The Economic and Demographic Transition, Mortality, and Comparative Development*†

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Abstract

We present a theory of the economic and demographic transition where adult longevity, child mortality, fertility and the education composition of the population are jointly determined. The model allows for an investigation of the determinants of underdevelopment traps as well as of the mechanism that leads to an endogenous exit out of the trap. We also study the different roles of exogenous reductions in mortality and of permanent differences in extrinsic mortality for comparative development. The theory delivers a series of novel predictions which are illustrated with a simple dynamic simulation of the model. These predictions are shown to be consistent with evidence using both time series data and cross-country panel data.

JEL-classification: E10, J10, J13, N30, O10, O40

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1 Introduction

Poor economic living conditions, high fertility, and high mortality characterize large parts of the world. In 1970, half of all countries were trapped in economic underdevelopment and had not undergone the demographic transition: life expectancy at birth was less than 55 years, average total fertility was around six children per woman and the share of population having completed at least secondary education was less than one out of five. By 2000, 40 percent of these countries were still trapped in conditions of underdevelopment, predominantly countries located in tropical areas prone to a large variety of infectious diseases. These observations raise several important questions that are still largely unsettled. What are the underlying forces behind economic and demographic underdevelopment traps? What is the role of life expectancy and the mortality environment? Why do many countries still remain trapped in poor living conditions today?

This paper addresses these questions by providing a micro-founded theory that allows for an investigation of the forces underlying the development trap by studying how the interactions between education decisions, fertility choices, and the mortality environment bring about the endogenous economic and demographic transition. We consider a simple overlapping generations framework in which heterogenous adults optimally decide about the number of children, about the amount of time invested in providing them with basic education, as well as about their own education. The choice of own education concerns the type of human capital they want to acquire, i.e. the extensive margin, and how much time to spend on its acquisition, i.e. the intensive margin. These individual decisions crucially depend on child mortality, adult longevity, as well as wages. Wages are determined in equilibrium and depend on the technological environment and the aggregate levels of human capital of the different types. The environment in terms of life expectancy and technology evolves endogenously over time and depends on the availability of human capital through intergenerational spill-overs.

The model predicts that fertility and education depend on adult longevity, child mortality and the level of technology. Greater longevity induces individuals to acquire more time intensive types of human capital, thereby spending more of their lifetime on education and less on working. The share of individuals acquiring skilled human capital is monotonically, but non-linearly, increasing in life expectancy, and also depends on the technological environment. Due to the equilibrium fertility differential between high-skilled and unskilled individuals, average fertility depends on the education composition of the population. Additionally, lower child mortality reduces gross fertility. The model dynamics exhibit a non-linear development path, characterized by a long phase of poor living conditions, eventually followed by a rapid endogenous economic and demographic transition to sustained growth, low mortality, as well as net reproduction rates below pre-transitional levels. The transition is the result of an ongoing bi-directional feedback process between mortality, fertility and technological change.

With regard to the determinants of development traps, the model predicts that the economic and demographic transitions take place only when a substantial fraction of the population optimally decides to acquire formal education and, accordingly, reduce fertility. Joint improvements in both the economic and the demographic environment are required in order to change the education composition of the population and trigger a transition. This also implies that changes in longevity, technology or schooling policies alone, even through external interventions, might not be sufficient to trigger the transition if they are not successful in inducing a change in the education composition. The results are illustrated by a simulation of the model dynamics, which is compared to time series data of the development path from several countries. The ultimate driving force of the rapid transition is the substantial change in the education composition, implying a monotone relationship between the share of skilled with mortality and fertility. Cross-country panel data display cross-sectional patterns which strongly support this prediction.

Concerning the role of mortality for development, the model delivers two sets of novel predictions. First, we investigate the consequences of exogenous improvements in life expectancy (due to, e.g., medical advances like the discovery of penicillin) on the prospective development of a country. The model predicts a non-linear effect of this variation whose size depends on the state of development at the time of the implementation of the improvement. The effect is smallest for countries where longevity is still very low or already very high. This novel prediction of a hump-shape relationship between mortality improvements and the change in education composition is supported by empirical evidence. Second, we explore the role of permanent differences in the extrinsic mortality environment which are related, for example, to the prevalence and diversity of infectious diseases. The model predicts that extrinsic mortality barely affects the cross-sectional correlations but crucially changes the timing of the transition. Countries with a harsher mortality environment experience a delayed transition from the development trap. Moreover, in these countries, exogenous improvements in life expectancy are less effective in triggering a take-off. Simulations of the model illustrate that even moderate heterogeneity in extrinsic mortality results in substantial differences in the persistence of development traps and the timing of the transition.

From a cross-sectional perspective, the model implies that differences in development across countries can be interpreted in terms of these countries being located at different points along their transition path. Since the transition is rapid, relatively few countries are observed during their transition, while most countries are observed either during the pre-transitional or during the post-transitional phase. Hence, the model predicts a bi-modal cross-country distribution of life expectancy, fertility, education composition, and income. This prediction is illustrated by kernel density estimates of the different variables obtained with simulations for different extrinsic mortality environments. These kernel density estimates are shown to closely resemble kernel density estimates obtained with cross-country panel data at different points in time.

This paper is related to several strands of the literature. The paper contributes to the studies of the endogenous phase transition from stagnation to modern growth. The seminal work by Galor and Weil (2000) studies the fertility transition within a representative agent framework modeling the trade-off between the quantity and quality of children, while abstracting from mortality.[1] Soares (2005) and Falcão and Soares (2007) investigate how exogenous reductions in mortality affect the value that parents attach to children and to their own labor force participation. Boucekkine, de la Croix, and Licandro (2003) and Strulik (2008) examine the role of exogenous reductions in mortality as a trigger for the economic transition.[2] Strulik (2004) and de la Croix and Licandro (2007), in turn, investigate endogenous investments in children's health and survival. Lagerlöf (2003) explores the role of population density and epidemics, and Cervellati and Sunde (2005) study the the interaction between adult longevity and the transition in the education composition. The focus on differential fertility is shared with the papers by Dahan and Tsiddon (1998), de la Croix and Doepke (2003, 2004) and Doepke (2004), which, however, do not consider the role of life expectancy. To the best of our knowledge, the theory presented in this paper is the first that studies an endogenous demographic transition, that is, a transition in fertility as well as mortality, together with an economic transition and the change in the education composition of the population. This way, the model allows to study the joint interactions between adult longevity, child mortality, gross and net fertility, education, and income, and their different roles for the transition out of the development trap.

Another main difference of this paper compared to the unified growth theories cited above is the focus on the cross-country implications of the model. The paper therefore also contributes to the literature on comparative development. Complementing models of development traps

[1] Other early contributions studying the role of fertility for long-term development along these lines include Kögel and Prskawetz (2001), Galor and Moav (2002), and Doepke (2004). See Galor (2005b) and Doepke (2007) for surveys of this literature.

[2] Boldrin and Jones (2002) consider exogenous changes in child mortality in a model of children caring for their parents to generate a transition in income growth and fertility.

that study the failure to adopt existing technologies or particular institutions, we focus on the interaction between the demographic environment and the education composition of the population as a major determinant for (under-)development.[3] The results suggest an important role of extrinsic mortality for the development path. The model also provides an economic rationale for why and how the bi-modality in the world distribution of life expectancy, and its change overtime, is linked to the bi-modalities in the distributions of fertility and education composition. This allows the model to rationalize earlier findings regarding twin peaks in fertility as well as in the world income distribution (see e.g. Azariadis and Stachurski, 2005, for an overview), and recent findings by Bloom and Canning (2007) concerning mortality traps.

Finally, the model adds to the recent debate about the empirical role of mortality and life expectancy for economic development. Recent studies have come to opposing conclusions concerning the causal effect of life expectancy on development. Several contributions have found evidence for adult life expectancy being the key determinant of human capital acquisition, and consequently for the observed income differences across countries, see Shastry and Weil (2003) and Soares (2005). Using cross-country panel data with extrinsic mortality, based on geographical environment and the prevalence of diseases, as an instrument for life expectancy, Lorentzen, McMillan, and Wacziarg (2008) find a positive causal effect of life expectancy on growth.[4] In contrast, Acemoglu and Johnson (2007) find no causal effect of life expectancy on long-run growth when using data from global health interventions and the introduction of new drugs like penicillin as an instrument for life expectancy. In contrast to previous theories, the model presented here can help discriminate between the role of permanent extrinsic differences in mortality, and the role of shocks to longevity along the development path, i.e. between the two types of empirical variation that have been used to identify the effect of life expectancy on development. The predictions of the model concerning the different roles of longevity for development indeed imply different results for empirical investigations based on these different identification strategies, thereby helping to reconcile the seemingly contradictory empirical findings.

By explicitly accounting for the interactions between life expectancy, fertility, and the education composition, the model can therefore generate and reconcile several stylized empirical patterns that have been difficult to rationalize thus far. The novel predictions for the dynamic

[3] See for example the recent debate on the role of institutions versus geography for development and development failures, including contributions by Glaeser et al. (2004), Rodrik et al. (2004), Acemoglu and Johnson (2007) and Sachs (2003).

[4] In related work, Weil (2007) uses microeconomic estimates to construct estimates of the aggregate effect of health and life expectancy on growth, and finds a significant effect.

and cross-sectional implications of the theory are shown to be consistent with evidence using both time series data and cross-country panel data.

The remainder of the paper is organized as follows. The theoretical framework is presented in Section 2, and the intra-generational equilibrium is derived and studied in Section 3. Section 4 studies the dynamics, and briefly discusses the robustness of our theoretical framework as well as the role of the different assumptions. Section 5 presents the main implications of the model for the development path and the economic and demographic transition, and discusses them in light of historical time series and contemporaneous cross-country panel data.

2 The Model

2.1 Population Structure and Timing

Time is continuous, denoted by $\tau \in \mathbb{R}^+$. The economy is populated by an infinite sequence of overlapping generations of individuals, which are denoted with subscript t, where $t \in \mathbb{N}^+$. A generation of individuals t, born at τ_t enjoys a childhood of length $k_t = k$ after which individuals turn adult. Reproduction is asexual and takes place once individuals become adults. Consequently, every generation is born $k_t = k$ periods after the birth of the respective previous generation.[5] Not all children of generation t survive childhood because of child mortality. The fraction of children surviving to adulthood is denoted by $\pi_t \in (0, 1)$. The timing of the model is illustrated in Figure 1.

Each generation t is formed by a continuum of individuals. At birth, every individual is endowed with ability $a \in [0, 1]$. The distribution of ability within a given generation of new-born individuals is denoted by $d(a)$ and for simplicity assumed to be uniform over the unit interval. Since child mortality affects every child the same way, the ability distribution of adults is also uniform.[6] During childhood individuals receive parental education (as will be discussed below).

[5] Instead of assuming a fixed frequency of births, the length of the time spell between the births of two successive generations, hence the timing of fertility, could be modelled as a function of the life expectancy of the previous generation. This would modify the results concerning population size, but would leave the main results concerning the economic and demographic transition unchanged. See Blackburn and Cipriani (2002) for a paper on long-term development that deals with changes in the timing of fertility, and Falcão and Soares (2007) for a model of how reductions in child mortality and adult longevity affect fertility and labor force participation differently.

[6] The assumption of a uniform distribution is for simplicity since the central results can be generated with any distribution $d(a)$ of ability a among the surviving adults, including the degenerate distribution in which all individuals are equally able. The *ex ante* distribution of innate ability or intelligence does not change over the course of generations. See also Galor and Moav (2002) for a model in which the ability distribution changes over

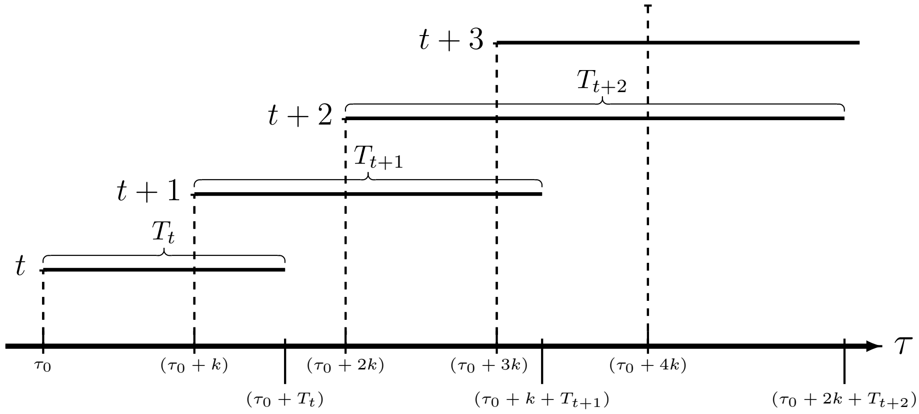

Figure 1: Timing of Events

Once adults (i.e. at age k) individuals make decisions concerning their own education, fertility and the time invested in raising their offspring. We abstract from issues of non-divisibility and threat the number of children as continuous choice variable, $n \in \mathbb{R}_0^+$. In order to highlight the mechanism we also restrict attention to a deterministic framework without sequential child birth.[7] A generation of adults consists of a continuum of agents with population size N_t, which is determined by size and fertility of the previous generation, as well as the survival probability of children. Adults of generation t face a (deterministic) remaining life expectancy T_t.

2.2 Preferences and Production Function

Adults care about their own lifetime consumption as well as about the potential lifetime income of their surviving offspring. This reflects the trade-off between the resources devoted to own consumption and to raising children. We denote by c_t^i the total lifetime consumption of an agent i of generation t, and by n_t^i the total number of this agent's offspring. Individual preferences are represented by a utility function which is strictly monotonically increasing, concave and satisfies the standard boundary conditions that insure interior solutions. In particular, lifetime utility is given by

$$U(c_t^i, y_{t+1}^i \pi_t n_t^i) = \left(c_t^i\right)^{(1-\gamma)} \left(y_{t+1}^i \pi_t n_t^i\right)^{\gamma} \text{ with } \gamma \in (0,1) \ , \tag{1}$$

the pass of generations.

[7]As investigated by Doepke (2005), accounting for the fact that in reality the number of children is discrete can affect the optimal choice if the parents have a precautionary demand for children. In the current framework the assumption of n being a continuous variable is only made for simplicity. Uncertainty giving rise to precautionary motives in fertility behavior is realistic, but strictly complementary to our analysis of fertility. Sequential fertility decisions would complicate the analysis without adding any additional insights.

where y_{t+1}^i is the (potential) lifetime income of a surviving offspring of individual i.[8] We abstract from discounting and life cycle considerations like the choice of the optimal consumption and savings path over the life cycle of an individual.[9]

Individual income y_t^i results from supplying human capital on a competitive labor market as studied below. A unique consumption good is produced with an aggregate production technology that uses all human capital available in the economy at any moment in time, i.e. embodied in all generations alive at that date, as the only factors of production. We consider two types of human capital. The first type is interpreted as high-quality human capital characterized by a higher content of abstract knowledge. We refer to this as *skilled* human capital and denote it by s. The second type is labelled *unskilled* human capital, denoted by u, and contains less intellectual quality, but more manual and practical skills that are important in performing tasks related to existing technologies. Apart from their different role in the production process, the main difference between the two types of human capital concerns the intensity with which they require time and ability in the education process.

The unique consumption good is produced with an aggregate production function with constant returns to scale. We adopt a simple formulation in which generation specific vintage technologies are identified by the total factor productivity A_t. A given generation t can only operate the respective technological vintage t.[10] In particular, generation t produces Y_t units of the consumption good using its stock of human capital, H_t^u and H_t^s, using the CES production function

$$Y_t = A_t \left[(1 - x_t) \left(H_t^u \right)^\eta + x_t \left(H_t^s \right)^\eta \right]^{\frac{1}{\eta}} \tag{2}$$

with $\eta \in (0, 1)$ and the relative production share $x_t \in (0, 1)$ $\forall t$.[11] Wage rates for each type

[8] This representation of preferences follows the standard way of modeling the trade-off between quantity and quality of offspring, see Becker and Barro (1988) and Galor and Weil (2000). The second component generates a link between generations that can be interpreted as a warm glow type of altruistic preferences.

[9] This formulation implies that individuals can perfectly smooth consumption as well as the utility from children over their life. At the same time, however, individuals cannot perfectly substitute utility from their own consumption with utility derived from their offspring.

[10] Human capital is inherently heterogenous across generations, because individuals acquire it in an environment characterized by the availability of different vintages of technologies. Human capital acquired by agents of a generation allows them to use technologies of the latest available vintage. This implies that a generation's stock of human capital of either type is not a perfect substitute of that acquired by older or younger generations, and is sold at its own price.

[11] Equivalently one could consider a production function with two sectors differing in their skill intensity. Inada conditions prevent corner solutions in human capital acquisition, but none of the results depends on this assumption.

of human capital are determined on competitive labor markets, and wages equal the respective marginal productivity,

$$w_t^s = \frac{\partial Y_t}{\partial H_t^s} \quad \text{and} \quad w_t^u = \frac{\partial Y_t}{\partial H_t^u} . \tag{3}$$

2.3 Human Capital

We model human capital production as the outcome of an education process that involves both the decisions of the individual as well as those of his parent. In particular, investments in education by the individual and by his parent are complementary inputs in the production of human capital.[12] Human capital acquisition involves a time intensive education process. We denote by e^{ij} the time devoted by an individual i to his own education for obtaining either type of human capital, unskilled or skilled, $j = u, s$. Let $r_{t-1}^i \in [0, 1]$ denote the fraction of lifetime of a parent i of generation $t-1$ spent in raising each of his children that survive childhood. The effect of parental preparatory education is reflected in the higher productivity of every unit of time spent by children in own education e^{ij} given a higher time investment of the parents which is given by

$$f\left(r_{t-1}^i, \cdot\right) , \tag{4}$$

with $f(0, \cdot) > 0$, $f_r(r, \cdot) > 0$, and $f_{rr}(r, \cdot) < 0$, where the function f may depend on other environmental parameters.[13] The larger the time spent raising children r_t the larger the impact on resulting human capital. The education process inherently differs between different types of human capital with respect to the time intensity of the education process and the effectiveness of ability. In particular, the larger the content of abstract knowledge incorporated in human capital the larger is the time required to master the building blocks and basic concepts of this type of human capital. This is captured by a fix cost $\underline{e}^j$ measured in time units, which an agent needs to pay when acquiring h^j units of human capital type $j = \{u, s\}$ with $\underline{e}^s > \underline{e}^u \geq 0$. Finally, the effectiveness of the education process depends on individual ability, which magnifies time investments in human capital j by a factor $m^j\left(a^i\right)$ with $\partial m^j\left(a^i\right)/\partial a^i \geq 0$. We assume that ability is relatively more important (and effective) when acquiring advanced skills. For analytical convenience we assume $m^s(a) = a$ while $m^u(a) = 1$.

These characteristics are formalized in the human capital production function

$$h^j(a) = \alpha^j \left(e_t^{ij} - \underline{e}^j\right) f\left(r_{t-1}^i, \cdot\right) m^j(a) \qquad \forall \;\; e \geq \underline{e}^j, \; j = u, s \tag{5}$$

[12]Modeling education as resulting from family and schooling inputs is in line with the canonical model of early child development, see, e.g., Todd and Wolpin (2003).

[13]Galor and Weil (2000) assume that technological progress g_t reduces the effectiveness of parental education so that $f(r, g)$. In this case g_t influences the optimal choice of basic education made by the parent.

and $h^j(a) = 0 \ \forall e < \underline{e}^j$ with $\alpha^j > 0$. In order to isolate the development effects related to the various dimensions of mortality and human capital formation, any links between generations through savings or bequests are excluded.[14]

This formulation of the education process implies that an individual i that has received an education r_{t-1}^i from his parent and acquires human capital of type j by investing an amount of e_t^{ij} in education can earn a total lifetime income of

$$y_t^{ij}(a) = y_t^j\left(a^i, r_{t-1}^i, e_t^{ij}\right) = w_t^j h^j\left(a, r_{t-1}^i, e_t^{ij}\right)\left(T_t - e_t^{ij}\right) . \tag{6}$$

For future reference we denote average human capital by $h_t^j := \left(H_t^j/N_t\right)$ for $j = \{u, s\}$ and the per capita income by $y_t := (Y_t/N_t)$.

3 Human Capital and Fertility

3.1 Individual Education and Fertility Decisions

Each generation t of individuals takes adult longevity T_t, the survival probability of children π_t and the level of technological advancement, as expressed by A_t and x_t, as given. In this Section we characterize the equilibrium formation of human capital for any given vector $\{T_t, \pi_t, A_t, x_t\}$.

The individual choice problem. Investment in own human capital, e_t^{ij}, as well as in rasing children, r_t^i, implies costs in terms of time that is not available for market work. The birth of each child, regardless of whether the child survives until adulthood or not, entails a cost in terms of a share b of lifetime, so that the total cost giving birth is given by bn_tT_t. Additionally, raising the πn surviving children involves costs in terms of foregone working time equal to $r_tT_t(\pi_t n_t)$. The only role of the consideration of a fix cost per birth this to ensure interior solution, as it implies an upper bound on the number of children born per individual. To simplify notation, and without loss of generality of any of the qualitative results presented below, we assume interior solutions and set $b = 0$ in the following.

Formally, the problem of an individual i with ability a born in generation t can be characterized as follows. The individual has to choose the type of human capital $j \in \{u, s\}$ he wants to acquire as well as the optimal education e_t^{ij}. The individual also chooses the number of offspring n_t^i and the time spent with each of them r_t^i. Since individuals have to choose their own type

[14] We also abstract from real resources as input for the human capital formation process, as well as issues related to capital market development and public provision of education. We return to this issue below.

of human capital as well as the optimal time in formal education, the problem implies a trade-off between acquisition of *own* human capital and fertility, in terms of number and education of offsprings. As consequence, own education, as well as quantity and quality of the children influence one another and must be treated as joint decisions.

Optimal choices are made under the lifetime budget constraint of an individual of generation t acquiring human capital j, taking life expectancy T_t, child survival probability π_t and the wage rates w_t^j and w_{t+1}^j as given. The vector of decisions that reflects the solution of the individual maximization problem is given by

$$\begin{aligned} \{j^*, e_t^{ij*}, n_t^{ij*}, r_t^{i*}\} &= \underset{\left\{n_t>0, r_t\in[0,1], e_t^{ij}\leq T_t, j=u,s\right\}}{\arg\max} U_t\left(c_t^i, \pi_t n_t^{ij} y_{t+1}^j\left(a, r_t^i, e_{t+1}^{ij}\right)\right) \\ \text{subject to:} &\quad c_t^i \leq (T_t\left(1 - r_t^i \pi_t n_t^{ij}\right) - e_t^j) w_t^j h_t^j\left(a^i, r_{t-1}^i, e_t^{ij}\right), \text{(5) and (6) for } j = u, s. \end{aligned} \tag{7}$$

In order to derive the optimal choices of an individual we proceed as follows. We first characterize the optimal education, fertility and child raising choices that maximize individual utility conditional on choosing to acquire a particular type of human capital $j = u, s$. We then identify the optimal education decision in terms of the type of human capital by comparing the indirect utility that each agent derives from acquiring the different types of human capital. The optimal individual choice h_t^{j*} is given by that type of human capital that offers the highest utility given the optimal choices of education time and fertility.

Education, fertility and child raising for j-type human capital. The optimization problem is strictly globally concave so that first order conditions uniquely identify the optimal choices made by any individual, conditional on the acquisition of a particular type of human capital. The optimal choices of education time and number of children for an individual of ability a acquiring human capital type j are given by the solution to the following optimization problem,

$$\{e_t^{ij*}, n_t^*, r_t^{i*}\} = \arg\max\left[\left(T_t\left(1 - r_t^i \pi_t n_t^{ij}\right) - e_t^{ij}\right) w_t^j h_t^j\left(a, r_{t-1}, e_t^{ij}\right)\right]^{(1-\gamma)}\left[y_{t+1}^i \pi_t n_t^{ij}\right]^{\gamma}. \tag{8}$$

The solution delivers the optimal education time and optimal fertility of agents acquiring human capital of type j. Substituting (5) into (8) and differentiating one gets the first order conditions for an interior optimum

$$e_t^{ij} = \frac{T_t\left(1 - r_t^i \pi_t n_t^{ij}\right) + \underline{e}^j}{2} \tag{9}$$

and

$$n_t^{ij} = \frac{\gamma\left(T_t - e_t^{ij}\right)}{T_t r_t \pi_t}. \tag{10}$$

These conditions imply that, ceteris paribus, having more children decreases the time invested in own education and vice versa. A higher fix cost $\underline{e}^j$ involved with the acquisition of skilled human capital requires a larger time investment in education, however. Furthermore the optimal number of children is decreasing with the time invested in each of them.

Concerning the optimal time spent on raising children, r_t^{i*}, the first order condition for the interior solution is given by,

$$(1-\gamma)\,T_t\pi_t n_t^{ij} - \gamma\left[T_t\left(1 - r_t^i\pi_t n_t^{ij}\right) - e_t^{ij}\right]\frac{\partial f\left(r_t^i,\cdot\right)}{\partial r_t^i}\frac{1}{f\left(r_t^i,\cdot\right)} = 0\,.$$

Making use of (10), this can be equivalently expressed as

$$\varepsilon_{f,r} \equiv \frac{\partial f\left(r_t^i,\cdot\right)}{\partial r_t^i}\frac{r_t^i}{f\left(r_t^i,\cdot\right)} = 1 \tag{11}$$

which implicitly defines the optimal decision concerning r.[15]

The individually optimal choices of e and n conditional on the acquisition of each type of human capital j are obtained solving the system of equations (9) and (10) given the optimal $r_t^{i*} = r_t^*$ implied by (11).

Proposition 1. *For any $\left\{w_t^j, T_t, \pi_t\right\}$, the vector of optimal education, fertility and time devoted to children of an individual deciding to acquire human capital of type $j = \{u, s\}$, $\left\{e_t^{j*}, n_t^{j*}, r_t^*\right\}$ is given by,*

$$n_t^{ij*} = n_t^{j*} = \frac{\gamma}{2-\gamma}\frac{T_t - \underline{e}^j}{T_t r_t^* \pi_t} \quad \textit{and} \tag{12}$$

$$e_t^{ij*} = e_t^{j*} = \frac{T_t\,(1-\gamma) + \underline{e}^j}{(2-\gamma)} \tag{13}$$

where r_t^* solves (11)*, for all i.*

From (12) this also implies a negative relationship between quantity and quality of children.

Individual Choice of Human Capital. In order to fully characterize optimal choices we now turn to the individual problem of choosing the type of education, s or u. This choice depends, among other things, on the level of wages which are determined in general equilibrium on the labor markets and which individuals take as given.

[15]Note that the formulation (5) implies that the time parents spend on the education of a child, r_t, improves the ability of the child in acquiring any type of human capital without creating a bias. In equilibrium it will be optimal to spend the same r_t on each offspring as shown below. This feature of the model also implies that the optimal choice of the type of education chosen by the children is unaffected by the time that their parents spent raising them. This neutrality of parental education depends on the assumed generalized Cobb-Douglas formulation of the production of human capital which represents a natural benchmark and greatly simplifies analytical tractability.

Using e_t^{s*} and e_t^{u*} from condition (13) and substituting into (5), one obtains the respective levels of human capital,

$$h_t^{j*}(a) = \alpha^j f(r_{t-1}, \cdot) \frac{(1-\gamma)\left(T_t - \underline{e}^j\right)}{(2-\gamma)} m^j(a) \qquad \text{for } j = u, s. \tag{14}$$

Conditions (12) and (13) imply that for any individual of ability a, there is a unique e_t^{j*} and level of fertility n_t^{i*} which maximize his lifetime utility conditional on acquiring a given type of human capital. The amount of h_t^s monotonically increases in a, however. This implies that individuals with higher ability have a comparative advantage in acquiring h^s. Consequently, the indirect utility enjoyed by acquiring s-type human capital, $U_t^{s*}(a)$ is strictly monotonically increasing in ability a, while U_t^{u*} does not depend on a. Hence, for any vector of wages, there exists a unique ability threshold $\widetilde{a}_t$ for which the indirect utilities of acquiring either types of human capital are equal. Denoting by $\alpha \equiv \alpha^u/\alpha^s$ the relative productivity of a unit of education time in the acquisition of the two types of human we have

Lemma 1. *For any $\{w_t^s, w_t^u, T_t, \pi_t\}$ there exists a unique $\widetilde{a}_t \in (0,1)$ given by,*

$$\widetilde{a}_t = \alpha \left(\frac{T_t - \underline{e}^u}{T_t - \underline{e}^s}\right)^{\frac{2-\gamma}{1-\gamma}} \frac{w_t^u}{w_t^s} \tag{15}$$

such that all agents with $a \leq \widetilde{a}_t$ optimally choose to acquire unskilled human capital u while all agents with $a > \widetilde{a}$ acquire skilled human capital s according to (14).

Proof. See Appendix. □

For any given distribution of abilities $d(a)$, the threshold $\widetilde{a}_t$ determines the fractions of the population of adults of a given generation that acquire skilled human capital. For any generation t, this fraction is unique and denoted by λ_t with

$$\lambda_t = \lambda(\widetilde{a}_t) := \int_{\widetilde{a}_t}^{1} d(a)\, da = (1 - \widetilde{a}_t) \quad \text{and} \quad (1-\lambda_t) = (1 - \lambda(\widetilde{a}_t)) := \int_0^{\widetilde{a}_t} d(a)\, da = \widetilde{a}_t\,, \tag{16}$$

where λ_t is a monotonic function of the threshold $\widetilde{a}_t$, and where the last equality for both shares follows from assuming a uniform distribution for $d(a)$. From (15) and (16), λ_t is increasing with the relative wage w_t^s/w_t^u; is decreasing with the relative fix cost necessary to master the basic knowledge $\underline{e}^s/\underline{e}^u$; and is increasing with adult life expectancy T_t, which facilitates the acquisition of high quality human capital even for individuals with lower ability.

3.2 The Effects of Mortality on Education Composition and Fertility

In the previous Section we have characterized fertility behavior and human capital acquisition as resulting from the decisions of both parent and child. Before studying the general equilibrium of the model and its dynamics, we briefly discuss the role of demographic and technological variables.

Conditional on the type of human capital acquired, condition (11) characterizes the trade-off between quantity of children and the time devoted to raise them, i.e. their quality. The properties of the level of r_t^* that optimally solves the quantity-quality trade-off are well studied in the literature. If the effectiveness of parental time spent in raising their children depends on technological progress, as in Galor and Weil (2000), then the optimal basic education supplied by parents changes with economic conditions. In that case r_t^{i*} is implicitly defined by (11) and depends on g_t with the negative effect of the rate of technological progress g_t reflecting an obsolescence effect.[16] Faster technical change implies a lower effectiveness of education, although a larger r_t tends to reduce this negative effect of a rapidly changing technological environment. Given these assumptions about $f(r_t, g_t)$ and by implicit differentiation of (11) this would imply

$$\frac{\partial r_t^*}{\partial g_t} > 0, \tag{17}$$

so that the key determinant of parental basic education of children is the rate of technological change g_t. Notice that this trade-off is unaffected by adult longevity, child survival probability, individual investment in education and the type of human capital acquired. As pointed out by Moav (2005) this implies that the effect of mortality on fertility does not work through a change in the quantity-quality trade-off.[17] For the moment, we keep assuming that $f(r_t, \cdot)$ does not depend on g_t which implies that the optimal investment in quality does not change overtime so that $r_t^* = r^*$ for every t. This allows to simplify illustration of the results by concentrating on the role of mortality and investments in own education and fertility.[18]

Optimal fertility and human capital decisions are primarily affected by the individuals' decision about how much time to spend on their *own* education. Conditional on the type of human capital acquired, a longer lifetime horizon T_t induces agents to spend more time in the acquisition of human capital of either type as can be seen from condition (13). From (12), and conditional on acquiring a certain type of human capital, longer life expectancy also leads to an increase in gross fertility.[19] This reflects an *income effect* of a longer life that increases both human capital

[16]Galor and Weil (2000) assume $f_r(.) > 0$, $f_g(\cdot) < 0$, $f_{rr}(.) < 0$, $f_{gg}(.) > 0$ and $f_{rg}(.) > 0$ for any $(r_{t-1}, g_t) \geq 0$. The assumption $f_{rg} > 0$ represents a sufficient but not necessary condition for $\partial r^*/\partial g > 0$.

[17]This implies that mortality plays a limited role for the phase transition in models based on the quantity-quality trade-off, see Doepke (2004), Hazan and Zoabi (2006). Recent work by Jones and Schoonbroodt (2008) studies the conditions under which the Barro-Becker model can reproduce the observed fertility changes in response to decreased mortality.

[18]We will return to the discussion of the role of a changing r_t^* below.

[19]These predictions are in line with empirical evidence. Bleakley and Lange (2006) have provided evidence for

acquisition and fertility by relaxing the lifetime constraint. From (12), child mortality does not affect own education choices but it affects (gross) fertility which is strictly monotonically decreasing in π for all individuals and for all types of human capital. This *substitution effect* due to changes in the relative price of consumption and children implies that lower child mortality is a key determinant of gross, but not net, fertility.[20]

While the income and substitution effects are at work independently of the type of human capital individuals decide to acquire, the model characterizes variations in the optimal choice of the *type* of human capital. The equilibrium share of the population deciding to be educated is a function of both economic and demographic conditions. Conditions (12) and (13) imply that the acquisition of skilled rather than unskilled human capital induces individuals to spend more time on skilled human capital, and to have a lower number of children, $e_t^{s*} > e_t^{u*}$ and $n_t^{s*} < n_t^{u*}$.[21] This *differential fertility* associated with the acquisition of formal education plays a key role in the model. Differential fertility emerges in the model since the acquisition of skilled human capital leads agents to substitute utility from the offspring with utility from own consumption. To illustrate this point consider the individual with ability $\widetilde{a}_t$ who, being indifferent, receives the same total lifetime utility acquiring either skilled or unskilled human capital. From (12), however, those acquiring unskilled human capital h^u enjoy relatively larger utility from surviving offspring, and relatively lower utility from own consumption. Conversely, acquiring h^s entails larger lifetime consumption and lower optimal fertility.[22]

Condition (15) implies that the threshold level of ability making an individual indifferent between both types of education, $\widetilde{a}_t$, is decreasing in adult longevity T. From (16) this induces a larger share of the population to optimally acquire formal education λ_t. The average gross

the causal effect of exogenous variation in disease environment and longevity on both education and fertility. See also the earlier work of Bleakley (2003, 2006). Behrman and Rosenzweig (2004) report evidence for a causal effect of health conditions during childhood, measured by birth weight, on schooling attainment. Weil (2007) contains for further references.

[20]Kalemli-Ozcan (2003), among others, studied this substitution effect as determinant of the drop in gross fertility. There it has also been shown that the existence of uncertainty and a precautionary demand motive for children would tend to reinforce this effect.

[21]This is in line with survey evidence from Brazil presented by Soares (2006) that higher adult longevity is associated with higher schooling and lower fertility.

[22]Also note that for any T the acquisition of skilled human capital implies a discretely larger time investment in education than the acquisition of unskilled human capital, and consequently a discretely lower lifetime devoted to work. This is consistent with evidence on the secular decline in lifetime working hours provided by Hazan (2006).

fertility rate is given by,

$$\begin{aligned} n_t^* &= (1-\lambda_t)n_t^{u*} + \lambda_t n_t^{s*} \\ &= \frac{\gamma}{2-\gamma}\left[(1-\lambda_t(T_t))\frac{T_t - \underline{e}^u}{T_t r^* \pi_t} + \lambda_t(T_t)\frac{T_t - \underline{e}^s}{T_t r^* \pi_t}\right] \\ &= \frac{\gamma}{2-\gamma}\frac{T_t - ((1-\lambda_t(T_t))\underline{e}^u + \lambda_t(T_t)\underline{e}^s)}{T_t r^* \pi_t}\,. \end{aligned} \tag{18}$$

By inducing a change in the skill composition of the population, and the associated differential fertility, life expectancy is a key determinant of average fertility.

The net fertility rate is given by

$$n_t^* \pi_t = \frac{\gamma}{2-\gamma}\frac{T_t - ((1-\lambda_t(T_t))\underline{e}^u + \lambda_t(T_t)\underline{e}^s)}{T_t r^*}, \tag{19}$$

which is negatively correlated with r^* and independent of π_t. Hence a drop in child mortality cannot deliver a drop in net fertility only through a quantity quality trade-off in the context of homogenous human capital, as was previously shown by Doepke (2005).

The effect of adult longevity on gross and net fertility is, in general, ambiguous. The income effect tends to raise gross and net fertility. Improvements in T, however, also imply a re-optimization on the extensive margin inducing more people to acquire formal education. This shift towards the acquisition of h^s is coupled with differential fertility and tends to reduce n_t^*. By investigation of (18) we have,

Proposition 2. *For any* $\{A_t, T_t, \pi_t, g_t\}$ *the average fertility rate is given by* (18) *with*

$$\frac{\partial n_t^*}{\partial \pi_t} < 0, \quad \frac{\partial n_t^*}{\partial T_t} \gtreqless 0 \text{ and } \frac{\partial n_t^*}{\partial r^*} < 0\,. \tag{20}$$

The previous Proposition states that an economy with a lower child mortality is characterized by lower gross fertility due to the *substitution* effect. Adult longevity has in principle an ambiguous effect on fertility due to the interaction between the *income* and the *differential fertility* effects. As a result, health in the form of adult longevity and child mortality affects gross and net fertility in the population both directly and indirectly. The direct effect induces a change in the intensive margin, i.e. in the optimal choice of education time and individual fertility conditional on the type of human capital acquired. The indirect effect concerns education choices at the extensive margin inducing individuals to acquire different types of human capital. The overall effect on the average (population wide) fertility rate depends on the relative strength of the different effects at work. Finally, an increase in the quality of the children is associated with a reduction in their number.

3.3 Life Expectancy and Education Composition in Equilibrium

The previous Section characterized optimal individual choices in partial equilibrium, that is conditional on market wages. We now study the equilibrium investment in human capital that is compatible with endogenously determined wages. The aggregate levels of the two types of human capital supplied by generation t are denoted by

$$H_t^u\left(\widetilde{a}_t\right) = N_t \int_0^{\widetilde{a}_t} h_t^u(a) d(a) da \qquad \text{and} \qquad H_t^s\left(\widetilde{a}_t\right) = N_t \int_{\widetilde{a}_t}^1 h_t^s(a) d(a) da \tag{21}$$

From (3), the ratio of wage rates which is determined in competitive markets is

$$\frac{w_t^u}{w_t^s} = \frac{1-x_t}{x_t}\left(\frac{H_t^s\left(\widetilde{a}_t\right)}{H_t^u\left(\widetilde{a}_t\right)}\right)^{1-\eta} . \tag{22}$$

The equilibrium is characterized by the unique threshold ability that splits the population into individuals acquiring skilled and unskilled human capital together with a unique vector of market wages. From (15), $\widetilde{a}_t$ is a mononically increasing function of w_t^u/w_t^s. Condition (22) in turn implies that the wage ratio depends on the ratio between the aggregate levels of human capital available in the economy. Also from (21), $H_t^s\left(\widetilde{a}_t\right)/H_t^u\left(\widetilde{a}_t\right)$ is a monotonically decreasing function in $\widetilde{a}_t$. Substituting (21) and the wage ratio (22) into the expression for the ability threshold (15), one obtains a unique equilibrium threshold $\widetilde{a}_t^*$ as a function of T_t. This function can be implicitly characterized by

$$\left(\frac{\left(1-\widetilde{a}_t^{*2}\right)^{1-\eta}}{\widetilde{a}_t^{*2-\eta}}\right)^{1-\gamma}\left[\left(\frac{1-x_t}{x_t}\right)^{1-\gamma}\left(\frac{1}{2}\right)^{(1-\eta)(1-\gamma)}\alpha^{\eta(1-\gamma)}\right]\left(\frac{T-\underline{e}^u}{T-\underline{e}^s}\right)^{1+\eta(1-\gamma)} = 1\,, \tag{23}$$

as is shown in the Appendix. From (16) it is possible to implicitly identify a unique $\lambda_t^* = (1-\widetilde{a}_t^*)$. For any vector of macroeconomic conditions $\{\pi_t, x_t, A_t\}$ and for any $T_t > \underline{e}^s$, there exists a unique equilibrium:

Proposition 3. *For any given generation t with $\{T_t \in [\underline{e}^s, \infty), \pi_t \in (0,1], A_t, x_t\}$, there exists a unique*

$$\lambda_t := 1 - \widetilde{a}_t^*$$

for which (15) and (22) hold. Accordingly there is a unique vector, $\left\{H_t^{j*}, w_t^{j*}, r_t^*, e_t^{j*}, n_t^{j*}, h^{j*}(a)\right\}$ for $j = u, s$ such that conditions (12), (13), (14), (15) *and* (22) *are jointly satisfied. The equilibrium share of skilled individuals λ_t^* is an increasing and S-Shaped function of T_t, with zero value at $T = \underline{e}^s$ and with zero slope for $T \longrightarrow \underline{e}^s$ and $T \longrightarrow \infty$.*

Proof. See Appendix. □

The previous proposition states that at each moment in time there exists a unique equilibrium in which optimal individual choices of education investments and fertility, the implied optimal individual levels of human capital, the corresponding population structure defined by the threshold λ_t and the resulting aggregate levels of human capital and wages are mutually consistent. The key state variables affecting λ_t are the relative productivity of the different skills, x_t, and adult longevity T_t. The share λ is monotonically increasing in T_t. In equilibrium, the larger the life expectancy, the more people optimally invest in the time-consuming human capital acquisition of h^s.

Despite being monotonic the effect of longevity on human capital is not linear. The effect of life expectancy on the ability threshold is stronger and more pronounced for intermediate values of T and λ. For low levels of T_t, the share of population investing in h^s is small due to the fix cost involved with acquiring h^s, which prevents a large part of the population from receiving sufficient lifetime earnings to be worth the effort. The larger the fix cost, the more pronounced is the concavity of the function implied by condition (23). In this situation, substantial increases in adult life expectancy are needed to give incentives to a significant fraction of individuals to acquire skilled rather than unskilled human capital. On the other hand, when a substantial share of the population acquires h^s, i.e. when the share skilled λ is very high, very large improvements in T are necessary to make even more individuals acquire h^s instead of h^u. This is due to the decreasing returns to human capital of either type, which drives down the relative wage w^s/w^u as consequence of the high supply of h^u. The wage effect, however, dampens the attractiveness of investing in h^s for the individuals with low ability, even though life expectancy is very high.

4 Dynamic Evolution of the Economy

In this Section we analytically characterize the dynamic evolution of the economy that results from endogenous changes in the central state variables for individual decisions, the mortality environment and the technological environment.

4.1 Mortality

A large body of evidence documents that environmental factors, in particular macroeconomic conditions, are crucial determinants of both child and adult mortality.[23] Mortality depends on

[23]Here we restrict attention to the evidence on the structural determinants of mortality and we abstract from other factors like, e.g., wars, plagues or epidemics. See, e.g., Lagerlöf (2003), Young (2005), Kalemli-Ozcan (2006) and Boucekkine et al. (2008) for papers that investigate these issues.

the level of development and availability of human capital, reflected in the sanitary conditions, medical technology, and the economic environment. Historical research documents that better knowledge about diseases and better technological conditions and public policies helped to avoid or cure diseases, thereby reducing mortality (see, e.g. Mokyr, 1993, Schultz, 1993 and 1999, and Easterlin, 1999). Empirically, income, wealth and particularly the level of education affect mortality and health, see Mirovsky and Ross (1998) and Smith (1999). The findings by van den Berg et al. (2006) show that macroeconomic conditions faced by individuals during early childhood have a causal effect on these individuals' longevity, even as adults. Also, better educated societies invent and use better drugs (Lichtenberg 2002, 2003).[24]

Child mortality and adult longevity appear to be affected by the macro environment in different ways. Cutler et al. (2006) provide a survey of recent findings about the different determinants of adult and child mortality. Their evidence suggests that the level of knowledge and the amount of human capital available in society at each moment in time is relatively more important for adult longevity than the level of development in terms of overall living conditions or per capita income. Adult longevity depends on the ability to cure diseases and is related to the level of medical knowledge, the availability of surgery and other medical treatments that allow to repair physical damage and delay the aging process.[25] Soares (2005, 2007) reports macroeconomic evidence that adult longevity is little affected by improvements in income or nutrition but is rather related to 'structural' factors that depend on the knowledge available in a society. Ricci and Zachariadis (2007) report additional evidence for the importance of an externality of education on longevity in that a country's tertiary education attainment rate affects individual longevity beyond individual education. On the other hand, empirical findings suggest that higher incomes, public health expenditures, but also access to electricity or vaccines, increase the probability of children to survive to adulthood, see e.g. Wang (2003) for a recent survey. Child mortality seems to depend primarily on the level of development at the time of birth of children, the possibility to avoid diseases, the availability of adequate and sufficient nourishment, and an environment that prevents or facilitates infectious diseases.[26]

[24]Further evidence that the aggregate income share spent on health care increases with aggregate income levels can be found in Getzen (2000) and Gerdtham and Jönsson (2000) and the references therein.

[25]Cutler et al. (2006) review the determinants of these patterns over history, across countries and across groups within countries and identify the implementation of scientific advance and technical progress (which is induced and facilitated by human capital) as ultimate determinant of health and mortality.

[26]These findings are also consistent with empirical evidence on the effect of maternal education on child health reported by Desai and Alva (1998) on the basis of data from Demographic and Health Surveys for 22 countries. Despite a strong positive correlation, they find little evidence for a causal effect of higher maternal education

We model changes in mortality over time as the result of macroeconomic externalities that link the level of development and the availability of human capital to adult longevity and child mortality. In line with the evidence, and in order to better highlight the relevant mechanisms, we consider a differential impact of human capital and income on adult and child mortality. In particular, adult longevity of generation t is linked to the human capital of its parent generation,

$$T_t = \Upsilon\left(\lambda_{t-1}\right) \ , \tag{24}$$

with $\partial T_t / \partial \lambda_{t-1} > 0$ and with $\Upsilon(0) = \underline{T} > 0$, where $\underline{T}$ reflects the baseline mortality environment faced by individuals.[27] We assume $\underline{T} > \underline{e}^s$, such that even at the minimum level of life expectancy it is possible to acquire both types of human capital which, as shown below, implies an interior solution.

The child survival probability π_t depends on the level of economic development at the time of birth, reflected by the average per capita income y_{t-1},

$$\pi_t = \Pi\left(y_{t-1}\right) \ , \tag{25}$$

with $\partial \pi_t / \partial y_{t-1} > 0$, and $\Pi\left(y_0\right) = \underline{\pi} > 0$.[28] This formulation entails a Malthusian element: a larger total income Y_{t-1} improves the probability of children reaching adulthood but a larger population size N_{t-1} deteriorates living conditions and therefore reduces child survival rates.[29]

Notice that this formulation implies that that improvements in both adult longevity and child survival involve no scale effects.[30] The precise functional forms of these relationships entail no consequences for the main results. Child survival probability π_t and adult longevity T_t faced by

on child health, but rather an indirect effect where education mainly reflects socioeconomic status and area of residence. In particular, access to clean, piped water and toilets has a larger immediate causal effect on health than maternal education.

[27]In the simulation below we we adopt a simple linear formulation $T_t = \underline{T} + \rho\lambda_{t-1}$ that implies a lower and an upper bound for adult longevity, where $\rho > 0$ is a parameter reflecting the strength of the positive externality in terms of the potential amount of time life can be extended by medical knowledge.

[28]In the simulation of the model presented below, we assume that

$$\pi_t = 1 - \frac{1-\underline{\pi}}{1+\left(qy_{t-1}\right)^{\mu}}$$

with $q > 0$, and with $\underline{\pi} \in (0,1)$ being the baseline survival probability in a non-developed society, in order to ensure that π_t is bounded between zero and one.

[29]Considerable evidence documents the negative effect of population density and urbanization on child mortality, especially during the early stages of the demographic transition, see e.g. Galor (2005b).

[30]Equivalently for our results, life expectancy and child survival probability could be related to average or total human capital or total income of the previous generation(s) as in Boucekkine, de la Croix, and Licandro (2002), or Blackburn and Cipriani (2002).

members of generation t could be related to both the average skilled human capital embodied in the parent generation, h^s_{t-1}, and to the per capita income of the parents generation y_{t-1} without changing the main results. Any monotonic relationship can be used without affecting the main mechanism.[31]

4.2 Technological Progress

Technological progress takes place in terms of the arrival of new vintages of technology with larger productivity together with the birth of a new generation. We consider skill biased technological change where skilled human capital h^s helps in adopting new ideas and technologies, and thus creates higher productivity gains than unskilled human capital h^u in the tradition of Nelson and Phelps (1966). We consider skill biased technical change which depends on the stock of human capital available in the economy. Empirical evidence provided e.g. by Doms et al. (1997) supports this feature which implies that the more individuals of a generation acquire skilled human capital the more attractive is the accumulation of skilled human capital for future generations. As a consequence, new technological vintages are characterized by larger TFP A. Using a simple vintage representation, advances in technology embodied in the latest vintage evolve according to:[32]

$$g_t = \frac{A_t - A_{t-1}}{A_{t-1}} = G\left(h^s_{t-1}, A_{t-1}\right) \ , \tag{26}$$

which implies,

$$A_t = \left[G\left(h^s_{t-1}, A_{t-1}\right) + 1\right] A_{t-1} \ . \tag{27}$$

[31]The quantitative features of the economic and demographic transitions depend on the precise formulation, however. Hence the differential roles of human capital and income per capita for child mortality and adult longevity have potentially important dynamic implications for the current debate on the exact timing of the fertility drop during the transition in the different countries, as discussed below.

[32]Also here, the specific functional form of this relationship is of little importance. Any specification implying a positive correlation between technological progress $(A_t - A_{t-1})/A_{t-1}$ and h^s_{t-1} would yield qualitatively identical results. In the simulations below, we adopt Jones' (2001) specification, which is a generalization of the original contribution of Romer (1990) allowing for decreasing returns,

$$A_t = \left(\delta \left(h^s_{t-1}\right)^{\psi} A^{\phi}_{t-1} + 1\right) A_{t-1} \ ,$$

where $\delta > 0$, $\psi > 0$, and $\phi > 0$. As will become clearer below, assuming exogenous technical change would be equivalent for the main results of the model. The only consequence of assuming exogenous advances would be shutting down an additional reinforcing feedback effect as the economy develops.

The skill bias feature is modelled assuming that the relative productivity of low-skilled human capital in production, x decreases with the level of technological advancement,

$$x_t = X(A_t) \text{ with } \frac{\partial X(A_t)}{\partial A_t} > 0\,. \tag{28}$$

Note that there are also no scale effects involved in the specification of technological progress. The crucial relation is between the level of development and the fraction of the previous generation of adults investing in skilled human capital.[33]

4.3 The Dynamic System

We now turn to the dynamic equilibrium of the economy. The economic and demographic transitions emerge from the interplay of individually rational behavior and macroeconomic externalities. The analysis of the full dynamic system must account for the evolution of all variables of interest.

The global dynamics of the economy are fully described by the trajectories of λ and the key state variables, T, π, A, x. The first element of the dynamic system is the intra-generational equilibrium relationship between λ_t and T_t implied by condition (23). The equilibrium share of population acquiring skilled human capital in each generation t, characterized by $\lambda_t := (1-\widetilde{a}_t^*)$, depends on life expectancy T_t and the technological environment, characterized by x_t. For notational brevity, denote this implicit equilibrium relationship

$$\lambda_t = \Lambda(T_t, x_t)\,, \tag{29}$$

which, from Proposition 3, is an increasing and S-Shaped function of T, and is defined for $T \in [\underline{e}^s, \infty)$.

Recalling condition (24), adult longevity T_t depends on the education composition of the population in terms of the share of skilled human capital λ_{t-1},[34]

$$T_t = \Upsilon(\lambda_{t-1})\;, \tag{30}$$

Notice that the level of skilled human capital can be expressed as

$$h_{t-1}^s = \alpha^s f(r^*, \cdot)\frac{(1-\gamma)}{2(2-\gamma)}(T_{t-1} - \underline{e}^s)\;\left[1-(1-\lambda_{t-1})^2\right]\;, \tag{31}$$

[33] In the simulations below we adopt the simple formulation $x_t = 1 - (A_0/A_t)^\chi$, with $\chi > 0$.

[34] Rather than adopting the simple linear formulation $T_t = \underline{T} + \rho\lambda_{t-1}^s$, one could adopt a more realistic formulation and relate T to h_{t-1}. This would leave the main results unaffected, but, from (31), would imply that Υ is a concave rather than linear function of λ_{t-1}.

where $[(1-\gamma)/(2-\gamma)](T_{t-1}-\underline{e}^s) = e_t^{s*} - \underline{e}^s$ is the optimal time investment in education acquiring skilled human capital. From (27), (28) and (31) the process of technological change is therefore given by

$$x_t = X(T_{t-1}, \lambda_{t-1}, x_{t-1}), \tag{32}$$

with X being an increasing function in all arguments.[35] Similarly, by the definitions of y_{t-1} and h_{t-1}^s which are functions of T_{t-1}, λ_{t-1} and x_{t-1}, we can rewrite (25) as

$$\pi_t = \Pi(\lambda_{t-1}, T_{t-1}, x_{t-1}). \tag{33}$$

The dynamic path of the economy is fully described by the sequence $\{\lambda_t, T_t, x_t, \pi_t,\}_{t\in[0,\infty)}$, resulting from the evolution of the nonlinear first-order dynamic system consisting of equations (29), (30), (32), (33):

$$\begin{cases} \lambda_t &= \Lambda(T_t, x_t) \\ T_t &= \Upsilon(\lambda_{t-1}) \\ x_t &= X(\lambda_{t-1}, T_{t-1}, x_{t-1}) \\ \pi_t &= \Pi(\lambda_{t-1}, T_{t-1}, x_{t-1}) \end{cases}. \tag{34}$$

In order to analyze the behavior of the dynamic system (34), notice the absence of any scale effect, in the dynamic path of λ ,T and x which does not depend on population size. This feature allows to characterize the evolution of these variables by restricting attention to equations (29), (30) and (32) since they do not depend on N and π. The evolution of this dynamic (sub-)system delivers the sequence $\{\lambda_t, T_t, x_t\}_{t=\in[0,\infty)}$ which, from (33) allows us to characterize the evolution of $\{N_t, \pi_t\}_{t=\in[0,\infty)}$. To illustrate the development dynamics consider the conditional system

$$\begin{cases} \lambda_t &= \Lambda(T_t, x_t) \\ T_t &= \Upsilon(\lambda_{t-1}) \end{cases}, \tag{35}$$

which delivers the dynamics of human capital formation and life expectancy conditional on the level of relative productivity x_t. Any steady state of system (35) is characterized by the intersection of the two loci Λ and Υ. From Proposition 3 and since $T(\lambda = 0) > \underline{e}^s$ and $T(\lambda = 1) < \infty$ the system (35) has always at least one and, due to the non-linearity of Λ, at most three steady state equilibria. Figure 2 illustrates the system (35) in the latter case. An unstable equilibrium is characterized by an intersection of the Υ-locus with the Λ-locus for intermediate levels of λ and T.

[35]Notice that from (28) $x_t = X(A_t)$ and from (27) we have $x_t = X(h_{t-1}^s, A_{t-1})$. Since X is invertible, $A_{t-1} = X^{-1}(x_{t-1})$. Finally from (31) we can write $x_t = X(T_{t-1}, \lambda_{t-1}, x_{t-1})$.

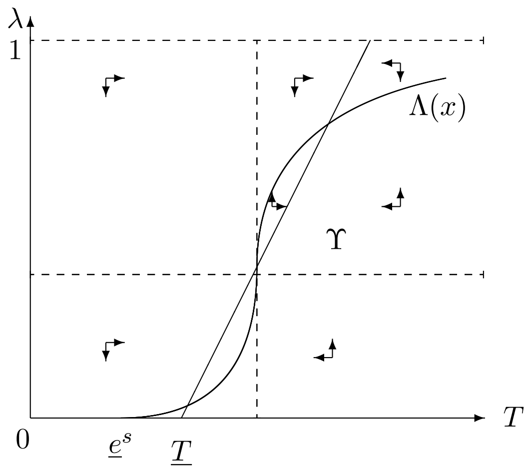

Figure 2: Phase Diagram of the Conditional Dynamic System

The state of technological development, as reflected by x_t crucially affects the relative returns for high-skilled human capital. The existence of endogenous skill biased technological change implies that the equilibrium relationship (29) changes over the course of generations. A larger x increases the returns to skilled human capital and the associated equilibrium fraction of individuals λ, so that $\partial\lambda_t/\partial x_t > 0$. Intuitively, the consequence of an increase in x is a counterclockwise shift of the S-Shaped Λ locus and a change of its shape that increases its concave part, leaving the Υ-locus unaffected.[36]

4.4 The Phase Transition

Consider a non-developed economy in which adult life expectancy T_0 and relative productivity x_0 are low.[37] Under these conditions, investing in h^s is relatively costly for a large part of the population as the importance of the fix cost for education, $\underline{e}^s$, is relatively large. This means that the convex part of the Λ locus is large and the conditional system is characterized by a unique steady state in which the fraction of individuals deciding to acquire h^s, λ_0 is small.

Endogenous skill biased technical change leads to a monotonic increase in the importance of skilled human capital for aggregate production.

Lemma 2. *The technology index A_t and the relative productivity of skilled human capital x_t increase monotonically over generations with $\lim_{t\longrightarrow\infty} A_t = +\infty$ and $\lim_{t\longrightarrow\infty} x_t = 1$.*

Proof. See Appendix. □

[36] In a more general specification that links T_t to h^s_{t-1} as mentioned in footnote 34, an increase in T_{t-1} would induce a clockwise shift of the Υ locus. This is the case since, for any λ_{t-1}, a larger T_{t-1} is associated with a higher time investment in education e_{t-1}, mirroring a change in both the extensive and intensive margin of human capital formation λ and e^s.

[37] As will become clear below, starting from this point is without loss of generality.

Before investigating the development path in details we first characterize the steady states of the system (35) for the limit cases of A_0 and $A_t \longrightarrow +\infty$ which reflect the conditions before and after the phase transition, respectively.

Proposition 4. *In an underdeveloped economy with $x_0 \simeq 0$, the system (35) exhibits a unique steady state with $\lambda \simeq 0$, $T \simeq \underline{T}$, and $\pi \simeq \underline{\pi}$, as well as net fertility*

$$n\underline{\pi} \simeq \frac{\gamma}{2-\gamma} \frac{\underline{T} - \underline{e}^u}{\underline{T}} \frac{1}{r^*} . \tag{36}$$

In a developed economy with $x_0 \simeq 1$, the system (35) exhibits a unique steady state with $\lambda \simeq 1$, $T \simeq \overline{T}$, and $\pi \simeq 1$, as well as net fertility

$$n \simeq \frac{\gamma}{2-\gamma} \frac{\overline{T} - \underline{e}^s}{\overline{T}} \frac{1}{r^*} . \tag{37}$$

Proof. See Appendix. □

In an underdeveloped economy, the equilibrium is characterized by a low level of technological development and poor living conditions. Both adult longevity and child survival probability are close to their minimum, $T_0 \simeq \underline{T}$ and $\pi_0 = \underline{\pi}$. From Proposition 3 the fraction of individuals optimally acquiring skilled human capital under these conditions is close to zero, $\lambda_0 \simeq 0$. Accordingly, average gross and net fertility correspond to those of unskilled individuals. The difference between gross and net fertility is sizable. In a developed economy, given the large adult longevity and technological level, almost all individuals acquire skilled human capital, $\lambda \simeq 1$. Correspondingly, aggregate fertility behavior mirrors that of skilled individuals. Since almost all children survive childhood gross and net fertility are almost identical. The level of life expectancy $\overline{T}$ is endogenously determined from (30) when $\lambda = 1$. Notice, however, that whether $\overline{T}$ is bounded or not entails no consequence for the main results. In fact, the existence of an upper bound to life expectancy is ultimately an empirical question that is the object of an ongoing debate in demography (see e.g. Vaupel, 1998 and Oeppen and Vaupel, 2002).

A long term decline in net fertility can be observed in the model even in the absence of precautionary demand for children if the reduction in fertility associated with the switch from unskilled to skilled human capital is large enough. Conditions (36) and (37) imply that net fertility declines whenever the total time spent on raising each child, relative to the time spent working, increases after the demographic transition. To make these conditions hold, the education cost of acquiring human capital must be large enough as compared to the increase in adult longevity. In this case the acquisition of skilled human capital involves a sufficiently larger fix cost of education $\underline{e}^s > \underline{e}^u$ which tends to reduce the number of years spent working.[38] The empirical evidence suggests that these conditions are very likely to hold in reality.[39]

The full dynamics of the economy exhibit a non-smooth process of development, characterized by an endogenous economic and demographic transition.

Proposition 5. [Economic and Demographic Transition] *The economy is characterized by the following phases in the process of development:*

(i) A (potentially very long) phase of stagnant development with little longevity, $T_0 \simeq \underline{T}$*, large child mortality* $\pi_0 \simeq \underline{\pi}$*, very few individuals acquiring human capital* h^s, $\lambda_0 \simeq 0$*, and large gross and net fertility rates as in* (36)*;*

(ii) A rapid transition involving substantial increases in T_t*,* π_t*,* λ_t*, income per capita* y_t*, the level of technological development reflected in* x_t *and* A_t*, and a reduction in gross and net fertility (possibly following an initial temporary increase);*

(iii) A phase of sustained growth in technology and income with long life expectancy $T_\infty \simeq \overline{T}$*, negligible child mortality* $\pi_\infty \simeq 1$*, and almost the entire population acquiring* h^s *human capital* $\lambda_\infty \simeq 1$*, and with low gross and net fertility rates as in* (37)*.*[40]

Proof. See Appendix. □

This phase transition is displayed in Figure 3. While the proof of this proposition is presented in the Appendix, the intuition behind this proposition can be described as follows.

(i) From Lemma 2, the process of skill biased technical change shifts the equilibrium locus (29) by increasing the incentives for the acquisition of skilled human capital. From Proposition 4, initially only a small fraction of the population is skilled so that the rate of technological change is small. Development is therefore characterized by an extended phase with low living standards,

[38] Notice that interpreting h^u as unskilled labor with $\underline{e}^u = 0$ and h^s as human capital involving $\underline{e}^s > 0$ this condition is satisfied for any increase in longevity. Moreover, the differential fertility effect would be reinforced by the usual change from quantity to quality of children as consequence of technological progress with $r^*_\infty > r^*_0$ as in Galor and Weil (2000).

[39] For instance concerning historical data for England and Wales, child mortality fell substantially from around 20 percent in the period 1550-1600 to less than 0.5 percent at the end of the 20th century. Adult longevity measured by life expectancy at the age of 30 experienced an increase from around 60 years to around 75 years (Data are from Wrigley and Schofield (1981) and UK national statistics). Considering that the acquisition of higher education is currently associated to a time investment, $\underline{e}^h$, of 10 to 15 years this can help rationalizing the reduction in fertility.

[40] It is important to note that the actual trajectory of the system depends on the initial conditions and cannot be precisely identified in general. Nevertheless the system moves generation by generation in the area of attraction of the locally stable steady state with low T and λ until this steady state disappears.

low adult longevity, high child mortality and large fertility. This is depicted in Figure 3(a). Furthermore, since a large part of the population is unskilled, any improvements in longevity and income directly funnel into higher average levels of fertility due to the income effect. As generations pass, productivity growth makes investing in h^s more profitable for individuals of any ability. As discussed above, the graphical consequence is a counterclockwise shift of the Λ locus and an increasing importance of its concave part. The dynamic equilibrium moves along Υ leading to improvements in longevity. During this early stage the feedback effects on mortality and technology are small and fertility remains high since child mortality is large and λ is low. After sufficiently many generations experiencing this early stage of sluggish development, Λ exhibits a tangency point, and eventually three intersections with Υ. From this point onwards the conditional dynamic system (35), exhibits also another stable steady state with larger T and λ, although the economy is trapped in the area of attraction of the initial, and locally stable, steady state with low T and λ. This is depicted in Figure 3(b).

(ii) The consecutive shifts of Λ eventually lead the initial steady state to lie at the tangency of the two curves and, thereafter, to disappear. At this point a unique globally stable steady state exists as shown in Figure 3(c). The following generations faces an adult longevity that is high enough to induce a substantially larger fraction of the population to acquire human capital h^s. This phase transition triggers a period of rapid development where λ increases fast within few generations, and T and π increase rapidly as well. The shifts in the Λ-locus accelerate accordingly. The transition lasts for a few consecutive generations. From Proposition 4 eventually fertility begins to fall during the transition, but from Proposition 2, due to the increase in T the income effect may lead to an intermediate, temporary increase in fertility.

(iii) Adult longevity and child survival probability converge asymptotically to $\overline{T}$, $\pi = 1$ and $\lambda \to 1$. Due to the composition change in the population and the associated differential fertility effect, average fertility declines. As consequence of endogenous growth mechanisms, economic development remains fast even though changes in adult longevity and human capital structure in the economy abate.

4.5 Discussion

Before investigating the empirical relevance of the theory we briefly comment on the role of some simplifying assumptions.

In the analysis we have deliberately neutralized the well-studied change in the quantity-quality trade-off linked to technological change. This is done to clarify the working of the

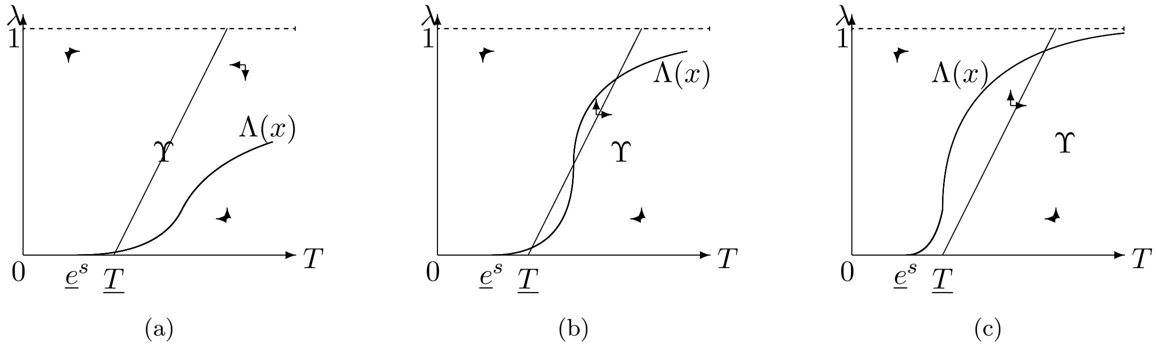

Figure 3: The Process of Development

model. The main results would be reinforced by the presence of a change in r^* as in the models in the spirit of Galor and Weil (2000), while leaving the qualitative dynamics of the system unchanged. In particular, r^* would be low before the take-off but would accelerate during and after the transition due to the larger technological change. This would imply a further reduction of gross fertility for all individuals (irrespective of the education they acquire) after the transition.

The proposed mechanism does not rely on scale effects or non-convexities like, e.g. subsistence consumption levels. Extending the model by adding a subsistence thresholds for consumption as in Galor and Weil (2000), Strulik (2008) and Dalgaard and Strulik (2008), would introduce additional Malthusian features by including a corner solution in the dynamics of development. Also, as shown by de la Croix and Doepke (2003) and de la Croix and Licandro (2007), the consideration of corner regimes increases the likelihood of observing a temporary increase in fertility at the onset of the transition before observing a drop after the transition.

The level of technology increases monotonically and deterministically over the course of generations. But this instrumental prediction is obviously not necessary for the main argument as long as productivity eventually increases enough to trigger the transition, i.e., to induce a sufficiently large fraction of the population to acquire skilled human capital.[41]

In the characterization of the transition dynamics we concentrated exclusively on technological change. There are a number of other variables which can trigger the transition, with potentially important implications for development policies. For example, the incentives for the acquisition of skilled human capital also depend on the relative productivity of the time invested in acquiring education. In fact, x and α are isomorphic in inducing a larger fraction of skilled individuals λ for any T. Hence endogenous improvements in the production technologies of

[41] Aiyar et al. (2006) propose a micro-foundation of the interactions between population dynamics and technological progress and regress in pre-industrial societies.

human capital (such as the creation of better schooling systems) or changes in the individual returns to education (for example due to improved health conditions) are complementary forces which may endogenously trigger the transition.[42]

In the model, all individuals face the same life expectancy, in particular irrespective of the type of education they acquire. Nonetheless there is evidence of a mortality differential related to own education which, despite being mild compared to the changes in average mortality during the transition, is observed in the last decades in countries that have completed the demographic transition.[43] The consideration of differential mortality would reinforce the predictions on the role of longevity while leaving the mechanism unchanged. This can be seen by considering the implicit characterization of the equilibrium threshold of ability in (23). A mark-up in adult longevity for individuals acquiring skilled human capital is equivalent to a reduction in the parameter measuring the fixed cost of education acquisition $\underline{e}^s$. Hence, introducing such a mark-up leaves the overall properties of the dynamic system unaffected. Technically, considering differential mortality would have two main implications. First, higher longevity would reinforce the incentives to acquire skilled human capital. This would modify the equilibrium locus (23) quantitatively by facilitating the acquisition of skilled human capital and making the S-shaped locus steeper for any given level of technology, but the qualitative features would be the same. Second, considering differential mortality would imply that the composition of the population is relevant not only for average fertility but also for average longevity. In particular, the observed increase in average longevity would be explained in part by changes in the education composition of the population.

5 Implications for Economic Development

In this section we present the empirical implications of the theory for development dynamics and comparative development. We do this by summarizing the implications of the analytical results presented in the last Section and illustrating them by a simulation of the model. The model

[42]See e.g. Galor et al. (2003) for a theory about the endogenous emergence of public schooling, and de la Croix and Licandro (2007) for a model with endogenous parental investments in health care and health capital of children.

[43]There is some contemporaneous evidence for a positive correlation between individual education and life expectancy, see, e.g., Kitigawa and Hauser (1973) for the U.S. and Cambois et al. (2001) for France. However, empirical findings also suggest that education differentials in mortality widened only in the last approximately 50 years, see e.g. Feldman et al. (1989), Pappas et al. (1993) and Preston and Elo (1995). The question about the existence of a causal impact of individual education on individual health and mortality, as well as the precise channel, is still a topic of a lively debate, see e.g., Kenkel (2001), Kilander et al. (2001) and Lleras-Muney (2005).

predictions are then confronted with historical time series data as well as with cross country panel data.

5.1 Benchmark Simulation

We illustrate the empirical predictions of the analytical results by simulating the model. The aim is to explore the theory's ability to generate the stylized patterns that are common to the development experiences of different countries, and not to calibrate the model to the development pattern of a specific country. To this end, we simulate the model with a single set of parameters, and then analyze the data generated by this simulation in comparison to both time series and cross-section data from actual statistics.[44]

The parameters for the simulation are chosen to reflect realistic values whenever possible, or to generate realistic values of some of the key moments of the model. The value $\underline{T}$ represents life expectancy that would arise in an underdeveloped economy where there is no education. In each country this baseline longevity may vary depending on exogenous characteristics which are related to e.g. climate, the presence of tropical diseases, etc. As a benchmark we consider $\underline{T}$=50 years.[45] Below we will explore the implications of different (exogenous) mortality environments by considering different levels of $\underline{T}$. The parameter ρ in the specification of adult longevity is chosen to generate an upper bound of 75 years. The fix time cost for obtaining skilled and unskilled education is 15 and 5 years, respectively. The lower bound for child survival is chosen to match average child mortality in pre and post transition countries. The model is capable of producing a deliberately long stagnancy period before the transition. The initial conditions and the technology parameters mainly affect the timing of the take-off. For illustrative purposes we set these parameters to generate a development path that is consistent with an initial period of stagnation. The parameter values are listed and discussed in Appendix A.2. We simulate the model for 400 generations. With a new cohort arriving every 5 years, the simulation can be interpreted as covering the years 500 - 2,500.

5.2 Model Dynamics and Stylized Facts

The development path generated by the model is depicted in Figure 4. Illustrating the predictions of Proposition 5, the model dynamics display an extended period of virtually stagnant

[44]In this respect, the purpose of our analysis differs from that of Manuelli and Sheshadri (2008), who calibrate an extended Barro-Becker framework to the US in order to investigate the role of differences productivity for explaining fertility differences between the US and Europe.

[45]Note that this is adult longevity conditional on having survived childhood.

development in income per capita, adult longevity, child mortality and fertility. At the same time, the share of skilled individuals remains at very low levels. Eventually the phase transition begins, with the fraction skilled λ starting to increase significantly, accelerating the improvements in adult longevity, child mortality, and income. This sets off the bi-directional virtuous circle that leads to an exit from the development trap, and to the country converging to a dynamic equilibrium with sustained income growth, large adult longevity, little child mortality and low fertility. In light of the model, the development patterns observed in average aggregate variables are related to, and the result of, the change in the composition of the population. Before the economic and demographic transition, only a very small share of the population acquires skilled education, while the vast majority of the population is illiterate. During the transition schooling enrolment rates grow fast. After few generations the vast majority of the population acquires higher education. Individual fertility for skilled and unskilled individuals intermediately increases during the demographic transition due to income effects, leading to a temporary hike in population growth. Eventually, however, gross and net fertility declines below pre-transitional levels as result of the change in education composition and the associated differential fertility.

A large body of empirical research in economics, economic history and demography has documented the economic and demographic transition. Common patterns emerge in the dynamics of the transitions in different countries, regardless of looking at historical transitions or more contemporaneous cases.[46] In Figure 5, we illustrate the typical pattern of economic and demographic transition using the data for Sweden, which is often referred to as a prototypical historical example.[47] After stagnation over the entire history, income per capita started increasing in the 19th Century, and quickly converged to a regime of modern growth. During the transition, life expectancy at birth and later ages improved while child mortality decreased substantially. Fertility, in gross and net terms, first increased slightly during the 19th Century and eventually dropped below the pre-transition levels.[48] Gross and net reproduction rates

[46]Detailed accounts of the evidence can be found in the book by Chesnais (1992), and the comprehensive surveys by Lee (2003), and Galor (2005b).

[47]The Data for GDP per capita is provided by the internet portal for historical Swedish statistics, `www.historia.se`. Life expectancy and fertility data are taken from Wrigley and Schofield (1981), Keyfitz and Flieger (1968). Population data are from the Swedish Central Statistical Office, `www.scb.se` and the internet portal for historical Swedish statistics, `www.historia.se`. Data on schooling enrolment have been constructed by de la Croix, Lindh, and Malmberg (2006). Missing values are obtained by linear intrapolation.

[48]Such an intermediate population growth is well documented in some historical cases, like England, as well as in some recent transitions observed in Latin America, Africa and Asia, see Dyson and Murphy (1985). In the model the increase in fertility due to the income effect of larger longevity can be sufficiently strong to generate a

eventually converged, as result of the drop in child mortality. Net fertility exhibited a marked decline compared to the pre-transition period (see also Lee (2003)). A large, and rapid, increase in primary and secondary school enrolment accompanied the growth take-off. The acquisition of at least some formal education was limited to a tiny fraction of the population before the transition, but became increasingly widespread.[49]

The historical transitions in other European countries and North America display very similar patterns, see Galor (2005b) for a detailed discussion. England was the first country to experience an economic and demographic transition, and the onset of the economic transition preceded that of Sweden by several decades. The development pattern in England is reported in Figure 6 and displays an increase in adult longevity, as measured in terms of life expectancy at age 30, which takes place considerably earlier than improvements in life expectancy at birth.[50] The education composition of the population has experienced a similarly dramatic change in England as in Sweden.[51] Other historical examples, and more recent experiences of transitions in other countries, exhibit roughly similar patterns. The most remarkable difference between historical and contemporaneous transitions concerns the speed of the phase transition, as well as the timing of changes in adult longevity and child mortality.[52]

sizeable temporary increase in gross and net fertility and a hump-shaped relationship between income and fertility.

[49]Extensive corresponding historical evidence is reported by Cipolla (1969), and Floud and McCloskey (1994), see also de la Croix et al. (2006) for more details on Sweden.

[50]GDP data is provided by Floud and McCloskey (1994) and Maddison (2006), literacy data is taken from Cipolla (1969). Life expectancy and fertility data are taken from Wrigley and Schofield (1981), Keyfitz and Flieger (1968) and the websites of the Office of National Statistics (`http://www.statistics.gov.uk`) and the Population Division of the Department of Economic and Social Affairs of the United Nations Secretariat (World Population Prospects: The 2004 Revision and World Urbanization Prospects: The 2003 Revision, `http://esa.un.org/unpp`). Missing values are obtained by linear intrapolation.

[51]Attention is usually placed on average years of formal education, which increased substantially in the Western World from close to zero to more than 11 years in the last 150 years, see Maddison (1991, Table 3.8). This view, however, neglects the fact that the acquisition of formal education and literacy in underdeveloped economies was limited to a small fraction of the population, whose general and scientific knowledge already reached remarkable levels long before the transition but it was highly concentrated in the hands of few. Literacy rates in most European countries were well below 20 percent, and concentrated among particular occupations, like traders, civil servants and clergy, which acquired education for several years see Mokyr (2002) and Stone (1969). For these occupations education, mostly in form of apprenticeships, already comprised more than 7 years in the fifteenth Century in Europe. Education very often included literacy acquisition and general training, and in some cases up to 12 years of compulsory education for some occupations as in Venice. See Cipolla (1969) for an extensive treatment of literacy and education in preindustrial Europe.

[52]In the model, the actual timing of the dynamics of the different variables during the transition depends on the relative strength of the different effects at work. In particular, adult longevity may increase while child mortality

By considering the endogenous change in the education composition and differential fertility, the model also provides a simple rationale for the link between increased longevity and the drop in gross and net fertility. This drop has been found difficult to rationalize in earlier models, which has led to the conclusion that the reduction in net fertility is unlikely to be caused by the mortality decline.[53]

The comparison of the simulation results and the actual time series data show that the model generates transition dynamics that are qualitatively in line with the stylized patterns in all dimensions. In particular, the model matches the non-linear evolution of the economic and demographic variables, the education composition of the population and the final drop in net fertility.

5.3 Differential Fertility and Education

Proposition 1 states that the acquisition of different types of human capital is associated with a fertility differential, which plays an important role for the population dynamics of the model. The theory predicts that the average fertility depends on the education composition of the population, and the final drop in net fertility is the result of the change in the share of the population acquiring skills combined with the fact that skilled individuals have a lower optimal fertility than unskilled individuals.

The demographic literature has documented substantial heterogeneity in individual fertility associated with different socio-economic backgrounds. This led Cochrane (1979) to state that the negative relationship between education and fertility is "one of the most clear-cut correlations in the social sciences". Even though historical time series data on differential fertility are dispersed, the available evidence documents that, historically as well as contemporaneously, women with higher education tend to exhibit lower fertility behavior before, during, and after the transition, see e.g. Bengtsson and Dribe (2006), Gutmann and Watkins (1990), Castro-Martín (1995), Rindfuss et al. (1996), and Mare (1997), as well as Caldwell (1999). For more recent periods several empirical studies point at a robust negative association between own education and fertility, especially for women. The level of educational attainment of parents is a key determinant of their fertility, with a strong negative correlation between the years of formal education and fertility.

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is still high in the early stages of the transition, consistent with the transition in England, where infant mortality began to fall much later than mortality at later ages. These differences have proved difficult to reconcile with previous theories, see, e.g., the discussions in Galor (2005b) and Doepke (2005).

[53] See Kalemli-Ozcan (2002, 2003), Doepke (2005) and Galor (2005a) for a discussion of this issue.

Skirbekk (2008) reports results of a meta-analysis of the large empirical literature in demography on the correlation between education and fertility in very different contexts. His data set contains the results from 902 samples reported in 136 different published research papers. The results show a strong and stable pattern of differential fertility, with lower fertility of individuals or groups with higher educational background. Figure 7 builds on data collected by Skirbekk and displays all available empirical estimates of relative fertility for different education (panel a) or occupation (panel b) groups across time and space that are currently known in the demographics literature. The vast majority of studies finds a negative effect of education on fertility (a dot below 0).[54]

In search of causality Osili and Long (2007) use data from a large scale "universal primary education program" in Nigeria and provides evidence for the robustness of the strong negative correlation when controlling for endogeneity and reverse causality by using appropriate instruments. The study reports a causal effect of one year of schooling reducing the number of births per woman by 0.26.

5.4 Cross-Sectional Predictions

The conditions of the development trap equilibrium in the model resemble the conditions observed in the non-developed world today: Life expectancy is low, child mortality high, while most of the population is unskilled, lacks elementary education and basic knowledge of hygiene and sanitation, and displays large fertility. If the underlying mechanism is valid, the theoretical predictions should not only hold within a given country across time. But it should be possible to observe its consequences also across different countries at different times, with the notion that these countries are in different phases of their, otherwise similar, transition process.

In the model the development trap arises because a small share of the population optimally decides to become skilled. The observed changes during the transition in terms of average human capital acquisition and fertility, are predominantly the outcome of substantial changes in the composition of the population. Hence, the main mechanism driving the results is based on the crucial role of an endogenously changing education composition, reflected by λ. The model predicts that all the variables of interest exhibit a monotonic correlation with λ. Reinterpreting the results of Proposition 5, we have,

[54]Note that in panel (b) relative occupation measures occupational status. It is well known that historically, social status was less strongly associated with education than in the recent history or nowadays. This might explain the negative slope of the regression line in panel (b). We are grateful to Vegard Skirbekk for useful discussions on these issues and for providing us with Figure 7.

Corollary 1. *There is a positive contemporaneous correlation between λ_t and life expectancy (T_t and π_t) over the course of development, and overall a negative contemporaneous correlation of λ with gross and net fertility.*

These novel predictions can be investigated by looking at the cross-sectional correlations between the main variables of interest, by simply pooling the data generated with the previous simulation and comparing the results to cross country panel data. As empirical counterpart of the population structure λ we use data from the data set constructed by Barro and Lee (1993, 2000). As measure for the share of skilled individuals λ we use the share of the total population with a minimum level of formal education, generated as 100 minus the percentage of "no schooling education" in the total population.[55] The other variables of interest are life expectancy at birth, child mortality before the age of one year, and total fertility in terms of births per woman.[56] In the following, we use all the countries for which we observe the variables of interest (life expectancy, infant mortality, adult mortality and total fertility) over the period 1960 or 1970 to 2000, which allows us to also look at changes in the patterns over a sufficiently long period of time.

Figure 8 presents the results implied by the thought experiment of re-interpreting the data produced by the previous dynamic simulation in a cross-sectional way. Figure 9 presents the respective picture arising from cross-country data. The patterns arising from the analysis of cross-sectional data are qualitatively very similar to those emerging from the simulation: adult longevity is strongly positively correlated with the share of skilled individuals, λ, while child mortality exhibits a clear negative correlation. There is a strong correlation between fertility and the education composition across countries. Fertility in countries with virtually no high skilled individuals is four times larger than in countries with universal lower secondary education, consistent with the differential fertility effect in the model. As the education composition changes to a larger λ, fertility drops substantially. Also the net replacement rate exhibits a negative relation with λ. Finally, the countries appear to have advanced in their transition, as can be inferred from the positive autocorrelation in λ with most observations above the 45 degree line.

Over the years 1970-2000, we observe a slight upwards shift in the data for life expectancy

[55] All results are qualitatively identical when using alternative measures. For example, we also used the fraction of the total population with at least completed lower secondary education, or restricted to different age cohorts such as, e.g., age 20-24 years, from a data set on education composition constructed by Lutz, Goujon, and Sanderson (2007). Detailed results are available upon request.

[56] The data sources are the UN Population Division World Mortality Report 2007, and the World Bank's *World Development Indicators*, respectively.

and a downwards shift in child mortality. Therefore there are improvements in the mortality environment conditional on the education composition across countries. Nonetheless, the cross-sectional pattern appears to be very stable over the observation period. Similarly, the relationship between fertility and the population share of skilled individuals appears to have changed little over time, with fertility having declined slightly conditional on λ. But this shift, which could be explained for example by the drop in child mortality or improved contraception, is small compared to the effect implied by the cross-sectional relationship with the education composition.

Overall, the data reveal surprisingly stable correlations between education composition, and mortality as well as fertility. In 2000, the demographic characteristics of the countries that reach a certain education composition in 2000 are comparable to the conditions observed in 1970 in countries that have reached the same education composition in 1970. This is noteworthy, since the relationships are obtained with a cross-section of countries with very different historical, geographical and institutional features that are not accounted for. Moreover, all relationships display small parallel shifts that might be attributed to e.g. the overall increase in living conditions and cross-country spill-overs of knowledge, but that quite remarkably leave the predicted correlations of interest virtually unchanged.

5.5 Life Expectancy and Development

In the empirical literature, the role of life expectancy for development has been investigated by exploiting exogenous changes in longevity on subsequent economic development, and by exploiting exogenous differences in the extrinsic mortality environment. We now discuss the predictions of the model on the role of longevity for development in these two dimensions. This can also help shedding some light on the recent debate about the apparently contradictory findings concerning the causal role of life expectancy for development mentioned in the introduction.

Longevity and Changes in Education along the Development Path. As a corollary of Proposition 5, the model predicts a non linear relationship between current longevity and the change in the composition of the population.

Corollary 2. *Along the development path, the correlation between longevity and the subsequent change in the education composition is hump-shaped. The longer the time horizon for subsequent changes in the education composition, the more pronounced is the hump and the lower is the longevity associated with the largest subsequent changes (the peak in the hump).*

The previous Corollary states that the observed change in λ depends on the level of initial longevity. Larger subsequent improvements should be observed over longer time intervals due to the compounding effects of the transition. The largest improvements are expected for intermediate levels longevity. In the short run this is due to the S-shape of the equilibrium locus Λ: changes in λ are largest in the intermediate range where the slope of Λ is steepest. From Propositions 3 and 5 the countries with largest improvements in the short run display lower increases in the future (due to the slow-down associated with the convergence process) while the countries with lower longevity display the largest overall improvements, because the scope for improvement is greatest for them. Therefore in the medium run the change in λ is relatively large for countries with intermediate longevity but it is relatively small for countries that are far from exiting the trap or for countries that have already undergone the transition.[57]

We illustrate these predictions using data from the same simulation as before and compare them to the patterns observed in the cross-country panel data used above. Figure 10 shows plots of the relationship between adult longevity in a particular year and the subsequent change in λ in the short run (5 years, panel (a)), in the medium-run (20 years, panel (b)), and in the long run (40 years, panel (c)) using the data from the previous simulation. Figure 11 displays the corresponding relationships using the cross-country panel data over different time horizons. The base year for (initial) life expectancy is 1960.[58] In order to highlight the curvature implied by the scatter plot, a quadratic trendline is added. Consistent with the predictions, the relationship between longevity and improvements in education is hump-shaped over all time horizons. Also, as expected, the largest increases in education is for countries with initial longevity in the mid-fifties in the short run, early fifties in the medium run and late forties when considering the longest horizon.

Exogenous Improvements in Life Expectancy and Changes in Education. We now consider the consequences of a permanent exogenous improvement in adult longevity T, for example due to the introduction of a new drug like penicillin, along the development path. This corresponds to the first type of exogenous variation in life expectancy that has been used in the empirical literature to identify the causal effect of life expectancy on development as mentioned above. In terms of the phase diagram displayed in Figure 2, this improvement corresponds to

[57]In the limit, if one considers an infinite time horizon the country displaying the largest improvements is the one with the lowest initial longevity.

[58]We consider 1960 as initial period to exploit the longest time horizon available. The results are similar when using different years as initial periods.

a rightward shift in the Υ-locus at some point during the development process. Repeating the same reasoning as before, the model predicts a non linear effect of this exogenous improvements in longevity: the effect is largest for countries in the intermediate range of initial longevity.

To study the prediction of a non linear effect of exogenous shocks to longevity we use the data of Acemoglu and Johnson (2007) who construct an instrument capturing changes in mortality that are presumably exogenous to the life expectancy and level of development of a country.[59] The instrument is based on the emergence and availability of new drugs and treatments like penicillin, the emergence of institutions that distributed and administered the new drugs, such as the WHO, and the increased international diffusion of medical knowledge in the post-World War II era. The measure of predicted mortality change is constructed as direct consequence of this innovation, representing the convergence from a country's level of life expectancy in 1940 to the world life expectancy frontier.

Figure 12 Panel (a) illustrates the non linear relationship between initial longevity and the change in education which is plotted for the subset of countries considered by Acemoglu and Johnson (2007).[60] Panel (b) illustrates the positive relationship between the predicted mortality change between 1940 and 1980 and the level of life expectancy in 1940.[61] Panel (c) documents the non-monotonic relationship between predicted mortality and the actual change in the education composition of the population between 1960 and 1980. The increase in the share of skilled individuals is largest on average for countries with intermediate predicted mortality improvements, i.e. with intermediate initial longevity, as is indicated by the quadratic regression line. The possibility of an inherently non-monotonic causal effect of life expectancy suggested by the model has not been taken into account in the existing empirical literature.[62]

Differences in the (Extrinsic) Mortality Environment. We next investigate the role of permanent differences in the mortality environment for the development process. There is evidence of a substantial heterogeneity in the extrinsic mortality environment across the world, which is largely exogenous and very persistent overtime (see Gallup et al. (1999)). The empirical

[59] Most importantly, they argue that this predicted mortality change is exogenous to the variable of interest in their context, namely income.

[60] Not all countries are included in this dataset, most notably the African countries are not included.

[61] This relationship reflects the fact that countries with initially low life expectancy at birth have the largest potential for reducing mortality due to the adoption of new drugs and jumping to the world life expectancy frontier as a consequence. As is discussed at length by Acemoglu and Johnson (2007), these effects are unlikely to be driven by changes in infant mortality, but rather reflect changes in life expectancy at later ages.

[62] This could explain the lack of any causal effect of life expectancy on growth found by Acemoglu and Johnson (2007), who restrict attention to a linear relationship.

work by Lorentzen et al. (2008) documents the role of the mortality environment, in terms of geography, climate and the occurrence of tropical diseases like malaria, for life expectancy.

In the model, differences in the extrinsic mortality environment are reflected in differences in $\underline{T}$ which represents the baseline mortality that would be observed in the economy in the absence of any human capital. From Figure 2, a harsher environment in terms of a lower $\underline{T}$ entails a Υ-locus, which is located further to the left over the entire history of development, while leaving the Λ-locus unaffected. This, in turn, implies that the tangency between the two loci that leads to the disappearance of the initial dynamic equilibrium and the transition (phase *ii* in Proposition 5) occurs later in time or, equivalently, for higher levels of A and x.

Corollary 3. *A lower baseline value for adult longevity,* $\underline{T}$ *implies, ceteris paribus, a later onset of the transition and, related to this, a higher level of economic development in terms of income or productivity at the onset of the transition.*

We illustrate the predictions about the role of extrinsic mortality environment by introducing heterogeneity in $\underline{T}$. Additionally to our benchmark simulation, we simulate four otherwise identical economies, which only differ with respect to their baseline adult longevity. We consider countries with (discrete) values for $\underline{T}$ between 46 and 49 together with the benchmark economy characterized by $\underline{T} = 50$. Figure 13 presents the cross-sectional patterns arising from pooling the data from these example economies. The simulated data closely resemble those obtained from one economy in Figure 8. The heterogeneity in terms of baseline longevity $\underline{T}$ varying from 46 to 50 years delivers some variation in adult longevity and fertility, but very little variation in the correlation between child mortality and λ or in the autocorrelation pattern of λ. Hence, accounting for heterogeneity in $\underline{T}$ entails little changes in the cross-sectional correlations, and the variables of interest are barely affected by differences in baseline mortality. This is also in line with the observation that despite the documented differences in extrinsic mortality the cross-country qualitative patterns displayed in Figure 9 appear very stable overtime.

Following Corollary 3, extrinsic mortality may nonetheless be important in affecting the timing of the exit from the development trap. Two countries that are identical in all dimensions except for $\underline{T}$ undergo the transition at different points in time. Consequently the development paths of these two countries look different from a comparative development view when observed at a point in time at which one country has already begun the transition while the other has not. Since, in the model, demographic development takes place over relative long periods, as the evolution of technology and mortality takes place in subsequent generations, this implies that the extrinsic mortality environment may be relevant to explain different levels of development in a comparative perspective over fairly long time horizons.

Figure 14 displays the time series of adult longevity for countries differing in terms of their extrinsic mortality. Panel (a) shows that even small increases in extrinsic mortality, i.e. moderate reductions in baseline adult longevity, delay the take-off from the trap by several generations. Panel (b) presents results for the alternative thought experiment keeping maximum longevity fixed.[63] The substantial delay in the transition introduced by a modest variation in baseline longevity $\underline{T}$ is unaffected by this. These results relate to the recent discussion in the literature concerning the role of geography for development prospects. They indicate that the extrinsic mortality environment can potentially play an important role for explaining the observed differences in development around the world by delaying the transition out of the development trap.

Also the second prediction concerning a higher level of income and technological development being needed at the onset of the transition to trigger the take-off in more unfavorable extrinsic mortality conditions appears to be consistent with the data. For example, at the onset of the transition in Sweden in 1800, life expectancy at the age of five years was a bit below 55 years. This is roughly comparable to that in those countries that have not entered the transition in the year 2000, with levels of life expectancy at age 5 of around 55-60 years. The level of GDP per capita in Sweden in 1800 (in reference prices for 2000 and in USD as of 2000) was around 890. GDP per capita in USD per 2000 in the countries that have not entered the transition in 2000, however, was mostly above 1000, in many cases above 2000.

5.6 Implications for Comparative Development

From Proposition 5 the development path is characterized by a long period of stagnation followed by a rapid transition to sustained growth. This is true for all countries irrespective of the time in which they exit the development trap. A direct corollary of this non-linear dynamic evolution is that, from a static cross country perspective, the distribution of all the variables is expected to be bi-modal. While intuitive, this implication is novel and has not been emphasized by any other model in the literature that we are aware of.

Corollary 4. *The cross-sectional distributions of adult longevity, child mortality, fertility and education are bi-modal, unless all countries are trapped or have completed the transition.*

[63] One implication that arises due to the specification of the intertemporal externality (see footnote 27) is that a variation in $\underline{T}$ implies also a variation in the level of $\bar{T}$ to which the economy converges asymptotically. In the simulation in panel (b) the variation in $\underline{T}$ is accompanied by a variation in the strength of the externality (the slope ρ) to keep the asymptotic level of longevity unchanged.

Consider again differences in extrinsic mortality across countries, which, for example, could be related to geographic characteristics like latitude. Distance from equator has been found a good predictor of extrinsic mortality, reflecting the fact that countries with a larger distance to the equator, with a more moderate climate, have a lower prevalence and diversity of infectious disease species, see e.g. Brown (1995), Gallup et al. (1999) and Guernier et al. (2004). Figure 15 illustrates the positive correlation between life expectancy at birth and latitude for the years 1960 and 2000. In 2000 virtually all of the pre-transitional countries (with longevity below about 60 years of life expectancy) are tropical with a latitude below 0.235.

To investigate the prediction of bi-modality in the distribution and examine the role of extrinsic mortality we simulate the benchmark economy for the levels of baseline mortality $\underline{T}$, assuming a uniform distribution on the range 46 to 50 in steps of 0.05, i.e. considering 81 identical model economies that differ only in terms of their baseline adult longevity $\underline{T}$. This allows to isolate the effects of heterogeneity in extrinsic mortality $\underline{T}$ on the development process.

We use the data generated from these simulations to compute the kernel density estimates of the distribution of key variables of the model at different points in time. The timing as well as the entire parametrization of this simulation is identical to that of the benchmark economy as in Figure 4 (with $\underline{T} = 50$). For ease of interpretation and later reference, we take 1950 and 2050 as years of observation of the cross-section of countries during the early and late phase of the transition, respectively. Figure 16 presents estimates of the kernel density of the variables of interest obtained from pooling all observations of the simulation of the 81 economies in the early phase and in the late phase. The simulated kernel distributions of adult longevity and life expectancy at birth for the early period, collected in the left column of the first two rows in Figure 16, are bi-modal with a larger mode at low levels of adult longevity or life expectancy. This reflects the fact that during the observation period most countries are still in the development trap. Analogous results emerge for all other variables of interest. The results look different for the late phase of the transition depicted in the right column. The distributions of adult longevity and life expectancy at birth are still bi-modal, while most probability mass is now contained in the mode at high longevity. Similarly, the distributions reveal large mass for high levels of education and low fertility, reflecting that the majority of observations correspond to countries that have already left the development trap.[64]

We can compare the estimated kernel distributions obtained from the simulated data to the

[64]Obviously, the precise shape of the distribution depends on the years used to define early and late stages of the development process. Clearly, taking the extremes, i.e. using a time frame such that all countries are still in the trap or such that all countries are already out of the trap, would lead to unimodal distributions.

corresponding kernel density estimates for the variables of interest from the cross-country panel data described before. Figure 17 presents these empirical distributions, where we use data from the 1970 and 2000 cross-section of countries to reflect early and late stages of development, respectively. The comparison of Figures 16 and 17 shows that the simulated kernel density estimates exhibit features which are qualitatively very similar to those that arise from cross-country data. The left column of Figure 17 shows that the distribution of all variables was roughly bimodal with large mass at low levels of life expectancy and education, and at high levels of child mortality and fertility in 1970, when the economic and demographic transition had not begun in many countries of the world. By the year 2000 many countries have left the development trap and undergone a transition. Yet, as has been shown more formally by Bloom and Canning (2007) for life expectancy, the distributions are still bi-modal in 2000, with a considerable number of countries still in the mortality trap.

The overall fit between the predicted and the real distributions is remarkably good, especially in view of the fact that the only source of variability in the simulations is the difference of few years in baseline longevity. The main qualitative difference between the simulated and real data distribution relates to the speed of the transition. This is true for particular variables and in general. For example, the distribution of infant mortality is bi-modal but already fairly thick at lower levels of infant mortality in 1970. The ongoing demographic transition pushes the distribution even further to the left by the year 2000. This suggests that, compared to the model economy, improvements in infant mortality were quicker than improvements in adult longevity by 1970. In terms of the general timing, the kernel density estimates of the simulated data for 1950 are roughly comparable to the ones in the data in 1970 while the estimates for the simulation in 2050 are roughly comparable to the estimates for real data in 2000. To interpret this observation notice that all parameters of the simulated economies are identical to the ones of the benchmark economy (e.g. Sweden or England) with a transition taking place in the nineteenth century. The only difference across countries is the extent of baseline mortality. Together with the findings of a proportional effect of baseline mortality on the timing of transition, illustrated in Figure 14, this implies that the speed at which countries exit the trap in the simulation is roughly constant over the full period. The comparison between the kernel density estimates obtained from simulated and cross-country data therefore suggests a faster transition out of the underdevelopment trap in the last fifty years in the countries of the world in comparison to the model prediction.

6 Concluding Remarks

This paper proposes a theory of the economic and demographic transition that is applicable to historical development experiences as well as to the contemporaneous heterogeneity of development across the world. The model presents an analytical characterization of the dynamic equilibrium path that exhibits a non-linear development process, featuring an endogenous transition from a development trap with low life expectancy, little education and large fertility to a modern growth regime. The theory delivers a set of novel predictions that are illustrated by a simple dynamic simulation. The implications of the theory appear broadly in line with empirical patterns that arise from time series and cross-country panel data. Overall, the model delivers qualitative predictions that closely resemble the data patterns in both the long term time series dimension, the cross-sectional dimension, and the changes in the cross-sectional data over time. This is noteworthy since the theory is driven by a fairly simple mechanism and the data are generated within a single simulation with a parsimonious parametrization.

Concerning the role of longevity the theory predicts a non linear effect of exogenous reductions of mortality for changes in education (and growth) which appear in line with empirical evidence. We illustrate the implications of the model concerning the permanent differences in the mortality environment by replicating the simulation with a controlled variation in baseline longevity only. We find that the main effect of differences in baseline mortality is to delay the timing of the take-off. The exercise suggests that differences in extrinsic mortality have the potential to explain part of the large differences in development outcomes across countries. Taken together, the model predictions suggest that the observation of numerous countries still being in a development trap today can potentially be explained by even moderate differences in the extrinsic mortality environment.

The dimension in which there is most room for improvement concerns the speed of the transition implied by the theory. The comparison between simulation and data reveals a faster transition in the cross-country data than what is predicted by the model. This acceleration is likely be the result of changes in other dimensions not incorporated in the model. For example, in reality improvements in technology and medical knowledge are likely to be coupled with substantial cross-country spill-overs in both dimensions, in particular in the last 40 years. This is likely to have affected the speed of the development process. Another dimension that is likely to be relevant is the (endogenous) evolution of institutions and public policies. Extending the model to investigate the role of these factors is a promising direction for future research.

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A Appendix

A.1 Proofs of Propositions

Proof of Lemma 1

The indirect utility from investment in each type of human capital is given by

$$\left[\left(T_t\left(1-r_t^{i*}\pi_t n_t^{ij*}\right)-e_t^{ij*}\right)w_t^j h_t^j\left(a,r_{t-1},e_t^{ij*}\right)\right]^{(1-\gamma)}\left[y_{t+1}^i\left(r^*\right)\pi_t n_t^{ij*}\right]^{\gamma}\,. \tag{38}$$

Recalling that $r_t^{i*}=r^*$, i.e., it does not depend on the type of human capital acquired, and comparing the indirect utility for investment in u and s we have $u_t^{u*}\gtrless u_t^{s*}$ which implies

$$\begin{aligned}&\left[\left(T_t\left(1-r^*\pi_t n_t^{iu*}\right)-e_t^{iu*}\right)w_t^u h_t^u\left(a,r^*,e_t^{iu*}\right)\right]^{(1-\gamma)}\left[n_t^{iu*}\right]^{\gamma}\\ \gtrless\ &\left[\left(T_t\left(1-r^*\pi_t n_t^{is*}\right)-e_t^{is*}\right)w_t^s h_t^s\left(a,r^*,e_t^{is*}\right)\right]^{(1-\gamma)}\left[n_t^{is*}\right]^{\gamma}\,.\end{aligned}$$

Substituting for n_t^{ij*}, e_t^{ij*} $h_t^j\left(a,r^*,e_t^{ij*}\right)$ from (12), (13) and (14) we have

$$\begin{aligned}&\left[\left(T_t\left(1-\frac{\gamma}{2-\gamma}\frac{T_t-\underline{e}^u}{T_t}\right)-\frac{T_t\left(1-\gamma\right)+\underline{e}^u}{2-\gamma}\right)w_t^u\alpha^u f\left(r^*,.\right)\frac{(1-\gamma)}{(2-\gamma)}\right]^{(1-\gamma)}\left[\frac{\gamma}{2-\gamma}\frac{T_t-\underline{e}^u}{T_t r^*\pi_t}\right]^{\gamma}\\ \gtrless\ &\left[\left(T_t\left(1-\frac{\gamma}{2-\gamma}\frac{T_t-\underline{e}^s}{T_t}\right)-\frac{T_t\left(1-\gamma\right)+\underline{e}^s}{2-\gamma}\right)w_t^s\alpha^s f\left(r^*,.\right)\frac{(1-\gamma)}{(2-\gamma)}a\right]^{(1-\gamma)}\left[\frac{\gamma}{2-\gamma}\frac{T_t-\underline{e}^s}{T_t r^*\pi_t}\right]^{\gamma}\end{aligned}$$

Due to the monotonicity of u_t^{h*} in ability, all agents with $a<\widetilde{a}_t$ optimally choose to acquire p, while those with ability $a>\widetilde{a}$ optimally choose to obtain h. Solving the previous expression as equality and rearranging we get the threshold (15),

$$\widetilde{a}=\frac{w^u}{w^s}\frac{\alpha^u}{\alpha^s}\left(\frac{T_t-\underline{e}^u}{T_t-\underline{e}^s}\right)^{\frac{2-\gamma}{1-\gamma}}\,. \tag{39}$$

Proof of Proposition 3

From conditions (14) and (21) we can derive the equilibrium average human capital of both types for generation t. Recalling that the distribution $d\left(a\right)$ is uniform we have,

$$h_t^u=\alpha^u f\left(r_{t-1},\cdot\right)\frac{(1-\gamma)}{(2-\gamma)}\left(T_t-\underline{e}^u\right)a\quad\text{and}\quad h_t^s=\alpha^s f\left(r_{t-1},\cdot\right)\frac{(1-\gamma)}{2(2-\gamma)}\left(T_t-\underline{e}^s\right)\left(1-a^2\right) \tag{40}$$

and the corresponding wage ratio,

$$\frac{w_t^u}{w_t^s}=\frac{1-x_t}{x_t}\left(\alpha\frac{\left(T_t-\underline{e}^s\right)}{\left(T_t-\underline{e}^u\right)}\frac{\left(1-a^2\right)}{2a}\right)^{1-\eta}\,. \tag{41}$$

Substituting the optimal human capital supplies (40) and the wage ratio (41) into condition (15), the equilibrium share of individuals with skilled human capital is implicitly characterized by,

$$\left(\frac{\left(1-\widetilde{a}_t^{*2}\right)^{1-\eta}}{\widetilde{a}_t^{*2-\eta}}\right)^{1-\gamma}\left[\left(\frac{1-x_t}{x_t}\right)^{1-\gamma}\left(\frac{1}{2}\right)^{(1-\eta)(1-\gamma)}\alpha^{\eta(1-\gamma)}\right]\left(\frac{T-\underline{e}^u}{T-\underline{e}^s}\right)^{1+\eta(1-\gamma)}=1\,,$$

as in condition (23). After some manipulations, one can derive,

$$(\mathcal{G}(\widetilde{a}_t^*)\mathcal{F}_t(x_t,\alpha)-1)\,T_t=\underline{e}^s\left(\mathcal{G}\left(\widetilde{a}_t^*\right)\mathcal{F}_t\left(x_t,\alpha\right)\right)^{1/(1+\eta(1-\gamma))}-\underline{e}^u \tag{42}$$

where

$$\mathcal{G}(\widetilde{a}_t^*)=\left(\left(1-\widetilde{a}_t^{*2}\right)^{1-\eta}/\widetilde{a}_t^{*2-\eta}\right)^{1-\gamma} \tag{43}$$

and

$$\mathcal{F}_t\left(x_t\right)=\left[\left(\frac{1-x_t}{x_t}\right)\left(\frac{1}{2}\right)^{(1-\eta)}\alpha^\eta\right]^{(1-\gamma)}. \tag{44}$$

Solving (42) for T_t one obtains the equilibrium relationship between life expectancy and the ability threshold for the extensive margin of human capital acquisition,

$$T_t=\frac{\underline{e}^s\left(\mathcal{G}\left(\widetilde{a}_t^*\right)\mathcal{F}_t\left(x_t\right)\right)^{-1/(1+\eta(1-\gamma))}-\underline{e}^u}{\left(\mathcal{G}(\widetilde{a}_t^*)\mathcal{F}_t\left(x_t\right)\right)^{-1/(1+\eta(1-\gamma))}-1}. \tag{45}$$

Equation (45) implicitly identifies the unique equilibrium share of the population that optimally decides to acquire skilled human capital in general equilibrium, $\lambda_t^*=1-\widetilde{a}_t^*$.

To simplify treatment express $\underline{e}^s=\Delta+\underline{e}^u$. Rewrite (45) as

$$\begin{aligned}&T_t\left(\left(\mathcal{G}(\widetilde{a}_t^*)\mathcal{F}_t\left(x_t\right)\right)^{-1/(1+\eta(1-\gamma))}-1\right)\\ =\;&\Delta\left(\mathcal{G}\left(\widetilde{a}_t^*\right)\mathcal{F}_t\left(x_t\right)\right)^{-1/(1+\eta(1-\gamma))}+\underline{e}^u\left(\left(\mathcal{G}\left(\widetilde{a}_t^*\right)\mathcal{F}_t\left(x_t\right)\right)^{-1/(1+\eta(1-\gamma))}-1\right).\end{aligned} \tag{46}$$

To simplify notation we omit the time index and we denote $\widetilde{T}:=T-\underline{e}^u$, $\widetilde{a}^*$ simply as a, $\mathcal{G}\left(\widetilde{a}^*\right)^{1/(1+\eta(1-\gamma))}:=\widetilde{\mathcal{G}}\left(a\right)$, $\mathcal{F}_t\left(x\right)^{1/(1+\eta(1-\gamma))}:=\widetilde{\mathcal{F}}\left(x\right)$ and $\sigma:=\frac{1-\gamma}{(1+\eta(1-\gamma))}\in(0,1)$. Notice that from (43) we have,

$$\widetilde{\mathcal{G}}\left(a\right)=\left(\frac{\left(1-a^2\right)^{1-\eta}}{a^{2-\eta}}\right)^\sigma. \tag{47}$$

Rearrange (46) to get, the equilibrium relationship between the ability share and life expectancy which can be expressed as,

$$\widetilde{T}=\frac{\Delta}{1-\widetilde{\mathcal{G}}\left(a\right)\widetilde{\mathcal{F}}_t\left(x\right)}. \tag{48}$$

This implies a negative relationship between $\widetilde{T}$ and a since,

$$\frac{\partial\widetilde{T}}{\partial a}=\Delta\frac{\frac{\partial\widetilde{\mathcal{G}}(a)}{\partial a}\widetilde{\mathcal{F}}_t\left(x\right)}{\left[1-\widetilde{\mathcal{G}}\left(a\right)\widetilde{\mathcal{F}}_t\left(x\right)\right]^2}=\widetilde{T}\frac{\frac{\partial\widetilde{\mathcal{G}}(a)}{\partial a}\widetilde{\mathcal{F}}_t\left(x\right)}{\left[1-\widetilde{\mathcal{G}}\left(a\right)\widetilde{\mathcal{F}}_t\left(x\right)\right]}<0. \tag{49}$$

Differentiating (47) we get

$$\frac{\partial\widetilde{\mathcal{G}}\left(a\right)}{\partial a}=-\sigma\widetilde{\mathcal{G}}\left(a\right)l\left(a\right)<0\,,\forall a\in[0,1] \tag{50}$$

where

$$l\left(a\right)=\frac{\left(2-\eta-\eta a^2\right)}{\left(1-a^2\right)a}>0\,. \tag{51}$$

Finally notice that since $\lambda=1-a$ equation (49) implies a positive relationship between the share of skill individuals λ and adult longevity T.

Now notice that, for any x, the function (48) is defined over the range $a \in (\underline{a}(x), 1]$ where[65]

$$\underline{a}(x) : \widetilde{\mathcal{G}}(a) \widetilde{\mathcal{F}}_t(x,) = 1 \,. \tag{52}$$

Applying calculus it can also be shown that $\partial \underline{a}/\partial x < 0$ with $\lim_{x \to 0} \underline{a}(x) = 1$ and $\lim_{x \to \infty} \underline{a}(x) = 0$. Accordingly for any x there exists a level $\overline{\lambda}(x) := 1 - \underline{a}(x) < 1$ which represents the maximum share of the population that at each moment in time would acquire skilled human capital in the case in which $T \to \infty$. This maximum share of skill agents is zero for $x \to 0$, is increasing in the productivity x and converges to one as $x \to \infty$,

$$\partial \overline{\lambda}(x) / \partial x > 0, \;\; \lim_{x \to 0} \overline{\lambda}(x) = 0, \;\; \lim_{x \to \infty} \overline{\lambda}(x) = 1 \,. \tag{53}$$

Notice that $\partial \widetilde{T}/\partial a = 0$ for $a = 1$ and $a = \underline{a}(x)$ which implies that $\partial \widetilde{T}/\partial \lambda = 0$ for $\lambda = 0$ and $\lambda = \overline{\lambda}(x)$. Since the function $\widetilde{T}$, (48), is continuous and strictly monotonically increasing with zero slope at $\lambda = 0$ and $\lambda = \overline{\lambda}(x)$ it must change concavity for some $\lambda \in \left[0, \overline{\lambda}(x)\right)$. To show that the function (48) is S-shape, i.e. that the there is a unique inflection point notice that $\widetilde{T}(a)$, being strictly monotonically decreasing, is invertible in the range $a \in (\underline{a}(x), 1]$. We show that there exists one and only one a for which the second derivative of this function equals zero and, accordingly, exists a unique λ at which the function changes concavity. Computing the second derivative of (48),

$$\frac{\partial^2 T(a)}{\partial a^2} = \Delta \frac{\frac{\partial^2 \widetilde{\mathcal{G}}(a)}{\partial a^2}\left[1 - \widetilde{\mathcal{G}}(a) \widetilde{\mathcal{F}}_t(x)\right] - 2\left[\frac{\partial \widetilde{\mathcal{G}}(a)}{\partial a}\right]^2 \widetilde{\mathcal{F}}_t(x)}{\left[1 - \widetilde{\mathcal{G}}(a) \widetilde{\mathcal{F}}_t(x)\right]^3} \tag{54}$$

and using (50) can be rewritten as

$$\frac{\partial^2 T(a)}{\partial a^2} = \Delta \frac{\widetilde{\mathcal{F}}_t(x) \widetilde{\mathcal{G}}(a) \left(l(a)^2 - \frac{\partial l(a)}{\partial a}\right)\left[1 - \widetilde{\mathcal{G}}(a) \Omega\right] + 2\left[\widetilde{\mathcal{G}}(a) l(a)\right]^2 \widetilde{\mathcal{F}}_t(x)^2}{\left[1 - \widetilde{\mathcal{G}}(a) \widetilde{\mathcal{F}}_t(x)\right]^3} \,.$$

The inflection point, that is the level of a at which $\partial^2 T(a)/\partial a^2 = 0$, is the solution of,

$$\widetilde{\mathcal{G}}(a)\left(\frac{\partial l(a)}{\partial a} - l(a)^2\right)\left[1 - \widetilde{\mathcal{G}}(a) \widetilde{\mathcal{F}}_t(x)\right] = 2\left[\widetilde{\mathcal{G}}(a) l(a)\right]^2 \widetilde{\mathcal{F}}_t(x) \,. \tag{55}$$

From (51), and computing $\partial l(a)/\partial a$, the previous expression can be simplified to get,

$$\frac{\partial l(a)/\partial a}{l(a)^2} - 1 = 2\left[\frac{\widetilde{\mathcal{G}}(a) \widetilde{\mathcal{F}}_t(x)}{\left[1 - \widetilde{\mathcal{G}}(a) \widetilde{\mathcal{F}}_t(x)\right]}\right] \,. \tag{56}$$

Standard calculus shows that the left hand side is a strictly increasing and continuous function of a and the right hand side is a strictly decreasing and continuous function in the range $a \in (\underline{a}, 1]$ so that, by intermediate function theorem, there is a unique level of a such that (54) equal zero. Accordingly there exists a unique inflection point λ.

Proof of Lemma 2

From Proposition 3 for any $T_t > \underline{e}^s$ and any $A_t > 0$, we have $\lambda_t^* > 0$ which, from (40), also implies $h_t^s > 0$ for all t. From (26) this implies $g_t > 0$ so that $A_t > A_{t-1}$ $\forall t$. From (27) it implies

[65] Since the denominator of (48) has a discontinuity at $\underline{a}$ and the function takes negative values for any $a \leq \underline{a}$.

that A_t increases monotonically overtime with $\lim_{t\to\infty} A_t = \infty$ for any $A_0 > 0$.[66] Accordingly from (28) we have $x_t > x_{t-1}$ with $\lim_{t\to\infty} x_t = 1$.

Proof of Proposition 4

Consider the equilibrium relationship linking $\widetilde{a}$ and T implied by (23). For any T, the equilibrium a is an implicit function of x. By implicit differentiation of (23) $\partial\widetilde{a}/\partial x < 0$ which implies that the equilibrium share of skilled agents is increasing in $x : \partial\lambda/\partial x > 0$ for any T. Graphically this implies that an increase in x moves the locus Λ upwards (counterclockwise). Furthermore, from (23) one sees immediately that, if $x = 0$, then $\widetilde{a} = 1$ and $\lambda = 0$ for all T. Notice also that if $x = 0$ then $\overline{\lambda}(0) \simeq 0$; $\forall T \in (\underline{e}^s, \infty)$ implying that a very small fraction of the population optimally acquires skilled human capital irrespective of T. Hence for $x \simeq 0$ the Λ is basically a flat line on the abscissa's axis. This implies that the loci Λ and Υ cross only once for $\lambda = 0$ and $T = \underline{T}$. Hence, from (19), the respective average fertility is given by n^u as implied by (12) evaluated at T. This immediately implies (36). Similarly if $x = 1$ then from (23), $\widetilde{a} = 0$ and $\lambda = 1$ (while $\overline{\lambda}(1) \simeq 1$). From (30) this implies that $T = \overline{T} \in (\underline{e}^s, \infty)$ such that $\overline{T} = \Upsilon\left(\overline{T}, 1\right)$ which implies (37).

Proof of Proposition 5

Consider first part *(i).* Take a A_0 sufficiently low such that $x_0 \simeq 0$ which implies, from Proposition 4, $\lambda_0 \simeq 0$, $T = \underline{T}$. In these conditions, from (2) and (40), the level of income per capita is, arbitrarily, low so that from (25) $\pi_0 \simeq \underline{\pi}$ with gross and net fertility characterized in (36). Next we characterized part *(iii).* Lemma 2 and Proposition 4 jointly imply that $A_\infty = \infty\,, x_\infty = 1$, $\lambda_\infty \simeq 1$, $T = \overline{T}$. Since $A_\infty = \infty$ and the fact that, from (18), average fertility n is bounded from above the per capita income grows unboundedly, $y_\infty = \infty$ which, from (25), implies $\pi_\infty \simeq 1$ and fertility given in (37). From part (i) and (iii) we know that the conditional system (35) is characterized by a unique steady state for A_0 and A_∞. Part *(ii)* follows directly from the fact that $\partial\lambda/\partial x > 0$ for any T as proved above. This implies a monotonic counterclockwise shift in the locus Λ making the locus steeper and a monotonic increase of λ for any T making the locus flatter. Similarly for any λ an increase in T leads to a clockwise shift in the locus Υ. This implies, in particular, that since the functions (29) and (30) are continuous it is always possible to identify a vectors (x', T') such that the two loci are tangent. In this situation the system is characterized by one stable and one unstable steady state with the latter characterizing the levels of T and λ. From the proof of Lemma 2 we know that $x_t > x_{t-1}$ which implies that in the next generation we would have $x > x'$. Under these conditions, the implied counterclockwise shift of Λ means that the conditional system is now characterized by a unique and stable steady state (part *(iii)*). At this stage the levels of T and λ, whose values were previously in line with (now disappeared) steady state, start converging toward the unique, and globally stable, steady state.

[66] With the simple function $A_t = \left(\delta\left(h^s_{t-1}\right)^{\psi} A^{\phi}_{t-1} + 1\right) A_{t-1}$ used in the simulation (with $\delta > 0$, $\phi > 0$, and $\psi > 0$) $A_t = d_{t-1}A$. Starting with any $A_0 > 0$ we can rewrite $A_t = \left(\prod_{i=1}^{t} d_{i-1}\right) A_0$, where $\left(\prod_{i=1}^{t} d_{i-1}\right) > 1$ and $\lim_{t\longrightarrow\infty}\left(\prod_{i=1}^{t} d_{i-1}\right) = \infty$.

A.2 Parametrization of the Model for Simulation

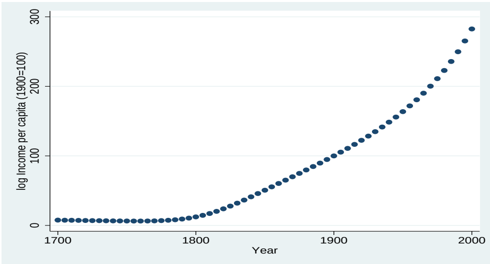
We simulate the model using a parametrization that reflects realistic values or generates realistic values of critical moments. As benchmark, we assume a lower bound to adult life expectancy in an undeveloped economy of $\underline{T} = 50$ years, which appears reasonable given the historical data presented before. Given the intergenerational externality $T_t = \underline{T} + \rho h^s_{t-1}$ as mentioned in footnote 27, we choose $\rho = 25$, which implies an upper bound of adult longevity, when substituting the respective values for $\lambda = 1$, of $\overline{T} = 75$ years.[67] The fix time cost for skilled education is assumed to be 15 years, the fix time cost of applied education 5 years. For lack of better data, the weight of children in the utility function, γ, is set to 1/2. One unit of time is assumed to be eight times as productive in acquiring skilled, compared to unskilled, human capital, with $\alpha = 8$ and $\alpha^u = 0.1$. To generate stable population patterns with a net fertility of 1 for a developed economy, we choose a time cost of raising a surviving child of 25/90. Given the formulation for child survival probability in footnote 28 and an initial aggregate gross fertility reflecting the fertility of the unskilled, we choose $\underline{\pi} = 0.648$ to generate an initial fertility that is 5/3 times that of a developed economy, roughly in line with the numbers of Figure 5.[68] We simulate the model for 400 generations, with new generations born at a frequency of five years.[69] The production function is specified as in condition 2 with $\eta = 0.4$. The technological environment parameterized as in footnote 32, with $\phi = 0.25$, $\psi = 0.9$, and $\delta = 0.015$. The production weight of skilled human capital is specified as $x_t = 1 - (A_0/A_t)^\chi$ with $\chi = 1.1$ and $A_0 = 2$.

[67] Alternatively, we investigated a specification in which adult longevity depends on the average human capital per capita of the previous generation. In this case, the specification implies a fix point of $\overline{T} = (T - \rho \cdot c \cdot \underline{e}^s)(1 - \rho \cdot c)$ with $c = \frac{\alpha^s(1-\gamma)}{2(2-\gamma)}$. The model behavior is qualitatively identical, but more difficult to characterize analytically as the Υ-locus also moves counterclockwise over time.

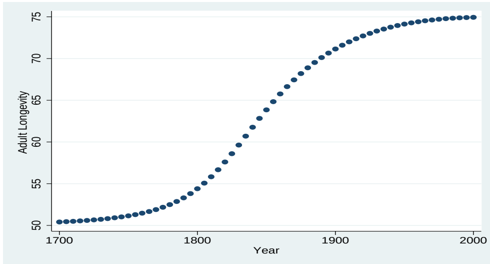
[68] The other parameters for the externality on child survival probability are chosen as $q = 0.04$ and $\mu = 1.5$. To add some realism, we additionally assume that 2.5 percent of the adult population does not reproduce.

[69] With the year 500 as the initial period, this reflects a horizon until 2500, which includes the period of economic and demographic transition.

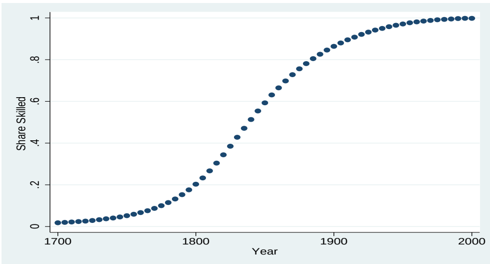
Figure 4: A Simulation of the Development Process



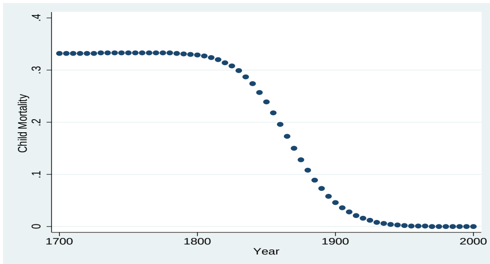
(a) log GDP per capita ($\ln y$)



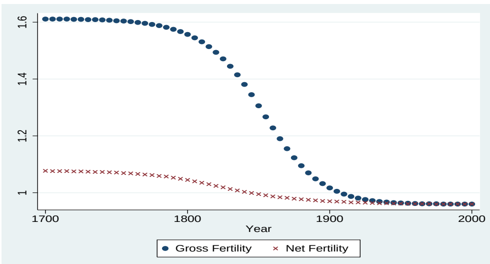
(b) Adult Life Expectancy (T)



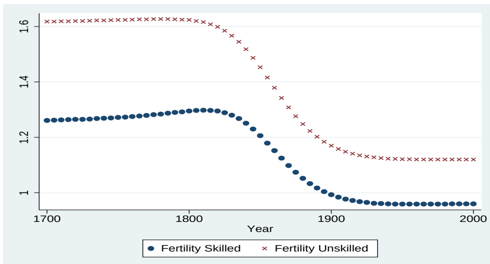
(c) Share Skilled (λ)



(d) Child Mortality

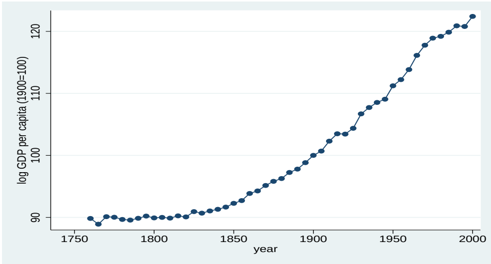


(e) Gross and Net Reproduction Rates (n, πn)

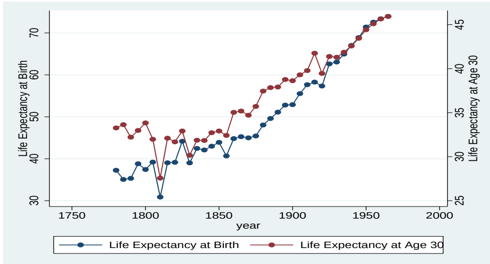


(f) Differential Fertility (n_H, n_L)

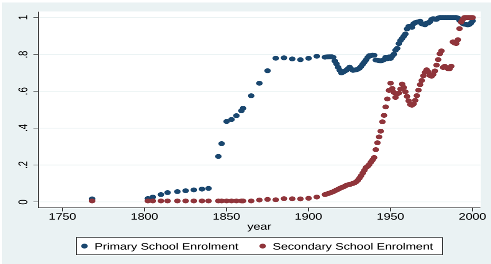
Figure 5: The Stylized Facts of Long-Run Development for Sweden



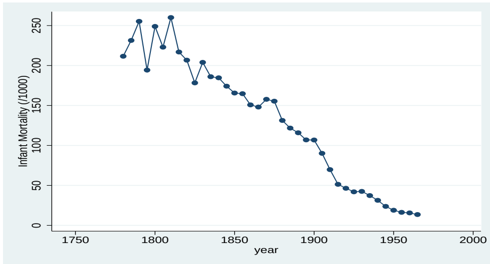
(a) log GDP per capita



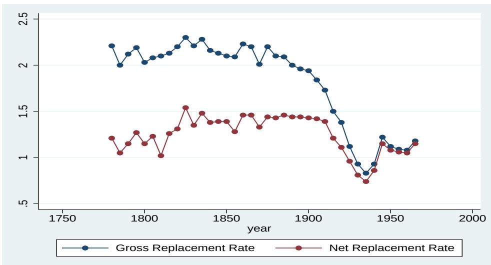
(b) Life Expectancy at Birth (left axis, lower graph) and Life Expectancy at Age 30 (right axis, upper graph)



(c) Primary and Secondary School Enrolment

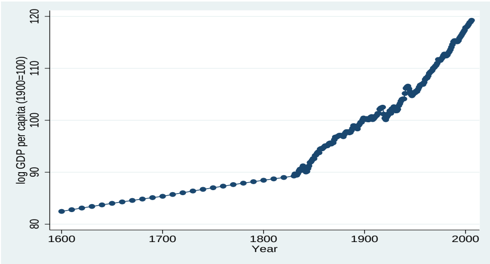


(d) Infant Mortality Rate

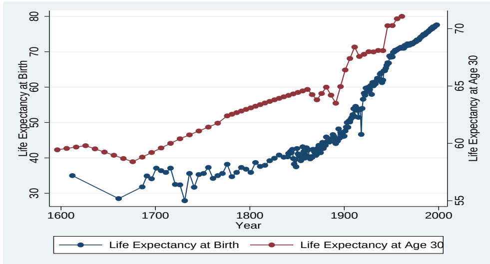


(e) Gross and Net Reproduction Rates

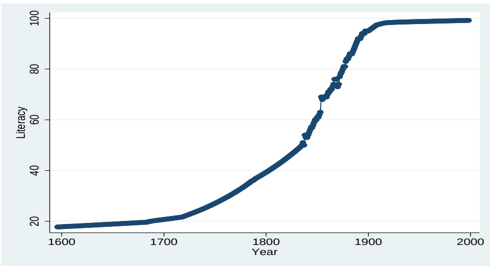
Figure 6: The Stylized Facts of Long-Run Development for England



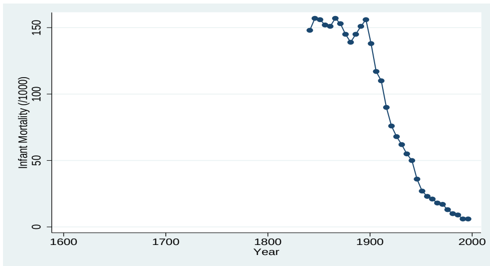
(a) log GDP per capita (U.K.)



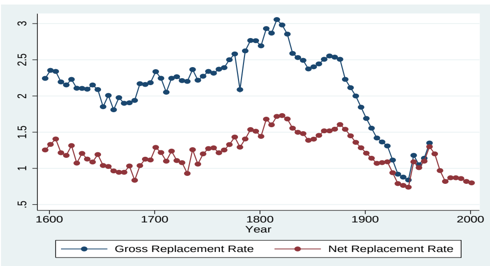
(b) Life Expectancy at Birth (left axis, lower graph) and Life Expectancy at Age 30 (right axis, upper graph) (England and Wales)



(c) Literacy Levels (England and Wales)

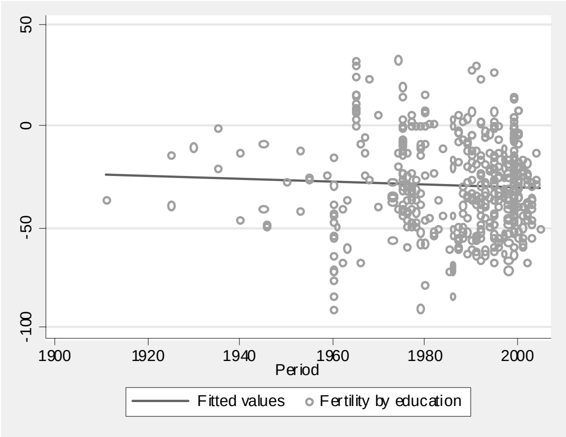


(d) Infant Mortality Rate

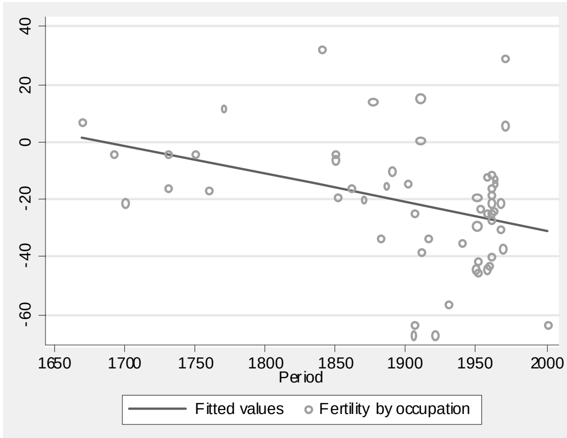


(e) Gross and Net Reproduction Rates (England and Wales)

Figure 7: Differential Fertility. Meta-Analysis of Studies in Demography by Skirbekk (2008)



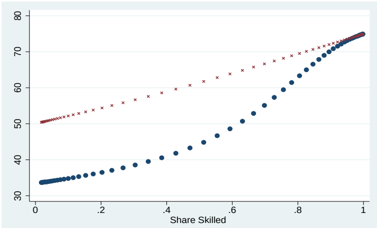
(a) Differential Fertility by Education



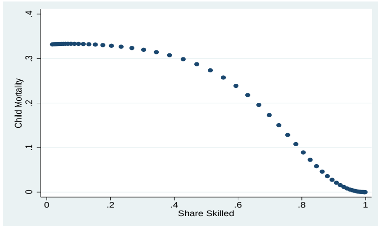
(b) Differential Fertility by Occupation

Data points reflect relative (gross) fertility of the highest education or occupation status group relative to the respective lowest group, $(n_{high} - n_{low})/n_{low}$, in each sample.

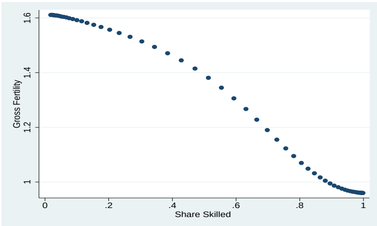
Figure 8: Cross-Sectional Implications: Simulation (one country)



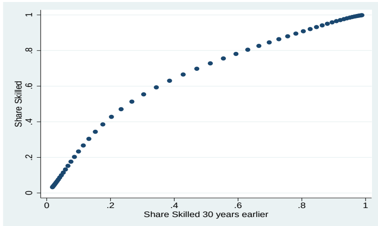
(a) Adult Longevity (x), Life Expectancy at Birth (o), and λ



(b) Child Mortality and λ

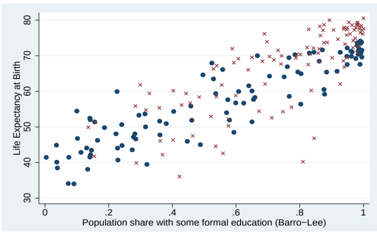


(c) Average Fertility and λ

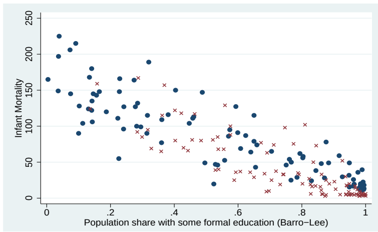


(d) λ_t and λ_{t-n} (n=30 yrs.)

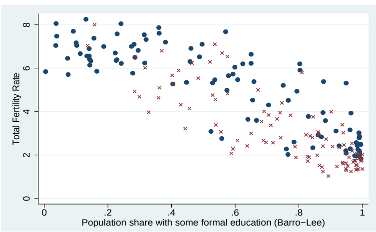
Figure 9: Cross-Sectional Implications: Data(λ = 1- population share with no schooling)



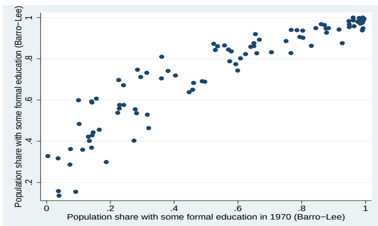
(a) Life Expectancy at Birth and λ, 1970 (o) and 2000 (x)



(b) Child Mortality and λ, 1970 (o) and 2000 (x)



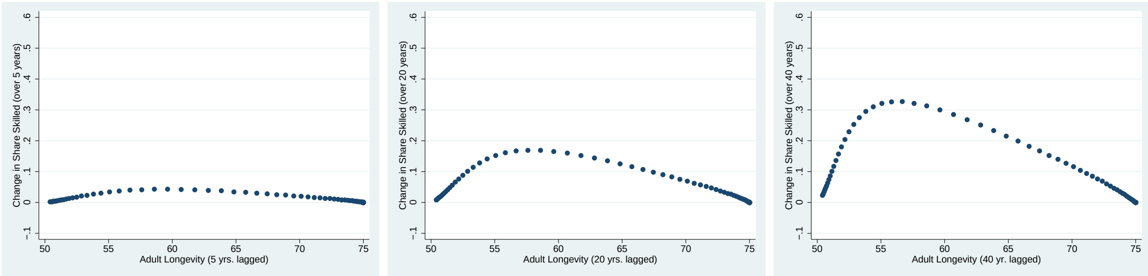
(c) Total Fertility Rate and λ, 1970 (o) and 2000 (x)



(d) λ 1970 and 2000

Data Sources: UN Population Division, World Mortality Report 2007, World Bank, World Development Indicators and Barro and Lee (2001).

Figure 10: Life Expectancy and Changes in Education Composition: Simulation

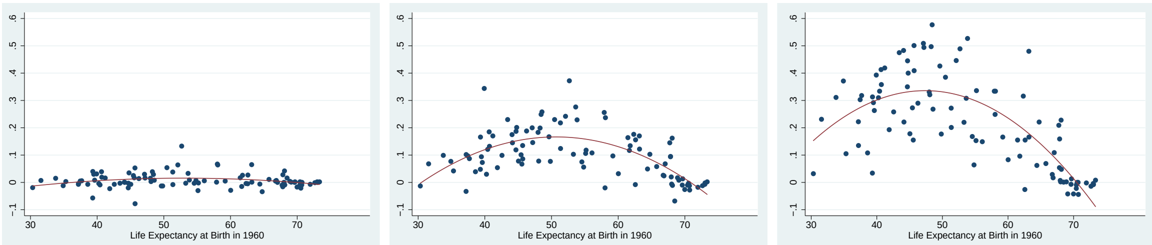


(a) Change in λ over 5 years

(b) Change in λ over 20 years

(c) Change in λ over 40 years

Figure 11: Life Expectancy and Changes in Education Composition: Data



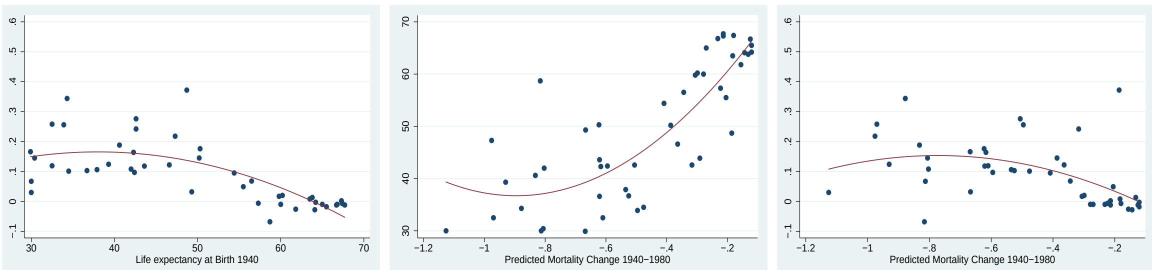
(a) Change in λ over 5 years (1960-1965)

(b) Change in λ over 20 years (1960-1980)

(c) Change in λ over 40 years (1960-2000)

Data Sources: UN Population Division, World Mortality Report 2007, and Barro and Lee (2001).

Figure 12: Exogenous Predicted Changes in Mortality in Education Composition: Data



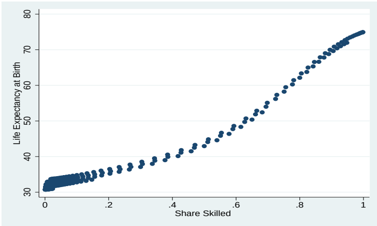
(a) Actual Life Expectancy in 1940 and Change in λ (1960-1980)

(b) Predicted Mortality Change 1940-1980 and Actual Life Expectancy in 1940

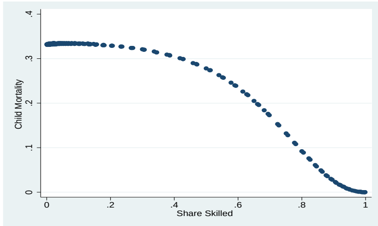
(c) Predicted Mortality Change 1940-1980 and Change in λ (1960-1980)

Data Sources: UN Population Division, World Mortality Report 2007, Acemoglu and Johnson (2007), Barro and Lee (2001).

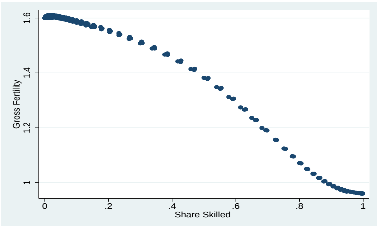
Figure 13: Cross-Sectional Implications: Simulation (5 countries, $\underline{T} \in \{46, 47, 48, 49, 50\}$)



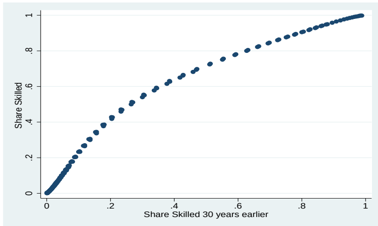
(a) Adult Life Expectancy and λ



(b) Child Mortality and λ

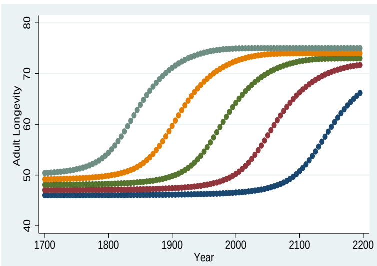


(c) Average Fertility and λ

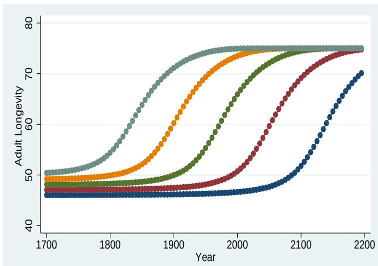


(d) λ_t and λ_{t-n} (n=30 yrs.)

Figure 14: Comparison of the Development Process in 5 Countries ($\underline{T} \in \{46, 47, 48, 49, 50\}$)

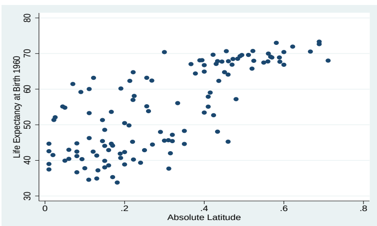


(a) Only Variation in $\underline{T}$

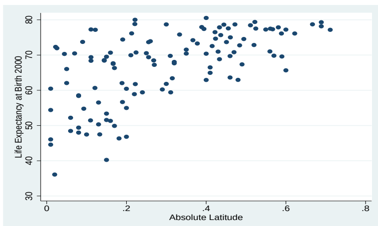


(b) Variation in $\underline{T}$ with fixed $\bar{T} = 75$

Figure 15: Geographic Latitude and Life Expectancy



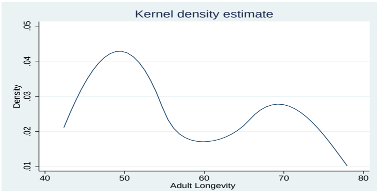
(a) Life Expectancy at Birth 1960



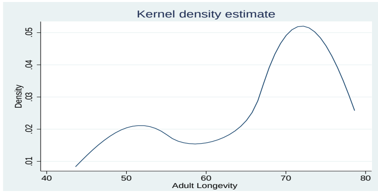
(b) Life Expectancy at Birth 2000

Data Sources: UN Population Division, World Mortality Report 2007, and CIA World Factbook.

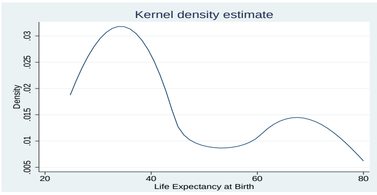
Figure 16: Cross-Sectional Implications: Simulation (81 countries, $\underline{T} \in [46, 50]$)



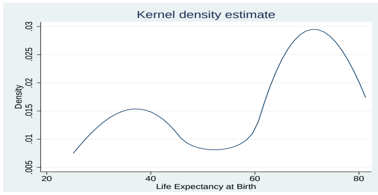
(a) Adult Longevity (early stages)



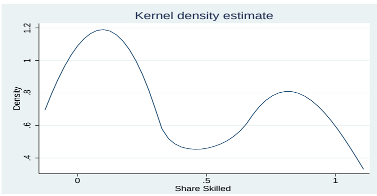
(b) Adult Longevity (late stages)



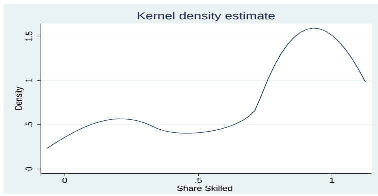
(c) Life Expectancy at Birth (early stages)



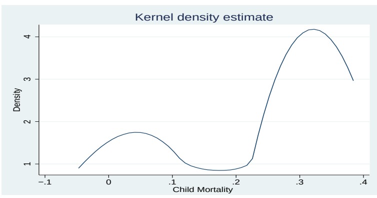
(d) Life Expectancy at Birth (late stages)



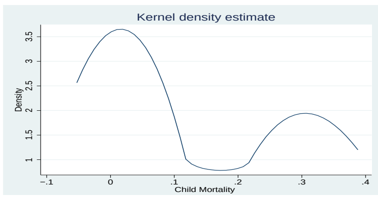
(e) Share Skilled (λ) (early stages)



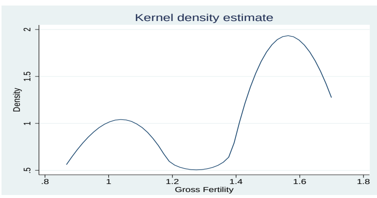
(f) Share Skilled (λ) (late stages)



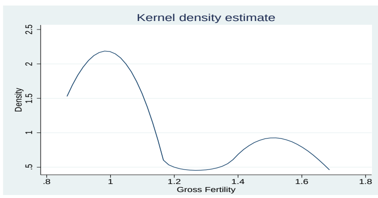
(g) Child Mortality ($1 - \pi$) (early stages)



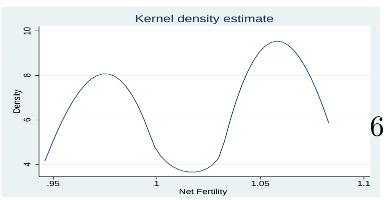
(h) Child Mortality ($1 - \pi$) (late stages)



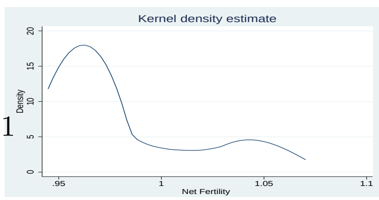
(i) Gross Fertility (early stages)



(j) Gross Fertility (late stages)

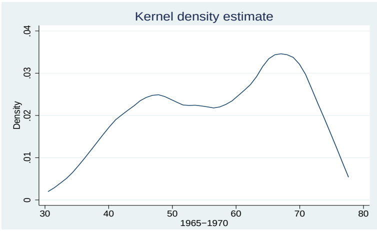


(k) Net Fertility (early stages)

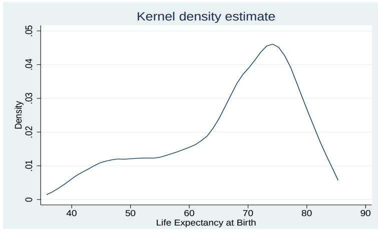


(l) Net Fertility (late stages)

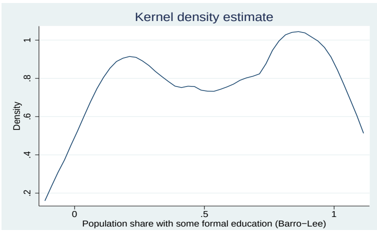
Figure 17: Cross-Sectional Implications: Data



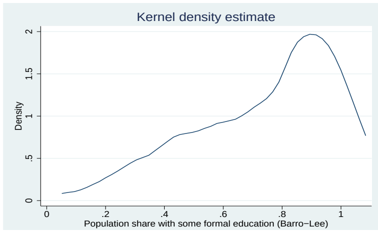
(a) Life Expectancy at Birth 1965-1970



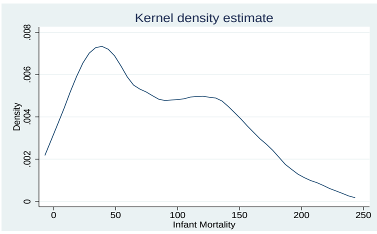
(b) Life Expectancy at Birth 2000-2005



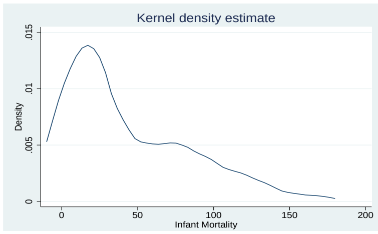
(c) Share with at least some schooling 1970



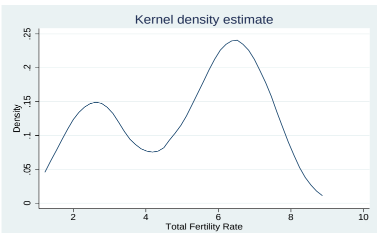
(d) Share with at least some schooling 2000



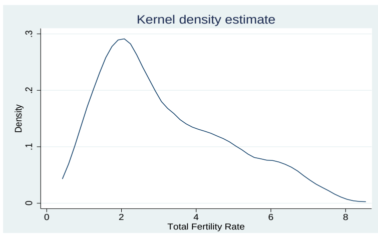
(e) Infant Mortality 1970



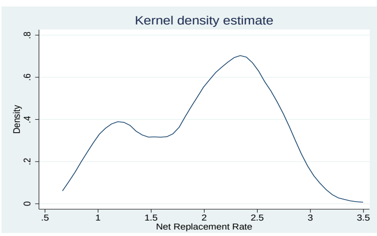
(f) Infant Mortality 2000



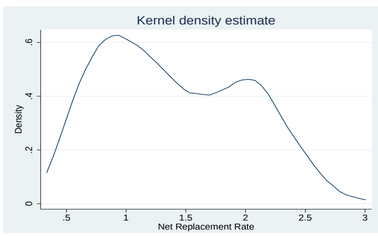
(g) Total Fertility Rate 1970



(h) Total Fertility Rate 2000



(i) Net Replacement Rate 1970



(j) Net Replacement Rate 2000

Data Sources: UN Population Division, World Mortality Report 2007, and World Bank, World Development Indicators and Barro and Lee (2001).