A Tractable Model of Precautionary Reserves or Net Foreign Assets*

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Abstract

We model the motives for residents of a country to hold foreign assets, including the precautionary motive that has been omitted from much of the previous literature as intractable. Our model captures many of the key insights from the existing specialized literature on the precautionary motive. Our economy exhibits a unique target value of assets, which balances impatience, prudence, risk, intertemporal substitution, and the rate of return in a convenient formula. We use the model to shed light on two topical questions: the "upstream" flows of capital from developing countries to advanced countries, and the long-run impact of resorbing global financial imbalances.

Keywords: Foreign Reserves, Net Foreign Assets, Sovereign Wealth Funds

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1 Introduction

The remarkable recent accumulation of foreign reserves in emerging economies has captured the attention of academics, policymakers, and financial markets, partly because reserve accumulation seems to have played a role in the development of global financial imbalances. A distinct (but probably related) puzzle is that national saving rates of the fast-growing emerging economies have been rising over time,[1] leading to surprising "upstream" flows of capital from developing to rich countries. And, just in the past year or two, sovereign wealth funds have begun to attract scrutiny as they have emerged as prominent actors in global capital markets.

A popular interpretation of all these trends is that they reflect precautionary saving against the risks associated with economic globalization.

Such an interpretation raises several questions. What are the main determinants of the demand for external assets? What are the welfare benefits of international integration, if it leads developing countries to export rather than import capital? How persistent will the recent increase in developing countries' demand for foreign assets prove to be? How does the precautionary motive for accumulating such assets interact with other motives?

This paper introduces a tractable model that can be used to analyze these questions and others. The model is a small-open-macroeconomy version of the model of individual precautionary saving developed by Carroll (2007), based on Tochè (2005) (see also Sargent and Ljunqvist (2000)). The model permits us to characterize the dynamics of foreign asset accumulation with phase diagrams that should be readily understandable to anyone familiar with the benchmark Ramsey model of economic growth, and to derive closed-form expressions that relate the target level of net foreign assets to fundamental determinants like the degree of risk, the time preference rate, and expected productivity growth. The model's structure is simple enough to permit straightforward calculations of welfare-equivalent tradeoffs between growth, social insurance generosity, and risk.

We then present two applications of our framework. First, we look at what the model has to say about the puzzling relation between economic development and international capital flows (especially the fact that faster-growing developing countries tend to export more capital). We show that this puzzle can be explained in our framework if economic development involves not only a pickup in productivity growth but also an increase in the degree of idiosyncratic risk (that is, risk that is borne by individuals, like unemployment spells that result in substantial lost wages). Second, we use a two-country version of the model to investigate the long-term impact on the United States and the rest of the world if the recent global financial imbalances were to be resorbed. We find that a decrease in the desired level of wealth in the rest of the world has a substantial negative impact on the global capital stock as well as the U.S. real wage.

[1] For evidence of causality from growth to saving, see Carroll and Weil (1994); Loayza, Schmidt-Hebbel, and Servén (2000); Attanasio, Picci, and Scorcu (2000); Hausmann and Rodrik (2005); Gourinchas and Jeanne (2007).

A central purpose of the paper is to distill the main insights of the complex literature that interprets capital flows through the lens of the precautionary motive.[2]

One strand of that literature looks at the effects of aggregate risk on domestic precautionary wealth. For example, Durdu, Mendoza, and Terrones (2007) present some estimates of the optimal level of precautionary wealth accumulated by a small open economy in response to business cycle volatility, financial globalization, and the risk of a sudden stop. They conclude that these risks are plausible explanations of the observed surge in reserves in emerging market countries.[3] Arbatli (2008) argues that precautionary motives associated with the possibility of sudden stops in credit can explain the dynamics of the current account in emerging economy business cycles. Fogli and Perri (2006) instead take the perspective of the U.S. and argue that the decrease in its saving rate can be explained partly by the moderation in the volatility of its business cycle.

Closer to this paper are the contributions that examine the impact of idiosyncratic risk on saving behavior. Mendoza, Quadrini, and Rios-Rull (2007) model the determination of capital flows in a closed world in which economies differ by their level of financial development (market completeness). They find that international financial integration can lead to the accumulation of a large level of net and gross liabilities by the more financially advanced region. Sandri (2008) presents a model in which growth acceleration in a developing country causes a larger increase in saving than in investment because capital market imperfections induce entrepreneurs not only to self-finance investment but also to accumulate precautionary wealth outside their business enterprise.

Several of our analytical results resonate with themes developed, or touched upon, in those papers (in particular, the importance of domestic financial development or social insurance for international capital flows). The main comparative advantages of our analysis are two. First, the insights are reflected in tractable analytical formulas. The impact of key variables can be analyzed using a simple diagram or closed-form expressions—although (as usual) analysis of transitional dynamics requires numerical solution tools (which we provide). Second, our model of prudent (Kimball (1990)) intertemporal choice is integrated with a standard neoclassical treatment of production (Cobb Douglas with labor augmenting productivity growth), so that the familiar Ramsey-Cass-Koopmans framework can be viewed as the perfect-insurance special case of our model. This allows us analyze the link between economic development and capital flows in a way that is directly comparable to the corresponding analysis in the standard model.[4]

[2]Precautionary accumulation is not the only interpretation of recent developments in international capital flows. For example, Caballero, Farhi, and Gourinchas (2008) present a model in which those flows are driven by countries' supply of (rather than demand for) assets.

[3]In contrast, Jeanne (2007)) and Jeanne and Rancière (2008) find that it is difficult to explain the build-up in emerging markets reserves as insurance against the risk of sudden stop.

[4]The models of Fogli and Perri (2006) and Mendoza, Quadrini, and Rios-Rull (2007) do not incorporate growth. The model of Sandri (2008) has economic growth in the transition dynamics toward a long-run steady state with no growth. By contrast, our model allows one to look at the impact of different long-run productivity growth rates.

2 Model

We consider a small open economy whose population and GDP grow at constant rates. A resident of this economy accumulates precautionary wealth in order to insure against the risk of unemployment, which results in complete and permanent destruction of the individual's human capital.[5,6] The saving decisions of our individuals aggregate to produce "net foreign assets" for the economy as a whole.[7]

2.1 Macroeconomic Assumptions

Domestic output is produced according to the usual Cobb-Douglas function:

$$\boldsymbol{P}_t = \boldsymbol{K}_t^{\alpha}(z_t \boldsymbol{L}_t)^{1-\alpha}, \tag{1}$$

where $\boldsymbol{K}_t$ is domestic capital and $\boldsymbol{L}_t$ is the supply of domestic labor. The productivity of labor increases by a constant factor G in every period,

$$z_{t+1} = \mathsf{G} z_t.$$

Capital and labor are supplied in perfectly competitive markets. Capital is perfectly mobile internationally, so that the marginal return to capital is the same as in the rest of the world,

$$\daleth + \alpha \frac{\boldsymbol{P}_t}{\boldsymbol{K}_t} = \mathsf{R}, \tag{2}$$

where $\daleth \equiv (1-\delta)$ is proportion of capital that remains undepreciated after production, and R is the worldwide constant risk-free interest factor. Thus, the capital-to-output ratio is constant and equal to

$$\frac{\boldsymbol{K}}{\boldsymbol{P}} = \frac{\alpha}{\mathsf{R} - \daleth}. \tag{3}$$

Labor is supplied by domestic workers. Each worker is part of a 'generation' born at the same date, and every new generation is larger by the factor Ξ than the newborn generation in the previous period. If we normalize to 1 the size of the generation born at $t = 0$, the generation born at t will be of size Ξ^t.

An individual's life has three phases: Employment, followed by unemployment, which terminates in death. Transitions to unemployment and to death follow Poisson processes with constant arrival rates. The probability that an employed worker will become unemployed is $\mho$ (while the probability of remaining employed is denoted as the cancellation of unemployment, $\cancel{\mho} \equiv 1 - \mho$). The probability that an unemployed individual dies before the next period is Φ; the probability of survival is denoted by the cancellation of death,

[5] Below, we explore the consequenses of introducing partial or complete insurance against unemployment risk.

[6] For the sovereign wealth fund interpretation of our model, this risk should be interpreted as reflecting a radical reduction in the purchasing power of the country's exports, e.g. a commodity price collapse for a commodity-based exporter.

[7] Our first appendix contains a list of our model's parameters and variables and their definitions, to aid the reader in keeping track.

$\not\Phi \equiv 1 - \Phi$. (Individuals are permitted to die only after they have become unemployed.) The employed population, $\mathcal{E}$, and the unemployed population, $\mathcal{U}$ thus satisfy the dynamic equations,

$$\begin{aligned} \mathcal{E}_t - \mathcal{E}_{t-1} &= \Xi^t - \mho\mathcal{E}_{t-1} \\ \mathcal{U}_t - \mathcal{U}_{t-1} &= \mho\mathcal{E}_{t-1} - \Phi\mathcal{U}_{t-1}. \end{aligned}$$

The first equation says that the net increase in the employed population is equal to the size of the newborn generation minus the flow of previously employed workers going to unemployment. The second says that the net increase in the unemployed population is equal to the number of newly unemployed workers minus the previously unemployed workers who exit life. It follows that the employed and unemployed populations are respectively given by

$$\begin{aligned} \mathcal{E}_t &= \frac{\Xi^{t+1}}{\Xi - \not\mho} \\ \mathcal{U}_t &= \frac{\mho\Xi^{t+1}}{(\Xi - \not\Phi)(\Xi - \not\mho)}. \end{aligned}$$

Total labor supply is the number of workers times the average labor supply per worker,

$$\boldsymbol{L}_t = \mathcal{E}_t \ell. \tag{4}$$

It then follows from (1) and (3) that in the balanced growth equilibrium capital and output grow by the same factor $\Xi\mathsf{G}$ in every period,

$$\frac{\boldsymbol{K}_{t+1}}{\boldsymbol{K}_t} = \frac{\boldsymbol{P}_{t+1}}{\boldsymbol{P}_t} = \Xi\mathsf{G}.$$

Finally, the real wage is equal to the marginal product of labor,

$$\mathsf{W}_t = (1-\alpha)\frac{\boldsymbol{P}_t}{\boldsymbol{L}_t},$$

which grows by the factor G in every period.

Perfect capital mobility means that residents and non-residents can hold domestic capital, and can hold foreign assets or issue foreign liabilities. The main variable of interest is $\boldsymbol{N}_t$, the aggregate net foreign assets of the economy at the end of period t. As a matter of accounting, the country's net foreign asset position is equal to the difference between the value of its total wealth and the domestic quantity of physical capital,

$$\boldsymbol{N}_t = \frac{\boldsymbol{S}_{t+1}}{\mathsf{R}} - \boldsymbol{K}_{t+1}, \tag{5}$$

where $\boldsymbol{S}_{t+1}/\mathsf{R}$ is the present discounted value at the end of period t of next period's total wealth (see the appendix). The dynamics of $\boldsymbol{S}_t$ are determined by the consumption/saving choices of individuals, to which we now turn.

2.2 The Microeconomic Consumer's Problem

Using lower-case variables for individuals, the period-t budget constraint of individual i relates current consumption $\boldsymbol{c}$ to current labor income and current and future wealth $\boldsymbol{s}$,[8]

$$\frac{\boldsymbol{s}_{t+1}(i)}{\mathsf{R}} + \boldsymbol{c}_t(i) = \boldsymbol{s}_t(i) + \overbrace{\varepsilon_t(i)\ell_t(i)\mathsf{W}_t}^{\text{labor income}}, \tag{6}$$

where ε is a dummy variable indicating the consumer's employment state. Everyone in this economy is either employed (state 'e'), in which case $\varepsilon = 1$, or unemployed (state 'u'), in which case $\varepsilon = 0$, so that for unemployed individuals labor income is zero.

Dropping the i index for convenience, we assume that the labor productivity ℓ of each individual worker who remains employed grows by a factor $\mathbf{X}$ every period because of increasing eXperience,

$$\ell_t = \mathbf{X}^t \ell_0, \tag{7}$$

where ℓ_0 is the labor supply of a newborn individual. $\mathbf{X}$ can be interpreted as the factor that governs the rate at which an individual's work skills improve, perhaps as a result of human capital accumulation, whereas G is the factor by which productivity grows in the economy as a whole, perhaps due to societal knowledge accumulation and technological advance (Mankiw (1995)). This means that for a consumer who remains employed, labor income will grow by factor

$$\Gamma \equiv \mathsf{G}\mathbf{X}.$$

Following Tochè (2005), unemployment means a complete and permanent destruction of the individual's human wealth: Once a person becomes unemployed, that person can never become employed again (i.e. if $\varepsilon_t = 0$ then $\varepsilon_{t+1} = 0$). Thus, unemployment could also be interpreted as retirement (we calibrate the model so that the average length of the working life is forty years). Employed consumers face a constant risk $\mho$ of becoming unemployed (regardless of their age; like death in Blanchard (1985)).

Consumers have a CRRA felicity function $\mathrm{u}(\bullet) = \bullet^{1-\rho}/(1-\rho)$ and discount future utility geometrically by β per period. We assume that unemployed workers have access to life insurance *a la* Blanchard (1985) and can convert their wealth into annuities. As shown in the appendix, the solution to the unemployed consumer's optimization problem is,

$$\boldsymbol{c}_t^u = \kappa^u \boldsymbol{s}_t^u, \tag{8}$$

where the u superscript now signifies the consumer's (un)employment status, and κ^u, the marginal propensity to consume for the perfect foresight unemployed Blanchardian

[8] A brief terminological rant: We generally call $\boldsymbol{s}$ 'wealth' rather than 'savings' because of the confusion induced by the words 'saving' and 'savings'; saving is a behavior (a flow; a choice not to spend some portion of current income) while savings is a stock of resources that result from past saving flows. Authors in this literature frequently misapply the stock word savings for the flow word saving or vice versa, so we prefer to use the word 'wealth' which unambiguously denotes a stock.

consumer, is given by

$$\kappa^u \;=\; 1 - \frac{(\beta \mathsf{R})^{1/\rho}}{\mathsf{R}/\cancel{\Phi}}. \tag{9}$$

We assume $\kappa^u > 0$, which is necessary for the unemployed consumer's problem to have a well-defined solution.

The Euler equation for an employed worker is,

$$(\boldsymbol{c}_t^e)^{-\rho} = \beta \mathsf{R} \left[\cancel{\mho}(\boldsymbol{c}_{t+1}^e)^{-\rho} + \mho(\boldsymbol{c}_{t+1}^u)^{-\rho} \right],$$

Now define nonbold variables as the boldface equivalent divided by the level of permanent labor income for an employed consumer, e.g. $c_t^e = \boldsymbol{c}_t^e/(\mathsf{W}_t \ell_t)$, and rewrite the consumption Euler equation as

$$1 = \frac{\beta \mathsf{R}}{\Gamma^\rho} \left\{ (1-\mho) \left(\frac{c_{t+1}^e}{c_t^e} \right)^{-\rho} + \mho \left(\frac{c_{t+1}^u}{c_t^e} \right)^{-\rho} \right\}. \tag{10}$$

Following Carroll (2004), it will be useful to define a 'growth patience factor':

$$\text{Þ}_\Gamma \;=\; \frac{(\beta \mathsf{R})^{1/\rho}}{\Gamma}, \tag{11}$$

which is the factor by which c^e would grow in the perfect foresight version of the model with labor income growth factor Γ. We will assume that the growth patience factor Þ_Γ is less than one

$$\text{Þ}_\Gamma \;<\; 1. \tag{12}$$

This condition—which Carroll (2004) dubs the perfect foresight 'growth impatience condition' (GIC)—ensures that a consumer facing no uncertainty is sufficiently impatient that his wealth-to-permanent-income ratio will fall over time. If $\beta \mathsf{R} = 1$, this condition is satisfied if individual labor income grows over time.

Using this, (10) can be written as

$$\left(\frac{c_{t+1}^e}{c_t^e} \right) \;=\; \text{Þ}_\Gamma \left\{ 1 + \mho \left[\left(\frac{c_{t+1}^e}{c_{t+1}^u} \right)^{\rho} - 1 \right] \right\}^{1/\rho}. \tag{13}$$

The budget constraint of an employed worker can be written, in normalized form, as

$$s_{t+1}^e = (\mathsf{R}/\Gamma)\left(s_t^e - c_t^e + 1\right). \tag{14}$$

Using this equation and $c_{t+1}^u = \kappa^u s_{t+1}^e$ to substitute out c_{t+1}^u from (13) (since a worker who becomes unemployed in period $t+1$ starts with wealth s_{t+1}^e), we have

$$c_{t+1}^e = \text{Þ}_\Gamma \cancel{\mho}^{1/\rho} c_t^e \left[1 - \mho \left(\frac{1-\kappa^u}{\kappa^u} \frac{c_t^e}{s_t^e - c_t^e + 1} \right)^{\rho} \right]^{-1/\rho}. \tag{15}$$

Equations (14) and (15) characterize the dynamics for the pair of variables (s_t^e, c_t^e). It is possible to show (see the Appendix) that those dynamics are saddle-point stable, and that the ratio of wealth to income, s_t^e, converges toward a positive limit, the target wealth-to-income ratio, denoted by $\check{s}$. Figure 1 presents the phase diagram.

Setting $c_{t+1}^e = c_t^e$ and $s_{t+1}^e = s_t^e$ in equations (14) and (15), simple manipulations give an explicit formula for the target wealth-to-income ratio,[9]

$$\check{s} = \left[\frac{\Gamma}{\mathsf{R}} - 1 + \kappa^u \left(1 + \frac{\text{Þ}_\Gamma^{-\rho} - 1}{\mho}\right)^{1/\rho}\right]^{-1}. \tag{16}$$

Here is the intuition behind the target wealth ratio: On the one hand, consumers are growth-impatient. This prevents their wealth-to-income ratio from heading off to infinity. On the other hand, consumers have a precautionary motive that intensifies more and more as the level of wealth gets lower and lower. At some point the precautionary motive gets strong enough to counterbalance impatience. The point where impatience matches prudence defines the target wealth-to-income ratio.

Expression (16) encapsulates several of the key economic effects captured by the model. The human wealth effect of growth is captured by the Γ and Þ_Γ terms. Increasing Γ will decrease the growth patience factor Þ_Γ and therefore reduce the target level of wealth. The human wealth effect of interest rates is similarly captured by the R term. An increase in the worker's patience (an increase in β and in the growth patience factor Þ_Γ) boosts the target level of wealth. Finally, an increase in unemployment risk increases the target level of precautionary wealth. Those comparative statics results can be summarized as

$$\begin{aligned} \frac{\partial \check{s}}{\partial \mho} &> 0, \\ \frac{\partial \check{s}}{\partial \beta} &> 0, \\ \frac{\partial \check{s}}{\partial \Gamma} &< 0. \end{aligned} \tag{17}$$

The response of the target asset ratio to the risk aversion parameter ρ is less straightforward. On the one hand, higher risk aversion enhances the demand for precautionary reserves. On the other hand, it also implies that consumption is less elastic intertemporally. We show in the appendix that if $\beta\mathsf{R} \leq 1$ then the target asset ratio increases with the level of risk aversion,

$$\frac{\partial \check{s}}{\partial \rho} > 0. \tag{18}$$

The response of $\check{s}$ to R is also ambiguous, reflecting the ambiguity of how wealth depends on the interest rate even in a deterministic setting. One can show that if $\rho \leq 1$, then the target level the wealth-to-income ratio increases with the interest rate. For $\rho > 1$, however, the sign of the response of $\check{s}$ to R could be positive or negative.

[9]See Carroll (2007) for an explicit derivation.

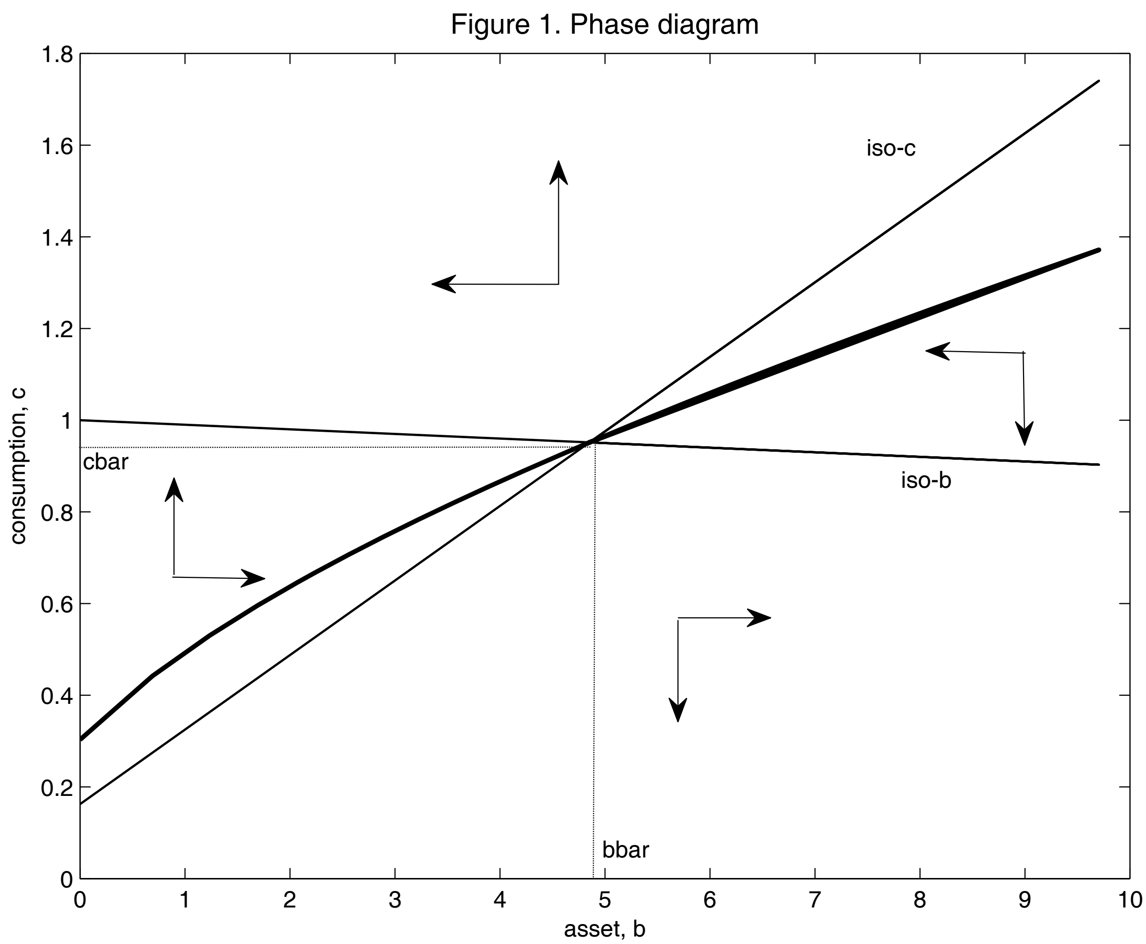

Figure 1: Phase Diagram

2.3 Foreign Assets

We now aggregate the individual balance sheets to find the country's aggregate net foreign assets. We first present a general formula that aggregates the resources of all generations of employed and unemployed workers. We then specialize this formula under two assumptions about the initial 'stake' of newborns in the economy. (We use 'stake' to designate a transfer received by newborns). In the model without stakes, the newborns do not receive any transfer and must accumulate wealth through their own frugality. The microeconomic consumer's problem, therefore, is the one we have described in the previous section. In the model with stakes, the newborns receive a transfer that puts their wealth-to-income ratio at par with the rest of the population. The main advantage of the model with stakes is that it is more tractable and yields a closed-form expression for the ratio of net foreign assets to GDP.

2.3.1 Aggregating Individual Wealths

First, let us focus on the wealth of the employed households. Simple computations (reported in the appendix) show that the ratio of employed workers' wealth to output is given by,

$$\frac{\boldsymbol{S}^e_t}{\boldsymbol{P}_t} = (1-\alpha)\left(1 - \underbrace{\frac{\not\mho \mathbf{X}}{\Xi}}_{\equiv \Lambda}\right)\sum_{n=0}^{+\infty} \Lambda^n s^e_{t,t-n}, \tag{19}$$

where $s^e_{t,t-n}$ is the wealth-to-income ratio at t of the workers born at $t-n$, and Λ is the factor by which the share of a generation in total labor supply shrinks every period. Equation (19), thus, says that the ratio of workers' wealth to output is the average of the individual wealth-to-labor-income ratios over the past generations, weighted by the share of each generation in total labor supply and by the share of labor income in total output.

Second, let us consider the wealth of the unemployed households (managed by the Blanchardian life insurance company). The aggregate wealth of unemployed households satisfies the dynamic equation,

$$\boldsymbol{S}^u_{t+1} = \mathsf{R}(\boldsymbol{S}^u_t - \boldsymbol{C}^u_t) + \mho \boldsymbol{S}^e_{t+1},$$

where the first term on the right-hand side reflects the accumulation of wealth by the unemployed households, and the second term is the wealth of the newly-unemployed households. The unemployed households consume a constant fraction of their wealth, $\boldsymbol{C}^u_t = \kappa^u \boldsymbol{S}^u_t$, so that the equation above can be rewritten,

$$\boldsymbol{S}^u_{t+1} = \mathsf{R}(1-\kappa^u)\boldsymbol{S}^u_t + \mho \boldsymbol{S}^e_{t+1}. \tag{20}$$

This equation fully characterizes the dynamics of the unemployed households' wealth ratio for a given path for the employed workers' wealth ratio.

Third, let us consider a steady state in which the wealth of the employed is a constant fraction of GDP, $\boldsymbol{S}^e/\boldsymbol{P}$. Then equation (20) and $\boldsymbol{P}_{t+1}/\boldsymbol{P}_t = \Xi\mathsf{G}$ imply that the ratio of wealth to GDP is also constant for unemployed households,[10]

$$\frac{\boldsymbol{S}^u}{\boldsymbol{P}} = \frac{\mho\Xi\mathsf{G}}{\Xi\mathsf{G} - \not{\Phi}(\beta\mathsf{R})^{1/\rho}}\frac{\boldsymbol{S}^e}{\boldsymbol{P}}. \tag{21}$$

The ratio of net foreign assets to GDP is obtained by subtracting domestic capital from domestic wealth. Using (3), (5), (21), $\boldsymbol{P}_{t+1}/\boldsymbol{P}_t = \Xi\mathsf{G}$, and $\boldsymbol{S}_t = \boldsymbol{S}_t^e + \boldsymbol{S}_t^u$, the ratio of net foreign assets to GDP is given by,

$$\frac{\boldsymbol{N}}{\boldsymbol{P}} = \frac{\Xi\mathsf{G}}{\mathsf{R}}\left(1 + \frac{\mho\Xi\mathsf{G}}{\Xi\mathsf{G} - \not{\Phi}(\beta\mathsf{R})^{1/\rho}}\right)\frac{\boldsymbol{S}^e}{\boldsymbol{P}} - \Xi\mathsf{G}\left(\frac{\alpha}{\mathsf{R} - \daleth}\right). \tag{22}$$

The only missing piece is the ratio of employed workers' wealth to GDP, $\boldsymbol{S}^e/\boldsymbol{P}$. We now present two ways of endogenizing this variable.

2.3.2 No Stake

The most natural assumption is that newborns enter the economy with zero wealth, and must save some of their income to ensure that they do not starve if they become unemployed. In this case, analysis must be performed using simulation methods, because households of different ages will have different ratios of wealth to income. (With a concave and nonanalytical consumption function, analytical aggregation cannot be performed.)

In this version of the model, each individual is faced with exactly the same problem as in section 2.2. Let us denote by $s^e(n)$ the level of normalized wealth held at the beginning of period n of the individual's life in the problem of section 2.2. We assume that the individual starts his life with zero wealth, $s^e(0) = 0$. In other words, $s^e(n)_{n=0,1,2,..}$ is the optimal time path of the individual's wealth. Then we can replace $s^e_{t,t-n}$ by $s^e(n)$ in equation (19),

$$S^e = \frac{\boldsymbol{S}^e}{\boldsymbol{P}} \;=\; (1-\alpha)(1-\Lambda)\sum_{n=0}^{+\infty}\Lambda^n s^e(n). \tag{23}$$

The ratio of workers' wealth to GDP is constant, and can be computed numerically based on the path $s^e(n)_{n=0,1,\ldots}$. Note that this ratio is lower than $(1-\alpha)\check{s}$, since it is a weighted average of $(1-\alpha)s^e(n)$, which converge toward $(1-\alpha)\check{s}$ from below.

2.3.3 A 'Stake' That Yields a Representative Agent

We now consider a version of the model in which an exogenous redistribution program guarantees that the behavior of employed households can be understood by analyzing the

[10]This expression assumes $\Xi\mathsf{G} > \not{\Phi}(\beta\mathsf{R})^{1/\rho}$. Otherwise $\boldsymbol{S}_t^u/\boldsymbol{P}_t$ grows without bound.

actions of a "representative employed agent." This will be achieved by the introduction of lump-sum transfers that ensure that the newborn individuals are endowed with the same wealth-to-income ratio that older generations already hold. This is explicitly *not* an inheritance, as we have assumed that individuals have no bequest motive and newborns are unrelated to anyone in the existing population. Our motivation is largely to make the model more tractable, rather than to represent an important feature of the real world; hence, we perform simulations designed to show that the characteristics of the model with no 'stake' are qualitatively and quantitatively similar to those of the more tractable model with a carefully chosen 'stake.'

The details of the model with stakes are given in the appendix. The transfer ensures that the workers have the same wealth-to-income ratio at all times.[11] Thus one can replace $s^e_{t,t-n}$ by s^e_t in equation (19), which gives,

$$S^e_t = \frac{\boldsymbol{S}^e_t}{\boldsymbol{P}_t} = (1-\alpha)s^e_t, \tag{24}$$

where S^e_t follows the same saddle-point dynamics as for a single agent (adjusted for the transfer).

One can show (see the appendix) that in the long run, s^e_t converges to

$$\check{\check{s}} = \left[\frac{\Gamma}{\mathsf{R}} - \frac{1}{2-\Lambda} + \kappa^u \left(1 + \frac{\text{Þ}_{\Gamma}^{-\rho} - 1}{\mho}\right)^{1/\rho}\right]^{-1} \tag{25}$$

so that (24) implies a closed-form expression for the ratio of workers' wealth to GDP,

$$\check{\check{S}} = \frac{\boldsymbol{S}^e}{\boldsymbol{P}} \;=\; (1-\alpha)\check{\check{s}}. \tag{26}$$

This expression can be plugged into equation (22) to find the ratio of net foreign assets to GDP.

It is interesting to compare formula (25) with the one that we obtained for an individual in the model without stakes—equation (16). Since $\Lambda < 1$ we have $\check{\check{S}} < \check{S}$. Thus equations (16) and (25) both predict that the ratio of wealth to GDP is lower than $(1-\alpha)\check{S}$, but in the new formula this comes from the fact that the target wealth-to-income ratio is lowered by the tax, rather than from the fact that the wealth-to-income ratio is lower for younger workers.

We will show below that the model with stakes provides a good approximation to the model with no stake. But the model with stakes has several advantages. First, the transition dynamics can be characterized using equation (24). In the model without

[11] More precisely, the transfer ensures that if all workers have the same ratio s^e in period t, then this is also true in period $t+1$. So one simply needs to assume that all workers had the same ratio s^e at some point in the past for this to be true in all periods. This would be the case, for example, if the country started with a first generation at some distant period in the past.

stakes the transition dynamics involve an infinite state space as the wealth-to-income ratio must be tracked separately for each generation. Second, the model with stakes gives a closed-form expression for the steady state ratio of foreign assets to GDP. This makes it possible to study analytically how the ratio of foreign assets to GDP depends on the exogenous parameters of the model. With formula (23), by contrast, such a study must rely on numerical simulations.

3 Calibration and Simulation

3.1 Benchmark Calibration and Sensitivity Analysis

Our benchmark calibration is reported in Table 1. The value for the unemployment probability, $\mho$, implies that a newborn worker expects to be employed for 40 years. The value for the probability of death, Φ, implies that the expected lifetime of a newly unemployed worker is 20 years.

Table 1

α	$\daleth$	Ξ	G	R	β^{-1}	$\mathbf{X}$	$\mho$	ρ	Φ
0.3	0.94	1.01	1.04	1.04	1.04	1.01	0.025	2	0.05

Note: For A Reminder of Parameter Definitions, See Appendix A

The phase diagram in Figure 1 was constructed under the benchmark calibration. The long-run levels of S^e and C^e are given by $\check{S} = 4.85$ and $\check{C} = 0.95$. The time paths for S^e_t and C^e_t are shown in Figure 2. The convergence to the targets is relatively rapid. The individual saves more than one third of his income on average in the first ten years of his life, after which his wealth-to-income ratio already exceeds two thirds of the target level. The wealth-to-income ratio reaches 99 percent of the target level after 40 years (the average duration of employment).

For the benchmark calibration we find: $\boldsymbol{K}/\boldsymbol{P} = 3$, $\boldsymbol{N}/\boldsymbol{P} = 0.084$ in the model with no stakes, and $\boldsymbol{N}/\boldsymbol{P} = 0.719$ in the model with stakes. These levels have the right order of magnitude (in view of the fact that most countries have a ratio of foreign assets to GDP between minus and plus 100 percent of GDP, based on the database of Lane and Milesi-Ferretti (2007)).

Figure 3 shows the sensitivity of the ratio of net foreign assets to GDP to changes in ρ, $\mho$, G and R. First, we observe that the model with stakes gives results that are higher than the model without stakes, but generally provides a good approximation for the variation of the net foreign assets with respect to the main parameters.

The variation with respect to risk aversion and the growth rate confirm theoretical properties derived earlier. The ratio of foreign assets to GDP is increasing with ρ, consistent with (18) since $\beta\mathsf{R} \leq 1$. The foreign assets ratio decreases with G, as predicted by

(17). The ratio of foreign assets to GDP also increases with the unemployment probability.[12] Finally, the foreign asset ratio is increasing with R, mainly because of the impact of higher interest rates in reducing the ratio of physical capital to output. The wealth-to-GDP ratio is not very sensitive to R, which is consistent with the ambiguity of the model prediction if $\rho > 1$.

3.2 Comparison with the Ramsey Model

The Ramsey model corresponds to the particular case where the economy is populated by one representative infinitely-lived worker ($\Xi = 1$ and $\mho = 0$). Thus, one might expect our model to yield the same results as the Ramsey model in the limiting case as unemployment risk goes to zero ($\mho \to 0$).

In fact this is not the case. The predictions of our model for net foreign assets and capital flows exhibit a discontinuity at $\mho = 0$. To see this, note that taking the limit of equation (16) gives

$$\lim_{\mho \to 0} \check{S} = 0,$$

so that the ratio of total domestic wealth to GDP goes to zero as the risk of unemployment becomes vanishingly small,[13]

$$\lim_{\mho \to 0} \frac{\boldsymbol{S}}{\boldsymbol{P}} = 0,$$

implying that the ratio of foreign assets to GDP is equal to minus the ratio of capital to output,

$$\lim_{\mho \to 0} \frac{\boldsymbol{N}}{\boldsymbol{P}} = -\frac{\boldsymbol{K}}{\boldsymbol{P}}. \tag{27}$$

The Ramsey model does not yield the same formula. If the unemployment risk is strictly equal to zero ($\mho = 0$), we must assume $\Gamma < \mathsf{R}$ for the intertemporal income of the worker to be well-defined and finite.[14] In this case income growth is the same at the individual level and at the aggregate level. We can also assume, without loss of generality, that $\mathbf{X} = 1$, so that $\Gamma = \mathsf{G}$. Then it is possible to show that the asymptotic ratio of total net foreign assets to GDP is given by,

$$\lim_{t \to +\infty} \frac{\boldsymbol{N}_t}{\boldsymbol{P}_t} = -\frac{\boldsymbol{K}}{\boldsymbol{P}} - \frac{1-\alpha}{1-\mathsf{G}/\mathsf{R}}. \tag{28}$$

(see the appendix).

Comparing (27) with (28) shows that the ratio of foreign assets to GDP is smaller in the Ramsey model. In fact, it is much smaller for plausible calibrations of the model.

[12]Note that the death probability Φ was adjusted to keep the total expected lifetime of an individual equal to sixty years, i.e., $\mho^{-1} + \Phi^{-1} = 60$.

[13]This results from equation (25) in the model with stakes. In the model with no stakes this results from the fact that $s^e(n) \leq \check{s}$ converges to zero for all n.

[14]Note that this condition is not satisfied by the benchmark calibration in Table 1.

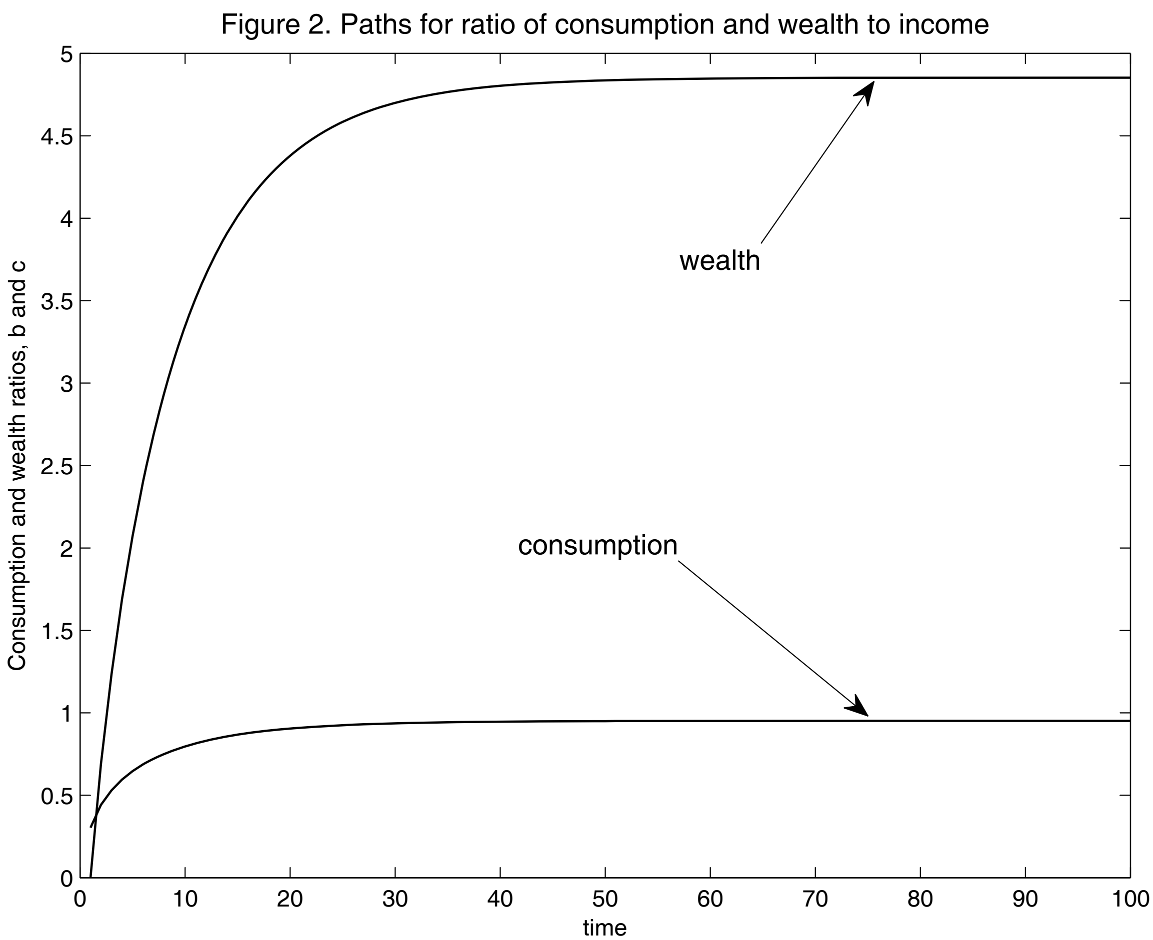

Figure 2: Transition Paths

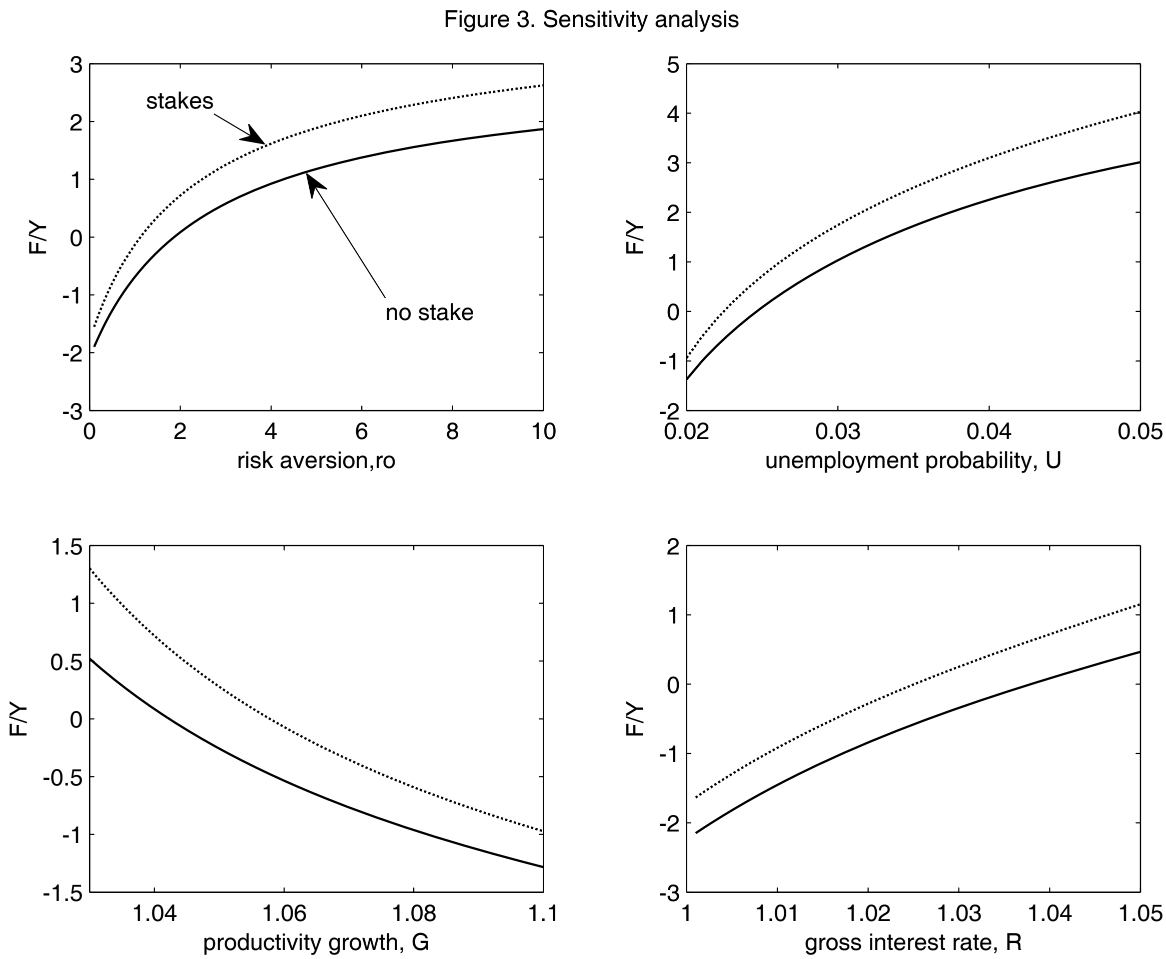

Figure 3: Sensitivity To Parameters

The ratio of gross foreign liabilities to GDP implied by the Ramsey model is close to 70 if $\mathsf{R} = 1.04$ and $\mathsf{G} = 1.03$, and goes to infinity as G converges to R from below. The growth impatience condition, which is necessary for the workers to have a finite target for their wealth to income ratio when they are vulnerable to unemployment, makes the infinitely-lived Ramsey consumer willing to borrow a lot against his future income.

The intuition for the discontinuity is that a consumer with CRRA utility will never allow wealth to fall to zero if there is a possibility of becoming permanently unemployed, because unemployment with zero wealth yields an infinitely negative level of utility (if $\rho > 1$). This is the reflection, in the international macroeconomic context, of a result long understood in the precautionary saving literature: Perfect foresight solutions are not robust to the introduction of large uninsurable noncapital income shocks, even if those shocks occur with low probability.

3.3 Social Insurance

The model assumes that the income of an unemployed worker falls to zero. This is a reasonable assumption for a country in which unemployed and retired workers receive no social transfer (i.e., in which there are no unemployment benefits and the retirement system is entirely based on capitalization). However, many countries have such transfers, and it is interesting to see their impact on foreign asset accumulation in our model. We consider now the consequences if the government creates a balanced-budget partial 'unemployment insurance' system.

Our definition of partial insurance starts by assuming that the 'true' labor income process is the one specified above, but the government interferes with this process by transferring to the workers who become unemployed in period t a multiple ς of the labor income that they would have received if they had remained employed. The social insurance of our model could be interpreted as an unemployment benefit or as a pay-as-you-go retirement benefit.

The wealth of a newly-unemployed worker now includes the payment from the insurance scheme, so that equation (8) becomes:

$$\boldsymbol{c}_t^u = \kappa^u(\boldsymbol{s}_t^u + \varsigma \mathsf{W}_t \ell_t) = \kappa^u(s_t^u + \varsigma)\mathsf{W}_t \ell_t.$$

We introduce social insurance in the model with stakes.[15] Going through the same steps as before, one can compute the target wealth-to-GDP ratio as

$$\check{\check{S}}(\varsigma) \;=\; \left\{1 - \varsigma\left[\frac{\mho \mathbf{X}}{\Xi} + \kappa^u\left(1 + \frac{\text{Þ}_\Gamma^{-\rho} - 1}{\mho}\right)^{1/\rho}\right]\right\}\check{\check{s}}, \tag{29}$$

where $\check{\check{s}}$ is the asset ratio without insurance, given by (25). The target wealth-to-income ratio is (linearly) decreasing with ς, as insurance provides a substitute to precautionary

[15] Introducing social insurance in the model without stakes raises no conceptual problems, but does not yield a closed-form solution.

wealth. The formula for $\boldsymbol{N}/\boldsymbol{P}$ remains (27), with the ratio of workers' wealth to GDP given by,

$$\frac{\boldsymbol{S}^e}{\boldsymbol{P}} = (1-\alpha)\grave{\check{S}}(\varsigma). \tag{30}$$

Figure 4 shows how the ratio of foreign assets to GDP, $\boldsymbol{N}/\boldsymbol{P}$, varies with ς. The ratio decreases from 0.72 when there is no insurance to negative values when ς exceeds 1 year of the worker's wage. The desired level of foreign assets is thus quite sensitive to the level of social insurance.

4 Applications

Although the model is very stylized, plausible calibrations can predict ratios of foreign assets to GDP that are close to the levels observed in the real world. This fact obviously does not constitute a test of the model (which would go beyond the scope of this paper), but it suggests that the quantitative implications of the model cannot be dismissed *prima facie* as irrelevant. This section illustrates how our framework can be applied by looking at two questions that have been discussed in recent policy debates and academic research: The relationship between economic development and capital flows, and the long-run consequences of resorbing global imbalances.

4.1 Economic Development and Capital Flows

Many observers have noted the paradox that international flows of capital have recently been going "upstream" from developing countries (especially in Asia and most notably China) to the United States. The case of China, which has caused so much consternation recently, is merely the latest and largest example of a long-established pattern: Over long time periods and in large samples of developing countries, the countries that grow at a higher rate tend to export more capital (see the evidence cited in footnote 1), a fact that is difficult to reconcile with the standard neoclassical model of growth (Carroll, Overland, and Weil (2000); Gourinchas and Jeanne (2007); Prasad, Rajan, and Subramanian (2007); Sandri (2008)). Can our model shed light on this puzzle?

4.1.1 Transitions

In this section we look at the correlation between economic growth and capital flows in a given country over time. We assume that the small open economy enjoys an economic "take-off," defined as a permanent increase in the growth rate of productivity. However, the rate of growth is not the only thing that increases at the time of the transition: The idiosyncratic unemployment risk rises too. An increase in idiosyncratic risk has been observed in many transition countries as they adopt market systems, a development

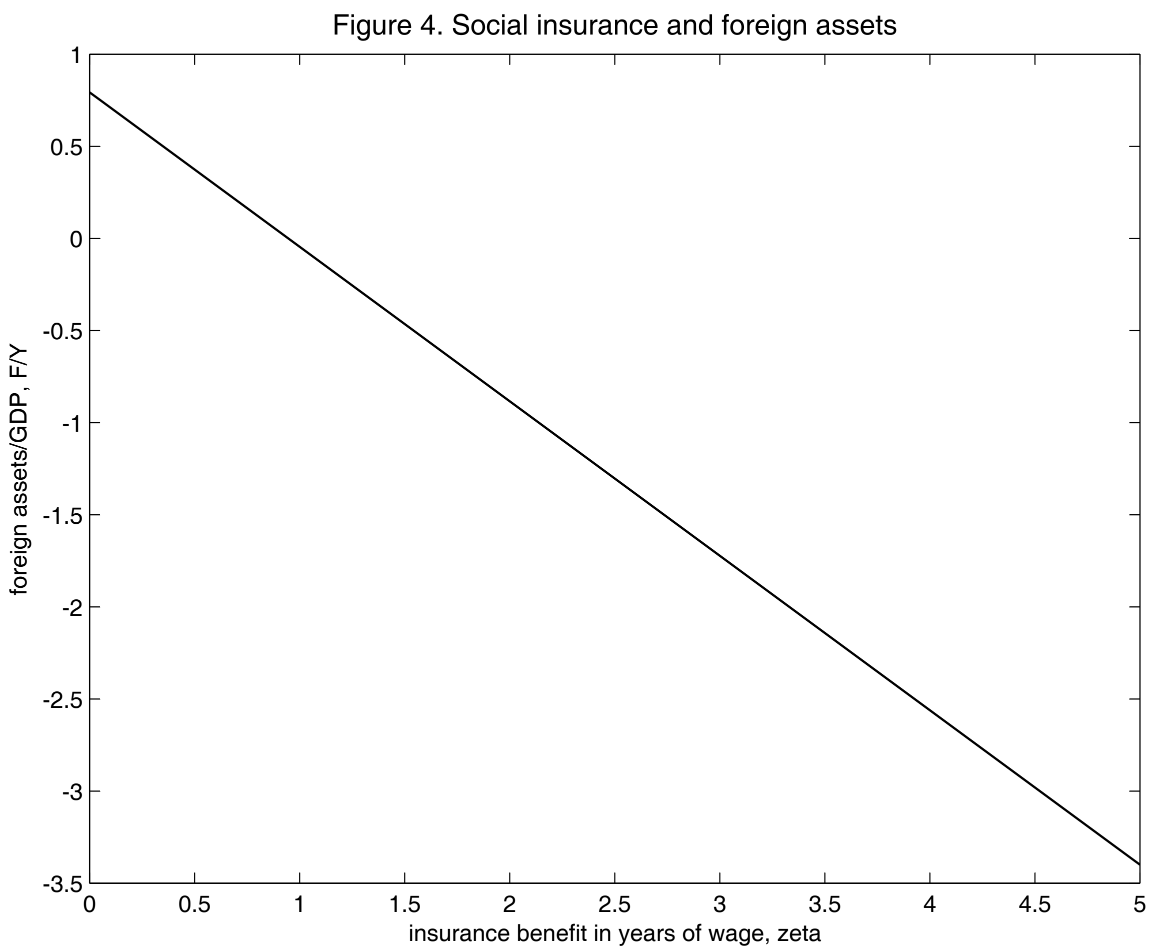

Figure 4: Social Insurance

that has not been associated, in most countries, with a corresponding increase in social insurance. The rise in idiosyncratic risk has been fingered as a reason for the very high saving rate in China (see, e.g., Chamon and Prasad (2008) and the references therein).

More formally, we assume that the economy starts from a steady state with constant levels for the productivity growth rate and the unemployment probability, G_b and $\mho_b$. At time 0, those variables unexpectedly jump to higher levels, $\mathsf{G}_a > \mathsf{G}_b$ and $\mho_a > \mho_b$. The subscripts b and a respectively stand for "before" and "after" the transition. The death probability is adjusted so as to keep the expected lifetime of an individual equal to 60 years.

Note that in order to benefit the domestic population, the transition must strictly increase the expected present value of an individual's labor income, given by

$$\sum_{n=0}^{+\infty} \mathsf{R}^{-n} \not\mho^{n} \ell_{t+n} \mathsf{W}_{t+n} = \frac{\mathsf{R}}{\mathsf{R} - \mathsf{G}\mathbf{X}\not\mho} \ell_t \mathsf{W}_t.$$

Thus one must have,

$$\mathsf{G}_a \not\mho_a > \mathsf{G}_b \not\mho_b. \tag{31}$$

The increase in the idiosyncratic risk, in other terms, should not be so large relative to the increase in the growth rate as to decrease workers' expected present value of labor income.

We consider the model with stakes, so that the transition dynamics for aggregate wealth can be derived from those for the representative agent. There is no social insurance. The appendix explains how the path of the main relevant variable can be computed. We are interested in whether capital tends to flow in or out of the country when the rate of growth picks up.

For the sake of the simulation, we assume that the growth rate increases from 2 percent to 6 percent in the transition, whereas the unemployment probability increases from 2 percent to 3 percent ($\mathsf{G}_b = 1.02$, $\mho_b = 0.02$, and $\mathsf{G}_a = 1.06$, $\mho_a = 0.03$). The other parameters remain calibrated as in Table 1.[16] Note that condition (31) is satisfied: indeed, the economic transition multiplies the expected present value of individual labor income by a factor 20. If the risk of unemployment did not increase with the transition, the expected net present value of labor income would become infinite.

Figure 5 shows the time paths for the growth rate, the ratio of net foreign assets to GDP and the ratio of capital outflows to GDP, with and without the increase in unemployment risk. Note that if unemployment risk increases, the growth rate takes time to converge to its new higher level because the rate of labor participation decreases over time, which dampens the acceleration of growth. The figure also shows that the increase in the idiosyncratic risk has a large impact on the desired level of net foreign assets in the long run—and thus on the direction of capital flows during the transition. If

[16] With $\mathsf{G} = 1.04$ and $\mho = 0.025$, our benchmark calibration is the average of the pre-transition and post-transition regimes.

the level of idiosyncratic risk remains the same, the pickup in growth lowers the long-run level of foreign assets from -23.9 percent to -135.6 percent of GDP, so that the higher growth rate is associated with a larger volume of capital inflows, both in the transition and in the long run. By contrast, if the level of idiosyncratic risk increases with growth, the long-run level of foreign assets increases to 69.7 percent of GDP, implying that higher growth is associated with capital outflows.[17] Thus, changes in the level of idiosyncratic risk have a first-order impact on the volume and direction of capital flows and may help explain the puzzling correlation between economic growth and capital flows that is found in the data.[18]

4.1.2 Steady States

We now look at what the model says about the steady-state correlation between growth and capital flows, rather than the correlation for a given country over time. The country exports capital if its net foreign asset position is positive ($\boldsymbol{N} > 0$), since the level of its net foreign assets increases over time with output. The ratio of capital outflows to output is given by,

$$\frac{\boldsymbol{N}_t - \boldsymbol{N}_{t-1}}{\boldsymbol{P}_t} = \frac{\boldsymbol{N}}{\boldsymbol{P}} \left(1 - \frac{1}{\Xi \mathsf{G}}\right). \tag{32}$$

On the one hand, with faster growth the target value of $(\boldsymbol{N}/\boldsymbol{P})$ will be smaller. On the other hand, a country that grows faster must export more capital to maintain a constant ratio of foreign assets to GDP (so the term in parentheses in (32) becomes larger).[19] Even if both initial and final values of $(\boldsymbol{N}/\boldsymbol{P})$ are positive, the sign of the relation between growth and net capital flows is theoretically ambiguous.

We calibrate the model with the pre-transition regime parameter values (i.e. with $\mathsf{G} = 1.02$ and $\mho = 0.02$). Figure 6 shows how the right-hand side of (32) varies with G under two different assumptions. The line "constant risk" shows the ratio of capital outflows to GDP if the only variable that changes is the growth rate. The line "increasing risk" is based on the assumption that the idiosyncratic risk increases linearly by 0.25 percent for every additional percent of growth. Points A, B, and C respectively correspond to the benchmark calibration, the pre-transition regime and the post-transition regime of the previous section.

Two findings stand out. First, if idiosyncratic risk does not increase with growth, the ratio of capital outflows to GDP is decreasing with growth. Second, if idiosyncratic

[17] The pattern shown in figure 5 is robust to plausible changes in the values of the parameters. For example, higher growth remains associated with capital outflows if the post-transition growth rate is 8 percent instead of 6 percent (keeping $\mho_a$ equal to 3 percent) or if the unemployment probability increases to 2.5 percent instead of 3 percent (keeping the post-transition growth rate equal to 6 percent).

[18] A similar point is made by Sandri (2008). In Sandri's model, the increase in the growth rate and in the level of idiosyncratic risk are jointly determined by the emergence of a class of entrepreneurs who invest in a risky technology.

[19] See Carroll (2000) for further discussion of the ability of precautionary models to generate a positive causality from growth to saving.

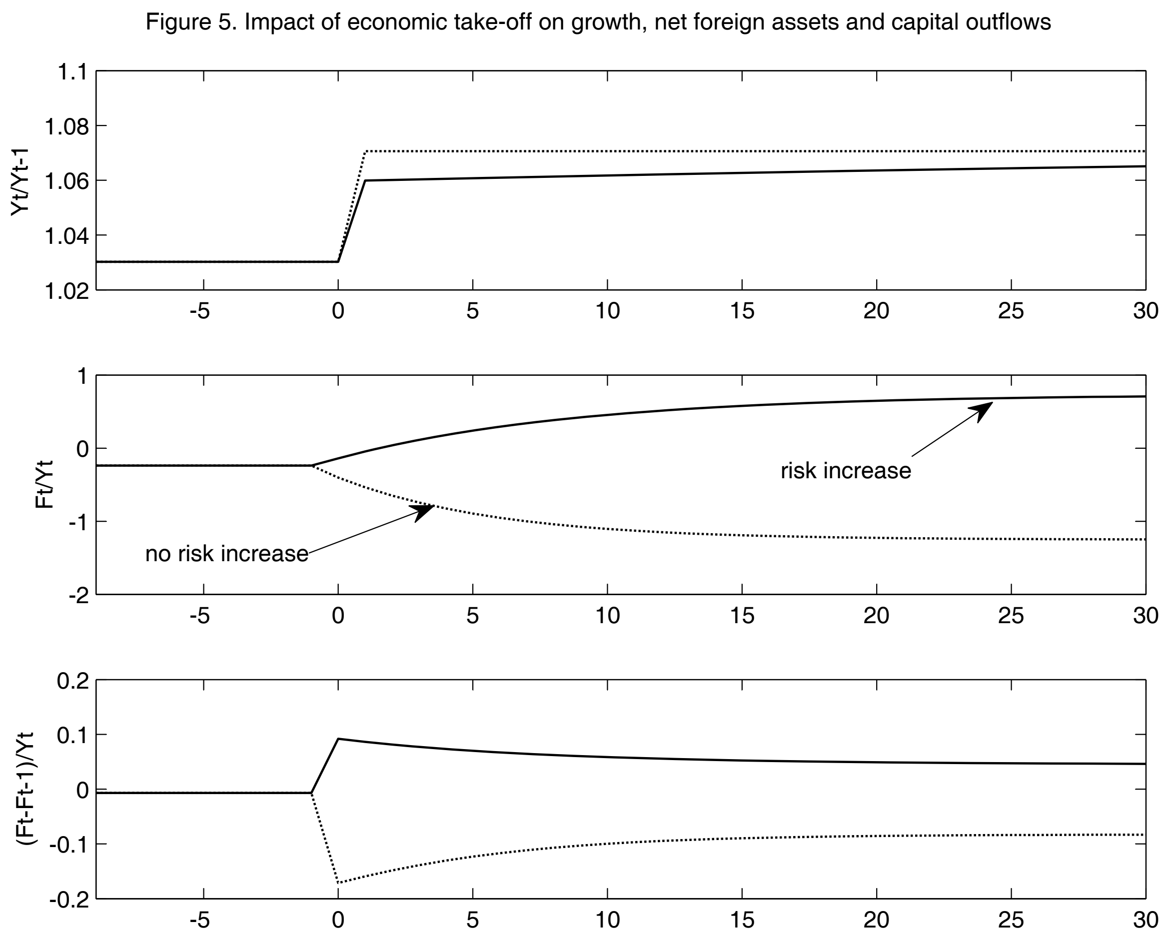

Figure 5: Transition Dynamics

risk increases with growth as we have specified, the ratio of capital outflows to output is positive, i.e., an increase in growth always causes the economy to export more capital (even if it grows at 10 percent per year). The relationship between the ratio of capital outflows to GDP and the growth rate is non-monotonic. Capital outflows increase (as a share of GDP) with the growth rate if the latter is lower than 6 percent. For higher levels of the growth rate the sign of the relationship is reversed.

4.2 Global Imbalances

The main counterpart for the accumulation of net foreign assets by developing countries has been the accumulation of net foreign liabilities by the United States. In a famous 2005 speech, Ben Bernanke hypothesized that the then-prevailing low level of world interest rates and high level of U.S. current account deficits could be due in part to this global "savings glut" (Bernanke (2005)). The U.S. authorities have subsequently argued (until the recent crisis) that an orderly resolution of global financial imbalances required the saving rate of Asian emerging market countries, most notably China, to decrease to more normal levels.

The small economy assumption is not appropriate for studying such large events. We present in this section a two-country general equilibrium version of the model that can be used instead. The model is solved only for the steady state equilibria, which means that we will be interested in the long-term consequences of particular policy experiments. We first look at a closed-economy version of the model.

4.2.1 Closed Economy

We consider an economy that has the same structure as the small open economy that we have considered so far, but is closed. Aggregate net foreign assets, thus, are equal to zero, which using (27) implies

$$\frac{1}{\mathsf{R}}\left(1+\frac{\mho\Xi\mathsf{G}}{\Xi\mathsf{G}-\cancel{\Phi}(\beta\mathsf{R})^{1/\rho}}\right)\frac{\boldsymbol{S}^{e}}{\boldsymbol{P}}=\frac{\alpha}{\mathsf{R}-\daleth}. \tag{33}$$

The left-hand side is the desired stock of wealth whereas the right-hand side is the desired stock of capital. The equality between the two endogenizes the steady-state interest rate. We assume that the desired stock of wealth comes from the model with stakes and social insurance, i.e., it is given by (30).

Figure 7 shows how the desired stocks of saving and of capital vary with the interest rate for the benchmark calibration and three different levels of social insurance $\varsigma = 0, 1$ and 2.[20] The desired level of capital is decreasing with the interest rate whereas the desired level of wealth is increasing with the interest rate. Note that the desired level

[20] We would obtain similar results by varying parameters other than the level of social insurance. We choose social insurance (as opposed to, say, taste parameters such as the level of risk aversion) because it is a policy variable that can be changed.

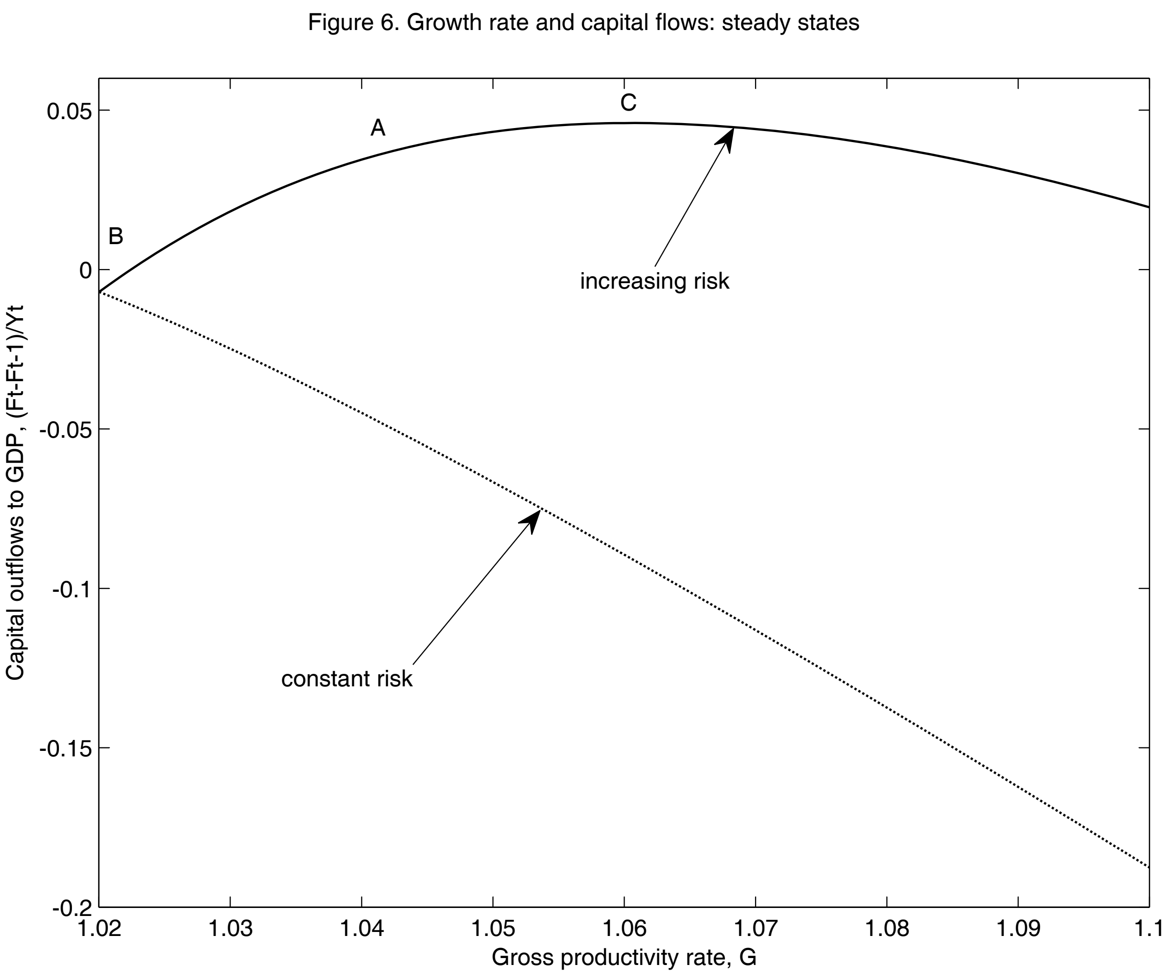

Figure 6: Capital Outflows

of capital is much more sensitive to the interest rate than the desired level of wealth. This implies that the decrease in desired wealth generated by higher social insurance is reflected almost one for one in a lower level of capital.

4.2.2 Long-term Impact of Reducing Global Imbalances

This section uses a two-country version of our model to investigate the long-run impact of a decrease in the desired stock of wealth outside of the United States. We consider a two-country world, where each country has the same structure as before. The two countries (denoted by h and f, respectively for "home" and "foreign") are identical, except for their sizes and their levels of social insurance (ς_h and ς_f). The shares of countries h and f in world output are respectively denoted by ω_h and ω_f. The two countries have the same growth rate, so that there is a well-defined balanced growth path in which each country maintains a constant share of global output.

The condition that global foreign assets must be equal to zero,

$$\boldsymbol{N}_h + \boldsymbol{N}_f = 0,$$

endogenizes the global interest rate R. Normalizing by the countries' GDP, this equation can be rewritten,

$$\omega_h \frac{\boldsymbol{N}_h}{\boldsymbol{P}_h} + \omega_f \frac{\boldsymbol{N}_f}{\boldsymbol{P}_f} = 0,$$

where for each country, $\boldsymbol{N}/\boldsymbol{P}$ is given by (27), with $\boldsymbol{S}^e/\boldsymbol{P} = (1-\alpha)\check{S}(\varsigma)$.

We consider the following experiment. Assume that the share of the home country in total GDP is 20 percent ($\omega_h = 0.2$ and $\omega_f = 0.8$), which is the right order of magnitude for the United States. Assume that $\varsigma_h > \varsigma_f$, implying that the home country has net liabilities because the desired ratio of wealth to GDP is lower at home than in the rest of the world. We assume the values $\varsigma_h = 1.5$ and $\varsigma_f = 0.75$, which implies $\mathsf{R} = 1.042$, $\boldsymbol{N}_h/\boldsymbol{P}_h = -0.512$ and $\boldsymbol{N}_f/\boldsymbol{P}_f = 0.128$ (the values of the other parameters remaining as in Table 1). The ratio of U.S. liabilities to GDP is higher than the current level (which is closer to 25 percent), but not implausible looking forward if the U.S. were to continue to maintain large current account deficits.

We then consider what would happen if global imbalances were resorbed as a consequence of a reduction in the desired wealth-to-income ratio in the rest of the world; this is achieved by increasing ς_f to the home level (from 0.75 to 1.5). Figure 8 shows the variation of the foreign assets and liabilities, as well as the global real interest rate and real wage (normalized by productivity). As expected, the net foreign assets of the home and foreign countries go to zero as the two countries converge to the same ratio of wealth to GDP. However, this convergence is achieved mainly by a decrease in global capital, which is reflected in an increase in the real interest rate (from 4.2 to 5.6 percent), and a decrease in the normalized real wage (by 5.4 percent).

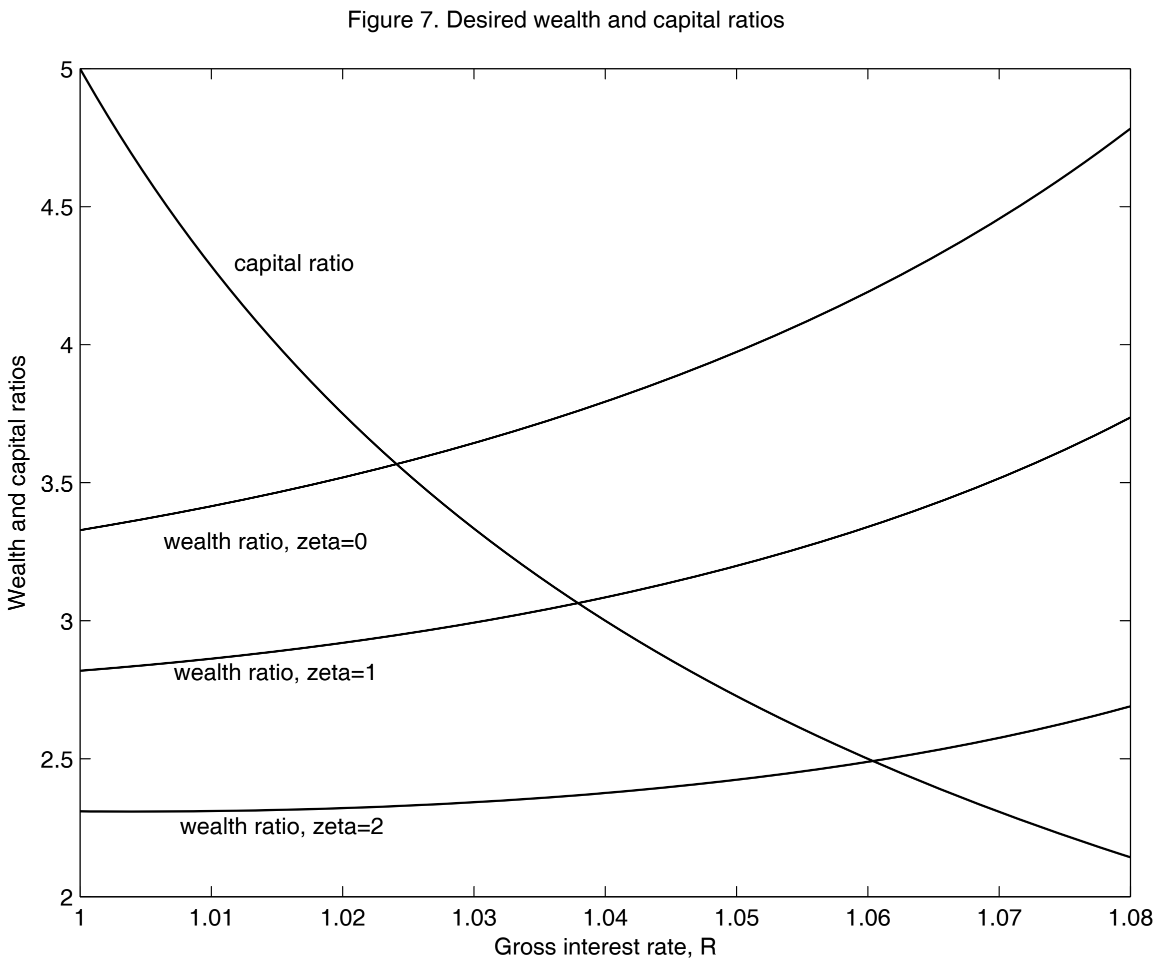

Figure 7: General Equilibrium

The decrease in the desired foreign level of wealth thus has a large negative impact on the real wage. The welfare effect is unambiguously negative for the home country. The long-run welfare impact is also negative in the foreign country, although not necessarily during the transition, as the generations that are alive at the time of the increase in social insurance benefit from consuming the accumulated net foreign assets. The home country enjoys an export boom during the transition, but this is associated with lower investment rather than higher output.

The intuition should be clear from the analysis of the closed economy in the previous section. The decrease in the desired level of foreign wealth raises the world interest rate, with little impact on the level of home wealth. Thus, it is reflected mainly in a decrease in the ratio of capital to output, which depresses the real wage.

5 Conclusion

This paper has presented a tractable model of the net foreign assets of a small open economy. The desired level of domestic wealth was endogenized as the optimal level of precautionary wealth against an idiosyncratic risk. We presented two applications of the model. The first concerned the relationship between economic development and capital flows. The second concerned the long-run global implications of reducing global imbalances by reducing the desired stock of saving outside of the United States.

Although very stylized, the model is able to predict plausible orders of magnitude for the ratio of net foreign assets to GDP. This being said, there are several dimensions in which the model could be made more realistic, probably at the expense of tractability. In particular, it would be interesting to know the exchange rate implications of a multi-goods extension of the model. (We anticipate that such an extension would show that a developing country that increases its desired level of foreign assets following economic liberalization will see a depreciation of its real exchange rate.) It would be also interesting to look at the impact of changes in the desired level of wealth on the price of assets other than currencies.

Our paper also has potential implications for future empirical work. To the best of our knowledge, the empirical literature has not looked at the impact of idiosyncratic risk and social insurance on net foreign assets in the context of a large country sample. The available evidence is anecdotal or focused on one country (e.g., Chamon and Prasad (2008)), or it is about financial development rather than social insurance (Mendoza, Quadrini, and Rios-Rull (2007)). It would be interesting to see if the predictions of our framework for net foreign assets can be tested with the available data.

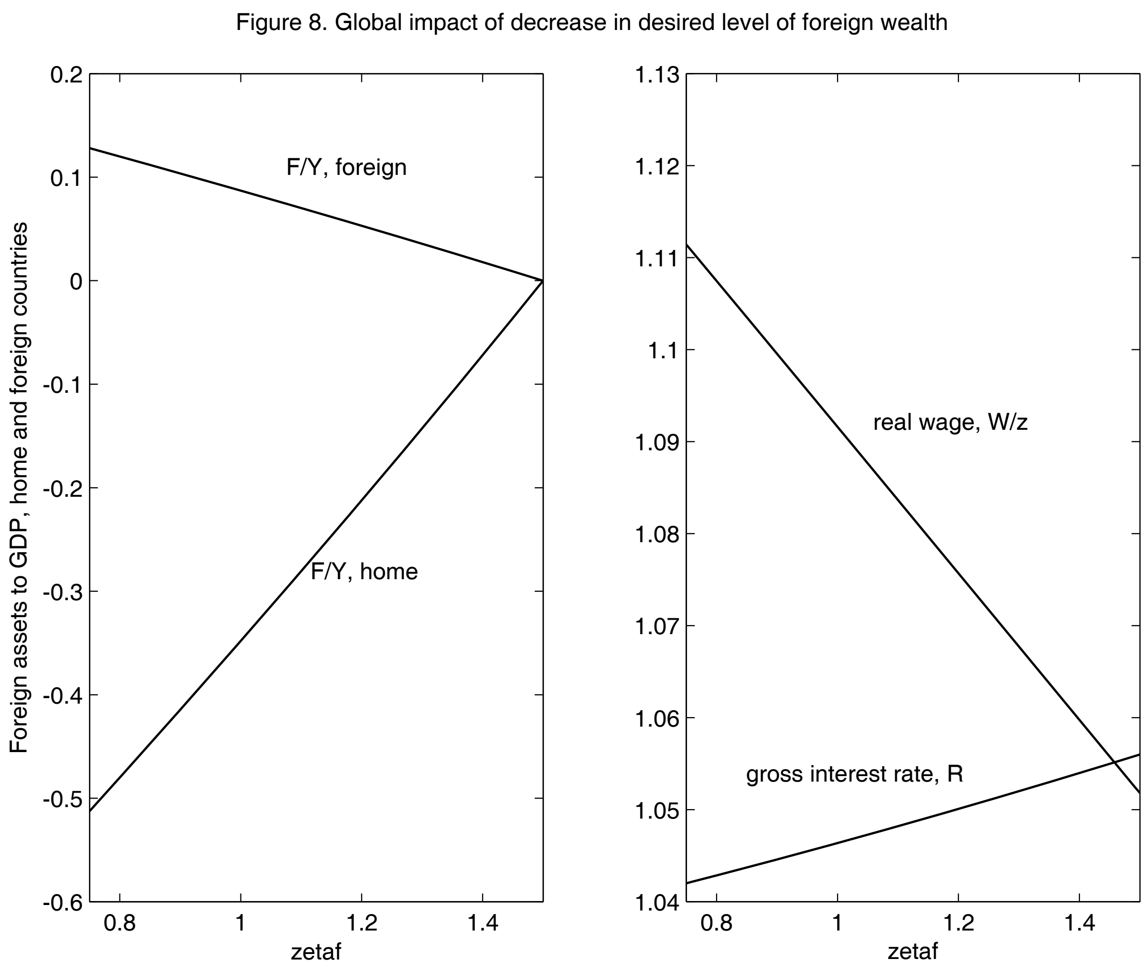

Figure 8: Global Imbalances

A Appendix

A.1 Key Model Parameters and Variables

We provide the following tables to aid the reader in keeping track of our notation.

Parameter	Definition
α	Capital's share in the Cobb-Douglas Production Function
$\daleth$	Depreciation Factor (Proportion Remaining After Depreciation)
Ξ	Population Growth Factor
G	Aggregate Productivity Growth Factor
R	Riskfree Interest Factor
β	Time Preference Factor
ρ	Coefficient of Relative Risk Aversion
ς	Severance Payment (In Years Of Income) Paid At Unemployment
$\mathbf{X}$	Individual (eXperience-based) Productivity Growth
ω_i	Weight (Share) Of Country i in World Income
$\mho$	Probability Of Employed Worker Becoming Unemployed
Φ	Probability of Death
τ	Tax Rate
χ	'Stake' In Version Of Model With Stakes
ε	Individual's Employment Status (1 if Employed; 0 if Not)

Some combinations of the parameters above are used as convenient shorthand:

Constant	Definition
$\not{\mho} \equiv 1 - \mho$	Period Probability Of Employed Worker Remaining Employed
$\not{\Phi} \equiv 1 - \Phi$	Probability of Survival (Not Dying)
$\not{\tau} \equiv 1 - \tau$	Proportion of Income Left After Taxation
$\Lambda \equiv \frac{\not{\mho}\mathbf{X}}{\Xi}$	Annual Shrinkage Of Old Generations' Share in $\boldsymbol{L}$
$\kappa^u \equiv 1 - \frac{(\beta\mathsf{R})^{1/\rho}}{\mathsf{R}/\not{\Phi}}$	Marginal Propensity to Consume for Unemployed Consumer
$\Gamma \equiv \mathsf{G}\mathbf{X}$	Labor Income Growth For Continuing-Employed Individual
$\text{Þ}_\Gamma \equiv \frac{(\beta\mathsf{R})^{1/\rho}}{\Gamma}$	Growth Patience Factor

Variable	Definition
C	Consumption
$\mathcal{E}$	Employed Population
I	Investment
K	Physical Capital Stock
L	Labor Supply
ℓ	Individual labor productivity per employed worker
N	Net Foreign Assets
P	GDP (‘Production’)
S	Total Wealth (Foreign and Domestic)
$\mathcal{U}$	Unemployed Population

Typeface	Meaning
Bold	Level of a Variable
Plain	Ratio Of The Variable To GDP or Labor Income
Uppercase	Aggregate Variable
Lowercase	Household-Level (Idiosyncratic) Variable

A.2 National Accounting

The aggregate budget constraint of residents can be written,

$$\frac{\boldsymbol{S}_{t+1}}{\mathsf{R}} + \boldsymbol{C}_t = \boldsymbol{S}_t + (1-\alpha)\boldsymbol{P}_t.$$

Using (2) this equation can be rewritten as,

$$\boldsymbol{C}_t + \boldsymbol{I}_t + (\boldsymbol{N}_t - \mathsf{R}\boldsymbol{N}_{t-1}) = \boldsymbol{P}_t,$$

where $\boldsymbol{I}_t = \boldsymbol{K}_{t+1} - \daleth\boldsymbol{K}_t$ is domestic investment, and $\boldsymbol{N}_t$ is given by (5). Using the fact that domestic output is either consumed, invested or exported, and defining $\boldsymbol{X}$ as net exports,

$$\boldsymbol{C}_t + \boldsymbol{I}_t + \boldsymbol{X}_t = \boldsymbol{P}_t,$$

so that net exports are equal to $\boldsymbol{X}_t = \boldsymbol{N}_t - \mathsf{R}\boldsymbol{N}_{t-1}$. By definition, the current account balance is equal to net exports plus the income on net foreign assets,

$$\text{Current Account}_t \equiv \boldsymbol{X}_t + (\mathsf{R}-1)\,\boldsymbol{N}_{t-1},$$

from which we can derive the balance-of-payments equation,

$$\text{Current Account}_t = \boldsymbol{N}_t - \boldsymbol{N}_{t-1}.$$

The current account balance is equal to the volume of capital outflows in period t.

A.3 The Consumption-Saving Problem of the Unemployed

An insurance company *a la* Blanchard (1985) provides each newly unemployed worker with an annuity, i.e., a consumption path that is conditional on the individual staying alive. The annuity contract maximizes the welfare of the individual conditional on the expected present value of his consumption being equal to his wealth. For a worker becoming unemployed at t it solves the problem,

$$\max \sum_{n=0}^{+\infty} \beta^n \not{\Phi}^n \mathrm{u}(\boldsymbol{c}^u_{t+n})$$

subject to

$$\sum_{n=0}^{+\infty} \mathsf{R}^{-n} \not{\Phi}^n \boldsymbol{c}^u_{t+n} \;=\; \boldsymbol{s}^u_t.$$

The Euler equation is,

$$\boldsymbol{c}^u_{t+n} = (\beta \mathsf{R})^{n/\rho} \boldsymbol{c}^u_t.$$

Using this expression to substitute out $\boldsymbol{c}^u_{t+n}$ from the expected present value constraint then gives,

$$\boldsymbol{s}^u_t = \sum_{n=0}^{+\infty} \mathsf{R}^{-n} \not{\Phi}^n \boldsymbol{c}^u_{t+n} = \frac{\boldsymbol{c}^u_t}{\kappa^u}.$$

A.4 Saddle-Point Stability

We first characterize the iso-s^e and iso-c^e loci in the space (s^e, c^e). Equation (14) implies that the iso-s^e locus is a line defined by,

$$c^e = 1 + \left(1 - \frac{\Gamma}{\mathsf{R}}\right) s^e.$$

Similarly, setting $c^e_{t+1} = c^e_t$ in equation (15) gives the following equation for the iso-c^e locus,

$$c^e = \left[1 + \frac{\Gamma}{\kappa^u \mathsf{R}} \left(1 + \frac{\text{Þ}_\Gamma^{-\rho} - 1}{\mho}\right)^{-1/\rho}\right]^{-1} (1 + s^e)\,.$$

The iso-c^e locus is an upward-sloping line which intersects the c^e-axis below the iso-s^e line. The iso-c^e line and the iso-s^e lines intersect in the positive quadrant (as indicated on Figure 1) if and only if $\check{s} > 0$. This is true because,

$$\frac{\Gamma}{\mathsf{R}} - 1 + \kappa^u \left(1 + \frac{\text{Þ}_\Gamma^{-\rho} - 1}{\mho}\right)^{1/\rho} > \frac{\Gamma}{\mathsf{R}} - 1 + \kappa^u = \frac{1}{\mathsf{R}} \left(\Gamma - (\mathsf{R}\beta)^{1/\rho}\right) > 0,$$

where the last inequality follows from the growth impatience condition (12).

Using equation (14), it is straightforward to see that s^e increases (decreases) if and only if (s^e, c^e) is below (above) the iso-s^e line. Equation (15) implies that c^e_{t+1} is decreasing with s^e_t. Therefore, c^e decreases if and only if (s^e, c^e) is in the region to the right of the iso-c^e locus. This is also the region below the locus, because this locus is upward-sloping. Thus, the phase diagram is as it is shown on Figure 1, and the dynamics for the pair (s^e_t, c^e_t) are saddle-point stable.

A.5 Comparative Statics

We characterize the variation of the target wealth-to-income ratio $\check{S}$ with respect to the risk aversion parameter ρ (the variations with respect to $\mho$, β and Γ being straightforward). We set $\tau = 0$ for simplicity.

The asymptotic levels of the consumption and wealth ratios satisfy the following equations,

$$\frac{\check{C}}{\check{S}} = \kappa^u \left(1 + \frac{\Gamma^\rho/\beta\mathsf{R} - 1}{\mho}\right)^{1/\rho}, \tag{34}$$

$$\frac{\check{C}}{\check{S}} = \frac{1}{\check{S}} + 1 - \frac{\Gamma}{\mathsf{R}}. \tag{35}$$

The question is how the RHS of the first equation varies with ρ. If it decreases with ρ, then a more risk averse worker would like his wealth to cover a higher multiple of his consumption, which is possible only if he accumulates more wealth (i.e., $\check{S}$ is higher).

The marginal propensity to consume κ^u may be increasing or decreasing with ρ. It is decreasing with ρ if and only if $\beta\mathsf{R} \leq 1$. In this case the unemployed worker has a consumption path $\boldsymbol{c}^u$ that decreases over time. Increasing ρ reduces the consumer's willingness to substitute consumption intertemporally, and thus reduces $\boldsymbol{c}^u_t$ (the consumption level of a worker becoming unemployed).

We then look at the variation in the logarithm of the term in parenthesis on the r.h.s. of equation (34), i.e., the function:

$$\rho \to \frac{1}{\rho} \log\left(1 + \frac{\Gamma^\rho/\beta\mathsf{R} - 1}{\mho}\right).$$

Differentiating this function shows that it is decreasing with ρ if and only if,

$$\log\left(1 + \frac{\Gamma^\rho/\beta\mathsf{R} - 1}{\mho}\right) > \log(\Gamma^\rho)\frac{\Gamma^\rho/\beta\mathsf{R}\mho}{1 + \frac{\Gamma^\rho/\beta\mathsf{R} - 1}{\mho}} \tag{36}$$

We then use the following property:

$$\forall \bullet > 0, \ \log(1 + \bullet) > \frac{\bullet}{1 + \bullet}.$$

This property follows from the facts that the two sides of the inequality are equal for $\bullet = 0$, and that the derivative of the right-hand side is strictly smaller than the derivative of the left-hand side.

Since $\Gamma^\rho/\beta\mathsf{R} - 1 > 0$, this property implies that inequality (36) is necessarily satisfied if,

$$\frac{\frac{\Gamma^\rho/\beta\mathsf{R}-1}{\mho}}{1+\frac{\Gamma^\rho/\beta\mathsf{R}-1}{\mho}} > \log(\Gamma^\rho)\frac{\Gamma^\rho/\beta\mathsf{R}\mho}{1+\frac{\Gamma^\rho/\beta\mathsf{R}-1}{\mho}},$$

or,

$$\frac{\Gamma^\rho}{\beta\mathsf{R}}\left(1+\log(\Gamma^\rho)\right) > 1,$$

which is true since $\Gamma^\rho/\beta\mathsf{R} = \text{Þ}_\Gamma^{-\rho} > 1$ and $\Gamma^\rho > 1$. We have shown that the term $\left(1+\frac{\text{Þ}_\Gamma^{-\rho}-1}{\mho}\right)^{1/\rho}$ is decreasing with ρ. The factor κ^u is also decreasing with ρ provided that $\beta\mathsf{R} \leq 1$. Hence, $\check{S}$ is increasing with ρ if $\beta\mathsf{R} \leq 1$.

A.6 Aggregating Individual Wealths

We now derive equation (19). The aggregate wealth of employed workers is given by,

$$\boldsymbol{S}_t^e = \sum_{n=0}^{+\infty} e_{t,t-n}\boldsymbol{s}_{t,t-n}^e$$

where $e_{t,t-n}$ is the number of employed workers born in period $t-n$, and $\boldsymbol{s}_{t,t-n}^e = s_{t,t-n}^e \mathsf{W}_t \ell_n$ is the level of wealth held by the representative worker in the generation born at $t-n$. Using $e_{t,t-n} = \Xi^{t-n}\mho^n$ and $\ell_n = \mathbf{X}^n \ell_0$ we have

$$\boldsymbol{S}_t^e = \Xi^t \ell_0 \mathsf{W}_t \sum_{n=0}^{+\infty} \Lambda^n s_{t,t-n}^e,$$

with Λ defined by (19). Using $\boldsymbol{P}_t = \mathsf{W}_t \boldsymbol{L}_t/(1-\alpha)$ the ratio of foreign assets to output can be written

$$\frac{\boldsymbol{S}_t^e}{\boldsymbol{P}_t} = (1-\alpha)\frac{\Xi^t \ell_0}{\boldsymbol{L}_t}\sum_{n=0}^{+\infty}\Lambda^n s_{t,t-n}^e. \tag{37}$$

Each individual has a labor endowment that increases at rate $\mathbf{X}$ until he becomes unemployed. Thus, in period t the generation born at $t-n$ supplies a quantity of labor equal to the number of workers from this generation who are still employed at t, times the labor supply per worker,

$$\boldsymbol{L}_{t,t-n} \;=\; \Xi^{t-n}\mho^n \ell_0 \mathbf{X}^n = \Xi^t \Lambda^n \ell_0.$$

Total labor supply, thus, is given by,

$$\boldsymbol{L}_t \;=\; \Xi^t \sum_{n=0}^{+\infty}\Lambda^n \ell_0 = \Xi^t \frac{\ell_0}{1-\Lambda}.$$

Using this expression to substitute out $\boldsymbol{L}_t$ from equation (37) then gives equation (19).

A.7 Model with Stakes

The period-t budget constraint of an individual is

$$\frac{\boldsymbol{s}_{t+1}}{\mathsf{R}} + \boldsymbol{c}_t + \chi_t = \boldsymbol{s}_t + \varepsilon_t \ell_t \mathsf{W}_t,$$

where χ_t is a lump-sum transfer. The transfer puts newborn individuals at the same net wealth-to-income ratio as the rest of the population, so for them

$$\chi_t = -\not\tau s_t^e \ell_0 \mathsf{W}_t.$$

For the other workers the transfer is a lump-sum tax that is proportional to their generation's wealth. For an employed worker born at $t-n$ the tax is,

$$\chi_t = \tau s_t^e \ell_0 \mathbf{X}^n \mathsf{W}_t.$$

In all periods of an individual's life, thus, the normalized budget constraint is given by,

$$s_{t+1}^e = \frac{\mathsf{R}}{\Gamma}\left(\overbrace{\not\tau}^{\equiv 1-\tau} s_t^e - c_t^e + 1\right), \tag{38}$$

which generalizes (14). Equation (15) is replaced by,

$$c_{t+1}^e = \text{Þ}_\Gamma \not\mho^{1/\rho} c_t^e \left[1 - \mho\left(\frac{1-\kappa^u}{\kappa^u}\frac{c_t^e}{\not\tau s_t^e - c_t^e + 1}\right)^\rho\right]^{-1/\rho}.$$

Going through the same steps as before one can find the following expression for the target wealth-to-income ratio,

$$\check{\check{\check{S}}} = \left[\frac{\Gamma}{\mathsf{R}} - \not\tau + \kappa^u\left(1 + \frac{\text{Þ}_\Gamma^{-\rho} - 1}{\mho}\right)^{1/\rho}\right]^{-1}. \tag{39}$$

The level of τ results from the following equality,

$$\Xi^t \not\tau s_t^e \ell_0 \mathsf{W}_t = \tau s_t^e \mathsf{W}_t \boldsymbol{L}_t.$$

The left-hand side is the flow of payment that is required to endow each newborn individual with the same ratio of net wealth to income as the rest of the population. The right-hand side is the proceeds of the tax on the employed workers. Using (4) to substitute out $\boldsymbol{L}_t$, this equation simplifies to $\not\tau = \tau/(1-\Lambda)$, which implies

$$\tau = \frac{1-\Lambda}{2-\Lambda}. \tag{40}$$

Using this expression to substitute out τ from (39) gives (25).

A.8 The Ramsey Model

The Ramsey model corresponds to the particular case where there is one representative infinitely-lived worker ($\Xi = 1$ and $\mho = 0$). In this case income growth is the same at the individual level and at the aggregate level. We can assume, without restriction of generality, that $\mathbf{X} = 1$, so that $\Gamma = \mathsf{G}$.

The individual's problem at time 0 is to maximize,

$$\sum_{t=0}^{+\infty} \beta^t \mathrm{u}(\boldsymbol{c}_t),$$

subject to the budget constraint,

$$\frac{\boldsymbol{s}_{t+1}}{\mathsf{R}} + \boldsymbol{c}_t = \boldsymbol{s}_t + (1-\alpha)\boldsymbol{p}_t,$$

where $\boldsymbol{p}_t = \mathsf{G}^t \boldsymbol{p}_0$ is the country's output. For the worker's discounted intertemporal income to be finite one must assume $\mathsf{G} < \mathsf{R}$.

Iterating on the budget constraint and using $\boldsymbol{c}_t = (\beta\mathsf{R})^{t/\rho}\boldsymbol{c}_0$ (from the Euler equation) and $\boldsymbol{p}_t = \mathsf{G}^t \boldsymbol{p}_0$ to substitute out consumption and output, we have

$$\begin{aligned}
\boldsymbol{s}_t &= \sum_{n=0}^{t-1} \mathsf{R}^{t-n}(1-\alpha)\boldsymbol{p}_n - \sum_{n=0}^{t-1} \mathsf{R}^{t-n}\boldsymbol{c}_n + \mathsf{R}^t \boldsymbol{s}_0, \\
&= (1-\alpha)\boldsymbol{p}_0 \frac{\mathsf{R}^t - \mathsf{G}^t}{1 - \mathsf{G}/\mathsf{R}} - \boldsymbol{c}_0 \frac{\mathsf{R}^t - (\mathsf{R}\beta)^{t/\rho}}{1 - (\mathsf{R}\beta)^{1/\rho}/\mathsf{R}} + \mathsf{R}^t \boldsymbol{s}_0.
\end{aligned}$$

For the transversality condition to be satisfied, $\boldsymbol{c}_0$ must be such that the terms in R^t cancel out in the expression above. Using this property to substitute out $\boldsymbol{c}_0$, the expression for $\boldsymbol{s}_t$ simplifies to,

$$\boldsymbol{s}_t = (1-\alpha)\boldsymbol{p}_0 \frac{(\mathsf{R}\beta)^{t/\rho} - \mathsf{G}^t}{1 - \mathsf{G}/\mathsf{R}} + \boldsymbol{s}_0 (\mathsf{R}\beta)^{t/\rho}.$$

The limiting wealth-to-output ratio is given by,

$$\begin{aligned}
\lim_{t\to+\infty} \frac{\boldsymbol{s}_t}{\boldsymbol{p}_t} &= \lim_{t\to+\infty} \frac{1-\alpha}{\mathsf{G}^t} \frac{(\mathsf{R}\beta)^{t/\rho} - \mathsf{G}^t}{1 - \mathsf{G}/\mathsf{R}} + \frac{\boldsymbol{s}_0}{\boldsymbol{p}_0} \frac{(\mathsf{R}\beta)^{t/\rho}}{\mathsf{G}^t}, \\
&= \lim_{t\to+\infty} (1-\alpha) \frac{\text{Þ}_\Gamma^t - 1}{1 - \mathsf{G}/\mathsf{R}} + \frac{\boldsymbol{s}_0}{\boldsymbol{p}_0} \text{Þ}_\Gamma^t \\
&= -\frac{1-\alpha}{1 - \mathsf{G}/\mathsf{R}}.
\end{aligned}$$

A.9 Social Insurance

Next we derive equation (30). The worker's normalized budget constraint is still given by (38), taking into account that the wage is taxed at rate τ_w to pay for the unemployment benefits,

$$s_{t+1}^e = (\mathsf{R}/\Gamma)\left(\not\tau s_t^e - c_t^e + \not\tau_w\right). \tag{41}$$

Equation (13) still applies, with $c_{t+1}^u = \kappa^u(s_{t+1}^e + \varsigma)$. Setting $s_{t+1}^e = s_t^e = \grave{\check{S}}$ and $c_{t+1}^e = c_t^e = \check{c}_n$ in equations (13) and (41) we obtain the following relationships between the long-run levels of the consumption and wealth ratios,

$$\check{c}_n = \left(1 + \frac{\text{Þ}_\Gamma^{-1/\rho} - 1}{\mho}\right)^{1/\rho} \kappa^u(\grave{\check{S}}(\varsigma) + \varsigma), \tag{42}$$

$$\check{c}_n = \not\tau_w + \left(\not\tau - \frac{\Gamma}{\mathsf{R}}\right)\grave{\check{S}}(\varsigma). \tag{43}$$

The tax rate τ_w must satisfy,

$$\tau_w \boldsymbol{L}_t \mathsf{W}_t = \mho \mathcal{E}_{t-1} \mathbf{X} \ell \varsigma \mathsf{W}_t.$$

The left-hand-side is the flow of tax receipts at time t. The right-hand-side is the amount needed to give the newly unemployed workers the same gross labor income as if they had remained employed. Thus one has,

$$\tau_w = \frac{\mho \mathbf{X}}{\Xi}\varsigma.$$

Using this expression and (40) to substitute out τ_w and τ from equations (42) and (43) and substituting out $\check{c}_n$ gives equation (30).

A.10 Transition Dynamics

Normalizing ℓ_0 to 1, the equation for the dynamics of aggregate labor supply is,

$$\boldsymbol{L}_t = \not\mho \mathbf{X} \boldsymbol{L}_{t-1} + \Xi^t,$$

implying that in steady state,

$$\boldsymbol{L}_t = \frac{\Xi^{t+1}}{\Xi - \not\mho \mathbf{X}}.$$

Up until period 0 (inclusive), the economy is in a steady growth path with $\mathsf{G} = \mathsf{G}_b$ and $\mho = \mho_b$, so that

$$\boldsymbol{L}_0 = \frac{\Xi}{\Xi - \not\mho_b \mathbf{X}}.$$

In period 0 it is announced that from period 1 onwards the productivity growth rate and the flow probability of unemployment jump to higher levels, G_a and $\mho_a$. Starting from $\boldsymbol{L}_0$, the dynamics of labor supply are given by,

$$\boldsymbol{L}_t = \not\mho_a \mathbf{X} \boldsymbol{L}_{t-1} + \Xi^t,$$

from which it is possible to compute the whole path $(\boldsymbol{L}_t)_{t\leq 0}$, as well as the gross rate of growth in labor supply, $\Lambda_t = \boldsymbol{L}_t/\boldsymbol{L}_{t-1}$. It follows from (2) that output is proportional to $z_t\boldsymbol{L}_t$. Hence the gross rate of output growth, $\gimel_t \equiv \boldsymbol{P}_t/\boldsymbol{P}_{t-1}$, is given by

$$\gimel_t = \mathsf{G}_a\Lambda_t$$

for $t \leq 1$. Using this expression we can compute the whole path $(\gimel_t)_{t\leq 1}$.

We now come to the ratios of net foreign assets and capital outflows to GDP, $\boldsymbol{N}_t/\boldsymbol{P}_t$ and $(\boldsymbol{N}_t - \boldsymbol{N}_{t-1})/\boldsymbol{P}_t$. Using the definition of $\boldsymbol{N}$ equation (5), we have

$$\frac{\boldsymbol{N}_t}{\boldsymbol{P}_t} = \gimel_{t+1}\left[\frac{(1-\alpha)s_{t+1}}{\mathsf{R}} - \frac{\boldsymbol{K}}{\boldsymbol{P}}\right],$$

$$\frac{\boldsymbol{N}_t - \boldsymbol{N}_{t-1}}{\boldsymbol{P}_t} = \frac{1-\alpha}{\mathsf{R}}\left(\gimel_{t+1}s_{t+1} - s_t\right) - (\gimel_{t+1} - 1)\frac{\boldsymbol{K}}{\boldsymbol{P}},$$

where $s_t = s_t^e + s_t^u$ is the ratio of aggregate wealth to aggregate labor income. The path for s_t^e is the individual convergence path for the model with stakes, where the initial condition s_0^e is given by (25) with $\mathsf{G} = \mathsf{G}_b$ and $\mho = \mho_b$. This gives us the whole path $(s_t^e)_{t\leq 0}$. As for s_t^u, the initial condition can be derived from equation (21),

$$s_0^u = \frac{\mho_b \Xi \mathsf{G}_b}{\Xi\mathsf{G}_b - \not{\Phi}_b(\beta\mathsf{R})^{1/\rho}} s_0^e.$$

The path for s_t^u can then be derived from equation (20), which can be rewritten in normalized form,

$$s_{t+1}^u = \frac{\not{\Phi}_a(\beta\mathsf{R})^{1/\rho}}{\gimel_{t+1}} s_t^u + \mho_a s_{t+1}^e.$$

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