Trade, Institutions, and Economies in Transition

Gal Hochman[*], Chrysostomos Tabakis[†], and David Zilberman[‡]

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Abstract

In the paper governments affect firms' profits by spending on public goods and by the introduction and enforcement of law. The elite in the paper may have close ties to the ruling government. One key question, which is investigated in the paper, is how opening to trade affects the market institutions and the amount of public spending on infrastructure. We find that opening an economy to trade improves the quality of its institutions and increases the attractiveness of its business environment. More specifically, the equilibrium institutions are stronger under trade in comparison to the one that would have prevailed in autarky. Finally, an open economy ends up with better infrastructure relative to a closed one.

JEL Classification Codes:F12, O19

Key words: Trade, autarky, public spending, market institutions, corruption, connected entrepreneurs

[*]UC Berkeley — Berkeley — CA, 94702 — email:galh@berkeley.edu

[†]New University of Lisbon — Lisbon — Portugal, 1099-032 — email:ctabakis@fe.unl.pt

[‡]UC Berkeley — Berkeley — CA, 94702 — email:zilber@are.berkeley.edu

1 Introduction

The quality of domestic institutions, such as property rights or rule of law, is a key determinant of long-run economic growth and can therefore help explain the under-performance of many developing countries (see, for instance, Acemoglu, Johnson, and Robinson, 2005a). Institutions are important in explaining income differences across countries. But it is still unclear what determines institutional differences across countries. [1] To this end, we investigate how international trade affects domestic institutions. A secondary but important question we address is how the former affects investment in public infrastructure, which is widely viewed as another critical factor for long-term economic performance (Tanzi and Davoodi, 1997; Francois and Manchin 2007).

We build an economic model in which the quality of market institutions and public infrastructure investment are both *endogenously* determined by a government that seeks to maximize real income. We compare the outcomes in both autarky and trade, and show numerically that opening an economy to trade leads to both higher investment in infrastructure and better domestic institutions. Therefore, our work suggests that international trade might have a beneficial (indirect) effect on long-term growth through the latter channels. Trade creates additional rents and supports stronger institutions, public spending is larger and the infrastructure burden is distributed more evenly across firms (i.e., less corruption). Trade increases the benefit from public spending on infrastructure, a public good that reduces variable costs. On the other hand, reducing corruption and improving market institutions decrease, on average, the cost from public spending to a firm. Different from existing literature, we introduce two policy choices; in addition to the choice of domestic institutions, the ruling government decides how much to spend

[1] Although some papers demonstrated that greater trade openness could raise institutional quality (e.g., Acemoglu, Johnson, and Robinson 2005b; and Levchenko 2007). Other recent papers demonstrated that in a country with weak political institutions, opening up to trade could make things worse with regard to both economic institutions and policies (e.g., B and B 2004; Segura-Cayuela 2006; and Stefanadis 2006).

on infrastructure. Furthermore, we focus on owners' ability to cope with corruption and bureaucracy, and not on the elite's ability to expropriate others.

To investigate the relationship between trade openness, institutions, and public spending on infrastructure, we assume in the paper that a (new) ruling government wants to reform the economy, and is considering the benefits from opening to trade. In the economy, some firm owners, denoted connected, have close ties to the ruling government. These connected firms benefit from a lower market-institutions coefficient, because the infrastructure burden is biased toward non-connected firm owners. Specifically, we assume heterogeneity at the firm owners' level: some firm owners, but not all, gain from flawed institutions and are denoted connected. Institutional quality is represented by a parameter that lies between 0 and 1, the market-institutions coefficient. A coefficient value of zero indicates that the economy is in total anarchy (e.g., contract enforcement is weak, there is extreme corruption, and political ties matter), whereas a coefficient value of 1 indicates that the economy is a perfect market economy (e.g., property rights are well defined, there is no corruption, and political ties do not matter). We assume that some (but not all) firms have political ties to the current government, which allows them for a *given* market-institutions coefficient to face a lower fixed cost of production, due perhaps to their ability to evade taxes or avoid the bureaucratic channels in order to carry out their everyday business. This introduces firm heterogeneity with respect to their cost structure. However, as the quality of domestic institutions increase (i.e., as the market-institutions coefficient rises), political ties matter less and firms' fixed cost of production converges to the same level.

Firms' fixed cost of production is also a function of infrastructure spending. The latter affects firms in two very different ways. On the one hand, it improves their productivity and thus their ability to compete internationally and export their products. On the other hand, it raises their fixed cost of production, since we implicitly assume that taxes will be increased in order to finance the augmented public spending. Of course, unless the market-institutions coefficient equals 1, firms share unequally this burden, since some of them can evade taxes through their political connections.

Our modelling of the production side of the economy relies heavily on Dixit and Stiglitz's (1977) model of monopolistic competition and is clearly inspired by the Melitz (2003) model of trade with heterogeneous firms and monopolistic competition, as well as Helpman, Melitz, and Yeaple (2004). Nevertheless, there are some major differences. As we discussed above, in our model, firms differ with respect to their fixed cost of production. However, firms *within* a country have the same productivity, because the overall firm productivity is a function of the economy's infrastructure – our model allows only for firm productivity differences *across* countries. In addition, we assume that firms face a fixed cost to foreign trade, which is also a function of their political ties to the government and the market-institutions coefficient. In other words, *given* a level of institutional quality, firms face heterogeneous fixed costs of exporting. Given these assumptions, in equilibrium, only the firms that are fairly politically connected find it optimal to engage in production. Furthermore, only the most connected of them export.

The timing of our game is as follows. The government of the transition economy initially picks the market-institutions coefficient so as to maximize real national income. Subsequently, it decides on public infrastructure spending with the same objective function in mind. Finally, production takes place and profits are realized. When setting the timing of the game we implicitly assumed that the ruling government can decide on public spending on infrastructure only after it sets the market institutions. Before deciding how much to spend, it needs to know how much it can collect from the firms. We explore the outcomes of these decisions in both autarky and free trade. We assume the trading partner of the economy is a perfect market economy with a market-institutions coefficient equal to 1. In other words, there is no firm heterogeneity in the latter economy. Although under autarky, and assuming public spending is partially financed by the firms themselves (e.g., lump sum taxes), the number of firms producing in equilibrium decreases with public spending on infrastructure. The number of firms, both those producing only domestically as well as those exporting, increase with infrastructure spending.

We also find numerically that both the endogenously chosen market-institutions coefficient and the level of public spending on infrastructure are higher in free trade than in autarky. Therefore, we identify two additional benefits from international trade, above and beyond the standard ones with respect to increased variety or a lower ideal price index faced by domestic consumers.

This work, therefore, differs from Segura-Cayuela (2006), who argued that part of the reason for poor growth performance may be related to the interaction between weak institutions and trade. This paper differs from that paper, since we focus on owners' ability to cope with corruption and bureaucracy, and not on the elite's ability to expropriate others. We show that trade may both increase welfare since it imports "strong" institutions, and increase potential profits since it lowers fixed costs for the majority of owners. It generates demand for further reforms in the market institutions, as well as demand for more infrastructure. It is interesting to note that Law and Demetriades (2006), using dynamic panel data techniques and data from 43 developing countries during the period 1978-2000, showed that openness, in terms of trade and capital flows, is particularly potent in promoting financial development in middle-income countries, but much weaker in low-income countries. Cheptea (2007) also finds support for a positive correlation between trade and institutions, focusing on two factors promoting CEE–EU trade integration: trade liberalization and institutional reforms.[2].

Do and Levchenko (2006), while assuming a fixed mass of firms, showed that openness may be detrimental to institution quality. They assumed heterogeneity with respect to productivity. This paper, unlike Do and Levchenko, models heterogeneity with respect to firm owners' ability to deal with bureaucracy and corruption and assumes institu-

[2]In general, trade openness is associated with better institutions in a cross-section of countries (Ades and Di Tella 1997, Rodrik et al. 2004, and Rigobon and Rodrik 2005, among others).

tions affect this ability. The current paper also allows for free entry, where the political environment allows us to relax the assumption that the number of firms is constant.[3] The two papers differ in the conditions required for us to observe that institution quality improves with the introduction of trade. Specifically, the regularity conditions, which link productivity and the ability to deal with corruption (i.e., with "weaker" institutions), are not required. Although both papers linked institutions with fixed costs, the interpretation given here differs from that given by Do and Levchenko, since we do not need to argue that corruption is correlated with productivity. To this end, Faccio (2006) showed that firms owned by politically connected entrepreneurs are less productive, on average.

The economic structure of this model is described in Section 2. The autarky equilibrium is depicted in Section 3, where the trade equilibrium in Section 4. The choice of public spending on infrastructure is derived and characterized in Section 5, and Section 6 characterizes the second endogenous variable, i.e., the market-institution coefficient. Section 7 discusses results and concludes.

2 The Model

We now describe the economic model, which includes production and consumption decisions, as well as the timing of the multi-stage game. First consumers are defined, then we move to production, define the market-institution coefficient, and conclude with the timing of the game.

[3]Note that the paper can easily be extended to adopt the approach of Benabou (2000), whereby wealthier owners carry larger weights with the government, and therefore increase the set of parameters for trade to deteriorate the economy's institutions.

2.1 Demand

Consider an economy consisting of two sectors. The first sector produces a homogeneous good z, while the second one produces a continuum of differentiated goods $x(v)$. The preferences of a representative consumer are captured by the following utility function:

$$U = \left[\int_{v \in V} x(v)^{\alpha} \, dv \right]^{\frac{\beta}{\alpha}} z^{1-\beta}, \tag{1}$$

where V is the set of available varieties of good x, and α, $\beta \in (0,1)$. Standard utility maximization results in the following demand functions (e.g., Do and Levchenko 2006):

$$x(v) = Ap(v)^{-\varepsilon} \text{ and} \tag{2}$$

$$z = \frac{(1-\beta) E}{p_z}, \tag{3}$$

where $A \equiv \frac{\beta E}{\int_{v \in V} p(v)^{1-\varepsilon} dv}$, E is total expenditure, $p(v)$ and p_z are the goods prices, and $\varepsilon = \frac{1}{1-\alpha} > 1$. Whereas β measures the share of expenditure spent on the differentiated goods, ϵ measures the elasticity of substitutions between varieties of good x.

2.2 Production

Labor is the only factor of production and is inelastically supplied at its aggregate level L. The homogeneous good z is produced with a linear technology of the form:

$$l = z. \tag{4}$$

We normalize p_z and thus, the wage to 1.

Moreover, in the differentiated sector, the mass of the potentially produced varieties is fixed and equals to N.[4] There is a continuum of firms, each of which can only produce a single variety with the following production technology:

$$l = af(F) + \frac{x}{F}, \tag{5}$$

[4] A similar assumption was used in Helpman, Melitz, and Yeaple (2004).

where F is the level of infrastructure in the economy. Firms share the same $f(F) > 0$, with $\partial f / \partial F > 0$, but have a different a and thus, a heterogeneous fixed cost of production. We can then index firms by a. Of course, any given firm is free not to produce and therefore, not to incur the fixed cost $af(F)$. All firms deciding to produce face a residual demand curve given by (2) and have the same marginal cost of production. Profit maximization yields the following pricing strategy for all firms:

$$p(a) = \frac{1}{F\alpha}. \tag{6}$$

Firm profit is then:

$$\pi(a) = p(a)\,x(a) - l(a) = A\left(\frac{1}{F\alpha}\right)^{1-\varepsilon}(1-\alpha) - af. \tag{7}$$

2.3 The Market-Institutions Coefficient

The fixed-cost parameter a characterizing each firm in the differentiated sector is given by:

$$a = a_0(1-\theta) + \theta, \tag{8}$$

where $\theta \in (0,1)$ is the market-institutions coefficient, with higher θ signifying higher domestic institutional quality, and a_0 represents the political ties of a given firm to the current government, with lower a_0 signifying closer ties to the latter. We assume that a_0 is uniformly distributed on $[0,2]$. Therefore, for a *given* θ, a firm with a lower a_0 (i.e., a firm with closer ties to the government) has to incur a lower fixed cost in order to engage in production. Intuitively, this could be due to the ability of such a firm to more easily evade taxes. However, as θ rises (i.e., as institutional quality improves), the political ties to the current government matter less and firms' fixed cost of production converges to $f(F)$.

2.4 Timing

We assume the (new) ruling government wants to reform the economy, and is considering the benefits from opening to trade. To this end, we compare two competing regimes: Autarky and trade. Given the regime choice, we assume the following multi-stage game:

- Stage 1: The government picks the market-institutions coefficient θ so as to maximize its objective function (to be defined below).
- Stage 2: The government picks the level of infrastructure F so as to maximize its objective function.
- Stage 3: Firms choose whether to actually engage in production or not. Production then takes place, and profits are realized.

This outlines the basic structure of our model. We solve our game recursively. Specifically, we first characterize the closed and open economy equilibria for a given θ and F. Subsequently, we turn to the government's problem in Stage 2 and solve for the optimal F under both autarky and trade: $F_A^\star$ and $F_T^\star$, respectively. We then compare the two solutions and show numerically that $F_T^\star > F_A^\star$. Last, we turn to the government's problem in Stage 1, and solve for the optimal θ under both autarky and trade: $\theta_A^\star$ and $\theta_T^\star$, respectively. We finally prove that under certain conditions, $\theta_T^\star > \theta_A^\star$.

3 Equilibrium in a Closed Economy

To determine the autarky equilibrium, we need to find the cutoff level of a, a_A, such that all firms above the latter decide not to engage in production. The cutoff firm makes zero profit:

$$\pi\left(a_A\right) = 0 \Rightarrow A\left(\frac{1}{F\alpha}\right)^{1-\varepsilon}(1-\alpha) - a_A f = 0. \tag{9}$$

Since only firms with $a \leq a_A$ operate in equilibrium, we have that:

$$\int_{v \in V} p(v)^{1-\varepsilon} dv = N \int_{\theta}^{a_A} \left(\frac{1}{F\alpha}\right)^{1-\varepsilon} dG(a) = \frac{N}{(F\alpha)^{1-\varepsilon}} G(a_A). \tag{10}$$

Thus, in equilibrium, $A = \frac{(F\alpha)^{1-\varepsilon}\beta E}{NG(a_A)}$. The zero-cutoff-profit condition can then be rewritten as:

$$\pi(a_A) = 0 \Rightarrow \frac{\beta E}{NG(a_A)}(1-\alpha) = a_A f. \tag{11}$$

The equilibrium value of E is obtained by imposing the goods market-clearing condition that expenditure must equal national income. Since there is no free entry in our model (i.e., the number of potential producers is fixed), total national income equals the sum of labor income plus the profits accruing to all active firms.[5] It follows that:

$$E = L + N \int_{\theta}^{a_A} \pi(a) \, dG(a). \tag{12}$$

Using equation (11), it is straightforward to show that:

$$E = L + Nf \frac{(a_A - \theta)^2}{4(1-\theta)}. \tag{13}$$

Combining now the equilibrium conditions (11) and (13), we get that:

$$a_{A_{1,2}} = \begin{cases} \frac{Nf\theta[1-\beta(1-\alpha)] - \sqrt{Nf\left[4L\beta(1-\alpha)(1-\theta)(2-\beta(1-\alpha)) + Nf\theta^2\right]}}{Nf[2-\beta(1-\alpha)]}, \\ \frac{Nf\theta[1-\beta(1-\alpha)] + \sqrt{Nf\left[4L\beta(1-\alpha)(1-\theta)(2-\beta(1-\alpha)) + Nf\theta^2\right]}}{Nf[2-\beta(1-\alpha)]} \end{cases}. \tag{14}$$

It is direct to show that $a_A = \frac{Nf\theta[1-\beta(1-\alpha)] - \sqrt{Nf\left[4L\beta(1-\alpha)(1-\theta)(2-\beta(1-\alpha)) + Nf\theta^2\right]}}{Nf[2-\beta(1-\alpha)]} < \theta$, which would imply no firm operate in equilibrium. We choose to focus on the more interesting (and standard) case where the equilibrium is interior, i.e., the firm with the highest fixed cost of production does not produce in autarky. Therefore, we have that:

$$a_A = \frac{Nf\theta[1-\beta(1-\alpha)] + \sqrt{Nf\left[4L\beta(1-\alpha)(1-\theta)(2-\beta(1-\alpha)) + Nf\theta^2\right]}}{Nf[2-\beta(1-\alpha)]}. \tag{15}$$

[5] Note that the number of actual producers is not exogenously fixed but is rather endogenously determined in equilibrium.

4 Equilibrium in an Open Economy

We now model trade between the home economy (H) described above and a foreign one (F) whose structure is similar to the home one except for two aspects: (i) the market-institutions coefficient θ equals 1, i.e., the foreign country is a perfect market economy with high institutional quality; and (ii) there is a large (unbounded) mass of potential entrants into the differentiated sector. We assume also that the two countries have the same endowment of labor L. Moreover, we maintain the assumption that z can be traded costlessly (implying that as long as both countries produce some z, wages in the two countries equal to 1), whereas firms in the differentiated sector need to incur two additional costs in order to export: a fixed per-period cost $af_X^i > 0$, where $i \in \{H, F\}$ and $a = a_0 (1 - \theta) + \theta$, and a per-unit transportation cost.[6] The former is measured in terms of units of labor and differs among firms in the home country. The latter is of the iceberg type, is common across firms, and is denoted by $\tau > 1$. τ represents the number of units of a particular variety that need to be shipped so that 1 unit arrives at destination. Without loss of generality, we assume that domestic and foreign firms in the differentiated sector face the same transportation costs.

Let us begin our analysis from the home country. To determine the open-economy equilibrium, we need to derive the cutoff values for production and exporting: a_D and a_X, respectively. Each firm's domestic pricing strategy is still $p_D^H(a) = \frac{1}{F^H \alpha}$ $\forall a$. Its resulting domestic profit is given by:

$$\pi_D^H(a) = A^H \left(\frac{1}{F^H \alpha} \right)^{1-\varepsilon} (1 - \alpha) - af^H. \tag{16}$$

[6] Regardless of whether an *active* firm decides to export or not, it still incurs the fixed cost af^i. Furthermore, note that $\theta = 1$ in the foreign country implies that all firms in its differentiated sector have the same fixed cost of production f^F and the same fixed cost of exporting f_X^F.

For the exporting firms, the prices in the foreign market are higher due to the transportation costs: $p_X^H(a) = \frac{\tau}{F^H \alpha}$ $\forall a$. Their profits from exporting are then equal to:

$$\pi_X^H(a) = A^F \left(\frac{\tau}{F^H \alpha}\right)^{1-\varepsilon} (1-\alpha) - a f_X^H. \tag{17}$$

The zero-cutoff-profit conditions are thus given by:

$$A^H \left(\frac{1}{F^H \alpha}\right)^{1-\varepsilon} (1-\alpha) \quad = \quad a_D f^H \text{ and} \tag{18}$$

$$A^F \left(\frac{\tau}{F^H \alpha}\right)^{1-\varepsilon} (1-\alpha) \quad = \quad a_X f_X^H. \tag{19}$$

Furthermore, our second equilibrium condition is that expenditure equals national income, or:

$$E^H = L + N^H \left(\int_\theta^{a_D} \pi_D^H(a)\, dG(a) + \int_\theta^{a_X} \pi_X^H(a)\, dG(a) \right). \tag{20}$$

Using (18) and (19), we can rewrite (20) as:

$$E^H = L + N^H \left(f^H \frac{(a_D - \theta)^2}{4(1-\theta)} + f_X^H \frac{(a_X - \theta)^2}{4(1-\theta)} \right). \tag{21}$$

We now turn to the foreign country. Here all the firms in the differentiated sector have identical cost structure. In addition, unlike the home country, there is free entry. Therefore, to derive the open-economy equilibrium, we need to obtain the equilibrium number of firms selling domestically and exporting: n_D^F and n_X^F, respectively. As before, the domestic and export prices equal to: $p_D^F = \frac{1}{F^F \alpha}$ and $p_X^F = \frac{\tau}{F^F \alpha}$, respectively. The corresponding profits from domestic sales and exports are:

$$\pi_D^F = A^F \left(\frac{1}{F^F \alpha}\right)^{1-\varepsilon} (1-\alpha) - f^F \text{ and} \tag{22}$$

$$\pi_X^F = A^H \left(\frac{\tau}{F^F \alpha}\right)^{1-\varepsilon} (1-\alpha) - f_X^F. \tag{23}$$

The zero-profit conditions are then:

$$A^F \left(\frac{1}{F^F \alpha}\right)^{1-\varepsilon} (1-\alpha) \quad = \quad f^F \text{ and} \tag{24}$$

$$A^H \left(\frac{\tau}{F^F \alpha}\right)^{1-\varepsilon} (1-\alpha) \quad = \quad f_X^F. \tag{25}$$

Furthermore, given there is free entry and thus no firm profits, the income-equals-expenditure condition is simply:

$$E^F = L. \tag{26}$$

Note that in equilibrium:

$$\begin{aligned}\int_{v^H \in V^H} p(v)^{1-\varepsilon}\, dv = N^H \int_{\theta}^{a_D} \left(\frac{1}{F^H \alpha}\right)^{1-\varepsilon} dG(a) + n_X^F \left(\frac{\tau}{F^F \alpha}\right)^{1-\varepsilon} = \\ = N^H \left(\frac{1}{F^H \alpha}\right)^{1-\varepsilon} \frac{a_D - \theta}{2(1-\theta)} + n_X^F \left(\frac{\tau}{F^F \alpha}\right)^{1-\varepsilon} \quad \text{and}\end{aligned} \tag{27}$$

$$\begin{aligned}\int_{v^F \in V^F} p(v)^{1-\varepsilon}\, dv = n_D^F \left(\frac{1}{F^F \alpha}\right)^{1-\varepsilon} + N^H \int_{\theta}^{a_X} \left(\frac{\tau}{F^H \alpha}\right)^{1-\varepsilon} dG(a) = \\ = n_D^F \left(\frac{1}{F^F \alpha}\right)^{1-\varepsilon} + N^H \left(\frac{\tau}{F^H \alpha}\right)^{1-\varepsilon} \frac{a_X - \theta}{2(1-\theta)}.\end{aligned} \tag{28}$$

It follows that:

$$A^H \equiv \frac{\beta E^H}{\int_{v^H \in V^H} p(v)^{1-\varepsilon}\, dv} = \frac{\beta E^H}{N^H \left(\frac{1}{F^H \alpha}\right)^{1-\varepsilon} \frac{a_D-\theta}{2(1-\theta)} + n_X^F \left(\frac{\tau}{F^F \alpha}\right)^{1-\varepsilon}} \quad \text{and} \tag{29}$$

$$A^F \equiv \frac{\beta E^F}{\int_{v^F \in V^F} p(v)^{1-\varepsilon}\, dv} = \frac{\beta E^F}{n_D^F \left(\frac{1}{F^F \alpha}\right)^{1-\varepsilon} + N^H \left(\frac{\tau}{F^H \alpha}\right)^{1-\varepsilon} \frac{a_X-\theta}{2(1-\theta)}}. \tag{30}$$

Using now equations (18), (19), (21), (24), (25), (26), (29), and (30), we obtain the following system of four equations and four unknowns:

$$\frac{(1-\alpha)\beta \left(L + N^H \left(f^H \frac{(a_D-\theta)^2}{4(1-\theta)} + f_X^H \frac{(a_X-\theta)^2}{4(1-\theta)}\right)\right)}{N^H \left(\frac{1}{F^H \alpha}\right)^{1-\varepsilon} \frac{a_D-\theta}{2(1-\theta)} + n_X^F \left(\frac{\tau}{F^F \alpha}\right)^{1-\varepsilon}} \left(\frac{1}{F^H \alpha}\right)^{1-\varepsilon} = a_D f^H, \tag{31}$$

$$\frac{(1-\alpha)\beta L}{n_D^F \left(\frac{1}{F^F \alpha}\right)^{1-\varepsilon} + N^H \left(\frac{\tau}{F^H \alpha}\right)^{1-\varepsilon} \frac{a_X-\theta}{2(1-\theta)}} \left(\frac{\tau}{F^H \alpha}\right)^{1-\varepsilon} = a_X f_X^H, \tag{32}$$

$$\frac{(1-\alpha)\beta L}{n_D^F \left(\frac{1}{F^F \alpha}\right)^{1-\varepsilon} + N^H \left(\frac{\tau}{F^H \alpha}\right)^{1-\varepsilon} \frac{a_X-\theta}{2(1-\theta)}} \left(\frac{1}{F^F \alpha}\right)^{1-\varepsilon} = f^F, \text{ and} \tag{33}$$

$$\frac{(1-\alpha)\beta \left(L + N^H \left(f^H \frac{(a_D-\theta)^2}{4(1-\theta)} + f_X^H \frac{(a_X-\theta)^2}{4(1-\theta)}\right)\right)}{N^H \left(\frac{1}{F^H \alpha}\right)^{1-\varepsilon} \frac{a_D-\theta}{2(1-\theta)} + n_X^F \left(\frac{\tau}{F^F \alpha}\right)^{1-\varepsilon}} \left(\frac{\tau}{F^F \alpha}\right)^{1-\varepsilon} = f_X^F. \tag{34}$$

Straightforward algebra reveals that:

$$a_D = \frac{f_X^F}{f^H}\left(\frac{F^H\tau}{F^F}\right)^{\varepsilon-1}, \tag{35}$$

$$a_X = \frac{f^F}{f_X^H}\left(\frac{F^H}{F^F\tau}\right)^{\varepsilon-1}, \tag{36}$$

$$n_D^F = (1-\alpha)\,\beta L\frac{1}{f^F} - N^H\left(\frac{F^H}{F^F\tau}\right)^{\varepsilon-1}\frac{\frac{f^F}{f_X^H}\left(\frac{F^H}{F^F\tau}\right)^{\varepsilon-1}-\theta}{2\,(1-\theta)}, \text{ and} \tag{37}$$

$$n_X^F = (1-\alpha)\,\beta\left(L + N^H\left(f^H\frac{\left(\frac{f_X^F}{f^H}\left(\frac{F^H\tau}{F^F}\right)^{\varepsilon-1}-\theta\right)^2}{4\,(1-\theta)} + f_X^H\frac{\left(\frac{f^F}{f_X^H}\left(\frac{F^H}{F^F\tau}\right)^{\varepsilon-1}-\theta\right)^2}{4\,(1-\theta)}\right)\right)\frac{1}{f_X^F}$$
$$- N^H\left(\frac{F^H\tau}{F^F}\right)^{\varepsilon-1}\frac{\frac{f_X^F}{f^H}\left(\frac{F^H\tau}{F^F}\right)^{\varepsilon-1}-\theta}{2\,(1-\theta)}. \tag{38}$$

Finally, it is interesting to note that $a_D > a_X \Leftrightarrow f_X^F f_X^H \tau^{2(\varepsilon-1)} > f^F f^H$, which is a standard assumption in the literature on trade with heterogeneous firms.[7]

5 Infrastructure investment

Investment in infrastructure reduces the distribution cost; it reduces the unit cost, which includes the cost of distributing the physical product. A government, therefore, can make firms more efficient by investing in infrastructure. On the other hand, investing in infrastructure increases the firms' fixed cost, which may reduce the number of active firms (investment in infrastructure is financed, partly, by the firms themselves). Infrastructure investment, therefore, may reduce variety, and therefore make consumers worse off. In this section we illustrate how these incentives play out when an economy moves from an autarky to a trade regime.

[7] See, for instance, Melitz (2003).

Given the market-institution coefficient, governments choose the amount invested in infrastructure (see Section 2.4). Where the amount invested does not depend only on the market-institution coefficient, but also on the chosen regime. We illustrate this by comparing investment in infrastructure under the two regimes. We resort to numerical analysis to support our arguments.

5.1 Autarky

We start with the autarky regime, and show how an increase in public spending on infrastructure F_A affects the cut-off value a_A and the ideal price index under autarky P_I^A. Although an increase in the amount invested in infrastructure reduces the unit cost, it increases the fixed costs, i.e., $\frac{df}{dF_A} > 0$. An increase in the amount invested in infrastructure under an autarky regime, therefore, decreases the number of active firms; namely, a_A decreases.

Lemma 1 *If the firms' fixed costs increase with infrastructure investment, then the cut-off value a_A decreases with F_A; namely, if $\frac{df}{dF_A} > 0$ then $\frac{\partial a_A}{\partial F_A} < 0$.*

To derive Lemma 1 we took the derivative of a_A with respect to F_A (see Appendix A). Although increasing the amount investment in infrastructure reduces the cut-off value a_A, the ideal price index P_I^A may increase or decrease as F_A increases. Recall that, on the one hand, the unit cost is lower, whereas on the other hand the fixed cost is higher and thus entry costs are higher.

Lemma 2 *The ideal price index p_I^A may decrease or increase, such that*

$$sign\left(\frac{\partial p_I^A}{\partial F_A}\right) = sign\left(\frac{(\varepsilon - 1)(\theta - a_A)}{F_A} - \frac{\partial a_A}{\partial F_A}\right).$$

Given that fixed costs increase with infrastructure investment, a necessary condition for the ideal price index p_I^A to decrease with F_A is that $a_A > \theta$; namely, that the marginal active firm is sufficiently not-connected to the government. Although increasing infrastructure increases fixed costs, it also decreases the unit cost. Then, when the benefit from lower unit costs outweighs the cost of higher fixed costs, the ideal price index decreases with infrastructure investment.

Proposition 1 *If infrastructure investment is funded, partly, by the firms themselves then increasing investment in infrastructure may cause the the ideal price index to decrease, provided the number of active firms is large enough. Moreover, if fixed costs do not increase with the level of infrastructure investment, then* $\frac{\partial p_I^A}{\partial F_A} < 0$.

The proposition follows from Lemma 1 and 2. See also the Appendix A. If fixed cost do not change with infrastructure investment, then sufficient condition for the ideal price index to decrease with F_A is

$$\theta < a_A = \frac{N_H f\theta\left[1-\beta\left(1-\alpha\right)\right] + \sqrt{N_H f\left[4L\beta\left(1-\alpha\right)\left(1-\theta\right)\left(2-\beta\left(1-\alpha\right)\right) + N_H f\theta^2\right]}}{N_H f\left[2-\beta\left(1-\alpha\right)\right]}.$$

But we showed in Appendix A that $\theta < a_A$. Therefore, if $\frac{df}{dF_A} \equiv 0$, then a_A does not depend on F_A and the ideal price index decrease with the level of investment in infrastructure.

5.2 Trade

A trade regime introduces an indirect, albeit efficient, means of producing the differentiated goods. Under a trade regime these goods can be produced abroad and imported. Investment in infrastructure, on the other hand, improves the efficiency of products produced domestically.

Let $\eta_H \equiv \frac{\partial f_H}{f^H} / \frac{\partial F_H}{F_H}$.

Lemma 3 *If the elasticity of substitution is sufficiently large then the domestic cut-off value increases with infrastructure investment; in other words, if* $(\varepsilon - 1) > \eta_H$ *then* $\frac{da_D}{dF_H} > 0$.

If the elasticity of substitution is large enough then, and different from an autarky regime, the domestic cut-off value a_D increases with the level of infrastructure investment. Increasing infrastructure investment improves firms' efficiency (it lowers the unit cost). Although increasing infrastructure investment also increases fixed costs, the reduction in the firms' unit cost more than compensate firms; the domestic cut-off value increases.

Let $\eta_{XH} \equiv \frac{\partial f_X^H}{f_X^H} / \frac{\partial F_H}{F_H}$.

Lemma 4 *If the elasticity of substitution is sufficiently large then the export cut-off value increases with infrastructure investment; in other words, if* $(\varepsilon - 1) > \eta_{XH}$ *then* $\frac{\partial a_X}{\partial F_H} > 0$.

When the elasticity of substitution is large enough, not only do the domestic cut-off value increases, but also the exporting cut-off value increases; the number of domestic firms increase, both the number of firms that sell domestically and the number of firms that export the good.

Lemma 5 *If the elasticity of substitution is sufficiently large then the number of foreign firms selling to the domestic market in country F decreases. Put differently, if* ε *is sufficiently large then* $\frac{\partial n_F^D}{\partial F_H} < 0$.

Lemma 5 follows from Lemma 4. If ε is sufficiently large then the number of firms in country F selling to consumers in country F declines. Improving infrastructure in country H reduces the unit cost of producing and distributing the final products, and makes the firms in country H more efficient. More efficient firms in country H translates to more competition in country F and therefore fewer domestic firms in country F.

Proposition 2 *A necessary condition for the ideal price index to decrease and for the market share of firms in country H to increase – both in country F and in country H – when the level of investment in infrastructure increases, is that the elasticity of substitution is large enough.*

The impact on firms in country F that export to country H is ambiguous: On the one hand, firms in country H are more efficient, whereas on the other hand, domestic firms in country H face larger fixed costs. Furthermore, and given that the elasticity of substitution ε is large enough, a necessary condition for the ideal price index to decrease is $\frac{\partial n_F^X}{\partial F^H} < 0$. Put differently, if the impact of infrastructure investment on firms' efficiency is large, then not only do the number of foreign firms exporting differentiated goods to country H decrease, but the ideal price index in country H decreases, i.e., real income in country H increases.

When the elasticity of substitution between different varieties is large, investing in infrastructure initially improves a country's ability to compete with foreign firms. However, as the amount invested in infrastructure increases further, the increment in firms' fixed costs becomes the dominant factor and governments that maximize real income will not further increase the amount invested in infrastructure.

5.3 The numerical analysis

The numerical section is used to illustrate how regime switching from autarky to trade not only increases the amount a government invests in infrastructure, but also improves the country's market institutions. The former is conclusion is derived in the current section, whereas the latter in Section 6. Trade creates incentives for a government, which maximizes domestic real income, to move closer to a market regime.

We first focus on the second stage where market institutions are already set, and assume governments maximize real income; that is

$$\underset{F}{Max} \; \{\Gamma^H = \frac{L}{p_I\left(F;\theta\right)}\}$$

We also assumed a linear relation between the current period fixed costs and the government's investment in infrastructure:

$$f = a + b \cdot F \tag{39}$$

where $a, b \geq 0$. We derive the equilibrium level of infrastructure investment under autarky and under trade regimes, while solving the equation numerically.

5.3.1 The autarky regime

In autarky, the ideal price index is

$$P_I^A = \left(N^H \cdot \left(F_A \cdot \alpha\right)^{\varepsilon-1} \frac{a_D - \theta}{2 \cdot (1-\theta)}\right)^{\frac{\beta}{1-\varepsilon}}$$

Then, because $\frac{\partial \Gamma^H}{\partial F_A} = \frac{-L^H}{P_I^{A2}} \frac{\partial P_I^A}{\partial F_A} = 0 \Rightarrow \frac{\partial P_I^A}{\partial F_A} = 0$, the first order condition can be rewritten as

$$\widetilde{F}_A\left(F_A\right) = \frac{f^H}{\partial f^H / \partial F_A} \cdot \frac{(\varepsilon-1)}{\Delta} \cdot \left(\left(2 \cdot \Delta + N^H \cdot f^H \cdot \theta^2\right) - \sqrt{N^H \cdot f^H \left(2 \cdot \Delta + N^H \cdot f^H \cdot \theta^2\right)}\right) \tag{40}$$

The autarky base mode	
N_H	5000
L	500
a	1
b	0.001

Table 1: The autarky base model

where $\Delta \equiv (2 \cdot L^{H} \cdot \beta \cdot (1-\alpha) \cdot (1-\theta) \cdot (2-\beta \cdot (1-\alpha)))$. We solve Eq. (40) numerically using the parameters depicted in Table 1, which imply that $\partial f^{H}/\partial F_A = 0.001$.

5.3.2 The trade regime

Under the trade regime, and different from the autarky regime, the ideal price index P_I^T in country H is affected by goods produced in country F.

$$P_I^T = \left(N^{H} \left(\frac{1}{F^{H}\alpha} \right)^{1-\varepsilon} \frac{a_D - \theta}{2\,(1-\theta)} + n_X^F \left(\frac{\tau}{F^F \alpha} \right)^{1-\varepsilon} \right)^{\frac{\beta}{1-\varepsilon}}$$

The ideal price index is not only a function of goods produced in country H (as is the case under the autarky regime), but also a function of goods imported from country F. Therefore, additional parameters need to be set (see Table 2). The solution to the first order condition of the government's maximization problem under the trade regime is used to derive a numerical solution for F_H; that is

$$\widetilde{F}_H\left(F^{H}\right) = \left[- \left(\frac{F^{F}}{\tau} \right)^{\varepsilon-1} \frac{2\,(1-\theta)}{N^{H}} \frac{dn_X^F}{dF^{H}} \frac{1}{(a_D-\theta)\,(\varepsilon-1)\,\frac{1}{F^H} + \frac{da_D}{dF^H}} \right]^{\frac{1}{\varepsilon-1}} \tag{41}$$

The trade base mode	
β	0.5
α	0.5
θ	0.4
τ	1.2
f_{XH}	3
f_{XF}	2.2
f_F	2
F_F	500

Table 2: The autarky base model

5.3.3 The numerical solution

The numerical exercise is depicted below, and the following conclusion is derived: Although in autarky a government does not invest in infrastructure, it will invest under trade. This outcome is robust to different scenarios. Trade "imports" infrastructure.

Equations (40) and (41) are solved using the parameters depicted in Tables 1 and 2. The outcome is no investment under the autarky regime, and an investment of 124.01 under the trade regime. Furthermore, although no firm is active under autarky, firms are active and exporting under trade. We choose to depict this extreme outcome because it highlights the benefit from opening to trade. Put differently, the base model contrasts the autarky regime, where no differentiated goods are produced because the marginal cost of production and distribution is very large, to a trade regime where the government invests in infrastructure and goods are produced both for the domestic market and for the foreign market.

The solution is also robust to different scenarios. To this end, we performed a sensitivity analysis with respect to a subset of parameters: α, θ, and a. Each time we changed one parameter, while maintaining the value of the other parameters at their base model level.

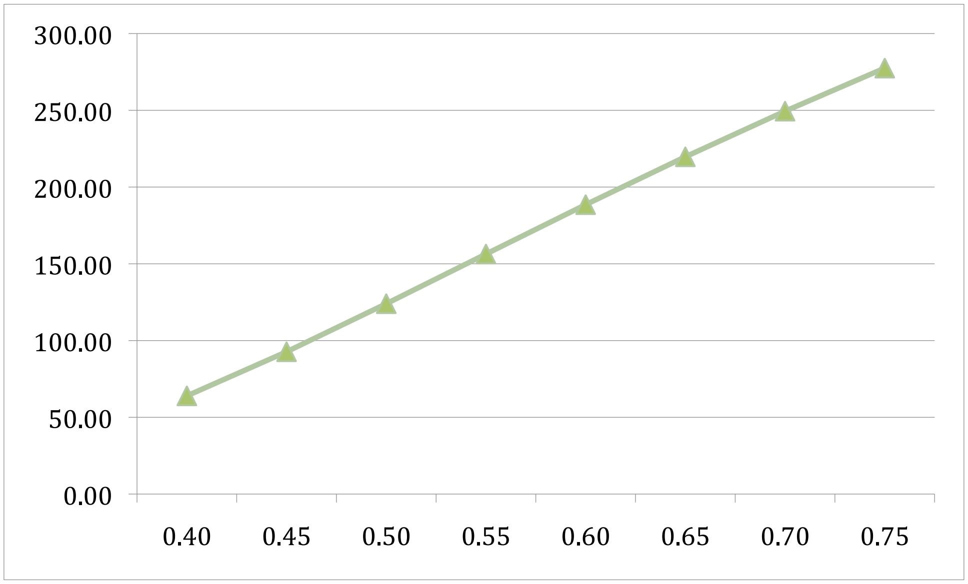
Sensitivity to changes in α. We solved Eq. (40) and (41) for $\alpha = \{0.1, 0.2, \ldots, 0.9\}$, where the larger α the larger the elasticity of substitution (recall that $\varepsilon = 1/(1-\alpha) > 1$). The higher the elasticity of substitution, the larger the amount the government invests in infrastructure. See Fig. 1(a), where the relation between α and the elasticity of substitution ϵ is depicted in Fig. 1(b).

Sensitivity to changes in θ. We solved Eq. (40) and (41) for $\theta = \{0.1, 0.2, \ldots, 0.9\}$, where θ measures the quality of the market institutions. As these institutions become stronger, the government increases the amount it invests. Put differently, when market institutions are weak connected investors free ride on the less connected investors, pay less fixed costs, get poorer infrastructure and produce with higher unit costs. Figure 2(a) depicts infrastructure investment as a function of θ; Fig. 2(b) depicts the domestic and export cut-off value as a function of θ. Not only investment in infrastructure increases with θ, but also the number of active firms.

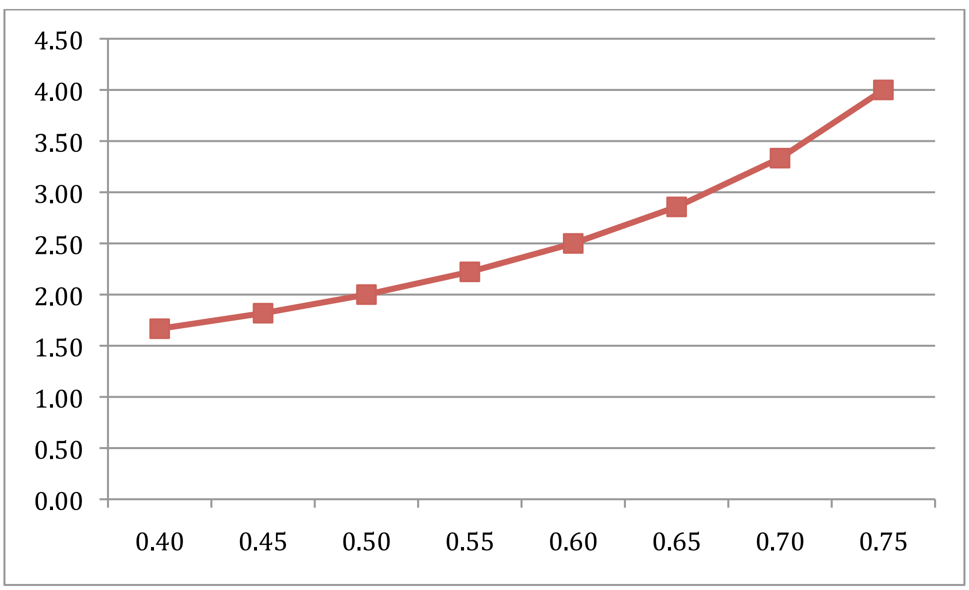
Sensitivity to changes in a. We solved Eq. (40) and (41) for $a = \{0.5, 0.6, \ldots, 1.5\}$, where a captures the part of the fixed cost not linked to the cost of infrastructure. The level of public investment in infrastructure increases, as fixed costs increase and the number of active firms decrease (see Fig. 3(a)). The number of firms exporting, however, increases. Figure 3(b) depicts the cut-off value, for firms producing for the domestic, as well as for the export market.

6 The market institution coefficient

Understanding how infrastructure investment affects real income is only part of the story. An important question, which should also be addressed, is how does a regime switch

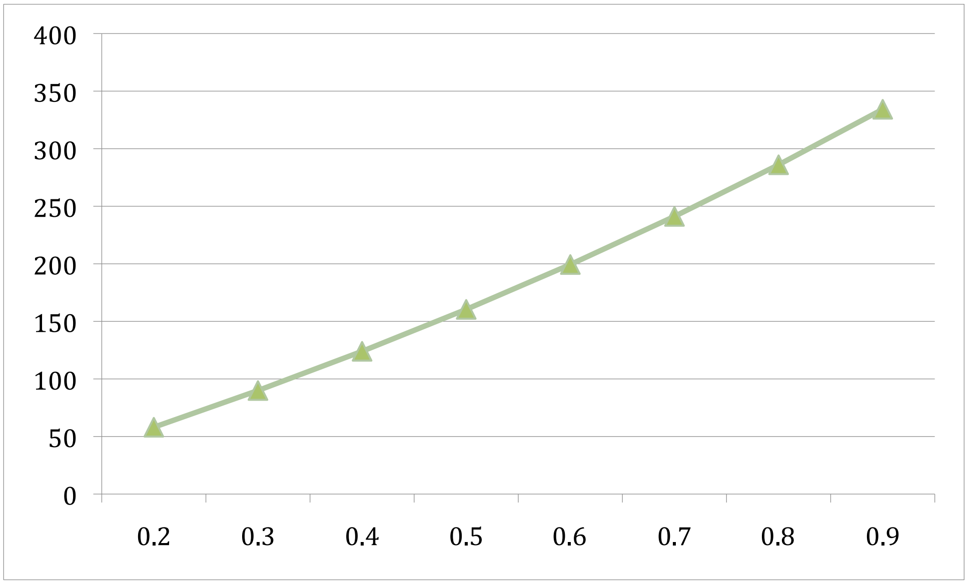


(a) Infrastructure investment and α

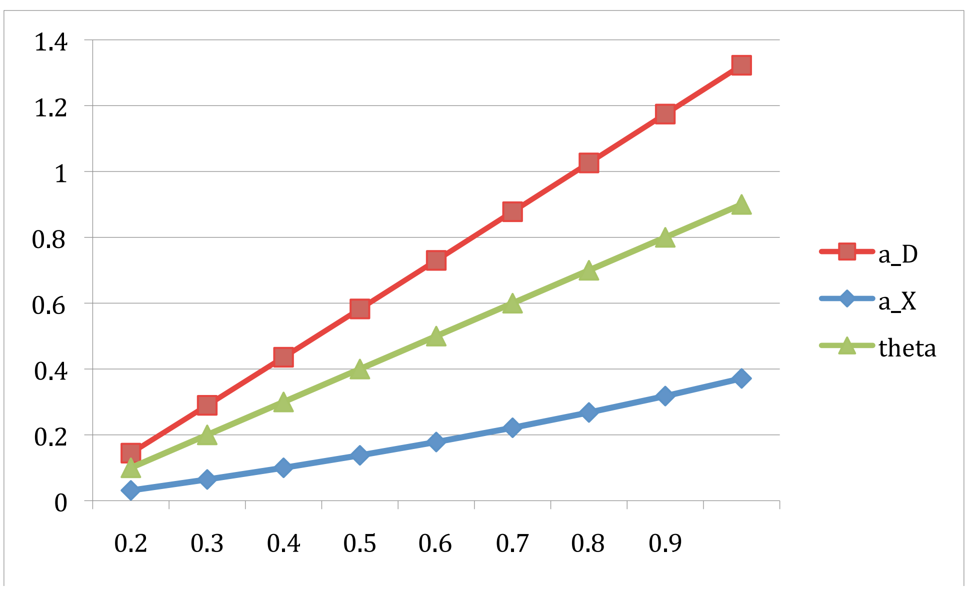


(b) α and the elasticity of substitution ϵ

Figure 1: Changing the value of α

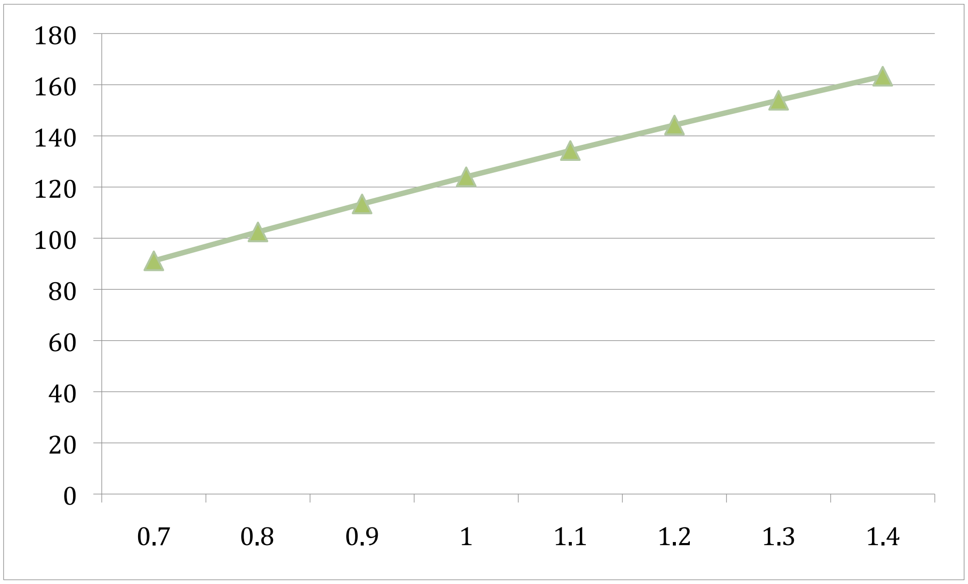


(a) Infrastructure investment and θ

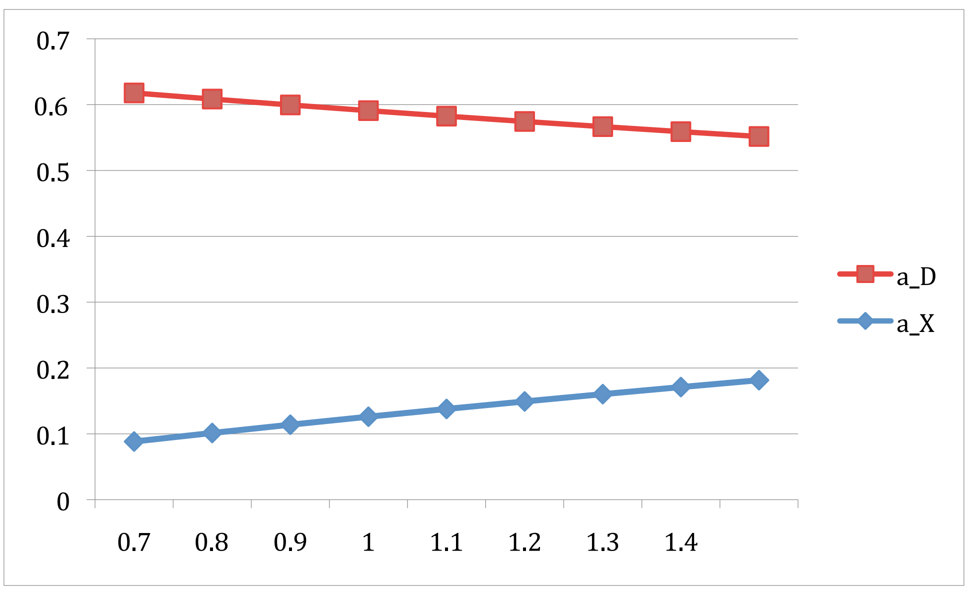


(b) The cut-off value and θ

Figure 2: Changing the value of θ



(a) Infrastructure investment and a



(b) The cut-off value and a

Figure 3: Changing the value of a

affect the market-institution coefficient. To this end, we take the derivative of the ideal price index with respect to the market-institution coefficient, and apply the envelope theorem (see Appendix B). We can then show that in autarky

$$\begin{aligned}\frac{dP_I^A}{d\theta} &= \frac{\beta}{1-\varepsilon}\left(\frac{N^H}{(F_A\alpha)^{1-\varepsilon}}\frac{(a_A-\theta)}{2(1-\theta)}\right)^{\frac{\beta}{1-\varepsilon}-1} \\ &\quad \frac{N^H}{2(1-\theta)(F_A\alpha)^{1-\varepsilon}}\left(\frac{a_A-1}{(1-\theta)}+\frac{\partial a_A}{\partial\theta}\right)\end{aligned}$$

whereas in trade

$$\begin{aligned}\frac{dP_I^T}{d\theta} &= \frac{\beta}{1-\varepsilon}\left(\left(\frac{1}{F^H\alpha}\right)^{1-\varepsilon}\left(\frac{a_D-\theta}{2(1-\theta)}\right)N^H+\left(\frac{\tau}{F^F\alpha}\right)^{1-\varepsilon}n_F^X\right)^{\frac{\beta}{1-\varepsilon}-1} \\ &\quad \left(\frac{N^H}{2(1-\theta)(F^H\alpha)^{1-\varepsilon}}\left(\frac{a_D-1}{(1-\theta)}+\frac{\partial a_A}{\partial\theta}\right)+\left(\frac{\tau}{F^F\alpha}\right)^{1-\varepsilon}\frac{\partial n_F^X}{\partial\theta}\right)\end{aligned}$$

Comparing the two expressions, we conclude that the optimal market institution coefficient under the trade regime is larger if n_F^X is sufficiently large. The reason is that $\frac{\partial a_A}{\partial\theta} > 0 > \frac{\partial n_F^X}{\partial\theta}$.

Proposition 3 *Switching from an autarky to a trade regime improves the market institutions when*

- *the number of foreign firms exporting to country H is large enough (n_X^F is sufficiently large), and/or*
- $0 < \frac{\partial n_F^X}{\partial\theta}$.

Put differently, trade not only introduces an efficient, albeit indirect method, to produce differentiated goods, but, given a sufficiently large volume of trade (i.e., number of foreign firms exporting differentiated goods to country H), it also improves market institutions.

7 Conclusion

This paper investigates the interactions between trade and corruption and shows that opening the economy to trade in goods, while assuming governments maximize real income, strengthens the market institutions and increases the amount invested in infrastructure.

We plan in the future to further understand the interactions between institutions and trade. We wish to extend the economic environment to more then two trading partners. This extension will help us investigate if a domino effect exists. In other words, when an economy improves its market institution does it lead in future periods to stronger institutions for its trading partners?

Another avenue we plan to investigate, is how the composition of the market-institution coefficient affects the equilibrium outcome. If indeed there are many combinations that yield a given market-institution coefficient, which one should be chosen? Should it be the coefficient with the least amount of political resistance? Put differently, in making recommendations to economies in transition, should the recommendation take into account political constraints.

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8 Appendices

8.1 Appendix A

8.1.1 Derivative with respect to the infrastructure investment – the autarky regime

$$\frac{\partial a_A}{\partial F_A} = \frac{1}{2\sqrt{\frac{4L\beta(1-\alpha)(1-\theta)}{Nf[2-\beta(1-\alpha)]} + \frac{\theta^2}{Nf[2-\beta(1-\alpha)]^2}}} \left(\frac{4L\beta(1-\alpha)(1-\theta)}{N[2-\beta(1-\alpha)]} + \frac{\theta^2}{N[2-\beta(1-\alpha)]^2} \right) \frac{-1}{f^2} \frac{df}{dF_A}$$

$$\frac{\partial P_I^A}{\partial F_A} = \frac{\beta}{\varepsilon-1} \left(N_H \left(F_A\alpha\right)^{\varepsilon-1} \frac{a_A-\theta}{2(1-\theta)} \right)^{\frac{\beta-(1-\varepsilon)}{1-\varepsilon}} N_H \left(F_A\alpha\right)^{\varepsilon-1} \frac{1}{2(1-\theta)} \left[(\varepsilon-1)\frac{1}{F}\left(\theta - a_A\right) - \frac{\partial a_A}{\partial F_A} \right]$$

$$\begin{aligned} a_A - \theta &= \frac{N_H f\theta[1-\beta(1-\alpha)] + \sqrt{N_H f\left[4L\beta(1-\alpha)(1-\theta)(2-\beta(1-\alpha)) + N_H f\theta^2\right]}}{N_H f[2-\beta(1-\alpha)]} - \theta \\ &= \frac{\sqrt{N_H f\left(4L\beta(1-\alpha)(1-\theta)(2-\beta(1-\alpha)) + N_H f\theta^2\right)} - f\theta N_H}{N_H f[2-\beta(1-\alpha)]} > \frac{N_H f\sqrt{\theta^2} - f\theta N_H}{N_H f[2-\beta(1-\alpha)]} = 0 \end{aligned}$$

8.1.2 Derivative with respect to the infrastructure investment – the trade regime

$$\begin{aligned} \frac{da_D}{dF^H} &= \left(\frac{\tau}{F^F}\right)^{\varepsilon-1} f_X^F \frac{d}{dF^H}\left(\left(F^H\right)^{\varepsilon-1}\frac{1}{f^H}\right) \\ &= \left(\frac{\tau}{F^F}\right)^{\varepsilon-1} f_X^F \left((\varepsilon-1)\left(F^H\right)^{\varepsilon-2}\frac{1}{f^H} - \frac{\left(F^H\right)^{\varepsilon-1}}{\left(f^H\right)^2}\frac{df^H}{dF^H}\right) \\ &= a_D\left((\varepsilon-1)\frac{1}{F^H} - \frac{1}{f^H}\frac{df^H}{dF^H}\right) \end{aligned}$$

if $(\varepsilon-1)\frac{f^H}{F^H} > \frac{df^H}{dF^H}$ $\left((\varepsilon-1) > \eta_H \text{ where } \eta_H = \frac{df^H}{f^H}/\frac{dF^H}{F^H}\right)$ then $\frac{da_D}{dF^H} > 0$.

$$\begin{aligned}
\frac{da_X}{dF^H} &= \left(\frac{1}{\tau F^F}\right)^{\varepsilon-1} f^F \frac{d}{dF^H}\left(\left(F^H\right)^{\varepsilon-1}\frac{1}{f_X^H}\right) \\
&= \left(\frac{1}{\tau F^F}\right)^{\varepsilon-1} f^F \left((\varepsilon-1)\left(F^H\right)^{\varepsilon-2}\frac{1}{f_X^H} - \frac{\left(F^H\right)^{\varepsilon-1}}{\left(f_X^H\right)^2}\frac{df_X^H}{dF^H}\right) \\
&= a_X\left((\varepsilon-1)\frac{1}{F^H} - \frac{1}{f_X^H}\frac{df_X^H}{dF^H}\right)
\end{aligned}$$

if $(\varepsilon-1)\frac{f_X^H}{F^H} > \frac{df_X^H}{dF^H}$ $\left((\varepsilon-1) > \eta_{XH} \text{ where } \eta_{XH} = \frac{df_X^H}{f_X^H}/\frac{dF^H}{F^H}\right)$ then $\frac{da_X}{dF^H} > 0$

$$\frac{dn_F^D}{dF^H} = -\left(\frac{1}{\tau F^F}\right)^{\varepsilon-1}\left(\frac{1}{2(1-\theta)}\right) N^H \left(F^H\right)^{\varepsilon-1}\left((\varepsilon-1)\frac{(a_X-\theta)}{F^H} + \frac{da_X}{dF^H}\right)$$

8.2 Appendix B: Deriving the change in the ideal price index

8.2.1 In autarky

$$\frac{dP_I^A}{d\theta} = \frac{\partial P_I^A}{\partial F_A}\frac{dF_A}{d\theta} + \frac{\partial P_I^A}{\partial\theta} \underset{\substack{\uparrow \\ \text{Using the envelope theorem}}}{=} \frac{\partial P_I^A}{\partial\theta}$$

$$= \frac{\beta}{1-\varepsilon}\left(\frac{N}{(F_A\alpha)^{1-\varepsilon}}\frac{(a_A-\theta)}{2(1-\theta)}\right)^{\frac{\beta}{1-\varepsilon}-1}\frac{N}{2(1-\theta)(F_A\alpha)^{1-\varepsilon}}\left(\frac{a_A-1}{(1-\theta)} + \frac{\partial a_A}{\partial\theta}\right)$$

8.2.2 In Trade

$$\frac{dP_I^T}{d\theta} = \frac{\partial P_I^T}{\partial\theta}$$

$$= \frac{\beta}{1-\varepsilon}\left(\left(\frac{1}{F^H\alpha}\right)^{1-\varepsilon}\left(\frac{a_D-\theta}{2(1-\theta)}\right)N^H + \left(\frac{\tau}{F^F\alpha}\right)^{1-\varepsilon} n_F^X\right)^{\frac{\beta}{1-\varepsilon}-1}\frac{N^H}{2(1-\theta)(F^H\alpha)^{1-\varepsilon}}\left(\frac{a_D-1}{(1-\theta)} + \left(\frac{\tau}{F^F\alpha}\right)^{1-\varepsilon}\frac{\partial n_F^X}{\partial\theta}\right)$$