Liquidity Risk Hedging

By Markus K. Brunnermeier and Motohiro Yogo*

Long-term bonds are exposed to higher interest-rate risk, or duration, than short-term bonds. Conventional interest-rate risk management prescribes that a firm structure the maturity of its liabilities in order to hedge the duration of its long-term assets (**?**). By doing so, the firm's assets and liabilities move in lockstep, and its net equity is shielded from (at least small) movements in interest rates. In situations in which the firm is constrained from achieving perfect interest-rate risk hedging, the conventional prescription is to match the duration of its liabilities to that of its assets as closely as possible.

In reality, a firm's assets are also exposed to sources of risk other than interest rates. These sources of risk can drive a wedge between the firm's financing cost and the riskless interest rate, and in extreme circumstances, the firm may be unable to borrow entirely. In particular, funding liquidity risk arises whenever the firm is unable to rollover its liabilities. Liquidity risk reflects the danger that a firm may be forced to liquidate its assets in a fire sale, even when it is socially optimal to continue funding the project until maturity.

In this paper, we develop a concept called liquidity risk hedging in analogy to interest-rate risk hedging. We find that the prescriptions of liquidity risk management can be substantially different from conventional interest-rate risk management. In particular, liquidity risk hedging can prescribe financing that is more "short term" than conventional interest-rate risk management. We provide first a three-period example where it is better to issue a one-

*Brunnermeier: Department of Economics, Princeton University, 26 Prospect Avenue, Princeton, NJ 08540 and NBER (e-mail: markus@princeton.edu); Yogo: Finance Department, The Wharton School, University of Pennsylvania, 3620 Locust Walk, Philadelphia, PA 19104 and NBER (e-mail: yogo@wharton.upenn.edu).

period bond first, rather than a two-period bond. If the firm is able to issue callable bonds, it is optimal to issue the longest maturity callable bond, despite the fact that callable bonds have lower duration than noncallable bonds of the same maturity. By financing the project through callable bonds, the firm retains the option of locking in long-term financing in the future should the prospects of the project become poor. By locking in long-term financing at that point, the firm minimizes rollover risk and delays inefficient liquidation for as long as possible.

Our analysis provides a possible justification for the dominance of callable bonds in the corporate bond market. As reported in **?**, the corporate bond market was almost entirely callable prior to 1985. By January 1985, only 382 of 5,755 corporate bonds with recorded prices in the Fixed Income Database were noncallable. By March 1995, 2,814 of 5,291 corporate bonds were noncallable. Although the share of noncallable bonds has steadily risen over time, callable bonds continue to be important in the corporate bond market.

1 Three-Period Example

We start with a simple three-period example, which captures the essential insights of the paper. We assume that the riskless interest rate at all maturities is constant and equal to zero in order to highlight the differences from conventional interest-rate risk management.

Consider a long-term project that costs \$100 in period 0. The project matures in period 3 and has payoffs given by the binomial model in Figure 1. Prior to maturity, information about the prospects of the project is revealed through the binomial model. The probability of an up/down movement is 0.5 at each node. The present value of the terminal payoff is \$103 in period 0, and it changes in increments of $\pm$\$2 from one period to the next.

The owner of the project, or the firm, has no initial wealth. Unspecified incentive constraints prevent the firm from issuing equity. The ability to issue three-period bonds would solve the financing problem trivially. Therefore, we assume that the firm can only issue

Period 0		Period 1		Period 2		Period 3
						(3a) 109
				(2a) 107	<	
		(1a) 105	<			(3b) 105
103	<			(2b) 103	<	
		(1b) 101	<			(3c) 101
				(2c) 99	<	
						(3d) 97

Figure 1: Present Value of the Project's Payoff in Period 3

one-period and two-period bonds. All bonds are held by a continuum of small risk-neutral debtholders of measure one, who can also invest in a riskless asset. Consequently, debtholders never accept investment opportunities with negative expected returns.

In order to succeed, the project requires continuous funding until maturity in period 3. If the firm is unable to rollover its debt prior to maturity, the project must be liquidated at a fire-sale (or market liquidity) discount of say \$0.16 per \$1 of initial investment. In the event of liquidation, a sequential servicing constraint binds so that only the first debtholders to queue receive the full face value of debt (?). This constraint can be thought of as a reduced form representation of a "rat race" to short-term financing during a credit crunch, which is analyzed in ?. The combination of the project's fire-sale discount (market illiquidity) and funding illiquidity leads to inefficient termination.

In period 0, consider two financing strategies. In a "long-short" strategy, the firm initially issues two-period bonds, then rolls them over as one-period bonds from period 2 to 3. In a "short-long" strategy, the firm initially issues one-period bonds, then rolls them over as two-period bonds from period 1 to 3. Conventional wisdom of interest-rate risk management would lead us to believe that the long-short strategy is better than the short-long strategy from the perspective of time zero. We now show that liquidity risk management prescribes otherwise.

Following the long-short strategy, the expected payoff of the project is only \$99 at node (2c). A debt-overhang problem prevents the firm from raising more than \$99. The project must therefore be liquidated for \$83 = \$99 - \$16 at the "liquidation node" (2c). In period 0, the expected payoff to the debtholder is at most $0.25 \times 107 + 0.5 \times 103 + 0.25 \times 83 = 99$. Therefore, the firm would not be able to raise the \$100 necessary to start the project. Note that for market liquidation inefficiencies smaller than \$ 16, inefficiency would also arise. In this case, however, the project will be started in period 0, it will be inefficiently liquidated at the "liquidation node" (2c).

In contrast, the short-long strategy leads to successful financing despite the severe maturity mismatch between assets and liabilities in period 0. In fact, the short-long strategy is the only way in which the firm can avoid inefficient liquidation in this example. At node (1b), the expected payoff of the project is \$101. The firm can therefore issue two-period bonds worth \$100, which ensures funding until maturity of the project. These two-period bonds sell at a discount relative to their face value of \$101, in order to compensate the debtholders for the possibility of default at node (3d). In period 0, the expected payoff to the debtholders is \$100.

To summarize, the optimal financing strategy of a long-term asset is to initially issue short-term liabilities, rather than long-term liabilities as conventional interest-rate risk management would prescribe. The firm should then hedge liquidity risk, that is lock in long-term financing, as soon as the project's net present value is sufficiently close zero. More precisely, the firm should issue the bond with maximum possible maturity whenever one reaches a node from which it is possible to reach a "liquidation node". By doing so, the firm delays inefficient liquidation for as long as possible. If the project's net present value is exactly zero, however, the firm is indifferent between continuing and liquidating. At that point, the firm effectively passes on the equity value of the firm to the debtholders. Therefore, the firm may not have sufficient incentive to hedge liquidity risk.

A simple way to implement the optimal financing strategy is through callable bonds. Suppose the firm can issue two-period callable bonds in addition to one-period and two-period noncallable bonds. Then in period 0, the firm should issue two-period callable bonds with the option to call the bond for \$100 in period 1. At node (1b), the firm should call the bond for \$100, then fund the project until maturity through two-period noncallable bonds. Alternatively, the firm may issue another callable bond at node (1b). However, the price at which the bond is callable in period 2 must be \$101, so the bond is effectively noncallable.

2 General Model

We now show that the insights of the three-period example extend to a very general discrete-time setting. The continuously compounded riskless interest rate at all maturities is constant and equal to r.

A project realizes a strictly positive random payoff of X_T at a random terminal date T. Let $X_t = \mathbf{E}_t[e^{-r(T-t)}X_T]$ be the present value of the terminal payoff discounted at the riskless interest rate, conditional on survival of the project until maturity. By the law of iterated expectations, the present value of the project is a martingale (i.e., $X_t = \mathbf{E}_t[e^{-r}X_{t+1}]$).

The project requires an initial investment of $D_0 < X_0$. A firm with no initial wealth must finance the project through debt. The firm may issue bonds of maturity up to M. All bonds are held by a continuum of small risk-neutral debtholders of measure one, whose continuously compounded expected return is always r. The project requires continuous rollover funding until maturity in order to realize its payoff. Otherwise, the project must be liquidated for $X_t - \Lambda$, where $\Lambda \gg 0$ reflects market illiquidity.

From a social welfare perspective, the project should always be financed until maturity. This is in contrast to many other corporate finance models in which early liquidation can be optimal. Therefore, the objective function of the firm coincides with that of the social planner, which is to minimize the expected cost of liquidation.

Suppose the project has not matured nor has it been liquidated through period t. Then the net present value of the project for the firm, conditional on issuing noncallable bonds of maturity m, is

$$
\begin{aligned}
V_t(m) &= \Pr(T \leq m)\mathbf{E}_t[e^{-r(T-t)}X_T | T \leq m] + \Pr(T > m)\mathbf{E}_t[e^{-rm}V_{t+m} | T > m] - D_t \\
&= \Pr(T \leq m)X_t + \Pr(T > m)\mathbf{E}_t[e^{-rm}V_{t+m}] - D_t \\
&= X_t + \Pr(T > m)\mathbf{E}_t[e^{-rm}(V_{t+m} - X_{t+m})] - D_t. \qquad (1)
\end{aligned}
$$

where D_t is the face value of debt that has to been agreed upon at time t and has be repaid at time $t+m$. (For simplicity we only consider zero-coupon bonds.)[1]

The choice of optimal maturity leads to the value function,

$$
V_t = \max_{m \in \{1,\ldots,M\}} V_t(m). \qquad (2)
$$

The form of the value function makes it clear that there are two state variables, X_t and D_t. The state variable D_t, which represents expected payoff to debt holders, is endogenous in that it depends on the history of financing policy.

2.1 Financing with Noncallable Bonds

Define $X_t^*(m)$ be the critical bound of X_t at time t such that issuance of noncallable bond of maturity m results in equity value of zero (i.e., $V_t = 0$). At this point, the face value of debt must be $D_{t+m} = \sup X_{t+m}$. The face value of debt is so high that debt is effectively equity. If the project fails to mature by period $t+m$, the firm has no incentive to keep operating. For simplicity, we assume that the project is liquidated at that point, that is $V_{t+m} = X_{t+m} - \Lambda$.

[1]The face value of the debt that expires at t, $D_{t,\text{previous}}$ determines the new face value D_t through he following equation

$$
\begin{aligned}
D_{t,\text{previous}} &= \Pr(T \leq m)\mathbf{E}_t[e^{-r(T-t)}(\min\{X_T, D_t\})] + \\
&\quad \Pr(T > m)\{\Pr(V(X_{t+m}) \geq \Lambda)D_t + \Pr(V(X_{t+m}) < \Lambda)(X_{t+m} - \Lambda)\}
\end{aligned}
$$

where we conjecture that $V_t(X_t)$ is a function of X_t.

Substituting in equation (1) and solving for the critical bound,

$$X_t^*(m) = \Pr(T > m)e^{-rm}\Lambda + D_t. \tag{3}$$

Note that this critical bound is decreasing in m. This implies that at the critical point for maturity m, the firm cannot issue a bond with shorter maturity. In other words, financing choice becomes limited as the present value of the project falls. In particular, when $X_t = X_t^*(M)$, the firm has no choice but to issue a M-period bond. Prior to that event, the funding strategy may be irrelevant, especially when the present value of the project far exceeds the required funding.

Equation (1) makes it clear that there is a potential tradeoff between issuing a bond of shorter maturity m and longer maturity $m + 1$. The bond with shorter maturity allows the firm to reoptimize financing sooner in period $t+m$, rather than period $t+m+1$. This option to refinance earlier may be valuable should the prospects of the project turn poor, and the firm would like to lock in financing earlier at $t + m$. This effect is captured through the term $\mathbf{E}_t[e^{-rm}(V_{t+m} - X_{t+m})]$ in the equation. The bond with the longer maturity locks in financing and ensures funding until period $m + 1$. This effect is captured through the term $\Pr(T > m)$ in the equation.

2.2 Financing with Callable Bonds

The form of the value function (2) makes it clear that callable bonds of maximum maturity M achieves the optimal policy implemented only through noncallable bonds. The callable bond has the best of both worlds, the ability to lock in financing until period M and the ability to reoptimize sooner depending on the prospects of the project. Through the use of callable bonds, the firm ensures the viability of the project for as long as possible. Note that when $X_t = X_t^*(M)$, the call price of the bond must be so high that the bond is effectively noncallable.

3 Discussion

3.1 Government Guarantees

In this subsection we return to our initial example, to illustrate a new form of risk-shifting through a suboptimal choice of debt maturity. At node (2c), each small debtholder tries to be among the first to obtain the full face value of debt, which causes a run on the firm that forces it to sell the project at a fire-sale price. What is striking about the event is that a shortfall in funding of a mere $1 causes a wealth destruction of $16 in present value. The government might therefore be tempted to provide a guarantee to debtholders at node (2c), thereby making the debt riskless. Such a guarantee creates a social value of $16, while costing the government $1 in present value.

However, such a guarantee distorts the ex-ante incentives of the firm. The presence of the government guarantee prevents the firm from choosing the socially efficient short-long strategy. Knowing that the government's guarantee will allow the firm to refinance at node (2c), the firm should optimally issue the one-period bond at node (1b). Alternatively, the firm should commit to the long-short strategy in period 0. Essentially, the firm triggers a wealth transfer from the public to the firm by exposing the project to liquidity risk. This is the standard risk shifting result arising from limited liability, presented here in a different context.