Technology adoption with exit in imperfectly informed equity markets

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Abstract

This paper focuses on the importance of equity markets in facilitating the exit of entrepreneurs investing in technology. Entrepreneurs’ willingness to invest and aggregate output is affected in two opposite ways. First, uncertainty about equity price or lack of market liquidity discourages technology adoption. This can explain slow technology adoption and limited participation by venture capitalists in underdeveloped equity markets. Second, imperfectly informed market participants rationally take fast adoption as a positive signal. The resulting increase of expected market value encourages technology adoption. Fast technology adoption is most probable if the quality of information is at an intermediate level.

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1 Introduction

There is a growing interest in channels through which more developed financial markets promote entrepreneurial and innovative activities and thereby long-term growth.¹ The benefits of more developed financial markets are most often analyzed through their positive impact on availability of external financing. This paper takes a different approach by emphasizing that in addition to providing funding, equity markets have an important role in facilitating ownership transfers from entrepreneurs investing in technology to managers running these firms once the technology is adopted. Good exit opportunities are also important for venture capitalists who can provide funding for investments in technology. The paper suggests a new mechanism that shows how the development of equity markets can determine the incentives to invest in technology and aggregate output even if credit constraints are not binding.

The paper focuses on technology adoption (or innovation) decisions made by risk averse entrepreneurs who sell their firms in the equity market. The main implications arise from

¹See Levine (2005) for a comprehensive review of theoretical and empirical literature on the relationship between finance and growth.
a double-sided information asymmetry. On the one hand, entrepreneurs are likely to have superior information about the fundamental value of their firms as compared to the average equity market participant. On the other hand, they do not know what information equity market participants will receive in the future.

The main results in this paper arise from two opposite forces affecting entrepreneurs’ incentives to invest in the newest and most expensive technologies. First, high uncertainty can discourage investment in the most advanced technologies – the "fear of unstable markets" force. Second, given that entrepreneurs have superior information about the value of their firms, their decision to invest in the newest technology becomes a positive signal to the market. This increases the expected market value of firms and encourages entrepreneurs to invest in such technology – the "adoption to signal" force. The quality of information in equity market (the degree of information asymmetry between entrepreneurs and investors) determines which of these two forces is predominant.

In less developed equity markets, when investors have very imprecise information, entrepreneurs choose to adopt technology slowly and copy older technologies instead of investing in the newest ones. Furthermore, underdeveloped equity markets can explain why foreign agents, who are able to reduce technology adoption costs, may not participate in projects they would find profitable in perfect equity market.

Fast technology adoption (i.e. investment in the newest technology) is most likely, when the quality of information investors have is at an intermediate level. In this case, investments in technology are still informative about the underlying productivity, while the negative effect through uncertainty is not pressing. As a result, entrepreneurs have the highest expected gains from investing. In fact, the gains from fast technology adoption are higher than in perfectly informed equity market. When quality of information is very good, there is no information asymmetry between entrepreneurs and potential buyers and both, the discouraging "fear of unstable markets" and the encouraging "adoption to signal" force disappear. The implied non-monotonic relationship between investments in technology or GDP growth and equity market development is consistent with correlations in transition economies and high and upper-middle-income countries (see Appendix A).

The non-monotonic relationship has implications for policies that aim for greater transparency. For example, policy makers can aim to develop institutions and laws that facilitate access to information. The paper considers a policy maker who has a full control over the quality information. If such policy maker aims to maximize the probability of fast technology adoption or output and wages of local agents, he would not choose full transparency. Setting

\footnote{Note that this is not a signalling model in the spirit of Spence. Entrepreneur’s adoption decision becomes a signal in the sense that it conveys information to investors.}
the quality of information "too high" would eliminate the gains from "adoption to signal".

In more developed markets, where the "adoption to signal" force is likely to be stronger than the "fear of unstable markets" force, the model predicts "overinvestment" in technology that leads to overpricing of equity and subsequently lower returns for investors. The paper provides a rational explanation for this pattern that is supported by empirical literature discussed in Section 4.

The basic model presented in this paper shows the mechanisms in a setting where the equity market is perfectly liquid (i.e. the number of investors trading in the equity market is very large). It also assumes that all investors are identical and less informed than entrepreneurs, who establish firms adopting technology. Further extensions show that the predictions of the model are robust to considering less liquid equity markets and the presence of some investors in the equity market, who are as well informed as the entrepreneurs. The lack of liquidity has an additional negative effect on incentives to invest in technology, called the "direct lack of liquidity" effect. When there is a limited number of informed investors in the equity market, the average quality of information among equity market participants also depends on the number of informed investors.

The model is also extended to analyze endogenous entry to entrepreneurship. It is shown that the aggregate impact of the main forces is reinforced. In equity markets, where "adoption to signal" force dominates the "fear of unstable markets" force (and "direct lack of liquidity" effect), there is more entry to entrepreneurship and faster growth compared to perfect equity market. While, if the negative forces dominate, there is less entry and slower growth.

The setup of the model relies on two crucial assumptions. First, an entrepreneur must sell his firm before it generates profits. The need to exit would emerge endogenously if some agents have a comparative advantage to be entrepreneurs rather than managers, as in Holmes and Schmitz Jr. (1990). Moreover, venture capitalists can be seen as agents who are skilled in judging whether it is worth investing in a particular technology. They are generally not constrained in credit markets and prefer to exit fast (Jovanovic and Szentes 2007). Lack of good exit opportunities is a major concern for these agents when assessing investments in developing countries (Lerner and Pacanins 1997). Figure 1 shows that venture capitalists perceive the concerns about successful exit to be a larger impediment than the lack of skilled workers or weak intellectual property laws. Among the less developed countries, Asia is often considered as one of the most attractive locations for venture capital (Aylward 1998, Survey 3).

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3 The venture capitalists surveyed are not necessarily investing in these regions. Important impediments that are excluded from Figure 1 are "Lack of quality deals that fit investment profile" and "Lack of knowledge and expertise of business environment" that are likely to be specific to a particular venture capitalist.
Figure 1: Impediments for venture capital investor (US and European respondents), data source: Deloitte Touche Tochmatsu and EVCA, 2006.

by Deloitte Touche Tochmatsu and EVCA 2006). While this region does not have more skilled labor than competing regions, it has more developed equity markets4.

This paper assumes that entrepreneurs sell their firms in the equity market. However, the suggested mechanism is valid more generally if agents deciding about investments in technology care about the future market value of their firms. This could be due to executive compensation packages that depend on equity prices (see Murphy 2002) or entrepreneurs' intentions to raise additional equity funds in the future.

The second crucial assumption relies on rational, but imperfectly informed equity market participants, whose expectations are affected by a noisy public signal as in Allen, Morris and Shin (2006) and Bacchetta and van Wincoop (2006). Such a public signal leads to the possibility that equity prices can deviate from the fundamental value of the firm, for example because of optimistic or pessimistic "market sentiment".5

The paper relates to the existing theoretical literature on the determinants of the speed of technology adoption. Differences in the speed of adoption could arise from the lack of skilled labor in certain countries that makes the frontier technologies inappropriate for these countries (e.g. Acemoglu 2002). While that argument is likely to be crucial in countries with

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4 According to WDI data for 1996-2004, the median share of the labor force with secondary education in Asia is 28.2% as compared to 33.3% in Latin America, 62.2% in transition countries that entered EU and 56.6% in other transition countries. At the same time, the median stock market capitalisation to GDP in these regions is 44.5%, 24.5%, 13.4% and 10.4%, respectively.

5 There is a large empirical literature on deviations of equity prices from their fundamentals and the impact of market sentiment (see e.g. Cutler, Poterba and Summers 1991, Lee, Shleifer and Thaler 1991, Jegadeesh and Titman 1993, Swaminathan 1991, Chan, Jegadeesh and Lakonishok 1996).
the lowest shares of educated labor force, it is unlikely to explain why there are differences among countries where the share of educated labor force is similar to that of developed countries (e.g. transition countries). In this paper, low productivity of the labor force, would also reduce the probability of fast technology adoption. However, it is shown how the speed of technology adoption can differ in countries with a similar labor force, because of the differences in the quality of information.

Obstacles to fast technology adoption can also arise from credit constraints (e.g. Gertler and Rogoff 1990, Aghion, Bacchetta and Banerjee 2004, Aghion, Comin and Howitt 2006). To emphasize the role of the equity market in providing exit opportunities, rather than access to funding, the paper abstracts from credit constraints. Credit constraints of local agents are unlikely to explain, for example, why foreign venture capitalists do not invest more in less developed countries with relatively skilled and inexpensive labor.

Closer to this paper are Bencivenga, Smith and Starr (1995) and Levine (1991) who show that the lack of liquidity in the equity market reduces the incentives to invest in technology adoption. However, explicit modeling of the equity market in the current paper allows me to isolate the negative effect of lack of liquidity from the non-monotonic effects that arise from imperfect information. As a result, this paper adds new mechanisms. Furthermore, to the extent that development of equity markets is likely to lead to both improvements in liquidity and in the quality of information, an innovative result in this paper is a non-monotonic relationship between equity market development and investments in technology. This is in contrast to the monotonic relationship suggested by the aforementioned papers.

This paper is also related to the literature on institutions (e.g. Parente and Prescott 1994), which assumes that weaker institutions increase the cost of technology adoption and imply slower technology adoption. Marimon and Quadrini (2006) model more specific frictions such as the interaction between start-up cost and limited contract enforceability that affects the incentives for new entries to the innovation sector. While additional institutional frictions (e.g. property rights, taxation, or other obstacles to establishing or running a firm) could be incorporated in the model, the two main forces found would still remain important.

Finally, the mechanisms discussed in this paper could apply to investments in general. This paper focuses on incentives for investments in technology for the following reasons. First, investment in technology is a driver of long-term growth (e.g. Romer 1990, Aghion and Howitt 1992) and is therefore likely to have a larger aggregate impact. Second, these investments are likely to require higher entrepreneurial skills and thus potential efficiency gains from ownership transfers are higher. Third, as venture capital has been shown to be a major source of funding for technology firms (see Kortum and Lerner 2000), the importance of good exit opportunities is likely to be more important for investments in technology than
investment in capital. Consistent with this, Appendix A shows that R&D expenditures are more strongly correlated with equity market development than investments.

The remainder of this paper is organized as follows. Section 2 presents the basic model, where all equity market participants are uninformed and the market is liquid. Section 3 presents a number of extensions. It shows the effects arising from lack of liquidity in the equity market, the presence of informed equity market participants and endogenous entry to entrepreneurship. Section 4 discusses empirically testable implications of the model and presents some suggestive evidence from existing empirical literature that is consistent with the model. Section 5 concludes.

2 The model

The model is a small open economy general equilibrium model with rational expectations. It builds on the endogenous growth literature with quality improvements of technology (e.g. Aghion and Howitt 1992, Aghion et al. 2006) and the rational expectations literature (e.g. Grossman 1976, Allen et al. 2006).

2.1 The setup

2.1.1 Consumers

The local economy is populated with overlapping generations of rational agents endowed with one unit of raw labor in each period. These agents work and invest in asset markets in the first period of their lives and consume only in the second period of their lives. The measure of local rational agents is \( \mu \). There is a measure \( \tilde{\mu} = \mu \) of similar overlapping generations of foreign agents endowed with exogenous wealth \( W^* \) in each period investing in local assets. All agents investing in the asset market are identical and called "investors".

In addition, some rational agents have special skills to be "entrepreneurs" and establish local monopolistic firms engaging in fast technology adoption.

All agents have mean-variance preferences

\[
U_t = E[c_{t+1}|\Omega_t] - \frac{\gamma}{2} \text{Var} \left( c_{t+1}|\Omega_t \right),
\]

where \( c_{t+1} \) is consumption, \( \Omega_t \) is the available information set in \( t \) and \( \gamma \) measures the extent of risk aversion.

None of the agents is borrowing or short-sales constrained. The assets traded are local equity and a foreign risk-free bond with a gross return normalized to one available with
ininitely elastic supply\textsuperscript{6}. The equity market consists of the shares of \( j \) local monopolistic firms that engage in technology adoption.

### 2.1.2 Final good production

The production side of the economy consists of a competitive final good production sector and an intermediate goods sector that also engages in technology adoption.

The price of the final good is normalized to one. The final good producers use raw local labor, \( L \), and \( j \) distinct intermediate goods. Each of these intermediate goods, \( x_t(j) \), is of quality \( A_t(j) \) \((j \in [0, 1])\). For example, the intermediate good, \( x_t(j) \), could be a computer designed to perform a particular task in the production line and the vintage of the computer, \( A_t(j) \), would determine how fast it will perform the task. Final good producers take the price of intermediate goods, \( p_{x,t}(j) \), and wages, \( w_t \), as given and solve

\[
\max_{L, x_t(j)} Y_t - w_t L - \int_0^1 p_{x,t}(j)x_t(j) dj, \tag{2}
\]

where the production function has constant returns to scale,

\[
Y_t = (\phi_t L)^{1-\alpha} \int_0^1 A_t^{1-\alpha}(j)x_t^\alpha(j) dj \tag{3}
\]

and \( \phi_t \) measures the productivity of the local labor force in using the technology.

This productivity is uncertain before actual production takes place (i.e. uncertainty about \( \phi_t \) resolves in period \( t \)) and can be decomposed into two parts

\[
\phi_t = \theta_t + u_t, \tag{4}
\]

where \( \theta_t \) is the explainable part of productivity that is uncorrelated across time and with any other shocks, and \( u_t \) is a mean zero unexplainable (i.e. pure noise) part of productivity that is uncorrelated with \( \theta_t \) and also uncorrelated across time and with any other shocks. The explainable part of productivity measures factors such as the quality of labor force, working and management culture, general institutional framework etc. The unexplainable part of productivity captures events that can be due, for example, to natural disasters, sudden disruptions in the production process and the general degree of uncertainty in the economy. The paper studies different distributional assumptions about these variables that will be specified in Sections 2.3, 3.1 and 3.2.

\textsuperscript{6}Allowing for a gross interest rate higher than one does not change the main results of the model.
2.1.3 Intermediate good production

The final good producer buys each intermediate good, \( x_t(j) \), from the local intermediate goods producers. Intermediate good producers in each sector \( j \) use one unit of final good to produce one unit of intermediate good.

The quality of intermediate goods, \( A_t(j) \), and competition in the intermediate goods sector depend on the investment decision of an entrepreneur in sector \( j \) two periods earlier (i.e. in period \( t - 2 \)). If such entrepreneur invested in fast technology adoption (or innovation) in \( t - 2 \), the quality of intermediate goods in period \( t \) is at the level of the current frontier, i.e. \( A_t(j) = A_t^* \). Old technology can be costlessly copied and slow technology adoption implies \( A_t(j) = A^*_{t-1} \). The technology adoption decision in period \( \tau \) is denoted by

\[
\tilde{1}_\tau(j) = \begin{cases} 
1 & \text{if entrepreneur invests in new technology in } \tau \text{ in sector } j \\
0 & \text{otherwise.} 
\end{cases} 
\] (5)

Fast technology adoption in period \( t - 2 \) gives intermediate goods producers monopolistic power\(^7\) in period \( t \). From period \( t + 1 \) onwards, the technological leader of period \( t \) is either overtaken by a new incumbent or free entry of firms adopting the old technology drives its profits to zero. Intermediate goods sector solves

\[
\begin{align*}
\max_{p_{x, t}(j), x_t(j)} & \quad \pi_t(j) = p_{x, t}(j)x_t(j) - x_t(j) \quad \text{st.} \quad p_{x, t}(j) = \frac{\partial y_t}{\partial x_t(j)} \text{ if } \tilde{1}_t-2(j) = 1 \\
\max_{x_t(j)} & \quad \pi_t(j) = p_{x, t}(j)x_t(j) - x_t(j) = 0 \text{ if } \tilde{1}_t-2(j) = 0.
\end{align*}
\] (6)

All intermediate goods depreciate fully in one period. Section 2.2 will show how uncertainty about the productivity of the labor force (\( \phi_t \)) translates into uncertainty about the future demand for intermediate goods and the profits of local monopolists.\(^8\)

Finally, assume that the frontier technology (\( A_t^* \)) that can be adopted (or invented) grows at an exogenous rate,

\[
g^* \equiv \frac{A^*_{t+1} - A_t^*}{A_t^*} \text{ for any } t
\] (7)
and the growth rate of technology is sufficiently high

\[
1 + g^* > \alpha^{-\frac{\alpha}{1-\alpha}} > 1.
\] (8)

\(^7\)Monopolistic power is justified by patent protection or by the fact that it takes time before copying the newest technologies becomes possible.

\(^8\)Differentiated intermediate goods are introduced only to justify the monopolistic power of the intermediate goods sector, which is necessary for entrepreneurs to have incentives to invest in fast technology adoption. As the uncertainty considered is aggregate, Section 2.2 shows that all firms are identical. Allowing for idiosyncratic uncertainty would complicate the model without eliminating the main mechanisms.
This condition guarantees that final goods sector always prefers to buy machines with a frontier technology ($A_t^*$) from a monopolist to buying cheaper machines from competitive intermediate goods producers who produce machines with old technology ($A_{t-1}^*$).

### 2.1.4 Technology adoption and information asymmetry

Each period $t$, an entrepreneur in sector $j$ decides whether to invest in fast technology adoption and establish a local monopolistic firm that is active in period $t + 2$.

The basic model assumes that for each intermediate good $j$, there is only one talented entrepreneur, whose effort is needed for technology adoption and who knows the explainable part of productivity in the final goods sector, $\theta_{t+2}$. Given that the entrepreneur must retire before his firm produces profits, he sells his firm in the equity market. This assumption captures the need for exit and ownership transfers. Each firm has one divisible share.

In addition to entrepreneur’s effort, fast technology adoption requires paying a fixed cost in final goods. The fixed cost of establishing a fast adopting firm is

$$I_t = \zeta A_{t+2}^*.$$  

The cost of fast technology adoption is assumed to be proportional to the quality of technology in the period in which the firm will be active and $\zeta$ measures how expensive fast technology adoption is.

If there is any cost for entrepreneurs born in $t$ to establish a firm that produces intermediate goods with quality $A_{t+1}^*$ in period $t + 2$, entrepreneurs never establish such firms. This is because the profit of such firm (6) and therefore its equity market value is always zero.

From (1), investment in fast technology adoption is optimal if

$$E[P_{t+1}(j)|\theta_{t+2}] - \gamma \frac{\gamma}{2} \text{Var}(P_{t+1}(j)|\theta_{t+2}) \geq I_t,$$

where $P_{t+1}(j)$ is the price of the share of firm $j$ is period $t + 1$.

As the main goal of this paper is to illustrate the importance of information asymmetry between entrepreneurs and investors, the basic model assumes that all investors trading in

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9 More specifically, this is a necessary and sufficient condition for none of the final goods producers to deviate and buy older technology. A firm that deviates, cannot afford to pay equilibrium wages to its workers. The proof is available upon request. As $0 < \alpha < 1 \Rightarrow \alpha - \frac{t\alpha}{1-t} > 1$

10 This assumption is relaxed in Section 3.3 that considers endogenous entry to entrepreneurship.

11 All entrepreneurs are also investors in the equity market. However, while entrepreneurs in $t$ care about $\phi_{t+2}$, investors care about $\phi_{t+1}$. Under CARA type utility, no borrowing or short-sales constraints and lack of serial correlation between shocks, the trading and technology adoption decisions are independent and investors and entrepreneurs can be seen as separate agents.
Figure 2: Timeline of events

period \( t + 1 \) are uninformed\(^{12}\) and obtain a noisy signal, \( \tilde{\theta}_{t+2} \), about the explainable part of productivity (\( \theta_{t+2} \)). They also observe the technology adoption decisions made one period earlier, \( \tilde{I}_t(j) \). The information set of uninformed investors is \( \Omega^U_{t+1} = \{ \tilde{\theta}_{t+2}, \tilde{I}_t(j); j \in [0, 1] \} \). While entrepreneurs born in \( t \) know \( \theta_{t+2} \), they do not know \( \tilde{\theta}_{t+2} \). Figure 2 summarizes the main mechanism and timing of the events.

\[ C_t + \int_0^1 x_t(j) dj + \int_0^1 \tilde{I}_t(j) I_t(j) dj = F_t + Y_t. \]  \hspace{1cm} (11)

\(^{12}\)Section 3.2 extends the analysis to consider the possibility that some investors are informed.
2.2 Identical profits and technology adoption decisions

If entrepreneurs invest in fast technology adoption in period \( t \), \( \tilde{I}_t(j) = 1 \), the optimal solution for the final goods sector (2) and intermediate goods sector (6) implies that the demand for an intermediate good is linear in labor productivity and quality of technology,

\[
x_{t+2}(j) = \left(\alpha^2\right)^{\frac{1}{\alpha}} \phi_{t+2} L A_{t+2}^* \tag{12}
\]

and the equilibrium profit in any sector \( j \) is

\[
\pi_{t+2}(j) = \pi_{t+2} = \Gamma A_{t+2}^* (\theta_{t+2} + u_{t+2}); \quad \Gamma \equiv \frac{1 - \alpha}{\alpha} \left(\alpha^2\right)^{\frac{1}{\alpha}} L. \tag{13}
\]

Because profits are the same across firms, all entrepreneurs make identical choices, i.e. \( \tilde{I}_t(j) = \tilde{I}_t \) for any \( j \). As a result, there is a continuum of monopolistic firms whose profits are perfectly correlated.

If \( \tilde{I}_t = 1 \), all these firms are traded in the equity market and modeling all these firms and their owners is equivalent to modeling one risky asset and one entrepreneur. Therefore \( P_{t+1}(j) = P_{t+1} \). If \( \tilde{I}_t = 0 \), the supply of equity in period \( t+1 \) is zero and consumers invest all their wage income (or wealth) in risk-free asset.

2.3 Equity prices

2.3.1 General results

The optimal technology adoption decision in period \( t \) can be solved backwards by first finding the equilibrium equity price in \( t+1 \) and then substituting it in the optimality condition for fast technology adoption (see equation (10)).

The basic model assumes that all investors (local and foreign) trading in the equity market are identical. Maximizing utility \( (1) \) subject to the individual investor’s budget constraint

\[
\hat{c}_{t+2} = [\pi_{t+2} - P_{t+1}] \hat{h}_t + \hat{W}_{t+1}; \quad \hat{W}_{t+1} \in \{W^*, w_{t+1}\}
\]

yields the optimal equity demand \( \hat{h}_t \) as

\[
\hat{h}_t = \frac{E \left[ \pi_{t+2} | \Omega_{t+1}^U \right] - P_{t+1}}{\gamma \text{Var} \left[ \pi_{t+2} | \Omega_{t+1}^U \right]}, \tag{14}
\]

where \( \Omega_{t+1}^U \) denotes the information set available to investors in trading period \( t+1 \). Using the worldwide equity market clearing condition, \( \mu \hat{h}_t + (\tilde{\mu} - \mu) \hat{h}_t = \tilde{\mu} \hat{h}_t = 1 \), the equilibrium
equity price becomes

\[ P_{t+1} = E [\pi_{t+2}\Omega_{t+1}] - \frac{\gamma}{\mu} \text{Var} [\pi_{t+2}\Omega_{t+1}] \]. \hspace{1cm} (15)

If the equity market is perfect, defined as no asymmetric information between entrepreneurs and investors (i.e. investors know \( \theta_{t+2} \)) and a liquid market (i.e. \( \mu \rightarrow \infty \)), it is clear from (13) and (15) that the equilibrium equity price is

\[ P^P_{t+1} = \Gamma A^*_{t+2} \theta_{t+2}. \] \hspace{1cm} (16)

### 2.3.2 Equity prices in liquid market

Given that this paper analyzes a small open economy, it is reasonable to assume that the number of uninformed foreign investors who can invest in the local equity is large as compared to the size of the local market. Therefore, assume a liquid market, where \( \mu \rightarrow \infty \).

Using then (13), (15) and \( E[u_{t+2}\Omega^U_{t+1}] = 0 \), the equilibrium equity price is:

\[ P_{t+1} = \Gamma A^*_{t+2} E [\theta_{t+2}\Omega^U_{t+1}]. \] \hspace{1cm} (17)

When the number of uninformed investors approaches infinity, the amount of equity each of them holds approaches zero. This forces the equilibrium risk premium, \( \frac{\gamma}{\mu} \text{Var} [\pi_{t+2}\Omega^U_{t+1}] \) to zero and the equity price corresponds to conditional expectations of uninformed investors.

Investors obtain a noisy public signal \( \tilde{\theta}_{t+2} = \theta_{t+2} + \epsilon_{t+2} \), where \( \epsilon_{t+2} \) is uniformly distributed in the interval \( \left[ -\frac{1}{\beta^\epsilon}, \frac{1}{\beta^\epsilon} \right] \) and \( \beta^\epsilon \) is a measure of the quality of information that investors have. They also know entrepreneurs’ technology adoption decision. Given that entrepreneurs have superior information (i.e. they know \( \theta_{t+2} \)), we can conjecture that their decision to invest in fast technology adoption, \( \tilde{I}_t = 1 \), implies that \( \theta_{t+2} \geq \tilde{\theta}_{t+2} \). This conjecture is verified in Section 2.4 where it is also shown that the threshold (\( \tilde{\theta}_{t+2} \)) is known to uninformed investors trading in \( t+1 \). Hence investors information set \( \Omega^U_{t+1} = \{ \tilde{\theta}_{t+2}, \theta_{t+2} \geq \tilde{\theta}_{t+2} \} \)

The technology adoption decision reveals information about \( \theta_{t+2} \) whenever \( \tilde{\theta}_{t+2} - \tilde{\theta}_{t+2} \leq \frac{1}{\beta^\epsilon} \). In such case \( \epsilon_{t+2}\tilde{I}_t = 1 \) is uniformly distributed in the interval \( \left[ -\frac{1}{\beta^\epsilon}, \tilde{\theta}_{t+2} - \tilde{\theta}_{t+2} \right] \) and expected productivity (and profit) is higher compared to the case where technology adoption decision is not informative \( (\tilde{\theta}_{t+2} - \tilde{\theta}_{t+2} > \frac{1}{\beta^\epsilon}) \), i.e.

\[ E[\theta_{t+2}\Omega_{t+1}] = \begin{cases} \tilde{\theta}_{t+2} + \frac{1}{\beta^\epsilon} \left( \frac{1}{\beta^\epsilon} - (\tilde{\theta}_{t+2} - \tilde{\theta}_{t+2}) \right) & \text{if } \tilde{\theta}_{t+2} - \tilde{\theta}_{t+2} \leq \frac{1}{\beta^\epsilon} \\ \tilde{\theta}_{t+2} & \text{if } \tilde{\theta}_{t+2} - \tilde{\theta}_{t+2} > \frac{1}{\beta^\epsilon} \end{cases}. \] \hspace{1cm} (18)

12
The condition $\tilde{\theta}_{t+2} - \tilde{\theta}_{t+2} \leq \frac{1}{\beta_\epsilon}$ can be written as $\epsilon_{t+2} \leq \bar{\omega}_{t+2} \equiv \frac{1}{\beta_\epsilon} - (\theta_{t+2} - \bar{\theta}_{t+2})$ and equilibrium equity price is:

$$P_{t+1} = \begin{cases} \Gamma A_{t+2}^* (\theta_{t+2} + \epsilon_{t+2} + \frac{1}{2} (\bar{\omega}_{t+2} - \epsilon_{t+2})) & \text{if } \epsilon_{t+2} \leq \bar{\omega}_{t+2} \\ \Gamma A_{t+2}^* (\theta_{t+2} + \epsilon_{t+2}) & \text{if } \epsilon_{t+2} > \bar{\omega}_{t+2} \end{cases}.$$  

Equity prices can deviate from the perfect equity market benchmark (16) for two reasons. First, the public signal investors receive can be incorrect, i.e. $\epsilon_{t+2} \neq 0$. Second, when investment in technology is informative ($\epsilon_{t+2} \leq \bar{\omega}_{t+2}$), entrepreneur’s decision to invest in fast technology adoption increases the market value of his firm as it implies that the explainable part of productivity $\theta_{t+2}$ is not too low. The main results in this paper arise from these two effects.

As a final note, the technology adoption choice is potentially uninformative in this setting only because of the distributional assumptions about the public signals. As the upper bound of the support of the uniform distribution is finite, investors who receive a very high public signal ($\tilde{\theta}_{t+2}$) know with certainty that entrepreneurs would have invested in the technology even if $\theta_{t+2}$ is at its lowest possible value. If such case is very unlikely, as in the case of normal distribution analyzed in Section 3.2, the technology adoption decision is always informative.

### 2.4 Technology adoption decision

Entrepreneurs’ technology adoption decision in period $t$ is based on their knowledge of the explainable part of productivity, $\theta_{t+2}$. There is uncertainty about the asset price in period $t+1$, because entrepreneurs do not know $\tilde{\theta}_{t+2}$ (and $\epsilon_{t+2}$).

Given (10) and results in Section 2.2, investment in technology pays off if

$$E [P_{t+1}|\theta_{t+2}] - \frac{\gamma}{2} \text{Var}(P_{t+1}|\theta_{t+2}) \geq \zeta A_{t+2}^*.$$  

**Proposition 1** Entrepreneurs choose to adopt technology fast ($A_{t+2} = A_{t+2}^*$) if the observable component of productivity satisfies $\theta_{t+2} \geq \tilde{\theta}_{t+2}$, where

$$\tilde{\theta}_{t+2} = \frac{\zeta}{\Gamma} - \frac{1}{2\beta_\epsilon} + \frac{\gamma \Gamma A_{t+2}^*}{24\beta_\epsilon^2}.$$  

**Proof.** Let us evaluate (20) it in the neighborhood of the threshold, i.e. $\theta_{t+2} = \tilde{\theta}_{t+2} + \Delta \theta_{t+2}$, where $\Delta \theta_{t+2} \to 0$\(^{13}\). In such case, fast technology adoption decision is always

\(^{13}\)The proof for the general case $-\frac{2}{\beta_\theta} < \Delta \theta_{t+2} < \frac{2}{\beta_\theta}$ is at author’s website and yields exactly the same threshold (21).
informative, because \( \epsilon_{t+2} \leq \omega_{t+2} \approx \frac{1}{\beta_t} \). Using (19), this implies that \( E[P_{t+1} | \theta_{t+2}] = \Gamma A_{t+2}^* \left( \theta_{t+2} + \frac{1}{2\beta_t} + \frac{1}{2} E[\epsilon_{t+2}] \right) = \Gamma A_{t+2}^* \left( \theta_{t+2} + \frac{1}{2\beta_t} \right), \) \( \text{Var}(P_{t+1} | \theta_{t+2}) = \Gamma^2 A_{t+2}^* \frac{1}{\beta_t} \text{Var}(\epsilon_{t+2}) = \Gamma^2 A_{t+2}^* \frac{1}{\beta_t^2}. \) Replacing these in (20) and rearranging proves the proposition. \( \blacksquare \)

It can be seen from above that the threshold depends on the variables and constants that are observable by all agents. Therefore, investors trading in period \( t+1 \) know the value of \( \bar{\theta}_{t+2} \). Proposition 1 thereby verifies the conjecture in Section 2.3.2.

**Corollary 1.1** In perfect equity market, the threshold simplifies to

\[
\bar{\theta}_{t+2}^P = \frac{\zeta}{\Gamma}. \tag{22}
\]

**Proof.** Using (16) in (20) gives (22). An alternative way to prove it uses the fact that in perfect equity market \( \beta_t \rightarrow \infty \). Using (21) \( \lim_{\beta_t \rightarrow \infty} \bar{\theta}_{t+2} = \frac{\zeta}{\Gamma}. \blacksquare \)

As long as information in equity market is not perfect, there are two opposite forces that affect the adoption decision: "fear of unstable markets" and "adoption to signal".

The "fear of unstable markets" force is captured by the term \( \frac{\Gamma A_{t+2}^*}{24\beta_t^4} \) in (21). Uncertainty about the price on exit has a negative effect on risk averse agents' incentives to invest in the frontier technology. This force arises from the possibility that the equity price deviates from the perfect equity market benchmark due to an error in the public signal (i.e. the first effect discussed at the end of Section 2.3.2). This force is strongest in underdeveloped equity markets, where quality of information \( (\beta_t) \) is low. The magnitude of this force monotonically decreases with improvements in the quality of information (i.e. \( \frac{\partial \Gamma A_{t+2}^*}{24\beta_t^4} / \partial \beta_t < 0 \)).

The "adoption to signal" term is captured by \( \frac{1}{2\beta_t} \) in (21). Entrepreneurs know that uninformed investors will take fast adoption as an indication of higher profitability and are willing to pay a higher price for it (19). Therefore, technology investment decision becomes a natural signal that increases entrepreneurs' incentives to invest in fast technology adoption (i.e. the second effect discussed at the end of Section 2.3.2). The possibility of gains from this remains despite the fact that uninformed investors are rational and aware of the force. This force is also strongest in underdeveloped equity markets, because the informativeness of the technology adoption decision in such markets is higher. It decreases with improvements in the quality of information (i.e. \( \frac{\partial \frac{1}{2\beta_t}}{\partial \beta_t} < 0 \)).

\( ^{14} \)More specifically, let us call \( E_{PL} \equiv E[P_{t+1} | \theta_{t+2}, \epsilon_{t+2} \leq \omega_{t+2}], \) \( E_{PH} \equiv E[P_{t+1} | \theta_{t+2}, \epsilon_{t+2} \geq \omega_{t+2}], \) \( V_{PL} \equiv \text{Var}(P_{t+1} | \theta_{t+2}, \epsilon_{t+2} \leq \omega_{t+2}), \) \( V_{PH} \equiv \text{Var}(P_{t+1} | \theta_{t+2}, \epsilon_{t+2} > \omega_{t+2}) \) and \( q \equiv \text{Pr}(\epsilon_{t+2} > \omega_{t+2}). \) Then using (19), the law of total expectations implies \( E[P_{t+1} | \theta_{t+2}] = E_{PL} + q(E_{PH} - E_{PL}) \) and \( \text{Var}(P_{t+1} | \theta_{t+2}) = V_{PL} + q(V_{PH} - V_{PL}) + p(1 - q)(E_{PH} - E_{PL})^2. \) Given that when \( \Delta \theta \rightarrow 0 \) \( p = \frac{\beta_t(\theta - \theta)}{2} \rightarrow 0 \) and \( \omega \rightarrow \frac{1}{\beta_t}, \) it is clear that \( E[P_{t+1} | \theta_{t+2}] = E_{PL} \approx E \left[ P_{t+1} | \theta_{t+2}, \epsilon_{t+2} \leq \frac{1}{\beta_t} \right] \) and \( \text{Var}(P_{t+1} | \theta_{t+2}) = V_{PL} \approx \text{Var} \left( P_{t+1} | \theta_{t+2}, \epsilon_{t+2} \leq \frac{1}{\beta_t} \right). \)
Corollary 1.2 The probability of fast technology adoption is higher than in perfect equity market, if quality of information is above a finite threshold

$$\beta_\epsilon \geq \bar{\beta}_\epsilon \equiv \frac{\gamma \Gamma A_t^{t+2}}{12}. \tag{23}$$

Proof. For any prior distribution of $\theta_{t+2}$, probability of fast technology adoption is higher than in perfect equity market iff $\bar{\theta}_{t+2} \leq \bar{\theta}^P_{t+2}$. Using (21) and (22) this implies $\frac{1}{2\beta_\epsilon} \geq \frac{\gamma \Gamma A_t^{t+2}}{24\beta_\epsilon^3}$. Simplifying this proves the corollary.

Corollary 1.2 has some interesting implications. While both forces decrease with the development of the equity market (increase of $\beta_\epsilon$), the "fear of unstable markets" tends to be relatively stronger compared to the "adoption to signal" force when the quality of information is low. Therefore, the model suggests that countries with underdeveloped equity markets are more likely to adopt technology slowly.

"Adoption to signal" force is relatively stronger, when quality of information is high. However, as the magnitude of both forces decreases with the quality of information, fast technology adoption is most likely when the quality of information is at an intermediate level. If the quality of information is very high, the potential gains are negligible.

To formalize the argument, suppose that a local policy maker’s objective is to maximize the probability of the fast technology adoption. As it will be shown in Section 2.5, such objective also maximizes output and wages in period $t+2$ and therefore increases the consumption of local agents benefiting from this. Maximizing the probability of fast technology adoption is equivalent to minimizing the threshold for fast technology adoption, i.e.

$$\beta^{opt}_\epsilon = \arg \min_{\beta_\epsilon} (\bar{\theta}_{t+2}), \tag{24}$$

where $\bar{\theta}_{t+2}$ is given by (21).

Corollary 1.3 If a policy maker has full control over the quality of information, he will set the quality of information at

$$\beta^{opt}_\epsilon = \frac{\gamma \Gamma A_t^{t+2}}{6}. \tag{25}$$

Proof. The proof is straightforward from (24).

This corollary suggests that the local policy maker does not choose full transparency ($\beta^{opt}_\epsilon \to \infty$). This is due to the "adoption to signal" force that enables fast technology adoption at a lower level of productivity than what would be possible in perfect equity market. It is important to point out that the seemingly suboptimal policy encouraging potentially "too fast" technology adoption is justified because the policy maker is local.
Only local agents that lose from "too fast" technology adoption are local investors. As local investors hold only a negligible share of local equity (namely they hold \(\mu h \approx 0\), given that \(\mu\) is finite, (13), (14) and (17), the additional opportunities of fast technology come at the expense of losses of foreign investors.\(^{15}\)

Figure 3 summarizes the main findings in this section. The further implications of Corollary 1.2 and 1.3 are the following. High risk aversion (\(\gamma\)) magnifies the "fear of unstable markets" force and therefore increases the minimum quality of information that guarantees that the "adoption to signal" force dominates the "fear of unstable markets" force. It also makes the optimal quality of information higher.

It is clear from (22) that if productivity were to remain constant at some \(\theta \geq \bar{\theta}\), a country can always keep up with fast technology adoption under perfect equity market. In imperfect equity markets, the impact of "fear of unstable markets" will increase with the level of technology, \(A_t^*+2\) (see equation (23)). This is because profits (13) and variance of profits are higher at higher technology levels. This implies, that there is a tendency towards persistently slow technology adoption over time, i.e. failing to adopt fast in one period makes it less likely to adopt fast in the following periods. To offset such tendency, policy makers would aim for higher transparency over time as it can be seen from (25).

If, in addition, we assume that the cost of fast technology adoption is an increasing function of the distance to the frontier as in Aghion et al. (2006) (e.g. \(\zeta = \bar{\zeta} \left( \frac{A_{t+1}}{A_t} \right)\), \(\zeta'() > 0\), makes the tendency towards persistently slow technology adoption stronger. It

\(^{15}\)Furthermore, even the policy maker would be "global", there are potential gains from encouraging faster technology adoption because of monopolistic distortions. Due to monopolistic distortions, social benefits of better technology are higher than private benefits.
is clear from (21) that failing to adopt technology fast once will also make it more costly to adopt fast in the following period.

Proposition 1 also implies that participation of foreign investors, who might be able to adopt frontier technology for a lower cost, i.e. \( \zeta^* < \zeta \), would increase the probability of fast technology adoption in a discrete manner. However, they would be affected by the same forces. Therefore, foreign venture capitalists or other foreign agents would not participate in technology adoption in countries where quality of information is low (underdeveloped equity markets). The model suggests that this outcome does not require these countries to have unskilled labor force or any other institutional frictions that reduce the productivity, \( \theta_t \).

2.5 Aggregate output, wages and local goods market clearing

The demand for intermediate goods in the case of fast technology adoption is given by (12). If technology is adopted slowly, the optimization problems of final good sector (2) and intermediate goods sector (6) implies

\[
x_t = \alpha^{\frac{1}{1-\alpha}} \phi_t L A^*_t - 1.
\]

Replacing the labor market clearing condition, \( \mu = L \), and demand for intermediate capital goods, (12) and (26) in the production function (3), the aggregate final good production is increasing in the level of technology and the productivity of the labor force:

\[
Y_t = \bar{I}_{t-2} \alpha^{\frac{2\mu}{1-\mu}} L A^*_t \phi_t + (1 - \bar{I}_{t-2}) \alpha^{\frac{\mu}{1-\mu}} L A^*_{t-1} \phi_t.
\]

(27)

From the first order condition of (2), the equilibrium wages are proportional to aggregate final good production:

\[
w_t = (1 - \alpha) \frac{Y_t}{\mu}.
\]

(28)

Under the assumption about the growth rate (8), output (and wages) are always higher when technology is adopted fast (i.e. \( \bar{I}_t = 1 \)). A violation of (8) would mean that there is no demand from the final good sector for intermediate goods with quality \( A^*_t \) in period \( t \).

To illustrate the effect of technology choices on growth rate, assume for a moment that the realized productivity stays constant \( \phi_t = \phi \). If technology is always adopted either slowly or fast, the growth rate of output \( (g_{y,t} = Y_t/Y_{t-1} - 1) \) equals to the growth rate of technology \( g_{y,t} = g^* \). Condition (8) also implies that if technology is adopted fast in period \( t \) and slowly in period \( t + 1 \), the growth rate is lower, i.e. \( g_{y,t} = \alpha^{\frac{\mu}{1-\mu}} - 1 < g^* \). If technology is adopted slowly in period \( t \) and fast in period \( t + 1 \), the growth rate is higher,
i.e. \( g_{y,t} = \alpha \frac{\sigma^2}{\eta} (1 + g^*) (1 + g^*) - 1 > g^* \). There is growth in the case of a switch from fast to slow technology adoption because of the assumption about competition in intermediate goods sector. In such a case, technology adopting firms lose monopolistic power and the deadweight loss caused by this, disappears in one period.

Finally, Appendix B confirms that local goods market clears every period, which completes the model.

3 Extensions

The basic model in Section 2 emphasized the importance of two main forces, "fear of unstable markets" and "adoption to signal", and the implied non-monotonic relationship between the quality of information in equity markets and technology investments (and output). This section demonstrates that these findings are robust, when considering a less restrictive framework, where the equity market is not perfectly liquid (Section 3.1), or some investors are as well informed as entrepreneurs (Section 3.2). The basic model also assumed that there is only one talented entrepreneur in each sector. It will be shown that free entry and competition for better quality of technology among entrepreneurs, reinforces the aggregate effect of the main forces (Section 3.3).

3.1 Illiquid equity market

While the assumption that the number of potential investors is large is reasonable for small open economies that have functioning local equity markets, it may not be realistic in all cases. For example, countries may use restrictions on foreign portfolio investments, or the trading costs may be high, or the local equity market may be too underdeveloped to attract enough foreign investors. In such cases, it is reasonable to consider the possibility that equity markets are not liquid (i.e. the number of investors \( \hat{\mu} \) is finite). The negative effect of lack of liquidity in the equity market has been identified by Bencivenga et al. (1995) and Levine (1991), who do not explicitly model the equity market. By explicitly modelling the equity market, this effect can be isolated from the effects that arise from information asymmetry between investors and entrepreneurs.

As in Section 2.3.2., information set of investors is \( \Omega_{t+1} = \{ \theta_{t+2}, \theta_{t+2} \geq \tilde{\theta}_{t+2} \} \). From (15), the equity price is affected by the risk premium \( \tilde{\mu} \text{Var}[\pi_{t+2} | \Omega_{t+1}] \). As profit (13) depends on the unexplainable component of productivity, \( u_{t+2} \), the variance of profits depends on the variance of \( u_{t+2} \). Assume that \( u_{t+2} \sim u \left[ \begin{array}{c} -\frac{1}{\beta_u}, \frac{1}{\beta_u} \end{array} \right] \). Using this, (13), (15), \( \tilde{\theta}_{t+2} = \theta_{t+2} + \epsilon_{t+2} \) and \( \omega_{t+2} = \frac{1}{\beta_e} - (\theta_{t+2} - \tilde{\theta}_{t+2}) \), the equilibrium equity price is
In addition to the two main effects (discussed in Section 2.3.2), that make equity prices to deviate from the perfect equity market benchmark (16), lack of liquidity tends to reduce equity prices because of the risk premium. If technology adoption decision is informative, there is a secondary effect in play: an error in public signal ($\epsilon_{t+2}$) moves the expected value and variance of profits in the same direction. From the point of view of an entrepreneur, this effect tends to reduce the variance of equity prices.

**Proposition 2** In an illiquid equity market entrepreneurs choose to adopt technology fast ($A_{t+2} = A_{t+2}^*$) if the observable component of productivity satisfies $\theta_{t+2} \geq \bar{\theta}_{t+2}^\mu$, where

$$\bar{\theta}_{t+2}^\mu = \frac{\zeta}{\Gamma} - \frac{1}{2\beta_\epsilon} + \frac{\gamma \Gamma A_{t+2}^*}{24\beta_\epsilon^2} \psi (\mu, \beta_\epsilon) + \frac{\gamma \Gamma A_{t+2}^*}{3\mu} \left[ \frac{1}{3\beta_\epsilon^2} + \frac{1}{\beta_u^2} \right],$$

where $\psi (\mu, \beta_\epsilon) \equiv \left[ \left( 1 - \frac{\gamma \Gamma A_{t+2}^*}{3\mu\beta_\epsilon} \right)^2 + \left( \frac{\gamma \Gamma A_{t+2}^*}{3\mu\beta_\epsilon} \right)^2 \frac{1}{15} \right] > 0$.

**Proof.** If $\theta_{t+2} = \bar{\theta}_{t+2} + \Delta \theta_{t+2}$ and $\Delta \theta_{t+2} \rightarrow \infty$, then $E [P_{t+1} | \theta_{t+2}] = \Gamma A_{t+2}^* \left( \theta_{t+2} + \frac{1}{2\beta_\epsilon} \right) - \frac{\gamma \Gamma A_{t+2}^*}{3\beta_\epsilon} \left[ \frac{1}{3\beta_\epsilon^2} + \frac{1}{\beta_u^2} \right]$ and $\text{Var} [P_{t+1} | \theta_{t+2}] = \frac{(\Gamma A_{t+2}^*)^2}{12\beta_\epsilon^2} \left[ \left( 1 - \frac{\gamma \Gamma A_{t+2}^*}{3\mu\beta_\epsilon} \right)^2 + \left( \frac{\gamma \Gamma A_{t+2}^*}{3\mu\beta_\epsilon} \right)^2 \frac{1}{15} \right]$. Replacing these in (20) and rearranging proves the proposition. ■

In an illiquid market three forces affect the incentives to invest in fast technology adoption. First, the "adoption to signal" force ($\frac{1}{2\beta_\epsilon}$) in an illiquid market is exactly the same as in the liquid market (21). As before, the opportunity to increase firm's market value, increases entrepreneur's incentives to invest.

Second, the discouraging "fear of unstable markets" force ($\frac{\gamma \Gamma A_{t+2}^*}{24\beta_\epsilon^2} \psi (\mu, \beta_\epsilon)$) can be stronger or weaker than in liquid equity market ($\frac{\gamma \Gamma A_{t+2}^*}{24\beta_\epsilon^2}$) depending on the values of $\mu$ and $\beta_\epsilon$. On the one hand, there is an additional source of uncertainty, which arises from the uncertainty about the risk premium ($\frac{\gamma}{\mu} \text{Var} [\pi_{t+2} | A_{t+1}]$). This tends to increase the magnitude of "fear on unstable markets". On the other hand, the positive correlation between the expected value and variance of profits, discussed above, reduces the variance of prices. We can see from

\footnote{See footnote 14. Calculating the variance uses the following relationship for any random variable $x$ and constants $a, b, c$: $\text{Var}(ax + bx^2 + c) = a^2 \text{Var}(x) + b^2 (E(x^4) - E^2(x^2)) + 2ab (E(x^3) - E(x)E(x^2))$. Notice also that $E (\varepsilon_{t+2}^4) = \frac{1}{5\beta_\epsilon^2}$ and $E (\varepsilon_{t+2}^3) = 0.$}
the definition of $\psi(\hat{\mu}, \beta_\epsilon)$ that the "fear of unstable markets" force tends to be stronger in illiquid market, if the market is highly illiquid (i.e. $\hat{\mu}$ is low).  

Third, there is a new force that can be called the \textbf{direct lack of liquidity} effect captured by the term $\frac{\gamma \Gamma A^*_t}{3\hat{\mu}} \left[ \frac{1}{3\beta^*_t} + \frac{1}{\beta^*_u} \right]$. Lack of liquidity has a negative effect on equity prices through the risk premium. The "direct lack of liquidity" effect is monotonically decreasing in the number of investors ($\hat{\mu}$) and the quality of information investors have ($\beta_\epsilon$). It is also the only force that is increasing in uncertainty about the unexplainable component of productivity ($\beta_u$). Other forces are not affected by $\beta_u$, because investors and entrepreneurs have exactly the same information about this component.

Without loss of generality, assume that $\frac{1}{\beta^*_u} = \rho \frac{1}{\beta^*_\epsilon} + 3\kappa$, where $\rho \geq 0$ and $\kappa \geq 0$. The first constant ($\rho$) measures the degree of interaction between the variance of explainable and unexplainable component of productivity. In a generally uncertain environment (low $\beta_u$), the degree of information asymmetry is likely to be higher (low $\beta_\epsilon$). The second constant ($\kappa$) measures the additional variance of unexplainable component of uncertainty that is not affected by the quality of information about the explainable component.

Consider the case, where there is no information asymmetry between entrepreneurs and investors, i.e. $\beta_\epsilon \rightarrow \infty$. From (30), the threshold in the case of no information asymmetry is

$$\bar{\theta}^\mu_{\epsilon \rightarrow \infty} = \lim_{\beta_\epsilon \rightarrow \infty} \bar{\theta}^\mu_{t+2} = \frac{\zeta}{\Gamma} + \frac{\gamma \Gamma A^*_t}{3\hat{\mu}} \kappa.$$  

(31)

It is higher than the threshold in perfect equity market (22) because of the negative effect of the risk premium on equity prices.

\textbf{Corollary 2.1} For any $\rho$ and $\kappa$, $\lim_{\beta_\epsilon \rightarrow \infty} \bar{\theta}^\mu_{t+2} \rightarrow \infty$ and there exists a finite $\bar{\beta}_\epsilon^\mu$ such that for any $\beta_\epsilon > \bar{\beta}_\epsilon^\mu$ the threshold for fast technology adoption $\bar{\theta}^\mu_{t+2} < \bar{\theta}^\mu_{t+2}$. If the sufficient condition $\rho \geq \frac{2}{3}$ holds, then $\bar{\beta}_\epsilon^\mu$ is unique such that for any $\beta_\epsilon \leq \bar{\beta}_\epsilon^\mu$, it holds that $\bar{\theta}^\mu_{t+2} \leq \bar{\theta}^\mu_{t+2}$ and $\bar{\beta}_\epsilon^\mu > \bar{\beta}_\epsilon$, where $\bar{\beta}_\epsilon$ is given by (23).

\textbf{Proof.} See Appendix C. \qed

Corollary 2.1 confirms that the non-monotonic relationship between the probability of fast technology adoption and quality of information in equity market is robust to the lack of liquidity.

If $\kappa = 0$, the threshold in the case of no information asymmetry (31) is the same as in perfect equity market (22). Therefore, technology adoption is more likely in imperfectly informed and illiquid market than in perfect market whenever $\beta_\epsilon > \bar{\beta}_\epsilon^\mu$. In such case, the

---

$^{17}$ $\psi(\hat{\mu}, \beta_\epsilon) > 1$ if and only if $\hat{\mu} < \frac{\gamma \Gamma A^*_t}{3\beta^*_\epsilon}$.  

$^{18}$ Notice that $\text{Var}(\epsilon_{t+2}) = \frac{1}{3\beta^*_t}$ and $\text{Var}(u_{t+2}) = \frac{1}{3\beta^*_u}$. Thus $\text{Var}(u_{t+2}) = \rho \text{Var}(\epsilon_{t+2}) + \kappa$.
positive "adoption to signal" force is stronger than both the "fear of unstable markets" force and the "direct lack of liquidity" effect together.

It can be seen from (31) that an increase of $\kappa$ reduces the the probability of fast technology adoption. If $\kappa$ is small, "adoption to signal" force is stronger than the negative forces for some range of $\beta_\epsilon$ in the interval $(\bar{\beta}_\epsilon^\mu, \infty)$. If $\kappa$ is large, "adoption to signal" never dominates both the "fear of unstable markets" force and the "direct lack of liquidity" effect.

Under the assumption that the variance of the unexplainable component of productivity is not too low compared to the explainable one, $\rho \geq \frac{2}{3}$, fast technology adoption in illiquid market is always less likely than in liquid market (because $\bar{\beta}_\epsilon^\mu > \bar{\beta}_\epsilon$).

Figure 4 illustrates this by comparing the probability of fast technology adoption in liquid and illiquid market. For the illiquid market, it plots two possibilities: first, the case where $\rho = 1$ and $\kappa = 0$ and second, the case where $\rho = 1$ and $\kappa > 0$.

Notice that $\rho \geq \frac{2}{3}$ is a sufficient but not a necessary condition for technology adoption in illiquid market to be less likely than in perfect market ($\bar{\beta}_\epsilon^\mu > \bar{\beta}_\epsilon$). This holds for a wider range of parameters $\kappa$ and $\rho$. It does not hold for all values of these parameters and $\hat{\mu}$ because of the presence of $\psi(\hat{\mu}, \beta_\epsilon)$ in (30). The ambiguity arises from the secondary effect through which the lack of liquidity can reduce the variance of prices (as discussed above, this is more likely if $\hat{\mu}$ is relatively high).

This section highlights the need to separate the effect of different improvements in equity market. In less developed equity markets, where both quality of information and liquidity are likely to be low, improvements in either of them increases the probability of fast technology adoption. In more developed equity markets, where both quality of information and liquidity
are likely to be relatively high, only further improvements in liquidity increase the probability of fast technology adoption.

### 3.2 Informed investors

The main model in Section 2 assumes that only entrepreneurs can be informed. In a more realistic setting, some investors are likely to be able and willing to acquire the same information. For example, local (institutional) investors or generally more sophisticated investors may face lower information costs compared to the average equity market participant. This section shows that the findings are robust to the presence of such investors.\(^{19}\)

Assume that there are \(\hat{\mu}^I\) informed investors trading in period \(t + 1\) equity market. The information set that is relevant for these investors is \(\Omega^I_{t+1} = \{\theta_{t+2}\}\).

The remaining \(\hat{\mu}^U = \hat{\mu} - \hat{\mu}^I\) investors are uninformed. These investors obtain a noisy signal, \(\tilde{\theta}_{t+2} = \theta_{t+2} + \epsilon_{t+2}\), where \(\epsilon_{t+2} \sim \mathcal{N}(0, \frac{1}{\beta_c})\). This is similar to Section 2 apart from the assumption that the public signal is now normally distributed. This allows to derive the equilibrium equity price analytically, which would not be possible when maintaining the assumption about uniform distribution. It will be shown shortly, that the distributional assumptions do not alter the main findings.

The existence of some informed investors in the equity market implies that uninformed investors obtain information about the productivity, \(\theta_{t+2}\), also from the equity price, \(P_{t+1}\). In order to prevent the equity price from being fully revealing (the Grossman and Stiglitz (1976) paradox), assume that in addition to the rational informed and uninformed investors, there are some noise traders who demand a random quantity \(s_{t+1} \sim \mathcal{N}(0, 1/\Gamma^2A^2_{t+2}\beta_s)\) of equity. Without loss of generality, it is assumed that noise traders do not receive wage income and do not have initial wealth\(^{20}\). The assumption that the variance of noise trading decreases with the quality of technology guarantees that the variance of the price signals uninformed investors receive does not increase over time\(^{21}\).

As in Section 2, it can be guessed and verified that \(\bar{I}_t = 1\) implies that \(\theta_{t+2} \geq \tilde{\theta}_{t+2}\). Therefore, the information set uninformed investors have is \(\Omega^U_{t+1} = \{\tilde{\theta}_{t+2}, P_{t+1}, \theta_{t+2} \geq \tilde{\theta}_{t+2}\}\).

Finally, assume that the unexplainable part of productivity \(u_{t+2} \sim \mathcal{N}(0, \frac{1}{\beta_u})\) and the variables \(\theta_{t+2}, \epsilon_{t+2}, s_{t+1}\) and \(u_{t+2}\) are uncorrelated with each other and over time.

---

\(^{19}\)The findings are also robust to endogenizing the number of informed investors. This extension is available at author’s website.

\(^{20}\)Given the CARA utility assumed, the split of wage income between noise traders and rational agents does not affect aggregate conditions and conclusions in the model.

\(^{21}\)Relaxing this assumption would tend to increase the tendency towards slow technology adoption over time discussed in Section 2.4. In such case, the quality of information uninformed investors have, would become worse over time and magnify the "fear of unstable markets" force.
The equilibrium equity price is derived in Appendix D. Focusing on the case where equity market is liquid in the sense that the number of uninformed investors is large, i.e. \( \hat{\mu}_t^U \to \infty \), it is shown that equity price equals to the expected profit by uninformed investors

\[
P_{t+1} = E \left[ \pi_{t+2} | \Omega_{t+1}^U \right] = \frac{\lambda b_{t+1}}{\sqrt{z_v}}.
\]

where

\[
z_v = \beta_\epsilon + \left( \frac{\hat{\mu}_t^U}{\gamma} \right)^2 \beta_s
\]

measures the quality of information uninformed investors have from both public and price signals, \( \bar{s}_{t+1} \equiv \frac{-\gamma A_{t+2}}{\mu^U \beta_\alpha s_{t+1}} \sim \mathcal{N} \left( 0, \left( \left( \frac{\hat{\mu}_t^U}{\gamma} \right)^2 \beta_s \right)^{-1} \right) \) is the error in price signals and \( \lambda b_{t+1} \) is the inverse Mills ratio that is always positive.\(^{22}\)

Comparing (32) with (19) shows similar effects to the ones discussed in Section 2.3.2. Equity prices can differ from those in perfect equity market (16) for two reasons. First, there can be errors in the signals investors receive (\( \epsilon_{t+2} \neq 0 \) and/or \( \bar{s}_{t+1} \neq 0 \)). The individual effect of these shocks depends on the relative precision of public and price signal. Second, entrepreneur’s decision to invest in fast technology adoption increases the market value of the firm. This is reflected in the presence of \( \lambda b_{t+1} > 0 \) in (32).

**Proposition 3** Entrepreneurs choose to adopt technology fast if the observable component of productivity satisfies

\[
\theta_{t+2} \geq \bar{\theta}_{t+2} \equiv \frac{\zeta}{\Gamma} - \frac{\Lambda_1}{\sqrt{z_v}} + \frac{\gamma A_{t+2}^* (1 - \Lambda_2)^2}{z_v},
\]

where \( \Lambda_1, \Lambda_2 > 0, \Lambda_2 < 1 \). The probability of fast technology adoption is higher than in perfect equity market if the quality of information is above a finite threshold

\[
z_v \geq \bar{z}_v \equiv \left( \frac{\gamma A_{t+2}^* (1 - \Lambda_2)^2}{\Lambda_1} \right)^{1/2}.
\]

**Proof.** See Appendix E.

Comparing Proposition 3 with Proposition 1 and Corollary 1.2 shows that all mechanisms discussed in Section 2 are robust to the inclusion of a limited number of informed investors.

\(^{22}\)\( \lambda b_{t+1} = \frac{\phi(b_{t+1})}{1 - \phi(b_{t+1})} \) where \( \phi(.) \) and \( \Phi(.) \) are standard normal p.d.f. and c.d.f., respectively and

\[
b_{t+1} \equiv \sqrt{z_v} \left( \bar{\theta}_{t+2} - \frac{\bar{z}_v}{z_v} \bar{\theta}_{t+2} - \frac{\bar{z}_v}{z_v} \bar{P}_{t+1} \right).
\]

See Appendix D.
As before, there is the encouraging "adoption to signal" force captured by the term \( \frac{\Delta t}{z_v} \) and the discouraging "fear of unstable markets" force captured by the term \( \gamma^t A_{t+2} (1-A_2)^2 \). Fast technology adoption is more likely than in perfect equity market, if the average quality of information \((z_v)\) is not too low. The quality of information high when either the number of informed investors \((\mu^I)\) is high or uninformed investors receive relatively precise public signals (i.e. \(\beta\) is high).

There are a few additional implications. The variance of unexplainable component of productivity \((1/\beta_u)\) does not affect the probability of fast technology adoption if there are no informed investors \((\hat{\mu}^I \rightarrow 0)\). However, with some informed investors, the higher variance of this component reduces the probability of fast technology adoption. This is because informed investors hold less equity, which reduces the informativeness of price signals and the overall quality of information in equity market \((z_v)\). There is an additional channel through which higher risk aversion \((\gamma)\) reduces the probability of fast technology adoption. First, as before the magnitude of the "fear of unstable markets" force is higher, because entrepreneurs care more about the uncertainty (the right hand side of (35) increases). Second, the average quality of information in equity market is lower, because informed investors hold less equity and price signals reveal less information (using (33), the left hand side of (35) decreases).

3.3 Endogenous entry to entrepreneurship

So far, the paper assumed that there is only one talented local entrepreneur in each sector of the economy, who has skills to adopt technology fast. In a more realistic environment, good prospects for technological improvements are likely to encourage entry of several firms that compete for being the best innovator. An illustrative example of this is the information technologies sector in the 1990s, when good prospects of developing new technologies lead to high entry rates of new firms. Many of these firms, were unsuccessful. This section considers the implications of free entry to entrepreneurship and competition for the best technology.

Assume that potential entrepreneurs differ in the quality of technology they can adopt. For example, they can differ in their innovative skills, or in the case of pure adoption of the existing frontier technology, they can adopt it with some loss (or improvement) in quality. Assume that the technology that can be adopted by an entrepreneur \(k\) is

\[
A_{t+2}^k = [1 + \hat{g} + \delta \eta_t^k] A_{t+1},
\]

(36)

where \(\hat{g}\) and \(\delta\) are positive constants and \(\eta_t^k\) is the idiosyncratic component of technological

\[23\text{Similarly to (8), assume that } 1 + \hat{g} > a^{-\frac{\sigma}{\alpha}}.\]
improvement that is drawn from uniform distribution in the interval \([0, 1]\). The latter is unknown before period \(t\).

Two dates just before period \(t\), in \(t - 2\Delta t\), each potential entrepreneur decides whether to pay an entry cost \(X_{t+1}\). At this stage, potential entrepreneurs have a private signal\(^24\)
\[\tilde{D}_{t+2} = \theta_{t+2} + d_{t+2},\]
where \(d_{t+2}\) has a uniform distribution in the interval \([-\frac{1}{\beta_d}, \frac{1}{\beta_d}]\). They also know the total number of entrants, \(N\). After paying the entry cost, entrepreneurs find out \(\theta_{t+2}\) and become able to adopt technology.

Assume further that \(-\frac{1}{\beta_d} \leq \tilde{D}_{t+2} - \tilde{\theta}_{t+2} \leq \frac{1}{\beta_d}\). If this assumption would not hold, it would be always or never optimal to invest in fast technology adoption and the problem would be uninteresting.

In \(t - \Delta t\), an entrepreneur \(k\) decides whether to invest an additional \(\psi \approx 0\)\(^25\) in order to find out \(\eta_{t+2}^k\) and apply for a patent that gives him monopolistic rights to produce intermediate goods with quality \(A_{t+2}^k\) in period \(t + 2\). At this stage, he knows \(\theta_{t+2}\).

Assume that only the best technology gets a patent by period \(t\). Therefore, among all entrepreneurs that apply for a patent, indexed with \(n \in \{1, \ldots, N\}\), an entrepreneur \(k\) will become the monopolist, only if \(A_{t+2}^k = \max\{A_{t+2}^1, \ldots, A_{t+2}^N\}\).

In period \(t\), there is one entrepreneur, who has obtained the patent and knows the quality of his technology \(A_{t+2}^k\). This entrepreneur decides whether to invest \(\zeta A_{t+2}^k\) in developing the technology or to quit. This stage is equivalent to the technology investment decision in the previous sections of the paper. Figure 5 summarizes the timing.

This game can be solved backwards. In period \(t\), the benefit of technology adoption for

\(^24\)If the signal would be public, it is reasonable to assume that it is also available in trading period \(t + 1\). In such case, initial owners would know something about the public signal investors receive and likely optimism \((\tilde{D}_{t+2} > \theta_{t+2})\) would tend to increase the probability of fast technology adoption. The paper abstracts from this channel.

\(^25\)This assumption is not crucial. The main difference is that if \(\psi > 0\), some entrepreneurs will quit in \(t - \Delta t\) if the initial signal was optimistic.

---

Figure 5: Timing of entry decisions
the entrepreneur who survives is

\[ U_t^k = \begin{cases} 
\Gamma A_{t+2}^k (\theta_{t+2} - \bar{\theta}_{t+2}), & \text{if } \theta_{t+2} \geq \bar{\theta}_{t+2} \\
0, & \text{if } \theta_{t+2} < \bar{\theta}_{t+2}
\end{cases} \]  

(37)
in all settings analyzed.\(^{26}\) Given this, we can derive the optimal decisions in earlier periods and the equilibrium number of entrants.

**Proposition 4** The equilibrium number of entrants \(N^*\) is decreasing in \(\theta_{t+2}\), and increasing in \(\tilde{D}_{t+2}, \tilde{g}, \delta\). If fast technology adoption is optimal \((\theta_{t+2} > \bar{\theta}_{t+2})\), the expected growth rate of technology is

\[ \frac{E[A_{t+2}|N^*, \theta_{t+2} \geq \bar{\theta}_{t+2} - A_{t+1}]}{A_{t+1}} = \tilde{g} + \delta \frac{N^*}{N^* + 1}. \]  

(38)

**Proof.** See Appendix F. \(\blacksquare\)

Proposition 4 implies that a decrease of the threshold for fast technology adoption \((\bar{\theta}_{t+2})\) leads to a higher number of entrants. A decrease in \(\theta_{t+2}\) increases the probability of fast technology adoption and the expected gains from technology investments. Given the results of the previous sections, the number of entrants is higher than in perfect equity market if and only if the "adoption to signal" force is stronger than the "fear of unstable markets" force (and the "direct lack of liquidity" effect, if the equity market is not liquid).

Proposition 4 also implies that the higher is the equilibrium number of entrants, the faster is the expected growth rate of technology, \(\frac{E[A_{t+2}|N^*, \theta_{t+2} \geq \bar{\theta}_{t+2} - A_{t+1}]}{A_{t+1}} = \frac{\delta}{(N^* + 1)^2} > 0\). This is because higher competition among entrants increases the expected quality of technology of the firm that survives and aggregate output.

Combining Proposition 4 with the findings in Section 2.4, 3.1 and 3.2, it is clear that the technology tends to develop faster than in perfect equity market if the "adoption to signal" force dominates the other forces. Similarly, it would develop more slowly, if the discouraging "fear of unstable market" (and "direct lack of liquidity") force dominates. Therefore, when there is free entry to entrepreneurship, the aggregate impact of the main forces discussed in this paper is amplified.

The other variables that lead to higher entry and growth are higher opportunities for technological improvement \((\tilde{g} \text{ and } \delta)\) and optimism among entrepreneurs (high \(\tilde{D}_{t+2}\)). The effect of optimism on growth would be absent if \(\theta > 0\), provided that the signal potential entrepreneurs receive remains private.

\(^{26}\) Assume that a firm with technology \(A_{t+2}^k\) survives. Then (20) implies that the benefit from developing a better technology is \(U_t^k = E[P_{t+1}|\theta_{t+2}] - \frac{1}{2} \text{Var}(P_{t+1}|\theta_{t+2}) - \zeta A_{t+2}^k\). Using \(E[P_{t+1}|\theta_{t+2}], \text{Var}(P_{t+1}|\theta_{t+2})\) from the proofs of Proposition 1 and 2 and from Appendix E and the thresholds (21), (30) and (34) gives (37). Notice that this holds when the difference \(\theta_{t+2} - \bar{\theta}_{t+2}\) is relatively small.
4 Empirical implications

This section highlights the testable empirical implications of the model and discusses suggestive evidence from the existing empirical literature. The main novel implications of the model are the "adoption to signal" force and the non-monotonic relationship between information in the equity market and output (or investments in technology).

"Adoption to signal" force

This force suggests that investors take investments in technology and pre-announcements of new products (i.e., announcements of a new product that are being developed, but not yet marketed) as a positive signal about the future value of the firm. This alone does not prove the presence of "adoption to signal" force, as investors may price such announcements correctly. However, in more developed equity markets (i.e., $\beta_e$ is high), where "adoption to signal" is likely to dominate the negative forces, the model predicts overinvestment in technology (compared to fundamental value) that is positively associated with overpricing of equity and subsequent lower returns from the equity market.27

Within this spirit, a good example of the "adoption to signal" force is the development of Windows Vista, that lead to temporary overpricing of Microsoft:

"Microsoft had 25 percent gain in its stock price in the six months before launching its Vista operating system. But after Vista became available to consumers in January - and got lukewarm response - the stock wilted. /../ The share price then climbed again, but is below its pre-launch peak" (Washington Post 01/07/2007)

Microsoft share had a cumulative abnormal return of 14 percent in the 6 months before the worldwide release of Windows Vista on January 30, 2007. In the 6 months after the release, the cumulative abnormal return was -5.8 percent.28

Some more indirect examples of a positive effect of investments in technology and new product pre-announcements on stock prices are the following:

"Stock price boosting succeeds so well because Wall Street is full of investors looking for a lower-priced stock that may develop a new electronics or space product and become a fast

27 Consider a simple example in the framework of Section 2. Assume that the shocks are at their mean value, i.e. $\epsilon_{t+2} = 0$ and $u_{t+2} = 0$. From (19) equity is overpriced $P_{t+1} = \Gamma A_{t+2}^* (\theta_{t+2} + \frac{1}{\sqrt{T}})$; from (13) and (19) investors get negative returns $\tau_{t+2} - P_{t+1} = -\Gamma A_{t+2}^* \frac{1}{\sqrt{T}}$, and from (21), (22) and (23) there is overinvestment in technology if $\theta_{t+2}^\epsilon - \frac{1}{\sqrt{T}} (\beta_e - \beta_\epsilon) < \theta_{t+2} < \theta_{t+2}^\epsilon$, where $\beta_e > \beta_\epsilon$ if "adoption to signal" dominates "fear of unstable markets" force.

28 Author’s calculations. The market index used is the S&P Composite Index. The specific event of the Vista announcement is hard to identify, because of frequent build releases, rumors, leaks, etc. For example, the release to manufacturing build (the final version of the code that is shipped to customers) was finished on November 1, 2006 and announced by Microsoft on November 8, 2006. Microsoft share had a cumulative abnormal return of 4.9 percent in the ±10-day window around November 8.
raising glamour stock. The spread of stock-option plans as a form of executive compensation has made stock minded men of many corporate bosses who once paid little attention to Wall Street" (TIME U.S. 01/02/1961)

"Compaq share price may well reach $120 or so over the next 6 to 12 months [at the time of the article, the share price was $98.25] /.../ Analysts point out an expected new-product announcement next week as an example of Compaq’s present strength" (The New York Times 14/04/94)

"There are two qualities you should always look for: innovation and valuation. It was the relentless focus in the network sector that made Cisco Systems a stock market icon in the late 1990s" (Newsweek 09/04/01)

The positive effect of new product pre-announcements on stock prices has also been documented in an empirical study by Mishra and Bhabra (2001). In particular, they show that pre-announcements have a positive impact on stock prices if they are accompanied with some evidence such as investments, R&D efforts, prototype or product demonstrations etc. Pre-announcements that lack such evidence, do not affect equity prices. This indicates that a commitment to develop a new product is taken as a positive signal by investors.

The "adoption to signal" force implies that the equity prices do not just increase, but equity is likely to be overpriced because of investments in technology. There is overwhelming empirical evidence of positive relationship between overpriced equity and investments\(^\text{29}\). However, an alternative reason for this could be that credit constrained and equity dependent firms issue more equity to finance their investments (e.g. Baker, Stein and Wurgler 2003). To test "adoption to signal" force, this channel should be controlled for.

A recent paper by Polk and Sapienza (2008) presents systematic evidence that is consistent with the "adoption to signal" force. They explicitly control for equity issuance channel and find that firms with abnormal investments are overpriced and subsequently have low stock returns. They find that this pattern is particularly strong for R&D intensive firms. It is also stronger for firms with shorter shareholder horizons. The "adoption to signal" force provides a rational explanation for this pattern.

Polk and Sapienza (2008) explain the pattern through a more behavioral argument. Namely, they argue that investors irrationally pay higher price for firms that make particular investments (i.e. investments in technology and R&D). Entrepreneurs, knowing this, will cater the market sentiment and choose to overinvest in such projects. The "adoption to signal" force provides a direct explanation why investors take an investment in technology as a positive signal and why they systematically pay too much for firms that overinvest

\(^{29}\)See for example, Morck, Shleifer and Vishny (1990), Blanchard, Rhee and Summers (1993), Gilchrist, Himmelberg and Huberman (2005).
in technology. These investments reveal noisy information about high future productivity. While investors know that the resulting increase of the market value gives some entrepreneurs an incentive to overinvest, they do not know whether a particular entrepreneur invested because of this or because of high future productivity.

The catering theory also relies on a stronger assumption than the one adopted in this paper. Namely, entrepreneurs should have superior information about both the value of their firm and the future sentiment in equity market, as opposed to only about the value of their firm, as it is assumed in the current paper.\textsuperscript{30}

\textit{Non-monotonic relationship between equity market development and output}

The model predicts a non-monotonic (concave) relationship between the level of equity market development and investments in technology (or output). In particular, at the low level of equity market development, an improvement either in the quality of information (increase of $\beta_i$) or in the liquidity of the equity market (increase of $\hat{\mu}$) increases investments in technology and growth. Conversely, if the information asymmetry is small, only improvements in liquidity are likely to increase investments in technology, while improvements in quality of information would reduce it.

As mentioned in the Introduction and shown in Appendix A, cross country correlations between a measure of equity market development and R&D investment is consistent with a potentially non-monotonic relationship. Furthermore, a recent paper by Rousseau and Wachtel (2005) shows that the relationship between broad measures of financial development and growth that was robust in the sample 1960-1989 is absent in the more recent 1990-2003 sample. In particular, the relationship is missing among the more developed countries, where the quality of information is likely to be higher in absolute terms and improved in the more recent years. However, the positive relationship is still present within poorer countries. As higher incentives to invest also imply higher demand for credit, this is an indirect indication for a potentially non-monotonic relationship.

A rigorous empirical test of the model should take into account the channels through which credit and equity markets provide funding for R&D investments, which the current paper abstracts from. The evidence suggests that retained earnings and debt are more important sources for financing corporate investments than equity issuance (see e.g. Rajan and Zingales 1995, Mayer and Sussman 2004). Furthermore, using macro data, papers by Levine and Zervos (1998), Rousseau and Wachtel (2000) and Beck and Levine (2004) find

\textsuperscript{30}This offers a potential test in order to separate the "adoption to signal" force from catering. The relationship between overinvestment in technology and equity overpricing should also be present at times where technology sector is not "popular" (e.g. at the end of technology stocks boom).
that stock market development is positively correlated with growth when controlling for access to credit.

While the existing literature that seeks for a monotonic relationship between equity market development and output makes a compelling case for the existence of the negative "fear of unstable markets" and "direct lack of liquidity" forces, it does not allow for a non-monotonic relationship. To the extent that broad measures of equity market development (e.g. market capitalization, value traded) are likely to be correlated with both, the quality of information and the number of potential investors, further empirical analysis should consider the possibility of a non-monotonicity.

A more direct test of the predictions of the model could aim to separate the negative lack of liquidity effect from the non-monotonic quality of information effect. This may be complicated because it requires good measures of both. In reality, the liquidity may be low precisely because the quality of information is low. It is also likely that the sample of countries (or firms), where there is "too little" information asymmetry between entrepreneurs and investors is very small or non-existing. The information asymmetry is likely to be present even in countries with well functioning equity markets, such as the United States.

5 Concluding remarks

This paper analyzes the effect of information imperfections in equity market on entrepreneurs' incentives to invest in technology and the resulting differences in the speed of technology adoption across countries. It argues that if firms that engage in technology adoption are sold in imperfectly informed equity markets, two main opposite forces arise: a negative "fear of unstable markets" force and a positive "adoption to signal" force.

The relative importance of these forces depends on the quality of information in equity markets. "Adoption to signal" is likely to be most influential in countries where equity markets are developed but not perfect, while "fear of unstable markets" should dominate in underdeveloped markets. The less precise are the signals on which uninformed traders base their decisions, the stronger are these forces. The importance of both forces falls with improvements in the quality of information. Still, the recent overpricing of the technology sector assets in the United States and other developed countries suggests that there is room for "adoption to signal" (which in this case it should be seen as "innovation to signal") even in developed countries.

The mechanisms analyzed in this paper affect both local entrepreneurs and foreign investors (such as venture capitalists) intending to invest in establishing new firms. Uncertainty about equity prices in markets where quality of information is low, can discourage
foreign investors from participating in projects where they could reduce the costs associated with adopting the frontier technology. The limited presence of venture capitalists in most developing countries is likely to reflect the weakness and instability of local equity markets.

The paper also shows that the main forces are robust to a number of extensions. If equity markets are illiquid, it creates an additional negative effect that reduces the incentives to invest in fast technology adoption. The relationship between the degree of information asymmetry and the probability of fast technology adoption remains nevertheless non-monotonic. The setup is also robust to the inclusion of some informed investors in equity market.

The main forces are magnified, when there is free entry to entrepreneurship. If the "adoption to signal" force is strong enough to dominate the other forces, both the number of firms that enter and the growth rate of economy is higher compared to the perfect equity markets. The opposite is true, if the "fear of unstable markets" and/or the "direct lack of liquidity" force is strong.

The paper also shows that a local policy maker would not aim for a full transparency (i.e. eliminating the information asymmetry between entrepreneurs and equity market participants) in order to maximize the chances of fast technology adoption. This is due to the "adoption to signal force" that has its strongest effect at an intermediate level of quality of information. The benefits of "too fast" technology adoption are present because it is likely to be largely financed by foreign investors. Furthermore, due to monopolistic distortions, the social benefits of better technology (though generating higher output and wages) are higher than private benefits (to the owners of the firms). Therefore, encouraging fast technology adoption is likely to be beneficial even in a closed economy.

The paper assumes that the firms are listed in the local stock exchange. Listing in a well established stock exchange (e.g. NASDAQ) can allow a firm to access a larger number of potential buyers. For the mechanisms analyzed in this paper to be valid, this assumption is not crucial, because the uncertainty is about the local economy. Listing in a foreign stock exchange is more likely to increase liquidity than eliminate information asymmetry, which is the driver of main results in this paper. Interpreting the equity market to be local is more natural, because for most firms from developing countries, the fixed costs associated with an initial public offering in NASDAQ are likely to be too high. Therefore, this possibility is only available for the most successful and innovative firms.

Furthermore, local firms could be sold directly to a strategic (foreign) owner. As long as the price the strategic owner pays for a firm reflects its market value, the mechanism suggested in this paper remains valid. If the local equity market is very underdeveloped and most firms are directly transferred between local agents, potentially both the low number of informed buyers and the lack of liquidity are likely to discourage fast technology adoption.
A Income per capita and R&D expenditure

Figure 6: Data sources: GDP, R&D expenditures and investments from WDI, World Bank; "Equity Size Index" from FSDI, World Bank and securities market index for transition countries: Transition Report, EBRD.

B Local goods market clearing

The consumption of each local agent from trading in the asset market is \( c_t = \bar{1}_{t-2} \hat{h}_{t-1} \pi_t + m_{t-1} \), where \( m_{t-1} \) is his risk-free asset holdings. Defining the aggregate equity demand by foreign agents as \( H^*_t = \bar{1}_{t-1} (\hat{\mu} - \mu) \hat{h}_t \) and using equity market clearing condition \( \bar{1}_{t-1} \hat{\mu} \hat{h}_t = \bar{1}_{t-1} \), the aggregate equity demand by local agents is \( \bar{1}_{t-1} \mu \hat{h}_t = \bar{1}_{t-1} (1 - H^*_t) \). Defining aggregate risk-free asset holdings by local agents as \( M_t \equiv \mu m_t \), their aggregate consumption

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31 The figures about transition countries exclude 5 transition countries that had a substantially lower initial PPP adjusted GDP per capita (below 3000 USD) in 1991. The remaining countries have a mean of 6600 USD and a standard deviation of 2000 USD.
from investment in equity and risk-free asset is $\mu c_t = \tilde{I}_{t-2} (1 - H_{t-1}^*) \pi_t + M_{t-1}$. Some local agents are entrepreneurs adopting technology. These agents consume $\tilde{I}_{t-1} P_t$, therefore aggregate consumption is $C_t = c_t + \tilde{I}_{t-1} P_t = \tilde{I}_{t-2} (1 - H_{t-1}^*) \pi_t + M_{t-1} + \tilde{I}_{t-1} P_t$.

Each young local agent receives wage income $w_t$ and the aggregate budget constraint is $\mu w_t = \tilde{I}_t (1 - H_t^*) P_t + M_t + \tilde{I}_t I_t$. The net inflow of goods from abroad is determined by the net inflow of equity and risk-free asset, $F_t = \left( \tilde{I}_{t-1} H_t^* P_t - \tilde{I}_{t-1} H_{t-1}^* \pi_t \right) + (M_{t-1} - M_t)$. From the above, we can find

$$C_t - F_t = \tilde{I}_{t-2} \pi_t + \mu w_t - \tilde{I}_{t-1} I_t. \quad (39)$$

From (12) and (26), the aggregate investment in intermediate good production $\int_0^1 x_t(j) dj = \tilde{I}_{t-2} (\alpha^2 \frac{1}{\gamma} \phi_i L A_t^* + (1 - \tilde{I}_{t-2}) \phi_i L A_{t-1}^*)$. Replacing this, $\mu = L$, (27), (28), (39) in to goods market clearing condition (11) gives $\tilde{I}_{t-2} \pi_t = \tilde{I}_{t-2} \frac{1 - \alpha}{\alpha} (\alpha^2 \frac{1}{\gamma} \phi_i L A_t^*)$ and holds by (4) and (13).

### C Proof of Corollary 2.1

The proof has three parts: 1) the relationship between $\theta_{t+2}^\mu$ and $\beta_\epsilon$ is non-monotonic, 2) for $\rho \geq \frac{2}{3}$, the threshold $\tilde{\beta}_\epsilon^\mu$ is unique and 3) for $\rho \geq \frac{2}{3}$, it holds that $\tilde{\beta}_\epsilon^\mu > \beta_\epsilon$.

1) The relationship between the probability of fast technology adoption and quality of information in equity markets is non-monotonic if there exist positive and finite values for $\beta_\epsilon$ such that $\theta_{t+2}^\mu < \lim_{\beta_\epsilon \rightarrow 0} \theta_{t+2}^\mu$ and $\theta_{t+2}^\mu < \lim_{\beta_\epsilon \rightarrow \infty} \theta_{t+2}^\mu = \theta_{t+2}^{\mu\kappa}$. From (30) it is clear that $\lim_{\beta_\epsilon \rightarrow 0} \theta_{t+2}^\mu \rightarrow \infty$ and $\lim_{\beta_\epsilon \rightarrow \infty} \theta_{t+2}^{\mu\kappa}$ is given by (31). Therefore, using (30) and (31) we need to show that $\theta_{t+2}^\mu < \theta_{t+2}^{\mu\kappa} \Leftrightarrow \frac{1}{2 \beta_\epsilon^\mu} > \frac{\gamma A_{t+2}^\mu}{24 \beta_\epsilon^\mu} \left[ \psi \left( \mu, \beta_\epsilon \right) + \frac{8(1+3\rho)}{3\mu} \right]$ for some values of $\beta_\epsilon$. Suppose that $\beta_\epsilon$ solves $\theta_{t+2}^\mu = \theta_{t+2}^{\mu\kappa}$, we can write

$$LHS \equiv \frac{12 \beta_\epsilon^\mu}{\gamma A_{t+2}^\mu} = \left( 1 - \frac{\gamma A_{t+2}^\mu}{3\beta_\epsilon^\mu} \right)^2 + \left( \frac{2 \gamma A_{t+2}^\mu}{3\beta_\epsilon^\mu} \right)^2 \frac{1}{15} + \frac{8(1+3\rho)}{3\mu} \equiv RHS. \quad (40)$$

From (40), it is clear that $LHS > 0$, $LHS = 0$, $LHS \rightarrow \infty$, $\frac{\partial LHS}{\partial \beta_\epsilon^\mu} = \frac{12}{\gamma A_{t+2}^\mu} > 0$ and $RHS > 0$, $RHS \rightarrow \infty$, $RHS = \frac{8(1+3\rho)}{3\mu}$ that is finite and $\frac{\partial RHS}{\partial \beta_\epsilon^\mu} = 2 \frac{\gamma A_{t+2}^\mu}{3\beta_\epsilon^\mu} \left( 1 - \frac{\gamma A_{t+2}^\mu}{3\beta_\epsilon^\mu} \frac{16}{15} \right)$. The latter implies that $\frac{\partial RHS}{\partial \beta_\epsilon^\mu} \leq 0$ if $\beta_\epsilon^\mu \leq \frac{2 \gamma A_{t+2}^\mu}{3\mu}$ and $\frac{\partial RHS}{\partial \beta_\epsilon^\mu} > 0$ if $\beta_\epsilon^\mu > \frac{2 \gamma A_{t+2}^\mu}{3\mu}$. From here we can conclude that there exists at least one and at most three finite and strictly positive $\beta_\epsilon^\mu$ that solve $\theta_{t+2}^\mu = \theta_{t+2}^{\mu\kappa}$.

Let us call the highest of these solutions $\bar{\beta}_\epsilon^\mu$. Given that in the case of $\theta_{t+2}^\mu < \theta_{t+2}^{\mu\kappa}$, $LHS > RHS$, there exists a positive and finite $\bar{\beta}_\epsilon^\mu$ such that for any $\beta_\epsilon > \bar{\beta}_\epsilon^\mu$ it holds that $\theta_{t+2}^\mu < \theta_{t+2}^{\mu\kappa}$. This proves the first part of the corollary.
2) A cubic function has one real solution and two complex solutions if its discriminant\(^{32}\) is strictly negative. The discriminant of (40) is \(-\left(\frac{\gamma A_t}{3}\right)^6 \frac{1}{\mu^6}Q\), where \(Q \equiv \frac{1}{102} \mu^3 + \frac{1}{4} (3 \rho - 1) \mu^2 + (11 \rho^2 - \frac{50}{3} \rho + \frac{127}{45}) \mu + \frac{128}{27} (1 + 3 \rho)^3\). This implies that the discriminant is negative if \(Q > 0\). While it does not hold for any values of \(\mu\) and \(\rho\), it holds for a wide set of parameters. We can show that if \(\rho \geq \frac{2}{3}\) is a sufficient condition by solving the following constrained minimization problem \(Q^* = \min Q\) st. \(\mu^3 + 1\), which gives \(Q^* \approx 32.4 > 0\). Therefore, if \(\rho \geq \frac{2}{3}\) the equation (40) has a unique real solution, \(\hat{\beta}_t^\mu\), and \(\hat{\theta}_t^\mu < \hat{\theta}_t^\mu\) for any \(\beta_t < \hat{\beta}_t^\mu\).

3) This can be proved by contradiction. Suppose that \(\hat{\beta}_t^\mu\) satisfies \(\hat{\beta}_t^\mu < 1\). Replacing \(\hat{\beta}_t^\mu = \frac{\gamma A_t}{12}\) from (23) in (40) gives \(\hat{\beta}_t^\mu = 1 + \frac{8}{\mu} \left(\frac{1 + 3 \rho}{3} - \frac{\hat{\beta}_t^\mu}{\hat{\beta}_t^\mu}\right) + \left(\frac{4 \hat{\beta}_t^\mu}{\mu^2 \hat{\beta}_t^\mu}\right)^2 \frac{16}{15} < 1\). For this to hold, it must be that \(1 + 3 \rho < \frac{\hat{\beta}_t^\mu}{\hat{\beta}_t^\mu}\). However, if \(\rho > \frac{2}{3}\) then \(1 + 3 \rho > 1\) and it must hold that \(\frac{\hat{\beta}_t^\mu}{\hat{\beta}_t^\mu} > 1\). This leads to a contradiction.

### D Equity market equilibrium in Section 3.2

Within the setting of Section 3.2, equity market clearing condition is:

\[
\hat{\mu}^I \hat{h}^I_{t+1} + \hat{\mu}^U \hat{h}^U_{t+1} + s_{t+1} = 1, \tag{41}
\]

where \(\hat{h}^I_{t+1}\) is equity demand by an investor of type \(i \in \{I, U\}\). Maximizing utility (1) subject to type \(i\) investor’s budget constraint, \(\hat{c}^i_{t+2} = [\pi_{t+2} - P_{t+1}] \hat{h}^i_t + \hat{W}^i_{t+1}\), yields the optimal equity demand as

\[
\hat{h}^i_{t+1} = \frac{E[\pi_{t+2} | \Omega^I_{t+1} - P_{t+1}]}{\gamma \operatorname{Var}(\pi_{t+2} | \Omega^I_{t+1})}. \tag{42}
\]

The information set that is relevant for informed investors is \(\Omega^I_{t+1} = \{\theta_{t+2}\}\). Therefore, if an investor is informed, then \(E[\pi_{t+2} | \Omega^I_{t+1}] = \Gamma \theta_{t+2} A^I_{t+2}\), \(\operatorname{Var}(\pi_{t+2} | \Omega^I_{t+1}) = \Gamma^2 A^2_{t+2} \frac{1}{\beta_a}\) and his optimal equity demand is

\[
\hat{h}^I_{t+1} = \frac{\Gamma \theta_{t+2} A^I_{t+2} - P_{t+1}}{\Gamma^2 A^2_{t+2} \frac{1}{\beta_a}}. \tag{43}
\]

Uninformed investors obtain a signal \(\hat{\theta}_{t+2} = \theta_{t+2} + \epsilon_{t+2}\), where \(\epsilon_{t+2} \sim \mathcal{N} \left(0, \frac{1}{\beta_a}\right)\). They also observe the equilibrium equity price. We can find the information revealed by the price equity by replacing \(\hat{h}^I_{t+1}\) into the equity market clearing condition (41), which gives

\[
\hat{\mu}^I \hat{W}_{t+1} + \hat{\mu}^U \hat{h}^U_{t+1} + s_{t+1} = 1. \tag{44}
\]

By rearranging this in to an observable and unobservable part from the point of view of an uninformed investor, we can define a price signal as

\[
\tilde{P}_{t+1} \equiv \frac{P_{t+1}}{\Gamma A_{t+2}} - \frac{\Gamma A_{t+2} \frac{1}{\mu^3}}{1 - \hat{\mu}^U \hat{h}^U_{t+1}} (1 - \hat{\mu}^U \hat{h}^U_{t+1}) = \theta_{t+2} + \tilde{s}_{t+1},
\]

\(^{32}\)Discriminant of \(ax^3 + bx^2 + cx + d = 0\) is \(-4b^3d + b^2c^2 - 4ac^3 + 18abcd - 27a^2d^2\).
where \( \tilde{s}_{t+1} \equiv \frac{\gamma A_{t+2}}{\mu A_{t+2}} s_{t+1} \sim \mathcal{N}(0, (\frac{\mu A_{t+2}}{\gamma})^2 \beta) \), because \( s_{t+1} \sim \mathcal{N}(0, \frac{1}{\mu A_{t+2}^2 \beta}) \). Defining \( z_v \) as in (33), the updated distribution \( \theta_{t+2} | \tilde{\theta}_{t+2}, \tilde{P}_{t+1}, \sim \mathcal{N}\left(\frac{\beta z_v}{z_v} \tilde{\theta}_{t+2} + \frac{z_v - \beta_s}{z_v} \tilde{P}_{t+1} + \frac{\lambda_{b+1}}{\sqrt{z_v}} \right) \).

Uninformed investors also get information from knowing entrepreneurs’ investment decision, i.e. \( \tilde{I}_t = 1 \) implies that \( \theta_{t+2} \geq \tilde{\theta}_{t+2} \).

Following pp. 899 in Green (2000) for the moments of truncated normal and (13) gives. \( E[\pi_{t+2}|\Omega_{t+1}^U] = E[\pi_{t+2}|\tilde{\theta}_{t+2}, \tilde{P}_{t+1}, \theta_{t+2} \geq \tilde{\theta}_{t+2}] = \Gamma A_{t+2}^* \left( \frac{\beta z_v}{z_v} \tilde{\theta}_{t+2} + \frac{z_v - \beta_s}{z_v} \tilde{P}_{t+1} + \frac{\lambda_{b+1}}{\sqrt{z_v}} \right) \) and \( \text{Var}(\pi_{t+2}|\Omega_{t+1}^U) = \text{Var}(\pi_{t+2}|\tilde{\theta}_{t+2}, \tilde{P}_{t+1}, \theta_{t+2} \geq \tilde{\theta}_{t+2}) = \Gamma^2 A_{t+2}^2 \left[ 1 - \frac{\lambda^2_{b+1} + b_{t+1} + 1}{z_v} \right] \), where

\[
b_{t+1} \equiv \sqrt{z_v} \left( \frac{\beta z_v}{z_v} \tilde{\theta}_{t+2} - \frac{z_v - \beta_s}{z_v} \tilde{P}_{t+1} \right) \text{ and } \lambda_{b+1} = \frac{\phi(b_{t+1})}{1 - \Phi(b_{t+1})} \text{ is an inverse Mills ratio at } b_{t+1} \text{ and } \phi(.) \text{ and } \Phi(.) \text{ are standard normal p.d.f. and c.d.f. respectively.} \]

Using the derived \( E[\pi_{t+2}|\Omega_{t+1}^U] \), \( \text{Var}([\pi_{t+2}|\Omega_{t+1}^U] \), \( E[\pi_{t+2}|\Omega_{t+1}^U] \), \( \text{Var}(\pi_{t+2}|\Omega_{t+1}^U) \) in (42) and plugging then \( \hat{h}_{t+1}^I \) and \( \hat{h}_{t+1}^U \) into equity market clearing condition (41), the equilibrium equity price is

\[
\tilde{P}_{t+1} = \frac{\theta_{t+2} + \frac{\beta z_v}{z_v} \tilde{\theta}_{t+2} + \frac{z_v - \beta s}{z_v} \tilde{P}_{t+1} + \frac{\lambda_{b+1}}{\sqrt{z_v}}}{\gamma \Gamma A_{t+2}^* + \frac{\mu_{t+1}^I}{\gamma \Gamma^2 A_{t+2}^2 \beta}} - 1 + s_{t+1}
\]

As the number of foreign uninformed investors \( \mu_{t+1}^I \rightarrow \infty, \mu_{t+1}^U \rightarrow \infty \) and the equity price equals to the expected profits by uninformed investors

\[
P_{t+1} = \Gamma A_{t+2}^* \left( \frac{\beta z_v}{z_v} \tilde{\theta}_{t+2} + \frac{z_v - \beta_s}{z_v} \tilde{P}_{t+1} + \frac{\lambda_{b+1}}{\sqrt{z_v}} \right) = E[\pi_{t+2}|\Omega_{t+1}^U].
\]

Using then \( \tilde{\theta}_{t+2} = \theta_{t+2} + \epsilon_{t+2} \) and \( \tilde{P}_{t+1} = \theta_{t+2} + \tilde{s}_{t+1} \) gives (32).

### E Proof of Proposition 3

Technology adoption decision is given by (20). Finding the moments \( E[P_{t+1}|\theta_{t+2}] \) and \( \text{Var}(P_{t+1}|\theta_{t+2}) \) is complicated by the fact that the equity price (32) includes \( \lambda_{b+1} \). While \( b_{t+1} \) is an observable constant for investors trading in \( t+1 \), it is not observable in period \( t \). Expressing \( b_{t+1} = \sqrt{z_v} \left( \tilde{s}_{t+2} - \theta_{t+2} - \frac{\beta z_v}{z_v} \epsilon_{t+2} - \frac{z_v - \beta_s}{z_v} \tilde{s}_{t+1} \right) \), it is clear that \( b_{t+1} \) has a normal distribution from the point of view of the entrepreneur who knows \( \theta_{t+2} \), but does not know \( \epsilon_{t+2} \) and \( \tilde{s}_{t+1} \). The moments of Mills ratio with normally distributed \( b_{t+1} \) are, to the best of
my knowledge, impossible to derive in closed form. However, it can be approximated. Let us focus on \(\theta_{t+2}\) in the neighborhood of \(\bar{\theta}_{t+2}\), i.e. \(\theta_{t+2} = \bar{\theta}_{t+2} + \Delta \theta_{t+2}\) and \(\Delta \theta_{t+2} \rightarrow 0\). In such a case, \(b_{t+1} = \sqrt{z_v} \left( -\frac{\beta_s}{z_v} \epsilon_{t+2} - \frac{z_v - \beta_s}{z_v} \bar{s}_{t+1} \right)\) and it is reasonable to approximate \(\lambda_{b_{t+1}}\) around \(b_{t+1} = 0\), where \(\epsilon_{t+2}\) and \(\bar{s}_{t+1}\) are at their mean value.

Using the first order Taylor approximation,

\[
\lambda_{b_{t+1}} \approx \lambda_{b_{t+1}=0} + \lambda'_{b_{t+1}=0} \sqrt{z_v} \left( -\frac{\beta_s}{z_v} \epsilon_{t+2} - \frac{z_v - \beta_s}{z_v} \bar{s}_{t+1} \right).
\]

Let us call \(\Lambda_1 \equiv \lambda_{b_{t+1}=0}\) and \(\Lambda_2 \equiv \lambda'_{b_{t+1}=0}\). As Mills ratio is always positive, and increasing function of \(b_{t+1}\), it holds that \(\Lambda_1, \Lambda_2 > 0\). Furthermore, left truncation Mills ratio is a convex function that is close to linear if \(b_{t+1} > 3\). In the linear area the slope is below 1, therefore \(\Lambda_2 < 1\). In fact, \(\Lambda_1 = \lambda_{b_{t+1}=0} = \frac{\phi(0)}{1-\Phi(0)} \approx 0.80\) and \(\Lambda_2 = \lambda'_{b_{t+1}=0} = \left( \frac{\phi(0)}{1-\Phi(0)} \right)^2 + \frac{\phi'(0)}{1-\Phi(0)} = \frac{(\phi(0))^2}{(1-\Phi(0))} \approx 0.64\), where \(\phi(\cdot)\) and \(\Phi(\cdot)\) are the p.d.f. and c.d.f. or standard normal respectively. The approximated equity price is

\[
P_{t+1} = \Gamma A^*_{t+2} \left( \theta_{t+2} + (1 - \Lambda_2) \left( \frac{\beta_s}{z_v} \epsilon_{t+2} + \frac{z_v - \beta_s}{z_v} \bar{s}_{t+1} \right) + \frac{\Lambda_1}{\sqrt{z_v}} \right).
\]

Given that \(\text{Var}(\epsilon_{t+2}) = \frac{1}{\beta_s^2}\) and \(\text{Var}(\epsilon_{t+2}) = \left( \frac{\mu_1 \beta_s}{\gamma} \right)^{-1} = z_v - \beta_s\), we can find \(E[P_{t+1}|\theta_{t+2}] = \Gamma A^*_{t+2} \theta_{t+2} + \Gamma A^*_{t+2} \frac{\Lambda_1}{\sqrt{z_v}}\) and \(\text{Var}(P_{t+1}|\theta_{t+2}) = \Gamma^2 A^*_{t+2}^2 (1 - \Lambda_2)^2 \frac{1}{z_v}\). Plugging these into (20) and rearranging gives (34).

Technology adoption is more likely than in perfect equity market if \(\bar{\theta}_{t+2} < \hat{\theta}_{t+2}^P = \frac{\zeta}{\Gamma}\). Using (34) this holds iff \(z_v > \left( \frac{\gamma \Gamma A^*_{t+2} (1 - \Lambda_2)^2}{\Lambda_1^2} \right)^2\).

## F Proof of Proposition 4

In \(t - \Delta t\), entrepreneurs know \(\theta_{t+2}\) and do not know their own and other entrepreneurs’ quality of technology. Let us assume first that \(\theta_{t+2} \geq \bar{\theta}_{t+2}\), so that further investments are optimal. Denote the event that an entrepreneur \(k\) will get the patent as

\[
S_k^t = \begin{cases} 
1 & \text{if } A^k_{t+2} = \max\{A^1_{t+2}, \ldots, A^N_{t+2}\} \\
0 & \text{otherwise.}
\end{cases}
\]

Also denote the information set with \(\Omega^N_{t-\Delta t} = \{\theta_{t+2} \geq \bar{\theta}_{t+2}, N\}\). Using (36) and (37), expected utility of \(k\) in \(t - \Delta t\) can be founds using the law of total expectations as

\[
E[U^k_t|\Omega^N_{t-\Delta t}] = \Gamma A_{t+1}\left(\theta_{t+2} - \bar{\theta}_{t+2}\right) \left(1 + \hat{\gamma} + \delta E[\eta^k_t|S_t^k = 1, \Omega^N_{t-\Delta t}] \right) \text{Pr}(S_t^k = 1|\Omega^N_{t-\Delta t}).
\]
Bayes’ rule implies that the density

\[
f \left( \eta_t^k | S_t^k = 1, \Omega_{t-\Delta t} \right) = \frac{f \left( \eta_t^k | \Omega_{t-\Delta t} \right) \Pr(S_t^k = 1 | \eta_t^k, \Omega_{t-\Delta t})}{\Pr(S_t^k = 1 | \Omega_{t-\Delta t})}.
\]

(44)

Using that \( \eta_t^k \sim u [0, 1] \) and independent of \( \theta_{t+2} \) and \( N \), \( f \left( \eta_t^k | \Omega_{t-\Delta t} \right) = 1 \). The probability of survival with a particular quality of technology \( \eta_t^k \) is \( \Pr(S_t^k = 1 | \eta_t^k, \Omega_{t-\Delta t}) = \prod_{n \neq k} \Pr(\eta_t^n < \eta_t^k) = (\eta_t^k)^{N-1} \). Therefore,

\[
\Pr(S_t^k = 1 | \Omega_{t-\Delta t}) = \int_0^1 (\eta_t^k)^{N-1} d\eta_t^k = \frac{1}{N},
\]

(45)

\[
E \left[ \eta_t^k | S_t^k = 1, \Omega_{t-\Delta t} \right] = \frac{\int_0^1 (\eta_t^k)^N d\eta_t^k}{\Pr(S_t^k = 1 | \Omega_{t-\Delta t})} = \frac{N}{N+1}.
\]

(46)

It is clear that, if \( \theta_{t+2} < \tilde{\theta}_{t+2} \), entrepreneurs would not make further investments and \( E \left[ U_t^k | \theta_{t+2} < \tilde{\theta}_{t+2}, N \right] = 0 \). Therefore,

\[
E \left[ U_t^k | \theta_{t+2}, N \right] = \begin{cases} 
\Gamma A_{t+1} \left( \theta_{t+2} + \tilde{\theta}_{t+2} \right) \left( \frac{1+\delta}{N} + \frac{\delta}{N+1} \right) & \text{if } \theta_{t+2} < \tilde{\theta}_{t+2} \\
0 & \text{if } \theta_{t+2} < \tilde{\theta}_{t+2}.
\end{cases}
\]

In period \( t-2\Delta t \), we can define the benefit of entry as \( BE(N) \equiv \frac{E[\theta_{t+2}, N] | \tilde{D}_{t+2}, N}{A_{t+1}} \). Using the results above and law of total expectations

\[
BE(N) = \Gamma \left( \frac{1+\delta}{N} + \frac{\delta}{N+1} \right) E \left[ \theta_{t+2} - \tilde{\theta}_{t+2} | \tilde{D}_{t+2}, \theta_{t+2} \geq \tilde{\theta}_{t+2} \right] \Pr \left( \theta_{t+2} \geq \tilde{\theta}_{t+2} | \tilde{D}_{t+2} \right)
\]

Given the distribution of \( \theta_{t+2} | \tilde{D}_{t+2} \sim u \left[ -\frac{1}{\beta_d}, \frac{1}{\beta_d} \right] \), the benefit of entry

\[
BE(N) = \Gamma \left( \frac{1+\delta}{N} + \frac{\delta}{N+1} \right) \frac{\beta_d}{4} \left( \tilde{D}_{t+2} - \tilde{\theta}_{t+2} + \frac{1}{\beta_d} \right)^2.
\]

If \( N-1 \) entrepreneurs have entered, the \( N^{th} \) entrant enters if \( BE(N) \geq 0 \). Ignoring the integer problem, equilibrium number of entrants, \( N^* \), solves \( BE(N^*) = \chi \). Taking a total derivatives of this the sensitivity of \( N^* \) with respect to parameters of the model

\[
\frac{dN^*}{dq} = -\frac{\partial BE(N^*)}{\partial \theta_{t+2}} / \partial N^*, \quad q = \{ \tilde{\theta}_{t+2}, \tilde{D}_{t+2}, \tilde{g}, \delta \}.
\]

It is clear that \( \frac{\partial BE(N^*)}{\partial \theta_{t+2}} < 0 \) and \( \frac{\partial BE(N^*)}{\partial \tilde{D}_{t+2}} > 0 \). Given that \( \tilde{D}_{t+2} - \tilde{\theta}_{t+2} \geq -\frac{1}{\beta_d} \),

\[
\frac{\partial BE(N^*)}{\partial \tilde{D}_{t+2}} = -\frac{BE(N^*)}{\tilde{D}_{t+2} - \tilde{\theta}_{t+2} + \frac{1}{\beta_d}} < 0 \quad \text{and} \quad \frac{\partial BE(N^*)}{\partial \tilde{D}_{t+2}} = \frac{BE(N^*)}{\tilde{D}_{t+2} - \tilde{\theta}_{t+2} + \frac{1}{\beta_d}} > 0.
\]

Therefore, \( \frac{dN^*}{d\theta_{t+2}} < 0 \) and \( \frac{dN^*}{d\tilde{D}_{t+2}}, \frac{dN^*}{d\tilde{g}}, \frac{dN^*}{d\delta} > 0 \). This proves the first part of Proposition 4.
For the second part, we need to find the expected quality of technology for the firm that survives. Assume that the equilibrium number of entrants in $N^*$, and firm indexed with $k$ survives. Then the expected quality of technology $E (A_{t+2}|N^*, \theta_{t+2} \geq \tilde{\theta}_{t+2}) = E (A_{t+2}|\Omega^{{N^*}_{t-\Delta t}})$ is given by

$$E (A_{t+2}|\Omega^{{N^*}_{t-\Delta t}}) = E (A_{t+2}^k|S_t^k = 1, \Omega^{{N^*}_{t-\Delta t}}) = A_{t+1} (1 + \hat{\gamma} + \delta E (\eta_t^k|S_t^k = 1, \Omega^{{N^*}_{t-\Delta t}}))$$

Using then $E (\eta_t^k|S_t^k = 1, \Omega^{{N^*}_{t-\Delta t}}) = \frac{N^*}{N^*+1}$ from (46) gives (38).

**References**


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