The Limits to Partial Banking Unions: A Political Economy Approach

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Online Appendix

A Appendix A – Proofs (For Online Publication)

A.1 Proof of Proposition 1

Consider the problem for the policymaker in country i with a partial banking union $(\tau, \underline{x})$. Let $\zeta^i \equiv \{x^i, g^i, g_1^i, b_1^i\}$ be the policies chosen by policymaker i, with $i \in \{D, F\}$. Policymaker D solves

$$\max_{\zeta^D} u(c^D(x^D, x^F)) + w(g^D) + \beta w(g_1^D), \tag{A1}$$

subject to

$$
\begin{align}
x^D + g^D &\leq e^D + \beta b_1^D + \tau, \tag{A2a}\\
x^D &\geq \underline{x} \tag{A2b}\\
g_1^D &\leq e^D - b_1^D, \tag{A2c}\\
b_1^D &\in \left[-e^D/\beta, e^D\right], \tag{A2d}\\
x^D &\leq \theta I^D. \tag{A2e}
\end{align}
$$

According to Assumption 1, constraint (A2e) does not bind.
Policymaker F solves

$$\max_{\zeta^F} u(c^F(x^F, x^D)) + w(g^F) + \beta w(g_1^F), \tag{A3}$$

subject to

$$x^F + g^F \leq e^F + \beta b - \tau, \tag{A4a}$$

$$g_1^F \leq e^F - b_1^F, \tag{A4b}$$

$$b_1^F \in \left[-e^F/\beta, e^F\right], \tag{A4c}$$

$$x^F \leq \theta I^F. \tag{A4d}$$

According to Assumption 1, constraint (A4d) binds for policymaker F.

By the Envelope Theorem, problem (A1) is strictly concave in e^D and τ, and problem (A3) is strictly concave in e^F and τ.

In choosing $(\tau, \underline{x})$, the supranational authority faces the following maximization problem:

$$\max_{\tau, \underline{x}} \{ \eta \left[u(c^D(x^D, x^F)) + w(g^D) + \beta w(g_1^D) \right]$$
$$+ (1 - \eta) \left[u(c^F(x^F, x^D)) + w(g^F) + \beta w(g_1^F) \right] \}$$

subject to

$$U^i(x^i, x^j, g^i, g_1^i) \geq U^i(x^{i0}, x^{j0}, g^{j0}, g_1^{i0}), \tag{A5}$$

where $i, j \in \{D, F\}, i \neq j$, and $\{x^{i0}, g^{i0}, g_1^{i0}\}$ denote the solution to policymaker $i's$ maximization problem when $\tau = 0$, $\underline{x} = 0$.

The supranational authority's objective function is a sum of utilities maximized in (A1) and (A3), so it is a strictly concave function of τ. Then, any solution to the supranational authority's problem that involves $\tau > 0$ implies a strict increase in the utility of the supranational authority. Given the participation constraints of the two governments, (A5), it follows that the utility of households in at least one country must increase.

A.2   Proof of Proposition 2

Assumption 1 guarantees that $\tau \leq e^F - e^{F*}$, where

$$e^{F*} = \frac{\theta I^F}{1 + \beta} + g^{F*} + \frac{r^{F*}}{1 + \beta},$$

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with g^{F*} and r^{F*} defined in Assumption 1. This means that full recapitalizations are provided in country F ($x^F = \theta I^F$) even if transfers are made to country D.

Step 1. *The policymakers' problem*

Consider a partial banking union with terms τ and $\underline{x}$. Let be the λ^D, ϑ^D, and $\beta\mu^D$ be the Lagrange multipliers on constraints (18a), (18b), and (18c), respectively. The first-order conditions to problem (17) when constraint (18b) binds and there is an interior solution lead to

$$(1 - \gamma^D)v'(r^D) = \gamma^D \sigma^D R u'(c^D), \tag{A6a}$$

$$r^D + x^D = \underline{x}, \tag{A6b}$$

$$g^D = g_1^D = e^D + \frac{\tau - \underline{x}}{1 + \beta}. \tag{A6c}$$

The maximization problem for policymaker F given $\{\tau, \underline{x}\}$ is to choose $\zeta^F = \{r^F, x^F, g^F, g_1^F, b^F\}$ to solve

$$\max_{\zeta^F} \left(1 - \gamma^F\right) v(r^F) + \gamma^F \left[u(c^F(x^F, x^D)) + w(g^F) + \beta w(g_1^F)\right] \tag{A7}$$

subject to

$$r^F + x^F + g^F \leq e^F + \beta b^F - \tau, \tag{A8a}$$

$$g_1^F \leq e^F - b^F, \tag{A8b}$$

$$b^F \in [\underline{b}^F, e^F], \tag{A8c}$$

$$x^F \leq \theta I^F, \tag{A8d}$$

where constraint (A8d) binds.

The first-order conditions for an interior solution for r^F and g^F imply

$$(1 - \gamma^F)v'(r^F) = \gamma^F w'(g^F), \tag{A9a}$$

$$g_1^F = g^F, \tag{A9b}$$

$$x^F = \theta I^F. \tag{A9c}$$

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Step 2. *The supranational authority's problem*

The supranational authority sets $\tau \geq 0$ and $\underline{x} \geq 0$ in order to maximize (9) given (10) and (11). The minimum reinvestment requirement is $r^D + x^D \geq \underline{x}$. Setting $\underline{x}$ at least equal to the policymaker's unconstrained choices is a weakly dominant strategy, so constraint (18b) holds with equality for policymaker D. The policymaker's utility from rents r^D and recapitalizations x^D is concave and additive, so a binding $\underline{x}$ implies $r^D \geq r^{D0}$ and $x^D \geq x^{D0}$. Then $v(r^D) \geq v(r^{D0})$, and (10) is satisfied as long as

$$U^D(x^{D0}, x^{F0}, g^{D0}, g_1^{D0}) - U^D(x^D, x^F, g^D, g_1^D) \leq \frac{(1 - \gamma^D)}{\gamma^D} \left[v(r^D) - v(r^{D0})\right].$$
(A10)

Step 3. *We show that if constraint (10) does not bind for some $\eta^C \in (0,1)$, then it does not bind $\forall \eta \geq \eta^C$.*

Assume there exists a value $\eta^C \in (0,1)$ at which (10) does not bind.

Case A. Corner solution for x^D.

Consider the case in which $\underline{x} = \underline{x}^* = \theta I^D + r^{D*}$, with r^{D*} defined implicitly by $(1 - \gamma^D) v'(r^{D*}) = \gamma^D \sigma^D R u'(\theta I^D, \theta I^F)$. In this case, the maximum recapitalization is achieved at $\eta^C : x^D = \theta I^D$.

Let ι denote the Lagrange multiplier on constraint (11). Then, the first order-condition that determines τ is

$$\eta w'(g^D)\frac{\partial g^D}{\partial \tau} = w'(g^F)\left(-\frac{\partial g^F}{\partial \tau}\right)(1 - \eta + \gamma^F \iota)$$
$$+ \iota(1 - \gamma^F)v'(r^F)\left(-\frac{\partial r^F}{\partial \tau}\right).$$
(A11)

Given this condition, applying the Envelope Theorem, an increase in η would increase τ, which is equivalent to increasing e^D, so

$$\frac{\partial U^D(x^D, x^F, g^D, g_1^D)}{\partial \eta} > 0,$$
(A12)
$$r^D = r^{D*},$$
(A13)

and the policymaker's utility is also increasing; hence, (10) does not bind

$\forall \eta \geq \eta^C.$

Case B: Interior solution for $\underline{x}$.

The first-order conditions to the supranational authority's maximization problem, in case of an internal solution $(\tau, \underline{x})$, are:

$$
\begin{aligned}
\left[\left(1 - \sigma^D\right) R u'(c^F)(1 - \eta + \gamma^F \iota) \right. & \\
\left. + \eta \sigma^D R u'(c^D) \right] \frac{\partial x^D}{\partial \underline{x}} &= \eta w'(g^D) \left(-\frac{\partial g^D}{\partial \underline{x}} \right) (1 + \beta),
\end{aligned}
\tag{A14}
$$

$$
\eta w'(g^D) \frac{\partial g^D}{\partial \tau}(1 + \beta) = w'(g^F) \left(-\frac{\partial g^F}{\partial \tau} \right)(1 + \beta)(1 - \eta + \gamma^F \iota)
$$

$$
+ \iota(1 - \gamma^F) v'(r^F) \left(-\frac{\partial r^F}{\partial \tau} \right).
\tag{A15}
$$

From (A6a)-(A6c), $\frac{\partial x^D}{\partial \underline{x}} > 0$, $\frac{\partial g^D}{\partial \underline{x}}(1 + \beta) = -1$, $\frac{\partial g^D}{\partial \tau} = \frac{1}{(1+\beta)}$. From (A9a), $0 < -\frac{\partial g^F}{\partial \tau} < 1$ and $0 < -\frac{\partial r^F}{\partial \tau} < 1$. Then, from (A14) and (A15), applying the Envelope Theorem, an increase in η implies $\frac{\partial \underline{x}}{\partial \eta} < 0$ and $\frac{\partial \tau}{\partial \eta} > 0$. So

$$
\frac{\partial U^D(x^D, x^F, g^D, g_1^D)}{\partial \eta} > 0.
\tag{A16}
$$

From (18b)

$$
\frac{\partial V^D(x^D, x^F, g^D, g_1^D)}{\partial \underline{x}} \leq 0.
\tag{A17}
$$

From (18a),

$$
\frac{\partial V^D(x^D, x^F, g^D, g_1^D)}{\partial \tau} > 0.
\tag{A18}
$$

Then, constraint (10) does not bind for $\eta > \eta^C$.

Step 4. *We show that if for some $\eta^B \in (0, 1)$ constraint (10) binds, then it binds $\forall \eta \leq \eta^B$.*

Since $\underline{x}$ is at least as high as policymaker $D's$ policy choices, $r^D \geq r^{D0}$. If (10) binds, then τ is inferred implicitly from this constraint as

$$
\gamma^D(1 + \beta)w \left(e^D + \frac{\tau - \underline{x}}{1 + \beta} \right) = (1 - \gamma^D)v(r^{D0}) + U^D(x^{D0}, x^{F0}, g^{D0}, g^{D0})
$$

$$
- (1 - \gamma^D)v(r^D(\underline{x})) - \gamma^D u \left(c^D(x^D(\underline{x}), \theta I^F) \right).
\tag{A19}
$$

Case A. There is a corner solution for $\underline{x} = \underline{x}^*$, with $\underline{x}^*$ defined in Step 3.

A decrease in η would not change the value of $\underline{x}$ nor the value of τ. Hence, (10) binds $\forall \eta < \eta^B$.

Case B. If $\underline{x} < \underline{x}^*$.

Constraint (10) binding implies that the first-order conditions (A14) and (A15) become

$$\left[\left(1 - \sigma^D\right)Ru'(c^F)(1 - \eta^B + \gamma^F \iota) + \eta^B \sigma^D Ru'(c^D)\right]\frac{\partial x^D}{\partial \underline{x}}$$

$$-\eta^B w'(g^D)\left(-\frac{\partial g^D}{\partial \underline{x}}\right)(1 + \beta) \geq 0, \tag{A20}$$

$$\eta^B w'(g^D)\frac{\partial g^D}{\partial \tau} + (1 - \eta)w'(g^F)\frac{\partial g^F}{\partial \tau} \leq 0. \tag{A21}$$

Then, a decrease in η keeps the constraint (10) binding.

Step 5 *We show that there exists $\eta^{B*} \in (0,1)$ such that constraint (10) binds for $\eta < \eta^{B*}$ and it does not bind for $\eta > \eta^{B*}$.*

If $\eta = 0$, the supranational authority maximizes the utility of the F households only, so τ is minimized and $\underline{x}$ is maximized given constraint (10). At $\eta = 0$, the first-order conditions to the supranational authority's problem are given by (A20) and (A21), with strict inequality for both. The left-hand side of (A20) is strictly decreasing in η, and the left-hand side of condition (A21) is strictly increasing in η. By the continuity of the utility functions it then follows that $\exists \eta^{B*} > 0$ such that (A20) and (A21) hold with equality.

Step 6. *We show there exists $\eta^* \in (0,1)$ such that $U^D(x^{D0}, x^{F0}, g^{D0}, g_1^{D0}) = U^D(x^D, x^F, g^D, g_1^D)$.*

From Step 5, there exists $\eta^{B*} \in (0,1)$ such that constraint (10) binds. Since $r^D \geq r^{D0}$, it follows that $U^D(x^{D0}, x^{F0}, g^{D0}, g_1^{D0}) \geq U^D(x^D, x^F, g^D, g_1^D)$.

If $\eta = 1$, the supranational authority maximizes the utility of the D households, so the transfer τ will be at the maximum level at which the participation constraint for the F government is satisfied. It then follows that $U^D(x^D, x^F, g^D, g_1^D) > U^D(x^{D0}, x^{F0}, g^{D0}, g_1^{D0})$ and $v(r^D) > v(r^{D0})$.

Given (A12) in case A and (A16) in case B, and the continuity of $U^D(x^D, x^F, g^D, g_1^D)$

it follows that there exists $\eta^* \in (0, 1)$ such that.

$$U^D(x^{D0}, x^{F0}, g^{D0}, g_1^{D0}) - U^D(x^D(\eta^*), x^F(\eta^*), g^D(\eta^*), g_1^D(\eta^*)) = 0. \quad \text{(A22)}$$

A.3 Proof of Corollary 1

The value of η^* satisfies

$$
\begin{aligned}
u(c^D(x^D(\eta^*), x^F)) + (1 + \beta)w(g^D(\eta^*)) &= u(c^D(x^{D0}, x^{F0})) \\
&\quad + (1 + \beta)w(g^{D0}). \quad \text{(A23)}
\end{aligned}
$$

From the supranational authority's first-order condition (A15), an internal solution for $(\tau, \underline{x})$ implies that

$$\eta^* \geq \frac{1}{1 + \dfrac{w'(g^D(\eta^*))}{w'(g^F(\eta^*))\left(-\frac{\partial g^F}{\partial \tau}\right)}}. \quad \text{(A24)}$$

Define $\Delta x \equiv x^D - x^{D0}$ and let Δg^D be implicitly given by

$$
\begin{aligned}
&u(c^D(x^{D0} + \Delta x, x^F)) + (1 + \beta)w(g^{D0} - \Delta g^D) \\
&= u(c^D(x^{D0}, x^F)) + (1 + \beta)w(g^{D0}). \quad \text{(A25)}
\end{aligned}
$$

So, $w'(g^D(\eta^*)) = w'(g^{D0} - \Delta g^D)$. When $x^D = \theta I^D$, $\Delta x^{D,MAX} \equiv \theta I^D - x^{D0}$, and $\Delta g^{D,MAX}$ is given implicitly by

$$
\begin{aligned}
&u(c^D(x^{D0} + \Delta x^{D,MAX}, x^F)) + (1 + \beta)w(g^{D0} - \Delta g^{D,MAX}) \\
&= u(c^D(x^{D0}, x^{F0})) + (1 + \beta)w(g^{D0}). \quad \text{(A26)}
\end{aligned}
$$

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If $\tau(\eta^*) > 0$, then $g^F(\eta^*) < g^{F0}$. So

$$\frac{w'(g^D(\eta^*))}{w'(g^F(\eta^*))\left(-\frac{\partial g^F}{\partial \tau}\right)} \leq \frac{w'(g^{D0} - \Delta g^{D,MAX})}{w'(g^F(\eta^*))\left(-\frac{\partial g^F}{\partial \tau}\right)}$$

$$\leq \frac{w'\left(g^{D0} - \frac{\Delta x^{D,MAX}}{1+\beta}\right)}{w'(g^{F0})}, \qquad (A27)$$

where $\frac{\Delta x^{D,MAX}}{1+\beta} > \Delta g^{D,MAX}$ given the concavity of $u(\cdot)$ and $w(\cdot)$. Then,

$$\frac{w'\left(g^{D0} - \frac{\Delta x^{D,MAX}}{1+\beta}\right)}{w'(g^{F0})} \leq \frac{w'\left(g^{D0} + \frac{x^{D0}}{1+\beta} - \frac{\theta I^D}{1+\beta}\right)}{w'(e^F - \frac{\theta I^F}{1+\beta})}$$

$$= \frac{w'\left(e^D - \frac{r^{D0}}{1+\beta} - \frac{\theta I^D}{1+\beta}\right)}{w'(e^F - \frac{\theta I^F}{1+\beta})}. \qquad (A28)$$

Then, from (A27),

$$\frac{w'(g^D(\eta^*))}{w'(g^F(\eta^*))\left(-\frac{\partial g^F}{\partial \tau}\right)} \leq \frac{w'\left(e^D - \frac{r^{D0}}{1+\beta} - \frac{\theta I^D}{1+\beta}\right)}{w'(e^F - \frac{\theta I^F}{1+\beta})} = \Phi(\theta), \qquad (A29)$$

and so

$$\eta^* \geq \frac{1}{1 + \Phi(\theta)}. \qquad (A30)$$

A.4 Proof of Corollary 2

The value η^* is defined as the value at which

$$u(c^D(x^D(\eta^*), x^F)) + (1+\beta)w(g^D(\eta^*)) = u(c^D(x^{D0}, x^{F0}))$$
$$+ (1+\beta)w(g^{D0}), \qquad (A31)$$

where $x^F = x^{F0} = \theta I^F$, given Assumption 1.

The effect of increasing e^F

Case A: Corner solution with respect to x^D $\left(x^D = \theta I^D\right)$.

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In this case, we have a corner solution with respect to x^D, so $\frac{\partial x}{\partial \eta^*} = \frac{\partial x}{\partial e^F} = 0$. Applying the Envelope Theorem in (A31), we obtain

$$\frac{\partial \eta^*}{\partial e^F} = -\frac{\partial \tau}{\partial e^F}\left(\frac{\partial \tau}{\partial \eta^*}\right)^{-1}. \tag{A32}$$

From (A15), applying the Envelope Theorem, $\frac{\partial \tau}{\partial e^F} > 0$ and $\frac{\partial \tau}{\partial \eta^*} > 0$, so

$$\frac{\partial \eta^*}{\partial e^F} < 0. \tag{A33}$$

Case B: Internal solution with respect to x^D ($x^D < \theta I^D$)

Applying the Envelope Theorem in (A31), we obtain

$$\frac{\partial \eta^*}{\partial e^F} = -\frac{\Gamma}{\Upsilon}, \tag{A34}$$

where

$$\Gamma \equiv \sigma^D R u'(c^D)\frac{\partial x^D}{\partial \underline{x}}\frac{\partial \underline{x}}{\partial e^F} + w'(g^D)\left(\frac{\partial \tau}{\partial e^F} - \frac{\partial \underline{x}}{\partial e^F}\right) \tag{A35}$$

$$\Upsilon \equiv \sigma^D R u'(c^D)\frac{\partial x^D}{\partial \underline{x}}\frac{\partial \underline{x}}{\partial \eta^*} + w'(g^D)\left(\frac{\partial \tau}{\partial \eta^*} - \frac{\partial \underline{x}}{\partial \eta^*}\right) \tag{A36}$$

From (A14), applying the Envelope Theorem, $\frac{\partial \underline{x}}{\partial e^F} > 0$.

From (A15), applying the Envelope Theorem, $\frac{\partial \tau}{\partial e^F} > 0$ and $\frac{\partial g^D}{\partial e^F} > 0$, so

$$\frac{\partial \tau}{\partial e^F} - \frac{\partial \underline{x}}{\partial e^F} > 0. \tag{A37}$$

Similarly, applying the Envelope Theorem in (A14) and (A15), $\frac{\partial \underline{x}}{\partial \eta^*} < 0$, $\frac{\partial \tau}{\partial \eta^*} > 0$, and

$$\frac{\partial \tau}{\partial \eta^*} - \frac{\partial \underline{x}}{\partial \eta^*} > 0. \tag{A38}$$

So

$$\Gamma > 0, \tag{A39}$$

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and

$$
\begin{aligned}
\Upsilon &= \left[w'(g^D) - \sigma^D R u'(c^D) \frac{\partial x^D}{\partial \underline{x}} \right] \left(-\frac{\partial \underline{x}}{\partial \eta^*} \right) + w'(g^D) \frac{\partial \tau}{\partial \eta^*} \\
&= \frac{1 - \eta^*}{\eta^*} \left(1 - \sigma^D \right) R u'(c^F) \frac{\partial x^D}{\partial \underline{x}} \left(-\frac{\partial \underline{x}}{\partial \eta^*} \right) + w'(g^D) \frac{\partial \tau}{\partial \eta^*} \\
&> 0
\end{aligned}
\tag{A40}
$$

Then,

$$
\frac{\partial \eta^*}{\partial e^F} < 0. \tag{A41}
$$

The effect of increasing γ^F

Case A: Corner solution with respect to x^D $\left(x^D = \theta I^D \right)$.

In this case, the corner solution implies $\frac{\partial \underline{x}}{\partial \eta^*} = \frac{\partial \underline{x}}{\partial \gamma^F} = 0$. Applying the Envelope Theorem in (A31), we obtain

$$
\frac{\partial \eta^*}{\partial \gamma^F} = -\frac{\partial \tau}{\partial \gamma^F} \left(\frac{\partial \tau}{\partial \eta^*} \right)^{-1}. \tag{A42}
$$

From (A15), applying the Envelope Theorem, $\frac{\partial \tau}{\partial \gamma^F} > 0$ and $\frac{\partial \tau}{\partial \eta^*} > 0$, so

$$
\frac{\partial \eta^*}{\partial \gamma^F} < 0. \tag{A43}
$$

Case B: Internal solution with respect to x^D $\left(x^D < \theta I^D \right)$

As above, applying the Envelope Theorem in (A31), we obtain

$$
\frac{\partial \eta^*}{\partial \gamma^F} = -\frac{\Xi}{\Upsilon}, \tag{A44}
$$

where

$$
\Xi \equiv \sigma^D R u'(c^D) \frac{\partial x^D}{\partial \underline{x}} \frac{\partial \underline{x}}{\partial \gamma^F} + w'(g^D) \left(\frac{\partial \tau}{\partial \gamma^F} - \frac{\partial \underline{x}}{\partial \gamma^F} \right). \tag{A45}
$$

From (A14) and (A15), applying the Envelope Theorem, $\frac{\partial \tau}{\partial \gamma^F} > 0$, $\frac{\partial \underline{x}}{\partial \gamma^F} > 0$

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and $\frac{\partial \tau - x}{\partial \gamma^F} > 0$, which in (A44) leads to

$$\frac{\partial \eta^*}{\partial \gamma^F} < 0. \tag{A46}$$

A.5 Proof of Corollary 3

The value η^* is defined in (A31) above. The effect of a change in $\left(-\alpha^F\right)$ on σ^D and σ^F is given by

$$\frac{\partial \sigma^D}{\partial \left(-\alpha^F\right)} = -\frac{\alpha^D z^D z^F}{[\alpha^D z^D + (1 - \alpha^F)z^F]^2} = -\frac{\sigma^D z^F}{I^D}, \tag{A47}$$

$$\frac{\partial \sigma^F}{\partial \left(-\alpha^F\right)} = \frac{-z^F(1 - \alpha^D)z^D}{[\alpha^F z^F + (1 - \alpha^D)z^D]^2} = -\frac{(1 - \sigma^F)z^F}{I^F}. \tag{A48}$$

Applying the Envelope Theorem in (A31), and using the above expressions, we obtain

$$\frac{\partial \eta^*}{\partial \left(-\alpha^F\right)} = \frac{\Psi}{\sigma^D Ru'(c^D)\frac{\partial x^D}{\partial \underline{x}}\frac{\partial \underline{x}}{\partial \eta^*} + w'(g^D)\left(\frac{\partial \tau}{\partial \eta^*} - \frac{\partial \underline{x}}{\partial \eta^*}\right)}, \tag{A49}$$

where

$$\begin{aligned}
\Psi \equiv\ & \sigma^D Ru'(c^{D0}) \left(-\frac{z^F}{I^D}x^{D0} + \frac{\partial x^{D0}}{\partial \left(-\alpha^F\right)} + \frac{(1 - \sigma^F)}{\sigma^D}\theta z^F \right) \\
& -\sigma^D Ru'(c^D) \left(-\frac{z^F}{I^D}x^D + \frac{\partial x^D}{\partial \left(-\alpha^F\right)} + \frac{(1 - \sigma^F)}{\sigma^D}\theta z^F \right) \\
& +(1 + \beta)w'(g^{D0})\frac{\partial g^{D0}}{\partial \left(-\alpha^F\right)} \\
& -w'(g^D) \left(\frac{\partial \tau}{\partial \left(-\alpha^F\right)} - \frac{\partial \underline{x}}{\partial \left(-\alpha^F\right)} \right) \\
& -\sigma^D Ru'(c^D)\frac{\partial x^D}{\partial \underline{x}}\frac{\partial \underline{x}}{\partial \left(-\alpha^F\right)}. \tag{A50}
\end{aligned}$$

Without a partial banking union, the first-order conditions to the

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policymaker's problem (17) without a banking union lead to

$$(1-\gamma^D)v'(r^{D0}) = \gamma^D \sigma^D R u'(c^{D0}), \qquad (A51)$$

$$(1-\gamma^D)v'(r^{D0}) = \gamma^D w'(g^{D0}). \qquad (A52)$$

Applying the Envelope Theorem in (A51) and (A52),

$$w''(g^{D0})\frac{\partial g^{D0}}{\partial(-\alpha^F)} = (\sigma^D R)^2 u''(c^{D0})\left(-\frac{z^F}{I^D}x^{D0} + \frac{\partial x^{D0}}{\partial(-\alpha^F)}\right.$$
$$\left. + \frac{(1-\sigma^F)\theta z^F}{\sigma^D}\right) - R u'(c^{D0})\frac{\sigma^D z^F}{I^D}, \quad (A53)$$

$$(1-\gamma^D)v''(r^{D0})\frac{\partial r^{D0}}{\partial(-\alpha^F)} = \gamma^D w''(g^{D0})\frac{\partial g^{D0}}{\partial(-\alpha^F)}. \qquad (A54)$$

From the budget constraint in country D :

$$\frac{\partial r^{D0}}{\partial(-\alpha^F)} + (1+\beta)\frac{\partial g^{D0}}{\partial(-\alpha^F)} + \frac{\partial x^{D0}}{\partial(-\alpha^F)} = 0. \qquad (A55)$$

We define

$$\Lambda^0 \equiv \frac{1}{(1-\gamma^D)v''(r^{D0})} + \frac{(1+\beta)}{\gamma^D w''(g^{D0})} + \frac{1}{\gamma^D(\sigma^D R)^2 u''(c^{D0})}, \qquad (A56)$$

and from the above conditions, we derive

$$\frac{\partial g^{D0}}{\partial(-\alpha^F)} = \frac{z^F}{w''(g^{D0})\Lambda^0}\left(\frac{1-\sigma^F}{\sigma^D}\theta - \frac{x^{D0}}{I^D} - \frac{u'(c^{D0})}{\sigma^D R u''(c^{D0})}\frac{1}{I^D}\right), \qquad (A57)$$

$$\frac{\partial x^{D0}}{\partial(-\alpha^F)} = \frac{z^F}{(\sigma^D R)^2 u''(c^{D0})\Lambda^0}\left(\frac{1-\sigma^F}{\sigma^D}\theta - \frac{x^{D0}}{I^D} - \frac{u'(c^{D0})}{\sigma^D R u''(c^{D0})}\frac{1}{I^D}\right)$$
$$+ \frac{u'(c^{D0})}{\sigma^D R u''(c^{D0})}\frac{z^F}{I^D} + \frac{z^F}{I^D}\left(x^{D0} - \frac{(1-\sigma^F)}{\sigma^D}\theta I^D\right) \qquad (A58)$$

With a partial banking union, when $x^D = \theta I^D$:

When the supranational authority's problem gives the corner solution $x^D = \theta I^D, \frac{dx^D}{d(-\alpha^F)} = 0$ and $\frac{dc^D}{d(-\alpha^F)} = 0$. If $\alpha^D + \alpha^F \le 1$, then $(1-\sigma^F)/\sigma^D \ge 1$.

A.12

Since $\frac{\partial x^D}{\partial(-\alpha^F)} < 0 \; \forall x^D \leq \theta I^D$, $\frac{\partial \underline{x}}{\partial(-\alpha^F)} > 0$. Applying the Envelope Theorem to the supranational authority's problem then leads to

$$\frac{\partial \tau}{\partial \left(-\alpha^F\right)} > 0. \tag{A59}$$

Using (A51), (A52) and (A57), Ψ can be simplified to

$$
\begin{aligned}
\Psi &= \sigma^D R u'(c^{D0})\theta \left(\frac{1-\sigma^F}{\sigma^D} - 1 - \frac{u'(c^{D0})}{\sigma^D R u''(c^{D0})}\frac{1}{I^D}\right)\left(1 - \frac{1}{\Lambda^0}\right) \\
&\quad - w'(g^D)\left(\frac{\partial \tau}{\partial\left(-\alpha^F\right)} - \frac{\partial \underline{x}}{\partial\left(-\alpha^F\right)}\right).
\end{aligned} \tag{A60}
$$

The upper bound on $\frac{\partial \tau}{\partial(-\alpha^F)} - \frac{\partial \underline{x}}{\partial(-\alpha^F)}$ is 0, it follows that $\Psi \geq 0$.

A.6   Proof of Proposition 3

The effect of a change in α^F on σ^θ and c^θ for fixed $\underline{x}$ and τ, and $x^F = \theta I^F$:

$$\frac{\partial \sigma^D}{\partial \alpha^F} = \frac{\alpha^D z^D z^F}{[\alpha^D z^D + (1-\alpha^F)z^F]^2} = \frac{z^F}{I^D}\sigma^D, \tag{A61}$$

$$\frac{\partial \sigma^F}{\partial \alpha^F} = \frac{(1-\alpha^D)z^D z^F}{[\alpha^F z^F + (1-\alpha^D)z^D]^2} = \frac{z^F}{I^F}\left(1-\sigma^F\right), \tag{A62}$$

$$\frac{\partial c^D(x^F, x^D)}{\partial \alpha^F} = R\left[\frac{\partial \sigma^D}{\partial \alpha^F}x^D - \frac{\partial \sigma^F}{\partial \alpha^F}x^F + \sigma^D\frac{\partial x^D}{\partial \alpha^F}\right] \tag{A63}$$

$$\frac{\partial c^F(x^F, x^D)}{\partial \alpha^F} = -R\left[\frac{\partial \sigma^D}{\partial \alpha^F}x^D - \frac{\partial \sigma^F}{\partial \alpha^F}x^F - \left(1-\sigma^D\right)\frac{\partial x^D}{\partial \alpha^F}\right], \tag{A64}$$

where, from the first-order conditions to the policymaker's problem,

$$\frac{\partial x^D}{\partial \alpha^F} = -\frac{z^F}{I^D}\frac{u'(c^D) + u''(c^D)R\left[\sigma^D x^D - \left(1-\sigma^F\right)\theta I^D\right]}{R\sigma^D u''(c^D) + \frac{1}{\gamma^D R\sigma^D}(1-\gamma^D)v''(r^D)}. \tag{A65}$$

Defining

$$\varpi \equiv \frac{(1-\gamma^D)}{\gamma^D\left(R\sigma^D\right)^2}\frac{v''(r^D)}{u''(c^D)} > 0, \tag{A66}$$

A.13

we can re-write the above as

$$\frac{\partial x^D}{\partial \alpha^F} = \frac{z^F}{\sigma^D I^D} \frac{1}{1+\varpi} \left(-\frac{u'(c^D)}{Ru''(c^D)} - \left[\sigma^D x^D - \left(1 - \sigma^F\right) \theta I^D \right] \right), \qquad \text{(A67)}$$

$$\begin{aligned}
\frac{\partial c^D(x^F, x^D)}{\partial \alpha^F} &= R\frac{z^F}{I^D} \left[\sigma^D x^D - \left(1 - \sigma^F\right) \theta I^D + \frac{\sigma^D I^D}{z^F} \frac{\partial x^D}{\partial \alpha^F} \right] \\
&= R\frac{z^F}{I^D} \frac{\varpi}{1+\varpi} \Big[\sigma^D x^D - \left(1 - \sigma^F\right) \theta I^D \\
&\quad + \sigma^D \left(-\frac{\gamma^D R\sigma^D u'(c^D)}{\left(1 - \gamma^D\right) v''(r^D)} \right) \Big]
\end{aligned} \qquad \text{(A68)}$$

$$\left(-\frac{u''(c^D)}{u'(c^D)} \right) \frac{\partial c^D(x^F, x^D)}{\partial \alpha^F} = \frac{z^F}{I^D} \frac{\left[\left(x^D - \frac{\left(1 - \sigma^F\right)}{\sigma^D} \theta I^D \right) \left(-\frac{v''(r^D)}{v'(r^D)} \right) + 1 \right]}{1+\varpi} \qquad \text{(A69)}$$

$$\begin{aligned}
&\frac{z^F}{I^D} - \left(-\frac{u''(c^D)}{u'(c^D)} \right) \frac{\partial c^D(x^F, x^D)}{\partial \alpha^F} \\
&= \frac{z^F}{I^D} \frac{1}{1+\varpi} \left(\varpi - \left(x^D - \frac{\left(1 - \sigma^F\right)}{\sigma^D} \theta I^D \right) \left(-\frac{v''(r^D)}{v'(r^D)} \right) \right) \qquad \text{(A70)}
\end{aligned}$$

Also,

$$\begin{aligned}
\frac{\partial c^F}{\partial \alpha^F} &= -R \left[\frac{\partial \sigma^D}{\partial \alpha^F} x^D - \frac{\partial \sigma^F}{\partial \alpha^F} x^F - \left(1 - \sigma^D\right) \frac{\partial x^D}{\partial \alpha^F} \right] \\
&= -R\frac{z^F}{I^D} \Big[\left(\sigma^D x^D - \left(1 - \sigma^F\right) \theta I^D \right) \left(1 + \frac{\left(1 - \sigma^D\right)}{\sigma^D} \frac{1}{1+\varpi} \right) \\
&\quad - \frac{\left(1 - \sigma^D\right)}{\sigma^D} \frac{1}{1+\varpi} \frac{u'(c^D)}{\left(-Ru''(c^D)\right)} \Big] \qquad \text{(A71)}
\end{aligned}$$

A.14

So, if $\alpha^D + \alpha^F \leq 1$, then also $\sigma^D + \sigma^F < 1$. Since $x^D \leq \theta I^D$, then

$$\frac{z^F}{I^D} - \left(-\frac{u''(c^D)}{u'(c^D)}\right) \frac{\partial c^D(x^F, x^D)}{\partial \alpha^F} > 0, \tag{A72}$$

and

$$\frac{\partial c^F}{\partial \alpha^F} > 0. \tag{A73}$$

Consider the first order conditions to the supranational authority's maximization problem when there is an interior solution for $(\underline{x}, \tau)$ with a binding $\underline{x}$ for policymaker D and an interior solution for $\{x^D, r^D, g^D\}$:

$$\eta \left[R\sigma^D u'(c^D(x^D, x^F)) \frac{\partial x^D}{\partial \underline{x}} - w'\left(e^D - \frac{\underline{x} - \tau}{1 + \beta}\right)\right]$$

$$+ (1 - \eta) \left[R\left(1 - \sigma^D\right) u'(c^F(x^F, x^D)) \frac{\partial x^D}{\partial \underline{x}} \right] = 0 ; \tag{A74}$$

$$\eta w'(e^D - \frac{\underline{x} - \tau}{1 + \beta}) + (1 - \eta) w'(g^F) \frac{\partial g^F}{\partial \tau} = 0 . \tag{A75}$$

Consider the effect of a change in α^F on

$$\Delta \equiv \eta \sigma^D u'(c^D) + (1 - \eta)\left(1 - \sigma^D\right) u'(c^F). \tag{A76}$$

$$\begin{aligned}
\frac{\partial \Delta}{\partial \alpha^F} &= \left[\eta u'(c^D) - (1 - \eta)u'(c^F)\right] \frac{\partial \sigma^D}{\partial \alpha^F} + \eta \sigma^D u''(c^D) \frac{\partial c^D}{\partial \alpha^F} \\
&\quad + (1 - \eta)\left(1 - \sigma^D\right) u''(c^F) \frac{\partial c^F}{\partial \alpha^F} \\
&= \eta u'(c^D)\sigma^D \left[\frac{z^F}{I^D} - \left(-\frac{u''(c^D)}{u'(c^D)}\right) \frac{\partial c^D}{\partial \alpha^F}\right] \\
&\quad - (1 - \eta)u'(c^F)\left(1 - \sigma^D\right) \left[\frac{\sigma^D}{\alpha^F} + \left(-\frac{u''(c^F)}{u'(c^F)}\right) \frac{\partial c^F}{\partial \alpha^F}\right] \tag{A77}
\end{aligned}$$

From (A72) and (A73), and since c^D, c^F, and x^D are bounded, it follows that there exist $\bar{\eta}$ and $\underline{\eta}$ such that:

- if $\eta < \underline{\eta}$, then $\frac{\partial \Delta}{\partial \alpha^F} < 0$, and applying the Envelope Theorem and ignoring

A.15

second order effects,

$$\frac{\partial \underline{x}}{\partial \sigma^F} < 0 \text{ and } \frac{\partial \tau}{\partial \sigma^F} < 0;$$

- if $\eta > \bar{\eta}$, then $\frac{\partial \Delta}{\partial \alpha^F} > 0$, and applying the Envelope Theorem and ignoring second order effects,

$$\frac{\partial \underline{x}}{\partial \sigma^F} > 0 \text{ and } \frac{\partial \tau}{\partial \sigma^F} > 0.$$

A.7 Proof of Proposition 4

Consider the case in which $x^D = \theta I^D$ and $x^F = \theta I^F$ (corner solutions for recapitalizations). Define $\phi \equiv \frac{z^D}{z^F}$ and consider a change in ϕ such that I^D does not change:

$$\frac{\partial z^F}{\partial z^D} = -\frac{\alpha^D}{1 - \alpha^F}, \text{ and}$$

$$d\phi = \frac{z^F + z^D \frac{\alpha^D}{1-\alpha^F}}{(z^F)^2} = \frac{I^D}{(1-\alpha^F)(z^F)^2} = \frac{1}{(1-\sigma^D)z^F}.$$

Re-writing σ^D and σ^F,

$$\sigma^D = \frac{\alpha^D \phi}{\alpha^D \phi + (1 - \alpha^F)}, \tag{A78}$$

$$\sigma^F = \frac{\alpha^F}{\alpha^F + (1 - \alpha^D)\phi}. \tag{A79}$$

A.16

The effect of the change in ϕ is:

$$\frac{\partial \sigma^D}{\partial \phi} = \frac{\alpha^D(1-\alpha^F)}{[\alpha^D\phi+(1-\alpha^F)]^2} = \frac{\sigma^D\left(1-\sigma^D\right)}{\phi}; \tag{A80}$$

$$d\sigma^D = \frac{\alpha^D}{[\alpha^D\phi+(1-\alpha^F)]^2}\frac{I^D}{(z^F)^2} = \frac{\alpha^D}{I^D}; \tag{A81}$$

$$\frac{\partial \sigma^F}{\partial \phi} = -\frac{\alpha^F(1-\alpha^D)}{[\alpha^F+(1-\alpha^D)\phi]^2} = -\frac{\sigma^F\left(1-\sigma^F\right)}{\phi}; \tag{A82}$$

$$d\sigma^F = -\frac{\sigma^F\left(1-\sigma^F\right)}{\phi\left(1-\sigma^D\right)z^F} = -\frac{\sigma^F\left(1-\sigma^F\right)}{\left(1-\sigma^D\right)z^D}; \tag{A83}$$

$$dI^F = -\alpha^F\frac{\alpha^D}{(1-\alpha^F)}+\left(1-\alpha^D\right) = \frac{1-\alpha^D-\alpha^F}{1-\alpha^F}. \tag{A84}$$

$$\gamma^D R\sigma^D u'(c^D) = (1-\gamma^D)v'(r^D), \tag{A85}$$

$$dx^D + dr^D = d\underline{x} = 0. \tag{A86}$$

$$\gamma^D R u'(c^D)\frac{\alpha^D}{I^D}+\gamma^D R\sigma^D u''(c^D)dc^D$$
$$+\gamma^D\left(R\sigma^D\right)^2 u''(c^D)dx^D + (1-\gamma^D)v''(r^D)dx^D = 0. \tag{A87}$$

At full recapitalizations, $c^D = z^D R$, so $dc^D = R$. Then,

$$dx^D = -\frac{1}{\sigma^D}\frac{\frac{u'(Rz^D)}{Rz^D u''(Rz^D)}+1}{1+\frac{(1-\gamma^D)v''(r^D)}{\gamma^D(R\sigma^D)^2 u''(c^D)}}. \tag{A88}$$

If

$$-\frac{u'(Rz^D)}{Rz^D u''(Rz^D)} < 1, \tag{A89}$$

then $dx^D < 0$ and then $d\underline{x} > 0$, since $\underline{x} = x^{D,MAX} + r^{D,MAX}$,

$$\gamma^D R\sigma^D u'(c^D(x^{D,MAX},x^{F,MAX})) = (1-\gamma^D)v'(r^{D,MAX}). \tag{A90}$$

A.17

Applying the Envelope theorem in the first-order condition for τ (ignoring second order effects) then leads to $d\tau > 0$.

If

$$-\frac{u'(Rz^D)}{Rz^D u''(Rz^D)} > 1, \tag{A91}$$

then $dx^D > 0$ and then $d\underline{x} < 0$. Applying the Envelope theorem in the first-order condition for τ then leads to $d\tau < 0$.

A.8 Proof of Proposition 5

Denote by $(\tau, \underline{x})$ the equilibrium supranational policy without fiscal rules. Consider introducing a fiscal rule $\overline{B}^D > b^D$ (i.e., that binds at $(\tau, \underline{x},)$). Denote by $(\tau^{FR}, \underline{x}^{FR})$ the optimal policy for the supranational authority when the fiscal rule is $\overline{B}^D$. Also, denote by (r^D, x^D, g^D, g_1^D) policymaker $D's$ utility maximizing policy choices under $(\tau, \underline{x})$ and no fiscal rules, and by g^F the public good provision in country F. Denote by $(\bar{r}^D, \bar{x}^D, \bar{g}^D, \bar{g}_1^D)$ the policy choices in country D under $(\tau^{FR}, \underline{x}^{FR})$ and fiscal rule $\overline{B}^D$, and by $\bar{g}^F$ the public good provision in country F.

Without the fiscal rule, the first-order condition (A15) implies that with an internal solution for $(\tau, \underline{x})$,

$$\eta w'(g^D) = (1 - \eta) w'(g^F) \frac{\partial g^F}{\partial \tau}, \tag{A92}$$

$$g^D = g_1^D \tag{A93}$$

With a binding fiscal rule $\overline{B}^D$,

$$\eta w'(\bar{g}^D) = (1 - \eta) w'(\bar{g}^F) \frac{\partial \bar{g}^F}{\partial \tau}, \tag{A94}$$

but $w'(\bar{g}_1^D) \leq w'(g_1^D)$, so

$$\eta w'(\bar{g}_1^D) \leq (1 - \eta) w'(\bar{g}^F) \frac{\partial \bar{g}^F}{\partial \tau}. \tag{A95}$$

A.18

Then, reducing $\overline{B}^D$ while keeping $(\tau, \underline{x},)$ constant increases the utility of the supranational authority. Since this holds true for all $\overline{B}^D > b^D$ and all $(\tau, \underline{x},)$, it implies that the supranational authority is maximized at $\overline{B}^D = b^D$.

A.9 Proof of Proposition 6

We first establish the following lemma:

Lemma 1 *There exists $\overline{\gamma}^F$ such that $\forall \gamma^F \geq \overline{\gamma}^F$, policymaker F provides full recapitalizations ($x^F = \theta I^F$) when domestic fiscal rules are in place.*

Proof. In section A.10 below. ∎

The existence of a binding fiscal rule only changes the equilibrium policies $\{r^D, x^D, g^D, g_1^D\}$ coming out of policymaker D's constrained maximization problem. It does not change the problem for the supranational authority.

Step 1. *The policymakers' problem*

With a binding debt limit $\overline{b}^D(\theta)$ in country D and a limit $\overline{b}^F(\theta)$ in country F, policymaker D's problem is

$$\max_{\{x^D, g^D, r^D, b^D\}} \left(1 - \gamma^H\right) v(r^D) + \gamma^H \left[u(c^D(x^D, x^F))\right.$$
$$\left. + w(g^D) + \beta w(e^D - b^D)\right] \tag{A96}$$

subject to

$$r^D + x^D + g^D \leq e^D + \beta b^D + \tau, \tag{A97}$$
$$r^D + x^D \geq \underline{x} \tag{A98}$$
$$b^D \leq \overline{b}^D. \tag{A99}$$

The first-order conditions with a binding rule $\underline{x}$ lead to

$$\gamma^D R \sigma^D u'(c^D (x^D, x^F)) = (1 - \gamma^D) v'(r^D), \tag{A100}$$
$$r^D + x^D = \underline{x}, \tag{A101}$$
$$g^D = e^D + \beta b^D - \underline{x} + \tau. \tag{A102}$$

The maximization problem for policymaker F facing debt limit $\bar{b}^F$ is

$$\max_{\{x^F, g^F, r^F, b^F\}} \left(1 - \gamma^F\right) v(r^F) + \gamma^F \left[u(c^F(x^F, x^D)) + w(g^F) + \beta w(e^D - b^F)\right]$$

$$(A103)$$

subject to

$$r^F + x^F + g^F \leq e^F + \beta b^F - \tau, \qquad (A104a)$$

$$g_1^F \leq e^F - b^F, \qquad (A104b)$$

$$b^F \leq \bar{b}^F, \qquad (A104c)$$

$$x^F \leq \theta I^F. \qquad (A104d)$$

Therefore, the above conditions (together with Assumption 1) imply

$$(1 - \gamma^F)v'(r^F) = \gamma^F w'(g^F), \qquad (A105a)$$

$$x^F = \theta I^F. \qquad (A105b)$$

In autarky, the policies $\left\{x^{D0}, g^{D0}, r^{D0}\right\}$ satisfy:

$$\gamma^D R \sigma^D u'(c^{D0}(\bar{x}^{D0}, \bar{x}^{F0})) = (1 - \gamma^D)v'(\bar{r}^{D0}), \qquad (A106a)$$

$$R \sigma^D u'(c^{D0}(\bar{x}^{D0}, \bar{x}^{F0})) = w'(\bar{g}^{D0}), \qquad (A106b)$$

$$\bar{x}^{D0} + \bar{g}^{D0} + \bar{r}^{D0} = e^D + \beta \bar{b}^D, \qquad (A106c)$$

$$\bar{g}_1^{D0} = e^D - \bar{b}^D. \qquad (A106d)$$

The policies $\left\{x^{F0}, g^{F0}, r^{F0}\right\}$ satisfy:

$$\gamma^D w'(\bar{g}^{F0}) = (1 - \gamma^D)v'(\bar{r}^{F0}), \qquad (A107a)$$

$$\bar{x}^{F0} = \theta I^F, \qquad (A107b)$$

$$\bar{x}^{F0} + \bar{g}^{F0} + \bar{r}^{F0} = e^F + \beta \bar{b}^F, \qquad (A107c)$$

$$\bar{g}_1^{F0} = e^D - \bar{b}^F. \qquad (A107d)$$

Steps 2-4 are analogous to the proof of Proposition 2.

A.20

It then follows that there exists $\eta^{**} \in (0,1)$ such that.

$$\overline{U}^D(\theta, \overline{b}^D, 0, 0) - \overline{U}^D(\theta, \overline{b}^D, \tau, \underline{x}) = 0. \tag{A108}$$

For η^* as defined in Proposition 2.

Let $b^D(\eta^*)$ denote the equilibrium debt at η^* under the partial banking union (with no fiscal rules). Consider a fiscal rule that marginally decreases the debt $b^D : \overline{b}^D = b^D - \varepsilon$, where $\varepsilon \to 0$. This has no effect on b^{D0}, since $b^D > b^{D0}$. So, at $\overline{b}^D$

$$\sigma^D R u'(c^D(x^{D0}, x^{F0})) \frac{\partial x^{D0}}{\partial \overline{b}^D} + w'(g^{D0}) \frac{\partial g^{D0}}{\partial \overline{b}^D} - \beta w'(g_1^{D0}) = 0. \tag{A109}$$

At η^* :

$$\sigma^D R u'(c^D(x^D, x^F)) + w'(g^D) - \beta w'(g_1^D)$$
$$= \sigma^D R u'(c^D(x^{D0}, x^{F0})) + w'(g^{D0}) - \beta w'(g_1^{D0}). \tag{A110}$$

So, by the Envelope Theorem, the change in η^* in response to a change of ε in b^D is:

$$\frac{\partial \eta^*}{\partial \overline{b}^D} = -\frac{\sigma^D R u'(c^D) \frac{\partial x}{\partial \underline{x}} \frac{\partial \underline{x}}{\partial \overline{b}^D} + w'(g^D) \left(\beta + \frac{\partial \tau}{\partial \overline{b}^D} - \frac{\partial x}{\partial \overline{b}^D}\right) - \beta w'(g_1^D)}{\sigma^D R u'(c^D) \frac{\partial x}{\partial \underline{x}} \frac{\partial \underline{x}}{\partial \eta^*} + w'(g^D) \left(\frac{\partial \tau}{\partial \eta^*} - \frac{\partial x}{\partial \eta^*}\right)}. \tag{A111}$$

From the first-order conditions to the supranational authority's problem, (A14) and (A15), $\frac{\partial x}{\partial \eta^*} < 0$, $\frac{\partial \tau}{\partial \eta^*} > 0$, $\frac{\partial \tau}{\partial \overline{b}^D} < 0$ and $\frac{\partial x}{\partial \overline{b}^D} > 0$.

As shown in the proof to Corollary 2, at η^*, the sign of the denominator in (A111) is positive.

At $\overline{b}^D \simeq b^D$, we have $g_1^D \simeq g^D$, so the numerator of (A111) simplifies to

$$\left(\sigma^D R u'(c^D) \frac{\partial x}{\partial \underline{x}} - w'(g^D)\right) \frac{\partial \underline{x}}{\partial \overline{b}^D} + w'(g^D) \frac{\partial \tau}{\partial \overline{b}^D} < 0 \tag{A112}$$

A.21

So

$$\frac{\partial \eta^*}{\partial \bar{b}^D} > 0. \tag{A113}$$

Then, when $\bar{b}^D = b^D - \varepsilon$, η^* decreases. This then implies $\eta^{**} \leq \eta^*$.

A.10 Proof of Lemma 1

Consider the rents r^{F*} and public good g^{F*} defined implicitly by:

$$w'(g^{F*}) = \sigma^F R u'(c^F(\theta I^F, \theta I^D)), \tag{A114a}$$
$$(1 - \gamma^F)v'(r^{F*}) = \gamma^F \sigma^F R u'(c^F(\theta I^F, \theta I^D)). \tag{A114b}$$

Let $\bar{b}^{F*}(\gamma^F) = \beta^{-1}\left(-e^F + \left(\theta I^F + r^{F*}(\gamma^F) + g^{F*}\right)\right)$. Then, by construction, policymaker F's maximization problem yields solutions $\left\{r^{F*}, g^{F*}, \theta I^F\right\}$. Assumption 1 guarantees that e^F is sufficiently large to allow for this. The rule $\bar{b}^{F*}(\gamma^F)$ gives the minimum budget in period 0 needed to obtain $x^F = \theta I^F$ when $x^D = \theta I^H$. A fiscal limit $\bar{b}^F > \bar{b}^{F*}$ is preferred by the F households if

$$w'(g^{F*})\frac{\partial g^{F*}}{\partial \bar{b}^{F*}} - \beta w'(g_1^{F*}) \geq 0, \tag{A115}$$

where

$$g_1^{F*} \equiv e^F - \bar{b}^{F*}(\gamma^F). \tag{A116}$$

From (A114a) and (A114b),

$$\gamma^F w'(g^{F*}) = (1 - \gamma^F)v'(r^{F*}), \tag{A117a}$$
$$\frac{\partial g^{F*}}{\partial \gamma^F} = 0. \tag{A117b}$$

and applying the Envelope Theorem in policymaker F's problem, we obtain

$$\frac{\partial r^{F*}}{\partial \gamma^F} = \frac{w'(g^{F*}) + v'(r^{F*})}{(1 - \gamma^F)v''(r^{F*})} < 0. \tag{A118}$$

The effect of increasing γ^F in (A115) is given by

$$w''(g_1^{F*})\frac{\partial r^{F*}}{\partial \gamma^F} > 0. \tag{A119}$$

For $\gamma^F \to 1$, $r^{F*} \to 0$, so (A115) holds since $w'(g^{F*}) > w'(g_1^{F*})$ for any binding fiscal rule. Then, $\exists \overline{\gamma^F} < 1$, such that (A115) is satisfied $\forall \gamma^F > \overline{\gamma^F}$.

A.11 Proof of Corollary 4

Let $(\tau, \underline{x})$ denote the equilibrium policy chosen by the supranational authority without fiscal rules, and by $(\tau^{FR}, \underline{x}^{FR})$ the equilibrium policy chosen by the supranational authority with fiscal rule $\overline{b}^D$ in country D and fiscal rule $\overline{b}^F$ in country F.

Consider country D. From the first-order conditions to the D government's problem, it follows that

$$\frac{\partial g^D}{\partial \overline{b}^D} > 0. \tag{A120}$$

Then, a decrease in debt from the non-binding value b^D to $\overline{b}^D$ in first-order condition (A15) implies an increase in τ to some $\tau^{FR} > \tau$.

From condition (A14) it follows that $\underline{x}^{FR} < \underline{x}$. Given the D government's first-order conditions, then

$$x^D(\underline{x}^{FR}) < x^D(\underline{x}) \tag{A121}$$

and

$$g^F(\tau^{FR}, \underline{x}^{FR}) < g^F(\tau, \underline{x}). \tag{A122}$$

Therefore, the utility of the Financing households is given by

$$\begin{aligned} U^F &= u(c^F(x^F, x^D(\underline{x}^{FR}))) + w(g^F(\tau^{FR}, \underline{x}^{FR})) \\ &\quad + \beta w(g_1^F(\tau^{FR}, \underline{x}^{FR})). \end{aligned} \tag{A123}$$

From policymaker F's problem, $u^F(c^F)$ is an increasing function of x^D,

A.23

$g^F = g_1^F$, and $w(g^F)$ is an increasing function of g^F. Then, U^F decreases if debt in country D is limited to $\bar{b}^D$.

A.12 Proof of Proposition 8

Denote the country D debt limits by $\bar{b}^D(\theta, 1) = \bar{b}^D$, and $\bar{b}^D(\theta, 0) = \bar{b}^{D0}$. The supranational authority sets $\underline{x}, \tau$ given $\bar{b}^D$. The minimum debt limit that can be set is $\bar{b}^{D,MIN} = 0$. Denote the policies in country D under the partial banking union by $\{\bar{r}^D, \bar{x}^D, \bar{g}^D, \bar{g}_1^D\}$.

Assume first a partial banking union with $\tau > 0$ and $\underline{x} \geq x^{D0} + r^{D0}$.

Consider the utility of policymaker D inside the partial banking union:

$$
\begin{aligned}
V^D &= \left(1 - \gamma^D\right) v(\bar{r}^D) + \gamma^D u(c^D\left(\bar{x}^D, \theta I^F\right)) \\
&\quad + \gamma^D w(\bar{g}^D) + \beta \gamma^D w(\bar{g}_1^D).
\end{aligned}
\tag{A124}
$$

The change in policymaker $D's$ payoff as $\bar{b}^D$ changes:

$$
\begin{aligned}
\frac{\partial V^D}{\partial \bar{b}^D} &= \left(1 - \gamma^D\right) v'(\bar{r}^D) \frac{\partial \bar{r}^D}{\partial \underline{x}} \frac{\partial \underline{x}}{\partial \bar{b}^D} \\
&\quad + \gamma^D \sigma^D R u'(c^D\left(\bar{x}^D, \theta I^F\right)) \frac{\partial \bar{x}^D}{\partial \underline{x}} \frac{\partial \underline{x}}{\partial \bar{b}^D} \\
&\quad + \gamma^D w'(\bar{g}^D) \left(\beta + \frac{\partial \tau}{\partial \bar{b}^D} - \frac{\partial \underline{x}}{\partial \bar{b}^D}\right) - \beta \gamma^D w'(\bar{g}_1^D).
\end{aligned}
\tag{A125}
$$

Applying the Envelope Theorem in (10) and (11), $\frac{\partial \tau}{\partial \bar{b}^D} < 0$, $\frac{\partial \underline{x}}{\partial \bar{b}^D} > 0$, and $\frac{\partial \tau - \underline{x}}{\partial \bar{b}^D} < 0$.

Let $\bar{b}^{D*}$ be the fiscal rule at which households in country D maximize utility. Then, if $\bar{b}^D \leq \bar{b}^{D*}$,

$$
\begin{aligned}
&\gamma^D \sigma^D R u'(c^D\left(\bar{x}^D, \theta I^F\right)) \frac{\partial \bar{x}^D}{\partial \underline{x}} \frac{\partial \underline{x}}{\partial \bar{b}^D} + \\
&\gamma^D w'(\bar{g}^D) \left(\beta + \frac{\partial \tau}{\partial \bar{b}^D} - \frac{\partial \underline{x}}{\partial \bar{b}^D}\right) - \beta \gamma^D w'(\bar{g}_1^D) \geq 0,
\end{aligned}
\tag{A126}
$$

and so

$$\frac{\partial V^D}{\partial \bar{b}^D} > 0. \tag{A127}$$

Therefore, the utility of policymaker D is lowest at $\bar{b}^{D,MIN}$.

Consider the case in which households in country D set fiscal rule $\bar{b}^{D,MIN}$ inside the partial banking union and debt limit b^{D0} outside the partial banking union (so no binding debt limit outside the partial banking union). The policies with fiscal rule $\bar{b}^{D,MIN}$ are denoted by $\left\{ \bar{r}^{D,MIN}, \bar{x}^{D,MIN}, \bar{g}^{D,MIN}, \bar{g}_1^{D,MIN} \right\}$

Then, policymaker D's participation constraint is given by

$$\left(1 - \gamma^D\right) v(\bar{r}^{D,MIN}) + \gamma^D u(c^D\left(\bar{x}^{D,MIN}, \theta I^F\right))$$
$$+ \gamma^D w(\bar{g}^{D,MIN}) + \beta\gamma^D w(\bar{g}_1^{D,MIN}) \geq \left(1 - \gamma^D\right) v(r^{D0})$$
$$+ \gamma^D u(c^D\left(x^{D0}, \theta I^F\right)) + \gamma^D w(g^{D0}) + \beta\gamma^D w(g_1^{D0}). \tag{A128}$$

The lowest value of γ^D at which the participation constraint is satisfied for the policymaker is

$$\underline{\gamma}^D = \frac{1}{1 + \Phi}. \tag{A129}$$

where

$$\Phi \equiv \frac{\overline{U}^D(\theta, b^{D0}, 0, 0) - \overline{U}^D(\theta, \bar{b}^{D,MIN}, \tau, \underline{x})}{v(\bar{r}^{D,MIN}) - v(r^{D0})}. \tag{A130}$$

If a partial banking union is formed with fiscal rule $\bar{b}^{D,MIN}$, then $\bar{r}^{D,MIN} > r^{D0}$, so $v(\bar{r}^{D,MIN}) > v(r^{D0})$. Also, if $\overline{U}^D(\theta, \bar{b}^{D,MIN}, \tau, \underline{x}) < \overline{U}^D(\theta, \bar{b}^{D0}, \tau, \underline{x})$, then $\Phi > 0$ and $\underline{\gamma}^D \in (0, 1)$; otherwise $\underline{\gamma}^D = 0$.

For $\gamma^D > \underline{\gamma}^D$, households can implement a fiscal rule above $\bar{b}^{D,MIN}$ inside the partial banking union and a fiscal rule below b^{D0} outside the partial banking union.

Notice that if the partial banking union with $\tau > 0$ is not formed (so $\tau = 0$), then the result follows immediately.

B   Appendix B – Alternative Fiscal Rules (For Online Publication)

B.1   Domestic Fiscal Rules That Do Not Anticipate the Partial Banking Union

It is worth noting how the results of the model differ if households were to myopically choose the fiscal rule without anticipating the partial banking union. This is relevant, since several domestic restrictions on government borrowing pre-date the increase in financial integration that would make cross-country transfers and recapitalizations a concern.[1]

The debt limit $\bar{b}^D(\theta)$ is chosen as the solution to the following problem:

$$\max_{\left\{\bar{b}^D, x^{D0}, g^{D0}, b^{D0}, r^{D0}\right\}} u(c^D(x^{D0}, x^{F0})) + w(g^{D0}) + \beta w(e^D - b^{D0}) \qquad \text{(B1)}$$

subject to

$$\gamma^D R\sigma^D u'(c^D(x^{D0}, x^{F0})) = (1 - \gamma^D)v'(r^{D0}), \qquad \text{(B2a)}$$

$$R\sigma^D u'(c^D(x^{D0}, x^{F0})) = w'(g^{D0}), \qquad \text{(B2b)}$$

$$r^{D0} + x^{D0} + g^{D0} \leq e^D + \beta b^{D0}, \qquad \text{(B2c)}$$

$$b^{D0} \leq \bar{b}^D. \qquad \text{(B2d)}$$

The problem for the F country is analogous.

The supranational authority must propose the transfer τ and reinvestment requirement $\underline{x}$ taking into account the debt limits $\bar{b}^D$ and $\bar{b}^F$ in each country. The problem it faces is

$$\max_{\tau, \underline{x}} \eta \overline{U^D}(\theta, \bar{b}^D, \tau, \underline{x}) + (1 - \eta)\overline{U^F}(\theta, \bar{b}^F, \tau, \underline{x}) \qquad \text{(B3)}$$

---

[1] See Budina et al. (2012).

subject to

$$(1 - \gamma^D)v(r^D) + \gamma^D \overline{U^D}(\theta, \bar{b}^D, \tau, \underline{x}) \geq (1 - \gamma^D)v(r^{D0})$$
$$+ \gamma^D \overline{U^D}(\theta, \bar{b}^D, 0, 0), \quad \text{(B4a)}$$
$$(1 - \gamma^F)v(r^F) + \gamma^F \overline{U^F}(\theta, \bar{b}^F, \tau, \underline{x}) \geq (1 - \gamma^F)v(r^{F0})$$
$$+ \gamma^F \overline{U^F}(\theta, \bar{b}^F, 0, 0). \quad \text{(B4b)}$$

Constraints (B4a) and (B4b) represent the participation constraints for the D and F governments, respectively. The participation constraints make it clear that the fiscal rules are set outside of the partial banking union, and therefore they remain in place even if the partial banking union is not accepted.

Analogous results to the case in which the partial banking union is anticipated are immediately obtained; however, one important difference emerges. Once the partial banking union is not anticipated, the strategic effect of choosing fiscal rules disappears. This may lead to higher welfare losses to country D households. The following result compares the loss in welfare from joining a partial banking union when fiscal rules are in place to the loss in welfare from joining a partial banking union when no fiscal rules are in place.

Corollary 1 *Consider a partial banking union that achieves full recapitalizations ($x^D = \theta I^D$). Then, there exists $\widetilde{\eta} \in (0, \eta^{**})$ such that $\forall \eta < \widetilde{\eta}$, having domestic fiscal rules in country D increases the welfare losses to households from joining a partial banking union.*

Proof. In section B.3.1. ∎

Fiscal rules may increase household welfare compared to having no fiscal rules, both with and without a banking union. What Corollary 1 shows is that the drop in welfare going into a partial banking union is higher when fiscal rules are in place. The result emerges because a low value of η means that the supranational authority allocates a high share of the bailout costs to country D. With a limited ability to borrow due to the fiscal rule, the D government must finance the spending on the banking sector by significantly reducing public good provision in period 0. This lowers the utility of D households,

and the welfare loss is higher than in the alternative scenario in which there are no fiscal rules.[2]

This case highlights the pitfall of domestic fiscal rules that do not anticipate a partial banking union: if the country carries a low weight at the supranational level, domestic constraints on spending increase the cost of implementing the agreement. The benefit of fiscal rules in terms of reducing rents is offset by the supranational transfers, which allow rents to increase. This creates a situation in which policymaker D still derives a higher relative benefit from the supranational agreement due to rent seeking, while the households face higher relative costs.

B.2 Domestic Fiscal Rules Non-contingent on θ

Consider the case in which $\Theta = [\underline{\theta}, \overline{\theta}]$ and domestic fiscal rules cannot be made contingent on the value of θ, and they are set without the anticipation of the partial banking union. The debt limit $\overline{b}^D$ for country D is set so as to maximize expected household utility:

$$\max_{\overline{b}^D} \mathbb{E}_\theta[u(c^D(x^D(\theta), x^F(\theta), \theta)) + w(g^D(\theta)) + \beta w(e^D - b^D(\theta))] \qquad \text{(B5)}$$

subject to

$$\gamma^D \sigma^D R u'(c^D(x^D, x^F, \theta)) = (1 - \gamma^D)v'(r^D(\theta)), \qquad \text{(B6a)}$$

$$\sigma^D R u'(c^D(x^D, x^F, \theta)) = w'(g^D(\theta)), \qquad \text{(B6b)}$$

$$w'(g^D(\theta)) = \mathbf{1}_{\{b^D < \overline{b}^D\}} w'(e^D - b^D(\theta)), \qquad \text{(B6c)}$$

$$r^D(\theta) + x^D(\theta) + g^D(\theta) \leq e^D + \beta b^D(\theta), \qquad \text{(B6d)}$$

$$b^D(\theta) \leq \overline{b}^D. \qquad \text{(B6e)}$$

---

[2]Corollary 6 discusses a comparison between a partial banking union and no banking union, with domestic fiscal rules in place in both cases. It can still be the case that household welfare in country D is higher in a banking union with fiscal rules compared to a banking union without fiscal rules.

Problem (B5) can be simplified by noticing that if $\bar{b}^D$ binds for some $\tilde{\theta} \in \Theta$, then it binds for all $\theta > \tilde{\theta}$. For $\Theta = [\underline{\theta}, \bar{\theta}] \subset \mathbb{R}$, problem (B5) can then be expressed as:

$$\max_{\bar{b}^D} \mathbb{E}_{\theta \geq \tilde{\theta}(\bar{b}^D)}[u(c^D(x^D(\theta), x^F(\theta), \theta)) + w(g^D(\theta)) + \beta w(e^D - \bar{b}^D)] +$$

$$\mathbb{E}_{\theta < \tilde{\theta}(\bar{b}^D)}[u(c^D(x^D(\theta), x^F(\theta), \theta)) + w(g^D(\theta)) + \beta w(e^D - b^D(\theta))], \quad (B7)$$

subject to (B6a)-(B6e).

In order to ensure that the objective in program (B7) is concave in $\bar{b}^D$, we make the following assumption about the government's utility from rent seeking:

Assumption 1 *For any set of feasible policies $\{x^D, g^D, r^D\}$ and $\theta \in \Theta$ that satisfy*

$$\gamma^D \sigma^D R u'(c^D(x^D, x^F, \theta)) = (1 - \gamma^D) v'(r^D), \qquad (B8)$$

$$\gamma^D w'(g^D) = (1 - \gamma^D) v'(r^D), \qquad (B9)$$

$$x^D + g^D + r^D \leq e^D(1 + \beta), \qquad (B10)$$

the following conditions are also satisfied:

$$\frac{u'''(c^D)}{(\sigma^D R) u''(c^D)^2} \geq \frac{\gamma^D}{(1 - \gamma^D)} \frac{v'''(r^D)}{v''(r^D)^2}, \qquad (B11)$$

$$\frac{w'''(g^D)}{w''(g^D)^2} \geq \frac{\gamma^D}{(1 - \gamma^D)} \frac{v'''(r^D)}{v''(r^D)^2}, \qquad (B12)$$

where $u'''(c^D)$, $w'''(g^D)$, and $v'''(r^D)$ denote the third derivatives of the utility functions.

We proceed to analyze the problem by establishing the following lemmas.

Lemma 2 *The objective function (B7) is strictly concave in $\bar{b}^D$ and the maximization problem has a unique solution $\bar{b}^{D*} \in [-e^D/\beta, e^D]$.*

Proof. In Section B.3.2. ∎

Lemma 3 *There exists $\theta^G \in \Theta$, $\theta^G < \bar{\theta}$ such that the debt limit imposed by the domestic fiscal rule is binding for policymaker D if $\theta \geq \theta^G$.*

Proof. In Section B.3.3. ■

The above lemmas establish that the solution to problem (B7) is unique, and that the fiscal rule is binding for a subset of the possible realizations of θ.

This setup captures the main trade-off of non-contingent fiscal rules: on the one hand, they limit the government's ability to engage in excessive spending in the first period; since part of first-period spending goes towards rents, the debt limit is beneficial to households because it reduces rents; on the other hand, fiscal rules limit government's ability to borrow in order to recapitalize banks in period 0.

For country F, we assume the analogous decision problem to (B7), such that debt limit $\bar{b}^F \leq e^F$ is set. The following Lemma ensures that $\forall \theta$, full recapitalization of F banks are performed even under the fiscal rule ($x^F(\theta) = \theta I^F$).

Lemma 4 *There exists $\overline{\gamma^F}^*$ such that $\forall \gamma^F \geq \overline{\gamma^F}^*$, policymaker F provides full recapitalizations ($x^F = \theta I^F$) when domestic fiscal rules are in place.*

Proof. In Section B.3.4. ■

Lemma 4 gives the equivalent result to that of Lemma 1.

Consider the supranational authority's problem with debt limits as described above. Analyzing the equivalent problem to problem (27), we obtain the following results.

Proposition 1 *For $\gamma^F \geq \overline{\gamma^F}^*$, there exists a threshold η^{***} such that a partial banking union under domestic fiscal rules achieves a Pareto improvement compared to no banking union whenever $\eta > \eta^{***}$.*

Proof. Analogous to the proof to Proposition 6. ■

The fiscal rules change the cost of funding recapitalizations, but they do not change the trade-off faced by the supranational authority between increasing

recapitalizations and reducing public good provision. Therefore, the intuition from the case without fiscal rules carries over to the case with fiscal rules.

We can also derive the equivalents of Corollaries 4 and 6.

Corollary 2 *Domestic fiscal rules in country D decrease the welfare of F households in the partial banking union, compared to the case without fiscal rules in country D.*

Proof. Same as the proof to Corollary 4. ∎

Finally, the equivalent of Corollary 6 holds even if the fiscal rule is not made contingent on θ. The proof is analogous to the proof to Corollary 6.

B.3 Proofs

B.3.1 Proof of Corollary 6

Consider a fiscal rule that sets a binding debt limit of $\bar{b}^D$, lower than b^{D0}, the debt chosen by the D government in the equilibrium without fiscal rules. Denote by $\left(r^D, x^D, g^D, g_1^D\right)$ the policies chosen by the D government given $(\tau, \underline{x})$ and no fiscal rules, by $\left(r^{D0}, x^{D0}, g^{D0}, g_1^{D0}\right)$ are the policies chosen by the D government without a banking union and without fiscal rules, by $\left(\overline{r^D}, \overline{x^D}, \overline{g^D}, \overline{g_1^D}\right)$ the policies chosen by the D government given policies $\left(\tau^{FR}, \underline{x}^{FR}\right)$ and fiscal rules $\left(\bar{b}^D, \bar{b}^F\right)$, and by $\left(\overline{r^{D0}}, \overline{x^{D0}}, \overline{g^{D0}}, \overline{g_1^{D0}}\right)$ the policies chosen by the D government without a banking union, but under fiscal rule $\bar{b}^D$ in country D.

From the proof to Proposition 2, Step 5, without fiscal rules, $\exists \eta^{B*} < \eta^*$ such that the participation constraint for policymaker D binds $\forall \eta < \eta^{B*}$. Given proof to Proposition 6, which is analogous to that of Proposition 2, $\exists \overline{\eta^{B*}} < \eta^{**}$, such that the participation constraint for policymaker D binds $\forall \eta < \overline{\eta^{B*}}$. Let $\widetilde{\eta^B} = \min\{\eta^{B*}, \overline{\eta^{B*}}\}$. Then, $\forall \ \eta < \widetilde{\eta^B}$,

$$(1 - \gamma^D)v(r^D) + \gamma^D U^D(x^D, x^F, g^D, g_1^D) = (1 - \gamma^D)v(r^{D0})$$
$$+ \gamma^D U^D(x^{D0}, x^{F0}, g^{D0}, g_1^{D0}), \quad \text{(B13)}$$

and

$$(1 - \gamma^D)v(\overline{r^D}) + \gamma^D U^D(\overline{x^D}, \overline{x^F}, \overline{g^D}, \overline{g_1^D}) = (1 - \gamma^D)v(\overline{r^{D0}})$$
$$+ \gamma^D U^D(\overline{x^{D0}}, \overline{x^{F0}}, \overline{g^{D0}}, \overline{g_1^{D0}}), \quad \text{(B14)}$$

where by Assumption 1 and the condition that $\gamma^F \geq \overline{\gamma^F}$ (described in Lemma 1), $x^F = x^{F0} = \overline{x^F} = \overline{x^{F0}} = \theta I^F$.

A binding fiscal rule decreases the outside option for the D government, since for $\overline{b}^D < b^{D0}$,

$$(1 - \gamma^D)v(r^{D0}) + \gamma^D U^D(x^{D0}, x^{F0}, g^{D0}, g_1^{D0}) >$$
$$(1 - \gamma^D)v(\overline{r^{D0}}) + \gamma^D U^D(\overline{x^{D0}}, \overline{x^{F0}}, \overline{g^{D0}}, \overline{g_1^{D0}}). \quad \text{(B15)}$$

Moreover, since the fiscal rules maximize D household utility,

$$U^D(\overline{x^{D0}}, \overline{x^{F0}}, \overline{g^{D0}}, \overline{g_1^{D0}}) > U^D(x^{D0}, x^{F0}, g^{D0}, g_1^{D0}). \quad \text{(B16)}$$

Consider the case in which $\underline{x}^{FR} = \underline{x}^* = \theta I^D + r^{D*}$, with r^{D*} defined implicitly by $(1 - \gamma^D)v'(r^{D*}) = \gamma^D \sigma^D R u'(\theta I^D, \theta I^F)$. In this case, the maximum recapitalization is achieved, so $x^D = \overline{x^D} = \theta I^D$. Conditions (B13) and (B14), together with $v(r^D) = v(\overline{r^D})$ lead to

$$U^D(\overline{x^{D0}}, \overline{x^{F0}}, \overline{g^{D0}}, \overline{g_1^{D0}}) - U^D(\overline{x^D}, \overline{x^F}, \overline{g^D}, \overline{g_1^D}) >$$
$$U^D(x^{D0}, x^{F0}, g^{D0}, g_1^{D0}) - U^D(x^D, x^F, g^D, g_1^D). \quad \text{(B17)}$$

B.3.2 Proof of Lemma 2

Denote by $U^{D0}(\theta)$ the value of the D household utility given the solution $\{r^{D0}(\theta), x^{D0}(\theta), g^{D0}(\theta), g_1^{D0}(\theta), b^{D0}(\theta)\}$ to policymaker D's maximization problem without the partial banking union and without the fiscal rule. Also, denote by $\overline{U^{D0}}(\theta, \overline{b}^D)$ the value of D household utility given the solution to policymaker D's maximization problem without a partial banking union, but with a fiscal rule $\overline{b}^D$. Finally, let $\widetilde{\theta}(\overline{b}^D)$ denote the value of θ at which $b^{D0} = \overline{b}^D$ (so

B.7

$g_1^{D0}(\widetilde{\theta}) = e^D - \bar{b}^D)$. Given $f(\theta)$ the p.d.f. for θ over $\Theta = [\underline{\theta}, \bar{\theta}]$, the function maximized by problem (B7) is

$$EU(\bar{b}^D) = \int_{\underline{\theta}}^{\widetilde{\theta}(\bar{b}^D)} U^{D0}(\theta) f(\theta) d\theta + \int_{\widetilde{\theta}(\bar{b}^D)}^{\bar{\theta}} \overline{U^{D0}}(\theta, \bar{b}^D) f(\theta) d\theta. \qquad (B18)$$

The function $\overline{U^{D0}}(\theta, \bar{b}^D)$ is a continuous and differentiable function of $\bar{b}^D$, since $u(c)$, $w(g)$ and $v(r)$ are continuously differentiable. Also, $\widetilde{\theta}(\bar{b}^D)$ is differentiable since it is a continuous function of $u(\cdot)$, $w(\cdot)$ and $v(\cdot)$, derived from the solution $\bar{b}^D$ to policymaker D's problem. Taking the first-derivative with respect to $\bar{b}^D$, we obtain

$$\begin{aligned}
\frac{\partial EU(\bar{b}^D)}{\partial \bar{b}^D} &= U^{D0}(\widetilde{\theta}(\bar{b}^D)) f(\widetilde{\theta}) \frac{\partial \widetilde{\theta}(\bar{b}^D)}{\partial \bar{b}^D} + \int_{\widetilde{\theta}(\bar{b}^D)}^{\bar{\theta}} \frac{\partial \overline{U^{D0}}(\theta, \bar{b}^D) f(\theta)}{\partial \bar{b}^D} d\theta \\
&\quad - \overline{U^{D0}}(\widetilde{\theta}(\bar{b}^D), \bar{b}^D) f(\widetilde{\theta}) \frac{\partial \widetilde{\theta}(\bar{b}^D)}{\partial \bar{b}^D}.
\end{aligned} \qquad (B19)$$

Notice that for $\theta = \widetilde{\theta}$, we have $U^{D0}(\widetilde{\theta}) = \overline{U^{D0}}(\widetilde{\theta}, \bar{b}^D)$, so

$$\frac{\partial EU(\bar{b}^D)}{\partial \bar{b}^D} = \int_{\widetilde{\theta}(\bar{b}^D)}^{\bar{\theta}} \frac{\partial \overline{U^{D0}}(\theta, \bar{b}^D) f(\theta)}{\partial \bar{b}^D} d\theta. \qquad (B20)$$

Then,

$$\frac{\partial^2 EU(\bar{b}^D)}{\partial \bar{b}^{D2}} = \int_{\widetilde{\theta}(\bar{b}^D)}^{\bar{\theta}} \frac{\partial^2 \overline{U^{D0}}(\theta, \bar{b}^D) f(\theta)}{\partial \bar{b}^{D2}} d\theta - \frac{\partial \overline{U^{D0}}(\widetilde{\theta}, \bar{b}^D) f(\widetilde{\theta})}{\partial \bar{b}^D} \frac{\partial \widetilde{\theta}(\bar{b}^D)}{\partial \bar{b}^D}. \qquad (B21)$$

But $\frac{\partial \overline{U^{D0}}(\widetilde{\theta}, \bar{b}^D) f(\widetilde{\theta})}{\partial \bar{b}^D} = 0$ since any increase in $\bar{b}^D$ would make the debt constraint $(b^D(\widetilde{\theta}) \leq \bar{b}^D)$ slack. Therefore,

$$\frac{\partial^2 EU(\bar{b}^D)}{\partial \bar{b}^{D2}} = \int_{\widetilde{\theta}(\bar{b}^D)}^{\bar{\theta}} \frac{\partial^2 \overline{U^{D0}}(\theta, \bar{b}^D) f(\theta)}{\partial \bar{b}^{D2}} d\theta. \qquad (B22)$$

Then

$$\frac{\partial^2 \overline{U^{D0}}(\theta, \bar{b}^D) f(\theta)}{\partial \bar{b}^{D2}} < 0 \Leftrightarrow \frac{\partial^2 EU(\bar{b}^D)}{\partial \bar{b}^{D2}} < 0. \tag{B23}$$

The change in household utility due to the change in the binding debt limit $\bar{b}^D$ is given by

$$\frac{\partial \overline{U^{D0}}(\theta, \bar{b}^D)}{\partial \bar{b}^D} = \sigma^D R u'(c^D) \frac{\partial x^D}{\partial \bar{b}^D} + w'(g^D) \frac{\partial g^D}{\partial \bar{b}^D} - \beta w'(e^D - \bar{b}^D). \tag{B24}$$

Then,

$$\begin{aligned}
\frac{\partial^2 \overline{U^{D0}}(\theta, \bar{b}^D)}{\partial \bar{b}^{D2}} &= \left[\left(\sigma^D R\right)^2 u''(c^D(x^D, x^F, \theta)) \left(\frac{\partial x^D}{\partial \bar{b}}\right)^2 \right. \\
&\quad \left. + w''(g^D) \left(\frac{\partial g^D}{\partial \bar{b}^D}\right)^2 + \beta w''(e^D - \bar{b}^D) \right] \\
&\quad + w'(g^D) \left(-\frac{\partial^2 r^D}{\partial \bar{b}^{D2}} \right) \tag{B25}
\end{aligned}$$

The first-order conditions to the Home government's problem give

$$\begin{aligned}
\gamma^D \sigma^D R u'(c^D(x^D, x^F, \theta)) &= (1 - \gamma^D) v'(r^D), \tag{B26a} \\
\gamma^D w'(g^D) &= (1 - \gamma^D) v'(r^D). \tag{B26b}
\end{aligned}$$

Then,

$$\begin{aligned}
\gamma^D \left(\sigma^D R\right)^2 u''(c^D(x^D, x^F, \theta)) \frac{\partial x^D}{\partial \bar{b}^D} &= (1 - \gamma^D) v''(r^D) \frac{\partial r^D}{\partial \bar{b}^D}, \tag{B27a} \\
\gamma^D w''(g^D) \frac{\partial g^D}{\partial \bar{b}^D} &= (1 - \gamma^D) v''(r^D) \frac{\partial r^D}{\partial \bar{b}^D}, \tag{B27b}
\end{aligned}$$

and

$$\frac{\partial x^D}{\partial \bar{b}^D} + \frac{\partial r^D}{\partial \bar{b}^D} + \frac{\partial g^D}{\partial \bar{b}^D} = \beta. \tag{B28}$$

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Combining the above conditions,

$$\frac{\partial r^D}{\partial \bar{b}^D} = \beta \left[1 + \frac{\left(1 - \gamma^D\right) v''(r^D)}{\gamma^D \left(\sigma^D R\right)^2 u''(c^D)} + \frac{\left(1 - \gamma^D\right) v''(r^D)}{\gamma^D w''(g^D)} \right]^{-1}. \tag{B29}$$

So

$$
\begin{aligned}
\frac{\partial^2 r^D}{\partial \bar{b}^{D2}} &= -\left(\frac{\partial r^D}{\partial \bar{b}^D}\right)^3 \frac{1}{\beta} \left(\frac{1 - \gamma^D}{\gamma^D} v''(r^D)\right)^2 . \\
&\cdot \left(\frac{\gamma^D}{1 - \gamma^D} \frac{v'''(r^D)}{v''(r^D)^2} \left(\frac{1}{\left(\sigma^D R\right)^2 u''(c^D)} + \frac{1}{w''(g^D)}\right) \right. \\
&\left. - \frac{u'''(c^D)}{\left(\sigma^D R\right) u''(c^D)^3} - \frac{w'''(g^D)}{w''(g^D)^3}\right).
\end{aligned} \tag{B30}
$$

By Assumption 1,

$$\frac{\partial^2 r^D}{\partial \bar{b}^{D2}} \geq 0. \tag{B31}$$

This, together with the concave increasing functions $u(c^D)$, $w(g^D)$ implies

$$\frac{\partial^2 \overline{U^{D0}}(\theta, \bar{b}^D)}{\partial \bar{b}^{D2}} < 0 \tag{B32}$$

and

$$\frac{\partial^2 EU(\bar{b}^D)}{\partial \bar{b}^{D2}} < 0. \tag{B33}$$

Given the strict concavity of the objective function, it follows that the maximization problem has a unique solution $\bar{b}^{D*} \in [-e^D/\beta, e^D]$.

B.3.3   Proof of Lemma 3

From the proof to Lemma 2, the first-order condition for the household expected utility maximization problem is given by

$$\int_{\tilde{\theta}(\bar{b}^D)}^{\bar{\theta}} \frac{\partial \overline{U^{D0}}(\theta, \bar{b}^D) f(\theta)}{\partial \bar{b}^D} d\theta = 0. \tag{B34}$$

B.10

Claim 1 *Consider the case in which* $\bar{b}^D < b^{D*}(\bar{\theta})$, *where* $b^{D*}(\bar{\theta})$ *is the level of debt at which the utility of D households is maximized when* $\theta = \bar{\theta} \equiv \max_\theta \Theta$.

Proof. *Let* $\bar{b}^D < b^{D*}(\bar{\theta})$. *Then,* $\forall \theta < \bar{\theta}$, $\frac{\partial \overline{U^{D0}}(\theta, \bar{b}^D)}{\partial \bar{b}^D} < 0$, *due to the concavity of* $\overline{U^{D0}}(\theta, \bar{b}^D)$. *Since it is set to maximize Home household utility,* $\bar{b}^D$ *is lower than the level of debt that maximizes policymaker D's utility when* $\theta = \bar{\theta}$.,*It follows that* $\tilde{\theta}(\bar{b}^D) < \bar{\theta}$. *So, for all nondegenerate probability distribution functions* $f(\theta)$, *we have*

$$\int_{\tilde{\theta}(\bar{b}^D)}^{\bar{\theta}} \frac{\partial \overline{U^{D0}}(\theta, \bar{b}^D)}{\partial \bar{b}} f(\theta) d\theta < 0. \tag{B35}$$

Then, $\bar{b}^D = b^{D*}(\bar{\theta})$ *cannot be the solution to (B34).* ∎

Since $\bar{b}^* < b^{D*}(\bar{\theta})$, it follows that $\exists \theta^G \in \Theta$ such that $\forall \theta \geq \theta^G$, $\overline{V^{D0}}(\theta, \bar{b}^D) < V^{D0}(\theta)$.

B.3.4   Proof of Lemma 4

From Lemma 1, $\forall \theta \in \Theta$, there exists $\overline{\gamma^F}(\theta)$ such that $x^F = \theta I^F$ $\forall \gamma^F \geq \overline{\gamma^F}(\theta)$. Then, it follows that $\overline{\gamma^F}^* = \max_\theta \{\overline{\gamma^F}(\theta)\}$.