

Identifying Preferences under Risk from Discrete Choices

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When studying consumption choices, economists have often relied on the abstraction of a representative agent. Such an agent can indeed be shown to exist and to replicate the aggregate consumers' demand under standard, but not necessarily convincing assumptions (Kirman (1992)). There was also a justifiable reluctance to introducing heterogeneous preferences, as such a step may seem ad hoc when trying to explain different consumption behaviors. The rise of empirical studies based on micro data has opened new perspectives. The micro-economic importance of uninsurable risks is now recognized, and threatens the foundations of the representative agent hypothesis often used in macroeconomics.. The continuing controversies surrounding the question of individual attitudes towards risk has motivated many empirical studies and observations; most of them conclude to a bewildering diversity of individual preferences¹

This paper proposes to check the conditions under which heterogeneous individual attitudes toward risk can be non-parametrically identified from discrete data on choices under risk. Our main result establishes that given data that is usually available (essentially market shares of the different options, plus the realizations of the final outcomes of agents), the analyst can recover the whole distribution of individual preferences if this can be indexed by a one-dimensional parameter, provided that a fairly weak single-crossing

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¹See Barsky et al., 1997; Cohen and Einav 2002; Guiso and Paiella 2008; Chiappori and Paiella 2007; and Choi et al 2007.

condition holds. We then discuss several applications of our general methodology.

1 Theory

We consider an economic situation in which a population of privately informed agents face a finite menu of prospects. In this section, we assume that for the econometrician this population appears homogeneous. Equivalently, the econometrician has data on a bigger population, including some variables X describing the population under study and the subgroup we consider here appears homogeneous because we control for the values of X .

The economic model Agents are heterogeneous; each agent is characterized by some parameter θ , his type, which is his private information. Because of asymmetric information, all agents are proposed the same finite menu $\mathcal{C} = \{c_1, \dots, c_n\}$. Each prospect c_i in this set can be thought of as a lottery (or a ‘bet’), which in practice may be an insurance contract, a job proposal, or any choice involving risky outcomes. There is a set of consequences $M \subset \mathbb{R}^k$, a typical element of which is a real vector m ; lotteries on this set are characterized by their c.d.f. F . By assumption, agents only care about consequences, so that we endow each agent with preferences represented by the utility functional $W(F, \theta)$. This functional may be expected utility:

$$W(F, \theta) = \int u(m, \theta) dF(m)$$

but we do not require this assumption; in fact, we will put much stress on identifying possible departures from expected utility in the population of agents.

When faced with the menu \mathcal{C} of prospects, agent θ associates a distribution $F(\cdot | c_i, \theta)$ to each feasible choice c_i , $i = 1, \dots, n$; note, in particular, that the (perceived) probability distribution induced by a given action may depend on the agent’s type, as would be the case for instance when riskiness is agent-specific. Given the agent’s preferences W , she will choose the option that gives the highest value to $V(c_i, \theta) = W(F(\cdot | c_i, \theta), \theta)$, which yields a choice function $C(\theta, \mathcal{C})$.

This model of choice is very general, indeed so much so that we cannot make much progress without specializing it. In this paper we discuss the one-dimensional case with single-crossing. So we take θ to be a scalar,

whose distribution we normalize to be uniform over $[0, 1]^2$; and we assume the existence of a complete ordering \prec of prospects such that:

Single Crossing Assumption (SC): for any $\theta < \theta'$ and any pair of prospects $c \prec c'$,

$$V(c, \theta) \leq V(c', \theta)$$

implies

$$V(c, \theta') \leq V(c', \theta')$$

The Single Crossing Assumption is standard. It states, in our context, that one can rank the prospects of the menu \mathcal{C} in such a way that agents with a higher type prefer prospects with a higher rank. One may, for instance, identify θ to a risk preference parameter, and rank prospects by increasing risk. Then our assumption requires, first, that the diversity of preferences be accounted for by a single parameter; and, second, that they can be ranked globally by decreasing dislike for risk. But our setting can accommodate other interpretations. For instance, in a non expected utility framework, θ could index some deformation of probability; alternatively, we may think of a model entailing robust control à la Hansen-Sargent (2007), and θ could then indicate the cost the agent put on probability distortions³

Without loss of generality, we may assume that the ranking characterized in the SC Assumption is the natural one - i.e., $c_i \prec c_j$ if and only if $i < j$. Note also that the single-crossing condition bears on a complex object, as the type θ may enter both preferences and distributions of consequences. We will discuss subcases below.

1.1 The Data

Ideally, we would like to recover the distribution of preferences $W(F, \theta)$ over the set of types. Obviously this requires at a minimum that we observe the same population repeatedly, as it makes choices in different menus. We index each such observation by the proposed menu \mathcal{C} , and we also assume that for each observation the econometrician observes

- the market share $s_i(\mathcal{C})$ of each prospect: so that we have

$$s_i(\mathcal{C}) = \Pr [C(\theta, \mathcal{C}) = i]$$

²If θ is distributed according to some C.d.f. Φ , then $\bar{\theta} = \Phi(\theta)$ is uniformly distributed on $[0, 1]$, and we can define $\bar{V}(c, \bar{\theta}) = V(c, \Phi^{-1}(\bar{\theta}))$.

³We could also forgo preference heterogeneity and focus on heterogeneity of risk for instance in a model of insurance under adverse selection.

- the empirical distribution of m , for each prospect:

$$G_i(m | \mathcal{C}) = E[F(m | c_i, \theta) | i = C(\theta, \mathcal{C})].$$

The assumption on the observability of market shares does not call for much discussion. On the other hand, a couple of remarks are in order here. First, the same population must be observed in different choice situations; this clearly requires that there exist an unmodelled, but exogeneous shock on the supply side. Think for example of the population of bettors in a horse race on Sundays, or of young male drivers in rural regions shopping for car insurance. In both cases it may be reasonable to assume that the distribution of preferences remains the same on each Sunday, or across regions. In the first case menus differ because different horses in different races have different winning probabilities. In the second case menus may vary from a region to another because loading factors differ, reflecting for instance various levels of competition across insurers by regions. Note that the econometrician does not need to model the precise reason why these menus differ—as long as she can make the case that the underlying process does not drive the preferences or the riskiness of agents at the same time.

Second, the equation on empirical distributions above requires that agents have correct beliefs when making their choices. For instance, if we analyze insurance contracts, we are requiring that an agent takes decisions based on the true distribution of claims conditional on (c_i, θ) . Such an assumption cannot be avoided: barring data on beliefs, most datasets could be rationalized by assigning strange beliefs to agents.

1.2 Identification

The beauty of single-crossing, as shown in hundreds of papers since Mirrlees (1971), is that it yields segmentation: given a menu \mathcal{C} , agents with a higher θ will choose prospects with a higher rank. Then we can define a sequence of numbers $\theta_i(\mathcal{C})$ for $i = 1, \dots, n - 1$ such that⁴:

- $\theta_0(\mathcal{C}) = 0$
- $\theta_n(\mathcal{C}) = 1$
- prospect $i \in \{1, \dots, n\}$ is chosen by the set of types

$$\theta \in [\theta_{i-1}(\mathcal{C}), \theta_i(\mathcal{C})].$$

⁴Recall that θ has support in $[0, 1]$.

In particular, the type $\theta_i(\mathcal{C})$ is defined by the property that such agents are just indifferent between choices c_i and c_{i+1} . By single crossing, all agents with a type above $\theta_i(\mathcal{C})$ strictly prefer c_{i+1} over c_i , while types below $\theta_i(\mathcal{C})$ have the opposite preference.

Since we normalized the distribution of types to be uniform in $[0, 1]$, the market share of prospect i among agents is:

$$s_i(\mathcal{C}) = \theta_i(\mathcal{C}) - \theta_{i-1}(\mathcal{C})$$

which can be rewritten as

$$\theta_i(\mathcal{C}) = \sum_{j \leq i} s_j(\mathcal{C})$$

We may thus assume that for each menu \mathcal{C} , the econometrician knows the indices of types indifferent between two consecutive prospects.

Similarly, we can rewrite the estimated distribution of consequences as

$$G_i(m | \mathcal{C}) = \frac{1}{s_i(\mathcal{C})} \int_{\theta_{i-1}(\mathcal{C})}^{\theta_i(\mathcal{C})} F(m | c_i, \theta) d\theta.$$

Now consider the subset of observations whose menu includes the prospect c_i . Assume that this subset is “large enough”, in the sense that the support of the distribution of $(\theta_{i-1}(\mathcal{C}), \theta_i(\mathcal{C}))$ is $[0, 1] \times [0, 1]$. Since, for each such observation, we know the values of the bounds in the integral, and the function to be integrated is the same, we can recover from the data the value of the function $F(m | c_i, \theta)$, for any m and any θ such that there exists an observation in which θ indeed chooses c_i .

Finally, we know that for any menu \mathcal{C} and for any i we have

$$W [F(\cdot | c_i, \theta_i(\mathcal{C})), \theta_i(\mathcal{C})] = W [F(\cdot | c_{i+1}, \theta_i(\mathcal{C})), \theta_i(\mathcal{C})]$$

Now fix a value for both the agent’s type and the prospect—say, $\bar{\theta}$ and \bar{c} . Consider the subgroup of observations for which $\bar{\theta}$ is indifferent between \bar{c} and an adjacent prospect—technically, the set of menus \mathcal{C} such that $c_i = \bar{c}$ and either $\theta_i(\mathcal{C})$ or $\theta_{i-1}(\mathcal{C})$ equals $\bar{\theta}$. Intuitively, each such observation provides a point on the indifference curve of θ that goes through $F(\cdot | c, \theta)$. Provided such observations display enough variation, one can identify the indifference curves of agent θ , and thus her preferences up to an increasing transform.

Our present goal is to introduce a general methodology. Consequently, we shall not attempt at a general statement of the necessary conditions for identification; it is much easier to illustrate them on specific examples. Suffices to say that, in many cases, the corresponding assumptions are fairly simple. Moreover, the single-crossing assumption can be tested, as well as other features of our model.

2 Applications

We now turn to a few selected examples.

2.1 Parimutuel Horse Betting

Let us start with parimutuel betting on horse races, which we study in much more detail in a companion paper (Chiappori et al., 2008). With betting, we typically do not have bettor-level data; on the other hand, we can observe odds of each horse in a very large number of races, along with the results of the race. The former gives us information on the preferences of bettors, as odds result from a simple market mechanism; and the latter informs us on the distribution of consequences of bets. To make this clear, consider the market for “win bets” in a race with n horses. Each bettor chooses on which horse he wants to bet a fixed amount. The odds is the net return from a winning bet: if a bettor bets \$1 on horse i , his final wealth is increased by $\$R_i$ if i wins, and reduced by \$1 if i loses.

Bets on races may be organized by bookmakers, as in Jullien-Salanié (2000); but parimutuel data are more useful for our purposes, as odds then mechanically reflect the market shares of horses. In parimutuel races, the money bet is placed in a pool; the organizers of the race (and/or the taxman) receive a share t known as the “take”, and the lucky bettors share what is left in the pool. Hence market shares (s_i) and odds (R_i) are related by the simple equality

$$R_i + 1 = \frac{1 - t}{s_i},$$

and the value of t can be deduced from the fact that $\sum_i s_i = 1$.

Observe also that if we properly control for observables, the winning probability of horse i in a race with odds (R_1, \dots, R_n) can only be a function of these odds, since bettors only care about consequences⁵. Hence this probability $p_i(R_1, \dots, R_n)$ can be computed as the empirical frequency of the event “horse i wins”, on the subset of races with such odds (assuming a large enough data set.)

We can now apply our theoretical work to horse races. The first step is to partition the set of races into different subsets; within each subset (e.g., races held on weekdays on a particular track), we assume that the population of bettors is the same in each race⁶

⁵This does not need to be true if odds are set by oligopolistic bookmakers, since they may set odds that depend on other considerations, such as the existence of other races. Under parimutuel betting, such a problem does not arise.

⁶We leave self-selection of bettors for further work.

Then we have to specify what agents know for each race; this is not an easy task here, as information on horses is typically dispersed across betters, and odds are not known until the beginning of the race since they result from the amounts bet. We rely here on a result by Gandhi (2008), which shows that if the pooled information of the agents pins down the actual probabilities, then there is a unique fully-revealing rational expectations equilibrium. Assuming that this equilibrium prevails, we obtain that beliefs will be correct in equilibrium, as if all betters knew the probabilities $p_i(R_1, \dots, R_n)$ and the odds (R_1, \dots, R_n) . Using the previous notation, we see that a menu the menu offered to betters is simply $\mathcal{C} = \{(p_1, R_1), \dots, (p_n, R_n)\}$ (each horse c_i being characterized by a probability of winning and a payment in case of success). The distribution of the final monetary outcome, $F(m | c, \theta)$, is simply a binomial distribution in general independent of θ^7 : given i and \mathcal{C} , $m = R_i$ with probability p_i , and $m = -1$ with probability $(1 - p_i)$.

As agents only care for consequences, we can thus write

$$V(c_i, \theta) = V(R_i, p_i, \theta).$$

Chiappori et al. (2008) then use a simple condition on V to ensure that single-crossing holds : agents with higher types θ must dislike risk less (in a precise sense that fits this non-expected utility framework). As a consequence, one can identify the whole distribution of preferences.

This application has several features that greatly simplify matters. First, the particular rules of parimutuel betting limits the need for data; odds allow to directly recover market shares and the take. Second, barring probability deformation, the distribution of consequences of a bet on a horse does not depend on the type θ of the bettor, but only on the odds of the horse R_i and on the probability p_i that the horse wins. As a consequence, it is very easy to identify the probability distribution on consequences from the data. Note, however, that more complex models can also be estimated in this framework; for instance, the decision criteria may entail non expected utility criteria that are not linear in probability, in which case the perceived distribution may depend on θ . Again, the reader is referred to Chiappori et al. (2008) for precise results and proofs.

2.2 Compensating Wage Differences

A very similar framework can be applied to the analysis of the statistical value of human life. Book I of the *Wealth of Nations* already emphasized

⁷Note, however, that the initial wealth of an agent can be part of his type θ .

that

[...] the wages of labour vary with the ease or hardship, the cleanliness or dirtiness, the honourableness or dishonourableness, of the employment.

A fortiori, Rosen (1988) emphasized that the wage differential needed to compensate a higher probability of accidental death allows to estimate the value of life. It has been extensively used subsequently (see for instance Viscusi and Alduy 2003). In all studies, however, homogeneity is a crucial prerequisite; i.e., identification relies on the rather doubtful assumption that all workers have the same monetary valuation of their life.

Our approach suggests that this assumption is simply not needed. Specifically, assume that agents are faced with a menu of jobs, each of which is defined by a wage and some parameters related to its riskiness; to keep things simple, let us simply assume that the risk is a fixed probability of a lethal accident, and that the latter can be estimated from data on the distribution of accidents in each job i . Also, let θ index the value that any given agent assigns to his life; that is, the value of agent θ 's life is some $v(\theta)$ with $v(0)$ corresponding to a minimum level (say zero) and $v(1)$ to some upper bound. Here, a job $c_i = (w_i, p_i)$ is fully defined by a wage w_i and a death probability p_i . The resulting probability distribution $F(m | c, \theta)$ is again a binomial distribution. An agent choosing the job i in the available menu \mathcal{C} receives $m = w_i$ with probability $1 - p_i$, and $m = -\theta$ with probability p_i . These parameters can readily be identified from available data on average casualty rate per job; also, the natural ranking is by increasing risk (and wage).

Considering now the estimation of preferences, we may start with the simplest case of risk neutral agents (which most of the existing literature focuses on); then $\theta_i(\mathcal{C})$ is defined by:

$$(1 - p_i) w_i + p_i v(\theta_i(\mathcal{C})) = (1 - p_{i+1}) w_{i+1} + p_{i+1} v(\theta_i(\mathcal{C}))$$

therefore

$$v(\theta_i(\mathcal{C})) = \frac{(1 - p_i) w_i - (1 - p_{i+1}) w_{i+1}}{p_{i+1} - p_i}$$

and the function v is exactly identified. Note that the single crossing condition is obviously satisfied in that case.

We may however go one step further, by assuming first that individuals are characterized by some common utility u that is concave in monetary rewards. The equation becomes:

$$(1 - p_i) u(w_i) + p_i v(\theta_i(\mathcal{C})) = (1 - p_{i+1}) u(w_{i+1}) + p_{i+1} v(\theta_i(\mathcal{C}))$$

Now, assume we can observe different menus in which the same agent $\bar{\theta}$ is pivotal between the same job (\bar{p}, \bar{w}) and different alternatives (p, w) . Then it must be the case that for all these alternatives, the expected utility $(1 - p) u(w) + pv(\bar{\theta})$ is constant (this is the 'indifference curve' mentioned above). In practice, if these various alternatives describe a wage/probability relationship of the form $p(w, \bar{\theta})$, then

$$v(\bar{\theta}) = \frac{(1 - \bar{p}) u(\bar{w}) - (1 - p(w, \bar{\theta})) u(w)}{p(w, \bar{\theta}) - \bar{p}}$$

and if we normalize $u(\bar{w})$ to be zero, we have that:

$$\frac{u(w)}{v(\bar{\theta})} = \frac{\bar{p} - p(w, \bar{\theta})}{1 - p(w, \bar{\theta})}$$

which identifies both u and v up to a common multiplicative constant and also generates strong overidentifying restrictions.

Finally, we may also introduce heterogeneous utilities by allowing u to depend on θ , although the single crossing condition will then have to be tested ex post.

The intuition of this example is clear. While the existing literature tends to pick up arbitrarily one type of risky job and estimate the statistical value of life from this unique source under the assumption of identical consumers, we suggest that additional mileage could be obtained from the respective "market shares" of various dangerous professions.

2.3 Insurance and portfolio choice

As a third possible application, consider the choice of an insurance contract—a topic already studied by Chiappori and Salanié (2008) and Cohen and Einav (2002) among others. Here, a contract is, in the simplest case, defined by a pair consisting of a premium and a fixed deductible. The distribution induced by a particular choice may be more complex than before, since it needn't be binomial: losses incurred by the agent can in principle take any value below the deductible - although several applied papers actually disregard this issue, certainly an acceptable simplification when the deductible is low. In any case, it can, as before, be identified from available data on accident realizations (and possibly conditional losses); this has actually been done in several papers.

The next step is to study individual demand for specific contracts. This task, again, has been performed in a few existing papers, although our approach introduces an important innovation. Specifically, existing work heavily relies on parametric assumptions on preferences (e.g., CARA expected

utility). However, it follows from the previous discussions that the shape of individual preferences as a function of the heterogeneity parameter θ can be non parametrically identified, by a procedure close to (although more general than) the one just described. Again, the crucial requirement is that similar agents must be faced with different menus of contracts, which implies a source of contract variation that is orthogonal to individual preferences. Fortunately, the existence of such variations seems well established empirically.

In the literature, two types of heterogeneity have been considered so far. One is heterogeneity in risks. In such frameworks, agents have identical preferences but different accident probabilities, and the choice of an insurance contract partly reflects information about idiosyncratic risk. The vast empirical literature on asymmetric information in insurance, starting with Chiappori and Salanié (2000), belongs to this class. Unlike the previous examples, in these models the distribution of probability over outcomes, $F(\cdot | c_i, \theta)$, does depend on θ (which is correlated with risk) as well as c_i (say, because of ex ante or ex post moral hazard). Estimating this relationship is often the difficult part of the empirical analysis; clearly, the availability of different menus is crucial here. On the other hand, once the function F has been recovered, estimating the (common) utility from contract choices is usually simpler. Alternatively, one can consider a model in which agents have homogeneous risks (at least conditional on the information available to the insurer),⁸ but differ in their preferences. Then F does not depend on θ (which is now interpreted, for instance, as a risk aversion parameter), and can readily be recovered from data on claims.⁹ Heterogeneity is mainly reflected in contract choices; the single crossing condition typically expresses that more risk averse agents always prefer more comprehensive coverage, a property most models will satisfy. Again, one can then try to non parametrically estimate the (heterogeneous) shape of individual preferences from available data.

An obvious limitation of our approach, in the case of insurance contracts, is the one dimensionality of the heterogeneity parameter θ . Strictly speaking, it requires either that individuals only differ in their risk or their risk aversion (but not both); or, if both dimensions are allowed to vary in the population, that there exists a perfect correlation between them, so that the resulting heterogeneity can be represented by one index only. While this setting is very restrictive, it is still possible to find interesting theoretical frameworks that are compatible with it. For instance, in Jullien, Salanié and Salanié

⁸Regarding automobile insurance, for instance, many insurers tend to believe that if anything, they have a much better knowledge of a client's risk than the client himself.

⁹However, the decision to file a claim may be endogenous, and possibly correlated with risk aversion; say Chiappori et al (2006) for a discussion.)

(2007), agents can exert an effort to reduce their accident probability, and heterogeneous attitudes to risk result in different effort levels, therefore different realized risks. In that context, a (one dimensional) distribution of risk aversion generates heterogeneity in both risk and contract choices conditional on risk.

Finally, the same logic can be extended beyond the strict confines of insurance. For instance, most work on portfolio choice uses continuous models, in which the proportions of the various asset classes can be freely varied. In many cases, however, the choice faced by individuals is much simpler. When electing a pension plan, for instance, many households are offered a small number of preexisting plans, and the choice is essentially discrete (even though continuous parameters can sometimes be varied in a given plan). Plans may differ in many respects: riskiness, liquidity, geographical coverage and others. To the extent that the heterogeneity between agents' preferences regarding these plans can be assumed to be one-dimensional (and satisfy the single crossing condition), our approach can be useful. Here, the 'consequences' can be any performance measure (expected return, volatility, beta,...) that can be estimated from the data; and the shape of (heterogeneous) preferences can be estimated, provided again that similar agents are faced with different menus of pension plans for exogenous reasons, say the employer's policy toward employees' retirement.

3 Conclusion

In this paper, we propose a general methodology for estimating heterogeneous preferences in situations of discrete choice under risk. Our key requirements are (i) that the existing heterogeneity can be summarized in a one-dimensional parameter, and (ii) that there exists some exogenous source of variation in the menus faced by the agents. Neither of these assumptions is innocuous. It should however be stressed that they typically allow a non parametric identification of the distribution of preferences (or risks); moreover, they are empirically testable, and actually generate strong overidentifying restrictions. Empirical work currently in progress should give us a better understanding of their relevance.

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