Financial Development and the Patterns of International Capital Flows*

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Abstract

We develop a tractable two-country overlapping-generations model and show analytically that the cross-country differences in financial development can explain three recent empirical patterns of international capital flows: financial capital flows from relatively poor to relatively rich countries while foreign direct investment flows in the opposite direction; capital in the net term flows from poor to rich countries; despite of its negative net positions on international investment, the United States receives a positive net investment income. We also analyze how the patterns of capital flows may reverse along the convergence process of a developing country.

According to Matsuyama (Econometrica, 2004), in the presence of credit market imperfections, financial market globalization may lead to a steady-state equilibrium in which fundamentally identical countries end up with different levels of per capita output. We show that this *symmetry-breaking* property depends crucially on the assumption of the fixed investment size of entrepreneurial projects.

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1 Introduction

Standard international macroeconomics predicts that capital should flow from capital-rich countries, where the marginal return on investment is low, to capital-poor countries, where the marginal return is high. Furthermore, there would be no difference between gross and net capital flows, as capital movements are unidirectional.

The patterns of international capital flows observed in the past 20 years, however, stand in stark contrast to these predictions (Lane and Milesi-Ferretti, 2001, 2006, 2007). First, since 1998, the average per-capita income of countries running current account surpluses has actually been below that of the deficit countries, i.e., net capital flows have been "uphill" from poor to rich countries (Prasad, Rajan, and Subramanian, 2006, 2007). Second, many developing economies, including China, Malaysia, and South Africa, are net importers of foreign direct investment (hereafter, FDI) and net exporters of financial capital at the same time, while developed countries such as France, the United Kingdom, and the United States follow the opposite pattern (Ju and Wei, 2007). Third, despite of the negative net positions on international investment since 1986, the U.S. has been receiving a positive net investment income until 2005 (Gourinchas and Rey, 2007; Hausmann and Sturzenegger, 2007; Higgins, Klitgaard, and Tille, 2007).

Recent research offers two main explanations to these empirical facts. Devereux and Sutherland (2009) and Tille and van Wincoop (2008a,b) focus on the risk-sharing that investors can achieve by diversifying investment globally. International portfolio investment is determined by the cross-correlation patterns of aggregate shocks hitting individual economies. These models do not distinguish between FDI and portfolio investment.

The other strand of literature focuses on the relevance of domestic financial market imperfections on the patterns of international capital flows (Aoki, Benigno, and Kiyotaki, 2007; Caballero, Farhi, and Gourinchas, 2008; Smith and Valderrama, 2008). Matsuyama (2004) shows that in the presence of credit market imperfection, financial market globalization may lead to a steady-state equilibrium in which fundamentally identical countries end up with different levels of per capita output, a result he calls "symmetry breaking". In the steady state, financial capital flows from the poor to the rich country. Mendoza, Quadrini, and Ríos-Rull (2009) analyze the joint determination of financial capital flows and FDI in a heterogeneous-agent model with idiosyncratic endowment and investment risks. The precautionary savings motive is the main driving force of two-way capital flows in their model. Ju and Wei (2007) show in a static model that when both FDI and financial capital flows are allowed, all financial capital leaves the country where credit market imperfections are more severe, while FDI flows into this country. Thus, capital mobility allows investors to fully *bypass* the underdeveloped financial system.

Our paper extends the second strand of literature and provides a tractable, two-country, overlapping-generations model to explain the three recent empirical facts. Our model builds on the notion that individuals in an economy differ in the productivity (Kiyotaki and Moore, 1997). From an efficiency perspective, it would be desirable to transfer all capital to the most productive individuals to maximize aggregate output. In that case, the rates of return on loan and equity capital would be equal to the maringal return on investment. Due to financial frictions, however, the most productive individuals are subject to borrowing constraints. The constraint on the aggregate credit demand has a general equilibrium effect, keeping the rate of return on loans (hereafter, the loan rate) lower and, thus, the rate of return on equity capital (hereafter, the equity rate) higher than the marginal return on investment.[1] Thus, financial frictions distort the two interest rates and generate an equity premium in this deterministic model.

Following Matsuyama (2004), we take the tightness of the borrowing constraints as a measure of a country's level of financial development. In a more financially developed country, credit contracts can be enforced and borrowers can be monitored more effectively. Thus, the most productive individuals can borrow more from financial institutions. The two countries in our model differ fundamentally only in the level of financial development. Under international financial autarky, the interest rates depends on two factors. First, a lower aggregate capital-labor ratio implies a higher marginal return on investment and a higher equity and loan rate. We call this the *neoclassical* effect, because it arises from the concavity of the neoclassical production function with respect to the capital-labor ratio. Second, for a given capital-labor ratio, a lower level of financial development implies a lower aggregate crdit demand, which leads to a lower loan rate and a higher equity rate. We call this the *credit-demand* effect of financial development. Under certain assumptions, financial frictions only distort the interest rates but not production efficiency in our model.[2] Thus, financial development affects the interest rates only via the credit-demand channel but not the neoclassical channel in the steady state. Be specific, the loan rate is higher while the equity rate is lower in the more financially developed country.

Under full capital mobility, with a larger and deeper credit market, the more financially developed country receives net capital inflows and becomes richer than the less financially developed country in the steady state. In other words, net capital flows are "uphill" from the poor to the rich country.[3] Given the cross-country interest rate differentials under

[1] The overlapping-generations framework together with certain assumptions ensures that the aggregate credit supply is perfectly inelastic to the loan rate. Thus, we can isolate the effect of financial frictions on the aggregate credit demand and on the interest rates. Caballero, Farhi, and Gourinchas (2008) assume that agents have a constant probability of death, which also ensures the perfectly inelastic credit supply.

[2] Our qualitative results hold in an extension where financial frictions distort production effeciency as well as interest rates (von Hagen and Zhang, 2009). However, the model then becomes less tractable.

[3] "Uphill" capital flows take place between two countries with the same level of financial development in Matsuyama (2004), while two countries differ in the level of financial development in our model.

international financial autarky, financial capital flows from the poor to the rich country, while FDI flows in the opposite direction. Since the rich country receives a higher return on its FDI than it pays on its foreign debts, it gets a positive net investment income despite its negative net position of international investment. Essentially, the more financially developed country "exports" its financial service in the form of two-way capital flows and receives a positive net reward. As our first contribution, we show that the cross-country differences in financial development can explain the three empirical facts.

Ju and Wei (2007) assume cross-country differences in terms of capital and labor endowment, financial development, corporate governance, and property right protection for generating two-way capital flows, while the cross-country differences in financial development are sufficient to generate two-way capital flows in our model. The static model of Ju and Wei (2007) is useful for analyzing the immediate impacts of capital account liberalization, but not for studying the transitional and long-run effects, while our overlapping-generations model facilitates the short-run and the long-run analyses.

We also analyze a more general and realistic scenario where one country is more financially developed and in the steady state, e.g., the developed country, while the other country is less financially developed and below its steady state, e.g., the developing country, before capital account liberalization. Here, the credit-demand effect and the neoclassical effect jointly determine the cross-country interest rate differentials. If the initial capital-labor ratio in the developing country is very low, the neoclassical effect dominates the credit-demand effect so that the loan rate in the developing country is higher than in the developed country under international financial autarky. Immediately after capital account liberalization, both financial capital and FDI flow into the developing country. Thus, there are one-way gross capital flows and "downhill" net capital flows. If the initial capital-labor ratio in the developing country is moderately low, the credit-demand effect dominates the neoclassical effect. Immediately after capital account liberalization, financial capital flows "uphill" but its magnitude is dominated by "downhill" FDI. Thus, there are two-way gross capital flows and "downhill" net capital flows. In both cases, capital account liberalization facilitates net capital inflows and speeds up capital accumulation, which increases the convergence rate of the developing country.

If the initial capital-labor ratio in the developing country is slightly below its steady-state value, "uphill" financial capital flows dominates "downhill" FDI immediately after capital account liberalization. Thus, there are two-way gross capital flows and "uphill" net capital flows. Net capital outflows hamper capital accumulation and eventually the developing country converges to the steady state with the capital-labor ratio lower than under international financial autarky. Thus, as our second contribution, we show that for a developing country, the patterns of capital flows may change or even reverse along its convergence process and capital mobility has opposite effects on capital accumulation and welfare at the different stages of its convergence process.

In our model, financial capital flows affect the owners of credit capital and equity capital differently and so do FDI flows. Liberalizing capital flows affects the intergenerational income distribution due to transitional effects. This way, our model explains why capital account liberalization often encounters both support and opposition in a given country.

Our model setting differs from Matsuyama (2004) in only one aspect. We assume that the mass of individuals in a country who can invest is fixed, while the investment size of each project is endogenously determined. Thus, changes in the aggregate investment takes place on the intensive margin instead of on the extensive margin as in Matsuyama (2004). We prove that under capital mobility, there exists a unique and stable steady state in the presence of credit market imperfections in our model. Thus, countries with identical fundamentals have the same and unique steady state under capital mobility. As our third contribution, we show that Matsuyama's *symmetry-breaking* property depends critically on the assumption of a fixed investment size of entrepreneurial projects and, thus, changes in the aggregate investment taking place only on the extensive margin.

The rest of the paper is structured as follows. Section 2 sets up the model under international financial autarky. Section 3 proves the properties of the steady state and the patterns of international capital flows under capital mobility. Section 4 concludes with the main findings. Appendix collects the technical proofs and relevant discussions.

2 The Model under International Financial Autarky

We use an overlapping-generations model closely related to Matsuyama (2004). The world economy consists of two countries, Home (H) and Foreign (F). There are two types of goods, a final good, which is internationally tradable and serves as the numeraire, and a capital good, which is not traded internationally. The price of the capital good in country $i \in \{H, F\}$ and period t is denoted by v_t^i. The final good can be either consumed or transformed into capital goods. At the beginning of each period, final goods Y_t^i are produced with capital goods K_t^i and labor L_t^i in a Cobb-Douglas fashion. Capital goods fully depreciate after production. Capital goods and labor are priced at their respective marginal products in terms of final goods. To summarize,

$$Y_t^i = \left(\frac{K_t^i}{\alpha}\right)^{\alpha} \left(\frac{L_t^i}{1-\alpha}\right)^{1-\alpha}, \qquad \text{where} \qquad \alpha \in (0,1), \tag{1}$$

$$v_t^i K_t^i = \alpha Y_t^i \qquad \text{and} \qquad w_t^i L_t^i = (1-\alpha) Y_t^i. \tag{2}$$

There is no uncertainty in the economy. In this section, we assume that capital flows are not allowed between the two countries.

In both countries, the population consists of two generations, the old and the young, which live for two periods each. There is no population growth and the population size of each generation in each country is normalized to one. Agents consume only when old. Young agents are endowed with a unit of labor which they supply inelastically to the production of final goods $L_t^i = 1$ at the wage rate w_t^i in period t. Each generation consists of two types of agents of mass η and $1-\eta$, respectively, which we call *entrepreneurs* and *workers*. Only young entrepreneurs are endowed with the productive projects and it takes one period to produce capital goods using final goods.

Consider any particular worker born in period t. With no other investment opportunity available to him[4], the worker lends his entire labor income inelastically to the credit market at a gross interest rate of r_t^i in period t to finance his consumption in period $t+1$,

$$c_{t+1}^{i,w} = w_t^i r_t^i. \tag{3}$$

Consider any particular entrepreneur born in period t. The entrepreneur invests i_t^i units of final goods into his project in period t and produces Ri_t^i units of capital goods in period $t+1$. Given the gross loan rate of r_t^i, he finances the investment i_t^i with the debt $z_t^i = i_t^i - w_t^i$ and the equity capital, w_t^i. Due to limited commitment problems, however, he can borrow only against a fraction of the project revenues,

$$r_t^i z_t^i = r_t^i (i_t^i - w_t^i) \leq \theta^i R i_t^i v_{t+1}^i. \tag{4}$$

Following Matsuyama (2004, 2007, 2008), we regard $\theta^i \in (0,1]$ as a measure of the level of financial development in country i. That is, θ^i is higher in countries with more sophisticated financial and legal systems, better creditor protection, and more liquid asset market. Thus, θ^i captures a wide range of institutional factors.[5] We assume that the two countries differ only in the level of financial development, i.e., $0 < \theta^H < \theta^F \leq 1$.

Let $\lambda_t^i \equiv \frac{i_t^i}{w_t^i}$ denote the investment-equity ratio of the entrepreneurial project and I_t^i denotes the aggregate project investment in country i and period t. Under international financial autarky, the credit market equilibrium condition,

$$\eta(i_t^i - w_t^i) = (1-\eta) w_t^i, \;\; \Rightarrow \;\; I_t^i = \eta i_t^i = w_t^i, \tag{5}$$

[4]Excluding workers from other savings alternatives facilitates the closed-form solution, but it may seem implausible. von Hagen and Zhang (2009) show that allowing workers to have other investment opportunity does not change our results qualitatively but the model becomes less tractable.

[5]The pledgeability, θ, can be argued in various forms of agency costs, e.g., costly state verification by Townsend (1979), inalienable human capital by Hart and Moore (1994), or unobservable project (effort) choices by Holmstrom and Tirole (1997). In order to compare our results with Matsuyama (2004), we minimize the deviation of our model setting from his by choosing this simplest form of borrowing constraints. The pledgeability of individual projects may depend on idiosyncratic features. As we focus on the aggregate implications of financial development, we assume that entrepreneurs investing in country i are subject to the same θ^i for simplicity.

implies that the aggregate labor income in period t is invested by young entrepreneurs. Thus, the investment-equity ratio is constant at $\lambda_t^i = \frac{1}{\eta}$ and the degree of financial development θ^i does not affect the aggregate investment. Intuitively, the aggregate credit demand is lower in the country with a lower level of financial development. Given the perfectly inelastic aggregate credit supply, the credit market clears at a lower loan rate.

After repaying the debt in period $t+1$, the entrepreneur gets $Ri_t^i v_{t+1}^i - r_t^i z_t^i$ as the return on equity capital, w_t^i. The equity rate is the rate of return on equity capital,

$$\Gamma_t^i \equiv \frac{Ri_t^i v_{t+1}^i - r_t^i z_t^i}{w_t^i} = Rv_{t+1}^i + (Rv_{t+1}^i - r_t^i)\frac{(1-\eta)}{\eta} \geq r_t^i. \tag{6}$$

Intuitively, for each unit of equity capital invested in the project, the entrepreneur gets Rv_{t+1}^i as the marginal return. Additionally, he can borrow $(\lambda_t^i - 1) = \frac{(1-\eta)}{\eta}$ units of debt which gives him an extra rate of return $(Rv_{t+1}^i - r_t^i)$. The term $(Rv_{t+1}^i - r_t^i)\frac{(1-\eta)}{\eta}$ captures the *leverage* effect. In equilibrium, the equity rate should be no less than the loan rate; otherwise, he would rather lend than borrow. The inequality in (6) is equivalent to $r_t^i \leq Rv_{t+1}^i$ and can be considered as his participation constraint.

If $r_t^i < Rv_{t+1}^i$, the entrepreneur borrows to the limit, i.e., he finances the investment i_t^i using $z_t^i = \frac{\theta^i Ri_t^i v_{t+1}^i}{r_t^i}$ units of debt and w_t^i units of equity capital in period t. After repaying the debt in period $t+1$, he gets $(1-\theta^i)Ri_t^i v_{t+1}$ as the project return. Given the investment-equity ratio at $\lambda_t^i \equiv \frac{i_t^i}{w_t^i} = \frac{1}{\eta}$, the equity rate has a closed-form solution,

$$\Gamma_t^i = \frac{(1-\theta^i)Ri_t^i v_{t+1}^i}{w_t^i} = \frac{(1-\theta^i)Rv_{t+1}^i}{\eta}. \tag{7}$$

Combining equations (6) and (7), we get a closed-form solution for the loan rate,

$$r_t^i = \frac{\theta^i Rv_{t+1}^i}{1-\eta}. \tag{8}$$

If $r_t^i = Rv_{t+1}^i$, the entrepreneur does not borrow to the limit. According to equation (6), the equity rate is equal to the loan rate, $\Gamma_t^i = r_t^i = Rv_{t+1}^i$. Lemma 1 summarizes the interest rate patterns with respect to the level of financial development.

Lemma 1. *Let $\bar{\theta} \equiv 1-\eta$. For $\theta^i \in (\bar{\theta}, 1]$, the borrowing constraints are not binding and $\Gamma_t^i = r_t^i = Rv_{t+1}^i$; for $\theta^i \in (0, \bar{\theta})$, the borrowing constraints are binding and $\Gamma_t^i = \frac{(1-\theta^i)Rv_{t+1}^i}{\eta} > Rv_{t+1}^i > \frac{\theta^i Rv_{t+1}^i}{1-\eta} = r_t^i$.*

Given the labor income w_t^i, the entrepreneur chooses the project investment i_t^i in period t to maximize his consumption when old in period $t+1$,

$$c_{t+1}^{i,e} = v_{t+1}^i Ri_t^i - r_t^i z_t^i = w_t^i \Gamma_t^i, \tag{9}$$

subject to the borrowing constraint (4) and the participation constraint (6). Note that only one of the two constraints can be strictly binding in equilibrium.

Since the aggregate labor income is invested in the entrepreneurial projects in period t, the aggregate output of capital goods available for production in period $t+1$ is

$$K_{t+1} = RI_t^i = Rw_t^i. \tag{10}$$

The market-clearing condition for final goods in period t is

$$C_t^i + I_t^i = Y_t^i, \tag{11}$$

where $C_t^i = \eta c_t^{i,e} + (1-\eta)c_t^{i,w}$ is the aggregate consumption of the old generation in period t. We measure the social welfare of the generation born in period t and country i using its aggregate consumption when old, C_{t+1}^i.

Definition 1. *Given the level of financial development θ^i, the market equilibrium in country $i \in \{H, F\}$ under international financial autarky is a set of allocations of workers, $\{c_t^{i,w}\}$, entrepreneurs, $\{i_t^i, z_t^i, c_t^{i,e}\}$, and aggregate variables, $\{Y_t^i, K_t^i, I_t^i, C_t^i, w_t^i, v_t^i, r_t^i, \Gamma_t^i\}$, satisfying equations* (1)-(5) *and* (9)-(11) *as well as Lemma 1.*

Since the size of the working population is normalized at one, the capital-labor ratio coincides with the aggregate capital stock. Thus, K_t^i also denotes the capital-labor ratio.

According to equations (1), (2), and (10), the model dynamics can be characterized by a first-order difference equation on the wage dynamics,

$$w_{t+1}^i = (1-\alpha)Y_{t+1}^i = \left(\frac{K_{t+1}^i}{\rho}\right)^\alpha = \left(\frac{Rw_t^i}{\rho}\right)^\alpha, \quad \text{where} \quad \rho \equiv \frac{\alpha}{1-\alpha}. \tag{12}$$

Given $\alpha \in (0,1)$, the phase diagram of wages is concave and starts from the origin. Its slope, $\frac{dw_{t+1}^i}{dw_t^i} = \alpha \left(\frac{R}{\rho}\right)^\alpha (w_t^i)^{\alpha-1}$, converges to $+\infty$ for $w_t^i \to 0$ and to 0 for $w_t^i \to +\infty$. Thus, there exists a unique and stable non-zero steady state with the wage at,

$$w_{IFA} = \left(\frac{R}{\rho}\right)^\rho, \tag{13}$$

where a variable with the subscript IFA denotes its steady-state value under international financial autarky. According to equations (12) and (13), the wage dynamics are independent of the level of financial development θ^i and, thus, the wage converges to the same steady state in the two countries. So do aggregate output and capital.[6]

According to Lemma 1, for $\theta^i \in [1-\eta, 1]$, the two interest rates are equal to the marginal return on investment, $r_t^i = \Gamma_t^i = Rv_{t+1}^i = R^\alpha \rho^{1-\alpha^2}(K_t^i)^{\alpha(\alpha-1)}$, depending negatively on the capital-labor ratio, K_t^i. Thus, the two interest rates are higher in the country

[6] The phase diagram of the capital-labor ratio is normally used to prove the existence, uniqueness, and stability of the steady state (Matsuyama, 2004). Our model dynamics can also be represented by the phase diagram of the capital-labor ratio $K_{t+1}^i = Rw_t^i = R\left(\frac{K_t^i}{\rho}\right)^\alpha$, which has the same concavity property as that of wages. For notational convenience, the phase diagram of wages is used in the following analysis.

with a lower capital-labor ratio. We call this the *neoclassical* effect, because it arises from the concavity of the neoclassical production function with respect to the capital-labor ratio. It is independent of the level of financial development.

For $\theta^i \in (0, 1-\eta)$, besides the neoclassical effect, the loan rate is affected positively by financial development, $r_t^i = Rv_{t+1}^i \frac{\theta^i}{(1-\eta)}$. Given the capital-labor ratio, the loan rate is higher in the country with a higher θ, reflecting the general equilibrium effect of the larger aggregate credit demand. We call this the *credit-demand* effect of financial development, $\frac{\theta^i}{(1-\eta)} \in (0,1)$. According to equation (6), besides the neoclassical effect, the equity rate is also affected by the leverage effect. Given the capital-labor ratio, the loan rate is higher in the country with a higher θ, which keeps the spread lower, $(Rv_{t+1}^i - r_t^i)$. Given the debt-equity ratio constant at $\frac{(1-\eta)}{\eta}$, the leverage effect is thus smaller. This way, the equity rate is affected negatively by financial development, $\Gamma_t^i = Rv_{t+1}^i \frac{(1-\theta^i)}{\eta}$.

The financial frictions in our model do not distort production efficiency but distort the two interest rates.[7] Through the distortions on the interest rates, financial frictions have a distributional effect on the welfare of borrowers (entrepreneurs) and lenders (workers).

The aggregate labor income is invested in the entrepreneurial projects in period t, $I_t^i = w_t^i = (1-\alpha)Y_t^i$, and the aggregate output of capital goods has the value of $v_{t+1}^i K_{t+1}^i = \alpha Y_{t+1}^i$ in period $t+1$. In the steady state, $Y_{t+1}^i = Y_t^i = Y^i$, and the marginal return on investment is $v^i R = \frac{v^i K^i}{w^i} = \frac{\alpha Y^i}{(1-\alpha)Y^i} = \rho$. Plugging it into Lemma 1, we get the steady-state patterns of interest rates, which is summarized in Proposition 1.

Proposition 1. *For $\theta^i \in (\bar{\theta}, 1]$, the two interest rates are independent of the level of financial development, $r^i = \Gamma^i = \rho$; for $\theta^i \in (0, \bar{\theta})$, the loan rate rises and the equity rate declines in the level of financial development, $r^i = \frac{\theta^i \rho}{1-\eta} < \rho < \Gamma^i = \frac{(1-\theta^i)\rho}{\eta}$.*

Figure 1 shows the steady-state patterns of output, wages, and interest rates, with the horizontal axis denoting $\theta \in (0, 1]$, where $\theta^U \equiv \bar{\theta} = 1 - \eta$. Since financial frictions in our model does not affect production efficiency, aggregate output is independent of θ^i and the marginal return on investment is constant at ρ. Thus, the cross-country loan rate (equity rate) differentials only depend on the credit-demand (leverage) effect.

According to Proposition 1, for $\theta^H \in [0, \bar{\theta})$ and $\theta^F \in (\theta^H, 1]$, the loan rate is lower while the equity rate higher in country H than in county F; for $\bar{\theta} \leq \theta^H < \theta^F \leq 1$, the borrowing constraints are not binding in the two countries so that the credit-demand effect and the leverage effect are muted. Then, the two interest rates coincide with the marginal return on investment, which is same in the two countries.

[7] The equity premium, $\Gamma_t - r_t > 0$, in the case of $\theta^i \in (0, \bar{\theta})$ arises from two factors, i.e., the difference in productivity and the binding borrowing constraints. For $\theta \in (0, \bar{\theta})$, the constraint on aggregate credit demand keeps the loan rate lower than the marginal return on investment. The equity premium is the reward to entrepreneurs' advantage in productivity. For $\theta \in (\bar{\theta}, 1]$, the unconstrained aggregate credit demand raises the loan rate to the marginal return on investment and the equity premium vanishes.

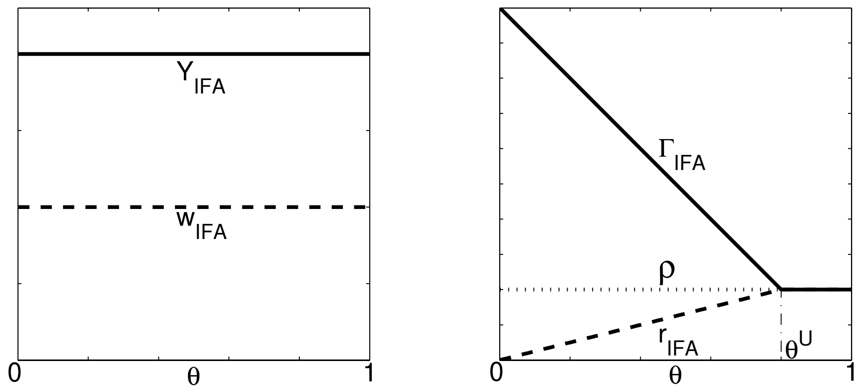

Figure 1: Steady-State Patterns under International Financial Autarky

3 The Model under International Capital Mobility

We consider three scenarios of capital mobility, *free mobility of financial capital* under which individuals are allowed to lend abroad but entrepreneurs are not allowed to make direct investment abroad, *free mobility of FDI* under which entrepreneurs are allowed to make direct investment abroad[8] but individuals are not allowed to lend abroad, and *full capital mobility* under which individuals are allowed to lend abroad and entrepreneurs are allowed to make direct investments abroad.

In subsections 3.1, 3.2, and 3.3, we assume that the two countries are in the steady state under international financial autarky before capital mobility is allowed from period $t = 0$ on. Thus, we analyze capital flows between two countries with the same initial per capita output. In subsections 3.4, we assume that country F is in the steady state while country H is below its steady state before capital mobility is allowed from period $t = 0$ on. This assumption allows us to analyze a more general and realistic case of capital flows between rich and poor countries, or between developed and developing countries.

Let Υ_t^i and Ω_t^i denote the aggregate outflows of financial capital and equity capital (FDI) from country i in period t, respectively, with negative values indicating capital inflows. Financial capital flows affect the domestic credit supply, $(1-\eta)w_t^i - \Upsilon_t^i$. Through affecting the aggregate equity capital for the domestic investment, $\eta w_t^i - \Omega_t^i$, FDI flows increase the aggregate credit demand in the host country and reduce that in the parent country.[9] With these changes, the analysis in section 2 carries through for capital mobility,

[8]Entrepreneurs can either bring their equity capital and projects abroad for investment or make equity investment in the foreign entrepreneurial project. The two alternatives are analytically equivalent in our model. Without the necessary skills, workers cannot make direct or equity investment abroad.

[9]In the case of debt default, the project liquidation value depends on the efficiency of the legal institution, the law enforcement, and the asset market in the host country. Thus, we assume that entrepreneurs making FDI borrow only from the host country and are subject to the borrowing constraints there. Alternatively, we can assume that entrepreneurs may borrow only in their parent country no matter

due to the linearity of the preferences, the projects, and the borrowing constraints.

Without loss of generality, we focus on the case of $0 < \theta^H < \theta^F \leq \bar{\theta}$, where the borrowing constraints in the two countries are binding in the steady state under the three scenarios of capital mobility. Appendix B provides a complete analysis of the bindingness of borrowing constraints for $\theta^i \in (0, 1)$, where $i \in \{H, F\}$.

3.1 Free Mobility of Financial Capital

The Cobb-Douglas production function implies,

$$v_{t+1}^i = (w_{t+1}^i)^{-\frac{1}{\rho}} \quad \text{and} \quad I_t^i = \frac{K_{t+1}^i}{R} = \frac{\rho}{R}(w_{t+1}^i)^{\frac{1}{\alpha}}. \tag{14}$$

Free mobility of financial capital equalizes the loan rates across the border, $r_t^H = r_t^F = r_t^*$. Given the domestic equity capital ηw_t^i, the aggregate domestic investment is,

$$I_t^i = \lambda_t^i \eta w_t^i = \frac{\eta w_t^i}{1 - \frac{\theta^i R v_{t+1}^i}{r_t^*}}. \tag{15}$$

Using equations (14) and (15) to substitute away I_t^i and v_{t+1}^i, we get

$$\eta w_t^i = \frac{\rho}{R}(w_{t+1}^i)^{\frac{1}{\alpha}} - \frac{\theta^i \rho}{r_t^*} w_{t+1}^i. \tag{16}$$

3.1.1 Existence, Uniqueness, and Stability of the Steady State

Proposition 2. *Given the world loan rate r_{FCF}^*, there exists a unique and stable non-zero steady state with the wage rate at $w_{FCF}^i = w_{IFA}\left[\eta + (1-\eta)\frac{r_{IFA}^i}{r_{FCF}^*}\right]^{\rho}$, where a variable with the subscript FCF denotes its steady-state value under free flows of financial capital.*

The solid line and the dash-dotted line in the left panel of figure 2 show the phase diagrams of wages under international financial autarky and under free mobility of financial capital, respectively, given a fixed world loan rate at $r_t^* = r_{IFA}^i$. In both cases, wages converge monotonically and globally to a unique steady state (point A).

Our model setting differs from Matsuyama (2004) in only one aspect. He assumes that the investment size of every project is fixed at $i_t^i = 1$, while the mass of individuals in a country who become entrepreneurs is endogenously determined. In contrast, we assume that the mass of entrepreneurs in a country is fixed at η, while the investment size of any project i_t^i is endogenously determined. The difference in assumptions makes a big difference in the property of the steady-state equilibrium. Matsuyama (2004) shows that,

where they invest, since the financial institutions in their parent country have better information on the credit record, social network, and business activities of the entrepreneurs. The realistic case should be a hybrid of these two. Our results hold under the two alternative assumptions.

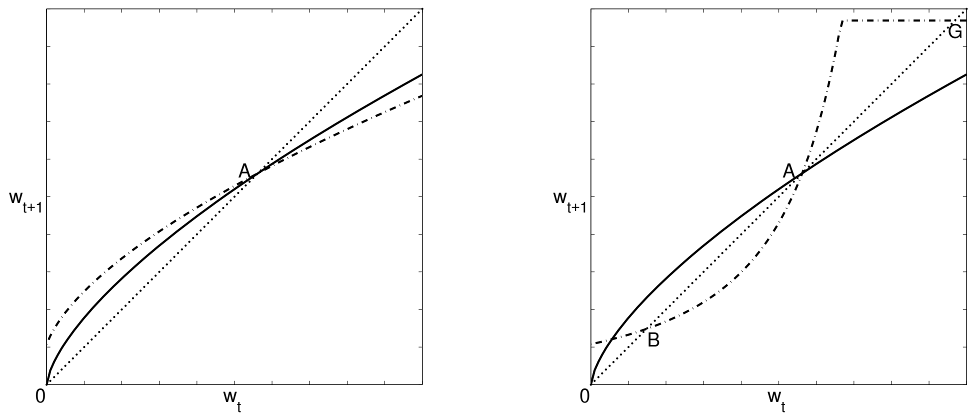

Figure 2: The Phase Diagrams of Wage

at a given world loan rate, free mobility of financial capital may lead to an equilibrium with multiple steady states. In contrast, there is a unique steady state in our model.

The borrowing constraints, if binding, take the same form in both models,

$$r_t^*(1 - \frac{w_t^i}{i_t^i}) = \theta^i R v_{t+1}^i = \theta^i R (w_{t+1}^i)^{-\frac{1}{\rho}}. \tag{17}$$

Lemma 2. *Given the world loan rate r_t^*, for $w_t^i \in [0, 1-\theta^i]$, the phase diagram of wages in Matsuyama (2004) described by $r_t^*(1 - w_t^i) = \theta^i R(w_{t+1}^i)^{-\frac{1}{\rho}}$ is strictly convex, and w_{t+1}^i monotonically increases in w_t^i with an intercept on the vertical axis at $w_{t+1}^i = \left[\frac{\theta^i R}{r_t^*}\right]^\rho$; for $w_t^i > 1 - \theta^i$, the phase diagram of wages is flat with $w_{t+1}^i = \left(\frac{R}{r_t^*}\right)^\rho$.*

The solid line in the right panel of figure 2 shows the phase diagram of wages under international financial autarky in Matsuyama (2004), which is identical as in our model. The dash-dotted line shows the phase diagram under free mobility of financial capital in his model, given a fixed world loan rate $r_t^* = r_{IFA}^i$. The phase diagram is convex for wages below a threshold value. Thus, the steady state at point A, which is stable under international financial autarky, becomes unstable under free mobility of financial capital, because the slope of the phase diagram at point A is larger than one. There are two stable steady states at points B and G. This implies that countries with the identical fundamentals (including θ) and, thus, the same steady state under international financial autarky may end up with different levels of per capita output under free mobility of financial capital. Thus, Matsuyama (2004) claims that in the presence of credit market imperfection, financial capital flows may result in the symmetry breaking.[10]

Intuitively, according to equation (17), given a world loan rate and a fixed size of project investment as in Matsuyama (2004), a marginal increase in the current wage reduces the credit demand of each borrower, $(1 - w_t^i)$, and the debt-investment ratio,

[10]The symmetry-breaking property depends on the specific value of the world loan rate and the steady-state equilibrium may be unique under other values of world loan rate. See Matsuyama (2004) for details.

$\frac{z_t^i}{i_t^i} = (1 - \frac{w_t^i}{i_t^i}) = (1 - w_t^i)$. More domestic individuals can borrow at the prevailing world loan rate and produce capital goods. The higher the initial wage level w_t^i, the lower the debt-investment ratio, the larger the expansion of the extensive margin of the aggregate investment, and consequently, the larger the increase in aggregate output and the wage in period $t+1$. This explains the convexity of the phase diagram of wages in his model and the possibility of the equilibrium with multiple steady states under capital mobility.

In contrast, given a constant world loan rate and a fixed mass of entrepreneurs in our model, a marginal increase in the current wage enables entrepreneurs to borrow and invest more. According to equation (17), the increase in i_t^i partially offsets the effects of the marginal increase in w_t^i on the debt-investment ratio, $\frac{z_t^i}{i_t^i} = (1 - \frac{w_t^i}{i_t^i})$, and then on w_{t+1}^i. The higher the initial wage level w_t^i, the smaller the expansion of the intensive margin of the aggregate investment, and consequently, the smaller the increase in the production of capital goods and the wage in period $t+1$. This explains the concavity of the phase diagram of wages and then the uniqueness of the steady state in our model.

3.1.2 Interest Rates and Capital Flows

Proposition 3. *There exists a unique world loan rate r_t^* that clears the world credit market every period. In the steady state, $r_{FCF}^* \in (\underline{r}^*, r_{IFA}^F)$, where $\underline{r}^* \equiv \frac{r_{IFA}^H + r_{IFA}^F}{2}$.*

Intuitively, given the steady-state loan rates in the two countries under international financial autarky, $r_{IFA}^H < r_{IFA}^F$, the steady-state loan rate under free mobility of financial capital lies between them.

Proposition 4. *Under free mobility of financial capital, if the borrowing constraints are binding in country i, $\Gamma_t^i = \frac{(1-\theta^i)\rho}{\eta} \frac{w_{t+1}^i}{w_t^i}$. In the steady state, $\Gamma_{FCF}^i = \frac{(1-\theta^i)\rho}{\eta} = \Gamma_{IFA}^i$.*

Given the binding borrowing constraints, entrepreneurs use $\frac{\theta^i R v_{t+1}^i}{r_t^*}$ units of loan and $w_t = \frac{(1-\theta^i) R v_{t+1}^i}{\Gamma_t^i}$ units of equity capital to finance each unit of investment in period t.

$$1 = \frac{\theta^i R v_{t+1}^i}{r_t^*} + \frac{(1-\theta^i) R v_{t+1}^i}{\Gamma_t^i} \quad \Rightarrow \quad \frac{1-\theta^i}{\Gamma_t^i} = \frac{1}{R v_{t+1}^i} - \frac{\theta^i}{r_t^*}. \tag{18}$$

Given θ^i, financial capital flows affect the equity rate in two ways. Consider country H. First, financial capital outflows raise the loan rate and the lower spread tends to reduce the equity rate. Second, financial capital outflow have a general equilibrium effect, i.e., all entrepreneurs reduce their project investment and the decline in the aggregate output of capital goods raises the price of capital good in period $t+1$. Due to the neoclassical effect, the equity rate tends to rise. In period $t=0$, the first effect dominates the second and the equity rate is lower than the steady-state level under international financial autarky. It is confirmed by the closed-form solution of the equity rate, $\Gamma_0^i = \frac{(1-\theta^i)\rho}{\eta} \frac{w_1^i}{w_0^i}$. Given the predetermined period-0 wage rate, financial capital outflows reduce the period-1 wage rate, $w_1^i < w_0^i = w_{IFA}$, and the equity rate is lower in period $t = 0$, $\Gamma_0^i < \frac{(1-\theta^i)\rho}{\eta} = \Gamma_{IFA}^i$. As the economy converges to the new steady state, the price of capital good rises further and the equity rate converges back the initial level, because the initial effect on the spread is fully offset by the neoclassical effect over time.

Proposition 5. *In the steady state, financial capital flows from country H to country F,* $\Upsilon_{FCF}^H > 0 > \Upsilon_{FCF}^F$, *where* $\Upsilon_{FCF}^i = (r_{FCF}^* - r_{IFA}^i)\frac{(1-\eta)w_{FCF}^i}{r_{FCF}^*}$ *and* $i \in \{H, F\}$.

In the steady state, financial capital outflows from country i are proportional to the steady-state loan-rate differentials under free mobility of financial capital and under international financial autarky. Since $r_{IFA}^H < r_{FCF}^* < r_{IFA}^F$, country H (F) has financial capital outflows (inflows).

3.1.3 Production and Welfare

From period $t = 0$ on, financial capital flows reduce (raise) the aggregate investment in country H (F). Thus, from period $t = 1$ on, aggregate output in country H (F) is higher (lower) than before period $t = 0$, $Y_t^H < Y_{IFA} < Y_t^H$.

Proposition 6. *From period* $t = 1$ *on,* $Y_t^H + Y_t^F < 2Y_{IFA}$.

Before period $t = 0$, aggregate production in the two countries is efficient and identical. From period $t = 0$ on, financial capital flows lead to the cross-border resource reallocation, which moves the world economy away from the efficient allocation. Due to the concave aggregate production with respect to the capital-labor ratio on the country level, the world output is lower than before period $t = 0$, according to the Jensen's inequality. This also explains the world output losses in Matsuyama (2004). More generally, this is a typical result of the theory of second best. Given domestic financial frictions, capital account liberalization causes financial capital flowing to the country with the higher loan rate rather than to the country with the higher marginal product of capital.

The welfare of entrepreneurs born in period t and country i is measured by their consumption when old, which is proportional to the labor income in period $t+1$, $c_{t+1}^{i,e} = w_t^i\Gamma_t^i = w_{t+1}^i\frac{(1-\theta^i)\rho}{\eta}$, according to Proposition 4. This reflects the joint effect of financial capital flows on the labor income and the equity rate. From period $t = 0$ on, due to financial capital flows, the aggregate investment in country H (F) is lower (higher) than its initial value and so is the wage in period $t+1$, $w_{t+1}^H < w_{IFA} < w_{t+1}^F$. From period $t = 0$ on, entrepreneurs born in country H (F) is worse (better) off than before period 0. Thus, entrepreneurs in the less (more) financially developed country have a strong incentive to oppose (support) policies favoring financial capital mobility.

Given the predetermined period-0 labor income w_0^i, workers born in country H (F) and period $t = 0$ are better (worse) off, $c_0^{i,w} = w_0^i r_0^*$, due to the rise (decline) in the loan rate, $r_{IFA}^H < r_0^* < r_{IFA}^F$. From period $t = 1$ on, financial capital flows affect the welfare of workers born in country H (F) and period t, $c_t^H = w_t^H r_t^*$ ($c_t^F = w_t^F r_t^*$), through the negative (positive) effect on the labor income, $w_t^H < w_{IFA} < w_t^F$ and the positive (negative) effect on the loan rate, $r_{IFA}^H < r_t^* < r_{IFA}^F$. The net welfare effect is ambiguous and depends on the levels of financial development in the two countries. Free mobility of financial capital also has an ambiguous welfare effect on the country level. Since free mobility of financial capital generates the world output losses in our model, its welfare implications on the world level is negative. Under free mobility of financial capital, it is impossible for any public transfer policy to achieve a world-level Pareto improvement, in comparison with the steady-state allocation under international financial autarky.

Proposition 7. *In comparison with the steady state under international financial autarky, free mobility of financial capital make entrepreneurs in country H (F) worse (better) off, while the welfare effects on workers and on the country level depend on the parameters.*

Table 1: The Long-Run Welfare Impacts on Workers

	$\kappa \in (-\infty, \frac{\theta^H+\theta^F}{2\theta^F}]$	$\kappa \in (\frac{\theta^H+\theta^F}{2\theta^F}, 1]$	$\kappa \in (1, \frac{\theta^F}{\theta^H}]$	$\kappa \in (\frac{\theta^F}{\theta^H}, \infty)$
$c_{FCF}^{H,w} - c_{IFA}^{H,w}$	+	+	?	-
$c_{FCF}^{F,w} - c_{IFA}^{F,w}$	-	?	+	+

Table 1 summarizes the long-run welfare impacts on workers under various parameter constellations, where $\kappa \equiv \frac{(\rho-1)(1-\eta)}{\eta}$ denotes the parameter combination.

For $\kappa \in (-\infty, \frac{\theta^H+\theta^F}{2\theta^F}]$, entrepreneurs (workers) born in country H lose (benefit) from financial capital flows in the long run as well as in the short run. Similar results exist for country F. Thus, free mobility of financial capital may have the opposite welfare effects on different individuals in the same country.

Proposition 8. *Workers of different generations born in the same country may be affected by financial capital flows in opposite ways during the transitional process from international financial autarky to free mobility of financial capital.*

Proof. Workers born in country H and period $t = 0$ are better off due to the higher loan rate. Financial capital outflows reduce the aggregate investment in country H. According to Table 1, for $\kappa \geq \frac{\theta^F}{\theta^H}$, the decline in the labor income dominates the rise in the loan rate in the long run so that workers are worse off in the long run. Workers of early and later generations born in country F are also affected in the opposite way. Thus, free mobility of financial capital may have opposite welfare effects across generations. □

3.2 Free Mobility of FDI

The analysis for free mobility of FDI resembles that for free mobility of financial capital. Here, we briefly summarize the main results and put the detailed analysis in appendix A.

Under free mobility of FDI, there exists a unique and stable steady state with the wage at $w^i_{FDI} = w_{IFA}\left[1-\eta+\eta\frac{\Gamma^i_{IFA}}{\Gamma^*_{FDI}}\right]^\rho$, where a variable with the subscript FDI denotes its steady-state value under free flows of FDI. In the steady state, the world equity rate is $\Gamma^*_{FDI} \in (\underline{\Gamma}^*, \Gamma^H_{IFA})$, where $\underline{\Gamma}^* \equiv \frac{\Gamma^H_{IFA}+\Gamma^F_{IFA}}{2}$; FDI flows from country F to country H, $\Omega^H_{FDI} < 0 < \Omega^F_{FDI}$, where $\Omega^i_{FDI} = (\Gamma^*_{FDI} - \Gamma^i_{IFA})\frac{\eta w^i_{FDI}}{\Gamma^*_{FDI}}$ and $i \in \{H, F\}$. The loan rate has a closed-form solution, $r^i_t = \frac{\theta^i\rho}{(1-\eta)}\frac{w^i_{t+1}}{w^i_t}$, with the steady-state value $r^i_{FDI} = \frac{\theta^i\rho}{1-\eta} = r^i_{IFA}$.

Aggregate production in the two countries is efficient and identical until period $t = 0$. From period $t = 0$ on, FDI flows raise (reduce) the aggregate investment in country H (F) and aggregate output in country H (F) is higher (lower) than before period $t = 0$. This way, FDI flows widen the cross-country output gap, which reduces the world output.

The welfare of workers born in period t and country i is measured by their consumption when old, which is proportional to the wage in period $t+1$, $c^{i,w}_{t+1} = w^i_t r^i_t = w^i_{t+1}\frac{\theta^i\rho}{(1-\eta)}$. This reflects the joint effect of FDI flows on the labor income and the loan rate in period t. From period $t = 0$ on, due to FDI flows, the aggregate investment in country H (F) is higher (lower) than its initial value and so is the wage in period $t+1$, $w^H_{t+1} > w_{IFA} > w^F_{t+1}$. From period $t = 0$ on, workers born in country H (F) is better (worse) off than before period 0. Thus, workers in the less (more) financially developed country have a strong incentive to support (oppose) policies favoring international mobility of FDI.

Given the predetermined period-0 labor income w^i_0, entrepreneurs born in country H (F) and period $t = 0$ are worse (better) off, $c^{i,e}_0 = w^i_0\Gamma^*_0$, due to the decline (rise) in the equity rate, $r^H_{IFA} < r^*_0 < r^F_{IFA}$. From period $t = 1$ on, FDI flows affect entrepreneurs born in country H (F) and period t, $c^H_t = w^H_t\Gamma^*_t$, $(c^F_t = w^F_t\Gamma^*_t)$ through the positive (negative) effect on the labor income, $w^H_t > w_{IFA} > w^F_t$ and the negative (positive) effect on the equity rate, $\Gamma^H_{IFA} > \Gamma^*_t > \Gamma^F_{IFA}$. The net welfare effect depends on the levels of financial development in the two countries. Free mobility of FDI also has an ambiguous welfare effect on the country level. Since free mobility of FDI generates the world output losses in our model, its welfare impacts on the world level is negative. Under free mobility of FDI, it is impossible for any public transfer policy to achieve a world-level Pareto improvement, in comparison with the steady-state allocation under international financial autarky.

3.3 Full Capital Mobility

Full capital mobility equalizes the loan rates and the equity rates across the border, respectively, $r^H_t = r^F_t = r^*_t$ and $\Gamma^H_t = \Gamma^H_t = \Gamma^*_t$. Using equation (14) to substitute away

v_{t+1}^i from equation (18), we get

$$(w_{t+1}^i)^{\frac{1}{\rho}} = \frac{R}{\rho}\left[\frac{(1-\theta^i)\rho}{\Gamma_t^*} + \frac{\theta^i \rho}{r_t^*}\right], \quad \text{where} \quad \frac{\partial w_{t+1}^i}{\partial \Gamma_t^*} < 0, \quad \frac{\partial w_{t+1}^i}{\partial r_t^*} < 0. \tag{19}$$

3.3.1 Existence, Uniqueness, and Stability of the Steady State

Proposition 9. *Given the world interest rates r_{FCM}^* and Γ_{FCM}^*, there is a unique and stable non-zero steady state with the wage at $w_{FCM}^i = w_{IFA}\left[\frac{(1-\theta^i)\rho}{\Gamma_{FCM}^*} + \frac{\theta^i\rho}{r_{FCM}^*}\right]^{\rho}$, where a variable with the subscript FCM denotes its steady-state value under full capital mobility.*

3.3.2 Interest Rates and Capital Flows

Before period $t = 0$, the loan rate is lower while the equity rate is higher in country H than in country F. From period $t = 0$ on, the initial cross-country interest rate differentials drive financial capital flowing from country H to country F while FDI flowing in the opposite direction. As a result, the loan rate adjusts from below (above) while the equity rate adjusts from above (below) to the world level in country H (F).

Proposition 10. *There exists the unique world loan rate and world equity rate that clear the world credit market and the world equity market every period. In the steady state, $\Gamma_{FCM}^* \in (\Gamma_{IFA}^F, \underline{\Gamma}^*)$ and $r_{FCM}^* \in (\underline{r}^*, r_{IFA}^F)$.*

Intuitively, given the steady-state loan rates and equity rates in the two countries under international financial autarky at $r_{IFA}^H < r_{IFA}^F$ and $\Gamma_{IFA}^H > \Gamma_{IFA}^F$ the steady-state loan rate and equity rate under free mobility of financial capital lie between them.

Proposition 11. *In the steady state, financial capital flows from country H to country F, $\Upsilon_{FCM}^H > 0 > \Upsilon_{FCM}^F$, FDI flows in the opposite direction, $\Omega_{FCM}^H < 0 < \Omega_{FCM}^F$, and net capital flows are from country H to country F, $\Upsilon_{FCM}^H + \Omega_{FCM}^H > 0 > \Upsilon_{FCM}^F + \Omega_{FCM}^F$, where $\Upsilon_{FCM}^i = (r_{FCM}^* - r_{IFA}^i)\frac{(1-\eta)w_{FCM}^i}{r_{FCM}^*}$, $\Omega_{FCM}^i = (\Gamma_{FCM}^* - \Gamma_{IFA}^i)\frac{\eta w_{FCM}^i}{\Gamma_{FCM}^*}$, and $i \in \{H, F\}$.*

In the steady state, Financial capital (FDI) outflows from country i are identical in the functional form as under free mobility of financial capital (FDI). Since $r_{IFA}^H < r_{FCM}^* < r_{IFA}^F$ and $\Gamma_{IFA}^H < \Gamma_{FCM}^* < \Gamma_{IFA}^F$, country H (F) has financial capital outflows (inflows) and FDI inflows (outflows). Since the credit market in country F has a larger capacity than that in country H, capital in the net term flows from country H to country F.

3.3.3 Production and Welfare

Proposition 12. *In the steady state, net capital flows keep aggregate output in country H (F) lower (higher) than its steady-state value under international financial autarky, $Y_{FCM}^H < Y_{IFA} < Y_{FCM}^F$. This way, net capital flows widen the cross-country output gap, which reduces the world output, $Y_{FCM}^H + Y_{FCM}^F < 2Y_{IFA}$.*

Note that net capital flows matter for the world output losses.

Proposition 13. *In the steady state, country F has a negative net position on international investment,* $\Upsilon^F_{FCM} + \Omega^F_{FCM} < 0$*, but receives a positive net investment income,* $(r^*_{FCM} - 1)\Upsilon^F_{FCM} + (\Gamma^*_{FCM} - 1)\Omega^F_{FCM} > 0$.

Given the positive equity premium, $\Gamma^*_t > r^*_t$, country F earns a higher return on its direct investment abroad than its pays out on foreign debt. Although the closed-form solutions of interest rates are not available, we can prove $r^*_{FCM}\Upsilon^i_{FCM} + \Gamma^*_{FCM}\Omega^i_{FCM} = 0$. Thus, The net investment income of country F,

$$(r^*_{FCM} - 1)\Upsilon^F_{FCM} + (\Gamma^*_{FCM} - 1)\Omega^F_{FCM} = -(\Upsilon^F_{FCM} + \Omega^F_{FCM}) = \Upsilon^H_{FCM} + \Omega^H_{FCM},$$

is fully financed by net capital outflow from country H. Intuitively, country F has a competitive advantage in financial intermediation. By exporting the financial service via two-way capital flows, it receives the positive net investment income.

Proposition 14. *In the steady state, due to the decline (rise) in the labor income and the equity rate in country H (F), entrepreneurs in country H (F) are worse (better) off than in the steady state under international financial autarky. In addition, country H (F) as a whole is worse (better) off.*

Full capital mobility is never an option for country H to make Pareto improvement upon the steady-state allocation under international financial autarky. In contrast, full capital mobility is a good option for country F to make Pareto improvement, if implemented with some appropriately designed public transfer policies. The non-zero net capital flows widen the cross-country output gap, which generates the world output losses. In this case, full capital mobility can never achieve Pareto improvement on the world level.

3.4 Capital Mobility between Initially Poor and Rich Countries

In this subsection, we assume that country F is financially developed, $\theta^F = \bar{\theta}$, and in the steady state, while country H is financially underdeveloped, $0 < \theta^H < \bar{\theta}$, and below the steady state, when capital mobility is allowed in period $t = 0$, $0 < w^H_0 < w^F_0 = w_{IFA}$. We analyze the patterns of capital flows in the three scenarios as well as how capital mobility affects the aggregate investment and output in country H in period $t = 0$.

3.4.1 Free Mobility of Financial Capital

Let us define a counterfactual case where the world economy is still under international financial autarky in period $t = 0$. It helps identify the cross-country interest rate differentials driving capital flows in period $t = 0$ in the actual case.

Consider the counterfactual case. Compared with the loan rate in country F, $r^F_{IFA} = \rho$, the loan rate in country H is affected positively by $K^H_0 < K^F_0$ via the neoclassical effect and negatively by $\theta^H < \theta^F$ via the credit-demand effect. There exists a threshold value $\tilde{K}^H_0 = \left(\frac{\theta^H}{\theta^F}\right)^{\frac{1}{(1-\alpha)\alpha}} K_{IFA} < K_{IFA}$. For $K^H_0 \in (0, \tilde{K}^H_0)$, the neoclassical effect dominates so that the loan rate in country H is higher than in country F. In the actual case, financial capital flows from country F to country H. "Downhill" capital flows reduce the cross-country output gap and generate the world output gains. For $K^H_0 \in (\tilde{K}^H_0, K_{IFA})$, the credit-demand effect dominates and financial capital flows from country H to country F, which widens the cross-country output gap and generates the world output losses.

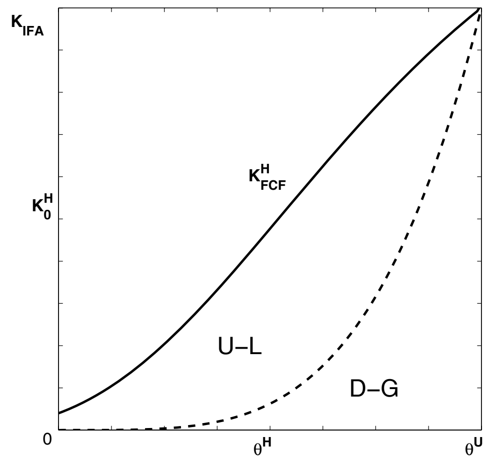

Figure 3: Financial Capital Flows between Initially Poor and Rich Countries

In figure 3, the dashed curve and the solid curve show $\tilde{K}^H_0$ and K^H_{FCF}, while the upper bound represents K_{IFA}. The horizontal axis denotes $\theta^H \in (0, \theta^U)$, and the vertical axis denotes the capital-labor ratio in country H and period $t = 0$. "**D-G**" refers to the region where financial capital flows "**D**ownhill" with the world output **G**ains, while "**U-L**" refers to the region where financial capital flows "**U**phill" with the world output **L**osses.

For a developing country, the capital-labor ratio is low at its early stage of economic growth. The loan rate under international financial autarky may be higher than the world loan rate. Under free mobility of financial capital, financial capital inflows speed up its capital accumulation. However, if its capital-labor ratio exceeds a threshold value $\tilde{K}^H_0$ so that its loan rate under international financial autarky falls below the world loan rate, financial capital mobility leads to capital outflows, which hampers the aggregate domestic investment. Eventually, the country converges to a steady state with the capital-labor ratio lower than that under international financial autarky, $K^H_{FCF} < K^H_{IFA}$. Thus, the patterns of financial capital flows may reverse along its convergence process. Furthermore, financial capital mobility has opposite effects on aggregate production at the different stages of its convergence process.

3.4.2 Free Mobility of FDI

Consider the counterfactual case. Compared with the equity rate in country F, $\Gamma_{IFA}^{F} = \rho$, the equity rate in country H is positively affected by $K_0^H < K_0^F$ via the neoclassical effect and by $\theta^H < \theta^F$ via the leverage effect. Thus, in period $t = 0$, the equity rate in country H is higher than in country F. In the actual case, FDI flows "downhill", which speeds up capital accumulation in country H. Eventually, country H converges to a steady state with the capital-labor ratio higher than that under international financial autarky.

In figure 4, the thick and thin solid curves represent K_{FDI}^H and K_{IFA}, respectively. The horizontal axis denotes $\theta^H \in (0, \theta^U)$, and the vertical axis denotes the capital-labor ratio in country H and period 0.

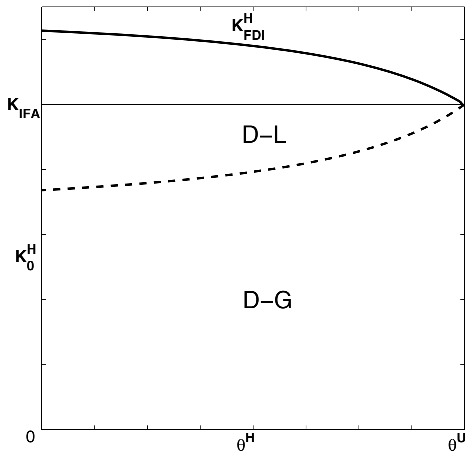

Figure 4: Free Mobility of FDI and The World Output

If K_0^H is far below K_{IFA} in period $t = 0$, FDI flows narrow the cross-country output gap and generate the world output gains in period $t = 1$. If K_0^H is slightly below K_{IFA} in period $t = 0$, due to FDI inflows, aggregate output in country H may exceed that in country F in period $t = 1$, which widens the cross-country output gap and generates the world output losses. In figure 4, the dashed line shows this threshold value $\tilde{K}_0^H$, "**D-G**" refers to the region where FDI flows "**D**ownhill" with the world output **G**ains, while "**D-L**" refers to the region where FDI flows "**D**ownhill" with the world output **L**osses.

3.4.3 Full Capital Mobility

Besides the first counterfactual case defined in subsection 3.4.1, let us define a second counterfactual case where only FDI flows are allowed from period $t = 0$ on.

Consider the second counterfactual case. Due to FDI flows, the loan rate in period $t = 0$ is higher (lower) in country H (F) than under international financial autarky. There exists a threshold value, $\hat{K}_0^H$ such that for $K_0^H = \hat{K}_0^H$, the loan rates in the two countries and period $t = 0$ coincide. In the actual case of full capital mobility, there are no financial capital flows in period $t = 0$. For $0 < K_0^H < \hat{K}_0^H$, due to the neoclassical effect, the loan rate in country H is higher than in country F in the second counterfactual case. In the actual case, financial capital and FDI flow "downhill". For $\hat{K}_0^H < K_0^H < K_{IFA}$, financial capital flows "uphill" while FDI flows "downhill" in the actual case.

Besides $\hat{K}_0^H$, there is another threshold value, $\tilde{K}_0^H > \hat{K}_0^H$. For $K_0^H \in (\hat{K}_0^H, \tilde{K}_0^H)$, "downhill" FDI flows dominates "uphill" financial capital flows and net capital flows in period 0 are "downhill". For $K_0^H \in (\tilde{K}_0^H, K_{IFA})$, "uphill" financial capital flows dominate "downhill" FDI flows and net capital flows are "uphill" in period 0.

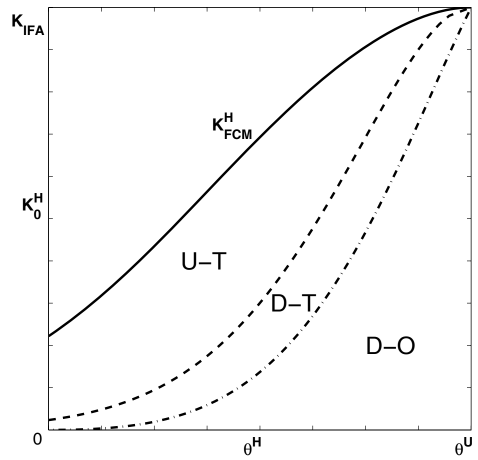

Figure 5: Full Capital Mobility between Initially Poor and Rich Countries

In figure 5, the dash-dotted line and the dashed line show $\hat{K}_0^H$ and $\tilde{K}_0^H$, respectively, while the upper bound of figure 5 represents K_{IFA}. The horizontal axis denotes $\theta^H \in (0, \theta^U)$, and the vertical axis denotes the capital-labor ratio in country H and period 0. **D-O**, **D-T**, **U-T** refer to the regions where capital in the net term flows "**D**ownhill" ("**U**phill") and financial capital and FDI flow in **O**ne (**T**wo) way(s). In regions **D-O** and **D-T** of figure 5, "downhill" net capital flows narrow the cross-country output gap, which generates the world output gains, while in region **U-T**, "uphill" capital flows widen the cross-country output gap, which generates the world output losses.

Consider a developing country with the capital-labor ratio in region **D-O** when it allows full capital mobility. Financial capital and FDI inflows speed up capital accumulation in the short run. As K_t^H moves sequentially into regions **D-T** and then **U-T**, financial capital flows change the direction from "downhill" to "uphill", and then, financial capital flows exceeds FDI flows so that net capital flows change the direction from "downhill" to "uphill". Eventually, the country converges to a new steady state with the capital-labor ratio, $K_{FCM}^H < K_{IFA}$. Thus, there is a tradeoff between the short-run benefit from enhanced capital accumulation and the long-run cost of lower output.

4 Conclusion

We develop a two-country, overlapping-generations model and show that the cross-country differences in financial development can explain three recent empirical facts. Intuitively, financial development can be considered as an endowment for a country, which does not change in the short run. In the country with more developed financial sector, the aggregate credit demand is, ceteris paribus, higher and the loan rate tends to be higher while the equity rate is lower under international financial autarky. Under full capital mobility, the cross-country interest rate differentials drive financial capital flows and FDI. Ceteris paribus, the country with competitive advantage in the financial sector "exports" its financial service by borrowing financial capital from abroad at a low interest rate and making foreign direct investment for a high rate of return. With a larger capacity of the credit market, this country becomes a net debtor in equilibrium. However, it receives a positive net investment income, because it receives a higher return on its foreign assets than it pays on its foreign liabilities. We also discuss how the patterns of capital flows change or even reverse along the convergence process of a developing country and how capital mobility affects its speed of convergence.

For simplicity, we take the level of financial development as given and analyze how the cross-country differences in financial development affect capital flows. An interesting question would be how various forms of capital flows affect financial development along the process of economic development. We leave this issue for future research.

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Appendix: not for publication

A Free Mobility of FDI

The equity rates are equalized across the border, $\Gamma_t^H = \Gamma_t^F = \Gamma_t^*$. According to the credit market equilibrium, domestic equity capital and investment in country i are

$$\eta w_t^i - \Omega_t^i = \frac{(1-\eta)w_t^i}{\lambda_t^i - 1} \text{ and } I_t^i = \lambda_t^i(\eta w_t^i - \Omega_t^i) = \frac{\lambda_t^i(1-\eta)w_t^i}{\lambda_t^i - 1} \quad \Rightarrow \quad \frac{\theta^i R v_{t+1}^i}{r_t^i} = \frac{(1-\eta)w_t^i}{I_t^i}.$$

Thus, the project-financing equation can be transformed into

$$1 = \frac{\theta^i R v_{t+1}^i}{r_t^i} + \frac{(1-\theta^i)R v_{t+1}^i}{\Gamma_t^*}, \quad \Rightarrow \quad 1 = \frac{(1-\eta)w_t^i}{I_t^i} + \frac{(1-\theta^i)R v_{t+1}^i}{\Gamma_t^*}. \tag{20}$$

Using equation (14) to substitute away v_{t+1}^i and I_t^i in equation (20), we get

$$(1-\eta)w_t^i = \frac{\rho}{R}(w_{t+1}^i)^{\frac{1}{\alpha}} - \frac{(1-\theta^i)\rho w_{t+1}^i}{\Gamma_t^*}. \tag{21}$$

A.1 Existence, Uniqueness, and Stability of the Steady State

Proposition 15. *There exists a unique and stable non-zero steady state with the wage rate at* $w_{FDI}^i = w_{IFA}\left[1 - \eta + \eta\frac{\Gamma_{IFA}^i}{\Gamma_{FDI}^*}\right]^\rho$*, where a variable with the subscript FDI denotes its steady-state value under free flows of FDI.*

The solid line and the dash-dotted line in figure 6 show the phase diagrams of wages under international financial autarky and under free mobility of FDI, respectively, given a fixed world equity rate at $\Gamma_t^* = \Gamma_{IFA}^i$. In both cases, the wage converges monotonically and globally to the unique steady state (point A).

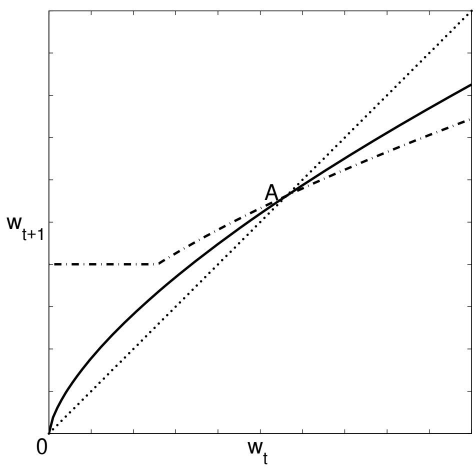

Figure 6: The Phase Diagram of Wages

A.2 Interest Rates and Capital Flows

Given that the world economy is initially in the steady state under international financial autarky, the equity rate is higher in country H than in country F. From period $t = 0$ on, entrepreneurs are allowed to make direct investment abroad. The initial cross-country equity rate differentials drive FDI unambiguously flowing from country F to country H and the equity rate in country H (F) adjusts from above (below) to the world level.

Proposition 16. *There exists a unique world equity rate that clears the world equity market every period. In the steady state,* $\Gamma^*_{FDI} \in (\underline{\Gamma}^*, \Gamma^H_{IFA})$*, where* $\underline{\Gamma}^* \equiv \frac{\Gamma^H_{IFA}+\Gamma^F_{IFA}}{2}$.

Proposition 17. *Under free mobility of FDI, if the borrowing constraints are binding in country* i, $r^i_t = \frac{\theta^i \rho}{(1-\eta)} \frac{w^i_{t+1}}{w^i_t}$. *In the steady state,* $r^i_{FDI} = \frac{\theta^i \rho}{1-\eta}$.

The proof resembles that of Proposition 4. In the steady-state, the equity-rate effect and the price-of-capital effect cancel out so that the loan rate is same as under international financial autarky.

Proposition 18. *In the steady state, FDI flows from country F to country H,* $\Omega^H_{FCF} < 0 < \Omega^F_{FCF}$*, where* $\Omega^i_{FDI} = (\Gamma^*_{FDI} - \Gamma^i_{IFA}) \frac{\eta w^i_{FDI}}{\Gamma^*_{FDI}}$ *and* $i \in \{H, F\}$.

In the steady state, FDI outflows from country i are proportional to the steady-state equity-rate differentials under free mobility of FDI and under international financial autarky. Since $\Gamma^H_{IFA} > \Gamma^*_{FCF} > \Gamma^F_{IFA}$, country H (F) has FDI inflows (outflows).

A.3 Production and Welfare

In the steady state, according to Propositions 15 and 16, $w^i_{FDI} = w_{IFA}\left[1-\eta+\frac{\eta\Gamma^i_{IFA}}{\Gamma^*_{FDI}}\right]^\rho$ and $\Gamma^* \in (\underline{\Gamma}^*, \Gamma^H_{IFA})$, imply $w^H_{FDI} > w_{IFA} > w^F_{FDI}$. Thus, aggregate output, proportional to the wage, is higher in country H than in country F, $Y^H_{FDI} > Y_{IFA} > Y^F_{FDI}$.

Proposition 19. *From period* $t = 1$ *on,* $Y^H_t + Y^F_t < 2Y_{IFA}$.

The proof follows that of Proposition 6. Since FDI flows widen the cross-country output gap, the world output is lower than under international financial autarky, due to the Jensen's Inequality. The welfare implications are discussed briefly in subsection 3.2 and summarized in Proposition 20.

Proposition 20. *In comparison with the steady state under international financial autarky, free mobility of FDI makes workers in country H (F) better (worse) off, while the welfare impacts on entrepreneurs and on the country level depend on the parameters.*

Table 2 summarizes the long-run welfare impacts on entrepreneurs under various parameter constellations, where $\mu \equiv \frac{(\rho-1)\eta}{(1-\eta)}$ denotes the parameter combination.

Table 2: The Long-Run Welfare Impacts on Entrepreneurs

	$\mu \in (-\infty, \frac{1-\theta^H+1-\theta^F}{2(1-\theta^H)}]$	$\mu \in (\frac{1-\theta^H+1-\theta^F}{2(1-\theta^H)}, 1]$	$\mu \in (1, \frac{1-\theta^H}{1-\theta^F}]$	$\mu \in (\frac{1-\theta^H}{1-\theta^F}, \infty)$
$c^{H,e}_{FDI} - c^{H,e}_{IFA}$	-	?	+	+
$c^{F,e}_{FDI} - c^{F,e}_{IFA}$	+	+	?	-

Proposition 21. *Entrepreneurs of different generations born in the same country may be affected by FDI flows in opposite ways during the transitional process from international financial autarky to free mobility of FDI.*

Proof. Entrepreneurs born in country H and period $t = 0$ are worse off due to the lower equity rate. FDI inflows increases the domestic investment in country H. According to Table 2, for $\frac{(\rho-1)\eta}{(1-\eta)} \geq 1$, and the rise in the labor income dominates the decline in the equity rate in the long run so that entrepreneurs are better off in the long run. Entrepreneurs of early and later generations born in country F are also affected in the opposite way. Thus, free mobility of FDI may have opposite welfare effects across generations. □

B Threshold Values under Capital Mobility

B.1 Free Mobility of Financial Capital

Proposition 22. *Given $\theta^H \in (0, \bar{\theta})$, there exists $\bar{\theta}^F_{FCF} \in (\bar{\theta}, 1 - \frac{\theta^H\eta}{1-\eta})$ as the function of θ^H such that for $\theta^F \in (\theta^H, \bar{\theta}^F_{FCF})$, the borrowing constraints are binding in both countries in the steady state; for $\theta^F \in (\bar{\theta}^F_{FCF}, 1]$, the borrowing constraints are binding in country H but not in country F and the economic allocation is same as that in the case of $\theta^F = \bar{\theta}^F_{FCF}$.*

Figure 7 illustrates these results. The horizontal and vertical axes denote $\theta^H \in (0, 1]$ and $\theta^F \in (0, 1]$, respectively.

For $\theta^H = \theta^F$, i.e., the parameters on the 45 degree line, the loan rates are same in the two countries under international financial autarky. For $\theta^H \in [\bar{\theta}, 1]$ and $\theta^F \in [\bar{\theta}, 1]$, i.e., the parameters in region A, the loan rates are equal to the marginal return on investment, which is same in the two countries, according to Proposition 1. In these two cases, there are no financial capital flows even if allowed. The curve splitting regions B and D represents the threshold value $\bar{\theta}^F_{FCF}$ as the function of θ^H described by equation (43). For the parameters on the curve, the equity rate in country F is equal to the world loan rate, $\Gamma^F_{FCF} = \frac{(1-\bar{\theta}^F_{FCF})\rho}{\eta} = r^*_{FCF}$. Similarly, the curve splitting region B' and D' represents the threshold value $\bar{\theta}^H_{FCF}$ as the function of θ^F. For the parameters on the curve, the equity rate in country H is equal to the world loan rate, $\Gamma^H_{FCF} = \frac{(1-\bar{\theta}^H_{FCF})\rho}{\eta} = r^*_{FCF}$.

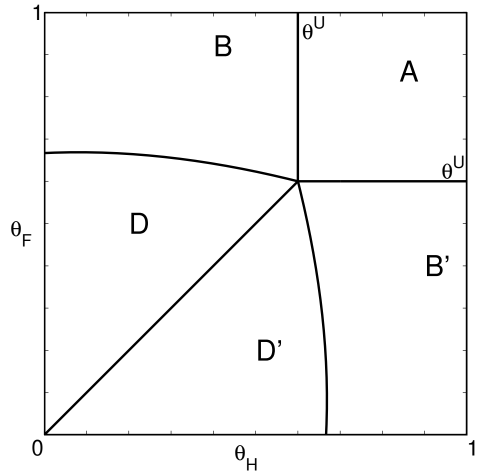

Figure 7: Free Mobility of Financial Capital: Threshold Values

Table 3: Financial Capital Flows and Equity Premium in the Steady State

Region	A	B	B'	D	D'
Υ^H	0	$\Upsilon^H(\theta^H) > 0$	$\Upsilon^H(\theta^F) < 0$	$(0, \Upsilon^H(\theta^H))$	$(\Upsilon^H(\theta^F), 0)$
$\Gamma^H - r^*$	0	$+$	0	$+$	$+$
$\Gamma^F - r^*$	0	0	$+$	$+$	$+$

Table 3 summarizes the steady-state patterns of financial capital flows and the equity premium in the five regions of figure 7. Note that $\Upsilon^F = -\Upsilon^H$. $\Upsilon^H(\theta^i)$ implies that given the parameters in region B and B', financial capital flows depend only on θ^i not on θ^m, where $i, m \in \{H, F\}$ and $i \neq m$. The borrowing constraints are strictly binding only if the equity premium is positive.

B.2 Free Mobility of FDI

Proposition 23. *If* $\eta \in \left[\frac{2^\rho}{1+2^{(\rho+1)}}, 1\right)$, *given* $\theta^H \in (0, \bar{\theta})$, *there exists* $\bar{\theta}^F_{FDI} \in (\bar{\theta}, 1)$ *as the function of* θ^H *such that for* $\theta^F \in (\theta^H, \bar{\theta}^F_{FDI})$, *the borrowing constraints are binding in country F in the steady state; for* $\theta^F \in (\bar{\theta}^F_{FDI}, 1]$, *the borrowing constraints are not binding in country F and the economic allocation is same as that in the case of* $\theta^F = \bar{\theta}^F_{FDI}$.

If $\eta \in (0, \frac{2^\rho}{1+2^{\rho+1}})$, *there exists* $\underline{\theta}^H$ *such that given* $\theta^H \in [\underline{\theta}^H, \bar{\theta})$, *there exists* $\bar{\theta}^F_{FDI} \in (\bar{\theta}, 1)$ *as the function of* θ^H *such that for* $\theta^F \in (\theta^H, \bar{\theta}^F_{FDI})$, *the borrowing constraints are binding in country F in the steady state; for* $\theta^F \in (\bar{\theta}^F_{FDI}, 1]$, *the borrowing constraints are not binding in country F and the economic allocation is same as that in the case of* $\theta^F = \bar{\theta}^F_{FDI}$. *Given* $\theta^H \in (0, \underline{\theta}^H)$, *the borrowing constraints are always binding in country F for* $\theta^F \in (\theta^H, 1]$.

Figure 8 illustrates these results in the cases of $\eta < \frac{2^\rho}{1+2^{\rho+1}}$ and $\eta > \frac{2^\rho}{1+2^{\rho+1}}$ respectively. The horizontal and vertical axes denote $\theta^H \in (0,1]$ and $\theta^F \in (0,1]$, respectively.

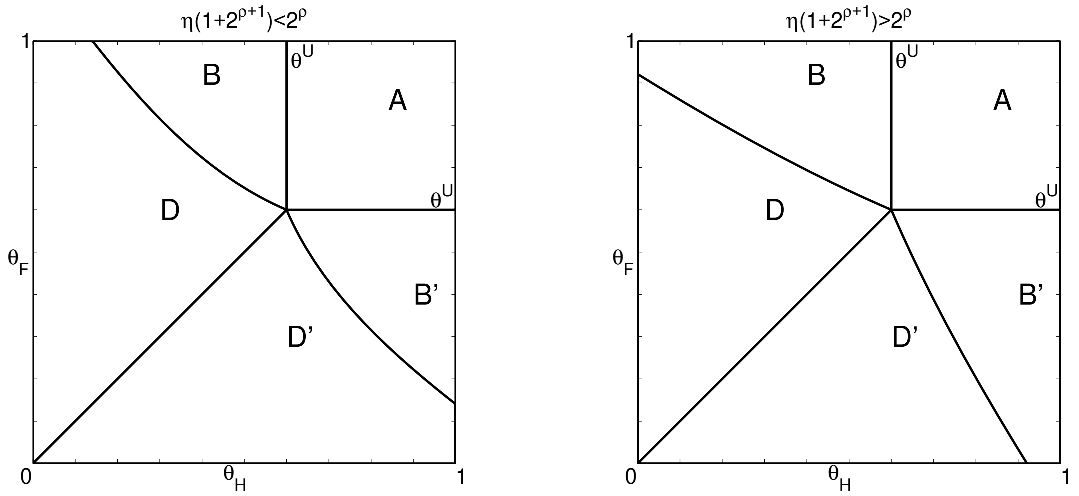

Figure 8: Free Mobility of FDI: Threshold Values

For $\theta^H = \theta^F$, i.e., the parameters on the 45 degree line, the equity rate is same in the two countries under international financial autarky. For $\theta^H \in [\bar{\theta}, 1]$ and $\theta^F \in [\bar{\theta}, 1]$, i.e., the parameters in region A, according to Proposition 1, the equity rates are equal to the marginal return on investment, which is same in the two countries. In these two cases, there are no FDI flows even if allowed. The curve splitting regions B and D represents the threshold value of $\bar{\theta}^F_{FDI}$ as the function of θ^H described by equation (44). For the parameters on the curve, the loan rate in country F is equal to the world equity rate $r^F_{FDI} = \frac{\bar{\theta}^F_{FDI}\rho}{1-\eta} = \Gamma^*_{FDI}$. Similarly, the curve splitting regions B' and D' represents the threshold value of $\bar{\theta}^H_{FDI}$ as the function of θ^F. For the parameters on the curve, the loan rate in country H is equal to the world equity rate, $r^H_{FDI} = \frac{\bar{\theta}^H_{FDI}\rho}{1-\eta} = \Gamma^*_{FDI}$.

Table 4: FDI Flows and Equity Premium in the Steady State

Region	A	B	B'	D	D'
Ω^H	0	$\Omega^H(\theta^H) < 0$	$\Omega^H(\theta^F) > 0$	$(\Omega^H(\theta^H), 0)$	$(0, \Omega^H(\theta^F))$
$\Gamma^H - r^*$	0	$+$	0	$+$	$+$
$\Gamma^F - r^*$	0	0	$+$	$+$	$+$

Table 4 summarizes the steady-state values of FDI flows and the equity premium in the five regions. Note that $\Omega^F = -\Omega^H$. $\Omega^H(\theta^i)$ implies that given the parameters in region B and B', FDI flows depend only on θ^i not on θ^m, where $i, m \in \{H, F\}$ and $i \neq m$. The borrowing constraints are strictly binding only if the equity premium is positive.

B.3 Full Capital Mobility

Proposition 24. *Given $\theta^H \in (\max\{1-2\eta, 0\}, 1-\eta)$, there exists a threshold value $\bar{\theta}^F_{FCM} = 2(1-\eta) - \theta^H$ such that for $\theta^F \in (\theta^H, \bar{\theta}^F_{FCM})$, the borrowing constraints are binding in both countries in the steady state; for $\theta^F \in (\bar{\theta}^F_{FCM}, 1]$, the world loan rate and equity rate are same as the marginal return to investment, $\Gamma^* = r^* = \rho$, in the steady state, the borrowing constraints are not binding in both countries, and the economic allocation is same as that in the case of $\theta^F = \bar{\theta}^F_{FCM}$.*

Figure 9 illustrates the results. The horizontal and vertical axes denote $\theta^H \in (0, 1]$ and $\theta^F \in (0, 1]$, respectively.

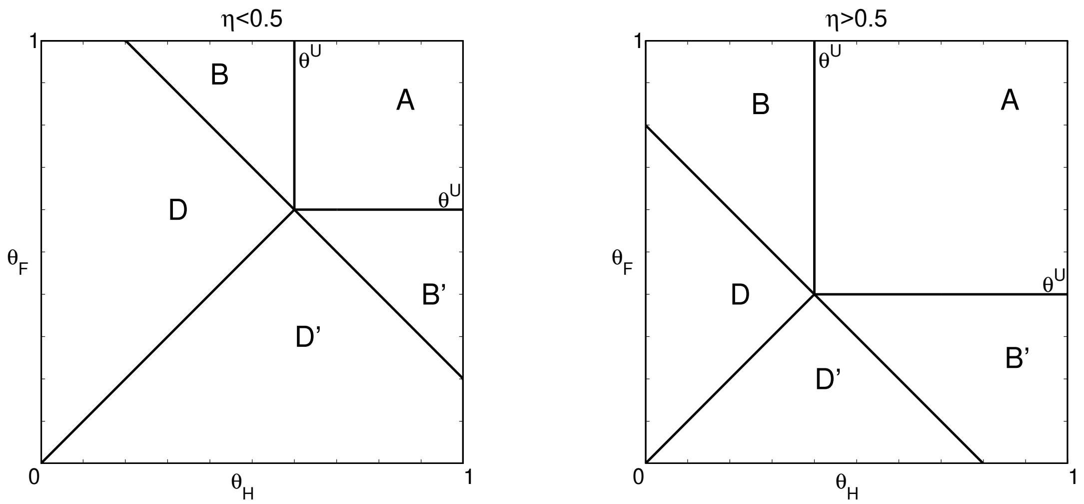

Figure 9: Full Capital Mobility: Threshold Values

For $\theta^H = \theta^F$, i.e., the parameters on the 45 degree line, the loan rates are same in the two countries under international financial autarky and so are the equity rates. For $\theta^H \in [\bar{\theta}, 1]$ and $\theta^F \in [\bar{\theta}, 1]$, i.e., the parameters in region A, according to Proposition 1, the loan rate and the equity rate under international financial autarky are equal to the marginal return on investment, which is same in the two countries, $r^i_{IFA} = \Gamma^i_{IFA} = \rho$. In these two cases, there are no financial capital flows or FDI even if allowed. The line splitting region B and D represents the threshold value of $\bar{\theta}^F_{FCM}$ as the function of θ^H, while the line splitting region B' and D' represents the threshold value of $\bar{\theta}^H_{FCM}$ as the function of θ^F. For the parameters on the two lines, the world loan rate is equal to the world equity rate, $r^* = \Gamma^* = \rho$.

Table 5 summarizes the steady-state patterns of capital flows and the equity premium in the five regions of figure 9. Note that $\Upsilon^F = -\Upsilon^H$ and $\Omega^F = -\Omega^H$. $\Upsilon^H(\theta^i)$ and $\Omega^H(\theta^i)$ implies that given the parameters in region B and B', financial capital flows and FDI depend only on θ^i not on θ^m, where $i, m \in \{H, F\}$ and $i \neq m$. The borrowing constraints are strictly binding only if the equity premium is positive.

Table 5: Capital Flows and Equity Premium in the Steady State

Region	A	B	B'	D	D'
Υ^H	0	$\Upsilon^H(\theta^H) > 0$	$\Upsilon^H(\theta^F) < 0$	$(0, \Upsilon^H(\theta^H))$	$(\Upsilon^H(\theta^F), 0)$
Ω^H	0	$\Omega^H(\theta^H) < 0$	$\Omega^H(\theta^F) > 0$	$(\Omega^H(\theta^H), 0)$	$(0, \Omega^H(\theta^F))$
$\Omega^H + \Upsilon^H$	0	0	0	$+$	$-$
$\Gamma^* - r^*$	0	0	0	$+$	$+$

C Proofs of Propositions

Proof of Propositions 2

Proof. Take the world loan rate r_t^* as given. According to equation (16), w_{t+1}^i can be considered as a function of w_t^i. For $w_t^i \in [0, \frac{(1-\theta^i)\rho}{R\eta}\left(\frac{R}{r_t^*}\right)^{\frac{1}{1-\alpha}}]$, take the first derivative of equation (16) with respect to w_t^i,

$$\eta = \left[\frac{\rho}{R\alpha}(w_{t+1}^i)^{\frac{1}{\rho}} - \frac{\theta^i \rho}{r_t^*}\right]\frac{dw_{t+1}^i}{dw_t^i}. \tag{22}$$

According to equation (16), for $w_t^i = 0$, there is a non-zero solution of $w_{t+1}^i = \left[\frac{\theta^i R}{r_t^*}\right]^\rho$. The slope of the phase diagram at the point $(0, \left[\frac{\theta^i R}{r_t^*}\right]^\rho)$ is $\frac{r_t^*\eta}{\theta^i} > 0$. In other words, $w_{t+1}^i \geq \left(\frac{\theta^i R}{r_t^*}\right)^\rho$. Thus, according to equation (22), the phase diagram of wages has the positive slope, $\frac{dw_{t+1}^i}{dw_t^i} > 0$. Take the second derivative of equation (16) with respect to w_t^i,

$$0 = \left[\frac{\rho}{R\alpha}(w_{t+1}^i)^{\frac{1}{\rho}} - \frac{\theta^i \rho}{r_t^*}\right]\frac{dw_{t+1}^i}{d^2 w_t^i} + \left(\frac{dw_{t+1}^i}{dw_t^i}\right)^2 \frac{1}{R\alpha}(w_{t+1}^i)^{\frac{1-2\alpha}{\alpha}}, \quad \Rightarrow \quad \frac{dw_{t+1}^i}{d^2 w_t^i} < 0.$$

Thus, the phase diagram of wages is concave for $w_t^i \in [0, \frac{(1-\theta^i)\rho}{R\eta}\left(\frac{R}{r_t^*}\right)^{\frac{1}{1-\alpha}}]$, and w_{t+1}^i monotonically increases in w_t^i with an intercept on the vertical axis at $w_{t+1}^i = \left[\frac{\theta^i R}{r_t^*}\right]^\rho$.

For $w_t^i > \frac{(1-\theta^i)\rho}{R\eta}\left(\frac{R}{r_t^*}\right)^{\frac{1}{1-\alpha}}$, the marginal return on investment is equal to the world loan rate, $Rv_{t+1}^i = r_t^*$, and, thus, entrepreneurs do not borrow to the limit. The phase diagram of wages $w_{t+1}^i = (v_{t+1}^i)^{-\frac{1}{\rho}} = \left(\frac{R}{r_t^*}\right)^\rho$ is flat and independent of w_t^i.

The phase diagram of wages is continuous and concave. It crosses the 45 degree line only once and from the left. There exists a stable and unique non-zero steady state. □

Proof of Lemma 2

Proof. Take the world loan rate r_t^* as given. For $w_t^i \in (0, 1-\theta^i]$ and $i_t^i = 1$, take the first and second derivatives of equation (17) with respect to w_t^i,

$$\frac{dw_{t+1}^i}{dw_t^i} = \frac{\rho r^*}{\theta^i R}(w_{t+1}^i)^{\frac{1}{\alpha}} > 0, \quad \text{and,} \quad \frac{dw_{t+1}^i}{d^2 w_t^i} = \frac{\rho r^*}{\theta^i R}\frac{1}{\alpha}(w_{t+1}^i)^{\frac{1}{\rho}}\frac{dw_{t+1}^i}{dw_t^i} > 0. \tag{23}$$

The phase diagram of wages is convex for $w_t^i \in (0, 1-\theta^i]$. By setting $w_t^i = 0$ in equation (17), we get the vertical intercept of the phase diagram of wages at $w_{t+1}^i = \left[\frac{\theta^i R}{r_t^*}\right]^\rho$. For $w_t^i > 1-\theta^i$, the marginal return on investment is equal to the world loan rate, $Rv_{t+1}^i = r_t^*$, and, thus, entrepreneurs do not borrow to the limit. The phase diagram of wages $w_{t+1}^i = (v_{t+1}^i)^{-\frac{1}{\rho}} = \left(\frac{R}{r_t^*}\right)^\rho$ is flat and independent of w_t^i. □

Proof of Proposition 3

Proof. The world loan rate is determined by the identity of financial capital flows, $\Upsilon_t^H + \Upsilon_t^F = 0$. We first prove the existence of a unique world loan rate clearing the world credit market every period and then derive the world loan rate in the steady state.

Suppose that the borrowing constraints are binding in country i. Given the predetermined w_t^i, equation (16) shows that w_{t+1}^i is a function of r_t^*. Take the first derivative of equation (16) with respect to r_t^*,

$$0 = \left[\frac{\rho}{R\alpha}(w_{t+1}^i)^{\frac{1}{\rho}} - \frac{\theta^i \rho}{r_t^*}\right]\frac{dw_{t+1}^i}{dr_t^*} + \frac{\theta^i \rho}{(r_t^*)^2}.$$

As shown in the proof of Proposition 2, $w_{t+1}^i \geq \left(\frac{\theta^i R}{r_t^*}\right)^\rho$ so that the term in the square bracket is positive. An increase in the world loan rate enhances financial capital outflows and reduces domestic investment. Thus, the wage in the next period declines, $\frac{dw_{t+1}^i}{dr_t^*} < 0$.

Capital outflows represent the gap between domestic savings and investment,

$$\Upsilon_t^i = w_t^i - I_t^i = w_t^i - \frac{\rho}{R}(w_{t+1}^i)^{\frac{1}{\alpha}} = (1-\eta)w_t^i - \frac{\rho}{r_t^*}\theta^i w_{t+1}^i. \tag{24}$$

The world credit market equilibrium implies

$$\Upsilon_t^H + \Upsilon_t^F = 0, \;\Rightarrow\; (1-\eta)(w_t^H + w_t^F) = \frac{\rho}{r_t^*}\left(\theta^H w_{t+1}^H + \theta^F w_{t+1}^F\right). \tag{25}$$

The loan rate in country H is lower than in country F before period $t = 0$, $r_{IFA}^H < r_{IFA}^F$. The world loan rate in period $t \geq 0$ must be $r_t^* \in (r_{IFA}^H, r_{IFA}^F)$. The proof is by contradiction. If $r_t^* > r_{IFA}^F > r_{IFA}^H$, w_{t+1}^i would be lower than under international financial autarky in both countries, as $\frac{dw_{t+1}^i}{dr_t^*} < 0$. Thus, equation (25) would not hold. The same argument applies to the case of $r_t^* < r_{IFA}^H < r_{IFA}^F$. Since w_{t+1}^i monotonically decreases with r_t^*, there is a unique solution $r_t^* \in (r_{IFA}^H, r_{IFA}^F)$ that clears the world credit market and financial capital flows from country H to F, given the predetermined w_t^H and w_t^F.

In the next step, we assume that there exists a unique world loan rate in the steady state and then prove its uniqueness.

Given r^*, Proposition 2 shows that the wage in the steady state is $w^i = \left(\frac{R}{\rho}\right)^\rho \left(\eta + \frac{\theta^i \rho}{r^*}\right)^\rho$. According to equation (24), financial capital flows in the steady state are

$$\Upsilon^i = w^i\left[(1-\eta) - \frac{\theta^i \rho}{r^*}\right] \quad\Rightarrow\quad \Upsilon^i = (r^* - r_{IFA}^i)\frac{(1-\eta)w^i}{r^*} \tag{26}$$

According to equation (25), the world loan rate in the steady state is determined by

$$\Upsilon^H + \Upsilon^F = 0, \quad \text{or,} \quad \frac{\frac{\theta^F \rho}{1-\eta} - r^*}{r^* - \frac{\theta^H \rho}{1-\eta}} = \frac{w^H}{w^F}, \quad \text{or,} \quad \frac{r^F_{IFA} - r^*}{r^* - r^H_{IFA}} = \left(\frac{\eta r^* + \theta^H \rho}{\eta r^* + \theta^F \rho} \right)^\rho. \qquad (27)$$

For $0 < \theta^H < \theta^F \leq \bar{\theta}$, the right-hand side of equation (27) is less than one,

$$\frac{r^F_{IFA} - r^*}{r^* - r^H_{IFA}} < 1, \quad \Rightarrow \quad r^* \in (\underline{r}^*, r^F_{IFA}).$$

Let $\aleph(r^*) \equiv \frac{r^F_{IFA} - r^H_{IFA}}{r^* - r^H_{IFA}} - 1$ and $\Re(r^*) \equiv \left[1 - \frac{(\theta^F - \theta^H)\rho}{\eta r^* + \theta^F \rho}\right]^\rho$ denote the left-hand and the right-hand sides of equation (27) as the functions of r^*. For $r^* \in (\underline{r}^*, r^F_{IFA})$,

$$\aleph'(r^*) < 0 < \Re'(r^*),$$
$$\aleph(r^* = \underline{r}^*) = 1 > \Re(r^* = \underline{r}^*),$$
$$\aleph(r^* = r^F_{IFA}) = 0 < \Re(r^* = r^F_{IFA}).$$

Thus, $\aleph(r^*)$ decreases while $\Re(r^*)$ increases monotonically in r^*; the two functions cross once and only once at $r^* \in (\underline{r}^*, r^F_{IFA})$. Therefore, there exists a unique steady state. □

Proof of Proposition 4

Proof. Under free mobility of financial capital, if the borrowing constraints are binding in country i, the equity rate is $\Gamma^i_t = \frac{(1-\theta)Rv^i_{t+1}}{1 - \frac{\theta R v^i_{t+1}}{r^*_t}}$. Using equations (14) and (15), we can rewrite the equity rate as $\Gamma^i_t = \frac{(1-\theta^i)Rv^i_{t+1}}{\left[\frac{\eta w^i_t}{\frac{\rho}{R}(w^i_{t+1})^{\frac{1}{\alpha}}}\right]} = \frac{(1-\theta^i)\rho}{\eta} \frac{w^i_{t+1}}{w^i_t}$. □

Proof of Proposition 5

Proof. See the proof of Proposition 3 and equation (26). □

Proof of Proposition 6

Proof. Let $a_t \equiv \frac{w^H_t + w^F_t}{2 w_{IFA}}$ and $b_t \equiv \frac{w^F_t - w^H_t}{2 w_{IFA}} + \frac{\Upsilon^H_t}{w_{IFA}}$, where $t = 0, 1, 2, 3, \ldots$. According to Proposition 5 and the aggregate resource constraint in country H, $0 < \Upsilon^H_t < w^H_{FCF}$, we get $b_t \in (0, a_t)$. In period $t \geq 0$, the aggregate project investment in country H and in country F are $I^H_t = w^H_t - \Upsilon^H_t = (a_t - b_t) w^i_{IFA}$ and $I^F_t = w^F_t + \Upsilon^H_t = (a_t + b_t) w^i_{IFA}$, respectively. Given the share of capital goods in the aggregate production, $\alpha \in (0,1)$, and $b_t \in (0, a_t)$, the world-average wage in period $t+1$ can be reformulated into a condensed form with the following property,

$$\frac{w^H_{t+1} + w^H_{t+1}}{2} = \left(\frac{R}{\rho}\right)^\alpha \left[\frac{(I^H_t)^\alpha + (I^F_t)^\alpha}{2}\right] \Leftrightarrow a_{t+1} = \frac{(a_t - b_t)^\alpha + (a_t + b_t)^\alpha}{2} < (a_t)^\alpha, \quad (28)$$

where the last inequality sign results from the Jensen's Inequality. The wage in period $t = 0$ is same in the two countries, $w_0^H = w_0^F = w_{IFA}$, and, thus, $a_0 = 1$. From period 0 on, financial capital flows are allowed. According to the inequality in equation (28), we get $a_1 < 1$. For $t = 1, 2, 3, ...$, given $b_t \in (0, a_t)$, we have $a_{t+1} < (a_t)^\alpha$ and, thus, the time series of a_t is below 1, or equivalently, $\frac{w_t^H + w_t^F}{2} < w_{IFA}$. Thus, the world output is smaller than before period $t = 0$, $Y_t^H + Y_t^F = \frac{w_t^H + w_t^F}{1-\alpha} < \frac{2w_{IFA}}{1-\alpha} = Y_{IFA}^H + Y_{IFA}^F$. □

Proof of Proposition 7

Proof. If the borrowing constraints are binding, the steady-state workers' consumption is

$$c^{i,w} = w^i r^* = \left(\frac{R}{\rho}\right)^\rho (r^*\eta + \theta^i \rho)^\rho (r^*)^{1-\rho},$$

$$\frac{d \ln c^{i,w}}{dr^*} = \frac{r^*\eta + \theta^i\rho - \theta^i\rho^2}{(r^*\eta + \theta^i\rho)r^*}.$$

As an analytical solution of the world loan rate is not obtainable, we provide sufficient conditions for the welfare changes as follows.

Let $\kappa \equiv \frac{(\rho-1)(1-\eta)}{\eta}$. Evaluate $\frac{d\ln c^{H,w}}{dr^*}$ at $r^* = r_{IFA}^H$ and $r^* = r_{IFA}^F$. For $\kappa \leq 1$, $\frac{d\ln c^{H,w}}{dr^*}|_{r^*=r_{IFA}^F} > \frac{d\ln c^{H,w}}{dr^*}|_{r^*=r_{IFA}^H} \geq 0$ implies that workers born in country H is better off in the long run than before period $t = 0$ since the positive loan rate effect dominates the negative wage effect; for $\kappa \geq \frac{\theta^F}{\theta^H}$, $\frac{d\ln c^{H,w}}{dr^*}|_{r^*=r_{IFA}^H} < \frac{d\ln c^{H,w}}{dr^*}|_{r^*=r_{IFA}^F} \leq 0$ implies that workers born in country H is worse off in the long run since the negative wage effect dominates; for $\kappa \in (1, \frac{\theta^F}{\theta^H})$, the numerical solutions are needed for the welfare evaluation.

Evaluate $\frac{d\ln c^{F,w}}{dr^*}$ at $r^* = \underline{r}^*$ and $r^* = r_{IFA}^F$. For $\kappa \leq \frac{\theta^H+\theta^F}{2\theta^F}$, $\frac{d\ln c^{F,w}}{dr^*}|_{r^*=r_{IFA}^F} > \frac{d\ln c^{F,w}}{dr^*}|_{r^*=\underline{r}^*} \geq 0$ implies that workers born in country F is worse off in the long run than before period $t = 0$ since the negative loan rate effect dominates the positive wage effect; for $\kappa \geq 1$, $\frac{d\ln c^{F,w}}{dr^*}|_{r^*=r_{IFA}^F} < \frac{d\ln c^{F,w}}{dr^*}|_{r^*=\underline{r}^*} \leq 0$ implies that workers born in country F is better off in the long run since the positive wage effect dominates; for $\kappa \in (\frac{\theta^H+\theta^F}{2\theta^F}, 1)$, the numerical solutions are required for the welfare evaluation.

Social welfare in country i in the steady state is

$$\begin{aligned} C^i &\equiv \eta c^{i,e} + (1-\eta)c^{i,w} = w^i[\eta\Gamma^i + (1-\eta)r^*] \\ &= \left(\frac{R}{\rho}\right)^\rho (r^*\eta + \theta^i\rho)^\rho (r^*)^{-\rho}[(1-\theta^i)\rho + (1-\eta)r^*], \end{aligned}$$

$$\frac{d\ln C^i}{dr^*} = \frac{\eta\rho}{r^*\eta + \theta^i\rho} - \frac{\rho}{r^*} + \frac{1-\eta}{(1-\theta^i)\rho + (1-\eta)r^*}.$$

Evaluate $\frac{d\ln C^H}{dr^*}$ at $r^* = r_{IFA}^H$ and $r^* = r_{IFA}^F$. For $\rho \in (0, \frac{\theta^H}{1-\eta}]$, $\frac{d\ln C^H}{dr^*}|_{r^*=r_{IFA}^F} > \frac{d\ln C^H}{dr^*}|_{r^*=r_{IFA}^H}$ ≥ 0 implies that the workers' welfare gains dominate the welfare losses of entrepreneurs and hence, country H as a whole benefits from free mobility of financial capital; for $\rho \in [\frac{\theta^F[\theta^H+\eta(\theta^F-\theta^H)]}{(1-\eta)\theta^H[1+(\theta^F-\theta^H)]}, \infty)$, $\frac{d\ln C^H}{dr^*}|_{r^*=r_{IFA}^H} < \frac{d\ln C^H}{dr^*}|_{r^*=r_{IFA}^F} \leq 0$ implies that both workers and entrepreneurs are worse off or the workers' welfare gains are dominated by the welfare losses of entrepreneurs and hence, country H as a whole loses from free mobility of financial capital; for $\rho \in (\frac{\theta^H}{1-\eta}, \frac{\theta^F[\theta^H+\eta(\theta^F-\theta^H)]}{(1-\eta)\theta^H[1+(\theta^F-\theta^H)]})$, the numerical solutions are required for the welfare evaluation.

Evaluate $\frac{d\ln C^F}{dr^*}$ at $r^*=\underline{r}^*$ and $r^*=r^F_{IFA}$. For $\rho \in (0, \frac{\frac{\theta^H+\theta^F}{2(1-\eta)}\left[\theta^F-\frac{\eta(\theta^F-\theta^H)}{2}\right]}{\theta^F\left[1-\frac{(\theta^F-\theta^H)}{2}\right]}]$, $\frac{d\ln C^F}{dr^*}|_{r^*=r^F_{IFA}} \geq \frac{d\ln C^F}{dr^*}|_{r^*=\underline{r}^*} \geq 0$ implies that both workers and entrepreneurs are worse off or the workers' welfare gains are dominated by the welfare losses of entrepreneurs and hence, country F as a whole loses from free mobility of financial capital; for $\rho \in [\frac{\theta^F}{1-\eta}, \infty)$, $\frac{d\ln C^F}{dr^*}|_{r^*=\underline{r}^*} < \frac{d\ln C^F}{dr^*}|_{r^*=r^F_{IFA}} \leq 0$ implies that the workers' welfare gains dominate the welfare losses of entrepreneurs and hence, country H as a whole benefits from free mobility of financial capital; for $\rho \in (\frac{\frac{\theta^H+\theta^F}{2(1-\eta)}\left[\theta^F-\frac{\eta(\theta^F-\theta^H)}{2}\right]}{\theta^F\left[1-\frac{(\theta^F-\theta^H)}{2}\right]}, \frac{\theta^F}{1-\eta})$, the numerical solutions are required for the welfare evaluation. □

Proof of Proposition 9

Proof. According to equation (19), w^i_{t+1} is determined only by Γ^*_t and r^*_t. The phase diagram of wages is flat and crosses the 45 degree line only once and from the left. □

Proof of Proposition 10

Proof. The world equity rate Γ^*_t is determined by the identity of FDI flows, $\Omega^H_t + \Omega^F_t = 0$ and the world loan rate r^*_t by $\Upsilon^H_t + \Upsilon^F_t = 0$. We first prove the existence of a unique world equity (loan) rate and clearing the world equity (credit) market every period. Then, we derive the world interest rates in the steady state.

According to the domestic credit market equilibrium and the Cobb-Douglas production function, FDI and financial capital flows are solved as

$$\lambda^i_t\left(\eta w^i_t - \Omega^i_t\right) = \frac{\lambda^i_t}{\lambda^i_t - 1}\left[(1-\eta)w^i_t - \Upsilon^i_t\right] = I^i_t = \frac{\rho}{R}(w^i_{t+1})^{\frac{1}{\alpha}}, \tag{29}$$

$$\Omega^i_t = \eta w^i_t - \frac{(1-\theta^i)\rho}{\Gamma^*_t} w^i_{t+1}, \tag{30}$$

$$\Upsilon^i_t = (1-\eta)w^i_t - \frac{\theta^i\rho}{r^*_t} w^i_{t+1}, \tag{31}$$

$$\Omega^i_t + \Upsilon^i_t = w^i_t - I^i_t = w^i_t - \frac{\rho}{R}(w^i_{t+1})^{\frac{1}{\alpha}}. \tag{32}$$

Given the world interest rates at Γ^*_t and r^*_t and the predetermined labor income w^i_t, w^i_{t+1} is uniquely determined under full capital mobility and so are Υ^i_t and Ω^i_t. Take first derivative of equations (30) and (31) with respect to the two interest rates, respectively,

$$\frac{d\Omega_t^i}{d\Gamma_t^*} = \frac{(1-\theta^i)\rho}{(\Gamma_t^*)^2} w_{t+1}^i - \frac{(1-\theta^i)\rho}{\Gamma_t^*}\frac{dw_{t+1}^i}{d\Gamma_t^*} > 0, \qquad \frac{d\Omega_t^i}{dr_t^*} = -\frac{(1-\theta^i)\rho}{\Gamma_t^*}\frac{dw_{t+1}^i}{dr_t^*} > 0,$$
$$\frac{d\Upsilon_t^i}{dr_t^*} = \frac{\theta^i\rho}{(r_t^*)^2} w_{t+1}^i - \frac{\theta^i\rho}{r_t^*}\frac{dw_{t+1}^i}{dr_t^*} > 0, \qquad \frac{d\Upsilon_t^i}{d\Gamma_t^*} = \frac{\theta^i\rho}{r_t^*}\frac{dw_{t+1}^i}{d\Gamma_t^*} > 0.$$

Υ_t^i and Ω_t^i increase monotonically in Γ_t^* and r_t^*. By contradiction, we can prove the existence and the uniqueness of the world interest rates as $\Gamma_t^* \in (\Gamma_{IFA}^F, \Gamma_{IFA}^H)$ and $r_t^* \in (r_{IFA}^H, r_{IFA}^F)$. Given w_t^i, the world interest rates Γ_t^* and r_t^* are uniquely determined by the two equilibrium conditions, i.e., $\Omega_t^H + \Omega_t^F = 0$ and $\Upsilon_t^H + \Upsilon_t^F = 0$,

$$\eta(w_t^H + w_t^F) = \frac{\rho}{\Gamma_t^*}\left[(1-\theta^H)R^\rho\left(\frac{1-\theta^H}{\Gamma_t^*} + \frac{\theta^H}{r_t^*}\right)^\rho + (1-\theta^F)R^\rho\left(\frac{1-\theta^F}{\Gamma_t^*} + \frac{\theta^F}{r_t^*}\right)^\rho\right],$$
$$(1-\eta)(w_t^H + w_t^F) = \frac{\rho}{r_t^*}\left[\theta^H R^\rho\left(\frac{1-\theta^H}{\Gamma_t^*} + \frac{\theta^H}{r_t^*}\right)^\rho + \theta^F R^\rho\left(\frac{1-\theta^F}{\Gamma_t^*} + \frac{\theta^F}{r_t^*}\right)^\rho\right].$$

In the next step, we assume that there exists a unique world loan rate and a unique world equity rate in the steady state and then prove that they are indeed unique.

In the steady state, $w_{t+1}^i = w_t^i$. According to equations (30) and (31), the equilibrium conditions of FDI and financial capital flows, $\Omega^H + \Omega^F = \Upsilon^H + \Upsilon^F = 0$, are rewritten as,

$$\frac{\eta - \frac{(1-\theta^F)\rho}{\Gamma^*}}{\frac{(1-\theta^H)\rho}{\Gamma^*} - \eta} = \frac{w^H}{w^F} = \frac{\frac{\theta^F\rho}{r^*} - (1-\eta)}{(1-\eta) - \frac{\theta^H\rho}{r^*}}, \tag{33}$$
$$\frac{(\theta^F - \theta^H)\rho}{(\rho - \eta\Gamma^*) - \theta^H\rho} - 1 = \frac{(\theta^F - \theta^H)\rho}{(1-\eta)r^* - \theta^H\rho} - 1, \tag{34}$$
$$(\rho - \eta\Gamma^*) - \theta^H\rho = (1-\eta)r^* - \theta^H\rho, \;\Rightarrow\; \Gamma^* = \frac{\rho}{\eta} - \frac{1-\eta}{\eta}r^*. \tag{35}$$

In the case of the binding borrowing constraints, $\frac{\partial \ln w^i}{\partial \theta^i} = \frac{\rho(\Gamma^* - r^*)}{r^* + \theta^i(\Gamma^* - r^*)} > 0$ implies $w^H < w^F$. According to equation (33),

$$\frac{\eta - \frac{(1-\theta^F)\rho}{\Gamma^*}}{\frac{(1-\theta^H)\rho}{\Gamma^*} - \eta} = \frac{\frac{\theta^F\rho}{r^*} - (1-\eta)}{(1-\eta) - \frac{\theta^H\rho}{r^*}} = \frac{w^H}{w^F} < 1, \quad \Rightarrow \quad \Gamma^* < \underline{\Gamma}^* \quad \text{and} \quad r^* > \underline{r}^*.$$

Thus, the steady-state values of the world equity rate and the world loan rate are $\Gamma^* \in (\Gamma_{IFA}^F, \underline{\Gamma}^*)$ and $r^* \in (\underline{r}^*, r_{IFA}^F)$, respectively.

Substitute equation (35) and $w^i = R^\rho\left[\frac{1-\theta^i}{\Gamma_t^*} + \frac{\theta^i}{r_t^*}\right]^\rho$ into equation (33), r^* solves,

$$\frac{(\theta^F - \theta^H)}{(1-\eta)\frac{r^*}{\rho} - \theta^H} - 1 = \left[1 - \frac{(\theta^F - \theta^H)}{\frac{\eta}{\frac{\rho}{r^*} - 1} + \theta^F}\right]^\rho. \tag{36}$$

Let $\aleph(r^*) \equiv \frac{(\theta^F - \theta^H)}{(1-\eta)\frac{r^*}{\rho} - \theta^H} - 1$ and $\Re(r^*) \equiv \left[1 - \frac{(\theta^F - \theta^H)}{\frac{\eta}{\frac{\rho}{r^*} - 1} + \theta^F}\right]^\rho$ denote the functions of r^* defined by the left-hand and the right-hand sides of equation (36). Given $\theta^F > \theta^H$ and

$r^* \in (\underline{r}^*, r_{IFA}^F)$, we get

$$\aleph'(r^*) < 0 < \Re'(r^*),$$
$$\aleph(r^* = \underline{r}^*) = 1 > \Re(r^* = \underline{r}^*),$$
$$\aleph(r^* = r_{IFA}^F) = 0 < \Re(r^* = r_{IFA}^F).$$

In the steady state, there exists a unique world loan rate $r^* \in (\underline{r}^*, r_{IFA}^F)$ that solves equation (36). So does the world equity rate, according to equation (35). $\square$

Proof of Proposition 11

Proof. See the proofs of Propositions 10 and 12. $\square$

Proof of Proposition 12

Proof. According to equations (30) and (31), the steady-state values of FDI and financial capital flows are $\Omega^i = (\Gamma^* - \Gamma_{IFA}^i)\frac{\eta w^i}{\Gamma^*}$ and $\Upsilon^i = (r^* - r_{IFA}^i)\frac{(1-\eta)w^i}{r^*}$, respectively. Since $r^* \in (\underline{r}^*, r_{IFA}^F)$, financial capital flows from country H to country F, $\Upsilon^H > 0 > \Upsilon^F$; since $\Gamma^* \in (\Gamma_{IFA}^F, \underline{\Gamma}^*)$, FDI flows in the opposite direction, $\Omega^H < 0 < \Omega^F$. The direction of capital flows is same as under free mobility of FDI and financial capital, respectively.

According to equations (32), net capital flows are $\Omega^i + \Upsilon^i = w^i\left[1 - \frac{\rho}{R}(w^i)^{\frac{1}{\rho}}\right]$ in the steady state. The identity of net capital flows $\Omega^H + \Upsilon^H + \Omega^F + \Upsilon^F = 0$ implies

$$\sum_{i\in\{H,F\}} w^i\left[1 - \frac{\rho}{R}(w^i)^{\frac{1}{\rho}}\right] = 0 \quad \Rightarrow \quad \left[1 - \frac{\rho}{R}(w^H)^{\frac{1}{\rho}}\right]\left[1 - \frac{\rho}{R}(w^F)^{\frac{1}{\rho}}\right] \leq 0. \tag{37}$$

If $\Gamma^* > r^*$, the borrowing constraints are binding and the steady-state wage is lower in country H than in country F, $w^H \leq w^F$. Thus, $1 - \frac{\rho}{R}(w^H)^{\frac{1}{\rho}} > 1 - \frac{\rho}{R}(w^F)^{\frac{1}{\rho}}$. According to equation (37), $1 - \frac{\rho}{R}(w^H)^{\frac{1}{\rho}} > 0 > 1 - \frac{\rho}{R}(w^F)^{\frac{1}{\rho}}$ and the net capital flows are from country H to country F in the steady state, $\Omega^H + \Upsilon^H > 0 > \Omega^F + \Upsilon^F$.

If $\Gamma^* = r^*$, the borrowing constraints are not binding and the steady-state wage is same in the two countries with zero net capital flows, $w^i = \left(\frac{R}{\rho}\right)^\rho$ and $\Omega^i + \Upsilon^i = 0$. Economic allocation is almost same as under international financial autarky except that the interest rates in country H are different, $\Gamma^* = \rho < \Gamma_{IFA}^H$ and $r^* = \rho > r_{IFA}^H$. $\square$

Proof of Proposition 13

Proof. According to equations (30) and (31), in the steady state, $r^*\Upsilon^i + \Gamma^*\Omega^i = w^i[(1 - \eta)r^* + \eta\Gamma^*] - \rho w^i$. According to equation (35), we get $r^*\Upsilon^i + \Gamma^*\Omega^i = 0$. This way, as a net debtor, $\Upsilon^F + \Omega^F < 0$, country F receives a positive net investment income, $NII^F \equiv (r^* - 1)\Upsilon^F + (\Gamma^* - 1)\Omega^F = 0 - (\Upsilon^F + \Omega^F) > 0$. Intuitively, the net interest income received by entrepreneurs from investing abroad, $|(\Gamma^* - 1)\Omega^F|$ dominates the net interest income paid to foreign workers, $|(r^* - 1)\Upsilon^F|$, due to the positive equity premium. $\square$

Proof of Proposition 14

Proof. According to Proposition 12, $w^H_{FCM} \leq w^i_{IFA} \leq w^F_{FCM}$. Given the world equity rate $\Gamma^*_{FCM} \in (\Gamma^F_{IFA}, \underline{\Gamma}^*)$, entrepreneurs born in country H (F) are worse (better) off in the long run than before period $t = 0$, due to the declines (increases) in the wage and in the equity rate, $c^{i,e} = w^i \Gamma^i$.

In the steady state, social welfare in country i is $C^i = \eta c^{i,e} + (1-\eta)c^{i,w} = w^i[\eta\Gamma^* + (1-\eta)r^*]$. According to equation (35), social welfare is proportional to the aggregate labor income, $C^i = w^i\rho$. Due to net capital flows, the aggregate investment in country H (F) is lower and so are the aggregate labor income and social welfare. □

Proof of Proposition 15

Proof. Take the world equity rate Γ^*_t as given. According to equation (21), w^i_{t+1} is considered as a function of w^i_t. For $w^i_t \in [0, \frac{\theta^i\rho}{R(1-\eta)}\left(\frac{R}{\Gamma^*_t}\right)^{\frac{1}{1-\alpha}})$, the marginal return on investment is equal to the world equity rate, $Rv^i_{t+1} = \Gamma^*_t$, and, thus, entrepreneurs do not borrow to the limit. The phase diagram of wages is flat at $w^i_{t+1} = \left(\frac{R}{\Gamma^*_t}\right)^{\rho}$, independent of w^i_t.

For $w^i_t \geq \frac{\theta^i\rho}{R(1-\eta)}\left(\frac{R}{\Gamma^*_t}\right)^{\frac{1}{1-\alpha}}$, take the first derivative of equation (21) with respect to w^i_t,

$$1-\eta = \left[\frac{\rho}{R\alpha}(w^i_{t+1})^{\frac{1}{\rho}} - \frac{(1-\theta^i)\rho}{\Gamma^*_t}\right]\frac{dw^i_{t+1}}{dw^i_t}. \tag{38}$$

For $w^i_t = \frac{\theta^i\rho}{R(1-\eta)}\left(\frac{R}{\Gamma^*_t}\right)^{\frac{1}{1-\alpha}}$, there is a non-zero solution $w^i_{t+1} = \left(\frac{R}{\Gamma^*_t}\right)^{\rho}$. The slope of the phase diagram at the point $(\frac{\theta^i\rho}{R(1-\eta)}\left(\frac{R}{\Gamma^*_t}\right)^{\frac{1}{1-\alpha}}, \left(\frac{R}{\Gamma^*_t}\right)^{\rho})$ is $\frac{\Gamma^*_t\eta}{\rho[\frac{1}{\alpha}-(1-\theta^i)]} > 0$. In other words, $w^i_{t+1} \geq \left(\frac{\theta^i R}{r^*_t}\right)^{\rho}$. Thus, according to equation (38), the phase diagram has the positive slope, $\frac{dw^i_{t+1}}{dw^i_t} > 0$. Take the second derivative of equation (21) with respect to w^i_t,

$$0 = \left[\frac{\rho}{R\alpha}(w^i_{t+1})^{\frac{1}{\rho}} - \frac{(1-\theta^i)\rho}{\Gamma^*_t}\right]\frac{dw^i_{t+1}}{d^2w^i_t} + \left(\frac{dw^i_{t+1}}{dw^i_t}\right)^2\frac{1}{R\alpha}(w^i_{t+1})^{\frac{1-2\alpha}{\alpha}}, \quad \Rightarrow \quad \frac{dw^i_{t+1}}{d^2w^i_t} < 0.$$

The phase diagram of wages is concave for $w^i_t > \frac{\theta^i\rho}{R(1-\eta)}\left(\frac{R}{\Gamma^*_t}\right)^{\frac{1}{1-\alpha}}$ and w^i_{t+1} monotonically increases in w^i_t.

The phase diagram of wages is continuous and concave. It crosses the 45 degree line only once and from the left. There exists a stable and unique non-zero steady state. □

Proof of Proposition 16

Proof. The world equity rate is determined by the identity of FDI flows, $\Omega^H_t + \Omega^F_t = 0$. We first prove the existence of a unique world equity rate clearing the world equity market every period and then derive the world equity rate in the steady state.

Suppose that the borrowing constraints are binding in country i. Given the predetermined w_t^i, equation (21) shows that w_{t+1}^i is a function of Γ_t^*. Take the first derivative of equation (21) with respect to Γ_t^*,

$$0 = \left[\frac{\rho}{R\alpha}(w_{t+1}^i)^{\frac{1}{\rho}} - \frac{(1-\theta^i)\rho}{\Gamma_t^*}\right]\frac{dw_{t+1}^i}{dr_t^*} + \frac{(1-\theta^i)\rho}{(\Gamma_t^*)^2}.$$

As shown in the proof of Proposition 15, $w_{t+1}^i \geq \left(\frac{R}{\Gamma_t^*}\right)^\rho$ so that the term in square brackets is positive. An increase in the world equity rate enhances FDI outflows and reduces the domestic investment. Thus, the wage in the next period declines, $\frac{dw_{t+1}^i}{d\Gamma_t^*} < 0$.

Capital outflows represent the gap between domestic savings and investment,

$$\Omega_t^i = w_t^i - I_t^i = w_t^i - \frac{\rho}{R}(w_{t+1}^i)^{\frac{1}{\alpha}} = \eta w_t^i - \frac{\rho}{\Gamma_t^*}(1-\theta^i)w_{t+1}^i. \tag{39}$$

The world equity market equilibrium implies

$$\Omega_t^H + \Omega_t^F = 0, \;\Rightarrow\; \eta(w_t^H + w_t^F) = \frac{\rho}{\Gamma_t^*}\left[(1-\theta^H)w_{t+1}^H + (1-\theta^F)w_{t+1}^F\right]. \tag{40}$$

The equity rate in country H is higher than in country F before period $t=0$, $\Gamma_{IFA}^H > \Gamma_{IFA}^F$. Given the predetermined wage w_t^H and w_t^F, the world equity rate in period t must be $\Gamma_t^* \in (\Gamma_{IFA}^F, \Gamma_{IFA}^H)$ and FDI flows from country F to country H. The proof is by contradiction similar as in the proof of Proposition 3.

In the next step, we assume that there exists a unique world equity rate in the steady state and then prove its uniqueness.

Given Γ^*, Proposition 15 shows the steady-state wage $w^i = \left(\frac{R}{\rho}\right)^\rho\left[(1-\eta) + \frac{(1-\theta^i)\rho}{\Gamma^*}\right]^\rho$. According to equation (39), FDI flows in the steady state are

$$\Omega^i = w^i\left[\eta - \frac{(1-\theta^i)\rho}{\Gamma^*}\right] \quad\Rightarrow\quad \Omega^i = (\Gamma^* - \Gamma_{IFA}^i)\frac{\eta w^i}{\Gamma^*}. \tag{41}$$

According to equation (40), the world equity rate in the steady state is determined by

$$\Omega^H + \Omega^F = 0, \;\Rightarrow\; \frac{\Gamma^* - \frac{(1-\theta^F)\rho}{\eta}}{\frac{(1-\theta^H)\rho}{\eta} - \Gamma^*} = \frac{w^H}{w^F}, \;\Rightarrow\; \frac{\Gamma^* - \Gamma_{IFA}^F}{\Gamma_{IFA}^H - \Gamma^*} = \left[\frac{\Gamma^* + \frac{1-\theta^H}{1-\eta}\rho}{\Gamma^* + \frac{1-\theta^F}{1-\eta}\rho}\right]^\rho. \tag{42}$$

For $\theta^H \in (0,\bar{\theta})$ and $\theta^F > \theta^H$, the right-hand side of equation (42) is larger than one,

$$\frac{\Gamma^* - \Gamma_{IFA}^F}{\Gamma_{IFA}^H - \Gamma^*} > 1, \quad \text{or} \quad \Gamma^* \in (\underline{\Gamma}^*, \Gamma_{IFA}^H).$$

Let $\aleph(\Gamma^*) \equiv \frac{\Gamma_{IFA}^H - \Gamma_{IFA}^F}{\Gamma_{IFA}^H - \Gamma^*} - 1$ and $\Re(\Gamma^*) \equiv \left[1 + \frac{(\theta^F - \theta^H)\rho}{(1-\eta)\Gamma^* + (1-\theta^F)\rho}\right]^\rho$ denote the left-hand and the right-hand sides of equation (42) as the functions of Γ^*. For $\Gamma^* \in (\underline{\Gamma}^*, \Gamma_{IFA}^H)$,

$$\aleph'(\Gamma^*) > 0 > \Re'(\Gamma^*),$$
$$\aleph(\Gamma^* = \underline{\Gamma}^*) = 0 < \Re(\Gamma^* = \underline{\Gamma}^*),$$
$$\aleph(\Gamma^* = \Gamma_{IFA}^H) \to \infty > \Re(\Gamma^* = \Gamma_{IFA}^H).$$

$\aleph(\Gamma^*)$ decreases while $\Re(\Gamma^*)$ increases monotonically in Γ^*; the two functions cross once and only once at $\Gamma^* \in (\underline{\Gamma}^*, \Gamma^H_{IFA})$. Thus, there exists a unique non-zero steady state. $\square$

Proof of Proposition 18

Proof. See the proof of Proposition 16 and equation (41). $\square$

Proof of Proposition 20

Proof. If the borrowing constraints are binding, the steady-state consumption of entrepreneurs is

$$c^{i,e} = w^i\Gamma^* = \left(\frac{R}{\rho}\right)^\rho \left[\Gamma^*(1-\eta) + (1-\theta^i)\rho\right]^\rho (\Gamma^*)^{1-\rho},$$
$$\frac{d\ln c^{i,e}}{d\Gamma^*} = \frac{\Gamma^*(1-\eta) + (1-\theta^i)\rho(1-\rho)}{[\Gamma^*(1-\eta) + (1-\theta^i)\rho]\Gamma^*}.$$

As the analytical solution of the world equity rate is not obtainable, we provide the sufficient conditions of welfare changes as follows.

Let $\mu \equiv \frac{(\rho-1)\eta}{(1-\eta)}$. Evaluate $\frac{d\ln c^{H,e}}{d\Gamma^*}$ at $\Gamma^* = \Gamma^H_{IFA}$ and $\Gamma^* = \underline{\Gamma}^*$. For $\mu \leq \frac{(1-\theta^H)+(1-\theta^F)}{2(1-\theta^H)}$, $\frac{d\ln c^{H,e}}{d\Gamma^*}|_{\Gamma^*=\Gamma^H_{IFA}} > \frac{d\ln c^{H,e}}{d\Gamma^*}|_{\Gamma^*=\underline{\Gamma}^*} \geq 0$ implies that entrepreneurs born in country H is worse off in the long run than under before period $t = 0$, since the negative equity rate effect dominates the positive wage effect; for $\mu > 1$, $\frac{d\ln c^{H,e}}{d\Gamma^*}|_{\Gamma^*=\underline{\Gamma}^*} < \frac{d\ln c^{H,e}}{d\Gamma^*}|_{\Gamma^*=\Gamma^H_{IFA}} \leq 0$ implies that entrepreneurs born in country H is better off in the long run since the positive wage effect dominates; for $\mu \in (\frac{(1-\theta^H)+(1-\theta^F)}{2(1-\theta^H)}, 1)$, the numerical solutions are required for the welfare evaluation.

Evaluate $\frac{d\ln c^{F,e}}{d\Gamma^*}$ at $\Gamma^* = \Gamma^H_{IFA}$ and $\Gamma^* = \Gamma^F_{IFA}$. For $\mu \leq 1$, we get $\frac{d\ln c^{F,e}}{d\Gamma^*}|_{\Gamma^*=\Gamma^H_{IFA}} > \frac{d\ln c^{F,e}}{d\Gamma^*}|_{\Gamma^*=\Gamma^F_{IFA}} \geq 0$, implying that entrepreneurs born in country F is better off in the long run since the positive equity rate effect dominates the negative wage effect; for $\mu \geq \frac{1-\theta^H}{1-\theta^F}$, we get $\frac{d\ln c^{F,e}}{d\Gamma^*}|_{\Gamma^*=\Gamma^F_{IFA}} < \frac{d\ln c^{F,e}}{d\Gamma^*}|_{\Gamma^*=\Gamma^H_{IFA}} \leq 0$, implying that entrepreneurs born in country F is worse off in the long run since the the negative wage effect dominates; for $\mu \in (1, \frac{1-\theta^H}{1-\theta^F}, 1)$, the numerical solutions are required for the welfare evaluation.

Social welfare of country i in the steady state is

$$C^i \equiv \eta c^{i,e} + (1-\eta)c^{i,w} = w^i[\eta\Gamma^* + (1-\eta)r^i]$$
$$= \left(\frac{R}{\rho}\right)^\rho \left[\Gamma^*(1-\eta) + (1-\theta^i)\rho\right]^\rho (\Gamma^*)^{-\rho}(\eta\Gamma^* + \theta^i\rho),$$
$$\frac{d\ln C^i}{d\Gamma^*} = \frac{(1-\eta)\rho}{\Gamma^*(1-\eta) + (1-\theta^i)\rho} - \frac{\rho}{\Gamma^*} + \frac{\eta}{\eta\Gamma^* + \theta^i\rho}.$$

Evaluate $\frac{d\ln C^H}{d\Gamma^*}$ at $\Gamma^* = \Gamma^H_{IFA}$ and $\Gamma^* = \underline{\Gamma}^*$. For $\rho \in (0, \frac{(2-\theta^H-\theta^F)[2-\theta^H-\theta^F+\eta(\theta^F-\theta^H)]}{2(1-\theta^H)[(2-(\theta^F-\theta^H)]}]$, $\frac{d\ln C^H}{d\Gamma^*}|_{\Gamma^*=\Gamma^H_{IFA}} > \frac{d\ln C^H}{d\Gamma^*}|_{\Gamma^*=\underline{\Gamma}^*} \geq 0$ implies that the welfare loss of entrepreneurs dominates the welfare gains of workers and hence, country H as a whole loses in the long run from free mobility of FDI; for $\rho \in [\frac{1-\theta^H}{\eta}, \infty)$, $\frac{d\ln C^H}{d\Gamma^*}|_{\Gamma^*=\underline{\Gamma}^*} < \frac{d\ln C^H}{d\Gamma^*}|_{\Gamma^*=\Gamma^H_{IFA}} \leq 0$ implies that both workers and entrepreneurs are better off or the workers' welfare gains dominate the welfare losses of entrepreneurs and hence, country H as a whole benefits in the long run from free mobility of FDI; for $\rho \in (\frac{(2-\theta^H-\theta^F)[2-\theta^H-\theta^F+\eta(\theta^F-\theta^H)]}{2(1-\theta^H)[(2-(\theta^F-\theta^H)]}, \frac{1-\theta^H}{\eta})$, the numerical solutions are required for the welfare evaluation.

Evaluate $\frac{d\ln C^F}{d\Gamma^*}$ at $\Gamma^* = \Gamma^H_{IFA}$ and $\Gamma^* = \Gamma^F_{IFA}$. For $\rho \in (0, \frac{1-\theta^H}{\eta}]$, $\frac{d\ln C^F}{d\Gamma^*}|_{\Gamma^*=\Gamma^H_{IFA}} > \frac{d\ln C^F}{d\Gamma^*}|_{\Gamma^*=\Gamma^F_{IFA}} \geq 0$ implies that the welfare gains of entrepreneurs dominates the welfare losses of workers and hence, country F as a whole benefits in the long run from free mobility of FDI; for $\rho \in [\frac{(1-\theta^H)[1-\theta^H-\eta(\theta^F-\theta^H)]}{\eta(1-\theta^F)(1+\theta^F-\theta^H)}, \infty)$, $\frac{d\ln C^F}{d\Gamma^*}|_{\Gamma^*=\Gamma^F_{IFA}} < \frac{d\ln C^F}{d\Gamma^*}|_{\Gamma^*=\Gamma^H_{IFA}} \leq 0$ implies that both workers and entrepreneurs are worse off or the welfare gains of entrepreneurs is dominated by the welfare losses of workers and hence, country F as a whole loses in the long run from free mobility of FDI; for $\rho \in (\frac{1-\theta^F}{\eta}, \frac{(1-\theta^H)[1-\theta^H-\eta(\theta^F-\theta^H)]}{\eta(1-\theta^F)(1+\theta^F-\theta^H)})$, the numerical solutions are required for the welfare evaluation. □

Proof of Proposition 22

Proof. According to Proposition 4, if the borrowing constraints are binding in the two countries under free mobility of financial capital, the steady-state equity rate $\Gamma^i = \frac{(1-\theta^i)\rho}{\eta}$ has the same form as under international financial autarky. Given $\theta^H \in (0, \bar{\theta})$ and $\theta^F = \bar{\theta}^F_{FCF}$, the equity rate in country F is equal to the world loan rate and the borrowing constraints are weakly binding in the steady state, $\Gamma^F = \frac{\rho(1-\bar{\theta}^F_{FCF})}{\eta} = r^*$. Thus, $\bar{\theta}^F_{FCF}$ is the solution to the following equation,

$$\frac{\bar{\theta}^F_{FCF} - \frac{1-\eta}{\eta}(1-\bar{\theta}^F_{FCF})}{\frac{1-\eta}{\eta}(1-\bar{\theta}^F_{FCF}) - \theta^H} = (1-\bar{\theta}^F_{FCF} + \theta^H)^\rho. \tag{43}$$

Let $\aleph(\bar{\theta}^F_{FCF}) \equiv \frac{\bar{\theta}^F_{FCF}-\theta^H}{\frac{1-\eta}{\eta}(1-\bar{\theta}^F_{FCF})-\theta^H} - 1$ and $\Re(\bar{\theta}^F_{FCF}) \equiv (1-\bar{\theta}^F_{FCF}+\theta^H)^\rho$ denote the left-hand and the right-hand sides of equation (43) as the functions of $\bar{\theta}^F_{FCF}$. For $\bar{\theta}^F_{FCF} \in (\bar{\theta}, 1-\frac{\theta^H\eta}{1-\eta})$,

$$\begin{aligned}
\aleph'(\bar{\theta}^F_{FCF}) > 0 > \Re'(\bar{\theta}^F_{FCF}),\\
\aleph(\bar{\theta}^F_{FCF} = \bar{\theta}) = 0 < (\eta+\theta^H)^\rho = \Re(\bar{\theta}^F_{FCF} = \bar{\theta}),\\
\aleph(\bar{\theta}^F_{FCF} = 1 - \frac{\theta^H\eta}{1-\eta}) \to +\infty > \left(\frac{\theta^H}{1-\eta}\right)^\rho = \Re(\bar{\theta}^F_{FCF} = 1 - \frac{\theta^H\eta}{1-\eta}).
\end{aligned}$$

Thus, $\aleph(\bar{\theta}^F_{FCF})$ monotonically increases while $\Re(\bar{\theta}^F_{FCF})$ monotonically decreases in $\bar{\theta}^F_{FCF}$; the two functions cross once and only once for $\bar{\theta}^F_{FCF} \in (\bar{\theta}, 1-\frac{\theta^H\eta}{1-\eta})$. Therefore, the threshold value of $\bar{\theta}^F_{FCF} \in (\bar{\theta}, 1-\frac{\theta^H\eta}{1-\eta})$ exists and is unique.

For $\theta^F \in (\bar{\theta}^F_{FCF}, 1]$, $\Gamma^F = r^*$ in the steady state and the borrowing constraints are not binding in country F. The economic allocation is same as in the case of $\theta^F = \bar{\theta}^F_{FCF}$. □

Proof of Proposition 23

Proof. If the borrowing constraints are binding in the two countries under free mobility of FDI, the steady-state loan rate $r^i = \frac{\theta^i \rho}{(1-\eta)}$ has the same form as under international financial autarky. Suppose that given $\theta^H \in (0, \bar{\theta})$ and $\theta^F = \bar{\theta}^F_{FDI}$, the borrowing constraints are binding and the loan rate in country F is equal to the world equity rate, $r^F = \frac{\theta^F \rho}{1-\eta} = \Gamma^*$. Substitute it into equation (42),

$$\frac{\eta - (1 - \bar{\theta}^F_{FDI})}{(1-\eta)(1-\theta^H) - \theta^F \eta} = (1 + \bar{\theta}^F_{FDI} - \theta^H)^\rho. \tag{44}$$

It can be shown for $\eta \in [\frac{2^\rho}{1+2^{\rho+1}}, 1)$, given $\theta^H \in (0, \bar{\theta})$, there exist a $\bar{\theta}^F_{FDI} \in (\bar{\theta}, 1)$ that solve equation (44). For $\eta \in (0, \frac{2^\rho}{1+2^{\rho+1}})$, there exists a $\underline{\theta}^H$ that solves equation (45),

$$\frac{\eta}{(1-\eta)(1-\theta^H) - \eta} = (2 - \theta^H)^\rho. \tag{45}$$

For $\theta^H \in [\underline{\theta}^H, \bar{\theta})$, there exists $\bar{\theta}^F_{FDI}$ that solves equation (44); for $\theta^H \in (0, \underline{\theta}^H)$, the borrowing constraints are always binding in country F for $\theta^F \in (\theta^H, 1]$. □

Proof of Proposition 24

Proof. Suppose that for $\theta^H \in (\max\{1-2\eta, 0\}, 1-\eta)$ and $\theta^F = \bar{\theta}^F_{FCM}$, the borrowing constraints are binding and the loan rate is equal to the equity rate in both countries. According to equation (35), $\Gamma^* = r^* = \rho$. The wage is same in the two countries, $w^i = \left(\frac{R}{\eta}\right)^\rho$. According to equation (33),

$$\frac{\frac{\theta^F \rho}{r^*} - (1-\eta)}{(1-\eta) - \frac{\theta^H \rho}{r^*}} = \frac{w^H}{w^F} = 1, \quad \Rightarrow \quad \bar{\theta}^F_{FCM} = 2(1-\eta) - \theta^H. \tag{46}$$

For $\theta^F \in (\bar{\theta}^F_{FCM}, 1)$, the borrowing constraints are not binding and the loan rate is equal to the equity rate at $\Gamma^* = r^* = \rho$. □