend only on the realization of mortgage defaults and provide limited liability to the underwriter. This setup, which allows information to be reveled over time, allows us to address a central issues in the market for MBS. Namely, how does the fact that mortgage performance occurs through time affect the contracting problem between the investors and the underwriter? Moreover, how much time is needed before the investors can completely payoff the underwriter while maintaining incentive compatibility?

Despite the apparent complexity of the contracting problem, the optimal contract takes a simple form: The investors receive the entire pool of mortgages at time zero and make a single lump sum transfer to the underwriter after a waiting period provided no default occurs. If a single default

[1] Recent empirical work in Keys, Mukherjee, Seru, and Vig (2008) finds that the securitized subprime loans have a higher default rate than loans held on portfolio. For an good description of the market for securitized subprime loans, see Ashcraft and Schuermann (2009).

[2] An alternative motivation for the relative impatience of underwriters are regulatory capital requirements which induce a preference for cash over risky assets.

occurs during the waiting period, the investors keep the mortgages, but no payments are made to the underwriter.

The timing and structure of the optimal contract is a trade-off between incentive provision through delaying payment and efficiency through accelerating payment. By making the payment for mortgages contingent upon an initial period of no default, the investors can provide incentives for underwriters to underwrite low risk mortgages since high risk mortgages will be more likely to default during the initial period. However, delaying payments past the initial waiting period is suboptimal since the underwriter is impatient.

The optimal contract can be implemented using a credit default swap (CDS) and a risk free bond. Under this implementation, the investors pay the underwriter for the pool of mortgages at time zero. In addition, a CDS on the pool of mortgages is issued by the underwriter and given to the investors. The proceeds from the sale of the pool of mortgages are invested in the risk-free bond, which is held as a capital requirement for the CDS until it expires. If default occurs before the expiration of the CDS, then the investors receive the bond and the underwriter receives nothing. Moreover, the maturity of the optimal contract is relatively short compared to the underlying mortgages, which is very appealing from a practical standpoint. For reasonable parameter values the optimal contract has a maturity of about one year.

Interestingly, the optimal contract calls for the underwriter to *pool* mortgages and include a bundled derivative security in the sale rather than sell each mortgage individually. By observing the timing of a single default, the investors learn about the quality of the remaining mortgages. As a result, the investors can infer the quality of the mortgages sooner by observing the entire pool rather than a single mortgage at a time, which we call the *information enhancement effect*. By making payment contingent upon the performance of the entire pool rather than each individual mortgage, it is possible to speed up payment to the underwriter while maintaining incentive compatibility. This result is not driven by any benefits from diversification.

Our findings are in stark contrast with the previous literature on security design with asymmetric information that primarily focuses on a static setting, e.g., DeMarzo (2005). We show that the timing of payments plays an extremely important role when the information about the underlying assets is revealed over time. In the dynamic setting of this paper, the optimal contract is about *when* the underwriter is paid, rather than what kind of piece of the underlying assets it retains.

Our paper relates most closely to the literature on optimal security design and asset backed securities (ABS). One approach of this literature is to treat the security design problem faced by an issuer of ABS is similar to the capital structure problem faced by a firm in the presence of asymmetric information over a fixed investment opportunity as characterized by Myers and Majluf (1984) giving rise to a pecking order theory of asset backed securities. Nachman and Noe (1994) present a rigorous frame work for when a given a security design minimizes mispricing due to asymmetric information, showing standard debt is optimal over a very broad class of security design problems. Building on the pecking order intuition, Riddiough (1997) shows that an informed issuer can increase her proceeds from securitization by creating multiple securities, or tranches, with differing levels of exposure to the issuers private information. Moreover, pooling assets that are not perfectly correlated can provide some diversification benefits and thus reduce the lemons discount.

Another approach to the optimal design of asset backed securities considers the role of costly signaling. The basic intuition of this approach is that an issuer of ABS can signal her private information by retaining a fraction of the issued security as in Leland and Pyle (1977). Building on this intuition, DeMarzo and Duffie (1999) presents a model of security design where an issuer minimizes the cost of signaling her private information by choosing a security design. Applying the security design framework of DeMarzo and Duffie (1999), DeMarzo (2005) explains the pooling and tranching structure of ABS. In his model, and issuer of ABS can signal her private information about a pool of assets by retaining a fraction of a security which is highly sensitive to that information. This signaling mechanism explains the multiclass, or tranched structure, of ABS.

Other studies of asset backed securities have focused on different types of asymmetric information. For example, Axelson (2007) considers a setting in which *investors* have superior information about the distribution asset cash flows. The author gives conditions for which pooling may be an optimal response to investor private information and for which single asset sale is preferred.

The literature on security design and ABS presented above utilizes mostly one period models of securitization. In contrast to the previous literature, we take a different approach by modeling mortgages which can default after some time has elapsed. This additional aspect allows us to show that the timing of payments from mortgage securitization can be a key incentive mechanism and that the duration of the optimal contract can be short while the duration of the mortgages is long.

Another important difference between our model and the literature is that there is little previous work on costly hidden actions in underwriting practices. The closest exception is Innes (1990), which considers a one period moral hazard model of security design. The moral hazard problem in underwriting practices is likely to be important in private securitization markets where both the quality of assets and the operations of issuers are extremely difficult to verify. Indeed, some empirical studies, such as Mian and Sufi (2009), suggest that “mispriced agency conflicts” may have played a crucial role in the current mortgage crisis. In addition, evidence presented in Keys, Mukherjee, Seru, and Vig (2008) suggests that securitization of subrpime loans led to lax lender standards, especially when there is “soft” information about borrowers which determines default risk but is not easily verifiable by investors.

2 The Model

2.1 Preferences, Technology, and Information

Time is infinite, continuous, and indexed by t. A risk-neutral agent (the underwriter[3]) originates N mortgages that she wants sell to a risk-neutral principal (the investors) immediately after origination. The underwriter has the constant discount rate of γ and the investors have the constant discount rate of r. We assume $\gamma > r$. This assumption could proxy for a preference for cash or additional investment opportunities of the underwriter (DeMarzo and Duffie 1999).

The underwriter may undertake an action $e \in \{0, 1\}$ at cost $C \cdot e$ at the origination of the pool of mortgages (t =0). This action is hidden from the investors and hence not contractable. Each mortgage generates constant coupon u until it defaults. Individual mortgages default according to an exponential random variable with parameter $\lambda \in \{\lambda_H, \lambda_L\}$ such that $\lambda = \lambda_L$ if $e = 1$ and $\lambda = \lambda_H$ if $e = 0$ and $\lambda_H > \lambda_L$. Upon default, all assets pay a lump sum recovery of $R < u/r$. All defaults are mutually independent conditional on effort. It may seem overly simplistic to assume that low underwriter effort leads to only high risk mortgages. A more realistic assumption is that law effort to leads to a mixture of both high risk and low risk mortgages. Such a setup would complicated the analysis as the mixture of two exponential distributions is not itself exponential. However, as we argue below, it does not add richness to the model to assume that low effort leads to a mixture of mortgage risk types.

A contract consists of transfers from the investors to the underwriter depending on mortgage defaults. Specifically, let D_t denote the total number of defaults that have occurred by time t and $\mathcal{F}_t$ the filtration generated by D_t. It will also be convenient to define the following sequence of stopping times

$$\tau_n = \inf\{t : D_t \geq n\}.$$

[3]Although we refer to the agent in our model as the underwriter, our setting applies equally well to other actors than mortgage underwriters. The defining characteristic of the agent in our model is that she can undertake costly hidden action to screen out high risk mortgages or assets

Formally, a contract is an $\mathcal{F}_t$-measurable process X_t giving the cumulative transfer to the underwriter by time t so that dX_t denotes the instaneous transfer to the underwriter at time t. Readers unfamiliar with this notation can think of X_t as a function of time t and all previous default times $\tau_n \leq t$ so that $dX_t = x(t, \tau_1, \ldots, \tau_n)dt$ for some function $x(\cdot)$. We restrict our attention to contracts that satisfyi the limited liability constraint $dX_t \geq 0$ and are absolutely integrable. The underwriter thus has the following utility for a given contract X_t and effort e

$$E\left[\int_0^\infty e^{-\gamma t} dX_t \big| e\right] - ce.$$

All integrals will be Stieljes integrals.[4]

2.2 Optimal Contracts

We assume that implementing high effort is optimal, hence the investors' problem is to maximize profits subject to delivering a contract that provides incentives to expend effort and a certain promised level of utility to the underwriter. We call a such contracts *incentive compatible*. Once we restrict our attention to incentive compatible contracts, the value the investors place on holding the mortgages is fixed since the contract cannot affect the distribution of mortgage defaults other than to guarantee that the underwriter only originates low risk mortgages. Hence, the investors maximize profits by choosing the incentive compatible contract with the lowest expected cost under their discount rate. In other words, the investor chooses the least costly incentive compatible contract. We state this formally in the following definition.

Definition 1 *Given a promised utility a_0 to the underwriter, a contract X_t is* optimal *if it solves the following problem*

$$b(a_0) = \min_{dX_t \geq 0} E\left[\int_0^\infty e^{-rt} dX_t \big| e = 1\right] \tag{I}$$

such that

$$E\left[\int_0^\infty e^{-\gamma t} dX_t \big| e = 0\right] \leq E\left[\int_0^\infty e^{-\gamma t} dX_t \big| e = 1\right] - C. \tag{IC}$$

and

$$a_0 \leq E\left[\int_0^\infty e^{-\gamma t} dX_t \big| e = 1\right] - C \tag{PC}$$

[4] Readers unfamiliar with Stieljes integrals may simply view this integral for an arbitrary integrand g as $\int g(t)dX_t = \int g(t)x(t)dt$ where $x(t)$ is the time t change in t. For a reference see Carter and Van Brunt (2000).

It is important to note that Definition 1 is equivalent to a definition where we hold the cost to the investor fixed and maximize the utility of the underwriter. As an alternative to Definition 1, we could fix the cost paid by the investor b and find the contract which maximizes the underwriters initial utility a_0. In the analysis that follows, we will find a one-to-one relationship between the cost paid by the investors and the value delivered to the underwriter; for any level of initial promised utility of the underwriter, we know the cost paid by the investors under the optimal contract and vice versa.

2.3 Solution

The contracting problem stated thus far amounts to solving an infinite dimensional optimization problem which at first glance seems quite complicated. In this section we will give a heuristic argument that transforms, subject to verification, our contracting problem to a simple problem of solving two equations for two unknowns by making a series of intuitive guesses about the form of the optimal contract. This argument follows from the observation that the optimal contract should feature the most front-loaded payment schedule which maintains incentive compatibility. It does not, however, constitute a rigorous proof of the main result. The proof requires the slightly more sophisticated, yet still quite simple, observation that a useful characterization of the class of incentive compatible contracts obtains by relating the expected underwriter value for the contract under high and low effort via a linear approximation of the Radon-Nikodym derivative of the respective probability measures.

We start by giving a heuristic derivation of the optimal contract when $N = 2$. This base case provides the basic intuition we will use throughout our solution. Some payment must be contingent on τ_1 and τ_2 to provide incentives to exert effort. If all payment occurs regardless of the realization of τ_1 and τ_2, then there is no incentive for the underwriter to exert effort. Specifically, the contract should reward the underwriter when τ_1 and τ_2 are relatively larger and punish the underwriter when τ_1 and τ_2 are relatively smaller since τ_1 and τ_2 are more likely to be large when the underwriter exerts effort. At the same time, the optimal contract should completely pay off the underwriter as quickly as possible to exploit the difference in discount rates of the investor and underwriter.

Given the intuition above, it is clear that the optimal contract will balance providing incentives with front loading payment. Observe that we can always take an arbitrary incentive compatible contract and move some payment that occurs later to an earlier date. To maintain incentive compatability, we must then move some payment that occurs earlier and move it to a later date. Doing so repeatedly should move all payment to a single date strictly later that $t = 0$. It is not yet clear that this process will result in reducing the cost of the contract, but we use it as a starting point. To that end, we focus on contracts of the form $dX_t = 0$ for $t \neq t_0$ and $dX_{t_0} = \mathbb{I}((\tau_1, \tau_2) \in \mathcal{A})y_0$ for some $t_0 \geq 0$ and set of events $\mathcal{A}$ chosen such that X_t is $\mathcal{F}_t$-measurable.[5] . We further suppose that both the participation and incentive compatibility constraints bind. We then have the following two equations

$$e^{-\gamma t_0} y_0 P((\tau_1, \tau_2) \in \mathcal{A}|e = 1) = a_0 + C \tag{1}$$

$$e^{-\gamma t_0} y_0 P((\tau_1, \tau_2) \in \mathcal{A}|e = 0) = a_0 \tag{2}$$

where $P(\cdot|e)$ denotes probability conditional on effort. Such a contract will cost the investors

$$e^{-rt_0} y_0 P((\tau_1, \tau_2) \in \mathcal{A}|e = 1) = e^{-(r-\gamma)t_0}(a_0 + c) \tag{3}$$

which is increasing in t_0 and independent of A. This implies that the optimal contract of this form will choose the set $\mathcal{A}$ to minimize t_0 subject to satisfying equations (1) and (2). To do so, we choose $\mathcal{A}$ such that $\mathbb{I}((\tau_1, \tau_2) \in \mathcal{A})$ depends *only* on τ_1 since any dependence on τ_2 necessarily increases t_0. Moreover, we guess that A should take on as simple structure as possible. So let $\mathcal{A} = \{\tau_1 \geq t_0\}$, then equations (1) and (2) reduce to the following

$$e^{-(\gamma+\lambda_L)t_0} y_0 = a_0 + C \tag{4}$$

$$e^{-(\gamma+\lambda_H)t_0} y_0 = a_0. \tag{5}$$

We can easily solve (4) and (5) for t_0 and y_0. We formally state the optimal contract in the following proposition.

Proposition 1 *An optimal contract X_t is given by*

$$dX_t = \begin{cases} 0 & \text{if } t \neq t_0 \\ y_0 \mathbb{I}(t_0 \leq \tau_1) & \text{if } t = t_0 \end{cases} \tag{6}$$

[5] $\mathbb{I}(\cdot)$ is the indicator function.

where

$$t_0 = \frac{1}{N(\lambda_H - \lambda_L)} \log\left(\frac{a_0 + C}{a_0}\right) \tag{7}$$

$$y_0 = \left(\frac{a_0 + C}{a_0}\right)^{\frac{\gamma + N\lambda_L}{N(\lambda_H - \lambda_L)}} (a_0 + C) \tag{8}$$

Moreover

$$b(a_0) = (a_0 + C)\left(\frac{a_0 + C}{a_0}\right)^{\frac{\gamma - r}{N(\lambda_H - \lambda_L)}} \tag{9}$$

The contract calls for no transfers from the investors to the underwriter to take place until the time t_0 given by equation (7). If the first default time $\tau_1 \geq t_0$ then the the contract calls for a payment of y_0 given by equation (8) at time t_0. Equation (7) is the product of two terms. The first term is the inverse of the difference between the arrival intensity of τ_1 given low effort and the arrival intensity of t_0 given high effort. The second term is the difference in the logs of the present value (gross of the cost of effort) of the contract from high effort and low effort respectively. Thus, t_0 is set so that the expected present value of a transfer of y_0 at t_0 under the underwriter's discount rate is exactly equal to $a_0 + C$ under high effort, and a_0 under low effort.

The cost of the contract to the investors is given by $b(a_0)$ in equation (9) and is the product of two terms. The first term is the expected present value of transfers under the discount rate of the underwriter. The second term adjusts this expected present value to the discount rate of the investors.

The intuition behind Proposition 1 is the following. On the one hand, delaying payment is costly due to the difference in the discount rates of the underwriter and investors. On the other hand, accelerating payment decreases the sensitivity of payments to the underwriter's choice of effort. These two forces imply that the optimal contract should feature the most accelerated payment structure that preserves the minimum sensitivity to effort choice needed to provide incentives.

To find the minimum sensitivity required to provide incentives, the proof of Proposition 1 makes use of the following useful observation. The expected underwriter value of the contract under low effort is related to the expected underwriter value of the contract under high effort via a change of measure. If we let $\mathbb{Q}$ be the measure induced by low effort and $\mathbb{P}$ be the measure induced by high effort, then an application of the monotone convergence theorem leads to the following equality.

$$E\left[\int_0^\infty e^{-\gamma t} dX_t | e = 0\right] = E\left[\int_0^\infty e^{-\gamma t} \pi(t) dX_t | e = 1\right] \tag{10}$$

where $\tilde{\pi}_t = \frac{d\mathbb{Q}}{d\mathbb{P}}$ is the Radon-Nikodym derivative of $\mathbb{Q}$ with respect to $\mathbb{P}$.[6]. Such a change of measure allows one to write the investors problem entirely in terms of conditional expectations with respect to $e = 1$. Unfortunately, direct calculation of $pi(t)$ is not possible, however we can write the following

$$E\left[\int_0^\infty e^{-\gamma t} dX_t | e = 0\right] \geq E\left[\int_0^\infty e^{-\gamma t} e^{-N(\lambda_H - \lambda_L)t} dX_t | e = 1\right] \tag{11}$$

$$\geq \frac{a_0}{a_0 + C} E\left[\int_0^\infty (N(\lambda_H - \lambda_L)(t_0 - t) + 1) e^{-\gamma t} dX_t | e = 1\right]. \tag{12}$$

Inequality (11) arises from approximating $\pi(t)$ and is verified in the appendix. Inequality 2.3 arises from the fact that $e^{-N(\lambda_H - \lambda_L)t}$ is convex in t and hence can bounded below by a linear function of t. Figure 1 gives graphical intuition of the argument that yields inequality .

[Figure 1 about here.]

Inequality (12) allows us to approximate the incentive compatibility constraint in terms of conditional expectations with respect to $e = 1$. The allows us to combine inequality the incentive compatibility constraint (IC) and participation constraint (PC) to get the following useful sufficient condition for an arbitrary contract to be incentive compatible

$$\frac{1}{a_0 + C} E\left[\int_0^\infty t e^{-\gamma t} dX_t | e = 1\right] \geq t_0. \tag{13}$$

Inequality (13) shows that the minimum duration (with respect to the risk adjusted discount rate) of any incentive compatible contract is exactly t_0, which turns out to be the duration of the optimal contract. Hence, the proof of Proposition 1 shows that optimal contracting problem we consider comes down to a duration minimization problem.

In order to find the minimum-duration contract we note that we can always improve on an arbitrary incentive compatible contract by delaying payment that occurs before t_0 and accelerating payment that occurs after t_0 until all payment occurs at t_0. Doing so reduces the duration of the contract. Eventually we will have all payment occurring at time t_0. The resulting contract is the optimal contract stated in Proposition 1.

To understand why the optimal contract only uses the information contained in the first default time as opposed including information contained in subsequent default times in the payment rule, it is useful to think of the investors's problem as a standard hypothesis testing problem in which there

[6]This is a slight abuse of notation, but the reader familiar with change of measure can think of π_t as the restriction of $\frac{d\mathbb{Q}}{d\mathbb{P}}$ to $\mathcal{F}_t$ the filtration generated by D_t

is a trade off between test power and the period of observation required to perform the test. At each point in time, the investors essentially test the null hypothesis H_0 : the underwriter chose $e = 1$ versus the alternative hypothesis H_1 : the underwriter chose $e = 0$. They then pay the underwriter based on the outcome of the test. However, they must choose the tests and payments to provide incentives to the underwriter. For the purpose of exposition, let us consider the following alternative to the optimal contract

$$dX_t = \tilde{y}_0 \mathbb{I}(\text{accept } H_0 \text{ given } \{D_s\}_{s \le \tilde{t}_0}, t = \tilde{t}_0),$$

where H_0 is accepted if $\{D_s\}_{s \le \tilde{t}_0} \in \mathcal{A}$ for some $\mathcal{A} \subset \mathcal{F}_{\tilde{t}_0}$. Incentive compatibility implies that this contract must correspond to a likelihood ratio test which accepts the null hypothesis if

$$\frac{P(\{D_s\}_{s \le \tilde{t}_0} \in \mathcal{A} | e = 1)}{P(\{D_s\}_{s \le \tilde{t}_0} \in \mathcal{A} | e = 0)} = \frac{a_0 + c}{a_0}. \tag{14}$$

Equation (14) illustrates a bit of classic principal-agent intuition in the optimal contracting problem we consider. Incentive compatibility imposes a tradeoff between the level and power of the likelihood ratio test used by the investors to determine payments to the underwriter. This implies that if the investors use a more powerful test than the test employed in the optimal contract, for instance a test which uses more information than the first default time, then the test must have a lower level than the optimal contract. Accordingly, such a test will have to use a longer period of observation than the optimal contract, in other words $\tilde{t}_0 \ge t_0$. Since all surplus in the model arises from front loading payments to the underwriter, any alternative of the above form will be suboptimal. Hence, by adding a time dimension to the way information is revealed in the model, we emphasize the trade off between the power of the test employed in the contract, and the amount of observation time needed to perform the test and preserve incentive compatibility.

Before continuing to discuss the implications and implementation of the optimal contract, we return to the discussion of the assumption that low effort leads to only high risk loans. On the one hand, this assumption can be relaxed so that low effort leads to a mixture of high risk and low risk mortgages, where the probability that a mortgage is high risk given low effort is ρ. Suppose $N = 1$, in this case, we can use the law of iterated expectations to get

$$
\begin{aligned}
E\left[\int_0^\infty e^{-\gamma t} dX_t \middle| e = 0\right] = E\Bigg[\mathbb{I}(\lambda = \lambda_H) E\left[\int_0^\infty e^{-\gamma t} dX_t \middle| e = 0, \lambda = \lambda_H\right] \\
+ \mathbb{I}(\lambda = \lambda_L) E\left[\int_0^\infty e^{-\gamma t} dX_t \middle| e = 0, \lambda = \lambda_L\right] |e = 0\Bigg] \\
= \rho E\left[\int_0^\infty e^{-\gamma t} dX_t \middle| \lambda = \lambda_H\right] + (1-\rho) E\left[\int_0^\infty e^{-\gamma t} dX_t \middle| \lambda = \lambda_L\right]
\end{aligned}
$$

Hence constraint (IC) would become

$$
\begin{aligned}
\rho E\left[\int_0^\infty e^{-\gamma t} dX_t \middle| \lambda = \lambda_H\right] + (1-\rho) E\left[\int_0^\infty e^{-\gamma t} dX_t \middle| \lambda = \lambda_L\right] \\
\leq E\left[\int_0^\infty e^{-\gamma t} dX_t \middle| \lambda = \lambda_L\right] - C. \quad (15)
\end{aligned}
$$

Rearranging (15) gives

$$
E\left[\int_0^\infty e^{-\gamma t} dX_t \middle| \lambda = \lambda_H\right] \leq E\left[\int_0^\infty e^{-\gamma t} dX_t \middle| \lambda = \lambda_L\right] - \frac{C}{\rho}. \quad (16)
$$

In light of (16), the optimal contract simply replaces c by $\tilde{C} = C/\rho$ in constraint IC when we allow for a mixture of risk types if the underwriter applies low effort in the single mortgage case. When $N > 1$ such a simplification is not available, but a similar optimal contract holds as we discuss in the Appendix.

On the other hand, it might not be too unrealistic to assume that low effort results in only high risk mortgages for the following reason. If the underwriter does not exert effort low risk borrowers will have to pay the same price for a mortgage as high risk borrowers. Under mild assumptions on competition in the market for underwriters, all low risk borrowers will go to an underwriter who exerts effort and can charge a lower price to low risk borrowers accordingly. Hence, if the underwriter does not exert effort, she will be left with only high risk type borrowers.

2.4 Implementation

In this section we show how to implement the optimal contract using a credit default swap (CDS) and a margin account. The implementation will refer to 3 classes of investors: Pool Buyers, CDS Broker, and CDS Buyers. These classes may be the same investors or may be separate. For the moment assume that the classes are separate and that the CDS Broker is not subject to a limited liability constraint and both the CDS Broker and Buyers are competitive.

Proposition 2 *Suppose all payments are common knowledge. The optimal contract can be implemented as follows:*

- *t=0*
  - *The Pool Buyers pay* $P_0 = y_0 e^{-(r+\lambda_L)t_0}$ *to the underwriter for the pool of mortgages.*
  - *The pool purchase price* P_0 *is placed in a margin account which continuously accrues interest at the risk free rate* r*.*
  - *The CDS Broker sells a CDS contract to the CDS Buyers with payout* y_0 *in the event that at least one mortgage defaults before time* t_0*.*
- $t > 0$
  - *The CDS Buyers pay a continuous premium*

    $$I = \frac{r\lambda_L y_0(1 - e^{-(r+\lambda_L)t_0})}{r - (r+\lambda_L)e^{-\lambda_L t_0} + \lambda_L e^{-(r+\lambda_L)t_0}}$$

    to the CDS Broker, the CDS Broker places

    $$I_u = \frac{r(1 - e^{-\lambda_L t_0})}{e^{rt_0} - 1} y_0$$

    into the underwriters margin account and keeps $I - I_u$*.*
  - *If* $\tau_1 < t_0$*, the underwriter pays* $P_0 e^{r\tau_1} + I_u(e^{r\tau_1} - 1)/r$ *and the CDS Broker pays* $y_0 - P_0 e^{r\tau_1} + I_u(e^{r\tau_1} - 1)/r$ *to the CDS Buyers and the CDS contract terminates.*
  - *If* $\tau_1 \geq t_0$*, the underwriter receives the balance of the margin account at* t_0 *and the CDS contract expires.*

Proof The proposed implementation yields the same net transfers as the optimal contract, so they are equivalent.

■

The implementation of the optimal contract given in Proposition 2 yields the following interpretation. The Pool Buyers would like to pay the underwriter for the pool at time zero to exploit the difference in discount rates, but doing so would fail to provide incentives to the underwriter to expend effort. In order to commit to high effort the underwriter sells a CDS contract to the CDS buyers on the pool value at a fair price given high effort. Observe that this is a credible commitment mechanism since low effort would make the insurance more valuable. However, due to the limited liability constraint, the underwriter cannot commit to having enough capital to cover the insurance if a default occurs before expiration. To overcome this problem the proceeds from the sale of the pool of mortgages along with all premium payments due to the underwriter are placed in a margin account, the balance of which is used to cover the payout on the CDS contract in the event that a default occurs before t_0. Any difference between the premium due to the underwriter I_u and the contracted premium I of the CDS is collected by the CDS Broker in exchange for the Broker paying the difference between the contracted payout y_0 of the CDS and the balance of the margin account.

Notice that the CDS Buyers and the Pool Buyers need not be the same entity. The crucial aspect of the implementation is that the contracted CDS premium I is common knowledge. In other words, the Pool Buyers can infer the effort level of the underwriter by observing I, where a low I corresponds to high effort.

Our implementation resembles the practice of "over-collateralization." In such a structure, the underwriter places additional mortages in a pool underlying a MBS so that the total face value of collateral is greater than stated face value of the pool. If after a certain period of time the pool is still performing according to expectations, the additional mortgages are returned to the underwriter. By making the return of the mortgages contingent on pool performance, and returning the mortgages at a single date, over-collateralization serves a similar role as the margin account in our implementation.

2.5 The benefits of pooling

One important feature of MBS is the process of "pooling" and "tranching." In this process, an issuer of an MBS first pools together many mortgages to form a collateral pool and then issues at least two derivative securities (tranches) on the collateral. This security design contrasts with individual loan sale in which an issuer simply sells each loan separately. Individual loan sale means that the transfers corresponding to the sale of one loan cannot effect the transfers corresponding to the sale of another. Hence, in the context of our model, individual loan sales correspond to a contract which is the sum of N individual contracts, each of which depends on only one mortgage. Let W_t denote an contract which calls for individual mortage sale, then

$$dW_t = \sum_{n=1}^{N} dV_t(n),$$

where $V_t(n)$ is measurable with respect to the filtration generated by the nth mortgage cash flows. Since each mortgage is independent and identically distributed after the underwriter chooses effort, it is natural to only consider individual loan sale contracts of the form $dV(n)_t = dV(m)_t$ for all n, m which implies that W_t is measurable with respect to the filtration generated by D_t. Thus, individual loan sale contracts are contained in the contract space we consider in the derivation of the optimal contract. This fact leads us to state the following important corollary to Proposition 2.

Corollary 1 *Pooling mortgages and bundling with a CDS contract is more efficient than individual loan sale.*

Notice that Corollary 1 does not depend on the number of mortgages, N, in the pool. Moreover, the benefits of pooling do not arise from risk diversification benefits. Other results in the literature, for example DeMarzo (2005) which considers an informed issuer selling multiple assets, attribute the benefits of pooling to the so called *risk diversification effect*. In his model, if some portion of the payoff from assets is unrelated to the private information of the issuer, then under mild assumptions on the distribution of this residual risk, the issuer can create a security with less risky payoffs than the pure pass through pool, in other words a senior "tranche." The issuer can then signal her private information by retaining a portion of the residual payoffs.

In our model, pooling is a consequence of providing incentives in the least costly manner. Returning to the intuition gained by viewing the contracting problem as a hypothesis testing problem, pooling in our model trades off the time it takes to implement the test with the loss in power from ignoring all defaults that occur after the first default. We call the decreased time required to implement the test the *information enhancement effect* of pooling. A similar concept is present, although not emphasized, in Diamond (1984). In that paper, a financial intermediary is punished according to an aggregate signal of entrepreneur loan portfolio performance in order to provide incentives to monitor. The main difference between that result on our result is the channel by which the cost of information revelation is decreased by pooling. In our model the key is reducing the time necessary to implement a given test as opposed to minimizing the total punishment necessary to provide incentives.

2.6 Comparing the optimal contract to alternative contracts

It is interesting to ask is how closely we can approximate the optimal contract using an alternative, and perhaps more standard, contract. To answer this question, we compare the optimal contract to two possible alternative contracts, one in which the underwriter retains a pure fraction of the mortgage pool, and one in which the underwriter retains a fraction of a "first loss piece.".

2.6.1 The optimal contract versus fraction of the mortgage pool

First we consider contracts in which the underwriter retains an fraction of the pool of mortgages and receives a lump sum transfer at time $t = 0$. Note that the total cash flow from the pool of mortgages at time t is $u(N - D_t)dt + RdD_t$. Hence a contract which calls for the underwriter to receive a time zero cash payment of K and retain a fraction α of the pool of mortgages must take the following form

$$dX_t = \begin{cases} \alpha K & t = 0 \\ \alpha(u(N - D_t)dt + RdD_t) & t \geq 0 \end{cases}. \tag{17}$$

It will also be useful to compute the expected present value of the contract under the underwriter's discount rate given high effort and low effort.

$$E\left[\int_0^\infty e^{-\gamma t} dX_t | e = 1\right] = K + N\alpha \frac{u + \lambda_L R}{\gamma + \lambda_L}$$
$$E\left[\int_0^\infty e^{-\gamma t} dX_t | e = 0\right] = K + N\alpha \frac{u + \lambda_H R}{\gamma + \lambda_H}$$

An optimal contract of the form given in (17) must make both the participation constraint and the incentive compatibility constraint bind. Hence we have the following system of equations

$$a_0 + C = K + \alpha N \frac{u + \lambda_L R}{\gamma + \lambda_L}$$
$$a_0 = K + \alpha N \frac{u + \lambda_H R}{\gamma + \lambda_H}.$$

Which we can solve to get

$$\alpha = \frac{C(\gamma + \lambda_H)(\gamma + \lambda_L)}{N(u - \gamma R)(\lambda_H - \lambda_L)}$$
$$K = a_0 - \frac{C(u + \lambda_H R)(\gamma + \lambda_L)}{(u - \gamma R)(\lambda_H - \lambda_L)}$$

The cost of providing such a contract, which we denote by $b^E(a_0)$, will then be

$$\begin{aligned} b^E(a_0) &= K + \alpha N \frac{u + \lambda_L R}{r + \lambda_L} \\ &= a_0 + C + \left(\frac{C(\gamma + \lambda_H)(\gamma - r)}{(u - \gamma R)(\lambda_H - \lambda_L)} \right) \left(\frac{u + \lambda_L R}{r + \lambda_L} \right) \end{aligned}$$

We assume that investors earn zero profits and calculate the implied promised value to the underwriter to get

$$a_0^E(N) = \left(N - \frac{C(\gamma + \lambda_H)(\gamma - r)}{(u - \gamma R)(\lambda_H - \lambda_L)} \right) \left(\frac{u + \lambda_L R}{r + \lambda_L} \right) - C.$$

Figure 2 shows the percentage gain in underwriter value for a range of values of the parameters λ_H and γ holding the other parameters fixed.

[Figure 2 about here.]

The efficiency gain from using the optimal contract over the alternative contract increases as either γ increases or λ_H decreases. As γ increases, the gains from securitization increase, and hence any improvement in the efficiency of the structure of securitization will have a larger effect on efficiency. As λ_H decreases, the severity of the moral hazard problem increases since we assume cost stays constant. That is, effort becomes harder to infer since λ_H and λ_L are closer in value, however the cost of effort stays the same, hence shirking becomes more attractive. As such, harsher incentives are needed to implement high effort and the optimal contract delivers larger efficiency gains over the alternative contract. The overall level of gains is high indicating that the equity-like contract is very inefficient. In fact, for some parameter values there does not exist and equity-like contract which yields positive profits to the investors and provides incentives for effort.

2.6.2 The optimal contract versus a "first loss piece"

The next alternative contract we consider is the so called "first loss piece." In the most simple form of this structure, the mortgages are pooled and two tranches are sold to investors, a senior tranche, and a junior tranche or first loss piece. The underwriter retains a sufficient fraction of the junior tranche to maintain incentive compatability. To define this contract we let Y_t and Z_t be the cumulative cash flow paid to the senior and junior tranches respectively by time t. The cash flow from the mortgages is distributed to the tranches according to the following rules

$$dY_t = (N - \max\{n, D_t\})udt + RdD_t \tag{18}$$

$$dZ_t = \max\{n - D_t, 0\}udt \tag{19}$$

for some $n < N$ which determines the size of junior tranche. The contract described by equations (18) and (19) states that when there have been less than n defaults, the payment to the senior tranche is constant and the junior tranche absorbs all defaults until retired. The underwriter retains a fraction α of the junior tranche as well as receiving the proceeds from the sale of the senior tranche and a $1 - \alpha$ fraction of the junior tranche.

The optimal contract of this form is the one featuring the smallest size of the first loss piece, n, where both the incentive compatibility and participation constraints bind and $\alpha \leq 1$. The value of the first loss piece of size n under the underwriters discount factor given $e = i$ for $i \in 0, 1$ is

$$E\left[\sum_{k=1}^{n}\int_0^{\tau_k} ue^{-\gamma t}dt | e = i\right] = \sum_{k=1}^{n}\frac{u\lambda_i}{\gamma}\int_0^{\infty}(1 - e^{-\lambda_i t})^{k-1}e^{-(N-k)\lambda_t}e^{-\lambda_i t}(1 - e^{-\gamma t})dt \tag{20}$$

$$= 1 - \frac{N!\lambda_i^n}{(N-n)!\gamma}\frac{1}{\prod_{k=0}^{n-1}(\gamma + (N-k)\lambda_i)}. \tag{21}$$

Hence the optimal first loss piece is given by the smallest n such that

$$\alpha = \frac{(N-n)!\gamma N}{!\lambda_i^n}\left(\frac{1}{\prod_{k=0}^{n-1}(\gamma + (N-k)\lambda_H)} - \frac{1}{\prod_{k=0}^{n-1}(\gamma + (N-k)\lambda_L)}\right)^{-1} \leq 1 \tag{22}$$

Again, we assume the investors are competitive and hence make zero profits. Figure 3 shows the gain in underwriter value (in basis points) from using the optimal contract versus the first loss piece structure.

[Figure 3 about here.]

The gain in efficiency from using the optimal contract versus the first loss piece depends on the parameters in the same way as the alternative contract in the previous section. However, the over level of gain is much smaller since the first loss piece is a more efficient contract. For parameter values with small gains from securitization and a small moral hazard problem, the first loss piece is a very good alternative to the optimal contract. However, when γ is large or λ_H is small, the gain from using the optimal contract is substantial, for example more 12 basis points when $\gamma = 7.5$ and $\lambda_H = .525\%$. In light of Figure 3, we can conclude that in many instances the first loss piece is a very effective incentive contract. The main reason for this conclusion is that the first loss piece, as presented above, accelerates payments relative to other contracts, such as the contract consider in the previous section. As such, any policy that hopes to improve incentives in the market for MBS while maintaining efficiency by requiring underwriters to retain a portion of the first loss piece must make sure that this position has a relatively short maturity.

3 Extensions

3.1 An Initial Capital Constraint

In this section we consider the optimal contracting problem when the underwriter faces an initial capital constraint. This case arises when the underwriter does not have sufficient internal capital to originate the mortgages. The structure of the contract remains largely unchanged, except for the addition of a transfer at $t = 0$.

Suppose the underwriter requires K in initial capital at $t = 0$ to originate the mortgages. Specifically we add the following constraint

$$dX_0 \geq K \tag{23}$$

to Definition 1. The following proposition states the solution to the optimal contracting problem.

Proposition 3 *Let a_0 be the promised value to the underwriter net of origination costs K. The optimal contract X_t that satisfies constraint (23) is given by $dX_0 = K$ and Proposition 1 for $t > 0$.*

Proof See appendix ■

The intuition behind Proposition 3 is essentially the same as for Proposition 1. After providing the initial required capital K, the contract makes all subsequent transfers dependent on the realization of the first default time to provide incentives to the underwriter to exert effort. Once a long enough period has passed before the first default time, it is optimal to completely pay off the underwriter due to the difference in discount rate between the investors and the underwriter.

One attractive feature of the optimal contract is it's relatively short maturity. Even for high values of K (the upfront capital required to originate the mortgages) the waiting period to required to provide incentives to the underwriter is short. Figure 4 shows t_0 as a function of λ_H for various levels of K holding other parameters constant.

[Figure 4 about here.]

When λ_H is relatively close to λ_L and K is relatively large, the moral hazard problem is relatively more severe, since the inference problem is more difficult and the continuation value of the underwriter is smaller. However, even for $K = 99.9\frac{N(u+\lambda_L R)}{r+\lambda_L}$ and $\lambda_H - \lambda_L = .5\%$, the maturity of the optimal contract is less than 2 years. When contrasted to the potentially infinitely lived mortgages of the model, this contract maturity is very short. This feature is appealling in practice since it implies that even when facing severe moral hazard problems, investors in MBS can enforce underwriter effort with a short-lived contract. In otherwords, even though a mortgage pool may last for 30 years, the underwriters position can be short-lived while still providing incentives to exert effort.

3.2 Partial Effort

The underwriter is endowed with an all-or-nothing effort technology in our model. In other words, the agent must choose a single effort level for each mortgage. Alternatively, we can consider a specification in which the underwriter can apply effort to some and not all of the mortgages. The optimal contract remains unchanged when we allow for such a deviation given a reasonable restriction on the cost of effort c detailed below. Specifically, suppose the underwriter can choose to apply effort to $n \leq N$ of the mortgages resulting in a pool of N mortgages in which n mortgages default according to an exponential distribution with parameter λ_L, and $N - n$ mortgages default according to an exponential distribution with parameter λ_H. We will refer to such a strategy as applying partial effort. We alter notation to let a partial effort strategy be denoted $e = n$ so that $e = 0$ corresponds to zero effort and $e = N$ corresponds to full effort. The cost of applying a partial effort strategy is given by $C = ce$. We modify Definition 1 to include incentive compatibility constraints for each possible partial effort strategy as follows

Definition 2 *Given a promised utility a_0 to the underwriter, a contract $\{x_n\}_{n=0}^N$ that implements $e = N$ is* optimal *if it solves the following problem*

$$b(a_0) = \min_{dX_t \geq 0} E\left[\int_0^\infty e^{-\gamma t} dX_t \big| e = n\right] \tag{24}$$

such that

$$E\left[\int_0^\infty e^{-\gamma t} dX_t \big| e = m\right] - c \cdot m \leq E\left[\int_0^\infty e^{-\gamma t} dX_t \big| e = N\right] - c \cdot N \qquad \textit{for all } m \tag{25}$$

and

$$a_0 \leq E\left[\int_0^\infty e^{-\gamma t} dX_t \big| e = N\right] - c \cdot N. \tag{26}$$

Proposition 4 *The optimal contract is the same as that of Proposition 1.*

Proof See appendix ■

Proposition 4 relies on the fact that under the contract detailed in Proposition 1 and the flexible effort technology, the underwriter does not gain more by deviating to a strategy in which she applies effort to some and not all of the mortgages than a strategy in which she applies zero effort. This implies that the original optimal contract satisfies the additional incentive compatibility constraints ruling out partial effort deviations since it satisfies the original incentive compatibility constraint ruling out the zero effort strategy. Moreover, the space of contracts that satisfy (25) is contained in the set of contracts which satisfy (IC) since constraint (IC) is contained in (25). Since the contract of Proposition 1 satisfies the additional constraints induced by partial effort strategies and is optimal over the larger space of contracts satisfying constraint (25), it is optimal over the smaller space of contracts satisfying the additional constraints.

3.3 Adverse selection

Throughout the above analysis, we have focused on a moral hazard setting in which the underwriter makes a hidden effort choice that affects the risk of the mortgages she sells to the investors. In this section, we show how our model can be altered to address an adverse selection problem in which the underwriter is endowed with mortgages with a given default risk and wishes to sell them to secondary market investors. This setting is similar to other papers in the literature, notably DeMarzo (2005). Our main result is that the optimal contract for an underwriter with low risk mortgages remains qualitatively unchanged.

In a standard adverse selection model of asset backed security design, the issuer, in our case the underwriter, has private information about the assets she wishes to sell. We model this by assuming that the underwriter has N mortgages to sell, all of which are either low risk or high risk where mortgage cash flows are given in section 2.1. We look for a separating equilibrium in which the underwriter can signal the quality of her mortgages by choosing a contract. Specifically, we look for a pair of contracts (X^H, X^L) such that an underwriter with high risk mortgages chooses the contract X^H, and an underwriter with low risk mortgages chooses the contract X^L. As such, we can view the choice of contract as the signal by the agent. Formally, we define the *separating equilibrium* as follows

Definition 3 *A separating equilibrium is a pair (X^H, X^L) such that*

1. *The underwriter chooses X^H when she has λ_H type mortgages,*

$$X^H \in \arg\max_X E\left[\int_0^\infty e^{-\gamma t} dX_t | \lambda_H\right] \tag{27}$$

2. *The underwriter chooses X^L when she has λ_L type mortgages*

$$X^L \in \arg\max_X E\left[\int_0^\infty e^{-\gamma t} dX_t | \lambda_L\right] \tag{28}$$

3. *Investors earn zero expected profits*

$$E\left[\int_0^\infty e^{-rt} dX_t^H | X^H\right] = \frac{N(u + R\lambda_H)}{r + \lambda_H} \tag{29}$$

$$E\left[\int_0^\infty e^{-rt} dX_t^L | X^L\right] = \frac{N(u + R\lambda_L)}{r + \lambda_L} \tag{30}$$

Since all gains from securitization arise by front loading payment to the underwriter, and investors are competitive and earn zero profits , we choose

$$dX_t^H = \begin{cases} \frac{N(u+\lambda_H)}{r+\lambda_H} & \text{for } t = 0 \\ 0 & \text{otherwise} \end{cases} \tag{31}$$

Of course separating equilibria with different X^H may exist, however, such equilibria will be Pareto dominated by equilibria with X^H described by equation (31).

In its current form, Definition 3 is not directly related to the contracting problem given in Definition 1. However we can draw a a relation between the two definitions by finding the *least cost sperating equilibrium*, which the is the equilibrium which satisfies Definition 3 and maximizes the expected payment to the underwriter when she has type λ_L mortgages. The least cost separating equilibrium thus solves the following problem

$$\max_X E\left[\int_0^\infty e^{-\gamma t} dX_t|\lambda_L\right] \tag{32}$$

$$\text{s.t.} E\left[\int_0^\infty e^{-\gamma t} dX_t|\lambda_H\right] \leq \frac{N(u+\lambda_H)}{r+\lambda_H}$$

$$E\left[\int_0^\infty e^{-rt} dX_t|\lambda_L\right] \leq \frac{N(u+\lambda_L)}{r+\lambda_L}$$

Problem (32) is the dual problem of (I) and thus we can apply Proposition 1 to get the following proposition.

Proposition 5 *The least cost separating equilibrium is given by* $dX_0^H = N\frac{u+\lambda_H R}{r+\lambda_H} = a_0^H$ *and* $dX_t^H = 0$ *for* $t > 0$*, and*

$$dX_t^L = \begin{cases} 0 & \text{if } t \neq t_0 \\ y_0 \mathbb{I}(t_0 \leq \tau_1) & \text{if } t = t_0 \end{cases}$$

where

$$t_0 = \frac{1}{N(\lambda_H - \lambda_L)} \frac{a_0^L}{a_0^H}$$

$$y_0 = e^{(\gamma + N\lambda_L)t_0} a_0^L.$$

where

$$a_0^L = \left(N\frac{u+\lambda_H R}{r+\lambda_H}\right)^{1/(1+\eta)} (a_0^H)^{\eta/(1+\eta)} \tag{33}$$

and

$$\eta = \frac{\gamma - r}{N(\lambda_H - \lambda_L)} \tag{34}$$

Proof Follows directly from Proposition 1. ■

In a static framework, such as that of DeMarzo (2005), the signal space is limited to the fraction the underwriter chooses to retain of some security backed by the mortgages. In our setting, the signal space is much richer since it includes any payment profile through time that is adapted to the information filtration generated by the cumulative default process of mortgages. Accordingly, the most efficient signal in our model uses timing as a central feature rather than the fraction of some high risk tranche retained by the underwriter.

4 Conclusion

This paper studies a model of mortgage securitization in a moral hazard setting which highlights an important aspect of contracting in mortgage markets, namely that information is reveled over time. We find that the optimal contract is a lump sum payment from the investors to the underwriter conditional on a period of no defaults. We give a simple implementation of the contract using a margin account and a CDS. By bundling a CDS with the mortgage pool, the underwriter can in effect signal high effort by selling insurance which is priced fairly conditional on high effort.

If we view the contracting problem as essentially a hypothesis testing problem, a natural trade-off between the power of the test used in the contract and the amount of time needed to implement the test. This intuition gives rise to three new findings. First, the timing of payments to the underwriter is a key mechanism providing incentive to the mortgage underwriter to exert effort. Second, the optimal contract maturity can be short while the mortgages are long lived. Finally, mortgage pooling, the process whereby mortgages are bundled to create a collateral pool, can arise from an *information enhancement effect*: by conditioning all payments on an aggregate signal taken from the entire pool of mortgages, rather than observing each mortgage individually, the optimal contract achieves the best possible trade-off between the testing power and the testing time.

In addition, we show that a large fraction of the surplus generated from securitization depends on the timing of payments to the underwriter by comparing the optimal contract to some realistic alternative contracts. An important policy implication of this conclusion is that forcing underwriters to retain a portion of the first loss piece may destroy the benefits of securitization, while a more efficient contract which uses payment timing as an incentive mechanism may not.

We consider three important extensions to the basic model. The main result is that the nature the our optimal contract is robust to altering the contracting problem in plausible ways. The first extension introduces an initial capital constraint that arises due to the underwriter lacking sufficient initial capital to originate the mortgages. Up to an important restriction on the parameters of the model, the qualitative features of the contract remain unchanged after introducing this constraint. The only significant difference being a time zero transfer from the investors to the underwriter exactly equal to the capital required to originate the mortgages. After time zero, the optimal contract is identical to the optimal contract without the initial capital constraint except for the magnitude and timing of the one time lump sum transfer. For such a contract to be optimal, we require that the promised value of the underwriter be greater than the capital required to underwrite the mortgages. This restriction guarantees that the underwriter will have sufficient continuation value for the contract after date zero to provide incentives.

The next extension we consider is a flexible effort technology. It is possible that the underwriter could apply costly underwriting practices to some and not all of the mortgages. For concave costs, our contract is robust to such an effort technology and remains optimal.

In the third extension, we show how our result can be adapted to an adverse selection setting. In this setting, we view the chose of structure of the contract as signal of the underwriter's private information. The problem then becomes finding the most efficient signal, and can easily be mapped into our original contracting problem.

The extensions we consider are certainly not exhaustive. For example, we could have considered risk averse underwriters or investors, correlation among mortgage defaults, or time varying default rates. These additional features would potentially change the specific lump sum form of the optimal contract. Regardless, the key contracting intuition will be unchanged. In each case, the dynamic nature of information revelation will cause the optimal contract to take the form of the best trade-off between test power and test timing.

A Appendix - Assumptions

A.1 Necessary conditions for the optimality of high effort

Assumption 1 *The parameters of the model satisfy*

$$N\frac{u+\lambda_L R}{r+\lambda_L} - \left(\frac{a_0(N)+c}{a_0(N)}\right)^{\frac{\gamma-r}{N(\lambda_H-\lambda_L)}}(a_0(N)+c) \geq N\frac{u+\lambda_H R}{r+\lambda_H} - a_0. \tag{35}$$

The left hand side of inequality (**??**) is the profit the investors receive if the contract implements high effort and is optimal and is the difference of two terms. The first term is the present value of the mortgages under the investors discount rate. The second term is the cost of the optimal contract. The right hand side of (**??**) investor profit from a contract which does not provide incentives to exert effort. If the parameters of the model violate Assumption (**??**), then it is not optimal for the investors to provide an incentive compatible contract. When no asymmetric information exists, high effort is efficient from the standpoint of the investors if

$$N\frac{u+\lambda_L R}{r+\lambda_L} - c \geq N\frac{u+\lambda_H R}{r+\lambda_H}$$

therefore when

$$\left(\frac{a_0(N)+c}{a_0(N)}\right)^{\frac{\gamma-r}{N(\lambda_H-\lambda_L)}}(a_0(N)+c)-a_0 > N\frac{u+\lambda_L R}{r+\lambda_L} - N\frac{u+\lambda_H R}{r+\lambda_H} > c$$

effort will be undersupplied relative to first best.

A.2 Low effort leads to a mixture of mortgage risk types.

As an to alternative basic model we present in the body of the paper. We could assume that low effort leads to a mixture of mortgage risk types, where a given mortgage is high risk with probability ρ and low risk otherwise. In this case the underwriters payoff from exerting low effort is

$$E\left[\int_0^\infty e^{-\gamma t} dX_t \big| e=0\right] = \sum_{n=0}^{N}\binom{N}{n}\rho^n(1-\rho)^{n-1}E\left[\int_0^\infty e^{-\gamma t} dX_t \big| n \text{ mortgages are high risk}\right].$$

So the incentive compatibility constraint is

$$\sum_{n=0}^{N}\binom{N}{n}\rho^n(1-\rho)^{n-1}E\left[\int_0^\infty e^{-\gamma t} dX_t \big| n \text{ mortgages are high risk}\right] \leq E\left[\int_0^\infty e^{-\gamma t} dX_t \big| e=0\right] - C.$$

Using similar intuition as the solution to the original contracting problem, both the incentive compatibility and participation constraints should bind and there should be an optimal contract of the form presenting in Proposition 1. Hence, we look for a solution $(\tilde{t}_0, \tilde{y}_0)$ to the following system of equations

$$\tilde{y}_0 e^{-(\gamma+N\lambda_L)\tilde{t}_0} = a_0 + c \tag{36}$$

$$\sum_{n=0}^{N}\binom{N}{n}\rho^n(1-\rho)^{n-1}\tilde{y}_0 e^{-(\gamma+n\lambda_H+(N-n)\lambda_L)\tilde{t}_0} = a_0 \tag{37}$$

If $\rho = 1$, then the solution is given by Proposition 1. If $\rho < 1$, suppose the parameters of the model satisfy $(1-\rho)a_0^{-1/N} \leq 1$ and let

$$q = 1-(1-\rho)a_0^{-1/N},$$

then a solution to equations (36) and (37) is given by

$$\tilde{t}_0 = \frac{1}{\lambda_H - \lambda_L}\left(\ln\left(\frac{1-q}{1-\rho}\right) - \ln\left(\frac{q}{\rho}\right)\right)$$
$$\tilde{y}_0 = e^{(\gamma+N\lambda_L)\tilde{t}_0}$$

One can then verify that this indeed is an optimal contract under the relaxed assumption on effort using techniques similar to the proof of Proposition 1. Note that although the precise functional forms of $\tilde{t}_0$ and $\tilde{y}_0$ are different than in the original optimal contract the basic structure is the same. The only difference is the added dependence on ρ. All else equal, a lower ρ corresponds to a higher deviation payoff , and hence a more severe moral hazard problem and a larger t_0. The dependence of the optimal contract maturity on the severity of the moral hazard problem is already present in the basic model, so that the additional complexity added by assuming that low effort can lead to a mixture of mortgage risk types does not increase the richness of the model.

B Appendix - Proofs

Proof of Proposition 1 Let X_t^* denote the proposed contract. First observe that X_t^* satisfies constraints (PC) since

$$\begin{aligned} E\left[\int_0^\infty e^{-\gamma t} dX_t^* \middle| e=1\right] &= E[e^{-\gamma t_0}\mathbb{I}(t_0 \le \tau_1)e^{(\gamma+N\lambda_L)t_0}(a_0+C)|e=1] \\ &= P(t_0 \le \tau_1|e=1)e^{N\lambda_L t_0}(a_0+C) \\ &= a_0 + c. \end{aligned}$$

Next observe that the proposed contract satisfies constraint (IC) since

$$\begin{aligned} E\left[\int_0^\infty e^{-\gamma t} dX_t^* \middle| e=0\right] &= E[e^{-\gamma t_0}\mathbb{I}(t_0 \le \tau_1)e^{(\gamma+N\lambda_L)t_0}(a_0+C)|e=0] \\ &= P(t_0 \le \tau_1|e=0)e^{N\lambda_L t_0}(a_0+C) \\ &= e^{-N(\lambda_H-\lambda_L)t_0}(a_0+C) = a_0. \end{aligned}$$

Now suppose X_t is an arbitrary incentive compatible contract. Let $X_t^0 = X_t(\omega_0)$ where $\omega_1 \in \{\tau_1 \ge t\}$. Note that $X_t^0 : \mathbb{R}^+ \to \mathbb{R}^+$ is not a random variable, rather it is a function of time which denotes the payment at time t given no defaults have yet occurred. We have

$$\begin{aligned}
E\left[\int_0^{\tau_1} e^{-\gamma t} dX_t^0 \middle| e=0\right] &= \int_0^{\infty} e^{-(\gamma+N\lambda_H)t} dX_t^0 \\
&= \int_0^{\infty} e^{N(\lambda_L-\lambda_H)t} e^{-(\gamma+N\lambda_L)t} dX_t^0 \\
&\geq e^{-N(\lambda_H-\lambda_L)t_0} \int_0^{\infty} \left(N(\lambda_H-\lambda_L)(t_0-t)+1\right) e^{-(\gamma+N\lambda_L)t} dX_t^0 \\
&= \frac{a_0}{a_0+C} E\left[\int_0^{\tau_1} \left(N(\lambda_H-\lambda_L)(t_0-t)+1\right) e^{-\gamma t} dX_t^0 \middle| e=1\right] \qquad (38)
\end{aligned}$$

where the second to last step follows from the fact that $e^{-N(\lambda_H-\lambda_L)t} \geq e^{-N(\lambda_H-\lambda_L)t_0}(N(\lambda_H - \lambda_L)(t_0-t)+1)$ since $e^{-N(\lambda_H-\lambda_L)t}$ is convex.

Now let $X_t^n(\mathbf{s}) = X_t \mathbb{I}(\omega_n)$ where $\omega_n \in \{s_1 = \tau_1, \ldots, s_n = \tau_n, t \leq \tau_{n+1}\}$ for $n = 1, \ldots, N$. For convenience let $\tau_{N+1} = \infty$. Note again that $X_t^n(\mathbf{s}) : \mathbb{R}^{n+} \times [s_n, \infty) \to \mathbb{R}^+$ is not a random variable, rather it is a function of time and the vector $\mathbf{s}$ which denotes the payment at time t given the first n defaults occurred at times given by the vector $\mathbf{s}$ and the $(n+1)$th default has not yet occurred. Let $\mathcal{A} = \{\mathbf{s} \in \mathbb{R}^{n+} | s_1 \leq s_2 \leq \cdots \leq s_n\}$ and $d\mathcal{A} = ds_n \cdots ds_1$. We have

$$\begin{aligned}
&E\left[\int_{\tau_n}^{\tau_{n+1}} e^{-\gamma t} dX_t^n(\tau_1, \ldots, \tau_n) \middle| e=0\right] \\
&= \frac{\lambda_H^n N!}{(N-n)!} \int_{\mathcal{A}} \int_{s_n}^{\infty} \exp\left(-(\gamma+(N-n)\lambda_H)t - \sum_{k=1}^{n} \lambda_H s_k\right) dX_t^n(\mathbf{s}) d\mathcal{A} \\
&= \frac{\lambda_H^n N!}{(N-n)!} \int_{\mathcal{A}} \int_{s_n}^{\infty} exp\left(-(\gamma+N\lambda_H)t + \sum_{k=1}^{n} \lambda_H (t-s_k)\right) dX_t^n(\mathbf{s}) d\mathcal{A} \\
&\geq \frac{\lambda_L^n N!}{(N-n)!} \int_{\mathcal{A}} \int_{s_n}^{\infty} exp\left(-(\gamma+N\lambda_H)t + \sum_{k=1}^{n} \lambda_H (t-s_k)\right) dX_t^n(\mathbf{s}) d\mathcal{A} \\
&\geq \frac{\lambda_L^n N!}{(N-n)!} \int_{\mathcal{A}} \int_{s_n}^{\infty} \exp\left(-(\gamma+N\lambda_H)t + \sum_{k=1}^{n} \lambda_L (t-s_k)\right) dX_t^n(\mathbf{s}) d\mathcal{A} \\
&\geq e^{-N(\lambda_H-\lambda_L)t_0} \frac{\lambda_L^n N!}{(N-n)!} \times \\
&\int_{\mathcal{A}} \int_{s_n}^{\infty} \left(N(\lambda_H-\lambda_L)(t_0-t)+1\right) \exp\left(-(\gamma+N\lambda_L)t - \sum_{k=1}^{n} \lambda_L (s_k-t)\right) dX_t^n(\mathbf{s}) d\mathcal{A} \\
&= \frac{a_0}{a_0+C} E\left[\int_{\tau_n}^{\tau_{n+1}} \left(N(\lambda_H-\lambda_L)(t_0-t)+1\right) e^{-\gamma t} dX_t^n(\tau_1, \ldots, \tau_n) \middle| e=1\right] \qquad (39)
\end{aligned}$$

where the second to last step again follows from the fact that $e^{-N(\lambda_H-\lambda_L)t}$ is convex.

Note that by construction we have

$$X_t|\{D_t\} = \sum_{n=0}^{N} X_t^n \mathbb{I}(\tau_n < t \leq \tau_{n+1}),$$

hence inequalities (38), (39) and the law of iterated expectations imply

$$E\left[\int_0^\infty e^{-\gamma t} dX_t|e=0\right] \geq \frac{a_0}{a_0+C} E\left[\int_0^\infty (N(\lambda_H - \lambda_L)(t_0 - t) + 1)e^{-\gamma t} dX_t|e=1\right]. \tag{40}$$

Combining constraints (IC) and (PC) with inequality (40) we are left with the following inequality

$$\frac{1}{a_0+C} E\left[\int_0^\infty t e^{-\gamma t} dX_t|e=1\right] \geq t_0 \tag{41}$$

Now consider the cost of this contract to the investors. Using a similar argument as above and the fact that $e^{-(\gamma-r)t}$ is convex we have

$$E\left[\int_0^\infty e^{-rt} dX_t|e=1\right] \geq e^{(\gamma-r)t_0}\left[E\left[\int_0^\infty (\gamma-r)(t-t_0)e^{-\gamma t} dX_t|e=1\right] + E\left[\int_0^\infty e^{-\gamma t} dX_t|e=1\right]\right]. \tag{42}$$

But inequality (41) implies that the first term on the right hand side of inequality (42) is greater than zero, which together with constraint (PC) implies

$$E\left[\int_0^\infty e^{-rt} dX_t|e=1\right] \geq e^{(\gamma-r)t_0}(a_0+C).$$

But $e^{-(\gamma-r)t_0}(a_0+C)$ is the cost to the investors of the proposed contract. So we have shown that the proposed contract costs less (or the same) to the investors than any alternative contract that satisfies (IC) and (PC), hence the proposed contract is optimal.

■

Proof of Proposition 3 We can rewrite the optimal contracting problem as follows

$$b(a_0) = \min_{dX_t \geq 0}\left\{K + E\left[\int_0^\infty e^{-rt} d\hat{X}_t|e=1\right]\right\}$$

such that

$$
\begin{aligned}
E\left[\int_0^\infty e^{-\gamma t} d\hat{X}_t | e=0\right] E\left[\int_0^\infty e^{-\gamma t} d\hat{X}_t | e=1\right] - c,& \\
a_0 \leq K + E\left[\int_0^\infty e^{-\gamma t} d\hat{X}_t | e=1\right] - c,& \\
d\hat{X}_t = dX_t - \mathbb{I}(t=0)K& \\
dX_0 \geq K&
\end{aligned}
$$

Thus the solution follows directly from Proposition 1.

■

Proof of Proposition 4 Suppose X_t is the contract detailed in Proposition 4. Let n be the implented level of effort and let m be some level of effort such that $0 < m < n$, we have

$$
\begin{aligned}
E\left[\int_0^\infty e^{-\gamma t} dX_t | e=m\right] - c \cdot m &= P(\tau_1 \geq t_0 | e=m) e^{-\gamma t_0} y_0 - c \cdot m \\
&= e^{-(m\lambda_L + (N-m)\lambda_H)t_0} e^{(N\lambda_L)t_0} (a_0 + c \cdot N) - c \cdot m \\
&= e^{-(N-m)(\lambda_H - \lambda_L)t_0} (a_0 + c \cdot N) - c \cdot m \\
&= \left(\frac{a_0 + c \cdot n}{a_0}\right)^{-\frac{N-m}{N}} (a_0 + c \cdot n) - c \cdot m \\
&= a_0 \left(\left(\frac{a_0 + c \cdot N}{a_0}\right)^{\frac{m}{N}} - \frac{c \cdot m}{a_0}\right) \\
&< a_0 \left(\frac{m}{N}\left(\frac{a_0 + c \cdot N}{a_0} - 1\right) + 1 - \frac{c \cdot m}{a_0}\right) \\
&= a_0 \\
&= E\left[\int_0^\infty e^{-\gamma t} dX_t | e=0\right]
\end{aligned}
$$

So that $e = 0$ is the most profitable deviation from $e = N$ given X_t. The result follows from Proposition 1

■

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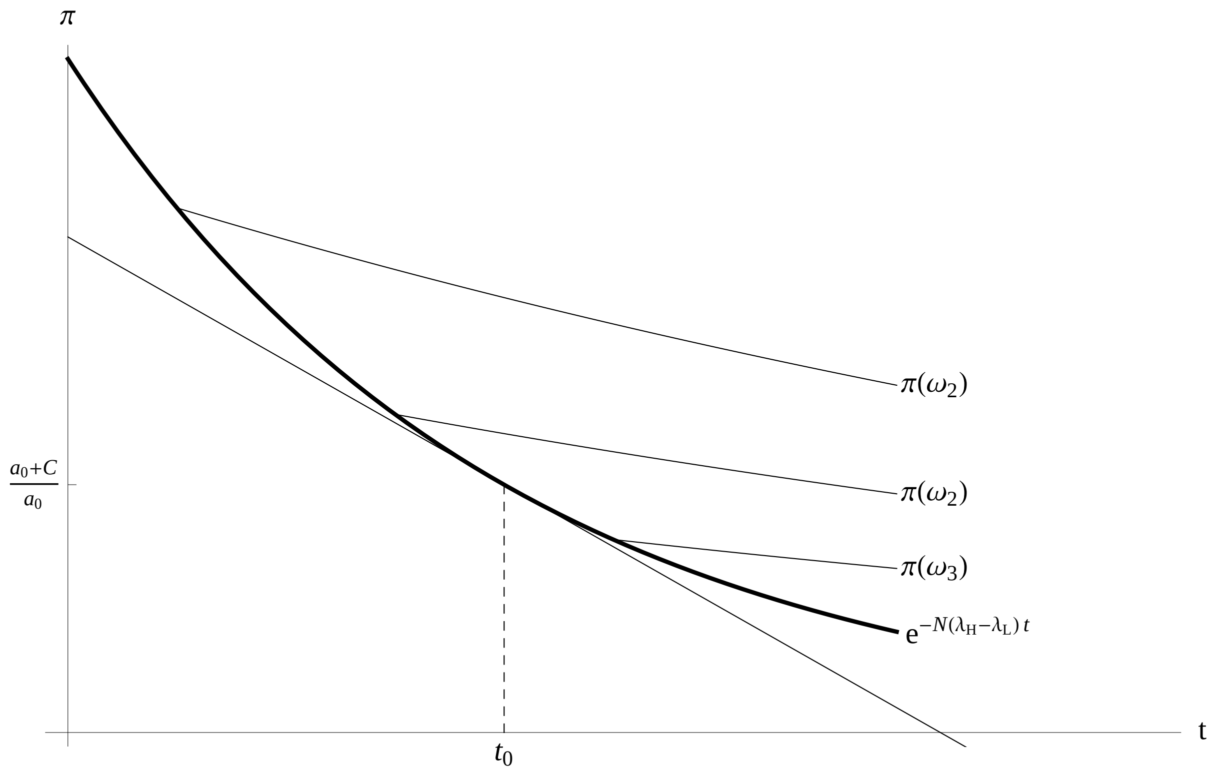

Figure 1: A plot of realizations of the Radon-Nikodym derivative $\pi(t)$ for different sample paths ω_1, ω_2, and ω_3. The thick curve is the lower bound on the Radon-Nikodym for all possible sample paths. The straight line is a linear approximation to the lower bound. Since the lower bound is convex, it dominates the straight line and we can use the straight line to find an inequality relating the expectations of underwriter value under high and low effort. This inequality is a useful characterization of the set of incentive compatible contracts.

Comparing the optimal contract to a fraction of the mortgage pool

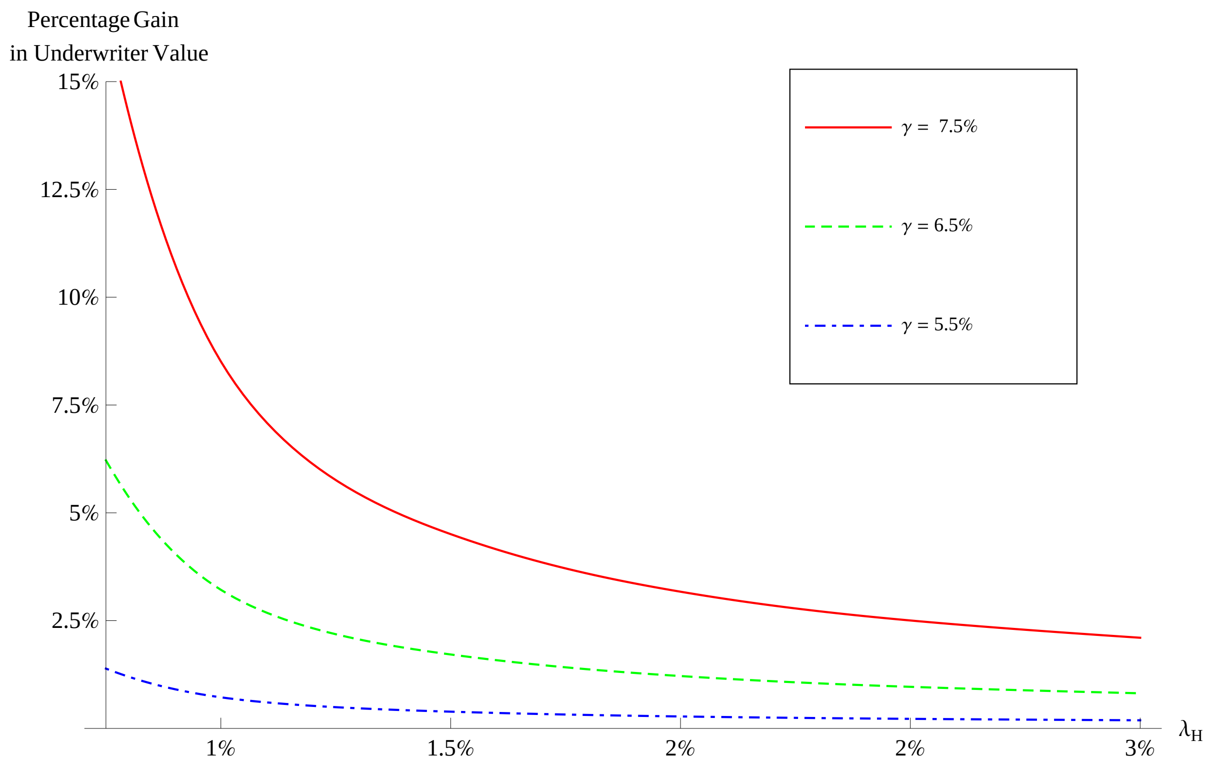

Figure 2: The percentage gain in underwriter value from using the optimal contract versus a contract in which the underwriter retains a fraction of the pool of mortgages. Parameter values : $r = 5\%$, $\lambda_L = .5\%$, $N = 100$, $R = (50\%)\frac{u}{r}$ and $c = (.25\%)\frac{Nu}{r}$

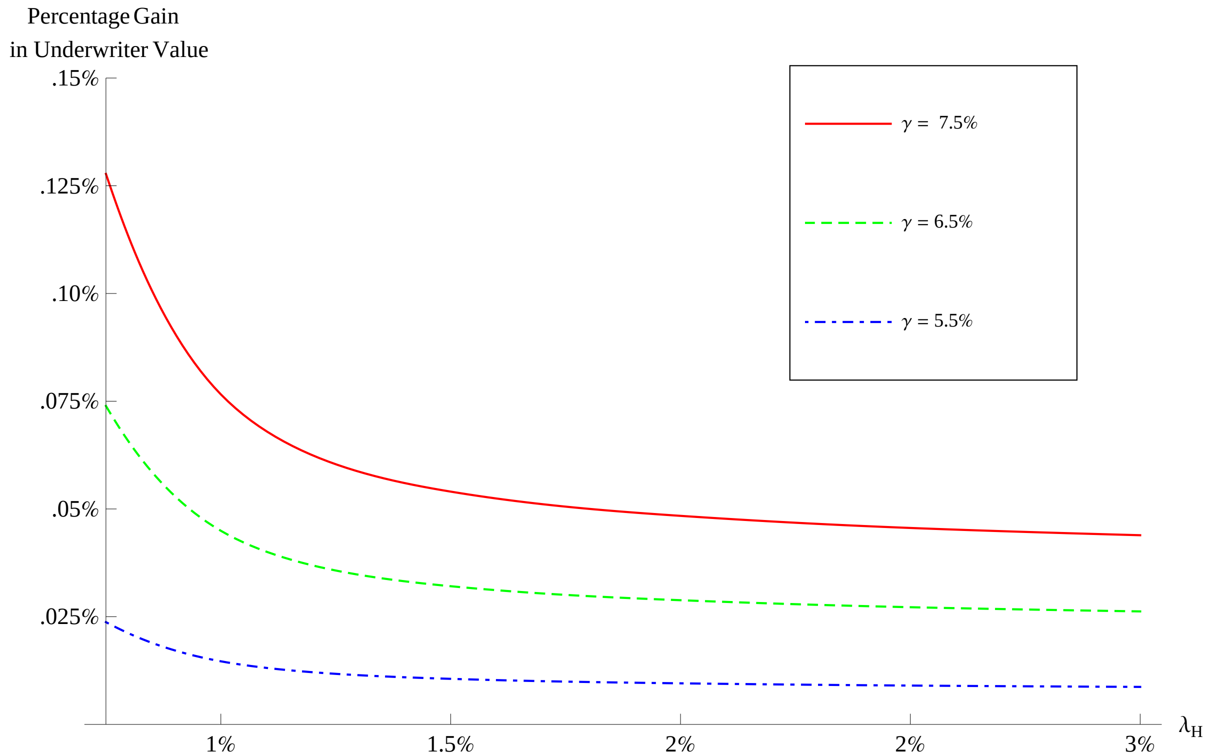

Figure 3: The percentage gain in underwriter value (reported in Basis Points) from using the optimal contract versus a contract in which the underwriter retains a fraction of the first loss piece. Parameter values : $r = 5\%$, $\lambda_L = .5\%$, $N = 100$, $R = (50\%)\frac{u}{r}$ and $c = (.25\%)\frac{Nu}{r}$

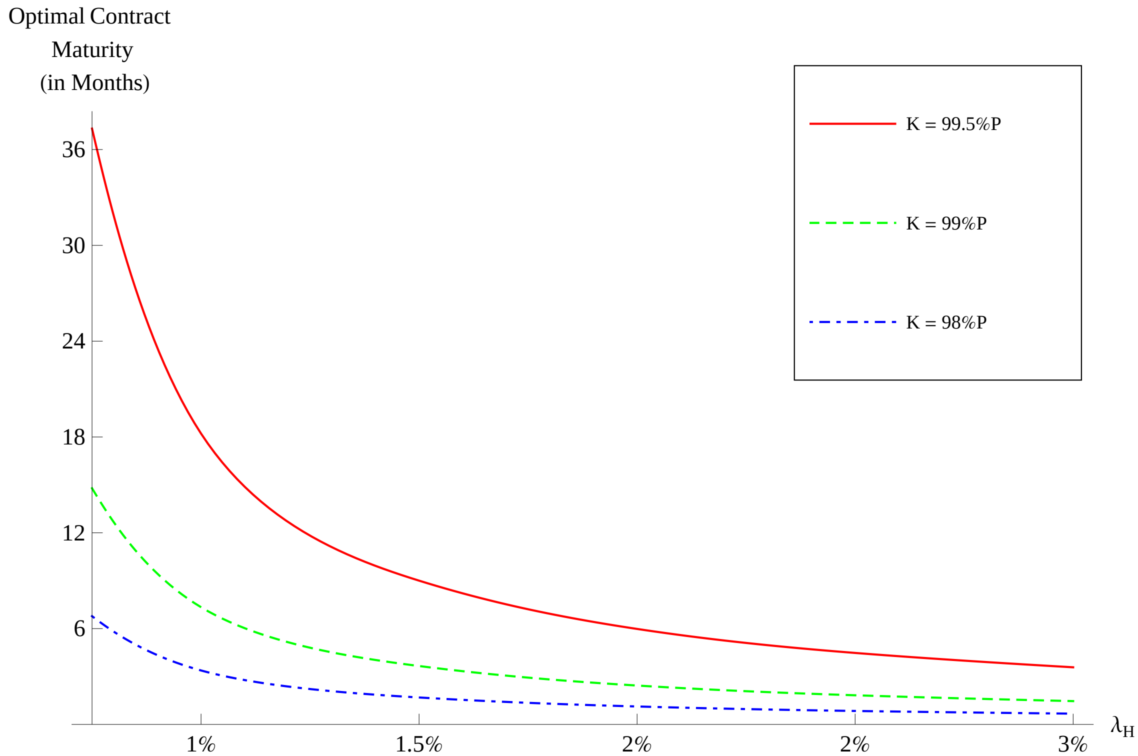

Figure 4: The maturity of the optimal contract t_0 for a range of parameters, where K is reported as a percentage price of the market value of the mortgage pool $P = \frac{N(u+\lambda_L R)}{r+\lambda_L}$. Parameter values $r = 5\%$, $\gamma = 6\%$ $\lambda_L = .5\%$, $N = 100$, $R = (50\%)\frac{u}{r}$ and $c = (.25\%)\frac{Nu}{r}$