A Theory of Brain Drain and Public Funding for Higher Education in the U.S.*

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Abstract

Many state policy makers are concerned about outmigration of college graduates. There is no consensus, however, in the empirical literature concerning the relationship between outmigration and university funding. We develop a theory of outmigration, brain drain, and university funding in the U.S. that explains the nature of this relationship. We account for heterogeneity in ability across individuals and education funding across states. We stress the role of scale economies in higher education and characterize college enrollment and brain drain within states with various degrees of economies of scale. Our results show that investing in higher education attracts college students. If a state does not benefit from increasing returns to scale in higher education, we find a positive relationship between public spending and out-mobility, but when the state enjoys economies of scale in education, we find that a negative relationship between public spending and out-mobility can arise in equilibrium.

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1 Introduction

On July 2, 1862, President Abraham Lincoln signed the Land Grant Act which established publicly funded universities in all states. The new state-funded universities greatly expanded access to higher education, especially for those of limited means (Goldin and Katz, 1999) and for minorities (Lowry, 2001).

Recently, however, state fiscal support for higher education has been falling, and post-secondary institutions have increased tuition to support their programs. From 1980 to 2005, real state appropriations for higher education per student have fallen an average of 20%, with only 6 states increasing funding. The largest decrease occurred in South Carolina, where higher education funding fell 53% (Grapevine, 2008). State budgets for higher education are competing with other funding needs, such as rising healthcare costs and an increased focus on primary and secondary education (Layzell, 2007). Institutions confronting funding shortfalls face difficult decisions about how to maintain quality in the face of declining revenue. Rutgers University, for example, has laid off employees and eliminated some college sports. They also increased tuition, closed some academic programs and decreased the number of courses taught (Rutgers, 2006).

The mobility of college graduates has been cited as a political reason for cutting university funding (Wirtz, 2003). In fact, the college educated display a high degree of geographic mobility: they often leave the state that subsidized their education. In the Baccalaureate and Beyond data set, 5 years after graduation, 44% of college graduates were living outside of the state where they attended college. Kodrzycki (2001) found that 37% of people with a bachelor’s degree changed their state of residence between 1979 and 1996, compared to 19% of those with a high school education or less. This figure rises to 45% for those with advanced degrees. A similar result is presented in Bound et al. (2004). They find that the elasticity between the stock of college graduates and the flow of college graduates (degrees granted) is 0.33: positive, but not large.

States are worried about outmigration of college graduates, and with good reason: losing college graduates is a drain on the local economy. New York is attempting to lure college graduates to the
state by improving the housing options for young professionals and the small business environment (Cornell University, 2007). In addition, a recent Wall Street Journal article reports that South Dakota, for example, has established a job placement service called “Dakota Roots” to convince expatriate former graduates of South Dakota colleges to return. In fact, in the Baccalaureate and Beyond data, 42% of graduates of South Dakota universities in 1992 were living outside the state in 1997.

Although many state policy makers are concerned about outmigration of college graduates, the relationship between outmigration of college graduates and university funding is unclear. Some empirical studies find that outmigration and university funding are negatively related, whereas others find the opposite. For example, Strathman (1994) estimated that a 1% increase in the outmigration of college graduates was associated with a 2.1% decrease in state university funding per student. Clotfelter (1976) reached a similar conclusion, but his results were not statistically significant. Some researchers find that outmigration and university funding are positively related. Polgreen (2008) finds that greater levels of university funding are associated with higher levels of outmigration – a one percentage-point increase in outmigration is associated with an increase of $43 of funding per student. Also, Suedekum (2003), in his study of less-well-developed areas of Europe, finds that university funding, especially when targeted directly to students, leads to outmigration: once students are educated, they migrate to areas where their skills are best rewarded, often to more-well-developed areas. Thus, previous studies have all consistently demonstrated a relationship between outmigration of college graduates and university funding in the U.S., but whether that relationship is positive or negative is unknown. A structured model is needed to determine this. To our knowledge, we are the first to develop such a theory.

Our paper builds upon theories of international brain drain. In the international context, brain drain is generally defined as the emigration of high-ability, skilled personnel from developing

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1State universities receive funding from state and federal sources, but in most cases, state funding is a more important part of their budget. For example, in 2004, 25% of revenues for the University of Iowa were from the state, while 0.3% were from the federal government (University of Iowa, 2008).
countries to more-developed and industrial nations. This migration primarily occurs as foreign-trained students choose to remain in the country where they were educated. While most brain-drain research focuses primarily on its consequences, two papers study its cause and are therefore more relevant to this topic (Kwok and Leland, 1982, and Miyagiwa, 1991). Kwok and Leland explain brain drain as a phenomenon arising from asymmetric information in the labor market while accounting for heterogeneity in ability within the group of skilled agents. Miyagiwa, however, presents an alternative theory that emphasizes increasing returns to scale in education as the main cause for brain drain. Following the latter, we carefully distinguish outmigration of skilled personnel from brain drain by embedding our analysis in a model that accounts for heterogeneity in ability within skill groups and for the possibility of economies of scale in higher education. We depart from Miyagiwa’s paper in several important ways: 1) We study domestic brain drain within the U.S., but brain drain across state boundaries differs from international brain drain: we define brain drain as the net emigration of college graduates (skilled personnel) from the state where they got their college degree to another state. 2) We account for different degrees of increasing and decreasing returns to scale to higher education among states. Within the U.S. most states benefit from economies of scale to higher education but to different degrees: some benefit more than others from positive spillover effects. 3) We allow for different education funding across states and derive theoretical results regarding the relationship between education funding, college enrollment, and brain drain.

Our results show that investing in higher education attracts college students. If a state benefits from increasing returns to scale (IRS) in higher education, we find that a negative relationship between public education spending and out-mobility can arise in equilibrium: high wages convince students to stay after graduation. However, if a state does not benefit from IRS in higher education, investing in public education will lead to brain drain: funding attracts students to college, but the relatively low wages in the decreasing-returns-to-scale (DRS) state are not sufficient to keep college

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2See Hanson (2001) for a thorough review of the human-capital spillovers and agglomeration literature.
graduates in state. We use our results to derive optimal policy prescriptions for states experiencing outmigration of college graduates.

The paper is organized as follows: Section 2 presents the general framework; we characterize college enrollment in section 3 and brain drain in Section 4; in Section 5 we discuss policy implications for states with different degrees of return to higher education; we conclude in Section 6.

2 General Framework

The economy is populated by measure-one, two-period-lived agents that differ by levels of latent ability, which is assumed to be distributed continuously on the interval $A = [0, 1]$ with a finite positive density function $f(a)$. If there is no higher education, all are unskilled, so the real wage for unskilled labor is $w_u = 1$. If there is higher education, then this will be reflected in earnings, such that $w_s(a) > w_u(a) = 1$, $\forall a \in A$. The returns to higher education for an individual of ability $a$ are given by

$$w_s(a) = g(\mu)a$$

with

$$\mu = \mu(a) = \int_a^1 f(z)dz,$$

where $\mu$ indicates the number of individuals who have received higher education.

**Assumption 2.1.** The return function satisfies the following conditions: $g(\mu) > 0$, $g'(\mu) > 0$, and $g(\mu) > g'(\mu)a$.

The term $g(\mu)$ captures the effect from positive spillovers of economies of scale: the higher the fraction of educated individuals in the economy, the higher the income for each educated individual. This is in line with findings that having a highly educated population creates positive externalities (spillovers). Rauch (1993) finds that the social marginal returns to increases in human capital
exceed the personal returns by a factor of 1.7. In addition, the productivity of professional work increases with an increase in the number of similar professionals concentrated in one location (Jaffe, 1989, Hoy, 1988, Dorfman, 1988). Workers share new skills, creating increased productivity (Lucas, 1988).

Note that \( \mu \) as a function of \( a \) represents the fraction of students with ability level higher than \( a \) that go to college. On an individual level, however, we assume that agents take \( g(\mu) \) as a parameter so that for a given value of \( \mu \), the net return to education is linear in \( a \). Once educated, an individual with a given ability level earns a proportionately higher level of income than another educated worker who possesses a lower ability level. Thus, the return function is consistent with the fact that higher-ability individuals will benefit more from a college education than lower-ability individuals. Education, however, comes with a cost, \( c > 0 \), that the agent has to pay. We assume that even if no one goes to college, there is an incentive for the agent with the highest ability level to enroll.

**Assumption 2.2.** The return function satisfies \( g(0) - c > 1 \).

With these specifications, the timing of the model is as follows: In the first period, the agent faces the decision whether to go to college or not. For an agent of ability \( a \) this decision is based on the wage differential between skilled and unskilled wages. If the agent chooses to enroll in college, he also decides where to go to college. Tuition levels (and thus university funding) will be the key in determining this decision. In the second period, the agent chooses whether or not to stay in the state where he graduated from college. While his decision in the first period is driven by skilled versus unskilled wages, the decision in the second period is guided by the difference in skilled wages net of taxes between the state where the agent went to college and the other state. This is in line with empirical findings that college-graduate mobility partially occurs because returns to higher education differ between states (e.g., Borjas, Bronars and Trejo, 1992; Farber and Newman, 1987). In our model, the incentive to leave the state or not depends on the degree to which the state benefits from increasing returns to scale to higher education.
3 Enrollment Under Heterogeneity in Public Funding

We first discuss the decision to enroll in college in a simplified economy, where there is no heterogeneity in terms of university funding, as in Miyagiwa (1991), then we will introduce differences in funding across states and reconsider the enrollment choice. Finally we will address the migration decision in the second period and link brain drain to university funding emphasizing the role of differences in economies of scale to higher education across states.

We turn now to the decision in the first period in a simple economy where there is a homogeneous cost to attend college, $c > 0$. Note that in this environment, if no one goes to college, $\mu = 0$, there is no incentive for the agent with the lowest ability level to enroll, since $w_s(0) − c < 1$. Higher education will be demanded by all individuals with ability higher than a marginal level $\bar{a}$ defined by $g(\mu)\bar{a} − c = 1$, as shown in the following proposition.

**Proposition 3.1.** There is a threshold $\bar{a}$ such that all individuals with ability $a > \bar{a}$ will enroll in college, and all individuals with $a < \bar{a}$ will not go to college.

*Proof.* Wlog we restrict ourselves to the case where no one goes to college, $\mu_0 = 0$; there is an incentive for the most gifted to enroll, i.e. $g(0) − c > 1$. Let $G_1(a) = g(0)a − c$, so $G_1(0) < 1 < G_1(1)$. Then there exists $\bar{a}_1$ such that $G_1(\bar{a}_1) = 1$ and $\mu_1 = \int_{\bar{a}_1}^1 f(z)dz = 1 − \bar{a}_1 > \mu_0$. Let $G_2(a) = g(\mu_1)a − c$, so $G_2(0) < 1 < G_2(1)$. Then there exists $\bar{a}_2$ such that $G_2(\bar{a}_2) = 1$ and $\mu_2 = \int_{\bar{a}_2}^1 f(z)dz = 1 − \bar{a}_2 > \mu_1$. By induction, there exists $\bar{a}_n$ such that $G_n(\bar{a}_n) = 1$ and $\mu_n = \int_{\bar{a}_n}^1 f(z)dz = 1 − \bar{a}_n$ with $\{\mu_n\}$ an increasing, bounded sequence on $[0, 1]$ and $\{\bar{a}_n\}$ a decreasing, bounded sequence on $[0, 1]$. Then there exist $\mu$ and $\bar{a}$ such that $\lim \mu_n = \mu$ and $\lim \bar{a}_n = \bar{a}$ with $g(\mu)\bar{a} − c = 1$ and $\mu = 1 − \bar{a}$.

Even if no one receives education, $\mu = 0$, there is an incentive for the most gifted, $a = 1$, to acquire education, but it never pays off for the least gifted to acquire higher education, regardless of the value of $\mu$. Proposition 3.1 delivers that the population gets divided into an educated (skilled)
and an uneducated (unskilled) labor force with \( \mu(\pi) = \int_1^\infty f(z)dz \), the fraction of agents who choose to go to college. In the case of the uniform distribution, \( \mu(\pi) = (1 - \pi) \).

Now we reconsider the enrollment choice in the case where the environment consists of two states that make decisions about university funding, with implications for either tuition subsidies or a higher quality of education. The agent of ability \( a \) faces three alternatives: go to college in state 1, go to college in state 2 or do not go to college at all. The agent’s location is irrelevant for his decision in this first period.

If university funding is the same in both states, by proposition 3.1, the same ability level as before, \( \pi \), will determine the enrollment decision with agents of ability \( a \) being indifferent between the two states. The proportion of students going to either state is the same with \( \mu_1(\pi) = \mu_2(\pi) = \frac{\mu(\pi)}{2} \). We assume that the two states have the same enrollment capacity, but the states do not need to be the same size.

We now consider the case where university funding differs between states. If state 1 provides a higher level of education funding, it can be funneled into either tuition subsidies for students or a higher quality of education. We consider the subsidy case with \( s_1 = c - c_1 \). The tuition the agent has to pay in state 1 is lower than the tuition in the other state, \( c_1 < c_2 = c \). The subsidy can be implemented in different ways: 1) The subsidy can be the same for all students regardless of ability level, \( s_1 \) is a constant \( \forall a \in A \), or the subsidy can be conditioned on ability level as in a merit based subsidy (higher ability students would pay less for their college education): \( s_1(a) \) with \( s_1'(a) > 0 \). Also, state 1 can impose a minimum requirement of ability to grant the subsidy: \( s_1(a) > 0, \forall a \geq \pi \) and \( s_1(a) = 0, \forall a < \pi \). The following proposition shows that no matter how the increased education funding is used, subsidizing education attracts more students.

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3 We assume there is no bias toward the student’s home state and no cost of mobility. If bias toward one’s home state exists, we assume that this bias is primarily driven by tuition differentials. In fact, Howard P. Tuckman (1970) found that college students are more likely to leave the state where they went to high school if in-state college tuition rates are high.

4 We do not distinguish between in-state- versus out-of-state-tuition because states that subsidize education will have an effect on the decisions of both in-state and out-of-state students even if they subsidize in-state students more heavily. In addition Groen and White (2004) find that both in-state and out-of-state students are equally likely to stay in state after graduation.
Proposition 3.2. Given that state 1 subsidizes education, regardless of the subsidy type, a higher fraction, \( \mu_1 \) will choose to go to college in state 1, \( \mu_1 = \mu(\overline{\alpha}_1) > \mu_2 = \mu(\overline{\alpha}_2) \). The fraction, \( \mu_1 \) increases with the subsidy.

Proof. By Proposition 3.1 there exist \( \overline{\alpha}_1 \) and \( \overline{\alpha}_2 \) such that all agents of type \( \alpha > \overline{\alpha}_1 \) go to college in state 1, and all agents of type \( \alpha > \overline{\alpha}_2 \) go to college in state 2. If state 1 gives a subsidy regardless of ability level, \( G(\alpha) = g(\mu(\alpha))a - c \) is the same as before. If state 1 chooses to condition the subsidy on ability level, the function is given by \( G(\alpha) = g(\mu(\alpha))a - c(\alpha) \), which is continuous and strictly increasing in \( a \) given \( c'(a) < 0 \), and \( g(\mu) > g'(\mu)a \). The two fractions that go to college are given by \( \mu_1 = \mu(\overline{\alpha}_1) = \int_{\overline{\alpha}_1}^{1} f(z)dz \) and \( \mu_2 = \mu(\overline{\alpha}_2) = \int_{\overline{\alpha}_2}^{1} f(z)dz \). Since \( c_1 < c \), it results that \( g(\mu_2)\overline{\alpha}_2 - c_1 > g(\mu_2)\overline{\alpha}_2 - c = 1 = g(\mu_1)\overline{\alpha}_1 - c_1 \). This implies \( \overline{\alpha}_1 < \overline{\alpha}_2 \), and the result follows.

For example, if state 1 subsidizes all students at the same rate and with a minimum ability requirement equal to the threshold in Proposition 3.1 \( s_1(\alpha) = s_1 \forall \alpha \geq \overline{\alpha} \), the tuition cost is lower in state 1 only for those who would have gone to college without any subsidy: \( c_1 < c \) for all \( \alpha \in [\overline{\alpha}, 1] \) and \( c_1 = c \) for all \( \alpha \in [0, \overline{\alpha}] \). State 1 can attract college students to the state without affecting the total number of college-educated individuals in the economy. In this particular case, our result implies that the same fraction of students attend college as in proposition 3.1 but all attend college in state 1: \( \mu_2 = 0 \) and \( \mu_1 = \mu(\overline{\alpha}) = (1 - \overline{\alpha}) \). We will consider this subsidy scheme for the rest of the paper. Results, however, are robust to alternative ways of implementing the subsidies.

4 Brain drain

In general, brain drain is an exodus of skilled personnel, in our case, college graduates. In the model, if there is no funding involved, from the previous discussion it follows that \( \mu_1 = \mu(\overline{\alpha})/2 \), and if state 1 subsidizes education, \( \mu_1 = \mu(\overline{\alpha}) \). Under these specifications, our definition of brain
drain (net emigration of college graduates from the state they got their degree) implies that brain drain occurs if more than $\mu_1/2$ of college graduates decide to leave state 1 post graduation.

4.1 The general problem

The subsidy in state 1, $s_1 = (c - c_1)$, is financed through lump-sum taxes collected in the post-college period from all skilled agents that live in state 1. Consequently there are higher taxes in state 1 relative to the other state (we normalize taxes in the other state to be 0). The local authority sets the tax each agent has to pay, $t_1 > 0$, to satisfy the budget constraint $(c - c_1)(1 + r)\mu = t_1\lambda(\mu)$ with $\lambda(\mu)$ representing the fraction of skilled agents that remain in the state after graduation. This satisfies $\lambda(0) = 0$ and $\lambda'(\mu) > 0$: the higher the fraction of college graduates, the higher the fraction of skilled labor remaining after college.

In period 2, the agent chooses whether or not to stay in state 1 after graduation. His decision depends on the earnings differential for skilled labor between the two states and the local tax, $t_1$. While the higher tax induces the college graduate to leave regardless of his ability level, the earnings differential, as given by $\frac{w_1(a)}{w_2(a)}$, is contingent on ability and hence will induce different effects across groups of agents.\(^5\)

There are 3 potential equilibria: one with no brain drain, $\lambda(\mu_1) \in [\frac{\mu_1}{2}, \mu_1]$, complete brain drain, $\lambda(\mu_1) = 0$, and incomplete brain drain, $\lambda(\mu_1) \in (0, \frac{\mu_1}{2})$. We show that in the case of economies of scale, incomplete brain drain is never possible: college graduates always have an incentive to leave state 1 given the higher taxes they have to pay to subsidize undergraduate education. Given increasing returns to education, however, there’s never an equilibrium where more than half of college graduates will choose to leave unless all of them leave.

Proposition 4.1. Given $\mu_1 = \mu(\pi)$, the fraction who enrolls in college in state 1, the fraction of

\(^5\)Note that we assume a self-financed scheme for higher education. In our model only highly educated people pay for their education, while the unskilled are not taxed. There is no loss of generality as the agent’s post-college decision is based on the earnings differential between the skilled in state 1 versus the skilled in state 2. Also, the focus in the paper is on education policies and brain drain, not on fiscal policies. Future research will extend this analysis to accommodate for both education and fiscal policies.
agents who stay is \( \lambda(\mu_1) \in \left[ \frac{\mu}{2}, \mu_1 \right] \) or \( \lambda(\mu_1) = 0 \).

**Proof.** Suppose, by contradiction, that there is incomplete brain drain. Then \( \lambda(\mu_1) < \mu_1 - \lambda(\mu_1) \) and since \( g \) is increasing it follows that \( g(\lambda(\mu_1)) - g(\mu_1 - \lambda(\mu_1)) < 0 \). Then for any \( a \), \( g(\lambda(\mu_1))a - g(\mu_1 - \lambda(\mu_1))a < 0 \). Since \( t_1 > 0 \), there is no agent who chooses to stay in state 1, i.e. \( \lambda(\mu_1) = 0 \). Contradiction. There is either complete or no brain drain, that is \( \lambda(\mu_1) = 0 \) or \( \lambda(\mu_1) \in \left[ \frac{\mu}{2}, \mu_1 \right] \). □

We find that a negative relationship between university funding and net outmigration can occur in equilibrium. High public spending attracts people to college and there is no brain drain as the number of agents that choose to leave after graduation does not outweigh the number of out-of-state college students. This is because positive spillovers increase the wages in state 1 offsetting the effect of higher taxes in state 1.\(^6\) We focus on the case when this occurs in equilibrium, and in the following theorem, we characterize the fraction of agents who choose to stay in state 1 and the fraction of agents who choose to leave.

**Theorem 4.2.** Assume there is no brain drain. There is a level \( \hat{\mu} \), such that for any \( a > \hat{\mu} \), agents will choose to stay state 1, and for any \( a < \hat{\mu} \), they choose to leave. For \( \hat{\mu} \), the agent is indifferent if \( \frac{w_1(\hat{\mu}) - t_1}{w_2(\hat{\mu})} = 1 \). The fraction of agents who leave post graduation is given by \( 1 - \lambda(\mu, \hat{\mu}) = \int_{\hat{\mu}}^{\mu} f(z)dz \).

(In the case of uniform distribution, \( 1 - \lambda(\mu, \hat{\mu}) = (\hat{\mu} - \mu), \) and the fraction who stays by \( \lambda(\mu, \hat{\mu}) = (1 - \hat{\mu}) \) with \( \hat{\mu} \in (0, 1] \).)

**Proof.** We know that less than half leave state 1, that is \( \lambda(\mu) > \frac{\mu}{2} \). Assume \( \lambda(\mu_1) < \mu_1 \), otherwise the conclusion is trivial. Then \( \lambda(\mu_1) > \mu_1 - \lambda(\mu_1) \) and \( g(\lambda(\mu_1)) - g(\mu_1 - \lambda(\mu_1)) > 0 \). Let \( a^* \) represent an agent who stays, i.e. \( g(\lambda(\mu)) - g(\mu - \lambda(\mu))a^* > t_1 \). For any \( a > a^* \), \( g(\lambda(\mu_1))a - g(\mu_1 - \lambda(\mu_1))a > g(\lambda(\mu_1))a^* - g(\mu_1 - \lambda(\mu_1))a^* > t_1 \), and \( a \) chooses to stay in state 1. Let \( a^{**} \) an agent who leaves, i.e. \( g(\lambda(\mu_1)) - g(\mu_1 - \lambda(\mu_1))a^{**} < t_1 \). For any \( a < a^{**} \), \( g(\lambda(\mu_1))a - g(\mu_1 - \lambda(\mu_1))a < g(\lambda(\mu_1))a^{**} - g(\mu_1 - \lambda(\mu_1))a^{**} < t_1 \), and \( a \) chooses to leave state 1. Hence there is a \( \hat{\mu} \in \left[ \mu, \frac{\mu}{2} \right] \)

\(^6\)Our findings are in line with those in the agglomeration literature. Acemoglu (2003) finds that increases in the number of skilled workers may cause firms to use those technologies that favor skilled workers, increasing their wages.
such that for any \( a > \hat{a} \), agents will choose to stay in state 1, and for any \( a < \hat{a} \), they choose to leave. From the continuity of \( g \), it follows that for \( \hat{a} \), the agent is indifferent if \( \frac{w_1(\hat{a}) - t_1}{w_2(\hat{a})} = 1 \). 

In other words, given economies of scale to higher education, there is an incentive for the college graduate to stay in state 1 given the large number of college graduates in the state. Higher taxes, however, counterbalance this effect. The lower ability college graduates earn lower returns on average and cannot afford to pay the higher taxes. They choose to leave state 1 because even though earnings may be lower, the absence of taxes makes them better off in the other state.

4.2 Decreasing returns to scale in higher education

In order to emphasize the role of economies of scale we consider the possibility of decreasing returns to scale (DRS) to higher education. In this case, the functional form is similar, but now \( g'(\mu) < 0 \), so increasing the number of college graduates in a state lowers rather than raises their wages. Consequently, their earnings will be decreasing in the number of college graduates. The following two propositions follow a path similar to those in the economies of scale (IRS) case.

**Proposition 4.3.** Consider the environment in section 2.1 (no subsidy is provided). There is a threshold \( \pi_{DRS} \) such that all individuals with ability \( a > \pi_{DRS} \) will enroll in college and all individuals with ability \( a < \pi_{DRS} \) will not enroll in college.

**Proof.** In the case \( g(0) - c < 1 \), the highest ability agent, \( a = 1 \), does not go to college. This implies that \( \forall a \), if \( g(0)a - c < 1 \), no one goes to college, i.e. \( \pi_{DRS} = 1 \).

In the case where \( g(0) - c > 1 \), let \( G(a) = g(\mu(a))a - c \) with \( G(0) = -c \) and \( G(1) = g(0) - c \). Hence \( G(0) < 1 < G(1) \), so by the Intermediate Value Theorem, there exists \( \pi_{DRS} \in (0,1) \) such that \( g(0)\pi_{DRS} - c = 1 \). In addition, since \( \mu(a) = \int_a^1 f(z)dz \) is continuous and decreasing in \( a \), and \( g'(\mu) < 0 \), \( G(a) \) is strictly increasing in \( a \), the threshold \( \pi_{DRS} \) is unique and the result follows.

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7This is in line with research by Iranzo and Peri (2007) who show that incentives to migrate (from Eastern Europe to Western Europe) differ for workers of different skills. They obtain this result using differences in TFP rather than differences in economies of scale in the earnings function.
Proposition 4.4. Regardless of the subsidy type, a higher fraction $\mu_{DRS}$, will choose to go to school in state 1, $\mu_{DRS} = \mu(\overline{a}_1) > \mu_2 = \mu(\overline{a}_2)$. The fraction $\mu_{DRS}$, increases with the subsidy.

Proof. The result follows from Propositions 4.3 and 3.2.

As before, we consider 3 potential equilibria: no brain drain, $\lambda(\mu_1) \in [\frac{\mu_1}{2}, \mu_1]$, complete brain drain, $\lambda(\mu_1) = 0$, and incomplete brain drain, $\lambda(\mu_1) \in (0, \frac{\mu_1}{2})$. The following proposition shows that the first equilibrium is never possible: either complete or incomplete brain drain arises in equilibrium. There is always an incentive to leave state 1 given the taxes that college graduates have to pay. Given decreasing returns to education, however, there is never an equilibrium where more than half of college graduates will choose to stay.

Proposition 4.5. Given $\mu_1 = \mu(\overline{a}_1)$, the fraction who enroll in college in state 1, the fraction of agents who stay is $\lambda(\mu_1) \in [0, \frac{\mu_1}{2})$.

Proof. Suppose, by contradiction, less than half of college graduates leave state 1, i.e. $\lambda(\mu_1) \geq \frac{\mu_1}{2}$. Then $\lambda(\mu_1) > \mu_1 - \lambda(\mu_1)$, and since $g$ is decreasing, it follows that $g(\lambda(\mu_1)) - g(\mu_1 - \lambda(\mu_1)) < 0$. Then for any $a$, $g(\lambda(\mu_1))a - g(\mu_1 - \lambda(\mu_1))a < t_1$. Hence agent $a$ chooses to leave state 1. Contradiction. There is either complete or incomplete brain drain, that is $\lambda(\mu_1) \in [0, \frac{\mu_1}{2})$.

Hence, when there are decreasing returns to education, we find a positive relationship between university funding and brain drain. High public spending causes brain drain as the number of agents that choose to leave after graduation outweighs the number of agents that were attracted to enroll in college from the other state. This is because the skilled wage differential between the two states is not enough to compensate skilled workers for the positive taxation in state 1. Only the most highly skilled can afford to stay. This is shown in the following theorem.

Theorem 4.6. There is a level $\hat{a}$, such that for any $a > \hat{a}$, agents will choose to stay in state 1, and for any $a < \hat{a}$, they choose to leave. For $\hat{a}$, the agent is indifferent as $\frac{w_1^{\hat{a}}(\hat{a}) - t_1}{w_2^{\hat{a}}(\hat{a})} = 1$. The fraction of agents who leave post graduation is given by $1 - \lambda(\overline{a}_1, \hat{a}) = \int_{\hat{a}}^{\overline{a}_1} f(z)dz$. (In the case of a
uniform distribution, $1 - \lambda(\hat{a}, \overline{a}_1) = (\hat{a} - \overline{a}_1)$, and the fraction who stays by $\lambda(\hat{a}, \overline{a}_1) = (1 - \hat{a})$ with $\hat{a} \in (\frac{1}{2}, 1]$.

**Proof.** We know that more than half of college graduates leave state 1, that is $\lambda(\mu_1) < \frac{\mu_1}{2}$. Assume $\lambda(\mu_1) > 0$, otherwise the conclusion is trivial. Then $\lambda(\mu) < \mu - \lambda(\mu)$ and $g(\lambda(\mu)) - g(\mu - \lambda(\mu)) > 0$. Let $a^*$ represent an agent who stays, i.e. $(g(\lambda(\mu)) - g(\mu - \lambda(\mu)))a^* > t_1$. For any $a > a^*$, $g(\lambda(\mu))a - g(\mu - \lambda(\mu))a > g(\lambda(\mu))a^* - g(\mu - \lambda(\mu))a^* > t_1$, and $a$ chooses to stay in state 1.

Let $a^{**}$ represent an agent who leaves, i.e. $(g(\lambda(\mu)) - g(\mu - \lambda(\mu)))a^{**} < t_1$. For any $a < a^{**}$, $g(\lambda(\mu))a - g(\mu - \lambda(\mu))a < g(\lambda(\mu))a^{**} - g(\mu - \lambda(\mu))a^{**} < t_1$, $a$ chooses to leave state 1. Hence there is a $\hat{a} \in (0, 1]$ such that for any $a > \hat{a}$, agents will choose to stay in state 1 and for any $a < \hat{a}$, they choose to leave. From the continuity of $g$, it follows that for $\hat{a}$, the agent is indifferent if $\frac{w_H^I(\hat{a}) - t_1}{w_O^I(\hat{a})} = 1$. \hfill \Box

We conclude that decreasing returns to scale in higher education do not affect all college graduates in the same way: when education funding rises, only the higher ability graduates can afford to remain in state 1. Those of lower ability and therefore lower wages will be better off in the other state, where taxes are lower. The next section considers the existence and different degrees of economies of scale relative to the other state and derives results both theoretically and empirically.

## 5 Implications for education policies

States vary in returns to higher education. Some benefit more than others from positive spillover effects. There are also states that do not experience economies of scale at all. In this section we extend our analysis to account for this variation. Specifically, we examine interactions between three pairs of states: both states are DRS; one state is IRS and another is DRS; both states are IRS.
5.1 DRS vs. DRS

Even if a state does not benefit from economies of scale to higher education, we have shown that it can attract more students if it chooses to subsidize college education. However, even with the increased funding, we find that brain drain always occurs in equilibrium. As a result, in the presence of brain drain, a DRS state should not subsidize higher education as people attracted to college in the DRS state will leave after graduation. This is true regardless of whether or not the other state benefits from economies of scale.\footnote{Proposition 4.5 shows this result when both states are in the DRS stage. However, these results also hold if the other state is in the IRS stage.}

If a DRS state liberally funds its universities, the college graduates who remain after graduation face higher taxes. So, only the higher ability graduates remain: they will have to support the cost of subsidizing the education of current college students. In fact, remaining agents end up paying more than double the amount of the subsidy they have received. This is shown in Corollary 5.1.

Corollary 5.1. In case of DRS any agent that remains in state 1 pays $t_1 > 2s_1$ and $\lambda'(s_1) > 0$.

Proof. In the case of DRS $t_1 = \frac{s_1(1+r)\mu}{\lambda} > \frac{s_1(1+r)\mu}{\mu'/2} > 2s_1(1 + r)$. Since $\lambda'(\mu) > 0$ and from proposition 3.2 $\mu'(s_1) > 0$, the result follows.

Corollary 5.1 implies that a higher subsidy level induces a higher fraction of agents to stay in the DRS state after graduation. However, even though the most skilled workers stay, and the number of them is increasing in the subsidy amount, it is still not worthwhile for a DRS state to increase university funding: the costs are greater than the benefits.

5.2 IRS vs. DRS

5.2.1 College Enrollment

We turn to the case where one state benefits from economies of scale but the other state does not. The effects on college enrollment are given by the following theorem.
Theorem 5.2. For any, \( s_1 \in [0, c) \), \( \mu(\overline{\pi}_{IRS}) > \mu(\overline{\pi}_{DRS}) \).

Proof. Let \( s_1 = c - c_1 \) arbitrarily chosen with \( c_1 \leq c \). As before, wlog we restrict ourselves to the case where if no one goes to college, \( \mu_0 = 0 \), there is an incentive for the most gifted to enroll, i.e. \( g(0) - c_1 > 1 \). Let \( G_1(a) = g(0)a - c_H \). Then \( G_1(0) < 1 < G_1(1) \). It results there exists \( \overline{\pi}_1 \) such that \( G_1(\overline{\pi}_1) = 1 = g(\mu_0)\overline{\pi}_1 - c_H \) and \( \mu_1 = \int_{\overline{\pi}_1}^1 f(z)dz = 1 - \overline{\pi}_1 \). By Proposition 4.3 there exists \( \{\overline{\pi}_n\} \) a decreasing, bounded sequence on \( [0, \overline{\pi}_1] \) with \( \lim \overline{\pi}_n = \overline{\pi}_{IRS} \in (0, \overline{\pi}_1) \). Let \( G_2(\overline{\pi}_1) = g(\mu_1)a - c_H < g(\mu_0)\overline{\pi}_1 - c_H = 1. \) Then \( G_2(\overline{\pi}_1) < 1 < G_2(1) \). Then there exists \( \overline{a}_2 \in (\overline{\pi}_1, 1) \) such that \( G_2(\overline{a}_2) = 1 \) and \( \mu_2 = 1 - \overline{a}_2 > \mu_1 \). By induction, there exists \( \overline{a}_n \) such that \( G_n(\overline{a}_n) = 1 \) and \( \mu_n = 1 - \overline{a}_n \) with \( \{\overline{a}_n\} \) an increasing, bounded sequence on \([\overline{a}_1, 1] \). Then there exist \( \mu_{DRS} \) and \( \overline{\pi}_{DRS} \) such that \( \lim \mu_n = \mu_{DRS} \) and \( \lim \overline{a}_n = \overline{\pi}_{DRS} \in (\overline{\pi}_1, 1) \) with \( g(\mu_{DRS})\overline{\pi}_{DRS} - c_H = 1 \) and \( \mu_{DRS} = 1 - \overline{\pi}_{DRS} \). So \( \overline{\pi}_{IRS} < \overline{\pi}_{DRS} \) for any given subsidy \( s_1 \in (0, c) \). \( \square \)

The implication of this result is that an IRS state can attract a higher fraction of students if the other state is in the DRS stage and neither state subsidizes education. If the DRS state decides to subsidize education, however, the IRS state should consider subsidizing it as well to insure that it continues to attract a higher fraction of students.

In addition, this result also implies that whenever one state is in the DRS stage and the other one in the IRS stage, it is never optimal for the DRS state to fund public education.

5.2.2 Brain Drain

When one state benefits from IRS, but the other state finds itself in a DRS stage, this induces a change in the return to higher education across the two states. The functional form is similar, but in the IRS state \( g'_1(\mu) > 0 \) whereas in the DRS state \( g'_2(\mu) < 0 \). There are few sub-cases to study: 1) \( |g'_1(\mu)| \leq |g'_2(\mu)| \), that is the rate of decline of the return to education function in the DRS state is at least as great as the rate of increase of the return to education function in the IRS state; 2) \( |g'_1(\mu)| > |g'_2(\mu)| \), that is the IRS in higher education grow at higher rate in state the IRS state...
than DRS declines in the DRS state. Intuitively, college graduates have an incentive to leave an IRS state, given the earnings differential net of taxes. If economies of scale increase at a lower rate in the IRS state than the decline in economies of scale in the DRS state, being a college graduate in the DRS state makes individuals better off: they can benefit from their minority status. This induces migration from the IRS state to the DRS state. However, when economies of scale in the IRS state increase at a faster pace, college graduates are better off in the IRS state, making the wage net of taxes high enough to attract them. So under case 2, graduates will stay in the IRS state, and state-level brain drain can be stopped.

**Proposition 5.3.** Given \( \mu_1 = \mu(\pi_{IRS}) \), the fraction who enrolls in college in state 1, the fraction of agents who stay is given by \( \lambda(\mu_1) = 0 \) if \( |g_1'(\mu)| \leq |g_2'(\mu)| \) and \( \lambda(\mu_1) = 0 \) or \( \lambda(\mu_1) \in \left[ \frac{\mu_1}{2}, \mu_1 \right] \) otherwise.

**Proof.** Assume \( |g_1'(\mu)| \leq |g_2'(\mu)| \). Let \( f(\lambda) = g_1(\lambda(\mu_1)) - g_2(\mu_1 - \lambda(\mu_1)) \). It results that \( f(\lambda) \leq 0 \). Then for any \( a \), \( g_1(\lambda(\mu_1))a - g_2(\mu_1 - \lambda(\mu_1))a \leq 0 \). Since \( t_1 > 0 \), there is no agent who chooses to stay in state 1, i.e. \( \lambda(\mu_1) = 0 \). If the other way around, it results that \( f(\lambda) > 0 \). By Proposition 4.1, it follows that there is either complete or no brain drain, that is \( \lambda(\mu_1) = 0 \) or \( \lambda(\mu_1) \in \left[ \frac{\mu_1}{2}, \mu_1 \right] \). 

The results above imply that a DRS state never finds it optimal to invest in public education, whereas an IRS state might find it worthwhile to do so. Assuming the extreme case of complete brain does not occur in equilibrium, an IRS state should consider investing in higher education only if the other state finds itself in a DRS stage with the return to education function declining at a lower rate than the rate of increase of the return to education function in the IRS state. In other words, earnings are rising so quickly in the IRS state that, even net of taxes, graduates do not have the incentive to migrate. It is not optimal, however, for the IRS state to invest in higher education if the other state is experiencing DRS with the return to education function declining at a rate equal to or greater than the rate of increase of the return to education function in the IRS state. In other words, wages are not rising fast enough in the IRS state to make up for the
increased taxes.

In addition, if the IRS state decides to subsidize education, each educated individual that remains in the IRS state will pay fewer taxes than if the state were in a DRS stage, as corollary 5.5 shows.

Corollary 5.4. For any \( s_H \in (0, c) \), \( \lambda(\pi_{IRS}) > \lambda(\pi_{DRS}) \).

Proof. It follows from Theorem 5.2, Proposition 4.5 and Proposition 4.1.

Corollary 5.5. In the case of IRS any agent that remains in state 1 pays \( t_1 < 2s_1 \) and \( \lambda'(s_1) > 0 \).

Proof. In the case of IRS \( t_1 = \frac{s_1(1+r)\mu}{\lambda} < \frac{s_1(1+r)\mu}{\mu/2} < 2s_1(1+r) \). Since \( \lambda'(\mu) > 0 \) and from proposition 3.2 \( \mu'(s_1) > 0 \), the result follows.

5.3 IRS vs. IRS: College Enrollment and Brain Drain

In this section we consider the situation in which both states are in the IRS stage, but one state has a higher rate of IRS than another state.

If both states 1 and 2 are in the IRS stage, returns to higher education in state 1 are given by \( ag_1(\mu) \) and in state 2 by \( ag_2(\mu) \) with \( g'_1(\mu) > 0 \) and \( g'_2(\mu) > 0 \). We consider two cases: \( g'_1(\mu) > g'_2(\mu) \), state 1 is a high IRS state relative to the other state, and \( g'_1(\mu) < g'_2(\mu) \), state 1 is a low IRS relative to the other state. The following proposition shows that when a state benefits from higher IRS relative to another state, it is able to attract more people to college than the lower-IRS state is. When a state has lower IRS relative to the other state, we show that the lower-IRS state is only able to attract more people to college with a high enough subsidy.

Assumption 5.6. We assume \( g_1(\mu) - g_2(\mu) > (g'_1(\mu) - g'_2(\mu))a \).

Proposition 5.7. A higher fraction will choose to go to school in state 1, \( \mu_1 = \mu(\overline{a}_1) > \mu_2 = \mu(\overline{a}_2) \) if state 1 subsidizes higher education at a sufficiently high rate.
Proof. For the case $g_1'(\mu) > g_2'(\mu)$, state 1 is a high IRS state relative to the other state the result follows from modified proof of Proposition 3.2. For the case $g_1'(\mu) < g_2'(\mu)$, state 1 is a low IRS relative to the other state, by Proposition 3.2, 

$$g_2(\mu_2)\bar{\sigma}_2 - c_1 > g_2(\mu_2)\bar{\sigma}_2 - c = 1 = g_1(\mu_1)\bar{\sigma}_1 - c_1.$$ 

For a low enough $c_1$, this implies $\bar{\sigma}_1 < \bar{\sigma}_2$, and the result follows.

Moreover, for the higher-IRS state, the relationship between public funding and net outmigration is negative; there is no brain drain arising in equilibrium. If the higher-IRS state decides to invest in higher education, it will not induce brain drain. If the lower-IRS state decides to subsidize education, however, it would not be able to keep college graduates in state: we find a positive relationship between public funding and net outmigration for low-IRS states. After they finish their education, lower-IRS-state students will be attracted to the higher wages in the higher-IRS state.

**Proposition 5.8.** Given $\mu_1 = \mu(\bar{\sigma})$, the fraction who enrolls in college in state 1, the fraction of agents who stay is $\lambda(\mu_1) \in \left[\frac{\mu_1}{2}, \mu_1\right]$ or $\lambda(\mu_1) = 0$ in case state 1 is a high IRS relative to the other state and $\lambda(\mu_1) \in [0, \frac{\mu_1}{2})$ otherwise.

Proof. For the case state 1 benefits of higher IRS, suppose, by contradiction, that there is incomplete brain drain. Then $\lambda(\mu_1) < \mu_1 - \lambda(\mu_1)$ and it follows that $g_1(\lambda(\mu_1)) - g_2(\mu_1 - \lambda(\mu_1)) < 0$. Then for any $a$, $g_1(\lambda(\mu_1))a - g_2(\mu_1 - \lambda(\mu_1))a < 0$. Since $t_1 > 0$, there is no agent who chooses to stay in state 1, i.e. $\lambda(\mu_1) = 0$. Contradiction. There is either complete or no brain drain, that is $\lambda(\mu_1) = 0$ or $\lambda(\mu_1) \in \left[\frac{\mu_1}{2}, \mu_1\right]$.

For the case state 1 has lower IRS relative to the other state, suppose, by contradiction, less than half of college graduates leave state 1, i.e. $\lambda(\mu_1) \geq \frac{\mu_1}{2}$. Then $\lambda(\mu_1) > \mu_1 - \lambda(\mu_1)$, and it follows that $g_1(\lambda(\mu_1)) - g_2(\mu_1 - \lambda(\mu_1)) < 0$. Then for any $a$, $g_1(\lambda(\mu_1))a - g_2(\mu_1 - \lambda(\mu_1))a < t_1$. Hence agent $a$ chooses to leave state 1. Contradiction. There is either complete or incomplete brain drain, that is $\lambda(\mu_1) \in [0, \frac{\mu_1}{2})$.

We have shown that if both states enjoy economies of scale in higher education, the state that
subsidizes education can attract more students. In addition, we find that a negative relationship between public funding and net outmobility can arise in equilibrium; it is worthwhile for a high-IRS-level state to subsidize education.

6 Conclusion

In this paper we develop a theory that explains empirical findings for domestic brain drain within the U.S. By linking brain drain to education funding, we build a framework to develop the optimal policy prescription for states experiencing an outmigration of college graduates.

Our results show that if heterogeneity in funding exists, the state that invests more in higher education attracts a higher fraction of students. However, outmobility of college graduates depends on whether the state is experiencing increasing returns to scale or decreasing returns to scale to higher education. If the state does not benefit from increasing returns to scale to higher education, we find a positive relationship between public funding and out-mobility of college graduates: the relatively low wages due to decreasing returns to scale cannot keep most graduates in state. However, when a state enjoys economies of scale to education, we find a negative relationship between public spending and out-mobility can arise in equilibrium, depending on the wages net of taxes.

We find that outmigration can lead to decreases in university funding. Similarly, Justman and Thisse (1997) show that exogenous student mobility leads to underinvestment in higher education. They show that a federal solution is needed. Our results could be used to support such a solution.

Further research is needed. Future work will examine the effects of both fiscal and education policy and the effects of different subsidy schemes in a dynamic framework.

References


