Monetary Policy and Inventory Investment

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Abstract

Declines in inventory investment account for a large fraction of the drop in output during a recession. But the relationship between monetary policy and inventories is unclear. Four main puzzles have been identified in the literature on monetary policy and inventory investment -- the mechanism puzzle, the sign puzzle, the timing puzzle, and the cost shock puzzle. First, the *mechanism* puzzle. Monetary policy changes the interest rate and should affect inventories, since the interest rate represents the opportunity cost of holding inventories. In fact, VAR studies find that monetary policy affects inventories. But 40 years of empirical literature on inventories has generally failed to find any significant effect of the interest rate on inventories. Second, the *sign* puzzle. Contractionary monetary policy raises the interest rate. An increase in the interest rate should **decrease** inventories through the increase in opportunity cost. VAR studies find that the short-term effect of contractionary monetary policy is to **increase** inventories. Third, the *timing* puzzle. Monetary policy induces transitory changes in the interest rate. The effect of monetary policy on the interest rate largely disappears within one year. But inventories begin to fall only after the transitory shock to the interest rate has largely dissipated. Fourth, cost shocks are a potential explanation for the stylized fact that production varies more than sales, but empirical work has found relatively little evidence that observable costs shocks affect inventories. We use simulations of a theoretical model based on learning and regime shifts in the real interest rate to address all four puzzles.

I. INTRODUCTION

Inventory investment tends to decline precipitously during recessions. Blinder and Maccini (1991) find that drops in inventory investment account for more than 80% of the fall in output during postwar recessions in the US. In their *Handbook of Macroeconomics* chapter, Ramey and West (1999) document the large declines in inventory investment during recessions across most of the G-7 countries.

This paper focuses on the role of inventories in the monetary policy transmission mechanism. Three main puzzles have been identified in the literature on monetary policy and inventory investment.

The first puzzle is the *mechanism* puzzle. Monetary policy changes the interest rate and should affect inventories, since the interest rate represents the opportunity cost of holding inventories. In fact, VAR studies find that monetary policy shocks affect inventories.[1] But 40 years of empirical literature on inventories has generally failed to find any significant effect of the interest rate on inventories. So how does monetary policy affect inventories?

In our theoretical model, the real interest rate is subject to persistent and transitory shocks. Firms don't react much to transitory shocks, but they do react to persistent shocks (regime changes). The previous 40 years of empirical inventory research primarily used econometric techniques that emphasized high-frequency variation in the data, where there is much transitory variation in the interest rate without corresponding variation in inventories – and much transitory variation in inventories (due to their role in

[1] See Bernanke and Gertler (1995), Christiano, Eichenbaum, and Evans (1996), and Jung and Yun (2005) for documentation of this fact.

buffering sales shocks) without corresponding variation in the interest rate. Empirical tests based on cointegration techniques, which emphasize low-frequency (long-run) movements in the variables, provide support for our model by showing a strong statistical relationship between the interest rate and inventories.

The second puzzle is the *sign* puzzle. Contractionary monetary policy raises the interest rate. An increase in the interest rate should **decrease** inventories through the increase in opportunity cost. VAR studies find that the short-term effect of contractionary monetary policy is to **increase** inventories.

Our solution to the puzzle is linked to the role of inventories in buffering demand (sales) shocks. Our empirical results show that demand shocks dominate the high-frequency movements in inventories. Sales drop rapidly in the first few months following a contractionary monetary policy shock. Inventories rise as they buffer negative sales shocks in the first few months following a contractionary monetary policy shock.

We test our solution to the sign puzzle by simulating the dynamic path of inventories in response to a monetary policy shock to assess whether the model produces the rise in inventories in the first few months after a contractionary shock that is observed in the actual data.

The third puzzle is the *timing* puzzle. Monetary policy induces transitory changes in the interest rate. The effect of monetary policy on the interest rate largely disappears within one year. But inventories begin to fall only after the transitory shock to the interest rate has largely dissipated.

Our solution to the timing puzzle works as follows. Because of learning, the Bayesian probabilities of being in a given interest rate régime respond slowly to a change in the interest rate (in simulations of our model). Although the effect of monetary policy on the interest rate tends to be short-lived, the effect on the probabilities is persistent. More than one third of the initial effect on the probabilities remains three years after the monetary policy shock.

The main elements of our theoretical model are learning and the behaviour of the real interest rate. The mean real interest rate tends to be highly persistent, with occasional large shifts. For example, the mean real interest rate was around 2% during the 1960s and early 1970s but negative from the mid-to late 1970s and much higher through much of the 1980s. As Garcia and Perron (1996) have shown, the behaviour of the real interest rate can be well captured by a Markov switching process with transitory fluctuations around persistent interest-rate regimes. Our model incorporates this stochastic process for the real interest rate into the optimization problem faced by the firm.

In the real world, no one posts a notice that the interest rate has shifted from a high-interest-rate regime to a low-interest-rate regime. Instead, firms must try to infer the expected path of interest rates from their best guess about the current interest rate regime. This best guess must be based on observable data, including current and past interest rates. Our theoretical model captures this by assuming that firms engage in a learning process.

The fourth puzzle is the *cost shock* puzzle. The standard inventory model with a convex production cost function predicts that firms should use inventories to smooth production relative to sales. But, many empirical studies find that production varies more than sales. Cost shocks are a leading theoretical explanation for this stylized fact. The problem is that the empirical studies tend to find little evidence that observable cost shocks affect inventories.

Our solution to the cost shock puzzle shares features of our solution to other puzzles. We argue that, because several types of shocks—sales, interest rate, cost-- are important for inventories, high-frequency techniques have difficulty detecting any particular shock, especially when there is a long lags between the realization of the shock and the impact on inventories. We deal with the problem in two ways. First, we use low-frequency techniques to uncover a significant effect of cost shocks on inventories. Second, we then use the structural parameters from the cointegrating regressions and simulations to decompose the channels through which monetary policy shocks affect inventories to isolate the effects of cost shocks alone.

The paper is organized as follows. Section II introduces the model. Section III describes how we identify monetary policy shocks and how we estimate the effect of monetary policy shocks on the Bayesian probabilities of being in a given interest rate régime. Section IV explains how we use the cointegrating regression for inventories to calibrate the model. Section V presents simulations of the effects of a monetary policy shock. Section VI illustrates the pure interest rate effect and the broad interest rate effects (through sales and through costs) during a particularly interesting episode of recent U.S. macroeconomic history, the Volcker disinflation. Section VII provides a summary and conclusion.

II. The Model

The Firm's Optimization Problem

Much of the inventory literature is based on the linear-quadratic inventory model, which emerged as a workhorse model that had enough subtlety to capture many of the complexities of the firm's inventory problem but that was tractable enough to be of genuine use in both theoretical and empirical research.[2] Unfortunately, serious problems arise with the standard linear-quadratic framework as a way of modeling the cointegrating relationships among the I(1) variables that drive inventories.[3] One important technical problem is that it is difficult to find stationary variables around which the nonlinear inventory Euler equation can be linearized. In this section, we introduce a new framework for the firm's inventory problem that is designed to smoothly incorporate I(1) variables. Our model includes production costs, adjustment costs, and inventory holding costs. A nice aspect of our model is that it carries over much of the tractability and many of the properties of the standard linear-quadratic framework. Most of the intuition that has been built up over decades of using the standard linear-quadratic model also carries over.

The representative firm is assumed to minimize the present value of its expected costs over an infinite horizon. Real costs per period consist of production costs, adjustment costs, and inventory holding costs.

Production costs, PC_t, are defined as

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[2] Some of the most brilliant minds in economics thought carefully about the complexities of the firm's inventory problem and developed the workhorse linear-quadratic inventory model. See especially Holt et al. (1960). See Blinder and Maccini (1991) and Ramey and West (1999) for surveys of the literature.
[3] Hamilton (2002) discusses some of these issues.

$$PC_t = Y_t^{\theta_1} W_t^{\theta_2} \tag{1}$$

with

$$\theta_1 > 1 \qquad \theta_2 > 0$$

where Y_t is real output, and W_t is an exogenous real cost shock, which we will associate with real input prices of variable factors of production. Observe that average production costs, J_t, are

$$J_t = \frac{PC_t}{Y_t} = Y_t^{\theta_1 - 1} W_t^{\theta_2}$$

and marginal production costs are $\theta_1 J_t$.

Adjustment costs, AC_t, are

$$AC_t = A\left(\frac{\Delta Y_t}{Y_{t-1}}\right) Y_{t-1} \tag{2}$$

with

$$A\left(\bar{y}\right) = A'\left(\bar{y}\right) = 0$$
$$A' \underset{<}{\overset{>}{-}} 0 \quad \text{as} \quad \frac{\Delta Y_t}{Y_{t-1}} \underset{<}{\overset{>}{-}} \bar{y}$$
$$A'' > 0$$

where $\bar{y}$ is then steady state growth rate of output. Adjustment costs, of course, capture the costs to the firm of changing output.

Inventory holding costs, HC_t, are

$$HC_t = \delta_1 \left(\frac{N_{t-1}}{X_t}\right)^{\delta_2} X_t + \delta_3 N_{t-1} \tag{3}$$

with

$$\delta_1 > 0 \qquad \delta_2 < 0 \qquad \delta_3 > 0$$

where N_t is the stock of finished goods inventories at the end of period t, X_t is the level of real sales, which is given exogenously. Inventory holding costs consist of two basic components. One, $\delta_1 \left(\frac{N_{t-1}}{X_t} \right)^{\delta_2} X_t$, captures the idea that, given sales, higher inventories reduce costs in the form of lost sales because they reduce stockouts. The other, $\delta_3 N_{t-1}$, captures the idea that higher inventories raise holding costs in the form of storage costs, insurance costs, etc.[4]

Let β_t be a variable real discount factor, which is given by $\beta_t = \frac{1}{1+r_t}$, where r_t denotes the real rate of interest. The firm's optimization problem is to minimize the present discounted value of expected total costs,

$$E_0 \sum_{t=0}^{\infty} \left[\prod_{j=0}^{t-1} \beta_j \right] C_t, \tag{4}$$

where

[4] These two components underlie the rationale for the quadratic inventory holding costs in the standard linear-quadratic model. The formulation above separates the components and assumes constant elasticity functional forms which facilitates log-linearization around constant steady states. See Bils and Khan (2000) for a model that deals with market structure issues and also utilizes a constant elasticity specification of the benefits of holding finished goods inventories, though the benefits are embodied on the revenue side of the firm.

Observe that (3) implies a "target stock" of finished goods inventories that minimizes finished goods holding costs. The target stock, N_t^*, is

$$N_t^* = -\left(\frac{\delta_3}{\delta_1 \delta_2} \right)^{\frac{1}{\delta_2 - 1}} X_t$$

so that the implied stock is proportional to sales. This is analogous to the target stock assumed in the standard linear-quadratic model. Note that the target stock is not the steady state stock of finished goods inventories. The steady state stock minimizes total costs in steady state whereas the target stock merely minimizes inventory holding costs.

$$C_t = PC_t + AC_t + HC_t$$
$$= Y_t^{\theta_1} W_t^{\theta_2} + A\left(\frac{\Delta Y_t}{Y_{t-1}}\right) Y_{t-1} + \delta_1 \left(\frac{N_{t-1}}{X_t}\right)^{\delta_2} X_t + \delta_3 N_{t-1} \tag{5}$$

subject to the inventory accumulation equation, which gives the change in inventories as the excess of production over sales,

$$N_t - N_{t-1} = Y_t - X_t. \tag{6}$$

The optimality conditions that result from this optimization problem are

$$E_{t-1}\left\{\beta_t \left[\beta_{t+1}\left(A\left(\frac{\Delta Y_{t+1}}{Y_t}\right) - \frac{Y_{t+1}}{Y_t} A'\left(\frac{\Delta Y_{t+1}}{Y_t}\right)\right) + \theta_1 Y_t^{\theta_1 - 1} W_t^{\theta_2} + A'\left(\frac{\Delta Y_t}{Y_{t-1}}\right) - \xi_t\right]\right\} = 0 \tag{7}$$

$$E_{t-1}\left\{\beta_t \left[\beta_{t+1}\left(\delta_2 \delta_1 \left(\frac{N_t}{X_{t+1}}\right)^{\delta_2 - 1} + \delta_3 - \xi_{t+1}\right) + \xi_t\right]\right\} = 0 \tag{8}$$

where ξ_t is the Lagrangean multiplier associated with the constraint (6) and $Y_t = N_t - N_{t-1} + X_t$.

To interpret the optimality conditions, ignore adjustment costs for simplicity, and eliminate the multiplier to reduce the optimality conditions to

$$E_{t-1}\beta_t \theta_1 Y_t^{\theta_1 - 1} W_t^{\theta_2} + E_{t-1}\left\{\beta_t \beta_{t+1}\left(\delta_2 \delta_1 \left(\frac{N_t}{X_{t+1}}\right)^{\delta_2 - 1} + \delta_3\right)\right\} = E_{t-1}\beta_t \beta_{t+1} \theta_1 Y_{t+1}^{\theta_1 - 1} W_{t+1}^{\theta_2}$$

Now, $E_{t-1}\beta_t\theta_1 Y_t^{\theta_1-1}W_t^{\theta_2}$ is the marginal cost of producing a unit of output today, $E_{t-1}\beta_t\beta_{t+1}\theta_1 Y_{t+1}^{\theta_1-1}W_{t+1}^{\theta_2}$ is the discounted marginal cost of producing a unit of output tomorrow, and $E_{t-1}\left\{\beta_t\beta_{t+1}\left(\delta_2\delta_1\left(\frac{N_t}{X_{t+1}}\right)^{\delta_2-1}+\delta_3\right)\right\}$ is the discounted marginal carrying costs of inventories, consisting of marginal holding costs plus the marginal interest charges which operate through the discounting process. To have a unit of output available for sale tomorrow, the Euler equation thus states that the firm should equate the marginal cost of producing a unit of output today and carrying it in inventories to the discounted marginal cost of producing the unit of output tomorrow.

In the Appendix, we show that linearizing the optimality conditions around steady state values yields a linearized Euler equation:

$$\begin{aligned}
&E_{t-1}\Big\{(\theta_1-1)\theta_1\bar{J}\left[\ln Y_t-\bar{\beta}\ln Y_{t+1}\right]+\theta_2\theta_1\bar{J}\left[\ln W_t-\bar{\beta}\ln W_{t+1}\right]+\bar{\beta}\psi\left[\ln N_t-\ln X_{t+1}\right]\\
&\qquad+\gamma\left[\bar{\beta}^2\Delta\ln Y_{t+2}-2\bar{\beta}\Delta\ln Y_{t+1}+\Delta\ln Y_t\right]+\theta_1\bar{J}r_{t+1}+c\Big\}=0
\end{aligned}\tag{9}$$

where $\bar{J}$ is steady state average production cost, $\theta_1\bar{J}$ is steady state marginal production cost, $\psi=(\delta_2-1)\delta_2\delta_1\left[\bar{R}_N\left(1-\bar{x}\right)\right]^{\delta_2-1}$ is steady state marginal inventory holding costs, $\bar{R}_N=\frac{\bar{N}}{\bar{X}}$ is the steady state inventory/sales ratio, $\gamma=A''\left(\bar{y}\right)$, $\bar{\beta}=\frac{1}{1+\bar{r}}$, $\bar{r}$ is the unconditional mean real interest rate, $\bar{x}$ is the steady state growth rate of sales, $\bar{y}$ is the steady state growth rate of output, and a bar above a variable denotes a steady state value.

A key innovation in Maccini, Moore and Schaller (2004) is to assume that the real interest rate follows a three-state Markov switching process.[5] This is consistent with empirical patterns in real interest rates—See Garcia and Perron (1996) and the empirical work in Maccini, Moore and Schaller (2004). Specifically, we assume that the real interest rate follows

$$r_t = r_{S_t} + \sigma_{S_t} \cdot \varepsilon_t \tag{10}$$

where $\varepsilon_t \sim$ i.i.d. N(0,1) and where $S_t \in \{1, 2, 3\}$ follows a Markov switching process. Let $r_1 < r_2 < r_3$, so that when $S_t = 1$ the real interest rate is in the low-interest-rate regime, when $S_t = 2$ the real interest rate is in a moderate interest rate regime, and when $S_t = 3$ the real interest rate is in a high-interest-rate regime. S_t and ε_t are assumed to be independent. Denote the transition probabilities governing the evolution of S_t by $p_{ij} = \text{Prob}(S_t = j \mid S_{t-1} = i)$. Collecting these probabilities into a matrix we have

$$P = \begin{bmatrix} p_{11} & p_{21} & p_{31} \\ p_{12} & p_{22} & p_{32} \\ p_{13} & p_{23} & p_{33} \end{bmatrix}.$$

We assume that the firm knows the structure and parameters of the Markov switching process but does not know the true real interest rate regime. The firm must therefore infer S_t from observed interest rates. We denote the firm's current probability assessment of the true state by π_t. That is,

$$\pi_t = \begin{bmatrix} \pi_{1t} \\ \pi_{2t} \\ \pi_{3t} \end{bmatrix} = \begin{bmatrix} \text{Prob}(S_t = 1 \mid \Omega_t) \\ \text{Prob}(S_t = 2 \mid \Omega_t) \\ \text{Prob}(S_t = 3 \mid \Omega_t) \end{bmatrix},$$

[5] For a comprehensive discussion of Markov switching processes, see Hamilton (1994, Chapter 22).

where the firm's information set, Ω_t, includes the current and past values of r_t. Here, π_{it} is the firm's estimate at date t of the probability that the real interest rate is in regime i.

To understand the learning process, consider how the firm uses its observation of the current real interest rate to develop its probability assessment, π_t. Beginning at the end of period t-1 the firm uses π_{t-1} together with the transition probabilities in P to form beliefs about the period t interest rate state *prior to observing* $\mathrm{r_t}$. That is the firm evaluates $\pi_{it|t-1} \equiv \mathrm{Prob}\left(\mathrm{S_t} = i \,|\, \Omega_{t-1}\right)$ for i = 1, 2, 3 using

$$\begin{bmatrix} \pi_{1t|t-1} \\ \pi_{2t|t-1} \\ \pi_{3t|t-1} \end{bmatrix} = P\pi_{t-1} \tag{11}$$

Once the firm enters period t and observes $\mathrm{r_t}$, it uses the prior probabilities from (11) together with the relevant conditional probability densities to update π_t according to Bayes' rule. Specifically,

$$\pi_{it} = \frac{\pi_{it|t-1} \cdot f(\mathrm{r}_t \,|\, S_t = i)}{\sum_{j=1}^{3} \pi_{jt|t-1} \cdot f(\mathrm{r}_t \,|\, S_t = j)} \quad \text{for } i = 1,2,3. \tag{12}$$

Thus, the firm uses Bayes' rule and its observations of the real interest rate to learn about the underlying interest rate regime.

Given π_{t-1}, the term $E_{t-1}\mathrm{r}_{t+1}$ in equation (9) can be computed as

$$E_{t-1}\mathrm{r}_{t+1} = \mathrm{r_v}' P^2 \pi_{t-1} = \gamma_1\pi_{1t-1} + \gamma_2\pi_{2t-1} + \gamma_3\pi_{3t-1} \tag{13}$$

where $\mathrm{r_v}' = [\mathrm{r_1}, \mathrm{r_2}, \mathrm{r_3}]$ and $\begin{bmatrix}\gamma_1 & \gamma_2 & \gamma_3\end{bmatrix} \equiv \mathrm{r_v}' P^2$. Since $\pi_{1t-1} + \pi_{2t-1} + \pi_{3t-1} = 1$ by definition, we can eliminate π_{2t-1} from the right hand side of (13) to obtain

$$E_{t-1}\mathrm{r}_{t+1} = (\gamma_1 - \gamma_2)\pi_{1t-1} + (\gamma_3 - \gamma_2)\pi_{3t-1} + \gamma_2 . \tag{14}$$

Now, to isolate the expected real interest rate in the linearized Euler equation, partition (9) so that

$$\begin{aligned} &E_{t-1}\Big\{(\theta_1 - 1)\theta_1 \overline{J}\left[\ln Y_t - \overline{\beta}\ln Y_{t+1}\right] + \theta_2\theta_1\overline{J}\left[\ln W_t - \overline{\beta}\ln W_{t+1}\right] + \overline{\beta}\psi\left[\ln N_t - \ln X_{t+1}\right] \\ &\quad + \gamma\left[\overline{\beta}^2\Delta\ln Y_{t+2} - 2\overline{\beta}\Delta\ln Y_{t+1} + \Delta\ln Y_t\right]\Big\} + \theta_1\overline{J}E_{t-1}r_{t+1} + c = 0 \end{aligned} \tag{15}$$

Then, substitute (14) into (15) to get

$$\begin{aligned} &E_{t-1}\Big\{(\theta_1 - 1)\theta_1 \overline{J}\left[\ln Y_t - \overline{\beta}\ln Y_{t+1}\right] + \theta_2\theta_1\overline{J}\left[\ln W_t - \overline{\beta}\ln W_{t+1}\right] + \overline{\beta}\psi\left[\ln N_t - \ln X_{t+1}\right] \\ &\quad + \gamma\left[\overline{\beta}^2\Delta\ln Y_{t+2} - 2\overline{\beta}\Delta\ln Y_{t+1} + \Delta\ln Y_t\right]\Big\} \\ &\quad + \theta_1\overline{J}\left[(\gamma_1 - \gamma_2)\pi_{1t-1} + (\gamma_3 - \gamma_2)\pi_{3t-1} + \gamma_2\right] + c = 0 \end{aligned} \tag{16}$$

We now derive the decision rule for optimal inventories that is implied by the firm's optimization problem. Assume now that sales and real input prices follow independent AR(1) processes and that the current information set of the firm includes lagged values of sales, input prices and interest rates. In the Appendix, we show that the linearized Euler equation, (16), may be written as a fourth-order expectational difference equation. Denote λ_1 and λ_2 as the stable roots of the relevant characteristic equation. We then show in the Appendix that the firm's actual inventory position is

$$\ln N_t = \Gamma_0 + (\lambda_1 + \lambda_2)\ln N_{t-1} - \lambda_1\lambda_2 \ln N_{t-2} + \Gamma_X \ln X_{t-1} + \Gamma_W \ln W_{t-1}$$

$$+ \Gamma_{\pi 1}\pi_{1t-1} + \Gamma_{\pi 3}\pi_{3t-1} \quad + \quad u_t \tag{17}$$

where

$$\Gamma_X \gtrless 0, \qquad \Gamma_W < 0, \qquad \Gamma_{\pi 1} > 0, \qquad \Gamma_{\pi 3} < 0$$

$u_t = \left(\frac{\bar{R}_Y}{\bar{R}_N}\right)\left(u_t^Y - u_t^X\right)$ is an inventory shock that consist of a production shock,

$u_t^Y = \ln Y_t - E_{t-1} \ln Y_t$, and a sales forecast error, $u_t^X = \ln X_t - E_{t-1} \ln X_t$.

III. MONETARY POLICY

First-Stage VAR

We identify monetary policy shocks by estimating a six-variable semi-structural VAR using the method of Bernanke and Mihov (1998). The variables in the VAR are divided into two blocks of three variables each. The non-policy or "macroeconomic" block, which is unrestricted, consists of the natural logarithms of real sales ($\ln X_t$), the GDP deflator, and real input prices ($\ln W_t$).[6] The policy block, which consists of total reserves, non-borrowed reserves, and the federal funds rate, is restricted using plausible assumptions about the market for bank reserves. These restrictions, together with the assumption that policy shocks only affect the macroeconomic variables after a one-month lag, are sufficient to identify the unobserved structural monetary policy shocks.

The VAR model is

[6] We found that the inclusion of input prices was sufficient to address the price puzzle and so do not add a commodity price index.

$$Z_t = \sum_{i=0}^{n} B_i Z_{t-i} + \sum_{i=0}^{n} C_i P_{t-i} + A^Z \mathrm{v}_t^Z \tag{14}$$

$$P_t = \sum_{i=0}^{n} D_i Z_{t-i} + \sum_{i=0}^{n} G_i P_{t-i} + A^P \mathrm{v}_t \tag{15}$$

where Z denotes the vector of macroeconomic variables and P denotes the vector of policy variables. $B_i, C_i, A^Z, D_i, G_i,$ and A^P are matrices, v_t^Z and v_t are vectors of mutually uncorrelated structural shocks. The assumption that policy variables have no contemporaneous affect on macroeconomic variables[7] requires that $C_0 = 0$.

Re-write equation (15) so that only lagged values of the policy variables appear on the right-hand-side. The result is

$$P_t = (I - G_0)^{-1} \sum_{i=0}^{n} D_i Z_{t-i} + (I - G_0)^{-1} \sum_{i=1}^{n} G_i P_{t-i} + \mathrm{u}_t \tag{16}$$

where,

$$\mathrm{u}_t = (I - G_0)^{-1} A^P \mathrm{v}_t . \tag{17}$$

Note that u_t, the vector of residuals from the policy-block VAR, is orthogonal to the residuals from the non-policy block. If the elements of $(I - G_0)^{-1} A^P$ are known we can use (17) to recover the unobservable structural shocks, v_t, from the observable policy-block residuals.

To obtain the restrictions necessary to identify the elements of $(I - G_0)^{-1} A^P$ Bernanke and Mihov (1998) consider the market for federal funds. Omitting time

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[7] This assumption is plausible for the monthly data used in this paper, and for the monthly and bi-weekly data in Bernanke and Mihov (1998) but would be less plausible for lower frequency data.

subscripts let u_{FFR} denote innovations in the federal funds rate, and let v^d denote exogenous shocks to the demand for total reserves. Innovations in total reserve demand, u_{TR}, are then given by

$$u_{TR} = -\alpha u_{FFR} + v^d \tag{18}$$

where $\alpha \geq 0$. Also, if u_{DISC} denotes innovations in the discount rate, then u_{BR}, which denotes innovations in the demand for borrowed reserves, is given by

$$u_{BR} = -\beta(u_{FFR} - u_{DISC}) + v^b \tag{19}$$

where v^b denotes exogenous shocks to the demand for borrowed reserves and where $\beta \geq 0$. Innovations in the demand for non-borrowed reserves, u^D_{NBR}, are by definition

$$u^D_{NBR} = u_{TR} - u_{BR}. \tag{20}$$

Use (18) and (19) to substitute for the terms on the right-hand side of (20), and assume that changes in the Fed's discount rate are infrequent and largely anticipated, so that $u_{DISC} = 0$. The result is

$$u_{FFR} = \left(\frac{-1}{\alpha+\beta}\right)u^D_{NBR} + \left(\frac{v^d}{\alpha+\beta}\right) - \left(\frac{v^b}{\alpha+\beta}\right). \tag{21}$$

Innovations in the supply of non-borrowed reserves, u^S_{NBR}, are governed by Federal Reserve policy. Let

$$u^S_{NBR} = \phi^d v^d + \phi^b v^b + v^s. \tag{22}$$

Here v^s is an exogenous shock to the supply of non-borrowed reserves. The policy parameters, ϕ^d and ϕ^b, describe how the Fed will react to shocks to the demand for total reserves and borrowed reserves, respectively. Consider two examples. If the Fed is targeting non-borrowed reserves, it will set $\phi^d = \phi^b = 0$ and, in so far as it is possible, hold the supply of non-borrowed reserves constant. If instead the Fed is targeting the federal funds rate, it will set $\phi^d = 1$ and $\phi^b = -1$. With $\phi^d = 1$, a positive shock to the demand for total reserves will be accommodated by an equal increase in the supply of non-borrowed reserves. A positive shock to the demand for borrowed reserves, holding total reserves constant, will cause a decline in the demand for non-borrowed reserves. With $\phi^b = -1$ this will be offset by an equal decline in the supply of non-borrowed reserves. Since $\mathrm{u}_{\mathrm{NBR}}^{\mathrm{D}} = \mathrm{u}_{\mathrm{NBR}}^{\mathrm{S}}$ in equilibrium use (22) to substitute for $\mathrm{u}_{\mathrm{NBR}}^{\mathrm{D}}$ in (21) to obtain

$$\mathrm{u}_{\mathrm{FFR}} = \left(\frac{1-\phi^d}{\alpha+\beta}\right)v^d + \left(\frac{-1}{\alpha+\beta}\right)v^s - \left(\frac{1+\phi^b}{\alpha+\beta}\right)v^b . \tag{23}$$

Combine equations (18), (22), and (23) to give equation (17) with $\mathrm{u}_{\mathrm{t}}' \equiv \begin{bmatrix} \mathrm{u}_{\mathrm{TR}} & \mathrm{u}_{\mathrm{NBR}} & \mathrm{u}_{\mathrm{FFR}} \end{bmatrix}$, $\mathrm{v}_{\mathrm{t}}' \equiv \begin{bmatrix} v^d & v^s & v^b \end{bmatrix}$, and

$$\left(I - G_0\right)^{-1} A^Z = \begin{bmatrix} \left(\frac{-\alpha}{\alpha+\beta}\right)\left(1-\phi^d\right)+1 & \left(\frac{\alpha}{\alpha+\beta}\right) & \left(\frac{\alpha}{\alpha+\beta}\right)\left(1+\phi^b\right) \\ \phi^d & 1 & \phi^b \\ \left(\frac{1-\phi^d}{\alpha+\beta}\right) & \left(\frac{-1}{\alpha+\beta}\right) & -\left(\frac{1+\phi^b}{\alpha+\beta}\right) \end{bmatrix}$$

Let $\hat{\Omega}_T$ denote the estimated variance-covariance matrix of the policy-block residuals. That is,

$$\hat{\Omega}_T = \left(\frac{1}{T-k}\right)\sum_{t=1}^{T}\hat{\mathrm{u}}_t\hat{\mathrm{u}}_t'$$

where $\hat{u}_t$ is the vector of policy-block residuals obtained by estimating the VAR in equations (14) and (15).[8] Since $\hat{\Omega}_T$ is a (3x3) symmetric matrix it has six unique elements. Next, note from (17) that

$$E(\mathrm{u_t u'_t}) = \left[\left(I - G_0 \right)^{-1} A^Z \right] E(\mathrm{v_t v'_t}) \left[\left(I - G_0 \right)^{-1} A^Z \right]'.$$

Since the elements of $\mathrm{v_t}$ are i.i.d. and mutually orthogonal by assumption, we can write

$$E(\mathrm{v_t v'_t}) = \begin{bmatrix} \sigma_d^2 & 0 & 0 \\ 0 & \sigma_s^2 & 0 \\ 0 & 0 & \sigma_b^2 \end{bmatrix}.$$

The matrix $E(\mathrm{u_t u'_t})$ is, of course, also (3x3) and symmetric. Equating $E(\mathrm{u_t u'_t})$ to $\hat{\Omega}_T$ therefore places six restrictions on the seven unknown structural parameters: $\alpha, \beta, \phi^d, \phi^b, \sigma_s^2, \sigma_d^2,$ and σ_b^2. At least one more restriction is needed to identify these parameters and, hence, the elements of $\left(I - G_0 \right)^{-1} A^P$.

Beranke and Mihov (1998) examine five alternative sets of identifying restrictions. Four of these sets impose two additional restrictions so that the model is over identified. For example, as explained above, the set of restrictions consistent with targeting the federal funds rate is $\phi^d = 1$ and $\phi^b = -1$. Bernanke and Mihov call their fifth set the "just identified" model as it imposes the single additional restriction that $\alpha = 0$. This restriction is motivated by Strongin's (1995) argument that the demand for total reserves is inelastic in the short run. Impulse-response functions show that a monetary policy shock has qualitatively similar effects under all five sets of restrictions. We

[8] We use a constant term plus 6 lags of the six variables, therefore k = 37.

therefore take the simplest approach to identification, the just identified model. We set $\alpha = 0$ and solve $E(\mathrm{u_t u_t'}) = \hat{\Omega}_T$ for the remaining six structural parameters.

To estimate the VAR we use monthly data from December 1961 through August 2004. In the macroeconomic block we obtain monthly observations of the GDP deflator using the state-space procedure of Bernanke, Gertler, and Watson (1997). [9] In the policy block, following Bernanke and Mihov (1998) we render total reserves and non-borrowed reserves stationary by measuring each as a ratio to a 36-month moving average of total reserves. [10]

We report the results of our estimation in Table 1. For the most part, our estimates are similar to Bernanke and Mihov's. The estimate of β is positive and indicting that an increase in the federal funds rate relative to the discount rate leads to an increase in the demand for borrowed reserves. However, our estimate of ϕ^b is not significantly different from zero, perhaps reflecting the declining role that borrowed reserves have played since the mid 1980s. The estimate of ϕ^d is positive and close to one. This suggests that the Fed dampens fluctuations in the federal funds rate by accommodating shocks to the demand for reserves.

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[9] This procedure uses several monthly series on prices to infer the unobserved monthly value of the GDP Deflator.

[10] There is a dramatic spike in the reserves data in the months of September and October 2001, following the September 11th attacks. We eliminate this spike by interpolating the series form August 2001 to November 2001.

Table 1

Parameter Estimates for the Just Identified Model

(assumes $\alpha = 0$).

β	ϕ^d	ϕ^b	σ_d	σ_b	σ_s
0.049 (0.026)	0.831 (0.047)	0.010 (0.141)	0.011	0.021	0.011

Estimates for the sample 1961:12 through 2004:8. Standard errors in parentheses.

Having identified the parameters that characterize the money market it is then possible to identify the monetary policy shocks by inverting equation (17) to obtain

$$\begin{bmatrix} v^d \\ v^s \\ v^b \end{bmatrix} = \left[\left(I - G_0\right)^{-1} A^z \right]^{-1} \begin{bmatrix} \mathrm{u}_{\mathrm{TR}} \\ \mathrm{u}_{\mathrm{NBR}} \\ \mathrm{u}_{\mathrm{FFR}} \end{bmatrix} \tag{24}$$

The middle row of this equation is

$$v^s = -\left(\phi^d + \phi^b\right)\mathrm{u}^{\mathrm{TR}} + \left(1 + \phi^b\right)\mathrm{u}^{\mathrm{NBR}} - \left(\alpha\phi^d - \beta\phi^b\right)\mathrm{u}^{\mathrm{FFR}} \tag{25}$$

Inserting the policy-block residuals for u^{TR}, $\mathrm{u}^{\mathrm{NBR}}$, and $\mathrm{u}^{\mathrm{FFR}}$ on the right-hand side of (25) yields the time series of monetary policy shocks, $\left\{v_t^s\right\}_{t=1}^T$.

The Link between Monetary Policy and the Probabilities

We use a straightforward procedure to estimate the effect of monetary policy shocks on the probabilities: we estimate a three-variable vector autoregression, with monetary policy shocks, π_1, and π_3 (using six lags of each variable).[11] Figure 1 shows the impulse response function of π_1 to a one-standard-deviation easing of monetary

[11] We do not include the probabilities in the Bernanke-Mihov VAR because there is too much collinearity between the probabilities and the interest rate.

policy. As the impulse response function shows, easing monetary policy increases the probability of the low interest rate state. The effect of monetary policy on π_1 is hump-shaped and peaks about six months after the shock. At the peak, a one-standard-deviation easing of monetary policy increases the probability of being in the low interest rate régime by about 0.044. The effect of monetary policy on π_1 is quite persistent, with more than half the peak effect still present two years after the shock.

Results are similar when we look at the effect of monetary policy on π_3. Loosening monetary policy reduces the probability of the high interest rate régime. The impulse response function is again hump-shaped, with an inflection point about six months after the shock. The ergodic probability of the high interest state is about 0.19. The 0.033 decrease in π_3 therefore represents a decrease of about 17% in the likelihood of the high real interest rate regime (relative to the ergotic probability). As in the case of the probability of the low interest rate regime, the effect of a monetary policy shock is quite persistent.

IV. CALIBRATION

The key to our calibration is the cointegrating regression for inventories. Using our model, it is possible to derive the mathematical relationships between the structural parameters and the coefficients of the cointegrating regression. This is of tremendous value for two reasons. First, decades of research based on stationary econometric techniques has produced little consensus on the structural parameters.[12] Second, recent

[12] See Ramey and West (1999) for a good survey of the empirical estimates based on the traditional, stationary econometrics.

work based on the cointegrating regression for inventories has produced evidence that the real interest rate and observable cost shocks have a statistically significant effect on inventories.[13]

Derivation of Cointegrating Vector

We show in the appendix that the log-linearized Euler equation can be written as

$$E_t\left\{\chi_{1t} + \ln N_t - b_X \ln X_t - b_W \ln W_t - b_1\pi_{1t-1} - b_3\pi_{3t-1}\right\} = 0 \tag{26}$$

where χ_{1t} is stationary, and where

$$b_X = \frac{-\left(1-\bar{\beta}\right)\left(\theta_1 - 1\right)\theta_1\bar{J}}{\bar{\beta}\psi} + 1 ,$$

$$b_W = -(1-\bar{\beta})\frac{\theta_2\theta_1\bar{J}}{\bar{\beta}\psi},$$

$$b_1 = -(\gamma_1 - \gamma_2)\frac{\theta_1\bar{J}}{\bar{\beta}\psi},$$

and

$$b_3 = -(\gamma_3 - \gamma_2)\frac{\theta_1\bar{J}}{\bar{\beta}\psi}.$$

Assume that $\ln X_t$, $\ln W_t$, π_{1t-1} and π_{3t-1} are I(1).[14] Equation (26) then implies that $\ln N_t$ is I(1) and that log inventories, log sales, the log of the cost shock, and the probabilities will be cointegrated with cointegrating vector

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[13] See Maccini, Moore, and Schaller (2004). Using traditional, stationary econometrics techniques, it has been difficult to find statistically significant evidence of a role for either the real interest rate or observable cost shocks.

$$\left[1, -b_X, -b_W \;, -b_1 \;, -b_3\right] \tag{27}$$

The Structural Parameters and the Cointegrating Regression

When the cointegrating vector is expressed in the form of a regression, $\ln X_t$, $\ln W_t$, π_{1t-1}, and π_{3t-1} will be on the right hand side of the equation, so their coefficients will have signs opposite to those shown in the cointegrating vector above. We estimate the cointegrating regression using Stock and Watson's (1993) DOLS procedure and report our estimates in Table 5 and Section V below.

Note from the definitions of b_W and b_3 that

$$\frac{b_W}{b_3} = \frac{1-\bar{\beta}}{\gamma_3 - \gamma_2}\theta_2 . \tag{28}$$

Since, $\bar{\beta}, \gamma_2$, and γ_3 are given from our estimates of the Markov switching model we invert (28) and use our estimates of b_W and b_3 to obtain a baseline value for θ_2 from

$$\theta_2 = \frac{\gamma_3 - \gamma_2}{1-\bar{\beta}} \frac{\hat{b}_W}{\hat{b}_3} . \tag{29}$$

Similarly, note from the definitions of b_X and b_3 that

$$\frac{1-b_X}{b_3} = \frac{1-\bar{\beta}}{\gamma_2 - \gamma_3}\left(\theta_1 - 1\right). \tag{30}$$

We invert (30) and use our estimates of b_X and b_3 to obtain a baseline value for θ_1 from

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[14] This is confirmed by unit root tests. Since π_{1t-1} and π_{3t-1} have a restricted range, one might wonder whether it is better to model them as I(0) or I(1). We note two points. First, in careful applied econometric research, variables with restricted ranges, such as the nominal interest rate, are modeled as I(1) variables when they are highly persistent. (See, e.g., Stock and Watson (1993) and Caballero (1994).) Second, unit root tests indicate that π_{1t} and π_{3t} are I(1).

$$\theta_1 = 1 + \left(\frac{1-\hat{b}_X}{\hat{b}_3}\right)\left(\frac{\gamma_2-\gamma_3}{1-\overline{\beta}}\right). \tag{31}$$

Finally, note from the definitions of b_3 and ψ that

$$b_3 = \frac{(\gamma_2-\gamma_3)\theta_1\overline{J}}{\overline{\beta}(\delta_2-1)\delta_2\delta_1\left[\overline{R}_N(1-\overline{x})\right]^{(\delta_2-1)}} \tag{32}$$

Rearranging (32), using the estimate of b_3, and imposing the normalization that $\delta_1 = 1$, we obtain

$$\frac{\hat{b}_3}{(\gamma_2-\gamma_3)}\overline{\beta}(\delta_2-1)\delta_2\left[\overline{R}_N(1-\overline{x})\right]^{(\delta_2-1)} = \theta_1\overline{J}\ . \tag{33}$$

With θ_1 determined from equation (31), Equation (33) gives a single restriction on the value of δ_2. We have assumed that $\delta_2 < 0$. We therefore search numerically over $\delta_2 \in (-\infty, 0]$ to find the value of δ_2 that satisfies (33).[15]

Using Estimates of the Markov Switching Model for the Real Interest Rate

$\overline{\beta}, \gamma_2$, and γ_3 are obtained from our estimates of the Markov switching model. We obtain estimates of the parameters of the stochastic process for the real interest rate, the elements of P and r_v, from our estimation of the three-state Markov-switching model. Those estimates are

$$P = \begin{bmatrix} p_{11} = 0.98 & p_{21} = 0.02 & p_{31} = 0.00 \\ p_{12} = 0.02 & p_{22} = 0.96 & p_{32} = 0.05 \\ p_{13} = 0.00 & p_{23} = 0.02 & p_{33} = 0.95 \end{bmatrix}$$

and

[15] Our numerical search shows that only one value of δ_2 satisfies (29).

$$r_v' \equiv \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} = \begin{bmatrix} -1.37 \\ 1.77 \\ 5.04 \end{bmatrix}.$$

Together these estimates imply that the unconditional mean of the monthly real interest rate is $\overline{r}=0.001$, which gives $\overline{\beta}=0.999$.

Stationary Ratios

$\overline{R}_N, \overline{R}_Y, \overline{x}$ and $\overline{J}$ are steady-state ratios. For $\overline{R}_N, \overline{R}_Y$, and $\overline{x}$ we use the sample average values of N_t/X_t, Y_t/X_t, and $\Delta X_{t+1}/X_t$, respectively, as reported in Table 2.

Table 2

Steady State Ratios

$\overline{R}_N$	$\overline{R}_Y$	$\overline{J}$	$\overline{x}$
0.468	1.001	0.734	0.00108

Note from $J_t = Y_t^{\theta_1} W_t^{\theta_2} / Y_t$ that $\overline{J}$ denotes the steady state value of average production costs. Based on data from the 1992 Census of Manufacturing, we estimate production costs to be 73.4% of total output and set $\overline{J}=0.734$.

Completing the Calibration

The only cost function parameter left to be determined is the parameter that governs adjustment costs, γ. Since the adjustment costs cannot be identified from the long-run equilibrium cointegrating relationship, for our baseline setting we assume that there are no adjustment costs and set $\gamma = 0$.

We have, therefore, used our estimates of the cointegrating regression (combined with estimates of the parameters of the Markov-switching model, and the empirical data on the steady- state ratios) to calibrate the structural parameters $\theta_1, \theta_2, \delta_1, \delta_2,$ and γ. The baseline values of the structural parameters of the cost function are reported in Table 3. The resulting values of the decision rule coefficients are reported in Table 4.

Table 3

Parameters of the Cost Function, Baseline Values

θ_1	θ_2	δ_1	δ_2	γ
41.936	59.321	1	-0.458	0

Table 4

Coefficients in the Decision Rule

λ_1	λ_2	Γ_X	Γ_W	$\Gamma_{\pi 1}$	$\Gamma_{\pi 3}$
0.944	0	0.021	-0.051	0.0016	-0.0011

V. THE PUZZLES

1. The Mechanism Puzzle

Monetary policy changes the interest rate and should affect inventories, since the interest rate represents the opportunity cost of holding inventories. Previous VAR studies find that monetary policy shocks affect inventories. However, an extensive empirical literature -- stretching over four decades and using a variety of empirical techniques -- has found little evidence that the interest rate affects inventories. This is the *mechanism puzzle*. If the interest rate doesn't affect inventories, how does monetary policy influence inventories? If the interest rate does affect inventories, why have more than 40 years of empirical studies failed to find the relationship?

In our theoretical model, the real interest rate is subject to persistent and transitory shocks. Transitory shocks have little effect on inventories, but firms do react to persistent shocks (regime changes). Until Maccini, Moore, and Schaller (2004), empirical inventory research primarily used traditional, stationary econometric techniques. These techniques tend to emphasize high-frequency movements in the data, where there is much transitory variation in the interest rate without corresponding variation in inventories – and much transitory variation in inventories (due to their role in buffering sales shocks) without corresponding variation in the interest rate.

In this subsection, we present new econometric evidence on the long-run determinants of inventories. Compared to the majority of previous empirical work on inventories, our innovation is to use nonstationary econometric techniques, specifically cointegration. Relative to Maccini, Moore, and Schaller (2004), we make two important innovations. First, we explore the effect of finite sample bias on the estimated coefficients in the cointegrating regression for inventories. Using simulations of our model, we find that standard cointegrating regressions can lead to large biases in the estimated coefficients. Second, we propose a simple, feasible approach that substantially reduces the bias. We use Monte Carlo simulations to verify that this approach works. Details are provided in an appendix.

Table 5
Estimated Cointegrating Regression

Constant	Time	X	π_1	π_3	W
1.573 (18.200)	0.000 (-39.170)	0.381 (4.530)	0.100 (11.267)	-0.036 (-5.377)	-0.897 (-5.781)

DOLS estimates of the cointegrating vector with (t-statistic).

Details to be incorporated in the notes to the table (or perhaps elsewhere): – Probabilities dated t-1 – New PRM Variable
Manufacturing finished goods, linearly detrended specification, new data, new sample period

Table 5 presents the estimated cointegrating regression. The key coefficients for the mechanism puzzle are those on π_1 and π_3, the probabilities that the economy is in the low and high real interest rate regime, respectively. Theory predicts that the coefficient on π_1 should be positive and the coefficient on π_3 should be negative, since a lower real interest rate reduces the opportunity cost of holding inventories. The data confirm both of these theoretical predictions. The coefficients on both π_1 and π_3 are significantly different from zero. Relative to Maccini, Moore, and Schaller (2004), our econometric approach provides stronger evidence for the statistical significance of π_1.

The solution to the mechanism puzzle therefore seems to run as follows. The real interest rate does affect inventories, but this relationship is obscured by transitory shocks when researchers use traditional, stationary econometric techniques.

2. The Sign Puzzle

Stimulative monetary policy reduces the interest rate. A decrease in the interest rate should **increase** inventories through the reduction in opportunity cost. VAR studies find that the short-term effect of stimulative monetary policy is to **decrease** inventories.[16] This is the *sign puzzle.*

To verify whether the sign puzzle exists in our data, we estimate a VAR similar to that described in Section III. The only difference is that we include inventories in the macroeconomic block. This allows us to calculate the impulse response function of inventories to a monetary policy shock (where the monetary policy shock, as elsewhere in this paper, is identified using the Bernanke-Mihov (1998) identification strategy). The response of inventories to a one-standard-deviation stimulative shock to monetary policy is shown in Figure 3. As found in other studies, our data show that the initial response of inventories is to decrease.

Does our calibrated model reproduce this negative initial response of inventories to a stimulative monetary policy shock? To address this question, we start with the decision rule for inventories implied by the model. In the absence of adjustment costs, the decision rule states that current inventories depend on last period's inventories, sales, input costs, and π_1 and π_3. To calculate the path of inventories in response to a monetary policy shock, we can simply substitute in the impulse response functions of sales, input

[16] This stylized fact is documented in Bernanke and Gertler (1995), Christiano, Eichenbaum, and Evans (1996), and Jung and Yun (2005).

costs, and the probabilities (using the Bernanke-Mihov specification described in Section III to identify monetary policy shocks and calculate the impulse response functions). We use the model to derive the structural parameters from the estimated coefficients of the cointegrating regression (as discussed in Section IV).

Figure 4 presents the theoretical response of inventories to a stimulative monetary policy shock, based on our model. As the figure shows, the initial response of inventories is to decrease, as in the empirical impulse response function in Figure 3. Intuitively, the key to understanding our model's success in matching the empirical sign puzzle is the role of inventories in buffering demand (sales) shocks. Demand shocks dominate the high-frequency movements in inventories. Sales rise in the wake of a stimulative monetary policy shock. Production does not respond immediately, so inventories fall as they buffer the positive sales shock.

3. The Timing Puzzle

Monetary policy induces transitory changes in the interest rate. The effect of monetary policy on the interest rate largely disappears within one year. But inventories begin to fall only after the transitory shock to the interest rate has largely dissipated.[17] The transitory effect of a monetary policy shock on the interest rate is illustrated in Figure 5 (which is based on our data and the Bernanke-Mihov VAR discussed above). Within nine months, the Fed funds rate returns to its pre-shock level. It is only many months later that inventories rise above their pre-shock level, as shown in Figure 3. The peak effect of the monetary policy shock on inventories occurs years after the shock. This is the *timing puzzle*: If monetary policy affects inventories through opportunity cost, why

[17] This stylized fact is documented in Bernanke and Gertler (1995), Christiano, Eichenbaum, and Evans (1996), and Jung and Yun (2005).

is the movement of inventories in the theoretically predicted direction delayed until well after the interest rate returns to its pre-shock level?

Regime switching and learning provide part of the explanation for the timing puzzle. Because of learning, the Bayesian probabilities of being in a given interest rate régime respond slowly to a change in the interest rate. Although the effect of monetary policy on the interest rate tends to be short-lived, the effect on the probabilities is persistent. This can be seen in Figure 1, where more than one-third of the effect of the monetary policy shock on π_1 is still present three years after the shock.

Production smoothing also plays a role. Some intuition may be helpful. In a production smoothing inventory model, the cost function is convex. This means that it is cheaper to produce at an intermediate level of output, rather than sometimes producing at low output and sometimes producing at high output. This makes the production level sticky: firms would prefer to produce at their usual (intermediate) output level, even when hit by transitory shocks. The more convex the cost function, the stickier output is.

In general, the previous literature has treated the interest rate as constant, so the standard intuition focuses on sales and cost shocks. Interest rate shocks work a bit differently, but much of the standard intuition still applies. An interest rate shock changes the desired long-run inventory level. However, changing output (away from the usual level) is expensive because of the convexity of the cost function. If firms recognize that the interest rate shock is purely transitory, they will adjust output (and therefore the stock of inventories) little, if at all. Because firms are reluctant to adjust output, the change in the stock of inventories is delayed.

Figure 6 illustrates how the combination of learning and production smoothing delays the response of inventories. As discussed above, the peak response of π_1 and π_3 occurs about six months after the monetary policy shock. Figure 6 shows the response of the inventories, based on the calibrated model, when the only variation comes from π_1, and π_3. (The other variables that enter the decision rule -- sales and input costs -- are held constant at their pre-shock levels.) With the combination of learning (which leads to a slow and persistent response of π_1, and π_3) and production smoothing (which stretches out the response of inventories to π_1, and π_3), the peak response of inventories in the calibrated model occurs about two years after the shock.

As noted above, it is the convexity of the cost function that spreads out the response of inventories to shocks. The convexity of the cost function is measured by the parameter θ_1. In Figure 7, we illustrate the effect of changes in θ_1 on the impulse response function for inventories. If we set θ_1 equal to half the value implied by the cointegrating regression estimates, the peak effect on inventories occurs seven months earlier.

Figure 7 illustrates another interesting point. In the existing inventory literature, it has been very hard to pin down the convexity of the cost function. In their survey paper, Ramey and West (1999) report a wide range of estimates. Using the cointegrating regression to calibrate θ_1, we obtain a value of θ_1 that leads to a theoretical impulse response function that is similar to the empirical impulse response function. If we move too far away from this value of θ_1, the theoretical impulse response function no longer matches the empirical impulse response function. As Figure 7 illustrates, using a value of

θ_1 that is 50% smaller leads to a peak response of inventories that is too large (about twice as large as the empirical peak response). Using a value of θ_1 that is much larger than the calibrated value from the cointegrating regression (e.g., 50% larger) leads to a theoretical impulse response function that no longer even qualitatively resembles the empirical impulse response function.

This point has both methodological and substantive economic implications. Methodologically, it suggests that good estimates of a well-specified cointegrating regression may provide a better technique for calibrating model parameters. Economically, it narrows the range of plausible estimates of the convexity of the cost function. For example, it rules out the possibility of a concave cost function.[18]

4. The Cost Shock Puzzle

In the standard inventory model, the firm tends to smooth production. When there is a transitory shock to sales, production changes less than sales. If these are the main type of shocks faced by the firm, production should vary less than sales. Empirically, however, a number of studies have shown that production varies more than sales. One explanation is cost shocks. A favorable cost shock tends to make it cheaper to produce now than in the future. Cost shocks are a leading explanation for the stylized fact that production varies more than sales. The problem with this explanation is that most empirical studies find that observable cost shocks have no significant effect on inventories. The literature has therefore tended to be pushed toward the somewhat uncomfortable position that unobservable cost shocks account for one of the leading

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[18] This is important, because, as Ramey (xx) points out, increasing returns to scale (i.e., a concave cost function) provide a potential explanation for some of the big stylized facts about inventories, such as the fact that the variance of output is greater than the variance of sales.

stylized facts about inventories.[19] The *cost shock puzzle* can be summarized as follows: Cost shocks play a useful theoretical role in explaining the behavior of inventories, but it is hard to find empirical evidence that observable cost shocks affect inventories.

We make some progress on the cost shock puzzle. The intuition for our results shares features of our earlier discussion of other puzzles. As we have seen, the convexity of the cost function implies that the effect of shocks is spread out over time. Theory suggests that several types of shocks -- sales, interest rate, and cost -- are important for inventories, making it difficult for older econometric techniques, which emphasize high-frequency variation in the data, to detect the effect of any particular shock, especially when there may be a long lag between the realization of the shock and its impact on inventories.

Econometric techniques that emphasize low-frequency movements in the data provide a more promising method of estimating the effect of cost shocks. We illustrate this in two ways. First, we use a cointegration approach to test whether observable input costs have a significant effect on inventories. Second, we use our theoretical model of inventories to extract structural parameters from the cointegrating regression and then decompose the channels -- sales, interest rate, and costs -- through which a monetary policy shock affects inventories, so that we can isolate the effect of costs.

In Section IV, we show that our theoretical model implies that real input costs appear in the cointegrating regression for inventories with coefficient b_W. Table 5 reports that the estimated value of b_W is -0.897, which is negative, as predicted by our model

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[19] This issue is discussed in greater detail in two major surveys of the inventory literature -- Blinder and Maccini (1991) and Ramey and West (1999).

(and, more generally, by economic theory). Table 5 also reports that the t-statistic on b_W is -5.781. Thus, the cointegrating regression provides statistically significant evidence that observable costs affect inventories.

Do observable costs play an economically important role in determining inventory movements? One way to address this issue is to use our model to calculate the impulse response function of inventories to a monetary policy shock. More precisely, we can use the model to shut down some of the channels through which monetary shocks affect inventories -- specifically, the sales and interest rate channels -- so that we can observe the cost channel in isolation. Figure 9 presents the impulse response function of real input costs to a stimulative monetary policy shock. The response of W is hump-shaped, with the peak response occurring about a year after the shock. Figure 10, which is based on our model, illustrates the effect of this change in real input costs on inventories (holding sales and the real interest rate constant). Because higher input costs discourage firms from holding inventories, inventories fall. Because of the convexity of the cost function, the maximal effect on inventories occurs more than a year after the peak in the response of W to the monetary policy shock. The magnitude of the pure cost effect is about half as large at its peak as the overall movement in inventories, but in the opposite direction. If input costs had no effect on inventories, the peak effect of a monetary policy shock on inventories would be about 50% larger. Relative to most of the previous literature, these results are striking. Based on our model (and on our calibration of the structural parameters, using the cointegrating regression), observable costs have an economically important effect on inventory movements.

VI. The Volcker Disinflation

(Forthcoming)

VII. Conclusions

(Forthcoming)

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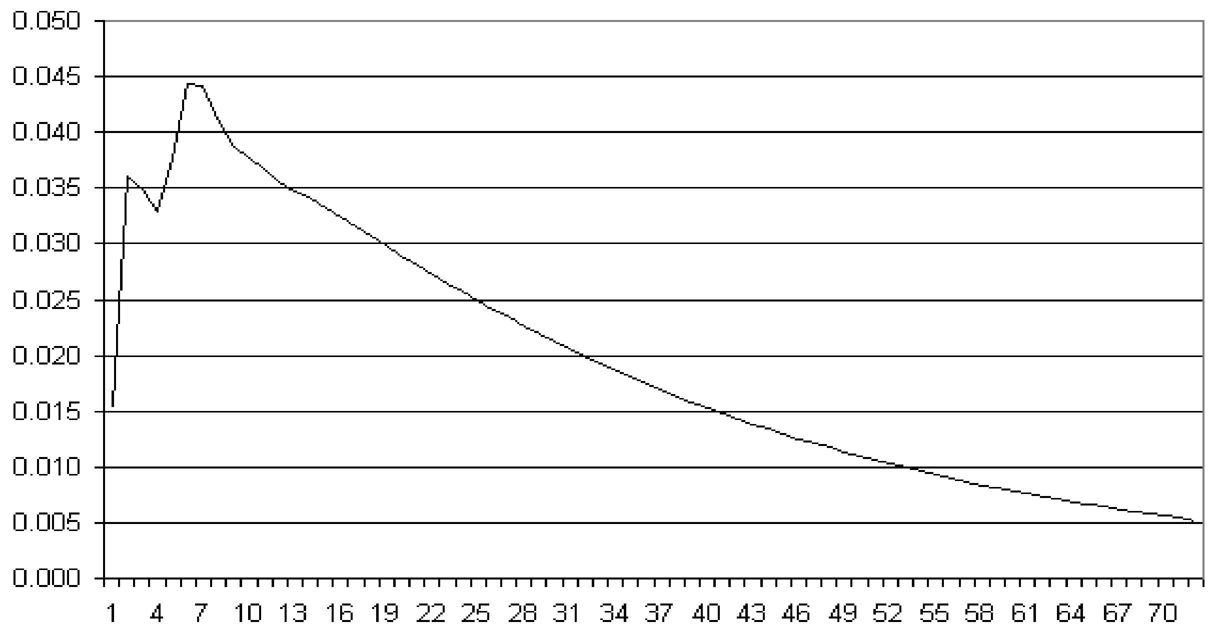
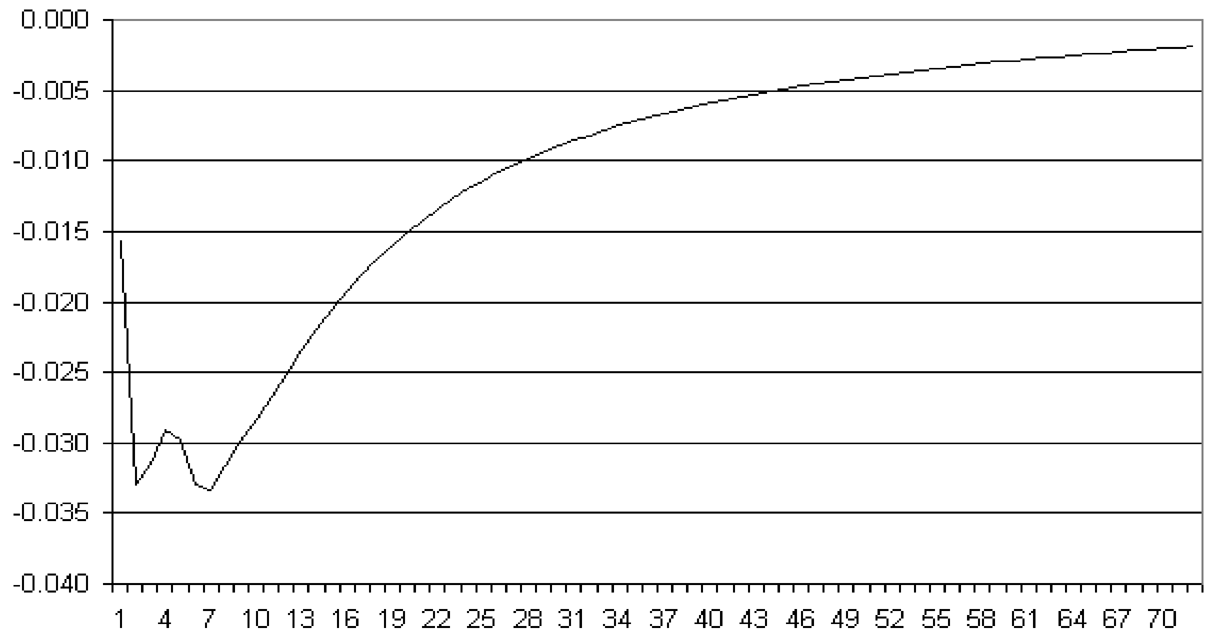
Figure 1
The Effect of a Monetary Policy Shock
on the Probability of the Low Interest Rate Regime



The solid line presents the impulse response function of π_1 (the probability of the low real interest rate regime, as perceived by the firm) to a one-standard-deviation stimulative monetary policy shock. The horizontal axis shows time in months.

Figure 2
The Effect of a Monetary Policy Shock
on the Probability of the High Interest Rate Regime



The solid line presents the impulse response function of π_3 (the probability of the high real interest rate regime, as perceived by the firm) to a one-standard-deviation stimulative monetary policy shock. The horizontal axis shows time in months.

Figure 3
Empirical Response of Inventories to a Stimulative Monetary Policy Shock

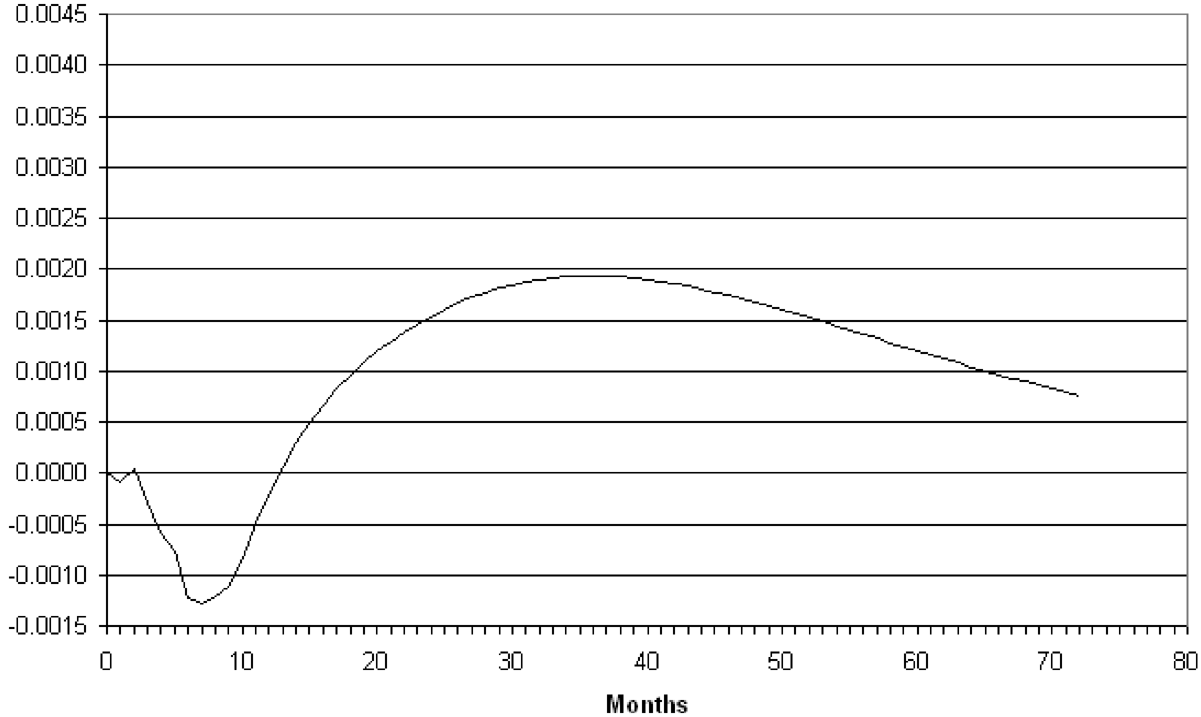


Figure 4
Theoretical Response of Inventories to a Stimulative Monetary Policy Shock

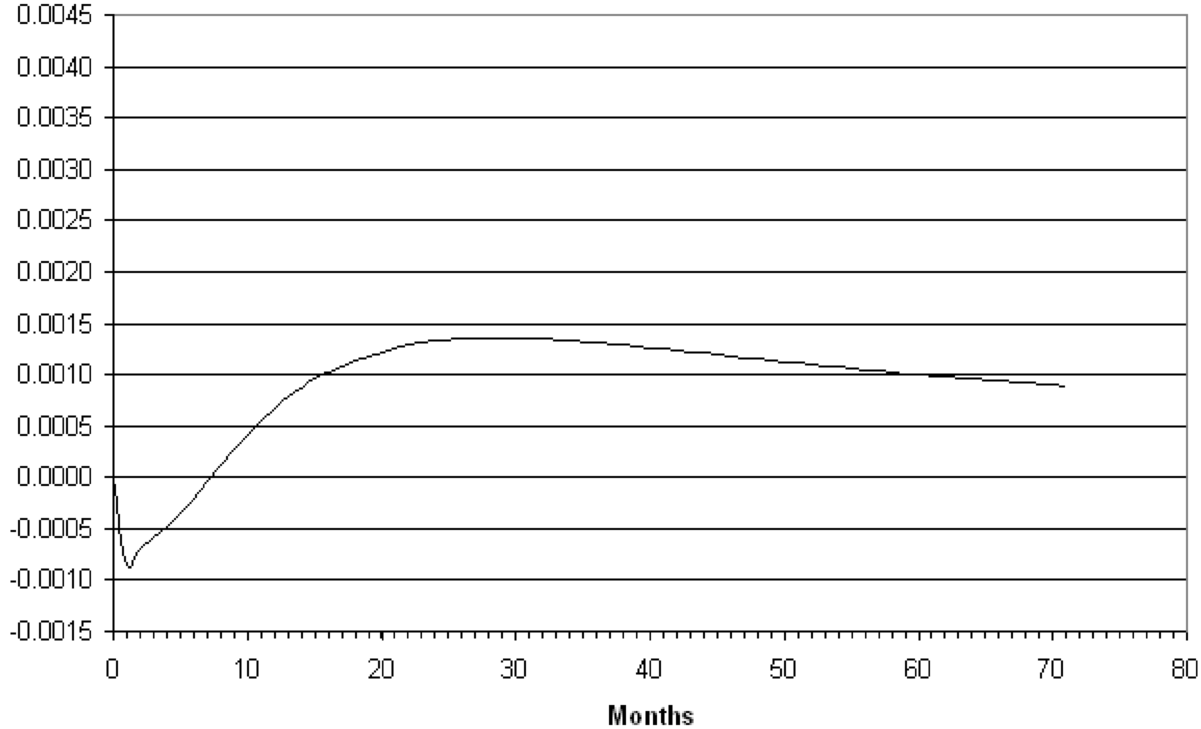


Figure 5
Empirical Response of the Fed Funds Rate to a Stimulative Monetary Policy Shock

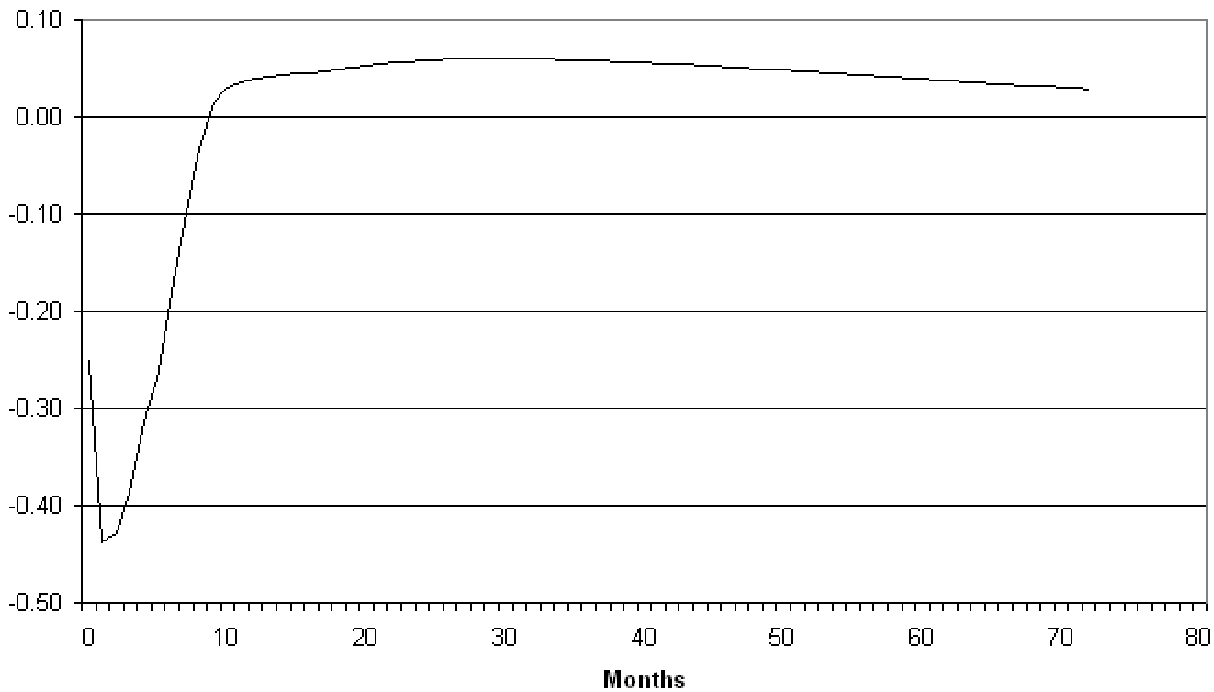


Figure 6
Theoretical Response of Inventories to a Stimulative Monetary Policy Shock
(Varying Only the Probabilities)

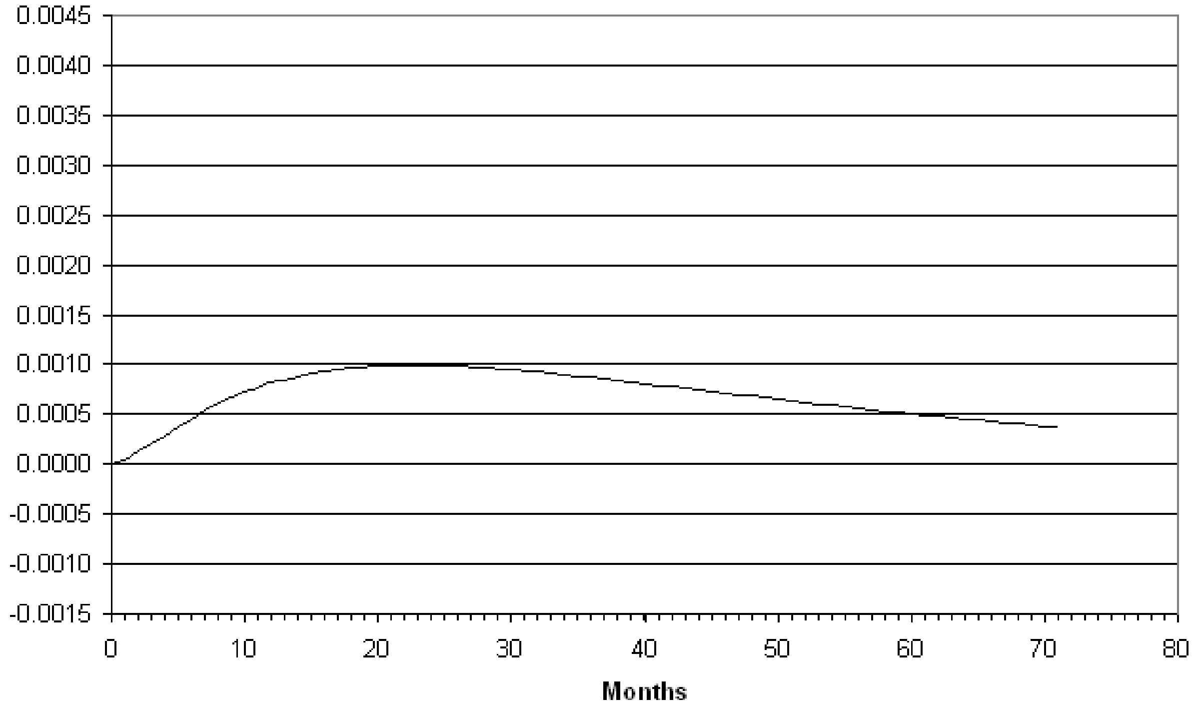
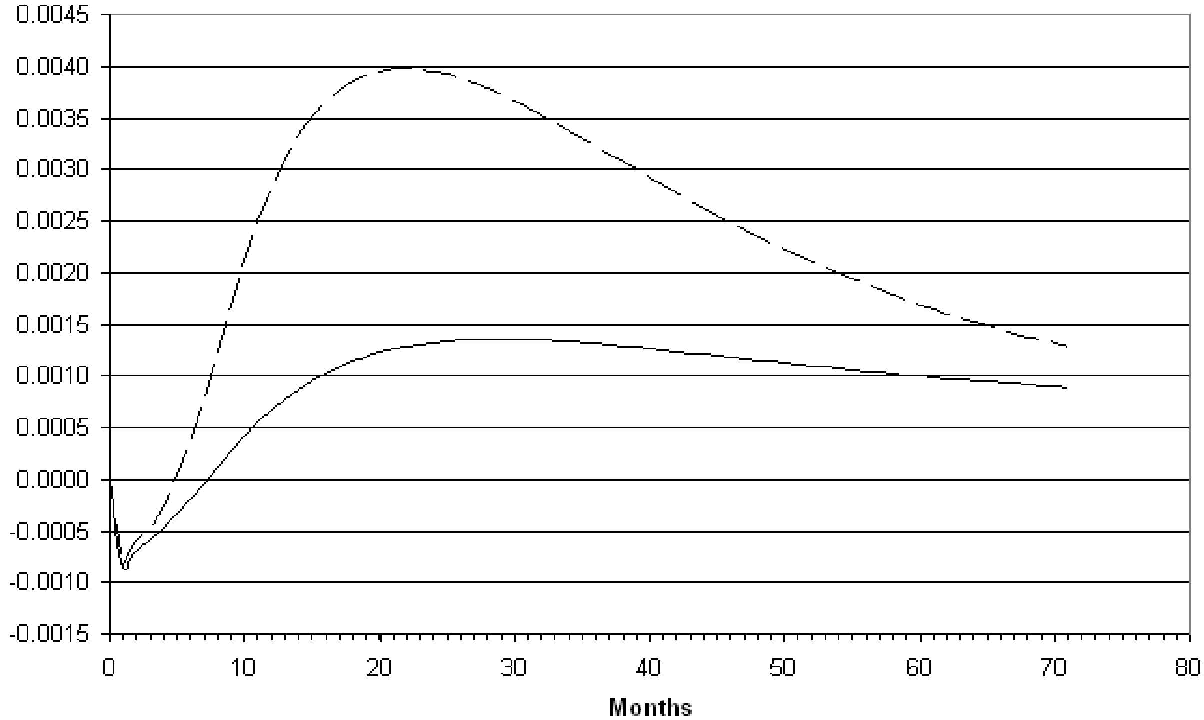


Figure 7
Theoretical Response of Inventories to a Stimulative Monetary Policy Shock
The Effect of the Convexity of the Cost Function



The dashed line shows the response of inventories based on setting θ_1 (the parameter that controls the convexity of the cost function) equal to 0.5 times the value obtained when the parameters are calibrated using the cointegrating regression. The solid line is based on setting θ_1 equal to the value obtained based on the cointegrating regression.

Figure 8
Empirical Response of Real Input Costs to a Stimulative Monetary Policy Shock

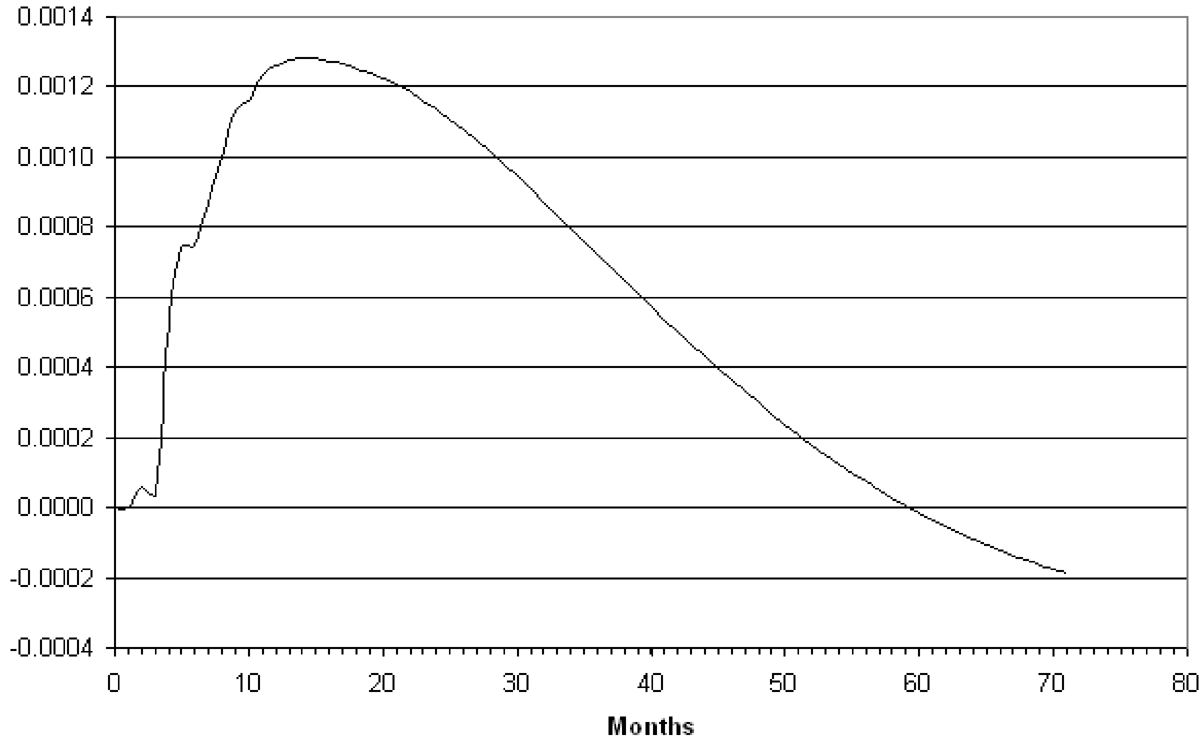


Figure 9
Theoretical Response of Inventories to a Stimulative Monetary Policy Shock
(Varying Only Real Input Costs)