1. Introduction

The federal government increasingly relies on subsidized credit guarantees in lieu of grants or other forms of assistance to targeted groups. In providing credit assistance, the government has a choice between direct lending where it funds loans directly via the Treasury, and guaranteed lending where funds are raised indirectly via private financial institutions in the capital markets. Whether guaranteed lending is an efficient way of subsidizing credit is an important question that we explore here in the context of the Small Business Administration's (SBA) 7(a) program.

The goal of the SBA 7(a) program is to lower the cost and improve the availability of credit to U.S. small businesses. In 2006, it guaranteed 82,000 loans totaling \$12 billion. The loans generally have an original maturity of 7 to 10 years, and interest is based on the prime rate plus a fixed spread. Qualified borrowers are able to obtain loans from SBA-certified private financial institutions, usually commercial banks, with the backing from a federal guarantee that insures the lender against 50 to 85 percent of credit losses.

Despite the benefit of a substantial federal guarantee and default experience that is no worse than that on seemingly comparable loans and bonds, 7(a) borrowers are charged rates that are no lower than on the comparable securities. In this paper we review the evidence that supports this conclusion, and evaluate several possible explanations for why borrowers are charged such seemingly high rates. One hypothesis is that high borrowing rates are a consequence of imperfect competition in the 7(a) lending market, making the loans a profitable line of business for participating lending institutions. In fact some banks originate large numbers of 7(a) loans, while others choose not to participate in the program at all, and in some regions there are very few SBA lenders.

To examine the hypothesis that high costs are due to limited competition, we first calculate a measure of SBA local lender concentration based on a Herfindahl index and then run a regression of interest rate spreads on Herfindahl index values, controlling for observable borrower and loan characteristics. We find no evidence that rate spreads increase with lender concentration, and in fact

consistently find the opposite. We then consider an alternative to the Herfindahl index, which is based on the number of loans originated by a particular lender. Based on this measure, we find that the largest lenders tend to charge higher spreads than smaller ones. Since large lenders are also likely to be administratively more efficient, this may reflect the greater market power of large lenders.

High borrowing rates for 7(a) borrowers may also be attributable to a higher cost of capital for 7(a) lenders than what one would expect with a full faith and credit federal guarantee. Past studies have shown that federally guaranteed obligations such as RTC bonds and student loans bear higher rates than standard Treasury securities despite full faith and credit federal guarantees, and that the rate differences can be substantial (Longstaff (2004), Lucas and Moore (2008)). These authors point to lower liquidity as a likely reason for the higher required rates of return than on comparable Treasury's.

In evaluating the rate charged on 7(a) loans, one has to take into account that 7(a) loan cash flows are risky, despite being guaranteed against default risk.¹ There is considerable uncertainty as to the timing of payments because of the possibility of default and prepayment, and due to movements in the prime rate. This can introduce systematic risk into cash flows even though direct losses from default are precluded. The additional layers of intermediation introduced by relying on banks and securitization markets also create may create additional costs that can elevate spreads.

To evaluate the cost of capital on the guaranteed portion of SBA loans, we develop a Monte Carlo model of the cash flows on such securities, taking into account historical default and prepayment behavior, and modeling interest rates using a two-factor Cox, Ingersoll and Ross (CIR) model. Discounting these cash flows at the risk-free rates along each path, we find the theoretical value of the cash flows at Treasury rates. By comparing these theoretical prices to actual secondary market prices of the securitized pooled and guaranteed portion of SBA loans, we can then infer the spread over Treasury rates necessary to equate the theoretical price with the market price. We find evidence that investors typically demand a spread between 100 and 200 bps over Treasury rates in the period for which we have

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¹ This is also true of other federally guaranteed loans such as FHA loans and student loans.

data (1/06 through 6/08), which explains part of why 7(a) loans cost borrowers so much despite the benefit of a federal guarantee.

The rest of the paper is organized as follows: Section 2 gives a brief overview of the 7(a) program. Section 3 presents evidence that 7(a) loans appear relatively expensive for borrowers, and also very profitable for lenders. Section 4 reports on how borrower characteristics, loan characteristics, and the concentration among SBA lenders are related to the cross-section of credit spreads. In Section 5 we develop the model to price SBA guaranteed loan pools, and compare model prices with market prices to infer the premium over Treasury rates demanded by investors. The likely profitability of SBA loans is reevaluated given this information. We conclude in Section 6 with a discussion of the policy implications of the findings.

2. SBA 7(a) Program Overview

In this section we briefly describe the most salient features of the 7(a) lending program to this analysis, since more complete descriptions are available elsewhere (e.g., CBO (2000) and Glennon and Nigro (2005)). In terms of total size, the program that has grown rapidly since 2001, with \$12 billion in principal disbursed on more than 82,000 loans in 2006. Still, 7(a) loans represent less than 5 percent of total U.S. small business borrowing. What we refer to as the 7(a) program actually has two parts, a regular program and an "Express" program. The Express program promises borrowers and lenders quick eligibility decisions, but provides a federal guarantee on only 50% of loan principal and has a stricter restriction on loan size. The Express program has grown to represent near 75 percent of loans guaranteed, but only 20 percent of dollar loan volume.

2.1 Borrowers

Businesses seeking 7(a) loans must satisfy a number of criteria. The most subjective is that they must establish that they have not been able to obtain credit elsewhere on "affordable terms." They also

must demonstrate an ability to repay the loans, for instance based on collateral, the amount of owners' equity, and the general quality of management.² More concretely, the business must be for-profit, small (the definition of which is based varies by industry), and independently owned and operated. Borrowers come from a wide variety of industries, with the largest three being retail trade, accommodation and food services, and manufacturing, which together account for about 50 percent of loans disbursed.

2.2 Loan Terms and Characteristics

Over 96 percent of 7(a) borrowers pay an interest rate set to a fixed spread over the prime rate. The prime rate is a floating rate that since 2000 has hovered around 3 percent above the overnight Fed Funds rate. The spread is capped by SBA regulation, but on most loans the spread cap does not appear to bind. There are also a variety of fees assessed by the SBA, some paid by the borrower directly and others by the lender. Borrowers pay a graduated one-time guarantee fee that ranges from 2 percent of principal for loans of less than \$150,000 to 3.5% for loans of \$1 million. Borrowers with long-term loans also face high prepayment penalties in the first three years of the loan's life, after which prepayment is a free option. Lenders pay the SBA an annual servicing fee of .545 percent on the guaranteed balance, although they bear most servicing costs themselves.

Loan maturity typically is 7 to 10 years, but varies with the purpose of the loan. Loans for real estate and equipment may extend to 25 years. The effective duration of loans is considerably shorter, both because the loans are amortizing and because of voluntary prepayments and defaults. Loans in the regular program can be for up to \$2 million, and in the Express program up to \$350,000.

The maximum percentage guaranteed varies with the loan size and program. In the regular program, for loans under \$150,000, SBA guarantees up to 85% and for loans above \$150,000 the maximum is 75%. On Express loans the maximum guarantee percentage is 50%. Loans do not always bear the maximum allowable guarantee, presumably to reduce the amount of fees paid to SBA. Upon

² The SBA assigns a risk rating to individual loans, but it is not a standard credit score and that data is proprietary.

default, a lender can recover from SBA the face value of the outstanding principal balance, and subsequently will receive a pro rata share of any recoveries. However, lenders sometimes choose not to exercise the guarantee when they expect to recover more from retaining the entire loan. Thus the SBA default statistics that we rely on are somewhat downward-biased estimates of the default rate experienced by the borrower.³

2.3 Lender Characteristics

SBA-backed loans are typically made by commercial banks, but some thrifts and finance companies also participate. Lenders classified by SBA as "preferred lenders" have the authority to determine eligibility in the Express program. For commercial banks, an advantage of participation is that it helps to satisfy Community Reinvestment Act requirements. As described in more detail in section 4, relatively few banks⁴ originate large numbers of 7(a) loans. Of 2017 distinct lending organizations identified for 2006, only 14 originated more than 1,000 loans each, and 880 originated only 1 or 2 loans. The identities of the 20 largest originators and their lending volume are shown in Table 1.

2.4 The Guaranteed Loan Pool Certificate Program (GLPCP)

To increase access to capital for 7(a) lenders, since 1995 the SBA has sponsored the Guaranteed Loan Pool Certificate Program (GLPCP). Under this program 7(a) lenders can sell the guaranteed portion of 7(a) loans to a Pool Assembler. The Assembler puts together pools of loans that are similar in terms of maturity, rate basis (e.g., monthly and prime-based), and rate spread. Each pool has a minimum of four underlying loans that total at least \$1 million in aggregate. The strictest restrictions on

³ For the purposes of calculating returns to lenders, this problem is mitigated by the fact that non-surrendered loans are those with relatively high recovery rates.

⁴ A bank for this tabulation consolidates over institutions governed by the same holding company, even if they operate in far flung geographic regions.

rate floors or ceilings on the individual loans are reported for the pool. The pools are sold via an auction to investors, and there is also a secondary market in the Certificates.

It appears that the securitization process does not introduce any significant counterparty risk into the lending process. Payments on the securities are centralized and made by Fiscal and Transfer Agent (FTA-Colson Associates), who is appointed by the government and acts as their agent. Payments are made on time even when default and prepayments occur. These payments are made by the FTA to investors in the form of advances. Lenders also do not bear counterparty risk either since they deal directly with the FTA.

The program has become an increasingly important source of financing, with a \$4.0 billion of guaranteed securities sold into pools in 2006, representing 42% of the total guaranteed amount of loans approved in that year. We have no evidence on whether securitized loans deviate systematically from a typical SBA loan in terms of default or prepayment behavior. We assume that there is no significant selection bias since they represent a large fraction of total originations. The market prices of these certificates is of interest in this analysis because they can be used to infer the cost of capital for the default-risk-free portion of 7(a) loans, as discussed in Section 5 below.

3. The Cost and Profitability of 7(a) Loans

Our inferences about the cost of SBA loans rely on comprehensive data obtained from SBA on all disbursed loans from 1988 to 2006. Associated with each loan is static information about the borrower (e.g., type and size of business), the lender (e.g., name, state), and loan terms (e.g., rate spread, basis, maturity, if collateral). There is also a record of prepayments and defaults. The SBA loan data is described in more detail in Appendix 1.

3.1 Cost to Borrowers

Average rate spreads and guaranteed shares on 7(a) loans are summarized in Table 2. The average spread over prime is 2.36%, based on all loans disbursed from 1988 and 2006. Over the sample

period the prime rate averaged 2.77% over the overnight Fed Funds rate⁵, and the level of the Fed Funds rate was roughly equivalent to the 1-year Treasury rates (see Figure 1). Thus on average borrowers paid a spread of about 5.13% over the 1-year Treasury rate, even though the average loan bore a federal guarantee on 70.6% of principal. The initial guarantee fee added about 40 bps to the effective rate paid by borrowers.⁶

To assess whether these spreads are in line with other credit obligations with comparable default risk, we draw on the analysis in CBO (2007), which uses the same SBA loan data. Figure 2 (reproduced from CBO (2007), Figure 4) shows that the spread charged SBA borrowers has been consistently higher than that for all commercial and industrial (C&I) loans by a margin of about two percentage points for regular 7(a) loans, and over three percentage points for Express loans.

Higher rates on SBA loans might be justified, even with a partial guarantee, if the probability of default is significantly higher than on other C&I loans. The most direct evidence that it is not higher comes from, Glennon and Nigro (2005), who find that, although medium-maturity loans originated under the SBA 7(a) loan guarantee program are targeted to small firms that fail to obtain credit through conventional channels, the default experience is comparable to that of a large percentage of loans held by larger commercial banks. Further evidence on default rates is found by comparing the cumulative default rates on 7(a) loans and public debt. CBO (2007) finds that the default experience on 7(a) loans falls between that on BB and BBB-rated bonds (see Figure 3, reproduced from Figure 9 in CBO (2007)). Figure 4 shows that overall default rates on 7(a) loans have fallen over time, and have stabilized at just less than 2 percent of principal outstanding since 1995. Netting the average recovery rate of between 40 and 50 percent suggests an annual loss rate to SBA of about 1 percent.

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⁵ Fed Funds is used as a reference point because since 2000 the prime rate has been fixed at 3% over Fed Funds.

⁶ This assumes a 2.5% guarantee fee, a 7-year initial maturity, and a discount rate of 7%.

3.2 Returns to Lenders

The value of 7(a) loans from a lender's perspective depends on the margin between revenues and costs. Revenues arise from borrower payments and proceeds from loan sales. Costs include the lender's weighted average cost of funds, servicing and other ongoing administrative costs, and fees paid to the SBA. Taking these costs into account, we do a simple back-of-the-envelope calculation in this section suggesting that 7(a) loans on average are quite profitable for banks.

For bank lenders, holding a 7(a) loan is like owning a portfolio of a default-risk-free government security and a risky small business loan. Banks' weighted average cost of capital for 7(a) loans should reflect these two components, and in a competitive market the advantage of the guarantee should be passed through to the borrower. The cost of funds is clearly different for the insured and uninsured portions of a loan. The insured portion is virtually default-risk free, suggesting that investors would require a return close to a short-term risk-free Treasury rate on a floating rate loan. (In Section 5 we present a more careful analysis of the cost of funds for the guaranteed portion.) On the uninsured portion, default experience suggests funding costs similar to those on BB to BBB-rated loans. Based on historical BB rate spreads, the gross funding rate on the risky portion is taken to be Treasury + 3.5 percent. This is also consistent with the spread between C&I loans and SBA loans in Figure 2. Based on an average 70.2% of guaranteed principal, the WACC is at a spread of 1.03% above Treasury.

The net spread between revenues and expenses is calculated by subtracting the weighted average cost of capital and other costs from interest payments received net of the expected default loss rate. Other expenses are assumed to be .75% annually for servicing and other administrative costs, and .545% annually for the SBA servicing fee on the guaranteed portion. Default losses are assumed to be 1.25% on the non-guaranteed portion, slightly higher than those suggested by SBA experience.⁸ Together, these

⁷ Consistent with this view, banks investing in GLPCP certificates have a 0% capital requirement against these holdings.

⁸ This is based on about a 1% default rate for the SBA, and a .25% adjustment for the loans that default but are entirely retained by the lender.

assumptions imply a net spread of 2.6%. The net spread measures profit as a percentage of loan principal. Another way to assess the significance of this margin is to ask what the breakeven cost of uninsured capital would be for the lender to break even according to this calculation. The break-even spread on uninsured capital is 12.4%, which is 8.85% more than the cost of uninsured capital suggested by market prices. Clearly this estimate is sensitive to the many assumptions made, but it does suggest that 7(a) lending is a profitable line of business.

4. Loan Spreads and Market Concentration

A possible explanation for the high spreads charged to borrowers is that lending institutions have market power that allows them to charge above market rates. As mentioned earlier, a relatively small number of banks are active originators of 7(a) loans, but many others make few or no such loans. Figure 5 shows the distribution of the number of loans originated by institution in 2006. The average number of loans originated is 39.6, but the median is only 3 loans. The distribution by dollar amounts of originations is similarly skewed to the left.

To explore the effect of local lender concentration on rates charged, we first define a measure of the regional competitiveness of SBA lending markets, and then regress rate spreads (over the prime rate) on this quantity and a set of control variables that could also affect the rate spread, including the percentage guarantee, loan maturity, loan size, and whether it is an Express loan. The cross-sectional data is pooled over the years 1988 to 2006. In the second set of regressions we only use the 2006 cohort, and account for market power by assigning lenders to 5 lender-distribution buckets, constructed with the number of SBA loans that lenders originated in 2006.

For construction of the Herfindahl index, a region is taken to be a Core Based Statistical Area (CBSA), which defines micropolitan and metropolitan areas. In each of these areas we look at the level of competition in the lending market for SBA loans. Using the borrower's zip code we are able to assign

each loan to a CBSA. Then for a given year and a given CBSA we determine the number and the dollar amount of loans originated by each lender. With this information we calculate a Herfindahl index for each CBSA and each year. The Herfindahl index for CBSA i at time t is given by:

$$H_t^i = \sum_{n=1}^{N_{i,t}} (s_{n,t}^i)^2$$
 (1)

where $s_{n,t}^i$ is market share of lender n, in CBSA i at time t. The Herfindahl index gives a number between 0 and 1, with 1 representing a monopolistic market and 0 a perfectly competitive market.

The SBA has enacted interest rate-spread limits and maximum loan guarantee which are applies to loan type, loan size and maturity. For loans with rates close to the ceiling, this could bias inferences about the effect of market power on rate spreads. The spread and guarantee limits are presented in Table 3. In the regressions of spreads on Herfindahl index we split the sample to incorporate these spread and guarantee caps, and run a regression for each subsample separately. In the regression of spread on lender's distribution we incorporate a dummy variable if the spread of the loan is within 25 basis point of the interest rate cap.

In table 4 we present the regression results, for tests of spreads on regional concentration. The sample is subdivided by whether the loan is regular or Express, by loan size, and by maturity. Control variables include the percentage guaranteed, the loan size, and maturity. All regressions suggest that the Herfindahl index does not explain higher interest rate spread. In fact, the regressions suggest that regions with more concentrated lending markets are associated with lower interest rate spreads, which seems counterintuitive. Perhaps borrowers cross city boundaries to look for affordable loans, and therefore regional concentration may not be the best way to capture market power. If that is the case, then higher Herfindahl index may be associated with smaller markets, and perhaps smaller lenders which have less market power.

For the 2006 cohort we use an alternative measure of lender market power, based on a set of indicator variables for the number of loans that a lender originated in that year. The categories are 0-9 loans, 10-99 loans, 100-499 loans, 500-999 loans, and at least 1000 loans. In addition, we introduce two other dummy variables, one to flag if a loan is express, and another to incorporate loans that are within 25 basis points of the limits shown on table 1.

The results in table 5 suggest that the largest lenders tend to charge higher interest rate spreads than smaller ones. In addition, controlling for other loan characteristics, Express loans charge an extra 127 basis points above Regular loans. The adjusted R² of the current specification shows a significantly higher explanatory power than in the previous specifications. This suggests that much of the unexplained variation in spreads that was captured by the intercept in previous specifications is accounted for by lenders' characteristics. In future drafts we will look at whether default and prepayment rates differ systematically by lender size, since these are alternative explanations for the higher rates charged.

5. Cost of Capital for Fully Guaranteed Loans

The analysis thus far suggests that borrowers pay a relatively high rate on SBA loans, and that the market power of larger lenders may be part of the explanation. We also know, however, that many banks choose to make very few or no 7(a) loans, suggesting that our estimate of profitability to lenders in Section 3.2 is likely too high. In this section we show that the apparent profitability of 7(a) lending is reduced by the fact that the cost of capital for fully guaranteed 7(a) loans is significantly above Treasury rates, as inferred from the pricing of securities in the Guaranteed Loan Pool Certificate Program (GLPCP).

5.1 Data

We were able to obtain a history of monthly market prices on 166 of GLPCP pools that were issued by Coastal Securities between January 2006 and May 2008. Coastal Securities is one of the leading dealers in SBA-backed securities, and actively markets GLPCP pools. Other than secondary

market certificate prices, the data set includes the origination date, final maturity, loan origination amount, and spread to prime. The average pool size at origination is \$10.3 million, with a standard deviation of \$12.2 million. The smallest pool size at origination is \$1.0 million, and the largest is \$95.2 million.

Our sample falls into a period with historically narrow credit spreads. Figure 6 shows that prior to the sample period and following the end of our sample (based on Bloomberg estimates and not our model), the yields on GLPCP securities were much higher. This suggests that usually there may be an even higher risk premium than what we infer in the analysis that follows.

5.2 Pricing Model

To estimate the risk premium priced into GLPCP securities, we implement a Monte Carlo Model to project the distribution of pool cash flows over the maximum life of the pool. The term structure of Treasury rates evolves stochastically according to a 2-factor CIR model following Kaplin, Sun and Jagannathan (2001). The Prime rate set to a 3 percent spread over the implied instantaneous Treasury rate. The model is described in more detail in Appendix 2.

Cash flows are affected by three sources of uncertainty: prepayments, defaults, and changes in the prime rate. Prepayments and defaults both trigger a terminal payment of outstanding principal and interest. Rate changes affect the interest payment in the next period, and also require a re-amortization of the remaining principal. Annual prepayment and default rates in each year after origination are based on average rates derived from the SBA loan data for each year in a loan's life. Both prepayment and default rates vary significantly over the life of a loan. Prepayment rates on medium- and long-term loans peak between 6% and 8% in years 3 to 6 and then steadily decline. Default rates peak in year 2 and 3 at between 2% and 3%, and drop off sharply thereafter. These patterns are reflected in the simulations, which employs draws from a uniform random number generator to determine whether a default or prepayment has occurred in a given month.

Uncertainty arising from changes in the prime rate, default, and prepayment potentially introduces systematic risk into the cash flows. The spread between Prime and Treasury, which we take to be fixed, in fact is expected to be countercyclical, since credit spreads increase in downturns. This imparts a slightly negative beta to the promised cash flows, which thereby get larger when times are bad. The sign on the systematic risk introduced by default and prepayment is not obvious. Both trigger an early return of principal, so the effect depends on the systematic risk of the triggering event, and whether the GLPCP securities tend to systematically priced at a discount or premium to par in a downturn. We do not try to quantify these potential effects of systematic risk. Instead, we look for the fixed percentage point spread over 1-month Treasury rates implied by the CIR model that equates the model price with the market price from the Coastal Securities data.

5.2 Results and Sensitivity Analysis

The results of the analysis are shown in Figure 7, which show the dispersion of implied risk premia across loan pools in each month in the sample. Note that the amount of price data increases over time, since new pools are entering the sample and older pools are not aging out of it. The overall average premium is 137 bps, with a standard deviation of 35 bps. An interesting question is how much of the variation in the premium can be explained by observable pool characteristics such as the original size and final maturity of the pool, and the spread to prime. This question, and additional sensitivity analysis, will be included in the next draft.

5.3 Implications for Profitability

The simple profitability calculation in Section 3.2 took the short-term Treasury rate as the cost of capital for the guaranteed portion of 7(a) loans. The analysis of this section suggests that the cost is higher by an average of 137 bps. Taking this into account in the cost of capital calculation decreases estimated profits as a percent of loan value from 2.6% to 1.6%, and decreases the breakeven cost of

uninsured capital from 17.6% to 14.3%. Hence, the significantly higher than risk-free cost of capital to finance the guaranteed portion of SBA 7(a) loans explains part, but not all, of why loans cost borrowers so much.

6. Summary and Conclusions

In this paper we have looked at two possible and complementary explanations for the high rates charged to borrowers in the 7(a) program. We find support in the data for both: (1) that large SBA lenders may be able to exert market power and thereby charge higher rates, and (2) that the cost of funding the guaranteed portion of SBA loans is significantly above Treasury's cost of funds. These findings suggest that there may be ways to modify the 7(a) program to reduce costs to borrowers and finance the program more efficiently.

The leading alternative to guaranteed lending is direct lending, where the government directly funds and originates loans. A rationale for involving private intermediaries in the origination process for government loan programs is when screening for credit quality is important. Although we did not try to assess the value of this function for the 7(a) program, clearly small business loans are risky and require judgment, suggesting an important screening role for the private sector. Nevertheless, it seems that the government could reduce program costs by purchasing the guaranteed portion of loans directly from bank lenders, and funding those purchases through the Treasury rather than through sponsorship of a private securitized loan market.⁹

If in fact large SBA lenders are able to exert market power in loan pricing, a further role for the federal government could be to try to increase competitiveness in the market, perhaps by introducing a more centralized loan application process that would force lenders to compete more directly for borrowers. This would be consistent with the general trend toward the increased reliance on credit

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⁹ This was the rationale in 1973 for consolidating federal borrowing for different programs through the Federal Financing Bank.

scoring and other types of hard information in small business lending, although local lenders may still have information advantages that make them the most efficient originators (citations TBA).

APPENDIX 1 – 7(a) Loan Data

The SBA administers 7(a) Program data in their Electronic Loan Information Processing System (ELIPS). This system incorporates static data – information that does not regularly change over the life of a loan – as well as loan transactions related to balance, purchase, recovery, and other activities. The current project relies on data from ELIPS to identify basic loan, size, maturity, lender information, borrower zip code, as well as to identify transactions related to purchases and recoveries to identify defaults and prepayments.

Static Loans File: The file consists of over 1.2m of SBA loans since the program has been in place, with one static record per loan. Each record contains a list of relevant information introduced at time of origination. It includes record of the following: date loan was taken out, loan amount, loan size, maturity date, SBA guarantee, interest rate (including the spread above prime rate), program type (Regular or Express), borrower and lender's zip code, lender's name, and characteristics of the business.

Transactions File: The file consists of all transactions for each individual loan in the static file. These transactions include disbursements, monthly loan payments (including fees), balance, SBA purchases to indicate default, and recoveries (when there is default). Based on the information contained in the transaction file it is possible to determine whether and when default and prepayment has occurred. The transactions file also provides full information about recoveries and other information which are important in determining the default and prepayment behavior of all SBA loans.

APPENDIX 2 – VALUING GLPCP SECURITIES

Overview of Securities and Underlying Risks

The GLPCP securities in our dataset amortize every month and pays interest at PRIME rate plus a fixed spread. The pool cash flows are subject to early termination in the form of default or prepayment. Either of these events triggers a final payment of outstanding principal and accrued interest. The full Government guarantee on these securities means that certificate holders do not lose any principal when default occurs. However, since these pools typically trade above par, investors may lose the premium above par when these pools terminate earlier than expected. Yet, it is unclear which portion of these risks is diversifiable and which portion is systematic. The approach used to address this question is to treat these risks as independent from interest rate risk, and use the empirical frequency to model these events. Furthermore, we perform a sensitivity analysis to infer that the magnitude of an increase in default (prepayment) frequency does not have a large impact on pool prices.

This appendix describes details of the Monte Carlo implementation used to model the distribution of pool cash flows and the value of the certificates, and is organized as follows; we first use a two-factor Cox-Ingersoll-Ross (CIR) model to simulate risk-neutral paths of interest rates used for credit-risk-free discounting and for determining the pool floating interest payments. We then model prepayment and default using the historic experience derived from 18 years of comprehensive data obtained from the SBA. We combine these three components to compute the value of each pool in our sample, and then devise an algorithm to measure the spread above Treasuries embedded in the market price of these securities.

A2.1 Interest Rates and Pool Amortization

We employ a two-factor CIR model to simulate paths of future Treasury rates. In the Jagannathan Kaplin Sun (JKS – 2003) version of the CIR model the instantaneous interest rate R(t) is the sum of a constant \overline{R} and the two state variables $y_i(t)$, for i = 1, 2:

$$R(t) = \overline{R} + y_1(t) + y_2(t)$$
 (1)

Each state variable follows an independent, mean-reverting, square root process along any period s, between the current time t and the maturity date T:

$$dy_i(s) = \kappa_i \left[\theta_i - y_i(s) \right] ds + \sigma_i \sqrt{y_i(s)} dW_i(s), \quad i = 1, 2.$$
 (2)

where W_i 's are standard and independent Brownian motions. Under the pricing (risk-neutral measure) the factors evolve according to:

$$dy_i(s) = \overline{\kappa}_i [\overline{\theta}_i - y_i(s)] ds + \sigma_i \sqrt{y_i(s)} dW_i^*(s), \ i = 1, 2$$
(3)

where

$$\frac{-}{\kappa_i} = \kappa_i + \lambda_i \tag{4}$$

and
$$\overline{\theta}_i = \frac{\kappa_i \theta_i}{\kappa_i + \lambda_i}$$
 (5)

The market price of risk, λ_i for each state variable is assumed to be linear. The price at current-time t of a zero coupon bond that pays \$1 at maturity date T is

$$P(t,T) = e^{-\bar{R}(T-t)} p_1(t,T) p_2(t,T)$$
(6)

where

$$p_i(t,T) = e^{A_i(t,T) - B_i(t,T)y_i(t)}$$
 (7)

$$A_{i}(t,T) = \frac{2\kappa_{i}\theta_{i}}{\sigma_{i}^{2}} \ln \left[\frac{2\gamma_{i} \exp\left[(\gamma_{i} + \kappa_{i})(T - t)/2\right]}{(\gamma_{i} + \kappa_{i})\left[\exp\left[\gamma_{i}(T - t)\right] - 1\right] + 2\gamma_{i}} \right]$$
(8)

$$B_{i}(t,T) = \frac{2(\exp\left[\gamma_{i}(T-t)\right]-1)}{(\gamma_{i}+\kappa_{i})\left[\exp\left[\gamma_{i}(T-t)\right]-1\right]+2\gamma_{i}}$$
(9)

and

$$\gamma_i = \sqrt{\overline{\kappa_i}^2 + 2\sigma_i^2} \tag{10}$$

The yield to maturity, YTM of a zero coupon bond maturing at T is

$$YTM(t,T) = \frac{-\ln P(t,T)}{T-t} \tag{11}$$

In our analysis we use the two-factor parameters estimated in JKS using weekly LIBOR rates of various maturities:

Table A2.1 – Two factor CIR parameters, estimated by JKS							
Factor	κ	θ	σ	λ			
1	0.3922	0.2727	0.0153	-0.00038			
2	0.0532	0.0162	0.0430	-0.05917			
$\overline{R} = -0.2289$							

Factor 1 has stronger degree of mean reversion and drives the gap between long and short rates. Factor 2 has higher volatility parameter and determines the long run rates.

In every period t in our sample we solve for the initial state variables, $y_1(t)$ and $y_2(t)$, by fitting two points in the yield curve from historic Treasury data, the three month T-Bill rate and the ten year Treasury bond rate. We perform this in two steps, first set the historic yields on left hand side of equation (11) to solve for P(t,T). Then use P and system (6) – (10) to solve for $y_1(t)$ and $y_2(t)$.

We need initial state variables to simulate the (monthly) discrete version of (3), with Monte Carlo paths that start at time t and end at T. Monthly discretization means that time step $h = \frac{1}{12}$. For any time s between t and t we have:

$$y_i(s+h) = y_i(s) + \overline{\kappa}_i[\overline{\theta}_i - y_i(s)]h + \sigma\sqrt{\max[y_i(s),0]}\sqrt{hW}$$
(12)

where W's are drawn from standard and independent normal random generator.

From (1) and the initial state variables we compute the initial short rate R(t). In turn cash flows arriving one month from t are discounted back to t with the factor

$$d(t,t+h) = \frac{1}{1 + R(t)h}$$
 (13)

Cash flows arriving at s+h are discounted back to t using factor

$$d(t,s+h) = \frac{d(t,s)}{1+R(s)h}$$
(14)

We assume that the prime rate is 3% above the short rate. Suppose a SBA guaranteed pool pays a fixed spread above prime, Δ . Then for the same Monte Carlo run as above the pool's floating interest rate is set to

$$\rho(s) = R(s) + 0.03 + \Delta$$

For a given balance B(s) the pool payment in the following month (when there is no termination) and new balance are set by the amortization schedule:

$$Pmt(s+h) = \frac{B(s)(\rho(s)h)}{1 - (1+\rho(s)h)^{s-(T-t)}}$$
(16)

and

$$B(s+h) = [1 + \rho(s)h]B(s) - Pmt(s+h)$$
(17)

A2.2 Default and Prepayment

A guaranteed pool is composed of several underlying SBA guaranteed loans. In practice these certificates experience partial termination when only a portion of the underlying loans default or prepay. However, some investors acquire single guaranteed SBA loans in the secondary market instead of pool certificates, and are therefore subject to full termination when the loan defaults or prepays. Since investors may diversify their holdings in these single loans, for valuation purposes we can model pools as if they were single loans by assuming that default or prepayment triggers full termination. In essence, the termination events in this section are reflected in simulations, which employ draws from a uniform random number generator to determine whether a loan has defaulted or prepaid.

We estimate default and prepayment frequencies as a function of loan age, using eighteen years of comprehensive data provided by the SBA. In period s the default and prepayment probabilities for loan j are expressed by $\pi_j^d(s)$ and $\pi_j^p(s)$.

In time step s of the Monte Carlo run where we draw W from eq. (12) we also draw two independent numbers from a uniform number generator U_s^d and U_s^p . Default on loan j is triggered if

 $U_s^d \leq \pi_j^d(s)$ and prepayment is triggered if $U_s^p \leq \pi_j^p(s)$. When either event occurs the pool terminates and investors receive the remaining balance plus any accrued interest. In those nodes Pmt in (16) becomes:

$$Pmt(s) = [1 + \rho(s-h)h]B(s-h)$$
 (16')

For valuation purposes it is convenient to set Pmt(u) = 0 for any u > s, where s is a termination time.

A2.3 Valuation

Using the interest rate behavior and termination behavior above we obtain pool prices at any time t. The only inputs for each pool are the origination date t_0 , the maturity date T, and the pool's fixed spread above the prime rate Δ . For a given Monte Carlo path indexed by n the price at time t of the pool j is

$$m_j^n(t) = \sum_{s=t+1}^T Pmt(s)d(t,s)$$
(17)

The model price of pool j at time t is computed as the average price of all simulated paths, and represented by

$$M_{j}(t) = \frac{1}{N} \sum_{n=1}^{N} m_{j}^{n}(t)$$
 (18)

In our simulations we use N=100,000.

A2.4 Market Prices, Spreads, and Sensitivity

In our sample, pool market prices are often lower than model prices $M_j(t)$, which suggests that investors discount pool cash flows at higher than credit-risk-free Treasury rates. Alternatively, investors may demand a premium for bearing any systematic termination-risk which may not have been accounted in our model. In this section we introduce a premium above Treasury rates to equate model prices to market prices. We also perform a similar exercise by introducing a multiplicative premium over default and prepayment probabilities to examine the price impact of changes in termination probabilities.

Define the discount premium ψ , and modify expression (13) to discount cash flows from t+h back to t and expression (14) to discount cash flows from s+h back to t:

$$d(\psi;t,t+h) = \frac{1}{1 + [R(t) + \psi]h}$$
(13')

and

$$d(\psi;t,s+h) = \frac{d(t,s)}{1 + [R(s) + \psi]h}$$
(14')

For each pool j and time t we solve for the ψ which equates $M_j(t)$ to market prices. The results are summarized in Figure 7, which shows the monthly dispersion of premium along our sample.

An alternative way to measure the premium demanded by pool investors is to introduce a multiplicative constant φ to enhance (or depress) the default or prepayment probabilities. For instance, if φ^d is a default probability premium then default on loan j is triggered at time s if $U_s^d \leq \varphi^d \pi_j^d(s)$ and

prepayment is triggered if $U_s^p \le \pi_j^p(s)$. Analogously, if φ^p is a prepayment probability premium then default on loan j is triggered at time s if $U_s^d \le \pi_j^d(s)$ and prepayment is triggered if $U_s^p \le \varphi^p \pi_j^p(s)$.

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Table 1: Top 20 SBA Lending Institutions in 2006

1 abic 1. 10p 20 0B	Loans per lender	\$ amount per lender
BANK - AMERICA NATL ASSOC	9,776	447,449,130
JPMORGAN CHASE BANK NATL ASSOC	6,567	695,544,516
RBS CITIZENS NATL ASSOC	4,905	299,151,862
WELLS FARGO BANK NATL ASSOC	4,448	666,003,115
CAPITAL ONE NATL ASSOC	4,159	206,557,962
U.S. BANK NATIONAL ASSOCIATION	4,093	460,909,009
WASHINGTON MUTUAL BANK	2,653	157,455,477
PNC BANK, NATIONAL ASSOCIATION	2,078	388,056,517
NATIONAL CITY BANK	1,947	162,624,308
BANCO POPULAR NORTH AMERICA	1,492	357,218,724
CIT SMALL BUS. LENDING CORP	1,380	792,124,943
MANUFACTURERS & TRADERS TR CO	1,289	144,180,273
HSBC BK USA NATL ASSOC	1,025	71,770,931
CITIBANK, N.A.	983	67,509,780
THE HUNTINGTON NATIONAL BANK	916	105,736,523
KEYBANK NATIONAL ASSOCIATION	892	139,012,935
COMPASS BANK	884	94,241,957
COMMERCE BANK NATL ASSOC	732	159,338,573
ZIONS FIRST NATIONAL BANK	712	84,877,789
CALIFORNIA BANK & TRUST	565	85,047,012

Table 2: Rate Spreads (bps) and Guarantee Percentages 1988-2006

_	Entire Sample	Regular 7 (a)	Express
Average* Spread (over Prime)	236.06	195.96	327.28
Average Guarantee	0.706	0.796	0.501

^{* -} simple average

TABLE 3 – MAXIMUM GUARANTEE AND INTEREST SPREAD CAPS

3a. MAXIMUM GUARANTEE

Loan Type	Loan Size	Maximum SBA Guarantee
Regular	\$0 - \$150K	85%
Regular	\$150K +	75%
Express	all	50%

3b. MAXIMUM INTEREST RATE SPREAD – EXPRESS LOANS

Loan Size	Maximum Interest Rate Spread
\$0 - \$50K	6.5%
\$50K +	4.5%

3c. MAXIMUM INTEREST RATE SPREAD – REGULAR LOANS

Loan Size	Maturity	Maximum Interest Spread
\$0 - \$25K	7yrs +	4.75%
\$0 - \$25K	0 – 7yrs	4.25%
\$25K – \$50K	7yrs +	3.75%
\$25K – \$50K	0 – 7yrs	3.25%
\$50K +	7yrs +	2.75%
\$50K +	0 – 7yrs	2.25%

TABLE 4 – The Effect of Regional Concentration on Interest Rate Spreads

	Loan Type	Loan Size	Maturity	Sample Size	Intercept	Herfindhl	Guarantee	Loan Size	Maturity	Adj Rsq
Express_Small	Express	0-\$50K	-	95,043	474.23	-55.3	-352.1	-0.00252	2.038	0.0894
					(20.05)	(-14.33)	(-7.49)	(-50.4)	(75.4)	
E		.		00.500	0.47.00	50.44	400.00	0.000004	0.404	0.0000
Express_Large	Express	\$50+	-	80,596	347.66 (24.19)	-58.41 (-15.43)		-0.000381 (-65.25)	0.481 (29.9)	0.0639
					(24.13)	(-13.43)	(-4.55)	(-03.23)	(23.3)	
Regular_Small_Short	Regular	0-\$25K	0-7yr	11,130	90.91	-22.76		0.000345	0.156	0.008
					(5.61)	(-5.18)	(7.03)	(1.57)	(2.95)	
Regular_Small_Long	Regular	0-\$25	7yrs +	1,351	33.83	-46.89	230.64	0.00064	-0.194	0.044
	-		-		(0.78)	(-4.52)	(4.67)	(1.24)	(-3.66)	
December Association	D I	#05 #50	0.7	00.005	00.44	00.54	454.00	0.00040	0.000	0.0450
Regular_Avg_Short	Regular	\$25-\$50	0-7yr	29,825	83.11 (9.25)	-36.51 (-14.59)		-0.00018 (-2.3)	0.089	0.0153
					(9.23)	(-14.59)	(13.39)	(-2.3)	(3.0)	
Regular_Avg_Long	Regular	\$25-\$50	7yrs +	8,124	75.46	-39.32	188.11	-0.00022	-0.119	0.0249
					(4.35)	(-9.04)	(9.81)	(-1.49)	(-5.45)	
Regular Large Short	Dogular	\$50-\$150	0.7.	76,315	134.7	-44.26	81.97	-0.00007	0.075	0.0139
Regular_Large_Short	Regulai	φου-φ1ου	0-7yr	76,315	(27.87)	(-28.0)		(-5.67)	(4.33)	0.0139
					(27.07)	(20.0)	(10.20)	(0.01)	(1.00)	
Regular_Large_Long	Regular	\$50-\$150	7yrs +	64,319	123.77	-48.35	108.98	0.000058	-0.07428	0.0218
					(23.79)	(-30.27)	(18.79)	(4.53)	(-13.38)	
Regular_Huge_Short	Regular	\$150+	0-7yr	41,144	180.32	-31.06	25.69	-0.00001	0.044	0.0069
Regulal_Huge_Short	Regulai	φ130+	0-7 yı	41,144	(34.38)	(-14.93)		(-5.11)	(2.17)	0.0069
					(04.00)	(14.55)	(4.10)	(3.11)	(2.17)	
Regular_Huge_Long	Regular	\$150+	7yrs +	167,310	181.36	-8.57	58.09	-0.000014	-0.116	0.0283
					(70.33)	(-8.79)	(18.41)	(-23.64)	(-46.81)	

TABLE 5 – The Effect of Lender Size on Interest Rate Spreads

	Estimates	t-stat
Guarantee	-44.44	-1.5
Loan Size	-0.00007	-19.0
Maturity (months)	0.382	22.8
Lender w/ 0 to 9 loans	96.27	4.1
Lender w/ 10 to 99 loans	42.95	1.8
Lender w/ 100 to 499 loans	137.097	5.8
Lender w/ 500 to 999 loans	119.97	7 5.1
Lender w/ 1000 + loans	189.61	8.1
Express Dummy	127.41	14.5
Interest Ceiling_dummy	252.63	156.0
Adjusted R-Squared	0.8687	7
Number of Observations	37,728	

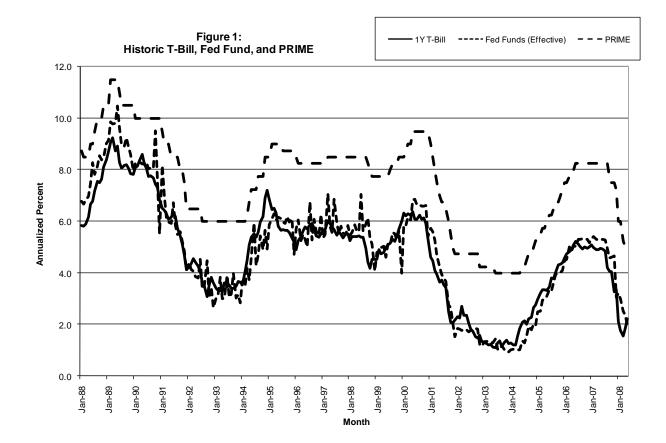
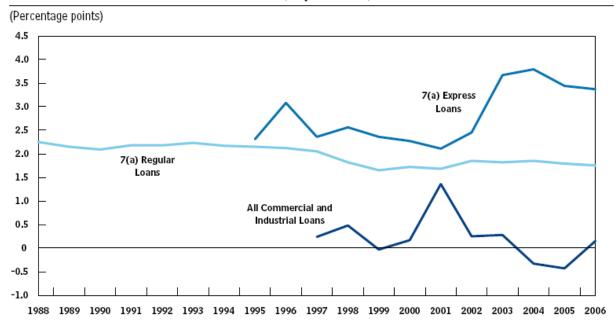


Figure 2:

Interest Rate Spreads over the Prime Rate for SBA 7(a) Loans and for All Commercial and Industrial Loans, by Cohort, 1988 to 2006

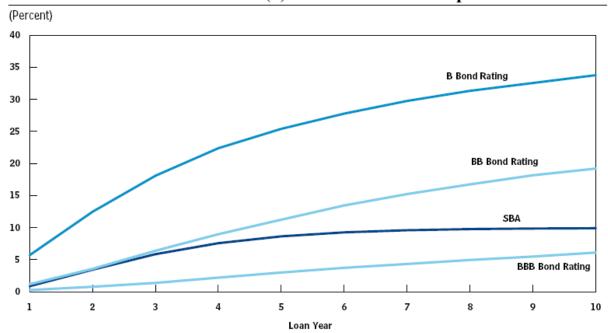


Source: Congressional Budget Office based on data from the Small Business Administration (SBA) and the Board of Governors of the Federal Reserve System.

Note: Interest rate spreads over the prime rate for all commercial and industrial loans are for loans made by commercial banks in amounts between \$100,000 and \$999,999.

Figure 3:

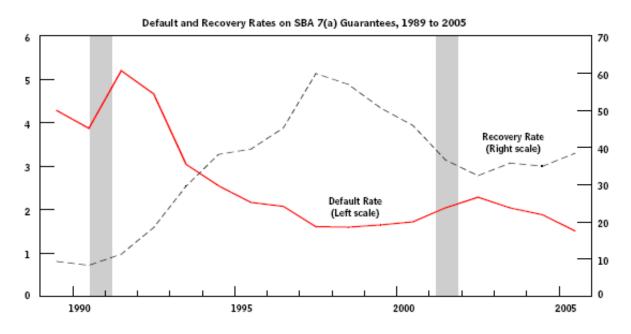
Cumulative Default Rates on SBA 7(a) Loans and on S&P Corporate Bonds



Source: Congressional Budget Office based on data from the Small Business Administration (SBA) and Standard & Poor's.

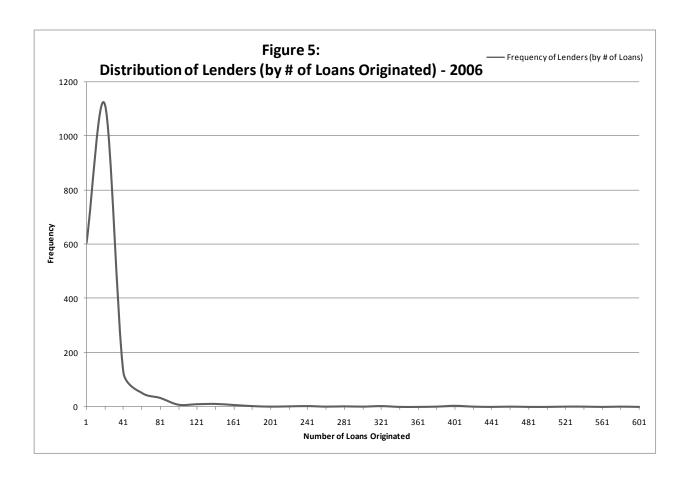
Note: The default rate is the number of defaulted loans as a percentage of the total number of loans in SBA's portfolio or the number of corporate bond issuances for each rating class as determined by Standard & Poor's.

Figure 4:

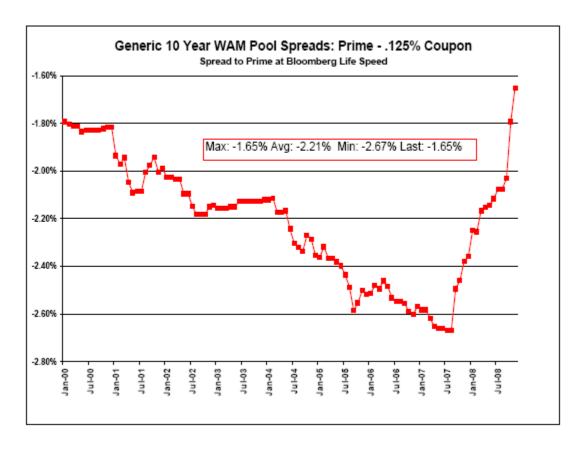


Source: CBO tabulations from SBA data.

Default rate is claims over total outstanding loan balances. Recovery rate is recoveries divided by total claims.







Source: Coastal Securities

 $http://www.coastalsecurities.com/sbamarketinfo/State\%\,20of\%\,20 the\%\,20 SBA\%\,20 Markets_20081203.pdf$

Note: Over this period Prime was 3% above overnight Fed Funds. Hence -2% on this scale is a yield of Fed Funds plus 1%.

