On Entrepreneurial Risk–Taking and the Macroeconomic Effects of Financial Constraints†

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October 15th, 2008

Abstract

This paper deals with credit market imperfections and idiosyncratic risks in a two–sector heterogeneous agent dynamic general equilibrium model of occupational choice. We focus especially on the effects of tightening financial constraints on macroeconomic performance, entrepreneurial risk–taking, and social mobility. Contrary to many models in the literature, our comparative static results cover a broad range for borrowing constraints, from an unrestrained to a perfectly constrained economy. In our baseline model, we find substantial gains in output, welfare, and wealth equality associated with credit market improvements. The marginal gains from relaxing constraints are largest for empirically relevant debt–equity ratios. Interestingly, the entrepreneurship rate and social mobility respond non–monotonically to a change in the tightness of financial constraints. The results crucially depend on the degree of income persistence and feedback effects in general equilibrium, where optimal firm sizes and the demand for credit are determined endogenously.

Keywords: CGE, occupational choice, financial constraints, wealth distribution
JEL classification: C68, D3 , D8, D9, G0, J24

†We are grateful to Robert E. Lucas, Ian P. King, Richard Rogerson, and Nancy L. Stokey for helpful comments, and thank seminar participants at the 2008 Southern Workshop in Macroeconomics, and on the 2007 / 2008 meetings of the Econometric Society, the Royal Economic Society, the Society for Computational Economics, the Society for Economic Dynamics, and the German Economic Association.

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1 Introduction

This paper examines the effects of financial constraints and idiosyncratic risks in a dynamic equilibrium model of occupational choice. We determine the qualitative and quantitative response of aggregate macroeconomic activity, social mobility, and the wealth distribution to credit market improvements.

Our analysis contributes to recent literature on dynamic stochastic heterogeneous agent general equilibrium models concerned with risk and distributional dynamics, for instance Quadrini (2000), Meh (2005, 2008), Boháček (2006, 2007) and Cagetti and De Nardi (2006a,b,c). All these contributions share as common feature that including entrepreneurship and occupational choice into a Huggett (1993) / Aiyagari (1994)–type economy provides a much better explanation of the empirically observed wealth inequality, especially in the upper tail of the distribution. The implications of our model regarding wealth inequality are in accordance with this strand of literature, although our focus is more directed towards the general effects of financial constraints and the question of how sensitive the macroeconomy as a whole responds to a reduction in credit market imperfections.

To this end we develop a model which is more closely related to modern growth theory. Regarding the role of entrepreneurship, we assume a more sophisticated sectoral structure than e.g. Boháček (2006) or Cagetti and De Nardi (2006a). We combine occupational choice under risk à la Kihlstrom and Laffont (1979) and Kanbur (1979a,b) with the two–sector approach of Romer (1990), but without endogenous growth. In each period of time, the risk–averse agents choose between between two alternative occupations. They either set up an enterprise in the intermediate goods industry which is characterized by monopolistic competition. Or, they supply their labor endowment to the production of a final good in a perfectly competitive market. Producers of the final good use capital and labor inputs, and differentiated varieties of the intermediate good. All households are subject to an income risk. Managerial ability and productivity as a worker follow independent random processes. Entrepreneurial activity is rewarded with a higher expected income.\(^1\) Similar to Lucas (1978), there is no aggregate risk.

The economic performance in the intermediate goods industry crucially depends on two factors: uncertainty and credit constraints. Business owners face an firm–specific productivity shock, and there are no markets for pooling risks. An entrepreneurs maximizes his profit if the business operates at the optimal firm size, which is endogenously determined and governed by market demand from the final goods industry. For an individual wealth too small to maintain the optimal firm size, the firm–owner would want to borrow the remaining amount on the credit market, where he might be subject to financial constraints. The individual demand for loans, too, is endogenous. If the entrepreneur is wealthy enough, he operates

\(^1\)See also Clemens (2006a,b, 2008) and Clemens and Heinemann (2006) for entrepreneurial risk–taking in a general equilibrium context.
his business at the profit-maximizing level and supplies the rest of his wealth to the capital market.

Due to the two-sector general equilibrium nature of our model, the optimal business size and the demand for credit are endogenously determined, and we do not have to fall back on fixed investment projects (or entry costs respectively) in order to analyze the effects of credit market frictions. There is no further portfolio choice in our framework. To this end, our approach draws a simple picture of the empirical result, stated by Heaton and Lucas (2000), that the entrepreneurial households’ business wealth on average constitutes a relevant fraction of their total wealth.

We are especially interested in the question of how tightening (or relaxing) financial constraints affects the macroeconomy regarding aggregate output, the sectoral allocation of capital and labor, factor prices, the income and wealth distribution, occupational choice, as well as the between-group mobility of households. Our comparative static analysis covers a broad range of values for borrowing constraints, from an unconstrained to a perfectly constrained economy. This is a novel approach since many models of the literature consider a fixed debt-equity ratio, or rest with a comparison of a complete vis-à-vis a specific incomplete market, or they focus on the no-credit market scenario.

The model is broadly consistent with macro data from industrialized countries. Naturally, the model cannot draw a realistic picture of the economy over the entire domain of financial constraints under consideration. For this reason, we define a benchmark economy with an empirically plausible debt-equity ratio, and calibrate the model to match macroeconomic key variables, for instance, for the U.S. and other OECD members.

We find substantial gains of relaxing constraints, amounting to 20% of aggregate output, and observe likewise improvements in average wealth holdings and welfare. The associated marginal gains of credit market improvements in the intermediate goods industry are substantial for debt-equity ratios in the empirically relevant domain and increasing.

The macroeconomic effects can primarily be attributed to an inefficient allocation of capital across sectors, and only secondly to adjustments in occupational choice, employment, and the entrepreneurship rate. In moving from an unconstrained to a completely credit-constrained economy, the average firm size drops down to a mere 50% of optimal business activity. Capital accumulation plays a twofold role in this context: On the one hand, it endows individuals with the wealth necessary to set-up and operate a firm. On the other hand, buffer-stock saving provides a self-insurance on intertemporal markets against the non-diversifiable income risk. Accordingly, we find that wealthier households are more likely to be

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2 It is a standard approach in the literature to assume entry costs, often combined with a decreasing returns technology, to generate some kind of ‘monopolistic’ market structure, which is an innate part of our model.
members of the entrepreneurial class than poorer ones and there is a marked concentration of wealth in the hands of entrepreneurs which is consistent with recent empirical findings (cf. Quadrini, 1999; Holtz-Eakin et al., 1994a). Upward mobility of entrepreneurs in our model is primarily accumulation driven. The riskiness of entrepreneurial incomes loses its importance for occupational choice once the household's income share generated from profits declines relative to his capital income. Nevertheless, in accordance with Hamilton (2000), many entrepreneurs of our model enter and persist in business despite the fact that they have lower initial earnings than average wage incomes.

Regarding empirical evidence, there is strong support in the literature for the hypothesis that financial constraints are an impediment to entering entrepreneurship; see Evans and Leighton (1989), Evans and Jovanovic (1989), Holtz-Eakin et al. (1994b), Blanchflower and Oswald (1998), Moskowitz and Vissing-Jørgensen (2002), and Desai et al. (2003). Gentry and Hubbard (2004) point out that external financing has important implications for individual investment and saving. This evidence is challenged by Hurst and Lusardi (2004), who find that the likelihood of entering entrepreneurship relative to initial wealth is flat over a large range of the wealth distribution and increasing only for higher wealth levels of workers.

The general equilibrium nature of our approach generates surprising and almost counter-intuitive results regarding the impact of financial constraints on occupational choice under risk. The entrepreneurship rate and social mobility respond non-monotonically to a change in the tightness of constraints. Because individual income persistence can be identified as an important determinant of occupational choice and mobility, we analyze two settings which only differ with respect to the serial correlation of the underlying idiosyncratic shocks. The entrepreneurship rate of the baseline model rises with an increase in credit market efficiency, which is in accordance with economic intuition. If, however, the persistence of entrepreneurial talent shocks is reduced, we predominantly observe a decline in the entrepreneurship rate. This outcome can primarily be attributed to the general equilibrium nature of our model.

Wealth inequality is not necessarily reduced, if we relax borrowing constraints. Regarding social mobility, the response to changes in credit availability also is non-monotonic. In general we find that workers and entrepreneurs with high individual productivity tend to remain in their present occupation, whereas low productivity individuals are more likely to switch professions.

Regarding the functional distribution of income, we find that credit constraints have a redistributive effect by raising the profit income share at the cost of capital incomes. The results indicate that the stochastic nature of the underlying idiosyncratic shocks also plays an important role for the explanation of the general equilibrium effects of credit market imperfections.

Recent contributions in this area of research suffer from several shortcomings which our approach aims to overcome. In Quadrini (2000), occupational choice
and the level of entrepreneurship is (more or less) entirely governed by the underlying productivity shocks. Li (2002) and Boháček (2006) discuss economies with a single sector of production which does not allow for factor movements between industries and therefore neglects factor substitution. In our model, producers of the intermediate and the final good are subject to competition, especially with respect to capital demand. Our approach does not have fixed entry costs (in terms of discrete investment projects) of entrepreneurship as in Ghatak et al. (2001), Fernández-Villaverde et al. (2003) or Clementi and Hopenhayn (2006). Instead, we have an endogenously determined optimal firm size and no discontinuities in individual credit demand. Occupational choice, entrepreneurial activity and performance crucially depend on monopoly profits, market shares and relative factor scarcity in the two sectors of production. Also different to Cagetti and De Nardi (2006a) or Kitao (2008), the entrepreneurs of our economy are essential for aggregate output. As will become obvious below, the interdependence of sectors is important for the general equilibrium results on occupational choice, between–group mobility and the income and wealth distribution, and contributes to the understanding of how financial constraints affect the macroeconomy.

The paper is organized as follows: Section 2 develops the two–sector model. We describe the equilibrium associated with a stationary earnings and wealth distribution. Since the formal structure of the model does not allow for analytical solutions, we perform numerical simulations of a calibrated model in order to examine the general equilibrium effects of an increase in the tightness of credit constraints. Section 3 gives information on the calibration procedure and related empirical evidence. Section 4 discusses the simulation results. Section 5 concludes. Technical details are relegated to the Appendix.

2 The Model

2.1 Overview

We consider a neoclassical growth model with two sectors of production. Drawing from Quadrini (2000) and Romer (1990), we consider a corporate sector with perfectly competitive large firms who hire capital and labor services and use an intermediate good in order to produce a homogeneous output which can be consumed or invested respectively. The intermediate goods industry (non–corporate sector) consists of a large number of small firms operating under the regime of monopolistic competition. Each firm in this sector is owned and managed by an entrepreneur. Both sectors of production are essential.

Market activity in the intermediate goods industry is constrained. In order to run the business at the profit–maximizing firm size, entrepreneurs either possess sufficient wealth of their own, or they need to compensate for their lack of equity by borrowing on the credit market, where they might be subject to borrowing
constraints. The two–sector setting allows us to endogenously relate financial constraints to individual characteristics and overall market activity.

The economy is populated by a continuum \([0, 1]\) of infinitely–lived households, each endowed with one unit of labor.\(^3\) In each period of time, individuals follow their occupation predetermined from the previous period and make a decision regarding their future profession, which is either to become producers of the intermediate good or to supply their labor services to the production of the final good. Labor efficiency as well as entrepreneurial productivity are idiosyncratic random variables. Regarding the associated income risk, we assume that wage incomes are less risky than profit incomes. There is no aggregate risk.

With respect to the timing of events, we assume that individual occupational choice takes place before the resolution of uncertainty. Once the draw of nature has occurred, entrepreneurs as well as workers in the final goods sector know their individual productivity. Those monopolists, who now discover their own wealth being too low to operate at the optimal firms size, will express their capital demand on the credit market, probably become subject to credit–constraints, and then start production. After labor and profit income is realized, the households decide on how much to consume and to invest. There is no capital income risk and no risk of production in the corporate sector.

### 2.2 Final Goods Sector

The representative firm of the final goods sector produces a homogeneous good \(Y\) using capital \(K_F\), labor \(L\), and varieties of an intermediate good \(x(i), i \in [0, \lambda]\) as inputs. Production in this sector takes place under perfect competition and the price of \(Y\) is normalized to unity. The production function is of the generalized CES–form\(^4\)

\[
Y = (K_F^{-1}L^{1-\gamma})^{1-\alpha} \int_0^\lambda x(i)^{\alpha} \, di, \quad 0 < \alpha < 1, \quad 0 < \gamma < 1.
\]

Each type of intermediate good employed in the production of the final good is identified with one monopolistic producer in the intermediate goods sector. Consequently, the number of different types is identical with the population share \(\lambda\) of entrepreneurs in the population. The number of entrepreneurs is determined endogenously through occupational choices of the agents, which will be described below. Additive–separability of (1) in intermediate goods ensures that the marginal product of input \(i\) is independent of the quantity employed of \(i' \neq i\). Intermediate goods are close but not perfect substitutes in production.

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\(^3\)It would be a straightforward extension of the model to include the life–cycle dimension, inter–generational altruism and bequests as in De Nardi (2004), Cagetti and De Nardi (2006a), or issues of firm growth, but this is beyond the scope of the present paper.

\(^4\)All macroeconomic variables are time–dependent. For notational convenience, we will drop the explicit time–notation unless necessary. If needed, the \(t\) symbol denotes next period variables.
The profit of the representative firm in the final goods sector, $\pi_F$, is given in each period by

$$\pi_F = Y - wL - (r + \delta)K_F - \int_0^\lambda p(i) x(i) \, di,$$

(2)

where $p(i)$ denotes the price of intermediate good $i$. We further assume physical capital to depreciate over time at the constant rate $\delta$, such that the interest factor is given by $R = 1 + r - \delta$. Optimization yields the profit maximizing factor demands consistent with marginal productivity theory

$$K_F = (1 - \alpha)\gamma \frac{Y}{r + \delta},$$

(3)

$$L = (1 - \alpha)(1 - \gamma)\frac{Y}{w},$$

(4)

$$x(i) = K^\gamma F L^{1-\gamma} \left( \frac{\alpha}{p(i)} \right)^{\frac{1}{1-\alpha}}.$$

(5)

The monopolistic producer of intermediate good $x(i)$ faces the isoelastic demand function (5), where the direct price elasticity of demand is given by $-1/(1 - \alpha)$. Condition (4) describes aggregate labor demand in efficiency units. Equation (3) is the final good sector demand for capital services.

### 2.3 Intermediate Goods Sector

The intermediate goods sector consists of the population fraction $\lambda$ of entrepreneurs who self-employ their labor endowment by operating a monopolistic firm. Each monopolist produces a single variety $i$ of the differentiated intermediate good by employing capital from own wealth and borrowed resources according to the identical constant returns to scale technology of the form

$$x(i) = \theta(i)_c k(i).$$

(6)

Firm owners are heterogeneous in terms of their talent as entrepreneurs. They differ with respect to the realization of an idiosyncratic productivity shock $\theta(i)_c$ which is assumed to be non-diversifiable and uncorrelated across firms. We will give more details on the properties of the shock below. Entrepreneurs hire capital after the draw of nature has occurred. The firm problem essentially is a static one. Under perfect competition of the capital market, the producer treats the rental rate to capital as exogenously given and maximizes his profit

$$\pi(k(i), \theta(i)_c) = p(i) x(i) - (r + \delta) k(i).$$

(7)

Utilizing the demand function for intermediate good type--$i$, (5), and the production technology (6), the optimal firm decision can be expressed in terms of the...
optimal firm size \( k(i)^* \), given by:

\[
k(i)^* = L \theta(i) e^{\frac{\gamma \theta_w}{(1-\gamma)(r+\delta)}} \left( \frac{\alpha \gamma}{r+\delta} \right)^{\frac{1}{1-\alpha}}.
\]  

(8)

Because capital demand takes place after the draw of nature has occurred, there is no individual capital risk and no under-employment of input factors. The optimal firm size increases with random individual productivity \( \theta(i) \), such that more productive business owners demand more capital on the capital market. The aggregate labor input in efficiency units determines the optimal firm size by means of the demand function for intermediate good type \( i \). Aggregate employment is a weighted average and depends on the size of the labor force \( 1-\lambda \), i.e. the population fraction of agents choosing the occupation of a worker, and the idiosyncratic shock on labor productivity \( \theta_w \). The larger the labor force \( 1-\lambda \), the higher—ceteris paribus—will be aggregate employment \( L \). This goes along with fewer monopolists in the intermediate goods industry, less competition, and a larger market share, as measured by the optimal firm size.

2.4 Incomes and Equilibrium Income Shares in the Unconstrained Economy

Households derive income from three sources: labor income, capital income and monopolistic profits. The technology parameters \( \alpha \) and \( \gamma \) determine the division of aggregate income among the three income sources in the absence of financial constraints on entrepreneurial activity. According to marginal productivity theory, we obtain from (1) a labor share of \( (1-\alpha)(1-\gamma) \) and a capital share of \( (1-\alpha)\gamma \). The remaining income share \( \alpha \) accrues to the two types of income generated in the intermediate goods sector, and splits on profits with \( \alpha(1-\alpha) \) and capital income with \( \alpha^2 \), respectively, such that the economy-wide capital share amounts to \( (1-\alpha)\gamma + \alpha^2 \).

2.5 Capital Market and Financial Constraints

Firms of the final goods sector and the intermediate goods industry differ with respect to access to financial markets. While the first are not constrained in their financing, the latter face greater difficulties in diversifying the risk from their entrepreneurial activities and, moreover, are subject to borrowing constraints. Entrepreneurs of the intermediate goods industry, who are wealth-constrained in operating their business at the optimal size (8), seek external financing from financial intermediaries. The credit market is imperfect with respect to lenders not being able to enforce loan-repayment due to limited commitment of borrowers (cf. Banerjee and Newman, 1993). In order not to default on loan contracts, borrowing amounts are limited, and individual wealth acts as collateral. We do not explicitly model financial intermediaries and assume that there is no difference between borrowing and lending rates.
In case of default, the financial intermediator is able to seize a fraction of the borrowers gross capital income \((1 + r) a(i)\). Alternatively, one could assume the entrepreneur’s profit income to act as collateral. The major difference between the two approaches is that, in the first case, borrowing amounts are entirely determined by the debtors individual wealth \(a(i)\), whereas in the second, they also depend on his entrepreneurial talent \(\theta(i)_e\), which might be private information.\(^5\)

The creditor will lend to the borrower only the amount consistent with the borrower’s incentive–compatibility constraint, such that it is in the borrower’s interest to repay the loan, and there is no credit default in equilibrium.

Let \(k(i) = a(i) + b(i)\) be the firm size an entrepreneur is able to operate at from own wealth \(a(i)\) and borrowed resources \(b(i)\). This operating capital \(k(i)\) is not necessarily equal to the optimal firm size \(k(i)_*\) determined in (8). An entrepreneur with individual wealth \(a(i)\) lower than \(k(i)_*\) will consider loans from the credit market \(k(i)_* - a(i)\). In case of \(k(i) < k(i)_*\) the firm faces a borrowing constraint. Incentive–compatibility requires a self–enforcing contract. It is never optimal for the borrower to default, if

\[
\pi(i) + (1 + r) a(i) \geq \pi(i) + b(i) (1 + r) + (1 - \phi) (1 + r) a(i)
\]

which reduces to

\[
b(i) \leq \phi a(i) . \tag{9}\]

The borrowing amount is limited such that the maximum possible loan is proportional to the borrowers individual wealth \(a(i)\). The parameter \(\phi\) is a measure for the extent to which a lender can use the borrower’s wealth income as collateral. Credit constraints become less tight with rising \(\phi\) and vanish for large \(\phi\). The limiting cases consequently reflect the two cases of either complete enforceability \((\phi \to \infty)\) or no enforceability \((\phi = 0)\), such that in the first case the borrower is considered solvent, whereas in the second one he is not.

Using the collateral constraint in the entrepreneurial budget constraint yields \(k(i) \leq (1 + \phi) a(i)\). The operating firm size \(k(i)\) of entrepreneur \(i\) with productivity \(\theta(i)_e\) and wealth \(a(i)\) can then be written as:

\[
k(\theta(i)_e, a(i)) = \min [k(i)_*, (1 + \phi) a(i)] . \tag{10}\]

The subsequent numerical analysis shows that the high–productivity entrepreneurs are more likely to be constrained than the low–productivity ones, because the optimal firm size and henceforth the capital demand increase in the productivity shock.

An entrepreneur, whose individual wealth exceeds the level needed to operate his business at the optimal firm size will lend the amount \(a(i) - k(i)_*\) on the capital market at the equilibrium interest rate. The supply side of the capital market

\(^5\)The qualitative implications for the model are identical under both assumptions. If borrowing amounts also depend on entrepreneurial productivity, the response to an increase in the tightness in financial constraints is smaller in magnitude. The results are available from the authors upon request.
altogether consists of those entrepreneurs whose wealth exceeds their individual optimal firm size and of workers, who supply their savings. On the demand side we have the credit-constrained entrepreneurs and firms from the final goods industry. From this follows immediately that the size of the intermediate goods industry relative to the final goods sector essentially depends on occupational choice and individual wealth accumulation, both determined endogenously in equilibrium.

2.6 Idiosyncratic Risks

In each period of time, workers are endowed with one unit of raw labor and are subject to an idiosyncratic shock $\theta_w$, affecting labor supply in efficiency units, and exposing each of them to an uninsurable income risk. We assume that labor productivity $\theta_w$ evolves according to a first-order Markov process with $h = 1, \ldots, H$ states, and $\theta_{w,h} > 0$. The transition matrix associated with the Markov process is $P_w$.

Entrepreneurial productivity $\theta_e$ also evolves according to a first-order Markov process with $h = 1, \ldots, H$ different states $\theta_{e,1}, \ldots, \theta_{e,H}; \theta_{e,h} > 0$, and transition probability $P_e$. Since agents can be either workers or entrepreneurs, it is possible to identify the occupational status of an agent with his productivity in the respective occupation. We assume worker productivities to be more evenly distributed than managerial skills, such that profit incomes in general are more risky than wage incomes. As is well-known from the literature, entrepreneurs on average are compensated with a positive income differential (aka ‘risk premium’) for bearing the production risk.

By modeling two distinct random processes for workers and entrepreneurs, we take into account that the two professions demand different talents, for instance specific managerial skills. We assume the processes $\theta_w$ and $\theta_e$ to be uncorrelated, such that for an individual the conditional expectation of entrepreneurial productivity is independent of the labor efficiency, if employed as a worker. A high productivity as a worker in the present does not necessarily indicate an equivalently high future productivity as an entrepreneur, if the individual should decide to switch between occupations in the next period. The associated probabilities are summarized in a $H \times H$ transition matrices $P_{j,j'}$ describing the transition from productivity state $\theta_{j,h}$ to state $\theta_{j',h'}$ for $h, h' = 1, \ldots, H$, $j = e, w$ and $j \neq j'$.

As the subsequent analysis will show, assuming different degrees of income persistence crucially affects the pattern of how occupational choice interacts with a change in the magnitude of financing constraints. If the income process is highly persistent, then e.g. lowly productive workers and entrepreneurs are more likely to be lowly productive in the future. The individual can infer from his present productivity how his future productivity in the same occupation probably will be.

We consider two different specifications regarding the Markov processes for entrepreneurial talent and worker efficiency respectively. By assuming a high (medium) serial correlation for entrepreneurs (workers) in our baseline model,
we are able to generate results which closely match empirical findings regarding the macroeconomic key variables and the wealth distribution. In the second setting we customize the persistence of entrepreneurial income processes to match those for workers, and focus on the effects on occupational choice and social mobility.

2.7 Intertemporal Decision and Occupational Choice

Each household \( i \) has preferences over consumption and maximizes discounted expected lifetime utility

\[
E_0 \sum_{t=0}^{\infty} \beta^t U[c(i)] \quad 0 < \beta < 1.
\]

\( E_0 \) is the expectation operator conditional on information at date 0 and \( \beta \) is the discount factor. Individuals are assumed to be identical with respect to their preferences regarding momentary consumption \( c(i) \) which are described by constant relative risk aversion

\[
U[c(i)] = \begin{cases} 
  \frac{c(i)^{1-\rho}}{1-\rho} & \text{for } \rho > 0, \rho \neq 1 \\
  \ln c(i) & \text{for } \rho = 1,
\end{cases}
\]

where \( \rho \) denotes the Arrow/Pratt measure of relative risk aversion.

Besides intertemporal consumption choice, the single household also makes a decision on his future occupation in each period, which is either to become a self-employed producer of an intermediate good in the monopolistically competitive market or to inelastically supply his labor services in efficiency units to the production of the final good. Occupational choice, once made, is irreversible in the same period.

Let \( V^w(a(i), \theta(i)_w) \) denote the optimal value function of an agent currently being a worker with wealth \( a \), who is in a given productivity state \( \theta_w \). If he decides to remain a worker, his productivity evolves according to the transition matrix \( P_w \) of the underlying Markov process. If, instead, he becomes an entrepreneur in the following period, he gets a new draw \( \theta_e \) from the invariant distribution of entrepreneurial productivities. The next period productivity is determined by the transition matrix \( P_{w,e} \) (see model calibration below).

There are no markets for pooling idiosyncratic risks. In addition to the financial constraints of the intermediate goods industry, we also have incomplete insurance markets. There is limited scope to which agents are able to smooth their intertemporal consumption flow by borrowing and lending. The standard approach of the literature is to assume that individual asset holdings are bounded from below. In what follows, we assume a lowest possible wealth level of \( g = 0 \).
The maximized value function for a typical individual currently being a worker is given by

\[ V^w(i, \theta(i)_w) = \max_{c(i) \geq 0, a(i') \geq a} \left\{ U[c(i)] + \beta \max_{q(i') \in \{0, 1\}} \left[ (1 - q(i'))E[V^w(a(i'), \theta(i')_w) | \theta(i)_w] + q(i')E[V^w(a(i'), \theta(i')_w)] \right] \right\} \]

s.t. \( a(i') = (1 + r) a(i) + \theta(i)_w w - c(i) \).

(11)

\( q \) is a boolean variable which takes on the values 0 or 1, depending on whether or not the agent decides to switch between occupations. \( r \) and \( w \) denote the equilibrium returns to capital and labor in efficiency units, which are constant over time for a stationary distribution of wealth and occupational statuses over agents. The optimal decision associated with the problem (11) is described by the two decision rules for individual asset holdings \( a(i')_w = A_w(a(i), \theta(i)_w) \) and the future professional state \( q(i')_w = Q_w(a(i), \theta(i)_w) \).

Let \( V^e(a(i), \theta(i)_e) \) denote the maximized value function of an entrepreneur with wealth \( a \) in productivity state \( \theta_e \), who faces a decision problem similar to those of a worker. If he decides to remain an entrepreneur, his productivity evolves according to the transition matrix \( P_e \) of the underlying Markov process. If, instead, he decides to switch between occupations by becoming a worker in the next period, his future productivity \( \theta'_w \) is determined by the transition matrix \( P_{e,w} \). With \( k(i)^* \) denoting the optimal firm size, the intertemporal problem of an entrepreneur can be written as

\[ V^e(a(i), \theta(i)_e) = \max_{c(i) \geq 0, a(i') \geq a} \left\{ U[c(i)] + \beta \max_{q(i') \in \{0, 1\}} \left[ (1 - q(i'))E[V^e(a(i'), \theta(i')_e) | \theta(i)_e] + q(i')E[V^e(a(i'), \theta(i')_e)] \right] \right\} \]

s.t. \( a(i') = (1 + r) a(i) + \pi(k(i), \theta(i)_e) - c(i) \)

\[ k(i) = \min[k(i)^*, (1 + \phi) a(i)] \]

\[ \pi(\theta(i)_e, k(i)) = p(x(i))x(\theta(i)_e, k(i)) - (r + \delta) k(i) \]

(12)

Again, \( q \) is a boolean variable, indicating the agent’s decision on leaving or remaining in his present occupation. The optimal decision is described by the decision rules for individual asset holdings \( a(i')_e = A_e(a(i), \theta(i)_e) \) and the future professional state \( q(i')_e = Q_e(a(i), \theta(i)_e) \).\(^6\)

\(^6\)Note that the value functions (11) and (12) may not be concave because of the boolean variable \( q \), indicating binary choice between occupations. Similar to Fernández-Villaverde et al. (2003), we
In general, our model generates the same implications for individual savings and wealth accumulation under risk as, for instance, discussed in Aiyagari (1994) or Huggett (1996). Similar to Quadrini (2000), we additionally consider occupational choice. Consequently, wealth accumulation plays a two-fold role: On the one hand, the shocks to worker efficiency and entrepreneurial productivity generate an income risk which households respond to with buffer-stock saving. On the other hand, higher wealth levels protect entrepreneurs against the danger of being subject to financial constraints. In terms of Sandmo (1970) there is only an income but no capital risk in our model, such that the share of risky incomes in total household income declines with growing wealth. Accordingly, the importance of risky profits providing negative incentives towards entrepreneurship fades for high levels of wealth.

2.8 Stationary Recursive Equilibrium

A stationary recursive competitive general equilibrium is an allocation, where equilibrium prices generate a distribution of wealth and occupations over agents which is consistent with these prices given the exogenous process for the idiosyncratic shocks and the agents' optimal decision rules.

We obtain aggregate labor supply by summing up individual labor supplies in efficiency units over the population fraction $1 - \lambda$ of workers. The stationary recursive equilibrium is a set of value functions $V^w(a, \theta_w)$, $V^e(a, \theta_e)$, decision rules $A^w(a, \theta_w)$, $Q^w(a, \theta_w)$ and $A^e(a, \theta_e)$, $Q^e(a, \theta_e)$, prices $w, r, p(i)$ and a distribution $\lambda, 1 - \lambda$ of households over occupations such that:

\begin{enumerate}[i]
  \item the decision rules $A^w(a, \theta_w)$, $Q^w(a, \theta_w)$ and $A^e(a, \theta_e)$, $Q^e(a, \theta_e)$ solve the workers' and entrepreneurs' problems (11) and (12) at prices $w, r, p(i)$,
  \item the aggregate demands of consumption, labor, capital and intermediate goods are the aggregation of individual demands. Factor and commodity markets clear at constant prices $w, r, p(i)$, where factor inputs are paid according to their marginal product.
  \item the stationary distribution $\Gamma$ of agents over individual wealth holdings, occupations and associated productivities is the fixed point of the law of motion which is consistent with the individual decision rules and equilibrium prices. The distribution $\lambda, 1 - \lambda$ of agents over occupations is time-invariant.
\end{enumerate}

The decision rules for workers, $A^w(a, \theta_w)$, $Q^w(a, \theta_w)$, and entrepreneurs, $A^e(a, \theta_e)$, $Q^e(a, \theta_e)$, together with the stochastic processes for individual labor productivity would like to stress that the dynamic programming algorithm underlying our computational modeling does not require concavity but monotonicity to converge to the true value function; see also Boháček (2007, fn. 4).  

\textsuperscript{7}See Appendix A for the equilibrium conditions of the discrete formulation of the model underlying the numerical simulations.
and entrepreneurial productivity, determine the stationary distribution \( \Gamma \) at equilibrium prices \( w, r \). The stationary distribution \( \Gamma \) governs the population share of entrepreneurs (i.e. the mass of firms in the intermediate goods sector), the efficiency units of labor supplied by workers, capital demand of the intermediate goods sector, and the aggregate capital supply, the latter equaling the mean of individual wealth holdings. Once the population share of entrepreneurs \( \lambda \) is derived, this together with the stationary distribution of entrepreneurial productivities determines the supply of intermediate goods (for details, see Appendix A).

3 Calibration

In order to evaluate the macroeconomic effects of changes in the tightness of financial constraints, our first step is to define a benchmark economy which matches standard macro data from OECD countries. We calibrate the model to replicate empirical observations regarding the functional and personal distribution of income and wealth, capital return, entrepreneurship rates, and social mobility. The benchmark value for the debt–equity ratio is set to \( \phi = 1 \), i.e., the maximum loan is limited to half the amount of the operating capital (cf. Evans and Jovanovic, 1989; Gentry and Hubbard, 2004). Table 1 summarizes the parameterization of the model and our calibration targets. We find that it is sufficient to mimic unlimited access to credit \( (\phi \rightarrow \infty) \) in our simulations by choosing the largest value for \( \phi = 1000 \), where virtually no entrepreneur is restrained.

We adopt a broad notion of entrepreneurship and consider an entrepreneur as someone, who owns and operates a small business, and who is willing to take risks, to be innovative, and to exploit profit opportunities (Knight, 1921; Schumpeter, 1930; Kirzner, 1973). Definitions of self–employment and entrepreneurial activity differ widely across countries.⁸ According to the OECD, self–employment encompasses “...those jobs, where the remuneration is directly dependent upon the profits derived from the goods and services produced. The incumbrants make the operational decisions affecting the enterprise, or delegate such decisions while retaining responsibility for the welfare of the enterprise.” (OECD, 2000, Ch. 5, p. 191). Our model generates self–employment business ownership rates around 20%, which is somewhat more at the upper range of values for OECD countries (including owner–managers), matching countries like New Zealand (20.8%), Italy (24.8%), or Spain (18.3%); see also the annual Global Entrepreneurship Monitor (GEM 2005, Minniti et al.) for data on total entrepreneurial activity.

Regarding preferences, we set the discount factor \( \beta \) and the coefficient of relative risk aversion \( \rho \) according to estimates from the literature, in order to generate equilibrium interest rates on safe assets around 3% which is consistent with empirical findings (cf. Mehra and Prescott, 1985; Obstfeld, 1994). The parameters of

⁸Often, the agricultural sector is excluded from the computation of entrepreneurship rates.
production technology, \( \alpha \) and \( \gamma \), are chosen such as to generate an equilibrium labor income share of 0.63 which matches empirical observations e.g. for the U.S. economy (King and Rebelo, 1999) or the average of EU 15. The corresponding capital and profit income shares of the unconstrained economy \((\phi \to \infty)\) are 0.16 and 0.21. PSID data report a income share for entrepreneurs of around 22%. The depreciation rate is fixed at 6%, which also is a standard choice in the literature.

The steady state of the simulated benchmark economy replicates the Gini coefficient of wealth inequality for the U.S. (PSID, 1989) but also matches OECD countries like Sweden, France, and Switzerland.

To take account of empirically observed income persistence, we assume that the processes for labor efficiency \( \theta_w \) and entrepreneurial productivity \( \theta_e \) are lognormal with normalized mean \( \ln \theta_w \sim \mathcal{N}( -\sigma_w^2/2, \sigma_w^2) \), \( \ln \theta_e \sim \mathcal{N}( -\sigma_e^2/2, \sigma_e^2) \) and AR(1) of the general form:

\[
\ln \theta_w' = (p_w - 1) \frac{\sigma_w^2}{2} + p_w \ln \theta_w + \sigma_w \sqrt{1 - p_w^2} \varepsilon,
\]

\[
\ln \theta_e' = (p_e - 1) \frac{\sigma_e^2}{2} + p_e \ln \theta_e + \sigma_e \sqrt{1 - p_e^2} \varepsilon,
\]

where \( \varepsilon \sim \mathcal{N}(0, 1) \). The process (13) is parameterized following Aiyagari (1994) with \( p_w = 0.6 \) and \( \sigma_w = 0.2 \). With respect to the entrepreneurial income process (14), our baseline model assumes a higher serial correlation \( p_e = 0.9 \) and a larger variance \( \sigma_e = 1.8 \) in order to reproduce the higher risk associated with entrepren-
eurial activity and to generate empirically plausible exit/entry rates and wealth inequality. We assume identical correlations $p_{e,w} = 0.6$ in our second setting to highlight that occupational choice and social mobility crucially depend on income persistence.

Entry rates into entrepreneurship equal exit rates in the stationary recursive equilibrium. Our model is calibrated to generate exit rates of around 4% of the population ($\approx 20\%$ of intermediate industry members) which consistent with the evidence reported by Quadrini (2000) but higher than the rates documented by Evans (1987) for the U.S., and also in the upper range of empirically plausible values for OECD countries (cf. Vale, 2006; Aghion et al., 2007).

The income processes are approximated with a five–state Markov chain by using the method described in Tauchen (1986). The transition matrices for individuals who decide to switch between occupations are derived from the stationary distributions of the Markov processes. The probability for a worker (entrepreneur) of ending up in a specific state of entrepreneurial (worker) productivity $\theta_{e,h}$ ($\theta_{w,h}$) is given by the stationary (unconditional) probabilities of this state. The algorithm for finding the equilibrium consists of three nested loops, starting from an initial guess on factor prices $w, r$ and employment $L$, then iterating until markets clear and the conditions of a stationary recursive equilibrium are met (see Appendix A).

4 Results

Our baseline model describes an economy where agents choose between two uncorrelated occupational lotteries with different degrees of income persistence. The benchmark economy is subject to moderate constraints on the credit market and calibrated such as to match empirical evidence for the U.S. and other OECD countries. We now proceed with investigating the effects of a change in the tightness of financial constraints on (a) inequality and the distribution of wealth, (b) on output, factor prices, and the factor income distribution, and (c) on occupational choice and social mobility.

A common finding for models with credit market imperfections is that the properties of the equilibrium often respond non-monotonically to parameter changes. If we look at the literature, we find models assessing the effects of credit market imperfections by assuming no credit market at all (e.g. Boháček, 2007). Other approaches compare imperfect to perfect markets. As Matsuyama (2007, p. 3) points out, there is no reason to believe that, first, the effects of an imperfect market equal those of no credit market, and second, the effects of improving credit markets are similar to those of completely eliminating market imperfections. Instead of discussing only a single case by assuming a predetermined magnitude of financial constraint, we vary the tightness of constraints in our simulations to cover the range.

---

9see Matsuyama (2007) and references therein.
Table 2: Wealth Distribution

<table>
<thead>
<tr>
<th></th>
<th>Top percentiles (in %)</th>
<th>Gini</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1%</td>
<td>5%</td>
</tr>
<tr>
<td>PSID 1994</td>
<td>22.6</td>
<td>44.8</td>
</tr>
<tr>
<td>SCF 1992</td>
<td>29.5</td>
<td>53.5</td>
</tr>
<tr>
<td>( \phi = 0 )</td>
<td>21.95</td>
<td>56.59</td>
</tr>
<tr>
<td>( \phi = 1.0 )</td>
<td>20.24</td>
<td>54.57</td>
</tr>
<tr>
<td>( \phi = 1000 )</td>
<td>20.16</td>
<td>56.16</td>
</tr>
</tbody>
</table>

Source: PSID and SCF data, Quadrini (2000, p. 6)

from no credit market (\( \phi = 0 \)) to an unconstrained market (\( \phi \to \infty \)).\(^{10}\) Although the value of \( \phi \) is fixed exogenously, the credit demand as well as the magnitude of rationing is determined endogenously and depends on firm specific factors, such as optimal business size (8), individual wealth, factor prices and the ability shock.

Regarding the comparative static results, we observe that the properties of the equilibrium respond sensitive to a change in serial correlation. To gain some understanding of the underlying forces at work, we contrast the baseline model with the case of identical serial correlation for the idiosyncratic shocks and find reversed implications of a change in the tightness of constraints for the equilibrium entrepreneurship rate and mobility.

Our analysis proceeds as follows: We first investigate to what extent our model is able to replicate empirical evidence on wealth distributions. We then examine how the presence of credit constraints affects the key macroeconomic variables, such as aggregate output, average firm size, factor prices and factor income shares as well as individual incomes, household wealth, and the degree of inequality, the latter measured by the Gini coefficient. In a next step, we analyze mobility between occupations.

4.1 Results for the Baseline Model

Distribution of Wealth and Business Size Table 2 reports the percentiles and Gini coefficients for household wealth computed from the PSID and SCF (Quadrini, 2000) and the associated values of our model economy for three different degrees of financial constraints, \( \phi \in \{0,1,1000\} \). Similar to related work by Quadrini (2000), Cagetti and De Nardi (2006a), and Boháček (2006) we find that introducing occupational choice into Aiyagari (1994)–type models of uninsurable shocks and borrowing constraints improves the prediction of wealth inequality, especially in the upper tail of the distribution. Our benchmark model economy (\( \phi = 1 \)) replicates the

\(^{10}\)Tables 2, 3 and 5 display only selected cases with values for \( \phi \in \{0,1,1000\} \). Figures 2 and 3 cover a broader range of values for \( \phi \) from our simulations.
Gini coefficient for the U.S. economy and closely matches the wealth distribution in the top percentiles.

As can be seen, the presence of financial constraints tends to increase the concentration of wealth at the top of the wealth distribution. Moreover, we observe an increase in overall wealth inequality, the Gini coefficient rising by 6.5 p.p.

Regarding the more general picture of the wealth distribution, our model generates results similar to those reported in the literature. Workers are more concentrated at lower wealth levels, and there exists a significant mass of wealthy entrepreneurs but also a comparably large share of poorer ones. This is in line with empirical findings by Gentry and Hubbard (2004), Hamilton (2000). Tables 3 and 5 show that on average an entrepreneur owns a multiple of the individual worker’s wealth, and that entrepreneurs more likely stem from upper wealth classes.

Figure 1 shows the cumulative distribution of firm sizes in the intermediate goods sector for $\phi \in \{0,1,\infty\}$. Each entrepreneur is able to operate his business at the optimal firm size (8) in case of unconstrained credit markets ($\phi \to \infty$). Consequently, we observe a stepwise CDF, each step corresponding to the optimal firm size associated with one out of the five underlying possible productivity states $\theta_{e,h}$.

Consider next the case $\phi = 1$, where entrepreneurs are able to acquire external financing up to maximum sum equal to their own wealth. Here, the operating firm size is bounded from above to twice the amount of individual wealth, which need not be the optimal firm size, especially, if the firm owner is highly productive. Recall at this point that the optimal firm size is endogenously determined; besides idiosyncratic random productivity also depending on factor prices, which in turn are determined by aggregate market activities and occupational choice in the general equilibrium.
The first observation is that the optimal firm sizes rise slightly for each possible state of entrepreneurial talent $\theta_{e,h}$. This increase in firm sizes can be ascribed to the factor price effect. Borrowing constraints prevent the efficient allocation of capital among sectors such that too much capital is employed in the production of the final good. This is associated with a decline in the real interest rate, which in turn raises the optimal firm size in the intermediate sector for each state of productivity.

The second, major observation in the credit–constrained economy is that there exists a positive mass of entrepreneurs between each two subsequent steps of optimal firm sizes, and the distribution is more concentrated at smaller firm sizes. Constraints become binding for many entrepreneurs, who have to operate their enterprise at a suboptimally low scale. Non–surprisingly, this effect is aggravated, if we reduce the availability of external financing to naught. For $\phi = 0$, steps in the CDF almost vanish, which means that more business owners are subject to constraints. The optimal levels of firm sizes for the different states of productivity rise even further, due to the factor price effect. In numbers, if we compare the unrestrained with the completely constrained economy, businesses in the entrepreneurial sector on average operate at 48% of their respective optimal firm size.

*Macroeconomic Effects*  
Table 3 and Figure 2 summarize the results for the macroeconomic key variables of the calibrated baseline model. The general picture reflects the outcome one would expect from credit market improvements. Aggregate output $Y$, aggregate wealth holdings $a$, factor prices $r,w$ and incomes, as well as welfare increases if we relax borrowing constraints.

Figure 2 shows that, except for wealth inequality, social mobility and the entrepreneurship rate, the response of the macroeconomic variables to a change in $\phi$ is monotonous. The overall loss in output in a perfectly constrained compared to the unconstrained economy ($\phi \to \infty$) lies at about 21%, and average wealth holdings decline by 23%. Tightening financial constraints goes along with a substantial drop in economic performance. The associated welfare loss, too, is substantial. The non–monotonic behavior of wealth inequality, social mobility and the entrepreneurship rate is systematic and will be discussed in more detail below.

We also see from Figure 2 that the response of output, wealth, factor prices, and welfare to a change in $\phi$ is concave. The marginal gains from improving credit markets are much higher for small values of $\phi$, especially in the range of debt–equity ratios between $0 < \phi < 2$, which is the empirically plausible domain. This interval accounts for more than three–quarter of the overall output loss associated with financial constraints.

Given the general equilibrium nature of the underlying model, one would expect several adjustments to take place following a reduction in external financing as borrowing constraints become more tight. If there is only limited or no capital demand from the intermediate goods industry, we observe a capital–relocation effect between sectors. More capital is employed in the final goods industry. This amounts
Table 3: Simulation Results

<table>
<thead>
<tr>
<th></th>
<th>Tightness of constraints</th>
<th>( \phi \to \infty )</th>
<th>( \phi = 1.0 )</th>
<th>( \phi = 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>entrepreneurship rate (%)</td>
<td></td>
<td>0.201</td>
<td>0.197</td>
<td>0.185</td>
</tr>
<tr>
<td>( \varnothing ) firm size total</td>
<td></td>
<td>0.964</td>
<td>0.839</td>
<td>0.699</td>
</tr>
<tr>
<td>( \varnothing ) credit rationing total</td>
<td></td>
<td>0.000</td>
<td>0.295</td>
<td>0.761</td>
</tr>
<tr>
<td>( \varnothing ) profits total</td>
<td></td>
<td>0.278</td>
<td>0.281</td>
<td>0.281</td>
</tr>
<tr>
<td>aggregate output ( Y )</td>
<td></td>
<td>0.253</td>
<td>0.232</td>
<td>0.200</td>
</tr>
<tr>
<td>capital input for ( Y ) ( K_R )</td>
<td></td>
<td>0.270</td>
<td>0.306</td>
<td>0.365</td>
</tr>
<tr>
<td>interest rate ( r )</td>
<td></td>
<td>0.042</td>
<td>0.028</td>
<td>0.008</td>
</tr>
<tr>
<td>factor price ratio ( w/(r+\delta) )</td>
<td></td>
<td>1.362</td>
<td>1.396</td>
<td>1.406</td>
</tr>
<tr>
<td>factor income shares labor</td>
<td></td>
<td>0.630</td>
<td>0.630</td>
<td>0.630</td>
</tr>
<tr>
<td>capital</td>
<td></td>
<td>0.149</td>
<td>0.131</td>
<td>0.110</td>
</tr>
<tr>
<td>profits</td>
<td></td>
<td>0.221</td>
<td>0.239</td>
<td>0.260</td>
</tr>
<tr>
<td>( \varnothing ) income workers</td>
<td></td>
<td>0.215</td>
<td>0.193</td>
<td>0.159</td>
</tr>
<tr>
<td>entrepreneurs</td>
<td></td>
<td>0.403</td>
<td>0.392</td>
<td>0.377</td>
</tr>
<tr>
<td>risk premium</td>
<td></td>
<td>0.399</td>
<td>0.542</td>
<td>0.825</td>
</tr>
<tr>
<td>( \varnothing ) wealth total</td>
<td></td>
<td>0.265</td>
<td>0.239</td>
<td>0.203</td>
</tr>
<tr>
<td>workers</td>
<td></td>
<td>0.111</td>
<td>0.083</td>
<td>0.048</td>
</tr>
<tr>
<td>entrepreneurs</td>
<td></td>
<td>0.876</td>
<td>0.871</td>
<td>0.888</td>
</tr>
<tr>
<td>wealth total</td>
<td></td>
<td>0.770</td>
<td>0.774</td>
<td>0.835</td>
</tr>
<tr>
<td>inequality workers ( (\text{Gini}) )</td>
<td></td>
<td>0.655</td>
<td>0.669</td>
<td>0.779</td>
</tr>
<tr>
<td>entrepreneurs</td>
<td></td>
<td>0.677</td>
<td>0.592</td>
<td>0.534</td>
</tr>
<tr>
<td>mobility</td>
<td></td>
<td>0.042</td>
<td>0.041</td>
<td>0.039</td>
</tr>
</tbody>
</table>

To shifting about 9.5% of the aggregate capital stock from the intermediate to the final goods sector over the entire range \( 0 \leq \phi < \infty \). The average excess demand for capital in the intermediate goods industry amounts to about 110% of the average firm size if credit markets are completely closed; see Table 3.

With diminishing marginal returns, the equilibrium interest rate \( r \), and accordingly the factor price for capital \( r + \delta \), decline by almost 3.5 p. p. in both sectors of the economy. Recalling that entrepreneurial households receive income from two sources, profits and capital incomes, the income share reflecting the user costs of capital declines for any given level of individual wealth, whereas the profit share rises. Altogether, we observe a shift in the functional income distribution from capital to profit incomes of 3.9 p.p. over the entire domain of \( \phi \).

The presence of credit constraints not necessarily implies that only those agents choose to become an entrepreneur, who have sufficient own wealth and borrowed resources to operate their business at the optimal firm size \( k^* \). These are the only firms who actually maximize their profits, whereas the constrained entrepreneurs are forced to operate at suboptimally small business sizes. Consequently, the average firm size in the intermediate goods industry decreases substantially as financial
constraints become more tight, and highly productive entrepreneurs are more af-

fected by the constraints than those with a low $\theta_e$. Figure 2a shows that in a com-
pletely constrained economy the average firm size only amounts to around 48% of
its optimal operating size.

The overall employment effect of tightening financing constraints is astonish-
ingly small. The entrepreneurship rate decreases by 1.6 p.p. over the entire range of
$\phi$. Credit constraints are an impediment to entrepreneurship, as is stated through-
out the theoretical and empirical literature; see e.g. Evans and Leighton (1989);
Evans and Jovanovic (1989); Blanchflower and Oswald (1998). Nevertheless, the
rather modest response of the entrepreneurship rate to an increase in credit ra-
tioning is quite surprising. Economic intuition would have suggested a more pro-
nounced reaction in occupational choice.

There are several factors explaining this result, which can mainly be traced back
to the general equilibrium nature of our approach. Credit constraints are only one
out of several determinants of occupational choice. First of all, competition between
the final and intermediate goods sector for capital determines the equilibrium in-
terest rate, the firm size and, most importantly, expected profits of the monopolistic
enterprises. The lower the user costs of capital, the higher are c.p. profits, which
increase if financial constraints become more tight (cf. Table 3). Borrowers benefit
from a decrease in the interest rate, whereas lenders and capital owners in gen-
eral loose for a given capital stock. Financial constraints affect especially business
owners with little wealth. Those, for whom the distribution of future profits be-
comes unfavorable, will exit the market, whereas other constrained entrepreneurs
(continue to) operate at a suboptimally low scale.

The reallocation of capital into the production of the final good induces an
increase in the demand for intermediate goods; see eq. (5). This, together with a
reduction in the interest rate, leads to an associated rise in expected entrepreneurial
profits, albeit by a mere 1.1%. The question is to what extent market exits due to
financial constraints are compensated by market entries due to the rise in expected
profits. The fraction of business owners, who are subject to constraints, increases
for smaller values of $\phi$. Here, the negative effects of financial constraints dominate
and the entrepreneurship rate decreases.

The choice of entering into entrepreneurship, however, is also determined by
opportunity costs in terms of foregone wage income and the expected earnings dif-
fferential (‘risk premium’) between the two types of income. Last, there is also the
question of income persistence, since households continuously decide between two
lotteries and possess (at least subjective) knowledge regarding the stochastic prop-
erties of the underlying shocks. If shocks are serially correlated, a low–productivity
worker is aware of the fact that being also lowly productive in the future is a more
probable outcome than otherwise. Consequently, he might be inclined to take his
chances with entrepreneurship, knowing that his current productivity as a worker
is not related to his future productivity as a business owner.
Figure 2: Macroeconomic Effects of a Change in $\phi$, Baseline Model
The risk premium on entrepreneurial activity more than doubles if we compare the unconstrained to the completely credit-constrained economy, thereby providing incentives towards entrepreneurship. A closer inspection of Figure 2 reveals the non-monotonic behavior of the entrepreneurship rate and social mobility. If we relax borrowing constraints, we first observe a sharp rise in the variables for small $\phi < 2$, but also a moderate decrease for large values of $\phi > 7.5$.

In the interval of values for $\phi$ between 4–7.5, the entrepreneurship rate is almost constant, which is also true for the wage rate and social mobility, although aggregate output and average firm sizes already display a more pronounced decrease. The environment under which occupational choice takes place does not undergo substantial changes in the range of intermediate values for $\phi$ between 4–7.5, thereby keeping the entrepreneurship rate comparatively stable. Once financial constraints become more severe ($\phi < 2$), we observe a more pronounced shift in employment.

Summing up: Whereas we have clear-cut substantial, negative effects of financial constraints on overall macroeconomic performance, we observe an ambiguous response in the entrepreneurship rate. Counteracting forces on occupational choice give rise to a non-monotonic behavior of the entrepreneurship rate. Increases in the tightness of financial constraints work against business ownership by limiting firm sizes to suboptimally low levels. On the contrary, changes in the real interest rate, expected profits, and the risk premium provide incentives towards entrepreneurship. As turns out, the structure of the occupational lottery, especially the degree of income persistence is crucial to the question of what force prevails in the end. We explore this issue more deeply in Section 4.2 for an alternative calibration of the model.

Regarding the wealth distribution, we first observe a sharp decline in the Gini coefficient for a rise in $\phi$ from 0.84 to 0.76, which is then followed by a gradual increase in overall wealth inequality back to a value of 0.77. This non-monotonic behavior of total wealth inequality can be explained, if we look at the within-
Table 4: Transition Probabilities for Households in a given Wealth Quintile and Productivity State

(a) Workers

<table>
<thead>
<tr>
<th>Tightness of constraints</th>
<th>$\phi = 1000$</th>
<th>$\phi = 1$</th>
<th>$\phi = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_{w,1}$</td>
<td>0  0  1  1  1</td>
<td>0  0  0.49 1  1</td>
<td>0  0  0  0.88 1</td>
</tr>
<tr>
<td>$\theta_{w,2}$</td>
<td>0  0  0.87 1  1</td>
<td>0  0  0.50 1  1</td>
<td>0  0  0  0.67 1</td>
</tr>
<tr>
<td>$\theta_{w,3}$</td>
<td>0  0  0  0  0.07</td>
<td>0  0  0  0  0.55</td>
<td>0  0  0  0  0.08</td>
</tr>
<tr>
<td>$\theta_{w,4}$</td>
<td>0  0  0  0  0</td>
<td>0  0  0  0  0.01</td>
<td>0  0  0  0  0.08</td>
</tr>
<tr>
<td>$\theta_{w,5}$</td>
<td>0  0  0  0  0</td>
<td>0  0  0  0  0</td>
<td>0  0  0  0  0</td>
</tr>
</tbody>
</table>

(b) Entrepreneurs

<table>
<thead>
<tr>
<th>Tightness of constraints</th>
<th>$\phi = 1000$</th>
<th>$\phi = 1$</th>
<th>$\phi = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_{e,1}$</td>
<td>–  –  1  1  1</td>
<td>–  –  1  1  1</td>
<td>–  –  1  1  1</td>
</tr>
<tr>
<td>$\theta_{e,2}$</td>
<td>–  –  1  1  1</td>
<td>–  –  1  1  1</td>
<td>–  –  1  1  1</td>
</tr>
<tr>
<td>$\theta_{e,3}$</td>
<td>–  –  1  1  1</td>
<td>–  –  1  1  1</td>
<td>–  –  1  1  1</td>
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<tr>
<td>$\theta_{e,4}$</td>
<td>–  –  0  0  0</td>
<td>–  –  0  0  0</td>
<td>–  –  0  0  0</td>
</tr>
<tr>
<td>$\theta_{e,5}$</td>
<td>–  –  0  0  0</td>
<td>–  –  0  0  0</td>
<td>–  –  0  0  0</td>
</tr>
</tbody>
</table>

Bars (–) indicate that 0% of entrepreneurs are in the associated wealth quintile.

Social Mobility We now discuss in more detail the question of how tightened financial constraints affect occupational choice and social mobility. Table 3 shows that an increase in the tightness of financial constraints leads to a decline in the entrepreneurship rate by 1.6 p.p. Between 3.9 to 4.2% of the population switch occupations in each period of time, which amounts to an exit rate of about 20% of all firms in the intermediate goods industry. Overall mobility declines, if constraints become more tight.

To understand mobility of entrepreneurs, we have to consider several factors. The future occupation is determined (a) by the present level of wealth, (b) the current draw of productivity governing present income, consumption and saving, (c) the choice between two lotteries with unconditional probabilities governing future income, consumption and saving, where the lottery over worker efficiencies is less risky than the lottery over entrepreneurial productivities, and (d) the expected market equilibrium of the next period, determining factor prices and factor income differentials.
We are especially interested in two dimensions of social mobility: Based on the steady state distribution of agents across occupations, we first focus on the individual level and determine the probabilities of transition between occupations in the next period for agents in a given wealth quintile and productivity state. The associated results are displayed in Table 4. Second, we take a more aggregate viewpoint and analyze the inflow of entrants into entrepreneurship and the workforce respectively by decomposing the entire flow with respect to the distributional characteristics of agents regarding wealth levels and productivity classes. This enables us to determine the probability with which switching households come from alternative (productivity-cum-wealth) backgrounds in the actual macroeconomic equilibrium.

If we look the transition probabilities of Table 4, the picture is striking for individuals, who currently are low and medium productivity entrepreneurs. All entrepreneurs of the lowest three productivity states change their occupation, even if they belong to the upper wealth quintiles. They, with certainty, will exit the market to seek employment as a worker in the next period. This results holds irrespective of the degree of constraint. There are no entrepreneurs in the lowest two wealth quintiles. Rich and highly productive business owners never change their occupation.

Regarding workers we find that generally rich workers, in particular rich and relatively unproductive workers switch occupations in the next period. The general mobility pattern is robust over different levels of $\phi$. The intuition behind this result is as already outlined above. Serially correlated shocks provide agents with a signal regarding future productivity. Since we assumed mutually uncorrelated processes for labor efficiency and entrepreneurial ability, a worker can infer from a low productivity today a probably low labor efficiency tomorrow, but this not necessarily indicates an equally low future ability as entrepreneur, which is given by the unconditional probability of states.

Poor workers will not enter into business ownership, independent of their productivity. This situation becomes more severe and even stretches to the third wealth quintile, if financial constraints become more tight. New entrepreneurs are mainly recruited among the group of wealthy workers. More tight credit constraints strikingly decrease the probabilities of becoming an entrepreneur for poorer workers, while the corresponding probabilities for richer workers (especially for those in the fourth and fifth quintile) increase. If financial constraints become more tight, the income premium for a poor worker entering entrepreneurship is smaller than for a rich worker. We observe an increasing spread in the distribution of premiums on profits over wages, which increases transition probabilities for rich workers.

While Table 4 provides us with information regarding the transition probability of agents who currently are in a certain wealth class and productivity state, Table 5 allows us trace back where switching households most likely come from in macroeconomic equilibrium in terms of productivity and wealth characteristics. A general insight from our model is that mobility over occupations is confined to agents who
Table 5: Decomposition of Flows: Probability of Wealth/Productivity-Class Background of Switching Households

(a) Workers

<table>
<thead>
<tr>
<th>Tightness of constraints</th>
<th>( \phi = 1000 )</th>
<th>( \phi = 1 )</th>
<th>( \phi = 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st 2nd 3rd 4th 5th 1st 2nd 3rd 4th 5th 1st 2nd 3rd 4th 5th</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \theta_{w,1} )</td>
<td>0 0 0.004 0.007 0.002</td>
<td>0 0 0.004 0.009 0.001</td>
<td>0 0 0 0.008 0.004</td>
</tr>
<tr>
<td>( \theta_{w,2} )</td>
<td>0 0 0.227 0.453 0.215</td>
<td>0 0 0.209 0.445 0.074</td>
<td>0 0 0 0.234 0.101</td>
</tr>
<tr>
<td>( \theta_{w,3} )</td>
<td>0 0 0 0 0.093</td>
<td>0 0 0 0 0.252</td>
<td>0 0 0 0.181 0.438</td>
</tr>
<tr>
<td>( \theta_{w,4} )</td>
<td>0 0 0 0 0</td>
<td>0 0 0 0 0</td>
<td>0 0 0 0 0.035</td>
</tr>
<tr>
<td>( \theta_{w,5} )</td>
<td>0 0 0 0 0</td>
<td>0 0 0 0 0</td>
<td>0 0 0 0 0</td>
</tr>
<tr>
<td>( \sum )</td>
<td>0 0 0.231 0.460 0.310</td>
<td>0 0 0.213 0.454 0.334</td>
<td>0 0 0 0.423 0.578</td>
</tr>
</tbody>
</table>

(b) Entrepreneurs

<table>
<thead>
<tr>
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<th>( \phi = 1000 )</th>
<th>( \phi = 1 )</th>
<th>( \phi = 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st 2nd 3rd 4th 5th 1st 2nd 3rd 4th 5th 1st 2nd 3rd 4th 5th</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \theta_{e,1} )</td>
<td>0 0 0.015 0.009 0.002</td>
<td>0 0 0.013 0.008 0.005</td>
<td>0 0 0 0.015 0.011</td>
</tr>
<tr>
<td>( \theta_{e,2} )</td>
<td>0 0 0.114 0.072 0.050</td>
<td>0 0 0.104 0.062 0.073</td>
<td>0 0 0 0.117 0.119</td>
</tr>
<tr>
<td>( \theta_{e,3} )</td>
<td>0 0 0.358 0.178 0.203</td>
<td>0 0 0.202 0.157 0.379</td>
<td>0 0 0 0.246 0.492</td>
</tr>
<tr>
<td>( \theta_{e,4} )</td>
<td>0 0 0 0 0</td>
<td>0 0 0 0 0</td>
<td>0 0 0 0 0</td>
</tr>
<tr>
<td>( \theta_{e,5} )</td>
<td>0 0 0 0 0</td>
<td>0 0 0 0 0</td>
<td>0 0 0 0 0</td>
</tr>
<tr>
<td>( \sum )</td>
<td>0 0 0.487 0.259 0.255</td>
<td>0 0 0.319 0.227 0.457</td>
<td>0 0 0 0.378 0.622</td>
</tr>
</tbody>
</table>

are not (or have not been) successful in their professions. Households continuously decide between two serially correlated lotteries. Generally speaking, they have little incentive to stay in their occupation, if they receive a low talent shock.

Table 5 shows that, independent of wealth levels, a new entrepreneur never is recruited from high productivity workers and vice versa. Likewise, we never observe a low-productivity worker from the lower wealth quintiles to make an entry into entrepreneurship. Mobility over occupations crucially depends on membership in wealth quintiles. New entrepreneurs have been rich workers with larger probability. Interestingly, new entrepreneurs most probably are recruited among those workers of the two lowest productivity states in the 4th wealth quintile. Here we observe a combination of two incentive effects: on the one hand, the poor future income prospects of a currently low productivity; on the other hand, the attractiveness of entrepreneurship due to the risk premium on entrepreneurial activity. Because members of the 5th wealth quintile receive a substantial share of their income from riskless capital investment, they are less exposed to income risk and therefore less likely to switch occupations, even if they are comparably unproductive. In
Table 6: Wealth Distribution

<table>
<thead>
<tr>
<th>Gini</th>
<th>1%</th>
<th>5%</th>
<th>10%</th>
<th>20%</th>
<th>30%</th>
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</thead>
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<tr>
<td>PSID 1994</td>
<td>22.6</td>
<td>44.8</td>
<td>59.1</td>
<td>75.9</td>
<td>85.9</td>
</tr>
<tr>
<td>SCF 1992</td>
<td>29.5</td>
<td>53.5</td>
<td>66.1</td>
<td>79.5</td>
<td>87.6</td>
</tr>
<tr>
<td>$\phi = 0$</td>
<td>18.48</td>
<td>50.60</td>
<td>70.10</td>
<td>86.24</td>
<td>93.70</td>
</tr>
<tr>
<td>$\phi = 1.0$</td>
<td>17.64</td>
<td>47.01</td>
<td>64.19</td>
<td>80.14</td>
<td>88.42</td>
</tr>
<tr>
<td>$\phi = 1000$</td>
<td>15.76</td>
<td>45.39</td>
<td>61.96</td>
<td>76.99</td>
<td>84.64</td>
</tr>
</tbody>
</table>

accordance with economic intuition, it is more likely to observe rich workers to switch occupations, if financial constraints become more tight.

Table 5b, displaying the probability that a new worker is recruited from a given wealth quintile and productivity level, has zero entries in the lower two wealth quintiles, because all entrepreneurs belong to the upper three wealth quintiles in the macroeconomic equilibrium. A rich and highly productive entrepreneur has no incentive to change occupations. Therefore, workers are only recruited from low levels of entrepreneurial productivity, and the probability is increasing, if the financial constraints become more tight.

4.2 Alternative Calibration of the Model

We want to conclude our numerical simulations with a short discussion of an alternatively calibrated model. The analysis is motivated by one of the major findings of the baseline model, namely the non–monotonic and in magnitude very modest response of the entrepreneurship rate and social mobility to tightening financial constraints. We show for an empirically equally plausible calibration that the entrepreneurship rate actually might increase in a more constrained economy. The implications for overall macroeconomic performance, however, remain unchanged.

A parameter crucial for the response of the entrepreneurship rate is the degree of serial correlation in the income processes. While entrepreneurial productivity shocks are assumed to be more persistent in the baseline model, we now postulate identical serial correlations for both, workers and entrepreneurs, $p_w = p_e = 0.6$. We additionally customize the rate of time preference to $\beta = 0.89$ and the variance of entrepreneurial shocks to $\sigma_c = 2.0$ to generate an empirically plausible interest rate and wealth inequality, and keep the remaining model parameters; see Table 1.

Table 6 shows that the new calibration also generates a wealth distribution which matches empirical evidence, for instance for the U.S., regarding the Gini coefficient but also with respect to the upper tail of the distribution.

Figure 3 displays selected results for the macroeconomic effects of an increase in the tightness of constraints under the new parameterization of the model. Table 7
in the Appendix gives a summary of the numerical results. We find that the general pattern of how macroeconomic performance responds to tightened constraints prevails. The effects are mostly in magnitude. The decline in aggregate output and average firm size is larger when compared to the baseline model. Aggregate output drops by 35% (vs. 21% in the baseline model) and completely constrained firms on average only make about 31% (vs. 48%) of the optimal firm size over the entire range of $\phi$. Instead of less competition in the intermediate goods industry, we observe an increase in the entrepreneurship rate in the constrained economy. This, however, comes at the cost of smaller market shares and lower average profits (~25% compared to the unconstrained economy).

Most important, the non-monotonic response of the entrepreneurship rate and social mobility is more pronounced. The entrepreneurship rate predominantly rises for an increase in financial constraints, and decreases moderately for very small values of $\phi$. Here, the factors encouraging entrepreneurship prevail over the negative effects of tightened constraints.

Likewise, social mobility increases for a rise in $\phi$, and only drops sharply for small values $\phi < 2$. For most values of $\phi$ tightened financial constraints not only increase the entrepreneurship rate of the economy but also the fluctuation between

Figure 3: Macroeconomic effects of a change in $\phi$, alternative calibration
occupations. Altogether, social mobility takes place at a much larger scale, more than twice as much agents switching occupations when compared to the baseline model. The implications regarding the transition probabilities between occupations are identical to the baseline model and therefore omitted.

5 Concluding Remarks

In this paper, we examined the effects of borrowing constraints and idiosyncratic risks on macroeconomic performance, wealth inequality, and social mobility in a two-sector heterogeneous agent dynamic general equilibrium model. Workers and firm owners are subject to idiosyncratic shocks. Entrepreneurship in the intermediate (non-corporate) goods industry is the riskier occupation. Our comparative static results cover a broad range for borrowing constraints, from an unrestrained to a perfectly constrained economy.

The stationary wealth distribution generated in the model is consistent with empirical findings. Entrepreneurial households own a substantial share of household wealth and their share increases throughout the wealth distribution.

Independent of the persistence of the idiosyncratic shocks, we find that tightening financial constraints is accompanied by substantial losses in aggregate output, consumption, wealth holdings, and welfare. The inefficient allocation of capital across sectors accounts for this result in the first place and only second the associated changes in employment and the entrepreneurship rate. To the extent firms of the intermediate goods industry are barred from participation in the credit market, more capital is employed in the final (corporate) goods sector. The associated decline in the interest rate causes a shift in the functional income distribution towards profit incomes.

The response of the macroeconomic variables to a change in credit availability is monotonous and concave. We find that the marginal gains of relaxing constraints are especially large for small (enforced) debt-equity ratios. This indicates that it is a worthwhile question to explore in more detail the marginal gains from credit market improvement, which at this point is left for future research.

The general equilibrium context of our model, where optimal firm sizes and the demand for credit are determined endogenously, gives rise to interesting implications regarding the change in the entrepreneurship rate and in social mobility, as we vary the degree of credit availability in the non-corporate sector. Contrary to economic intuition, there is only a slight difference in magnitude between the entrepreneurship rates of the completely constrained and the unrestrained economy. The overall employment effects of relaxing financial constraints are very small. We also observe a non-monotonic response in both the entrepreneurship rate and in social mobility to changes in the tightness of financial constraints.

This result can primarily be attributed to two factors: first, the general equilibrium nature of our approach. While financial constraints deter entrepreneurs from
operating at the optimal firm size, they also induce capital flows between sectors which ultimately end up in lower interest rates and higher expected profits. Second, the degree of income persistence. Workers and entrepreneurs with high individual productivity tend to remain in their present occupation, whereas low productivity individuals are more likely to switch between professions. Regarding exit and entry rates into entrepreneurship, we find that a lower persistence of entrepreneurial shocks generally increases between-group mobility and also raises the entrepreneurship rate over a broad range of financial constraints.

So what are the general implications of our model? The extent, to which the industry is subject to financial constraints and firm sizes as well as production possibilities are limited is more important for the macroeconomy as a whole, than the question of how many entrepreneurs there are and how occupational choice responds to a change in the tightness of constraints. Even an increase in the entrepreneurship rate (i.e. the number of businesses in the market) is not sufficient to prevent or mitigate the overall loss in output, consumption and welfare, if the access to external financing becomes increasingly difficult.

There are many important issues this paper does not address. The model lacks a fully micro–founded formulation of credit constraints and a more detailed modeling of financial intermediation. Also, testing the robustness of results with respect to varying attitudes towards risk is left for future research. So far, we assume worker efficiency and entrepreneurial ability to be uncorrelated. However, as already Cagetti and De Nardi (2006a) pointed out, it is difficult to measure correlated talent shocks in the data, which to some extent justifies our approach.

References


Fernández-Villaverde, Jesús, Galdón-Sánchez, José Enrique, and Carranza, Luis (2003), Entrepreneurship, Financial Intermediation and Aggregate Activity.


A Computational Issues

The state space of wealth is approximated by a grid of \( N \) wealth levels \( a_n \) for \( n = 1, \ldots, N \) with \( a_1 = a \) and \( a_N = \bar{k} \). The macroeconomic equilibrium is recursively computed. We start with an initial guess on factor prices \( \tilde{w}, \tilde{r} \), and the equilibrium level of employment in efficiency units \( \tilde{L} \). Let \( \mu = \{ \tilde{w}, \tilde{r}, \tilde{L} \} \) denote the vector of the initial guesses. From this first solution trial we obtain factor proportions in the final goods sector according to the marginal productivity conditions. The underlying production technology implies

\[
\tilde{K}^\gamma_F = \tilde{L}^\gamma (\tilde{w} \tilde{r} + \delta)^{1-\gamma}.
\]

Moreover,

\[
(\tilde{K}^\gamma_F, \tilde{L}^1) \gamma^{1-\alpha} \text{ equals } \tilde{L} \left( \frac{\tilde{w}}{\tilde{r} + \delta} \right)^\gamma.
\]

Let \( k_{n,h}(\mu) \) denote the firm size an entrepreneur with productivity \( \theta_{e,h} \) and wealth \( a_n \) is able to operate at for a given extent of borrowing constraints and the initial guess \( \mu \). His profit is given by

\[
\pi_{n,h}(\mu) = \alpha (\theta_{e,h}, k_{n,h}(\mu))^\alpha \tilde{L} \left( \frac{\tilde{w}}{\tilde{r} + \delta} \right)^\gamma - (\tilde{r} + \delta) k_{n,h}(\mu) .
\]

Let \( A^n_w(\mu) \) and \( Q^n_w(\mu) \) as well as \( A^n_e(\mu) \) and \( Q^n_e(\mu) \) for \( n = 1, \ldots, N \) and \( h = 1, \ldots, H \) denote the optimal policies associated with the discrete formulation of the optimization problems (11) and (12) for the given initial guess \( \mu \) on prices and employment. We characterize agents by their wealth holdings \( a_n \), their occupational status \( \zeta \), where \( \zeta \in \{ w, e \} \) and their current productivity state \( \theta_h, h = 1, \ldots, H \).
Knowing the policy functions and transition matrices for the underlying productivity shocks, we are able to compute a stationary distribution and the probability for an agent to have wealth $a_n$, occupational status $\zeta$ and productivity state $\theta_h$. Let $\Psi_{n,\zeta,h}(\mu)$ denote the respective probability for $n = 1,\ldots,N$, $\zeta = \{w,e\}$ and $h = 1,\ldots,H$ given the initial guess $\mu$. These probabilities can now be used to compute aggregate quantities. The aggregate capital stock (i.e. mean wealth holdings) can be determined as:

$$K(\mu) = \sum_{n=1}^{N} \sum_{\zeta \in \{w,e\}} \sum_{h=1}^{H} \Psi_{n,\zeta,h}(\mu) a_n$$

The population share of entrepreneurs results as

$$\lambda(\mu) = \sum_{n=1}^{N} \sum_{h=1}^{H} \Psi_{n,\zeta,h}(\mu)$$

while labor supply in efficiency units is given by

$$L(\mu) = \sum_{n=1}^{N} \sum_{h=1}^{M} \Psi_{n,\zeta,h}(\mu) \theta_{w,h}$$

Capital demand of the intermediate goods sector can be computed as:

$$K^D_I(\mu) = \sum_{n=1}^{N} \sum_{h=1}^{H} \Psi_{n,\zeta,h}(\mu) k_{n,h}(\mu)$$

The supply of capital to the final goods sector is thus given by $K^S_F(\mu) = K(\mu) - K^D_I(\mu)$. Finally, the initial guess $\mu$ together with the production decision of entrepreneurs generates an aggregate output of

$$Y(\mu) = \left( K^I_I \frac{1}{1-\gamma} \right)^{1-\alpha} \sum_{n=1}^{N} \sum_{h=1}^{H} \Psi_{n,\zeta,h}(\mu) (\theta_{e,h} k_{n,h})^\alpha$$

The initial solution guess represents an equilibrium only if the following conditions hold:

(i) Labor supply in efficiency units must equal the initial guess, i.e.

$$L(\mu) = \tilde{L}$$  \hspace{0.5cm} (A.1)

(ii) Labor demand in the final goods sector equals labor supply:

$$L(\mu) = (1-\alpha)(1-\gamma) \frac{Y(\mu)}{\tilde{w}}$$  \hspace{0.5cm} (A.2)

(iii) Capital demand in the final goods sector equals supply of capital to the final goods sector:

$$K^S_F(\mu) = K(\mu) - K^D_I(\mu) = (1-\alpha) \frac{Y(\mu)}{\tilde{r} + \delta}$$  \hspace{0.5cm} (A.3)

The algorithm for finding the equilibrium values consists of three nested loops over $\tilde{L}$, $\tilde{w}$ and $\tilde{r}$. The first loop iteratively computes the value $\tilde{L}$ which meets condition (A.1) for given factor prices $\tilde{w}$ and $\tilde{r}$. Then, factor prices $\tilde{w}$ and $\tilde{r}$ are adjusted according to the resulting excess demands for labor and capital according to conditions (A.2) and (A.3). The whole procedure is repeated until the equilibrium conditions (A.1) to (A.3) are satisfied, except for a tolerably small approximation error.

To implement the algorithm, we used the programming language C++. The underlying source code and the data are available from the authors upon request.
### Table 7: Simulation results

<table>
<thead>
<tr>
<th>⋆</th>
<th>Tightness of constraints</th>
<th>φ → ∞</th>
<th>φ = 1.0</th>
<th>φ = 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>entrepreneurship rate (%)</td>
<td></td>
<td>0.193</td>
<td>0.207</td>
<td>0.207</td>
</tr>
<tr>
<td>⋆</td>
<td>credit rationing</td>
<td>total</td>
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</tr>
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<td>⋆</td>
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<td>total</td>
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<td>final goods sector</td>
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<td>capital input for Y</td>
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<td>0.207</td>
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<td>interest rate</td>
<td>r</td>
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<td>0.017</td>
<td>0.003</td>
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<td>0.637</td>
<td>0.596</td>
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<td>0.630</td>
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<td></td>
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<td>wealth</td>
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