Supplemental Appendix

Buy Big or Buy Small? Procurement Policies, Firms' Financing, and the Macroeconomy

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A Data details

This section provides details on the main and complementary datasets used in the paper.

A.1 Main datasets

The main datasets we use are the following:

1. **Public procurement data scraped from the *Agencia Estatal Boletín Oficial del Estado*(BOE)**. Time coverage: 2000-2013. Relevant variables used (at the annual frequency):

    (a) Name of the firm to which the contract has been awarded.

    (b) Date when the contract has been awarded.

    (c) Value of the contract awarded.

    (d) Type of procedure used to award the contract.

According to Spanish law, all procurement contracts above a certain threshold awarded by public institutions must be published in official bulletins.[1] If the contract is awarded by the central government, the information on this contract must be published in the *Agencia Estatal Boletín Oficial del Estado* (BOE), which is the official bulletin of the central government of Spain. In contrast, if the entity that awards the contract is a regional government or a municipality, the information about this contract can alternatively be published at their respective regional or local bulletin. We construct a novel dataset on Spanish public procurement contracts by scraping the BOE website over the 2000-2013 period. Each contract provides information on the type of contract (kind of good or service provided), the awarding institution, the type of procedure used to allocate the contract, and the firm(s) that won the contract. In total, we scraped approximately 130,000 projects over 2000-2013, with a total value of about 165 billion euros. On average, our micro-level data represent around 10% of total public procurement as measured in National Accounts. Despite diffrences in levels, the data closely track the overall evolution of public procurement over time, showing an increase between 2000 and 2009 followed by a decline from 2010 to 2013.

2. **Firms' balance sheet and income statement data from the Bank of Spain**. Time coverage: 2000-2013. Relevant variables used (at the annual frequency):

    (a) Firm's total sales.

    (b) Firm's value added.

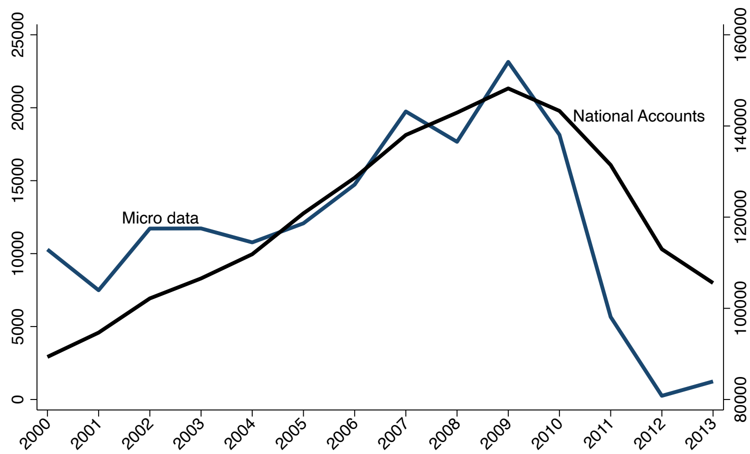
    (c) Firm's fixed assets.

    (d) Firm's number of employees.

    (e) Firm's age.

[1] The thresholds above which the contract must be advertised in official bulletins depend on the type of contract. In the case of supplies and services, for example, the threshold is 60,000 euros.

Figure A.1. Evolution of Public Procurement in Spain, 2000-13



Notes: This figure shows the evolution of public procurement in Spain over 2000-13 (in millions of Euros). The blue line ("Micro data", left y-axis) is computed by aggregating the individual projects scraped from the BOE, https://www.boe.es/. The black line ("National accounts", right y-axis) is measured from Spanish national accounts. To calculate government procurement in national accounts, we follow the OECD definition of government procurement and compute it as *government intermediate consumption + government gross fixed capital formation + social transfers in kind.*

(f) Firm's sector of activity (4-digit NACE Rev. 2 code).

We use the balance sheets and income statements of the quasi-universe of Spanish companies between 2000 and 2013, a dataset that is maintained by the Banco de España and taken from the Spanish Commercial Registry. For each firm and year, this dataset includes information on the firm's name, fiscal identifier, sector of activity (4-digit NACE Rev. 2 code), age, net operating revenue, material expenditures, number of employees, labor expenditures, total fixed assets, and total assets. The final sample covers around 85-90% of non-financial firms for all size categories in terms of both turnover and number of employees. See Almunia et al. (2018) for a detailed description of this dataset and its coverage.

3. **Credit registry from the Bank of Spain**. Time coverage: 2000-2013. Relevant variables used (at the annual frequency):

   (a) Firm's outstanding total *commercial and industrial (C&I)* loans.

   (b) Firm's outstanding *commercial and industrial (C&I)* loans collateralized by hard assets.

   (c) Firm's outstanding *commercial and industrial (C&I)* loans not collateralized by hard assets.

   (d) Firm's loan applications.

The *Central de Información de Riesgos* (CIR) is maintained by the Banco de España in its role as primary banking supervisory agency, and contains detailed monthly information on all outstanding loans over 6,000 euros to non-financial firms granted by all banks operating in Spain since 1984. Given the low reporting threshold, virtually all firms with outstanding

Table A.1. Descriptive evidence from the final merged dataset, year 2006

	mean		**25th pctile**		**50th pctile**		**75th pctile**	
	Proc	NoProc	Proc	NoProc	Proc	NoProc	Proc	NoProc
Age	20.41	10.96	12.00	5.00	17.00	10.00	24.00	15.00
Employment	73.47	12.76	16.00	3.00	45.00	6.00	154.00	12.00
Sales	8.97	1.20	1.15	0.10	4.22	0.28	16.89	0.86
Procurement/Sales	0.33	0.00	0.03	0.00	0.10	0.00	0.28	0.00
Fixed Assets	3.81	0.86	0.21	0.04	0.82	0.14	3.58	0.50
Credit	2.52	0.57	0.11	0.03	0.49	0.09	2.23	0.30
Coll. Credit (share)	0.15	0.29	0.00	0.00	0.00	0.00	0.15	0.74

Notes: This table presents summary statistics from our merged dataset for the year 2006, separately for firms with at least one procurement contract (n = 2,409) vs. the rest of the firms (n = 406,263). The variable Employment measures the number of full-time workers employed by the firm; the variable Sales is just firm's revenue measured in millions of euro; Procurement/Sales measures the value of all the procurement projects awarded to a firm in a given year divided by total revenue in that year. Assets measures the value of fixed assets; Credit measures the value of all firm's outstanding loans in millions of euro; Coll. Credit (share) is the share of Credit collateralized against firm's assets; age measures the age of the firm. We winsorize the 1% tails of all variables.

bank debt appear in the CIR. There are three main types of loans classified by "class": *commercial and industrial loans (C&I)*, *trade finance*, and *leasing*. Throughout our paper, we only consider the first, and hence abstract from any change in firms' overall credit coming from changes in trade credit. We believe that considering *commercial and industrial loans (C&I)* only is appropriate for two reasons. First, they account for more than 90% of the overall loan value in Spain. And second, we can decompose them into the two types of borrowing we have in our model. In particular, for each loan within that category, we have information on whether the loan is collateralized by hard assets (e.g., real estate, deposits) or not. We classify the former ones as "collateral credit" and the latter ones as "non-collateral credit". Ivashina et al. (2022) provide more details on the different loan types and their characteristics.

Loan applications. Besides the information on outstanding loans, we also have information about loan applications at the firm-bank level. The construction of this dataset is as follows. Spanish banks can request information about a firm whenever this firm "seriously" approaches them to obtain credit.[2] Because banks already have information about the firms with which they have a credit relationship, banks only request information on firms that have never received a loan from them or that ended the credit relationship before the current request. By matching the loan applications with the information on outstanding loans from CIR, we can infer whether the loan was granted or not.

Banks versus Cajas. Public savings banks (cajas) represented a large share of overall credit in Spain over our sample period, especially before the 2008-09 Spanish banking crisis. However, as discussed in Santos (2017), cajas operated under a different institutional framework than "regular" commercial banks, and were often controlled by local politicians.

[2]The Law stipulates that a bank can not request information about the firm without its consent, which indicates the seriousness of the approach

Delgado et al. (2007) explain the main features of the Spanish banking system, focusing on the differences in behavior of commercial banks and public savings banks.

A.2 Additional datasets

We also use a number of additional datasets throughout the paper:

4. **Small sample of projects with information on bidders**. The BOE website does not provide the identity of the firms that competed for the project but did not win. This is a limitation of our dataset because it does not allow us to construct a well-defined control group. To overcome this limitation, we construct a sample of procurement projects for which we have detailed information about the awarding process. Although we did not find any government agency that provided information about the awarding process during our main sample period (2000-2013), we could identify around 50 agencies that started providing detailed information about their projects starting in 2013. Putting all these agencies together, we were able to uncover the identity of the firms competing for the same projects as well as their final rankings for around 1,000 contracts over the 2013-2016 period. We were able to scrape information on the specific bidders' score in the auction for around half of the contests.

5. **Firms' credit scores**. This information is based on internal credit assessments of private Spanish non-financial corporations conducted by Bank of Spain. These assessments estimate the probability of default using the most recent available financial statements and assign each firm a score accordingly. The main purpose of these assessments is to provide Spanish banks with information about firms' financial reliability. Gavia et al. (2020) provide a detailed description of the so-called Banco de España in-house credit assessment system.

6. **Universe of procurement contracts in Spain**. Time coverage: 2018-2022. To have a comprehensive estimate of the actual number of firms active in procurement, we make use of a recently available dataset provided by the Spanish Government, *Plataforma de Contratación del Sector Público (PLACSP)*, where all types of public agencies in Spain are required to upload their procurement activity. Although some information is available since 2013, compliance was only strong starting in March 2018, when introducing information on all contracts became compulsory by law.[3] We use information for the years 2018, 2019, 2021, and 2022, when the coverage is already high and firms' participation not directly affected by COVID.

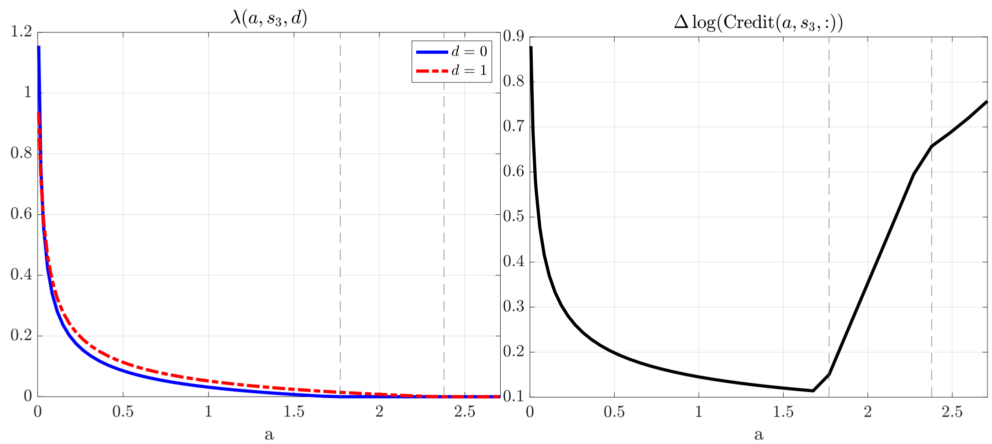
B Heterogeneous effects of credit growth

We investigate the heterogeneous effects of procurement on firms' credit growth. Before looking at the data, we first shed light on this relationship using our model's calibrated version. In panel (a) of Figure B.1, we show how the Lagrange multiplier associated with

[3] In particular, that law ("Ley 9/2017, de 8 de noviembre, de Contratos del Sector Público") was aimed to adopt the European Parliament directives 2014/23/UE and 2014/24/UE.

the financial constraint, $\lambda(s, a, d)$, changes across firms with different levels of net-worth, a.[4] We show this relationship for firms with procurement, i.e., $d = 1$, and without procurement, i.e., $d = 0$. As discussed in Section III.C of the main text, higher net-worth (a) firms are less constrained, and, conditional on a, procurement firms ($d = 1$) are more likely to be constrained because of their higher demand. The dashed lines divide the graph into three regions. The region to the left is one in which firms, with and without procurement, are financially constrained. In the middle region, only firms with procurement are constrained. In the region to the right, all firms are unconstrained.

Figure B.1. Heterogeneous effects of credit (calibrated model)



(a) Lagrange multiplier λ **(b)** change in log credit

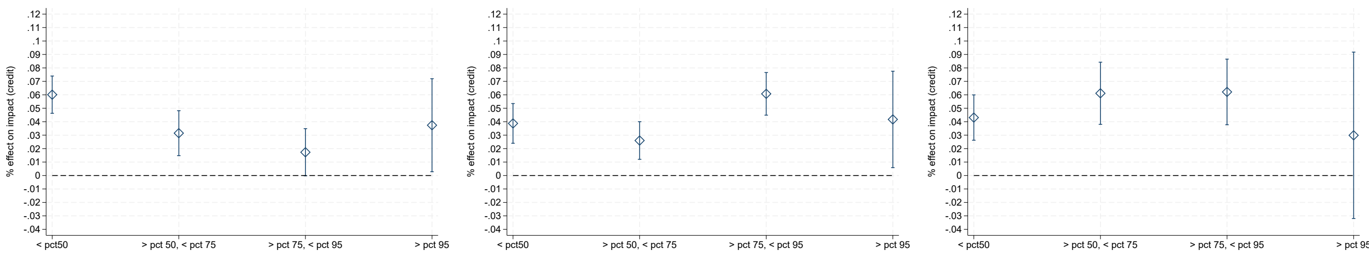
Notes: Panel (a) shows the Lagrangian multiplier associated to the financial constraint, λ, for firms with different levels of net-worth in our model, both for firms with and without procurement ($d = 0$ and $d = 1$). The dotted lines organize the graph in three regions. In Panel (b) the effect on impact, i.e., $h = 0$, for different levels of firms' financial constraints as proxied by a (and implicitly given by λ).

In panel (b), we show the change in credit on impact ($h = 0$), of a firm becoming active in procurement ($d = 0 \to d = 1$), as a function of net worth a, i.e., the inverse of financial constraints all else equal. The main takeaway from this graph is that the model predicts a non-monotonic relationship between firms' financial constraints and the effect of procurement on credit. When the financial constraint is binding both before and after the procurement shock (left region), the model predicts that less financially constrained firms (higher a and hence lower λ) exhibit a smaller increase in credit when becoming active in procurement. However, the impact for firms in the other two groups (when transitioning from unconstrained to constrained or remaining unconstrained) increases with net worth. As a result, the model exhibits the u-shaped relationship between net credit growth and net worth.

In Section IV.B we discuss the intuition for why unconstrained firms exhibit a larger increase in credit due to the procurement shock. The idea is that these firms, precisely because they are unconstrained, can expand freely by increasing credit. On the contrary,

[4] We produce this graph fixing the level of firms' productivity at the middle point in our productivity grid, i.e., s_3. For other levels of s, the graph would simply be an identical, scaled, version of Figure B.1.

Figure B.2. Heterogeneous effects of procurement on credit (data)



(a) distribution of fixed assets **(b)** distribution of leverage **(c)** distribution of age

Notes: This figure shows the effect on impact, i.e., $h = 0$ (as well as its 90% confidence intervals), of public procurement on firms' credit for different percentiles of the distribution of assets (panel a), leverage (panel b), and age (panel c). Standard errors clustered at the firm level.

even if procurement allows them to borrow more, constrained firms remain constrained, limiting the amount of credit they can raise after the demand shock implied by procurement.

We next look at the heterogeneous effects of procurement on firms' credit in the data. Similarly to Section II.A in the main text, we show the impact effect, i.e., $h = 0$ (as well as its 10% confidence intervals), of public procurement on firms' credit for different percentiles of the distribution of assets (panel a), leverage (panel b), and age (panel c). See Figure B.2. As in the model, the relationship between proxies for firms' financial constraints and the change in credit is non-monotonic. With fixed assets and leverage as proxies, the empirical evidence is reasonably consistent with the u-shaped relationship predicted by the model.

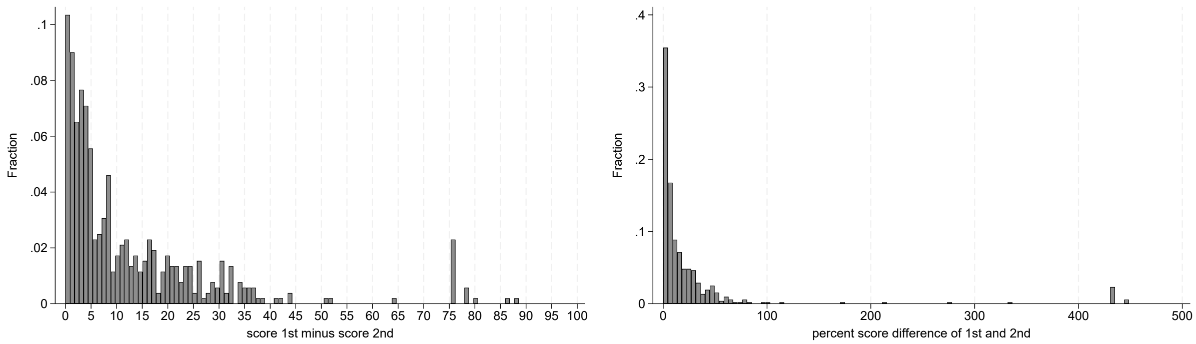
C Additional empirical evidence

This Appendix provides additional evidence to support our baseline results that measure the impact of a firm obtaining a procurement contract on its credit growth. To simplify the analysis and given that the largest impact that we find in our local projection regressions is in the initial period, we focus on static regressions of the following form:

$$\Delta \log l_{it} = \alpha_i + \alpha_{st} + \beta_1 \text{PROC}_{it} + \beta_2 \log l_{it-1} + \varepsilon_{it}, \tag{C.1}$$

where t may represent a year or a year-quarter depending on the frequency of the data we use. As such, the dependent variable $\Delta \log l_{it}$ is either the annual or quarterly growth of credit (loans) of firm i between $t-1$ and t defined as $\Delta \log l_{it} \equiv \log l_{it} - \log l_{it-1}$. The regressor PROC_{it} is a dummy variable that takes value one if the firm obtained a procurement contract in t. We include the firm's lagged credit at $t-1$ to control for the fact that firms with large outstanding loan volumes may mechanically have less room for credit growth than firms with smaller outstanding loan levels. The baseline annual regressions control for firm-level and sector×time fixed effects, but given the different datasets that we exploit, we will be able to control for additional fixed effects, such as firm×year fixed effects when using quarterly data, additional control variables, as well as moving beyond simple OLS as described below.

Figure C.1. Distribution of differences in 1st and 2nd place procurement contest scores



(a) Score differential **(b)** Percent differential

Notes: This figure plots the distributions of the difference in scores of 1st and 2nd place firms in procurement contests. Scores are out of a possible 100. Panel (a) presents the difference between 1st and 2nd place, while panel (b) plots the percentage difference of scores between 1st and 2nd place.

C.1 Effects on impact using bidders information

We use the sample of procurement projects where we have information on all bidders as well as the final ranking. Doing so allows us to run regressions analogous to equation (C.1), except that we can identify the association between a firm's ranking in a given auction and its ensuing credit growth.

Figure C.1 panel (a) presents a histogram of the difference in scores for winning and runner-up firms in these contests. The distribution is skewed with a few outliers with large score differentials, but otherwise a large fraction of the sample has score differentials less than 10 out of 100 points, with the majority of these observations with a score differential of 5 points or less. Panel (b) next shows the distribution of the percentage change differential between winners and runner-ups, where we normalize the absolute difference in score by that of the 2nd place firm. The distribution is quite dispersed, but with roughly twenty five percent of the contests decided by a percentage difference less than or equal to 3%.

Table C.1 presents credit growth regressions for a variety of sub-samples based on procurement contest characteristics, where we only report the coefficient on the procurement dummy for regressions using total credit (A), non-collateralized credit (B), and collateralized credit (C). The only difference in specifications is how we constrain the sample as we keep the set of fixed effects as stringent as possible across all regressions as well as including the lagged value of log credit (not reported).

Column (1) presents estimates from our baseline specification that includes the whole sample of projects (with no information on bidders) with quarterly data. In Column (2), we use the bidders sample to run regressions at the monthly frequency, where the dependent variable is quarter-over-quarter credit growth rates. We then zoom into characteristics of the sample based on the placement and scores in the procurement contest, as well as whether the auction was close or not. Column (3) next runs regressions including only winners and runners-ups in the sample. The number of observations does not drop dramatically relative to

Table C.1. Higher frequency credit growth regressions: whole sample and auctions

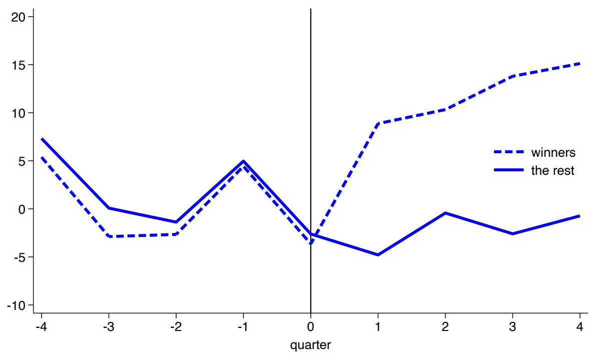
	(1)	(2)	(3)	(4)	(5)
(A) Total credit	0.007	0.073	0.073	0.115	0.124
	(0.002)	(0.028)	(0.028)	(0.042)	(0.059)
Observations	458,450	8,310	6,264	6,555	715
R-squared	0.493	0.360	0.335	0.389	0.292
(B) Non-collateralized credit	0.009	0.080	0.081	0.120	0.127
	(0.002)	(0.031)	(0.031)	(0.047)	(0.054)
Observations	435,079	8,110	6,128	6,357	678
R-squared	0.497	0.368	0.350	0.395	0.348
(C) Collateralized credit	0.001	-0.011	-0.012	0.026	0.195
	(0.003)	(0.028)	(0.027)	(0.017)	(0.131)
Observations	151,809	2,690	2,037	2,321	260
R-squared	0.517	0.357	0.367	0.358	0.315
Firm-Year FE	Yes	Yes	Yes	Yes	Yes
Sector-Quarter FE	Yes	No	No	No	No
Year-Month FE	No	Yes	Yes	Yes	Yes
Auction FE	No	Yes	Yes	Yes	Yes
Sample	All	Auction	1st & 2nd	Scores	1st & 2nd
Close auction	No	No	No	No	Yes (<3%)

Notes: This table presents firm-level regression estimates using quarterly data for all firms and monthly data for a sub-sample of observations with information on the procurement auctions. Columns (1)-(5) present the estimated coefficient on the procurement dummy. Column (1) presents baseline results using all possible firms in the dataset. Column (2) includes a subset of all firms with auction information, Column (3) includes only 1st and 2nd place firms, Column (4) includes all firms with contests reporting scores, and Column (5) includes only 1st and 2nd place firms in "close" contests, where the percentage difference in scores is less than 3%. Standard errors are clustered by firm and sector×year (Column 1) and by firm and auction×year (Columns 2-5).

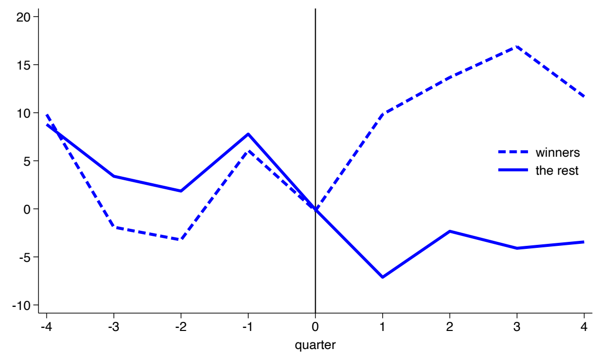
sample (2) given that most of the contests for which we have bidder information only include two bidder firms. Still, it is reassuring that the coefficient on the procurement dummy does not differ substantially from the baseline sample's. Column (4) constrains the samples to all bidder firms that are in procurement contests with recorded scores for all participants. This constraint reduces the sample by roughly one quarter, but the procurement dummy coefficient is relatively similar to the baseline for all types of loans. Finally, Column (5) considers "close" auction outcomes, defined as those located in the first quartile of the score percent differential, which implies a percentage difference in the scores between the first and second place firms of at most 3%. Looking at Column (5), we see that the estimated procurement coefficient is approximately the same magnitude as our baseline estimates and is statistically significant, both for the all-loans and non-collateralized-loans regressions.

Pre-trends for winners vs. the rest. We also check to see if the evolution of credit growth differs for winners of procurement contracts relative to the rest of the sample. Graphically, the right panel in Figure C.2 shows the average growth of credit without collateral of firms that win a procurement project in quarter 0 before and after winning the project, and compares it to the rest of firms. Again, there is a similar evolution of credit growth before procurement (parallel trends) and a clear (and persistent) divergence after that.

Figure C.2. Credit Growth: bidders sample



(a) Total credit



(b) Non-collateral credit

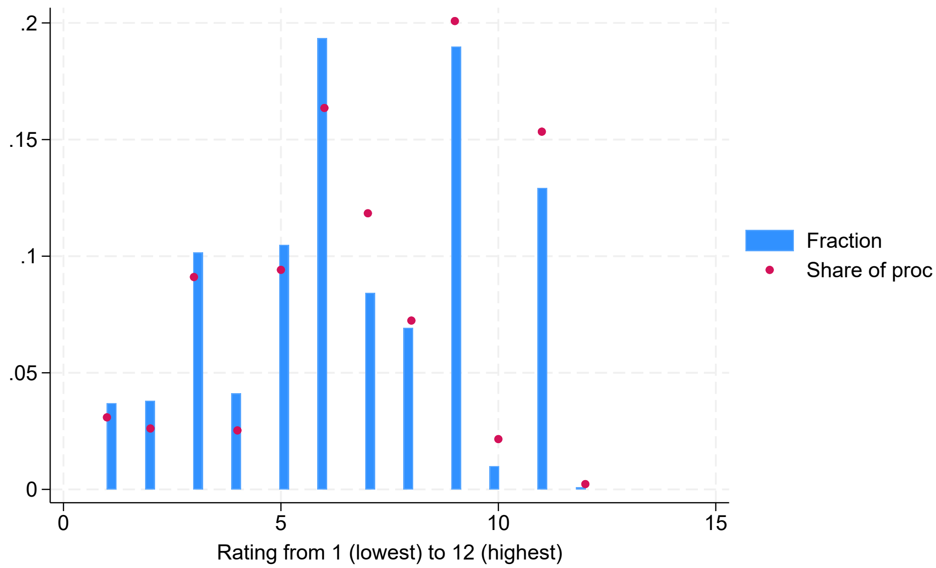
Notes: These graphs plot the evolution of the average change in credit for winning vs. non-winning firms, before and after the quarter in which the auction takes place (Quarter=0). The left panel is for all credit. The right panel is for non-collateral credit only.

C.2 Effects on impact using credit ratings data

We next turn to using information on firm credit scores and consider the possibility that a firm's credit quality may impact their procurement status. To that end, we use internal internal credit assessments of private Spanish non-financial corporations calculated by the Bank of Spain. These assessments estimate the probability of default using the most recent available financial statements and assign each firm a score accordingly. One of the main purposes of these assessments is to provide Spanish banks with information about firms' financial reliability. We merged these credit assessments with our main dataset.

After merging this information with our main dataset, we first examine the distribution of procurement winners conditional on credit ratings in Figure C.3. The blue bars plot the fraction of firms with a given rating, which ranges from 1 to 12, with 12 being the best credit rating, across all firms that have at least one contract of procurement over the sample period. Note that a firm's rating can change over time. The red circles then plot the share of procurement contracts won by firms in a given credit rating bin over the sample period. We would expect red circles to fall exactly at the top of the blue bars if procurements were won uniformly across firms regardless of their credit score. Instead, if we think that firms that have lower credit ratings, thus a lower chance of being able to borrow from a bank (ignoring the potential lending rate that might price the additional risk), are the ones that make more of an effort to win a procurement contract, we would expect to see the red circles above the

Figure C.3. Distribution of procurement contracts and credit ratings



Notes: This figure presents the distribution of credit ratings over all firms in the sample (the blue bars), along with the share of procurements that firms with a given credit rating receive over the whole sample period ('Share of proc,' pink dots).

blue bars for lower credit rating bins. Figure C.3 actually shows the opposite, where it is the higher-rated firms that have a greater proportion of procurement contracts over the sample period. In this respect, the potential for a selection problem seems to work in the opposite direction to be a concern, at least based on credit ratings.

We explore the impact of taking ratings into account by comparing subsets of firms based on their credit ratings. Given sample-size considerations for the regressions, we divide the sample based on four groups of rating: 1-3 (Group 1), 4-6 (Group 2), 7-9 (Group 3), and 10-12 (Group 4). Therefore, we would be evaluating the impact of procurement on firms with very low credit ratings in the first group of ratings 1-3, for example. Further, to avoid potential endogeneity issues, we use lagged credit growth dummy regressions for our analysis – both to construct groups for sub-sample analysis and when using as controls. Table C.2 presents the regression for total credit growth based on the four sub-samples, including both firm and sector×year fixed effects. The coefficient on the procurement dummy is positive for all groups and significant for Groups 1 through 3. Significance disappears for the last group of highest rated firms, though it should be noted that this sub-sample has the smallest number of observations. Obtaining a procurement contract increases the credit growth of high-risk firms relative to low-risk firms, while a procurement effect persists even for higher-rated firms.

C.3 Propensity score matching, ex-ante credit growth analysis, and generalized causal forests

Our next set of regressions analyze for potential selection effects, whereby firms with low access to credit may have an incentive to select into procurement. We follow three approaches

Table C.2. Credit growth regressions for sub-sample of firms based on lagged credit ratings

	(1) Group 1	(2) Group 2	(3) Group 3	(4) Group 4
Procurement$_t$	0.042	0.048	0.058	0.019
	(0.021)	(0.013)	(0.015)	(0.030)
Log(Credit$_{t-1}$)	-0.345	-0.367	-0.388	-0.429
	(0.018)	(0.011)	(0.012)	(0.020)
Observations	9,279	20,523	20,952	7,031
R-squared	0.610	0.537	0.508	0.553
Firm FE	Yes	Yes	Yes	Yes
Sector × Year FE	Yes	Yes	Yes	Yes

Notes: This table presents firm-level regression estimates splitting observations based on lagged firm credit ratings in a given year; 1-3 (Group 1), 4-6 (Group 2), 7-9 (Group 3), and 10-12 (Group 4). Standard errors are clustered by firm and year×sector.

to analyze this issue. The first models the procurement treatment, while the second approach conditions on firms' ex ante credit growth to analyze potential heterogeneous treatment effects. Finally, we combine the two approaches.

Our first approach to handle the procurement selection issue is to follow a version of the empirical setup suggested by Hebous and Zimmermann (2021), who estimate a propensity score matching (PSM) model to allow the possibility of selection in the procurement contract. Similarly to our work, their procurement measure is based on the realization of a contract winner with no quasi-experimental design to exploit, so their methodology is a "second-best" approach to measure the impact of winning the contract for a set of procurement and non-procurement firms that are similar based on observables. We estimate PSM regressions guided by the predictions of our theoretical model as well as controlling for other possible firm level variable. Specifically our empirical setup conditions on the following firm-level lagged variables in predicting credit growth: procurement participation, log credit, log assets, growth of sales to the private sector, leverage, log age, and credit rating dummies. We report these results below as "Model 3" in Table C.4 and describe the results there.

Our second approach that model firms' ex ante growth relies on two sets of analysis. The first set of results is obtained by following work such as Bursztyn et al. (2019),[5] who perform regression analysis in sub-samples that are constructed based on predicting ex ante outcome variables – in our case, we predict firm-level credit growth based on the set of lagged variables described in the previous paragraphs along with a set of fixed effects. Table C.3 presents the results. Column 1 reports our "baseline regression," including as explanatory variables the procurement dummy and the rest of the covariates. Column (2) reports the results from running the same regression but using single contract firms only. Columns (3) and (4) show the results from running the regression splitting the sample between firms above and below the average ex-ante predicted credit growth. For completeness, column (5) shows the results of the "first stage," which we use to predict ex-ante credit growth.

[5] It is important to note, that unlike the cited study, we are not relying on a randomized control experiment.

The main takeaway is that the procurement effect remains positive across all specifications. However, its statistical significance varies across sub-samples. In particular, while the effect is positive and significant at conventional levels for firms with high ex-ante predicted credit growth, it is smaller and not significant for firms with low predicted credit growth.

Table C.3. Procurement impact on credit growth: sample split based on "ex-ante" credit growth

	(1) baseline	(2) one-contract	(3) high Δ Credit	(4) low Δ Credit	(5) pre-proc.
Procurement$_t$	0.044	0.051	0.081	0.038	
	(0.010)	(0.017)	(0.023)	(0.032)	
Procurement$_{t-1}$	0.010	0.004	0.010	0.026	
	(0.011)	(0.020)	(0.025)	(0.036)	
Log (Credit$_{t-1}$)	-0.406	-0.443	-0.466	-0.397	-0.549
	(0.009)	(0.013)	(0.015)	(0.026)	(0.021)
Log (Asset$_{t-1}$)	0.157	0.163	0.175	0.104	0.130
	(0.016)	(0.023)	(0.029)	(0.049)	(0.046)
Priv.Sales Δ_{t-1}	0.022	0.014	0.009	0.021	-0.017
	(0.008)	(0.012)	(0.015)	(0.026)	(0.027)
Log (Leverage$_{t-1}$)	-0.003	-0.002	-0.002	-0.006	-0.002
	(0.001)	(0.002)	(0.002)	(0.003)	(0.003)
Log (Age$_{t-1}$)	0.040	0.002	0.004	-0.000	-0.079
	(0.035)	(0.048)	(0.064)	(0.096)	(0.097)
Observations	43,388	19,619	13,426	5,250	7,746
R-squared	0.424	0.467	0.507	0.512	0.562
Lagged credit rating FE	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes
Sector $\times$ Year FE	Yes	Yes	Yes	Yes	Yes

Notes: Column (1) shows the "baseline regression", including as explanatory variables the procurement dummy and the rest of the covariates mentioned above. Column (2) shows the same regression but focusing on firms that obtain only one contract over the sample period. Columns (3) and (4) show the results from running the regression splitting the sample between firms above and below the average ex-ante predicted credit growth. Column (5) shows the results of the "first stage", which we use to predict ex-ante credit growth. Standard errors are double clustered by firm and sector×year level. This table is reproduced in Appendix C.

We also estimate several versions of a causal forest model using the generalized random forests (GRF) tools developed by Tibshirani et al. (2024) – see Athey and Wager (2019) for an application. The underlying estimation strategy is based on extending a conditional average treatment effect model (CATE), which is usually applied in a random control trial setting. In that case, procurement treatment would be considered random. However, the model can be extended to allow for estimation of the propensity of treatment. In either case, after controlling for confounding factors to predict the outcome variable (i.e., credit growth) and treatment (i.e., procurement), the last step is to estimate a residual-on-residual regression, which yields a consistent estimate of the average treatment effect (Robinson, 1988).

The key here is that machine learning methods, e.g., random trees models, can be used to condition on the confounding variables in a flexible manner without needing to impose a functional form (e.g., linearity) for the impact of these variables nor a particular criterion to assign units into groups (e.g., firms above and below the average). In our particular application, we can use these models to control for factors that predict ex ante credit growth independent of the procurement treatment using the lagged firm-level variables described above.

Table C.4 presents estimation results for the following estimation models:

1. **Causal forest model with fixed treatment propensity.** We estimate the random trees within clusters, where a cluster is defined at the 4-digit sector×year level. A single forest with 500 trees in total is estimated with sampling clustered at the sector×year level. We set the propensity to 0.25 to match the frequency of observed procurement in our dataset.

2. **Causal forest model with estimated treatment propensity.** We estimate the random trees within clusters, where a cluster is defined at the 4-digit sector×year level. A single forest with 500 trees in total is estimated with sampling clustered at the sector×year level.

3. **PSM estimation model.** The treatment propensity is estimated using firm-level variables described above and 2-digit×sector year FE to predict procurement treatment.

Table C.4. PSM and causal forests model estimations of procurement effect

Model	Procurement$_t$	Observations	Cluster/FE
1	0.046	46,001	Sector×Year Cluster
	(0.007)		
2	0.042	46,001	Sector×Year Cluster
	(0.008)		
3	0.044	43,388	Sector×Year FE
	(0.007)		

As we can see from Models 1 and 2 in Table C.4, the causal forest models with our expanded set of controls yield procurement effects on credit growth in line with our baseline estimations in the paper as well as the treatment model estimated using a more "plain vanilla" PSM model. The estimated procurement effects are similar across specifications. The coefficients range between 0.042 and 0.046 across the causal forest models and the PSM approach following Hebous and Zimmermann (2021). Overall, the results from these regressions and the sub-sample analysis yields point estimates for the impact of procurement on credit growth that are very close to our baseline OLS results presented in the main text.

C.4 Loan applications

In this Section we ask whether firms are able to use their procurement contracts to access credit more easily at the extensive margin. A unique piece of information contained in the Banco de España's credit register allows us answer this question: the information on the loan application process for firms and banks. In particular, we can see whether a firm has applied to a given bank and whether the loan application has been accepted or rejected throughout our sample period. We use this information to help identify an increase in firms' borrowing capacity. To do so, we run regressions at the firm-bank level and relate the probability of firms obtaining a loan to whether they have received a procurement contract using the following linear probability specification:

$$\text{Loan granted}_{ibt} = \alpha_{ib} + \alpha_{bt} + \alpha_{st} + \beta \text{PROC}_{it} + \varepsilon_{ibt} \tag{C.2}$$

where the variable 'Loan granted' is a dummy variable equal to 1 when firm i receives a loan from bank b in quarter t conditional on the firm applying for it during that same quarter. We include firm×bank fixed effects, α_{ib}, which implies that we are identifying the coefficient β on the procurement variable via the variation within a firm-bank relationship over time. We further control for overall bank credit supply in a given period with bank×quarter fixed effect α_{bt}, and for macroeconomic events with sector×quarter fixed effects α_{st}.

Table C.5. Probability of a New Loan and Procurement

	All firms	
	(1)	(2)
PROC_{it}	0.023	0.022
	(0.008)	(0.011)
Observations	36,783	26,870
R-squared	0.395	0.628
Firm × Bank FE	Yes	Yes
Bank × Quarter FE	No	Yes
Sector × Quarter FE	No	Yes

Notes: Results from estimating the relationship between loan participation and procurement participation (PROC) by regression (C.2) with firms obtaining at least one procurement project over 2000-13 using quarterly data. Standard errors clustered by firm×bank.

Table C.5 shows the results from running this regression. We include only firm×bank fixed effects in column (1), and add the time-varying bank and sector fixed effects in column (2). Overall, regardless of the specification, the probability of receiving a bank loan conditional on having applied for it increases by approximately 2 percent in the quarter that a firm wins a procurement project.

C.5 Effects by contest type

We recover information on the type of procurement contest for our main sample of procurement contracts. Using that information, we run our baseline regression based on the types of procurement contests in which a firm participates. Table C.6 presents the estimated coefficient for the procurement dummy in our baseline regression. We divided the samples into firm-year observations for firms that participate at least once in a type of contest. Columns (1) and (2) presents results for firms that participate in "Abierto" and "Concurso" contests, respectively. These procurement contests are competitive open-bid competitions. Column (3), "Negociado", represents procurement contracts where the governmental institution approaches targeted firms and negotiates bilaterally.

Table C.6. Credit growth regression by contest type

	(1)	(2)	(3)
(A) Total credit	0.051	0.051	0.042
	(0.007)	(0.024)	(0.013)
Observations	89,547	7,628	23,538
R-squared	0.285	0.332	0.296
(B) Non-collateralized credit	0.062	0.054	0.029
	(0.007)	(0.027)	(0.014)
Observations	89,064	7,672	23,457
R-squared	0.279	0.328	0.291
(C) Collateralized credit	0.003	0.003	0.045
	(0.010)	(0.050)	(0.023)
Observations	30,073	1,927	6,819
R-squared	0.305	0.421	0.356
Firm FE	Yes	Yes	Yes
Sector-Year FE	Yes	Yes	Yes
Contest	Abierto (competitive)	Concurso (competitive)	Negociado (not competitive)

Notes: This table presents firm-level regression estimates splitting observations based on the type of procurement contest that a firm participates. Columns (1) and (2), 'Abierto' and 'Concurso' are competitive open-bid competitions, while column (3), 'Negociado,' represents procurement contracts where the governmental institution approaches targeted firms and negotiates bilaterally. Standard errors are clustered by firm and sector×year.

Table C.6 reveals several interesting findings. First, the majority of contests that underlie our baseline regressions are competitive. Second, the procurement dummy coefficient is positive and significant for all types of contests, but is larger in magnitude for competitive competitions (Columns 1 and 2). Finally, these patterns hold for non-collateralized credit growth, though the coefficient is marginally significant for the 'Negociado' sub-sample of firms. Therefore, our baseline results appear to be driven by competitive procurement competitions.

C.6 Effects by bank type

We exploit *bank-firm loan-level* data, as is commonly used in the credit register literature, to examine whether the procurement shock has a differential impact across loan growth

Table C.7. Bank-firm level credit growth regressions

	(1) All	(2) Commercial	(3) Cajas
Panel A. Total credit			
Procurement_t	0.022	0.020	0.026
	(0.004)	(0.004)	(0.007)
$\text{Log}(\text{Credit}_{t-1})$	-0.304	-0.313	-0.296
	(0.005)	(0.006)	(0.007)
Observations	9,241,155	4,490,151	4,750,841
R-squared	0.362	0.361	0.364
Panel B. Non-collateralized credit			
Procurement_t	0.024	0.022	0.026
	(0.004)	(0.004)	(0.006)
$\text{Log}(\text{Credit}_{t-1})$	-0.336	-0.341	-0.329
	(0.004)	(0.005)	(0.006)
Observations	6,905,429	3,564,005	3,341,274
R-squared	0.372	0.372	0.374
Panel C. Collateralized credit			
Procurement_t	0.006	0.006	0.007
	(0.006)	(0.007)	(0.008)
$\text{Log}(\text{Credit}_{t-1})$	-0.251	-0.244	-0.256
	(0.013)	(0.012)	(0.015)
Observations	2,728,143	1,043,246	1,684,310
R-squared	0.362	0.367	0.362

Notes: This table presents bank-firm level regression estimates for all loans and two sub-samples of lenders: commercial banks and saving banks ('cajas'). All regressions include bank×firm, bank×year, and sector× year fixed effects. Standard errors are double clustered by firm, bank, and year×sector.

across types of banks. The idea is to make sure that our results are not driven by cajas. These regressions results are reported in Table C.7. Given the granularity of these data, we are able to control for several additional levels of fixed effects relative to our baseline firm-level regressions, including (1) *bank*×*firm* FE that control for (non-time varying) bank-firm relationships, (2) *bank*×*year* FE, and *sector*×*year* FE. Therefore, we are capturing variation in loan growth within a given bank-firm pair over time. The results in the three panels of Table C.7 reinforce our main reduced-form finding: the procurement shock is positively correlated to credit growth, and particularly for non-collateralized credit. Furthermore, in examining the coefficients for the three types of banks included in the sample for estimation across the three panels, we see that the estimated procurement coefficient does not vary dramatically for the "All," "Commercial" or "Caja" bank samples.

Table C.8. Credit growth cross-sectional regression using the new 2018–19 sample

Panel A. Procurement dummy

Credit	Total	collateral	non-coll	non-coll	non-coll	non-coll	non-coll
Firms	All	All	All	Q1	Q2	Q3	Q4
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Procurement_t	0.069	0.004	0.072	0.061	0.048	0.041	0.032
	(0.004)	(0.005)	(0.004)	(0.008)	(0.007)	(0.007)	(0.008)
$\text{Log}(\text{Credit}_{t-1})$	-0.058	-0.000	-0.074	-0.150	-0.133	-0.111	-0.083
	(0.001)	(0.001)	(0.001)	(0.002)	(0.002)	(0.002)	(0.002)
Av. dep var.	-0.027	-0.144	0.016	-0.021	0.025	0.035	0.026
Observations	410,656	157,786	304,894	75,835	75,829	75,830	75,832
R-squared	0.028	0.002	0.034	0.070	0.063	0.051	0.042
Sector FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Panel B. (Procurement value)/Sales

Credit	Total	collateral	non-coll	non-coll	non-coll	non-coll	non-coll
Firms	All	All	All	Q1	Q2	Q3	Q4
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Procurement_t	0.185	0.008	0.184	0.144	0.128	0.162	0.119
	(0.016)	(0.024)	(0.018)	(0.031)	(0.034)	(0.032)	(0.037)
$\text{Log}(\text{Credit}_{t-1})$	-0.060	-0.003	-0.073	-0.154	-0.135	-0.113	-0.084
	(0.001)	(0.001)	(0.001)	(0.002)	(0.002)	(0.002)	(0.002)
Av. dep var.	-0.022	-0.146	0.019	-0.016	0.028	0.038	0.028
Observations	387,581	142,910	294,863	73,610	73,609	73,611	73,610
R-squared	0.029	0.002	0.033	0.070	0.064	0.053	0.043
Sector FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Notes: This table presents firm-level regression estimates using the procurement dummy and procurement-to-sales ratio. Panel A present results using the procurement dummy as a regressor, while panel B uses the procurement value-to-sales ratio. Standard errors are robust.

C.7 Firm-level heterogeneity of estimates

We have accessed more recent procurement data that are available electronically and which covers the universe of Spanish procurement contracts and are available **here**. Although the platform includes information since 2013, compliance was only strong starting in March 2018, when introducing information on all contracts became compulsory by law.[6] Although the Bank of Spain's credit register has undergone some changes since last time we used it, we were able to obtain the main information we use in our benchmark regressions (i.e., total credit, collateralized credit, and non-collateralized credit) for the years 2018 and 2019. Therefore, we can use credit growth by type of credit 2018–19 as our dependent variable after merging it with the universe of procurement contracts.

Table C.8 presents cross-sectional regressions of credit growth regressed on a procurement

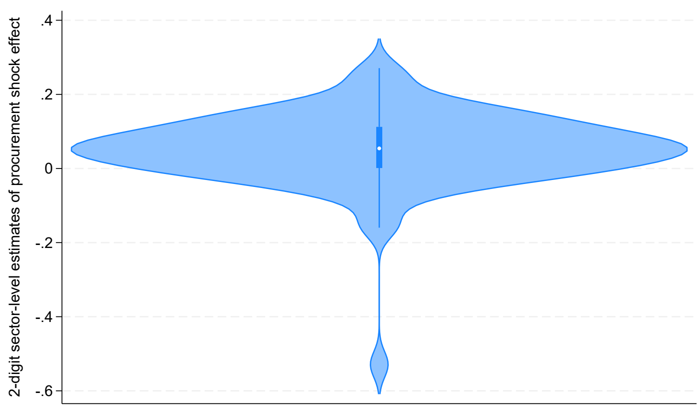
[6] In particular, that law ("Ley 9/2017, de 8 de noviembre, de Contratos del Sector Público") was aimed to adopt the European Parliament directives 2014/23/UE and 2014/24/UE.

dummy (panel A) or the procurement-to-sales ratio (panel B), along with lagged log credit and sector-level fixed effects. Column (1) presents results for all firms and total credit, while columns (2) and (3) present the estimates for collateral and non-collateral credit, respectively. Finally, columns (4)-(7) cut the sample into quartiles based on firm size (log assets) in the case of non-collateral credit regressions in column (3). Both sets of regressions reveal similar results. The effect of procurement on firm's credit is systematically positive and significant across the entire firm size distribution, both when using the dummy and the continuous.

C.8 Sector-level heterogeneity of estimates

We run regressions sector-by-sector, at the 2-digit sectoral level, to ensure a sufficient number of firm-level observations, to estimate the procurement effect. In total, this yields 82 betas, which we summarize in a violin plot in Figure C.4. While there certainly is variation in the estimated effects across sectors, the mean and median betas are 0.052 and 0.054, respectively. So, we are comfortable that the average effect that we estimate in our full panel regression is representative of the sample.

Figure C.4. Violin plot of the estimated effect of a procurement shock on credit growth across 2-digit sectors



Notes: This figure plots the cross-sector distribution of estimated coefficients of the procurement dummy for our baseline credit growth regressions run at the 2-digit sector level. We plot a "violin" of the overall distribution along with an interquartile range of estimates (dark blue bar), the median sector estimate (white dot), and cross-sectional standard deviation tails (dark blue lines).

C.9 The intensive margin of procurement

In this Section, we ask how the value of procurement relative to a firm's total sales impact credit growth. That is, we examine the impact of the intensive margin of procurement on credit growth. To do so, we replace the procurement shock variable in (C.1) with the procurement value-to-sales ratio, where we sum the values of all available procurement contracts that a firm wins in a given year. We also include firms' credit ratings as additional controls. Table C.9 presents the regression results by loan type. The estimated coefficient on the procurement ratio is qualitatively the same as our baseline regressions using the procurement shock variable: the coefficient is positive and significant for the sample of all loans (column

1) and non-collateralized loans (column 2), while being positive but insignificant for the collateralized loan sub-sample (column 3).

Table C.9. The intensive margin of procurement

	Type of credit		
	(1) Total	(2) Non-collateral	(3) Collateral
(Proc. value/Sales)$_t$	0.103	0.131	0.016
	(0.029)	(0.032)	(0.046)
Log(Credit$_{t-1}$)	-0.364	-0.378	-0.336
	(0.007)	(0.007)	(0.012)
Observations	67,464	63,548	20,548
R-squared	0.379	0.376	0.426
Firm FE	Yes	Yes	Yes
Sector-Year FE	Yes	Yes	Yes
Rating FE	Yes	Yes	Yes

Notes: This table presents firm-level regression estimates using the procurement-to-sales ratio as an explanatory variable. Standard errors are clustered by firm and sector×year.

D A microfoundation for government demand

In this Appendix, we present a microfoundation for our stylized model of firm competition for public sector demand. Competition for procurement projects is a multidimensional process, where public agencies assess the offered price, different measures of project quality, and compliance with technical requirements. Our microfoundation incorporates these aspects of reality but leverages a key simplification: quality and price competition are separated into two different stages, with the former being an intertemporal dynamic choice and the latter a static problem. This is what makes the model tractable.

In the economy there is a continuum of varieties indexed by $i \in [0, 1]$, each produced by a different firm. A government has a budget e and a continuum of size M of identical and independent agencies, which want to procure one variety each. The direct conditional utility or valuation of an agency $j \in [0, M]$ buying quantity x_{jig} of any variety i is given by:

$$\tilde{U}_{ji} = \mathbf{1}_{[q_i \geq \bar{q}]}\, x_{jig}\, \epsilon_{ji} \tag{D.1}$$

where $\mathbf{1}_{[q_i \geq \bar{q}]}$ is an indicator of whether the perceived quality q_i of the firm's project is above some required level $\bar{q}$ identical across agencies and ϵ_{ji} is a random taste shock for variety i in agency j, reflecting heterogeneous preferences for varieties across agencies. We can think of the quality q_i as the assessment by public officials that the offered good aligns with the technical requirements of the government. Each agency has an (identical) budget e/M and the price of variety i offered to the public agencies is p_{ig}. Hence, the agency j conditional demand for variety i is $x_{ig} = e/Mp_{ig}$ (it does not depend on j because of symmetry across agencies), which can be used to obtain the indirect utility function:

$$\tilde{V}_{ji} = \mathbf{1}_{[q_i \geq \bar{q}]} \left(e/Mp_{ig} \right) \epsilon_{ji} \tag{D.2}$$

Firms compete for contracts in two stages. In the first stage (see Section D.1), they make risky investments b_i into improving the perceived quality q_i of their variety. The varieties that succeed in getting at least a level of quality $\bar{q}$ form the set $\mathrm{I}_g \subset [0, 1]$ of measure $m_g < 1$ available to government agencies. Next period (see Section D.2), the producers of varieties in the set I_g compete for all agencies' demands along the price margin. We think of the budget e, the measure m_g, and the quality threshold $\bar{q}$ as policy variables, while the set I_g is the result of these policies plus the actions by firms in their competition for contracts.

D.1 First stage: quality competition

The quality threshold $\bar{q}$ is mandated by the government, and it internalizes the competition for contracts because in equilibrium the fraction of firms whose quality q_i is above the threshold $\bar{q}$ must equal the measure m_g of the set I_g.

We assume that the investment technology to improve perceived quality is given by $q_i = \tilde{q}_i/u_i$, where $\tilde{q}_i$ is chosen by the firm and u_i is a shock capturing the riskiness in this technology. The shock u_i may account for true uncertain returns to improving project quality or the idiosyncratic subjectivity of public officials in assessing each project. If we assume that u_i follows an exponential distribution with parameter λ_U, the probability of success for firm i with quality choice $\tilde{q}_i$ is given by:

$$\mathbf{P}[q_i \geq \bar{q}] = \mathbf{P}\left[u_i \leq \tilde{q}_i/\bar{q}\right] = 1 - e^{-\lambda_U \frac{\tilde{q}_i}{\bar{q}}} \tag{D.3}$$

Let's assume that to produce quality $\tilde{q}_i$ requires an investment $b_i = \left[\tilde{q}_i/\eta_0 \eta_2^{d_i} s_i^{\eta_3}\right]^{1/\eta_1}$, with $\eta_0 > 0$, $\eta_1 \in [0, 1)$, $\eta_2 \geq 0$, and $\eta_3 \geq 0$, which potentially allows for current procurement suppliers ($d_i = 1$) and more productive firms (if $\eta_3 > 0$) to fulfill the government's requirements at a lower cost. Inverting the cost function, the probability for producer i to pass the threshold $\bar{q}$ is given by,

$$\mathbf{P}[q_i \geq \bar{q}] = 1 - e^{-\frac{1}{\bar{q}} \lambda_U \eta_0 b_i^{\eta_1} \eta_2^{d_i} s_i^{\eta_3}} \tag{D.4}$$

Defining $\bar{p} \equiv \lambda_U \eta_0/\bar{q}$ we get the probability function 15 that we use in the quantitative exercise of the paper.

This microfoundation treats η_1, η_2, and η_3 as technology parameters, but in the paper we perform counterfactual exercises where these parameters are treated as policy parameters. There are two different ways of seeing η_1, η_2, and η_3 as policy parameters. First, the government can change the quality requirements for procurement goods, the trade-offs between different aspects of the application, or the complexity of the whole application process, which will change the parameters of the cost function. Alternatively, and focusing on η_2 and η_3, the government may set a lower quality threshold $\bar{q}$ for firms with procurement experience $d_i = 1$ and higher firm productivity s_i. In that case, one can define $\bar{q}(d_i, s_i) \equiv \nu_2^{d_i} s_i^{\nu_3} \bar{q}$, and properly redefining the parameters in (D.4) means that η_2 and η_3 are the product of underlying cost and policy parameters.

D.2 Second stage: price competition

The second stage follows standard steps in discrete choices with extreme value shocks, see for instance Anderson et al. (1992). Given the set I_g (with the varieties fulfilling $q_i \geq \bar{q}$) and the indirect utility (D.2), the agency j buys the variety $i \in \mathrm{I}_g$ that yields the highest value $\tilde{V}_{ji}$, with the choice depending on the prices of all varieties in the set I_g as well as on the idiosyncratic preference shocks ϵ_{ji}. Let's assume that $\ln \epsilon_{ji}$ are $i.i.d$ and drawn from a Gumbel distribution with dispersion parameter η. Then, the probability for an agency j of purchasing variety i is given by:

$$\mathrm{Pr}_i = \frac{e^{\left[\frac{1}{\eta}(\ln e/M - \ln p_{ig})\right]}}{\int_{i' \in \mathrm{I}_g} e^{\left[\frac{1}{\eta}\left(\ln e/M - \ln p_{i'g}\right)\right]} \mathrm{d}i'} = \frac{1}{m_g} \frac{p_{ig}^{-1/\eta}}{P_g^{-1/\eta}} \tag{D.5}$$

where $P_g \equiv \left[\frac{1}{m_g} \int_{i' \in \mathrm{I}_g} p_{i'g}^{-1/\eta} \mathrm{d}i'\right]^{-\eta}$ is an average price index of all varieties and Pr_i does not depend on j as all agencies are identical.

Hence, the total demand y_{ig} of variety i faced by its producer is given by:

$$y_{ig} = M \, \mathrm{Pr}_i \, x_{ig} \tag{D.6}$$

That is, the demand y_{ig} is given by the mass M of agencies times the fraction Pr_i of them selecting variety i times the amount x_{ig} each agency purchases when selecting variety i. Using the expressions for x_{ig} and Pr_i above gives:

$$y_{ig} = \frac{1}{m_g} \frac{e}{p_{ig}} \left(\frac{p_{ig}}{P_g}\right)^{-1/\eta} \tag{D.7}$$

which is a downward-sloping isoelastic demand for each variety i. To map this formulation into the one in Section III.A, we can redefine $P_g Y_g \equiv e$ as the government budget and $\sigma_g \equiv 1 + 1/\eta$ as the price elasticity such that:

$$y_{ig} = \left(\frac{p_{ig}}{P_g}\right)^{-\sigma_g} \frac{Y_g}{m_g} \qquad \text{or} \qquad p_{ig} = B_g y_{ig}^{-1/\sigma_g} \tag{D.8}$$

where $B_g \equiv m_g^{-1/\sigma_g} P_g Y_g^{1/\sigma_g}$. Note also that the price index P_g defined above is as in equation 3 and that, due to constant returns to scale, the aggregator of Y_g defined in equation 2 is consistent with total expenditure in varieties e being equal to $P_g Y_g$. The intuition for this equivalence is straightforward. If the different agencies have very homogeneous preferences across varieties (low η), that shows up as a very high elasticity of substitution across varieties (high σ_g) and price increases generate a large drop in demand.

E Details on the static production problem

In this Appendix we characterize analytically the solution of the static production problem. First, in Section E.1 we rewrite the FOC of the problem using the actual functional form for the revenue functions. In Section E.2 we derive the results that serve to restrict the parameters ϕ_p and ϕ_g such that the problem is well-behaved. Then, focusing on the case $\sigma_p = \sigma_g$, we characterize the solutions of the production problem in Section E.3, while in Section E.4 we study the static effect of a procurement shock, that is, the differences in allocations and profits between a firm with $(s, a, d = 1)$ and a firm with $(s, a, d = 0)$.

E.1 The FOC

We start rewriting the FOC of the static production problem as follows. Adding up equations 10, defining $k \equiv k_p + k_g$, and using $\frac{\partial p_p y_p}{\partial k} = \frac{\partial p_p y_p}{\partial k_p}\frac{k_p}{k}$ and $\frac{\partial p_g y_g}{\partial k} = \frac{\partial p_g y_g}{\partial k_g}\frac{k_g}{k}$ we obtain

$$\text{MRPK} \equiv \frac{\partial(p_p y_p + p_g y_g)}{\partial k} = u\left(\frac{r+\delta+\lambda}{1+\lambda\phi_p}\right) + (1-u)\left(\frac{r+\delta+\lambda}{1+\lambda\phi_g}\right) \tag{E.1}$$

where we have defined $u \equiv \frac{k_p}{k}$ as the share of capital in the private sector. That is, the revenue marginal product of capital (MRPK) for the whole firm, holding u constant, is a weighted average of the capital costs in the two sectors, with the weights given by the shares of capital used in each sector. Likewise, and defining $n \equiv n_p + n_g$, we can obtain a similar expressions for labor,

$$\text{MRPN} \equiv \frac{\partial(p_p y_p + p_g y_g)}{\partial n} = w$$

which tells us that the marginal revenue product of labor at the firm level must equal the wage. It will be useful to substitute the actual revenue functions in equations 9-10 to obtain the model FOC as,

$$(1-\alpha)\left(\frac{\sigma_p - 1}{\sigma_p}\right)\frac{p_p y_p}{n_p} = w \tag{E.2}$$

$$(1-\alpha)\left(\frac{\sigma_g - 1}{\sigma_g}\right)\frac{p_g y_g}{n_g} = w \tag{E.3}$$

$$\alpha\left(\frac{\sigma_p - 1}{\sigma_p}\right)\frac{p_p y_p}{k_p} = \frac{r+\delta+\lambda}{1+\phi_p\lambda} \tag{E.4}$$

$$\alpha\left(\frac{\sigma_g - 1}{\sigma_g}\right)\frac{p_g y_g}{k_g} = \frac{r+\delta+\lambda}{1+\phi_g\lambda} \tag{E.5}$$

Finally, using equations (E.2) and (E.3), it is easy to show that the labor shares are constant in each sector and given by $wn_p/p_p y_p = (1-\alpha)(\sigma_p - 1)/\sigma_p$ and $wn_g/p_g y_g = (1-\alpha)(\sigma_g - 1)/\sigma_g$. Hence, we can write revenues net of wage expenses as constant fractions of revenues:

$$p_p y_p - wn_p = [1-(1-\alpha)(\sigma_p - 1)/\sigma_p]\, p_p y_p \tag{E.6}$$

$$p_g y_g - wn_g = [1-(1-\alpha)(\sigma_g - 1)/\sigma_g]\, p_g y_g \tag{E.7}$$

where $[1-(1-\alpha)(\sigma_p - 1)/\sigma_p] \in (0,1)$ and $[1-(1-\alpha)(\sigma_g - 1)/\sigma_g] \in (0,1)$ as long as $\sigma_p > 1$ and $\sigma_g > 1$ respectively. Hence, one can rewrite the borrowing constraint as,

$$k \leq \phi_a a + \tilde{\phi}_p p_p y_p + \tilde{\phi}_g p_g y_g \tag{E.8}$$

with $\tilde{\phi}_p = \phi_p\,[1-(1-\alpha)(\sigma_p - 1)/\sigma_p]$ and $\tilde{\phi}_g = \phi_g\,[1-(1-\alpha)(\sigma_g - 1)/\sigma_g]$

E.2 Some preliminary results

Lemma 1 *The terms* $\frac{r+\delta+\lambda}{1+\lambda\phi_p}$ *and* $\frac{r+\delta+\lambda}{1+\lambda\phi_g}$ *describing the cost of capital for the production of the private sector and the public sector goods respectively, are (a) strictly below* $1/\phi_p$ *and* $1/\phi_g$ *respectively, (b) increasing in* λ*, and (c) strictly above* $r+\delta$ *when* $\lambda > 0$*, if and only if* $\phi_p < (\delta + r)^{-1}$ *and* $\phi_g < (\delta + r)^{-1}$ *respectively.*

Proof: Part (a) is straightforward:

$$\frac{r+\delta+\lambda}{1+\lambda\phi_p} < \frac{1}{\phi_p} \Leftrightarrow \phi_p (r+\delta+\lambda) < (1+\lambda\phi_p) \Leftrightarrow \phi_p (r+\delta) < 1 \Leftrightarrow \phi_p < (r+\delta)^{-1}$$

For part (b) note that

$$\frac{\mathrm{d}}{\mathrm{d}\lambda}\left(\frac{r+\delta+\lambda}{1+\lambda\phi_p}\right) \propto (1+\lambda\phi_p) - \phi_p (r+\delta+\lambda) > 0 \Leftrightarrow \phi_p (r+\delta) < 1 \Leftrightarrow \phi_p < (r+\delta)^{-1}$$

Finally, part (c) is proved by noting that $\frac{r+\delta+\lambda}{1+\lambda\phi_p}$ equals $r+\delta$ whenever $\lambda = 0$ and its derivative w.r.t. λ is positive, see part (b). The same arguments apply for $\frac{r+\delta+\lambda}{1+\lambda\phi_g}$. ■

Lemma 2 *The optimal unconstrained capital for the private and the public sector respectively cannot be self-financed through its own revenues if* $\phi_p \frac{\sigma_p}{\sigma_p - 1} \frac{r+\delta}{\alpha} < 1$ *and* $\phi_g \frac{\sigma_g}{\sigma_g - 1} \frac{r+\delta}{\alpha} < 1$ *respectively.*

Proof: The optimal unconstrained solution for the private sector capital is given by equation (E.4) when $\lambda = 0$, which implies $\frac{p_p y_p}{k_p} = \frac{\sigma_p}{\sigma_p - 1} \frac{r+\delta}{\alpha}$. When $\phi_p \frac{\sigma_p}{\sigma_p - 1} \frac{r+\delta}{\alpha} < 1 \Leftrightarrow \frac{\sigma_p}{\sigma_p - 1} \frac{r+\delta}{\alpha} < \phi_p^{-1}$, which means that $\frac{p_p y_p}{k_p} < \phi_p^{-1} \Leftrightarrow k_p > \phi_p p_p y_p \Rightarrow k_p > \phi_p (p_p y_p - w n_p)$, that is, the optimal unconstrained capital for the private sector, k_p, cannot be self-financed through its own revenues. The proof for the public sector capital is analogous by use of the FOC (E.5). ■

Proposition 1 *Entrepreneurs with zero net worth are financially constrained if both* $\phi_p \frac{\sigma_p}{\sigma_p - 1} \frac{r+\delta}{\alpha} < 1$ *and* $\phi_g \frac{\sigma_g}{\sigma_g - 1} \frac{r+\delta}{\alpha} < 1$.

Proof: Note that if both $\phi_p \frac{\sigma_p}{\sigma_p - 1} \frac{r+\delta}{\alpha} < 1$ and $\phi_g \frac{\sigma_g}{\sigma_g - 1} \frac{r+\delta}{\alpha} < 1$, then following Lemma 2 both $\phi_p (p_p y_p - w n_p) < k_p$ and $\phi_g (p_g y_g - w n_g) < k_g$. Adding them up leads to $\phi_p (p_p y_p - w n_p) + \phi_g (p_g y_g - w n_g) < k$, which implies that the capital of the unconstrained solution cannot be financed through revenue based constraints and hence entrepreneurs with zero net worth are constrained. ■

Lemma 3 *The terms* $\phi_p \frac{\partial p_p y_p}{\partial k_p}$ *and* $\phi_g \frac{\partial p_g y_g}{\partial k_g}$ *are positive and strictly smaller than one for all firms if and only if* $\phi_p < (r+\delta)^{-1}$ *and* $\phi_g < (r+\delta)^{-1}$ *respectively.*

Proof: Given the revenue function, $\phi_p \frac{\partial p_p y_p}{\partial k_p} > 0$ is straightforward. To show $\phi_p \frac{\partial p_p y_p}{\partial k_p} < 1$ just note that the FOC for k_p in equation 10 plus Part (a) in Lemma 1 tell us that $\frac{\partial p_p y_p}{\partial k_p} < \phi_p^{-1}$ iff $\phi_p < (r+\delta)^{-1}$. The proof for y_g is identical. ■

Lemma 4 *The terms $\phi_p \frac{\partial p_p y_p}{\partial k_p}$ and $\phi_g \frac{\partial p_g y_g}{\partial k_g}$ increase with λ if and only if $\phi_p < (r+\delta)^{-1}$ and $\phi_g < (r+\delta)^{-1}$ respectively.*

Proof: The FOC for k_p establishes that $\partial p_p y_p / \partial k_p = (r+\delta+\lambda)/(1+\phi_p \lambda)$, see equation 10. Multiplying both sides by ϕ_p we obtain $\phi_p \partial p_p y_p / \partial k_p = (r+\delta+\lambda)/(\phi_p^{-1}+\lambda)$, and the derivative of $(r+\delta+\lambda)/(\phi_p^{-1}+\lambda)$ with respect to λ has the same sign as $(\phi_p^{-1}-(r+\delta))$, which is positive whenever $\phi_p < (r+\delta)^{-1}$. The proof for y_g is identical. ■

Lemma 5 *The difference between the terms $\phi_p \frac{\partial p_p y_p}{\partial k_p}$ and $\phi_g \frac{\partial p_g y_g}{\partial k_g}$ has the same sign as the difference between ϕ_p and ϕ_g.*

Proof: The FOC for k_g and k_p in equation 10 allows us to write,

$$\phi_p \frac{\partial p_p y_p}{\partial k_p} > \phi_g \frac{\partial p_g y_g}{\partial k_g} \;\Leftrightarrow\; \frac{r+\delta+\lambda}{\phi_p^{-1}+\lambda} > \frac{r+\delta+\lambda}{\phi_g^{-1}+\lambda} \;\Leftrightarrow\; \phi_p > \phi_g$$

■

Lemma 6 *Labor choices in each sector increase monotonically with the capital choices in that same sector.*

Proof: Equations 9 determine the optimal labor demand in each sector. Totally differentiating them with respect to capital and labor gives

$$\frac{\mathrm{d}n_p}{\mathrm{d}k_p} = -\frac{\partial^2 (p_p y_p)/\partial n_p \partial k_p}{\partial^2 (p_p y_p)/\partial n_p^2} > 0 \quad \text{and} \quad \frac{\mathrm{d}n_g}{\mathrm{d}k_g} = -\frac{\partial^2 (p_g y_g)/\partial n_g \partial k_g}{\partial^2 (p_g y_g)/\partial n_g^2} > 0$$

as the revenue functions are concave in labor (negative denominator), and capital and labor are complements in the Cobb-Douglas production function (positive numerator). ■

E.3 Characterizing the solutions

We now analyze the production problem for the case $\sigma_p = \sigma_g = \sigma$. We do it for firms with procurement ($d=1$), with the case ($d=0$) easy to derive from it by trivially setting $B_g = 0$.

E.3.1 Unconstrained firms

We can obtain closed-form solutions to all objects of interest. We specify how these objects vary with the state variables a and s in Proposition 2 below.

Policy functions. With $\lambda = 0$ the FOC for k_p and n_p become $\frac{\partial p_p y_p}{\partial k_p} = r+\delta$ and $\frac{\partial p_p y_p}{\partial n_p} = w$. These two equations define the optimal factor demands for the private sector $k_p^*(s,a,d)$ and $n_p^*(s,a,d)$. In particular, one gets $\frac{\sigma-1}{\sigma}\alpha \frac{p_p y_p}{k_p} = r+\delta$ and $\frac{\sigma-1}{\sigma}(1-\alpha)\frac{p_p y_p}{n_p} = w$ and substituting for the revenue function yields the optimal demands for capital and labor in the private sector

$$k_p^*(s,a,d) = \left(\frac{\sigma_p - 1}{\sigma_p} B_p\right)^{\sigma} \left(\frac{\alpha}{r+\delta}\right) \left(\frac{s}{c(r,w)}\right)^{\sigma-1} \tag{E.9}$$

$$n_p^*(s,a,d) = \left(\frac{\sigma_p - 1}{\sigma_p} B_p\right)^{\sigma} \left(\frac{1-\alpha}{w}\right) \left(\frac{s}{c(r,w)}\right)^{\sigma-1} \tag{E.10}$$

where $c\left(r,w\right)\equiv\left[\left(\frac{r+\delta}{\alpha}\right)^{\alpha}\left(\frac{w}{1-\alpha}\right)^{1-\alpha}\right]$ is the optimal marginal cost of producing one unit of output for a firm with $s=1$. The factor demands for the public sector are identical, just replacing B_p by B_g. Hence, adding up the sector-specific factor demands we obtain the aggregate factor demands,

$$k^*\left(s,a,d\right)=\left(\frac{\sigma-1}{\sigma}\right)^{\sigma}\left(B_p^{\sigma}+B_g^{\sigma}\right)\left(\frac{\alpha}{r+\delta}\right)\left(\frac{s}{c(r,w)}\right)^{\sigma-1} \tag{E.11}$$

$$n^*\left(s,a,d\right)=\left(\frac{\sigma-1}{\sigma}\right)^{\sigma}\left(B_p^{\sigma}+B_g^{\sigma}\right)\left(\frac{1-\alpha}{w}\right)\left(\frac{s}{c(r,w)}\right)^{\sigma-1} \tag{E.12}$$

The ratio of both equations gives us that

$$\frac{k^*\left(s,a,d\right)}{n^*\left(s,a,d\right)}=\frac{\alpha}{1-\alpha}\frac{w}{r+\delta} \tag{E.13}$$

which states that the optimal capital to labor ratio is the same for all firms. This is also true for the capital to labor ratio in each sector, which equals the aggregate one. Finally, let's define $u^*\left(s,a,d\right)\equiv k_p^*\left(s,a,d\right)/k^*\left(s,a,d\right)$ as the optimal share of capital into the private sector. Then, we can write

$$u^*\left(s,a,d\right)=\frac{B_p^{\sigma}}{B_p^{\sigma}+B_g^{\sigma}}=\left(1+\left(\frac{B_g}{B_p}\right)^{\sigma}\right)^{-1} \tag{E.14}$$

with $n_p^*\left(s,a,1\right)/n^*\left(s,a,1\right)$ and $y_p^*\left(s,a,1\right)/y^*\left(s,a,1\right)$ being also equal to $u^*\left(s,a,1\right)$.

Profit and revenue functions. Next, note that profits are given by $\pi=p_py_p+p_gy_g-\left(r+\delta\right)k-wn$, which given the condition for the optimal choice of capital can be written as $\pi^*=\frac{1}{\sigma}\left(p_p^*y_p^*+p_g^*y_g^*\right)$ or $\pi^*=\frac{1}{\sigma-1}\frac{r+\delta}{\alpha}k^*$. Substituting the optimal capital demand in equation (E.11) we obtain

$$\pi^*\left(s,a,d\right)\;=\;\left(\frac{B_p^{\sigma}+B_g^{\sigma}}{\sigma}\right)\left[\left(\frac{\sigma-1}{\sigma}\right)\frac{s}{c(r,w)}\right]^{\sigma-1} \tag{E.15}$$

Finally, it can also be shown that $k^*=\frac{\alpha}{r+\delta}\frac{\sigma-1}{\sigma}\left(p_p^*y_p^*+p_g^*y_g^*\right)$ and $n^*=\frac{1-\alpha}{w}\frac{\sigma-1}{\sigma}\left(p_p^*y_p^*+p_g^*y_g^*\right)$.

Proposition 2 *For the unconstrained problem, the factor demands $k^*\left(s,a,d\right)$ and $n^*\left(s,a,d\right)$, firm size $y^*\left(s,a,d\right)$, and profits $\pi^*\left(s,a,d\right)$ all increase monotonically with productivity s and are invariant with the net worth a. The same is true for the sector-specific factor demands and output. Finally, the relative factor and output shares across sectors are independent of both s and a and are only determined by the relative sectoral demands B_p/B_g.*

Proof: Equations (E.9)-(E.15) plus the production function show this trivially. ■

E.3.2 Constrained firms

The problem for constrained firms does not deliver closed-form solutions for the policy and profit functions. However, one can still characterize how they vary with the state variables a and s.

Policy functions. The optimal solution is fully described by the FOC in equations 9-10 plus the financial constraint 11. To characterize the policy functions, we totally differentiate the binding borrowing constraint 11 with respect to the endogenous choices k_p, k_g, n_p, n_g and the exogenous variables a and s in turn, which gives,

$$\left(1-\phi_p\frac{\partial p_p y_p}{\partial k_p}\right)\mathrm{d}k_p+\left(1-\phi_g\frac{\partial p_g y_g}{\partial k_g}\right)\mathrm{d}k_g \quad = \quad \phi_a \mathrm{d}a \tag{E.16}$$

$$\left(1-\phi_p\frac{\partial p_p y_p}{\partial k_p}\right)\mathrm{d}k_p+\left(1-\phi_g\frac{\partial p_g y_g}{\partial k_g}\right)\mathrm{d}k_g \quad = \quad \left(\phi_p\frac{\partial p_p y_p}{\partial s}+\phi_g\frac{\partial p_g y_g}{\partial s}\right)\mathrm{d}s \tag{E.17}$$

where all the terms dn_p and dn_g disappear due to the FOC in equations 9. If $\phi_p = 0$ and $\phi_g = 0$ we are in the case without earnings-based collateral constraints where $dk/da = \phi_a$ and $dk/ds = 0$, as higher net worth allows the firm to operate with more capital but higher productivity does not. With at least one of $\phi_p > 0$ or $\phi_g > 0$, we have $dk/ds > 0$, that is, constrained firms with higher productivity operate with more capital. Also, with at least one of $\phi_p > 0$ or $\phi_g > 0$, the derivative $dk/da > \phi_a$ because of the multiplier effect: the extra capital obtained by an increase in net worth a allows for an increase in revenues, which allows further borrowing. We show this in Proposition 7 below, but before we need to characterize how optimal choices vary with λ and how λ varies with the state variables a and s.

Proposition 3 *More constrained firms (firms with higher λ) have a lower capital to labor ratio in each sector.*

Proof: Dividing equations (E.2) and (E.4) we obtain

$$\frac{k_p}{n_p}=\frac{\alpha}{1-\alpha}\frac{w}{(r+\delta+\lambda)\,/\,(1+\lambda\phi_p)} \tag{E.18}$$

and dividing equations (E.3) and (E.5) an analogous expression obtains for k_g/n_g. Given Lemma 1, the capital to labor ratio declines with λ in both sectors and hence it is lower for more financially constrained firms. ■

Proposition 4 *More constrained firms (firms with higher λ) have (a) a capital to labor ratio that is relatively higher in the sector with more pledgeable revenues, and (b) relative factor use larger in the sector with more pledgeable revenues. Instead, if $\phi_g = \phi_p$, relative factor use is as in the case with non-binding financial constraints.*

Proof: To prove (a) divide equation (E.18) by its counterpart in the procurement sector to obtain

$$\frac{k_p/n_p}{k_g/n_g}=\frac{1+\lambda\phi_p}{1+\lambda\phi_g} \tag{E.19}$$

such that the relative capital to labor ratio increases with λ in the sector with higher ϕ. To prove (b), use the set of FOC in (E.2)-(E.5) and the explicit functional form for the revenue functions to write

$$\frac{u}{1-u}=\frac{k_p}{k_g}=\left(\frac{B_p}{B_g}\right)^{\sigma}\left(\frac{1+\lambda\phi_p}{1+\lambda\phi_g}\right)^{\alpha(\sigma-1)+1} \tag{E.20}$$

that is, the allocation of capital across sectors depends on the relationship between ϕ_g and ϕ_p as well as on the relative demand B_p/B_g. In particular, the relative use of capital increases with λ in the sector with higher ϕ. Combining equations (E.19) and (E.20) we see that the same is true for labor. Finally, with $\phi_g = \phi_p$ we see that equation (E.20) is equivalent to equation (E.14). ■

Proposition 5 *Holding s constant, more constrained firms (firms with higher λ) produce less output $y(s,a,d)$ and use less capital $k(s,a,d)$ and less labor $n(s,a,d)$. Furthermore, their output and their factor demands are lower in both sectors.*

Proof: Let's combine the FOC (E.2) and (E.4) with the demand equation $p_p y_p = B_p y_p^{1-1/\sigma}$ and the production function $y_p = s k_p^{\alpha} n_p^{1-\alpha}$ to obtain the expression,

$$\frac{\sigma-1}{\sigma} B_p y_p^{-1/\sigma} = \frac{1}{s}\left[\frac{(r+\delta+\lambda)/(1+\lambda\phi_p)}{\alpha}\right]^{\alpha}\left[\frac{w}{1-\alpha}\right]^{1-\alpha} \tag{E.21}$$

which requires equalizing the marginal revenues in the private sector (left-hand-side) to its marginal cost (right-hand-side). By virtue of Lemma 1 the marginal cost increases with λ whenever $\phi_p < (\delta+r)^{-1}$, so y_p must fall with λ. Finally, Lemma 6 states that capital and labor demand move in the same direction. Hence, if $y_p = s k_p^{\alpha} n_p^{1-\alpha}$ falls with λ so must both k_p and n_p. The case for y_g is analogous, and the cases for y, k, and n follow. ■

Proposition 6 *The derivative of $\lambda(s,a,d)$ with respect to net worth a is negative, while the derivative of $\lambda(s,a,d)$ with respect to productivity s is positive as long as $a > 0$ and zero if $a = 0$.*

Proof: We start by noting that the optimal choices of k_p and k_g (and so $k = k_p + k_g$) can be expressed as functions of s and λ only. To see this for k_p, one can use equation (E.21), and substitute the production function $y_p = s k_p^{\alpha} n_p^{1-\alpha}$ and the capital to labor ratio in equation (E.18) to obtain:

$$k_p = \left(\frac{\sigma-1}{\sigma}B_p\right)^{\sigma}\left(s\left[\frac{\alpha}{(r+\delta+\lambda)/(1+\lambda\phi_p)}\right]^{\alpha}\left[\frac{1-\alpha}{w}\right]^{1-\alpha}\right)^{\sigma-1}\left[\frac{\alpha}{(r+\delta+\lambda)/(1+\lambda\phi_p)}\right] \tag{E.22}$$

The same goes for k_g. Second, using equation (E.18) we see that also n_p and n_g are functions of s and λ. Hence, the borrowing constraint 7 can be seen as a function of s, a, and λ. Given this, we start characterizing the derivative of λ with respect to a. To do so, we totally differentiate the borrowing constraint with respect to a and λ (holding s fixed) to obtain:

$$\frac{\partial k}{\partial \lambda}\mathrm{d}\lambda = \phi_a \mathrm{d}a + \left(\phi_p \frac{\partial p_p y_p}{\partial k_p}\frac{\partial k_p}{\partial \lambda} + \phi_g \frac{\partial p_g y_g}{\partial k_g}\frac{\partial k_g}{\partial \lambda}\right)\mathrm{d}\lambda$$

where all the terms $\partial n_p/\partial\lambda$ and $\partial n_g/\partial\lambda$ disappear due to FOC 9. This leads to

$$\frac{\mathrm{d}\lambda}{\mathrm{d}a} = \frac{\phi_a}{\left(1-\phi_p\frac{\partial p_p y_p}{\partial k_p}\right)\frac{\partial k_p}{\partial\lambda} + \left(1-\phi_g\frac{\partial p_g y_g}{\partial k_g}\right)\frac{\partial k_g}{\partial\lambda}} \tag{E.23}$$

Because the terms in parentheses in the denominator are strictly between 0 and 1 (Lemma 3) and the derivatives of k_p and k_g with respect to λ are both negative (Proposition 5), then $\frac{\mathrm{d}\lambda}{\mathrm{d}a} < 0$. Therefore, other things equal, wealthier entrepreneurs are less constrained. Next, we totally differentiate the borrowing constraint with respect to s and λ (holding a fixed) to obtain:

$$\begin{aligned}\frac{\partial k}{\partial s}\mathrm{d}s + \frac{\partial k}{\partial \lambda}\mathrm{d}\lambda &= \left(\phi_p \frac{\partial p_p y_p}{\partial k_p}\frac{\partial k_p}{\partial s} + \phi_g \frac{\partial p_g y_g}{\partial k_g}\frac{\partial k_g}{\partial s}\right)\mathrm{d}s + \left(\phi_p \frac{\partial p_p y_p}{\partial s} + \phi_g \frac{\partial p_g y_g}{\partial s}\right)\mathrm{d}s \\ &+ \left(\phi_p \frac{\partial p_p y_p}{\partial k_p}\frac{\partial k_p}{\partial \lambda} + \phi_g \frac{\partial p_g y_g}{\partial k_g}\frac{\partial k_g}{\partial \lambda}\right)\mathrm{d}\lambda\end{aligned}$$

Note that on the right-hand side we have two effects of s on the borrowing capacity. First, the indirect effect of s on earnings through the induced changes in the choices of k_p and k_g. Second, the direct effect of s on earnings for given capital choices. This allows us to write,

$$\frac{\mathrm{d}\lambda}{\mathrm{d}s} = \frac{\left[\phi_p \frac{\partial p_p y_p}{\partial s} + \phi_g \frac{\partial p_g y_g}{\partial s}\right] - \left[\left(1 - \phi_p \frac{\partial p_p y_p}{\partial k_p}\right)\frac{\partial k_p}{\partial s} + \left(1 - \phi_g \frac{\partial p_g y_g}{\partial k_g}\right)\frac{\partial k_g}{\partial s}\right]}{\left(1 - \phi_p \frac{\partial p_p y_p}{\partial k_p}\right)\frac{\partial k_p}{\partial \lambda} + \left(1 - \phi_g \frac{\partial p_g y_g}{\partial k_g}\right)\frac{\partial k_g}{\partial \lambda}} \tag{E.24}$$

We have seen above that the denominator is negative. To obtain the sign of the numerator, we proceed as follows. The first term can be written as

$$\sum_{j=p,g} \phi_j \frac{\partial p_j y_j}{\partial s} = \frac{\sigma - 1}{\sigma} \sum_{j=p,g} \phi_j \frac{p_j y_j}{s} = \frac{1}{\alpha} \sum_{j=p,g} \phi_j \frac{\partial p_j y_j}{\partial k_j}\frac{k_j}{s}$$

where we have used the revenue function to replace $\partial p_p y_p/\partial s = \frac{\sigma-1}{\sigma} p_p y_p/s$ in the first equality and $p_j y_j = \frac{\partial p_j y_j}{\partial k_j} k_j \frac{1}{\alpha}\frac{\sigma}{\sigma-1}$ in the second one. The second term in the numerator of equation (E.24) can be written as,

$$\sum_{j=p,g}\left(1 - \phi_j \frac{\partial p_j y_j}{\partial k_j}\right)\frac{\partial k_j}{\partial s} = (\sigma - 1)\sum_{j=p,g}\left(1 - \phi_j \frac{\partial p_j y_j}{\partial k_j}\right)\frac{k_j}{s}$$

where we have used the solution for k_j in equation (E.22) to substitute the marginal effect of s on k_j (holding λ constant) by the average effect. Putting both parts together, we can simplify the numerator of equation (E.24) as

$$\begin{aligned}\sum_{j=p,g} \phi_j \frac{\partial p_j y_j}{\partial s} - \sum_{j=p,g}\left(1 - \phi_j \frac{\partial p_j y_j}{\partial k_j}\right)\frac{\partial k_j}{\partial s} &= \frac{\sigma - 1}{s}\sum_{j=p,g}\left[\frac{1}{\alpha(\sigma-1)}\phi_j \frac{\partial p_j y_j}{\partial k_j} - \left(1 - \phi_j \frac{\partial p_j y_j}{\partial k_j}\right)\right]k_j \\ &= \frac{\sigma - 1}{s}\sum_{j=p,g}\left[\frac{1 + \alpha(\sigma - 1)}{\alpha(\sigma - 1)}\phi_j \frac{\partial p_j y_j}{\partial k_j} - 1\right]k_j = -\frac{\sigma - 1}{s}\phi_a a\end{aligned}$$

whose sign is the opposite of the sign of a. To see this last equality, we go to the borrowing constraint in (E.8) to write

$$k = \phi_a a + \sum_{j=p,g}\frac{1 + \alpha(\sigma - 1)}{\alpha(\sigma - 1)}\phi_j \frac{\partial p_j y_j}{\partial k_j}k_j$$

where we have replaced $p_j y_j = \frac{\partial p_j y_j}{\partial k_j} k_j \frac{1}{\alpha}\frac{\sigma}{\sigma-1}$. Hence, we can write equation (E.24) as

$$\frac{\mathrm{d}\lambda}{\mathrm{d}s} = -\frac{(\sigma-1)\phi_a a/s}{\left(1-\phi_p \frac{\partial p_p y_p}{\partial k_p}\right)\frac{\partial k_p}{\partial \lambda} + \left(1-\phi_g \frac{\partial p_g y_g}{\partial k_g}\right)\frac{\partial k_g}{\partial \lambda}} \tag{E.25}$$

which proves $\mathrm{d}\lambda/\mathrm{d}s > 0$ for $a > 0$ and $\mathrm{d}\lambda/\mathrm{d}s = 0$ for $a = 0$. Finally, note that putting together equations (E.23) and (E.25) we can write

$$\frac{\mathrm{d}\lambda}{\mathrm{d}s} = -(\sigma-1)\frac{a}{s}\frac{\mathrm{d}\lambda}{\mathrm{d}a}$$

■

Proposition 7 *The derivative of $k(a,s,d)$ w.r.t. net worth a is positive and bounded below by $dk(a,s,d)/da \geq \phi_a$, with strict inequality whenever at least one of ϕ_p or ϕ_g is strictly positive. The derivative of $k(a,s,d)$ w.r.t. productivity s is positive and bounded below by $dk(a,s,d)/ds > (\phi_p \partial p_p y_p/\partial s + \phi_g \partial p_g y_g/\partial s)$ whenever at least one of ϕ_p or ϕ_g is strictly positive, and zero otherwise.*

Proof: By the chain rule

$$\frac{\mathrm{d}k}{\mathrm{d}a} = \frac{\partial k}{\partial \lambda}\frac{\partial \lambda}{\partial a}$$

Now, $\partial k/\partial\lambda < 0$ (Proposition 5) and $\partial\lambda/\partial a < 0$ (Proposition 6) so $\mathrm{d}k/\mathrm{d}a > 0$. Next, using equation (E.23) we can write,

$$\frac{\mathrm{d}k}{\mathrm{d}a} = \frac{\phi_a}{\left(1-\phi_p \frac{\partial p_p y_p}{\partial k_p}\right)\frac{\partial k_p}{\partial \lambda}/\frac{\partial k}{\partial \lambda} + \left(1-\phi_g \frac{\partial p_g y_g}{\partial k_g}\right)\frac{\partial k_g}{\partial \lambda}/\frac{\partial k}{\partial \lambda}} \geq \phi_a \tag{E.26}$$

If $\phi_p = \phi_g = 0$ the denominator is 1 and the derivative is just ϕ_a. However, if at least one of ϕ_p and ϕ_g is strictly larger than zero, then the denominator is a linear combination of objects that are positive and (at least one of them) strictly smaller than one (Lemma 3), and hence the derivative is strictly larger than ϕ_a. This proves the part related to a. As for the effects of s, when $\phi_p = \phi_g = \phi$, equation (E.17) becomes

$$\left(1-\phi\frac{\partial p_p y_p}{\partial k_p}\right)\left(\frac{\mathrm{d}k_p}{\mathrm{d}s} + \frac{\mathrm{d}k_g}{\mathrm{d}s}\right) = \phi\left(\frac{\partial p_p y_p}{\partial s} + \frac{\partial p_g y_g}{\partial s}\right).$$

where we have used the two FOCs for capital in 10 to set $\partial p_p y_p/\partial k_p = \partial p_g y_g/\partial k_g$. Since $k = k_p + k_g$, we have

$$\frac{\mathrm{d}k}{\mathrm{d}s} = \frac{\phi\left(\frac{\partial p_p y_p}{\partial s} + \frac{\partial p_g y_g}{\partial s}\right)}{1-\phi\frac{\partial p_p y_p}{\partial k_p}}$$

If $\phi = 0$, then $\partial k/\partial s = 0$. If $\phi > 0$, then Lemma 3 implies that the denominator is strictly positive and strictly smaller than one and hence

$$\frac{\mathrm{d}k}{\mathrm{d}s} > \phi\left(\frac{\partial p_p y_p}{\partial s} + \frac{\partial p_g y_g}{\partial s}\right) > 0$$

Finally, we need to show the effect of s on k when $\phi_p \neq \phi_g$. Equation (E.17) describes the total differential of the borrowing constraint with respect to k_p, k_g, and s. The right-hand side of equation (E.17) is positive as, for fixed input choices, an increase in productivity s raises output. On the left-hand side, the terms in parentheses are between 0 and 1 (Lemma 3), which means that at least one of dk_p/ds or dk_g/ds has to be positive, but this does not say anything about the effect of s on $k = k_p + k_g$. To obtain the sign of dk/ds recall that we have defined $u \equiv k_p/k$ such that $k_p = uk$, and note that u only depends on λ, see equation (E.20). Then, by the product rule,

$$\frac{\mathrm{d}k_p}{\mathrm{d}s} = u\frac{\mathrm{d}k}{\mathrm{d}s} + k\frac{\mathrm{d}u}{\mathrm{d}\lambda}\frac{\mathrm{d}\lambda}{\mathrm{d}s} \tag{E.27}$$

If $a = 0$ then Proposition 6 states that $\mathrm{d}\lambda/\mathrm{d}s = 0$, and $\mathrm{d}k_p/\mathrm{d}s = u\,\mathrm{d}k/\mathrm{d}s$ and likewise $\mathrm{d}k_g/\mathrm{d}s = (1-u)\,\mathrm{d}k/\mathrm{d}s$. Replacing these into the left-hand side of equation (E.17) gives us $\mathrm{d}k/\mathrm{d}s > 0$. In particular, we get

$$\frac{\mathrm{d}k}{\mathrm{d}s} = \frac{\left(\phi_p\frac{\partial p_p y_p}{\partial s} + \phi_g\frac{\partial p_g y_g}{\partial s}\right)}{u\left(1-\phi_p\frac{\partial p_p y_p}{\partial k_p}\right) + (1-u)\left(1-\phi_g\frac{\partial p_g y_g}{\partial k_g}\right)} > \phi_p\frac{\partial p_p y_p}{\partial s} + \phi_g\frac{\partial p_g y_g}{\partial s}$$

For $a > 0$ we can show that $\mathrm{d}k/\mathrm{d}s > 0$ by contradiction. We can write $\mathrm{d}k/\mathrm{d}s = \mathrm{d}k_p/\mathrm{d}s + \mathrm{d}k_g/\mathrm{d}s$, and using equation (E.17) to replace $\mathrm{d}k_g/\mathrm{d}s$ we obtain

$$\frac{dk}{\mathrm{d}s} = \left(\phi_p\frac{\partial p_p y_p}{\partial s} + \phi_g\frac{\partial p_g y_g}{\partial s}\right)\left(1-\phi_g\frac{\partial p_g y_g}{\partial k_g}\right)^{-1} + \left[1-\left(1-\phi_p\frac{\partial p_p y_p}{\partial k_p}\right)\left(1-\phi_g\frac{\partial p_g y_g}{\partial k_g}\right)^{-1}\right]\frac{\mathrm{d}k_p}{\mathrm{d}s} \tag{E.28}$$

The first term in the right-hand side is positive. Let's assume $\phi_g > \phi_p$ (the proof for the reverse inequality is symmetrical). With $\phi_g > \phi_p$, the term in square brackets is negative (Lemma 5) and, according to equation (E.27), we have that $\mathrm{d}k_p/\mathrm{d}s < u\mathrm{d}k/\mathrm{d}s$ (as $\mathrm{d}\lambda/\mathrm{d}s > 0$ due to Proposition 6 and $\mathrm{d}u/\mathrm{d}\lambda < 0$ due to equation (E.20)). If $\mathrm{d}k/\mathrm{d}s \leq 0$ then $\mathrm{d}k_p/\mathrm{d}s < 0$, the second term in the right-hand side of the equation above is positive and hence $\mathrm{d}k/\mathrm{d}s > 0$, which is a contradiction. Therefore, it cannot be that $\mathrm{d}k/\mathrm{d}s \leq 0$ and it must be $\mathrm{d}k/\mathrm{d}s > 0$. Regarding the lower bound, we can rewrite (E.17) as

$$\frac{dk}{\mathrm{d}s} = \left(\phi_p\frac{\partial p_p y_p}{\partial s} + \phi_g\frac{\partial p_g y_g}{\partial s}\right) + \phi_p\frac{\partial p_p y_p}{\partial k_p}\frac{\mathrm{d}k_p}{\mathrm{d}s} + \phi_g\frac{\partial p_g y_g}{\partial k_g}\frac{\mathrm{d}k_g}{\mathrm{d}s} \tag{E.29}$$

Sufficient conditions for $\mathrm{d}k/\mathrm{d}s > \left(\phi_p\frac{\partial p_p y_p}{\partial s} + \phi_g\frac{\partial p_g y_g}{\partial s}\right)$ are that either (a) the second term in the right-hand side of equation (E.29) is positive or (b) the second term in the right-hand side of equation (E.28) is positive. When $\phi_g > \phi_p$, equation (E.27) gives $\mathrm{d}k_g/\mathrm{d}s > (1-u)\mathrm{d}k/\mathrm{d}s > 0$ (since $\mathrm{d}(1-u)/\mathrm{d}\lambda > 0$, equation (E.20) and $\mathrm{d}\lambda/\mathrm{d}s > 0$, Proposition 6). Then, if $\mathrm{d}k_p/\mathrm{d}s \geq 0$, both derivatives are non-negative with at least one positive and (a) is satisfied. If instead $\mathrm{d}k_p/\mathrm{d}s < 0$ then (b) is satisfied because the term in square in brackets in the second term in the right-hand side of equation (E.28) is negative with $\phi_g > \phi_p$, so the whole second term is positive. ■

Corollary 1 *The derivatives w.r.t. net worth a of capital demands $k_p(s,a,d)$, $k_g(s,a,d)$, labor demands $n_p(s,a,d)$, $n_g(s,a,d)$, $n(s,a,d)$, and output supplies $y_p(s,a,d)$, $y_g(s,a,d)$, $y(s,a,d)$ are all positive.*

Proof: By the chain rule we can write $\mathrm{d}k_p/\mathrm{d}a = (\partial k_p/\partial\lambda)(\partial\lambda/\partial a)$. The FOC for capital imply that $\partial k_p/\partial\lambda < 0$ (Proposition 5), and Proposition 6 tells that $\partial\lambda/\partial a < 0$, so it must be that $\partial k_p/\partial a > 0$. The proof for $\partial k_g/\partial a > 0$ is identical. Given this, Lemma 6 proves the case for $n_p(s,a,d)$ and $n_g(s,a,d)$, and $n(s,a,d)$ follows. Given that both production factors in each sector increase with a so must output $y_p(s,a,d)$ and $y_g(s,a,d)$ in each sector, and so must total output $y(s,a,d)$. ■

Corollary 2 *If $\phi_p = \phi_g$ (or $\phi_p \neq \phi_g$ and $a = 0$), the derivatives w.r.t. productivity s of capital demands $k_p(s,a,d)$, $k_g(s,a,d)$, labor demands $n_p(s,a,d)$, $n_g(s,a,d)$, $n(s,a,d)$, and output supplies $y_p(s,a,d)$, $y_g(s,a,d)$, $y(s,a,d)$ are all positive. If $\phi_p \neq \phi_g$ (and $a > 0$), then these derivatives are also positive, except for the ones of the sector with lower ϕ, which could go either way.*

Proof: Equation (E.20) tells us that k_p/k_g is constant whenever $\phi_p = \phi_g$, which means that k_p and k_g comove with k. Hence, given that $\mathrm{d}k/\mathrm{d}s > 0$ (Proposition 7), it must also be that $\mathrm{d}k_p/\mathrm{d}s > 0$ and $\mathrm{d}k_g/\mathrm{d}s > 0$. The marginal revenue product of labor in each sector is equal to the wage, see equation 9. Given that whenever s increases so do k_p and k_g, equation 9 requires $\mathrm{d}n_p/\mathrm{d}s > 0$ and $\mathrm{d}n_g/\mathrm{d}s > 0$ and hence $\mathrm{d}n/\mathrm{d}s > 0$. Finally, given that all production factors increase with s in all sectors, so do output in each sector and total output. For $\phi_p \neq \phi_g$ and $a = 0$ the same goes through because s does not change λ (Proposition 6) and hence k_p/k_g does not change. Instead, with $\phi_p \neq \phi_g$ and $a > 0$, s increases λ (Proposition 6), which moves the ratio k_p/k_g towards the sector with higher ϕ (Proposition 4). This implies that capital, labor, and output in that sector increase, but there is nothing preventing capital, labor, and output in the other sector to decline. ■

Corollary 3 *The derivative of $dk(s,a,d)/da$ w.r.t. λ is positive whenever at least one of ϕ_p or ϕ_g is strictly positive, and zero otherwise.*

Proof: With $\phi_g = \phi_p$ equation (E.26) becomes:

$$\frac{\mathrm{d}k(s,a,d)}{\mathrm{d}a} = \phi_a \left(1 - \phi_p \frac{\partial p_p y_p}{\partial k_p}\right)^{-1} \tag{E.30}$$

The term $\phi_p \frac{\partial p_p y_p}{\partial k_p}$ increases with λ (Lemma 4), and so does $\mathrm{d}k(s,a,d)/\mathrm{d}a$ if $\phi_g = \phi_p > 0$. For the case $\phi_g \neq \phi_p$, both terms $\phi_p \frac{\partial p_p y_p}{\partial k_p}$ and $\phi_g \frac{\partial p_g y_g}{\partial k_g}$ in equation (E.26) increase with λ, but a complete proof also needs to characterize how the terms $(\partial k_j/\partial\lambda)/(\partial k/\partial\lambda)$ change with λ. To do so, we go back to the solution of k_p in equation (E.22), and analogously for k_g, to write,

$$\frac{\partial k_j}{\partial\lambda} = k_j \frac{\partial \log k_j}{\partial\lambda} = -\nu k_j \frac{1 - \phi_j(r+\delta)}{1 + \phi_j\lambda} \frac{1}{r+\delta+\lambda} = -\nu k_j \frac{A_j(\lambda)}{r+\delta+\lambda}$$

where we have defined $\nu \equiv 1 + \alpha(\sigma - 1)$ and

$$A_j(\lambda) \equiv 1 - \phi_j \frac{r + \delta + \lambda}{1 + \phi_j \lambda} = \frac{1 - \phi_j(r + \delta)}{1 + \phi_j \lambda}$$

Note that under Lemma 1, $A_j \in (0, 1]$. Therefore, we can write

$$\frac{\partial k_j/\partial\lambda}{\partial k/\partial\lambda} = \frac{k_j A_j(\lambda)}{\sum_{\ell=p,g} k_\ell A_\ell(\lambda)}$$

Finally, because $\frac{\partial p_j y_j}{\partial k_j} = \frac{r+\delta+\lambda}{1+\phi_j\lambda}$ we can rewrite equation (E.26) as

$$\frac{\mathrm{d}k}{\mathrm{d}a} = \frac{\phi_a}{H(\lambda)}, \qquad H(\lambda) \equiv \frac{\sum_{j=p,g} k_j A_j(\lambda)^2}{\sum_{j=p,g} k_j A_j(\lambda)}$$

Hence, we need to show that $H'(\lambda) \leq 0$. Let

$$S_m \equiv \sum_{j=p,g} k_j A_j^m.$$

Then $H = S_2/S_1$. Since

$$\frac{\partial \log A_j}{\partial \lambda} = -\frac{1 - A_j}{r + \delta + \lambda},$$

we have

$$S_m' = -\frac{m S_m + (\nu - m) S_{m+1}}{r + \delta + \lambda}.$$

Therefore,

$$H'(\lambda) = -\frac{S_1 S_2 + (\nu - 2) S_1 S_3 - (\nu - 1) S_2^2}{(r + \delta + \lambda) S_1^2}.$$

The numerator can be rewritten as

$$S_1(S_2 - S_3) + (\nu - 1)(S_1 S_3 - S_2^2).$$

This expression is non-negative because $A_j \in (0, 1]$ implies $S_2 \geq S_3$, $\nu > 1$, and $S_1 S_3 - S_2^2 = k_p k_g A_p A_g (A_p - A_g)^2 \geq 0$. Hence $H'(\lambda) \leq 0$, and so

$$\frac{\partial}{\partial \lambda}\left(\frac{\mathrm{d}k}{\mathrm{d}a}\right) = -\phi_a \frac{H'(\lambda)}{H(\lambda)^2} \geq 0.$$

The inequality is strict whenever at least one of ϕ_p or ϕ_g is strictly positive; if $\phi_p = \phi_g = 0$, then $A_p = A_g = 1$, $H(\lambda) = 1$, and $\mathrm{d}k/\mathrm{d}a = \phi_a$ is constant in λ. ■

Corollary 4 *The derivative $dk\,(s, a, d)\,/da$ is negative w.r.t. a whenever at least one of ϕ_p or ϕ_g is positive (and it is zero otherwise), and positive w.r.t. s whenever $a > 0$ and at least one of ϕ_p or ϕ_g is positive (and it is zero otherwise).*

Proof: The proof of Corollary 3 above shows that

$$\frac{\mathrm{d}k}{\mathrm{d}a} = \frac{\phi_a}{H(\lambda)}$$

with $H'(\lambda) \leq 0$, strictly if at least one of ϕ_p or ϕ_g is strictly positive. Importantly, dk/da has no direct dependence on s or a once λ is held fixed. Hence, by the chain rule, we can write

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}a}\left(\frac{\mathrm{d}k}{\mathrm{d}a}\right) &= \frac{\partial}{\partial\lambda}\left(\frac{\mathrm{d}k}{\mathrm{d}a}\right)\frac{\mathrm{d}\lambda}{\mathrm{d}a} \\
\frac{\mathrm{d}}{\mathrm{d}s}\left(\frac{\mathrm{d}k}{\mathrm{d}a}\right) &= \frac{\partial}{\partial\lambda}\left(\frac{\mathrm{d}k}{\mathrm{d}a}\right)\frac{\mathrm{d}\lambda}{\mathrm{d}s}
\end{aligned}$$

Let's start with a. By Corollary 3, the first term on the right-hand side is positive if at least one of ϕ_p or ϕ_g is positive (and zero otherwise), while by Proposition 6, the second one is negative. Therefore, dk/da is weakly decreasing in a, and strictly decreasing whenever at least one of ϕ_p or ϕ_g is strictly positive. Regarding s, we just need to note that $\mathrm{d}\lambda/\mathrm{d}s > 0$ by Proposition 6 if $a > 0$ and zero otherwise, so we have that $\mathrm{d}k/\mathrm{d}a$ is weakly increasing in s. The increase is strict whenever $a > 0$ and at least one of ϕ_p or ϕ_g is strictly positive. ■

Profit function. We can obtain the derivatives of the profit function $\pi(s,a,d)$ as

$$\frac{\partial\pi(s,a,d)}{\partial a} = \left[\frac{\partial p_p y_p}{\partial k_p} - (r+\delta)\right]\frac{\partial k_p(s,a,d)}{\partial a} + \left[\frac{\partial p_g y_g}{\partial k_g} - (r+\delta)\right]\frac{\partial k_g(s,a,d)}{\partial a} \tag{E.31}$$

$$\begin{aligned}
\frac{\partial\pi(s,a,d)}{\partial s} &= \left[\frac{\partial p_p y_p}{\partial k_p} - (r+\delta)\right]\frac{\partial k_p(s,a,d)}{\partial s} + \left[\frac{\partial p_g y_g}{\partial k_g} - (r+\delta)\right]\frac{\partial k_g(s,a,d)}{\partial s} \\
&+ \frac{\partial p_p y_p}{\partial s} + \frac{\partial p_g y_g}{\partial s}
\end{aligned} \tag{E.32}$$

Using the FOC for k_p and k_g given by equations 10 to substitute out the terms in square brackets of equation (E.31), and next using equation (E.16) we obtain

$$\frac{\partial\pi(s,a,d)}{\partial a} = \phi_a\lambda(s,a,d) \tag{E.33}$$

likewise, doing the same thing with equations 10, (E.32), and (E.17) we obtain

$$\frac{\partial\pi(s,a,d)}{\partial s} = (1+\phi_p\lambda(s,a,d))\frac{\partial p_p y_p}{\partial s} + (1+\phi_g\lambda(s,a,d))\frac{\partial p_g y_g}{\partial s} \tag{E.34}$$

Proposition 8 *The derivatives of $\pi(s,a,d)$ w.r.t. net worth a and productivity s are positive.*

Proof: The derivatives of the profit function with respect to a and s are given by (E.33) and (E.34). These derivatives are positive because $\lambda(s,a,d) > 0$ for constrained agents and $\frac{\partial p_p y_p}{\partial s} > 0$ and $\frac{\partial p_g y_g}{\partial s} > 0$ (see the revenue functions). ■

Profits increase with a because more net worth allows to increase capital and hence profits. Profits increase with s for two reasons. First, there is the direct increase of revenues with s for given capital. Second, if $\phi_p > 0$ and/or $\phi_g > 0$ the increase in revenues with s allows to increase capital, which in turn increases profits. We can also show, Corollary 5 below, that the marginal value of net worth decreases with net worth a and increases with productivity s.

Corollary 5 *The derivative of $\partial\pi\left(s,a,d\right)/\partial a$ is negative with respect to a, and is positive with respect to s when $a > 0$ and zero when $a = 0$.*

Proof: Using (E.33) we can write the second derivatives as, $\frac{\partial^2\pi(s,a,d)}{\partial a^2} = \phi_a \frac{\partial\lambda(s,a,d)}{\partial a}$ and $\frac{\partial^2\pi(s,a,d)}{\partial a \partial s} = \phi_a \frac{\partial\lambda(s,a,d)}{\partial s}$, with $\partial\lambda/\partial a < 0$ and $\partial\lambda/\partial s \geq 0$ (Proposition 6). ■

E.4 A procurement shock

Finally, in this Section we analyze how firm choices change upon arrival of a procurement project for the case $\sigma_p = \sigma_g = \sigma$. To do so, we compare the choices of firms in the $(s,a,1)$ state with firms in the $(s,a,0)$ state.

E.4.1 Unconstrained firms

For unconstrained firms, the increase in total capital is given by $\frac{k^*(s,a,1)}{k^*(s,a,0)} = 1 + \left(\frac{B_g}{B_p}\right)^{\sigma}$, as $k^*\left(s,a,0\right) = k_p^*\left(s,a,1\right)$, that is, the amount of capital used in the private sector for the unconstrained firm with a procurement project equals the capital stock it was using without procurement. This means that unconstrained firms do not change their private sector operations and increase their capital stock to meet the extra demand. The increase in capital $k^*\left(s,a,1\right) - k^*\left(s,a,0\right)$ is given by $\left(\frac{B_g}{B_p}\right)^{\sigma} k^*\left(s,a,0\right)$. Because $k^*\left(s,a,0\right)$ increases with s and is independent from a, so does the capital increase with procurement. We can also see that the value of a procurement contract increases with firm productivity s and is independent from firm net worth a. This can be seen by use of the expression $\pi = \frac{1}{\sigma-1}\frac{r+\delta}{\alpha}k$, which implies that $\pi^*\left(s,a,1\right) - \pi^*\left(s,a,0\right)$ is proportional to the capital increase $k^*\left(s,a,1\right) - k^*\left(s,a,0\right)$. This could have also been seen by using equation (E.15) for $d = 1$ and $d = 0$, which allows to express $\pi^*\left(s,a,1\right) - \pi^*\left(s,a,0\right) = \left(\frac{B_g^{\sigma}}{\sigma}\right)\left[\left(\frac{\sigma-1}{\sigma}\right)\frac{s}{c(r,w)}\right]^{\sigma-1}$.

E.4.2 Constrained firms

For financially constrained firms, the effects of a procurement shock are more intricate, depending on the size of ϕ_g relative to ϕ_p and the net worth of the firm. Let's start by stating, in Lemma 7 and Lemma 8, two results that relate the private sector spillovers of a procurement shock to the change in the severity of the financial constraints.

Lemma 7 *A procurement shock generates a negative private sector spillover if and only if the procurement shock makes the firm more constrained, that is, $y_p\left(s,a,1\right) < y_p\left(s,a,0\right) \Leftrightarrow \lambda\left(s,a,1\right) > \lambda\left(s,a,0\right)$.*

Proof: The FOC for optimal k_p is given by equation (E.4), whose right-hand side increases with λ (see Lemma 1), so more constrained firms have a higher marginal revenue product of capital k_p. Proposition 5 establishes that this happens with lower k_p, lower n_p, and lower y_p. Hence, $y_p(s,a,1) < y_p(s,a,0) \Leftrightarrow \lambda(s,a,1) > \lambda(s,a,0)$. ■

Lemma 8 *A procurement shock generates a negative private sector spillover for constrained firms if and only if the chosen production for the public sector cannot be self-financed, that is, if and only if* $\phi_g[p_g(s,a,1)\, y_g(s,a,1) - wn_g(s,a,1)] < k_g(s,a,1)$

Proof: A private sector negative spillover of procurement means that $y_p(s,a,1) < y_p(s,a,0)$. Given that capital and labor in each sector co-move together (Lemma 6), we have that a private sector negative spillover requires both $k_p(s,a,1) < k_p(s,a,0)$ and $n_p(s,a,1) < n_p(s,a,0)$. Let's define $R_p(s,a,d) \equiv p_p(s,a,d)\, y_p(s,a,d)$ and $R_g(s,a,d) \equiv p_g(s,a,d)\, y_g(s,a,d)$. The choices of capital for constrained firms, with and without procurement, are given by

$$\begin{aligned} k_p(s,a,0) - \phi_p\left(R_p(s,a,0) - wn_p(s,a,0)\right) &= \phi_a a \\ k_p(s,a,1) - \phi_p\left(R_p(s,a,1) - wn_p(s,a,1)\right) &= \phi_a a - \left[k_g(s,a,1) - \phi_g\left(R_g(s,a,1) - wn_g(s,a,1)\right)\right] \end{aligned}$$

Importantly, the left-hand side of these equations increases with k_p. To see how, first note that the derivative of this term w.r.t. k_p is equal to $1 - \phi_p \frac{\partial p_p y_p}{\partial k_p}$, as the terms with n cancel out due to the FOC in equation 9. Now, $\phi_p \frac{\partial p_p y_p}{\partial k_p} < 1$ according to Lemma 3, so the derivative is positive. Hence, $k_p(s,a,1) < k_p(s,a,0)$ (a private sector negative spillover of procurement) implies

$$k_p(s,a,1) - \phi_p\left[R_p(s,a,1) - wn_p(s,a,1)\right] < k_p(s,a,0) - \phi_p\left[R_p(s,a,0) - wn_p(s,a,0)\right]$$

which requires $k_g(s,a,1) > \phi_g\left[R_g(s,a,1) - wn_g(s,a,1)\right]$. ■

Given this, we can now analyze the effect of a procurement shock in two special cases. First, when there are no earnings-based financial constraints, a procurement shock always generates a negative private sector spillover, as any given collateral a has to be split between the public and private sector markets, making the firm more financially constrained.

Proposition 9 *When* $\phi_g = 0$ *and* $\phi_p = 0$, *if* $\phi_a > 0$ *and* $a > 0$ *a procurement shock does not change firm size, that is,* $k(s,a,1) = k(s,a,0)$, *makes the firm more constrained, that is,* $\lambda(s,a,1) > \lambda(s,a,0)$, *and generates a negative private sector spillover, that is,* $y_p(s,a,1) < y_p(s,a,0)$.

Proof: We can write the borrowing constraint for $d = 0$ and $d = 1$ firms as

$$\begin{aligned} k_p(s,a,0) &= \phi_a a \\ k_p(s,a,1) + k_g(s,a,1) &= \phi_a a \end{aligned}$$

so it must be that $k_p(s,a,1) + k_g(s,a,1) = k_p(s,a,0)$ and hence firm size does not change with procurement. The FOC for k_g, equation (E.5), states that $k_g(s,a,1) > 0$, hence it must be $k_p(s,a,1) < k_p(s,a,0)$. Lemma 7 then implies that $\lambda(s,a,1) > \lambda(s,a,0)$. ■

Things are different in the other simple case, when there are earnings-based financial constraints but no collateral constraints (or, in the general case for firms with $a = 0$). In this case, a procurement shock makes the firm less financially constrained if $\phi_g > \phi_p$, allowing it to grow and generating a positive private sector spillover. However, whenever $\phi_g < \phi_p$ the firm becomes more financially constrained, there is a negative private sector spillover, and the firm may even shrink in size.

Proposition 10 *When $\phi_a = 0$ (or $\phi_a > 0$ but $a = 0$) and $\phi_g > \phi_p$, a procurement shock increases firm size, that is, $k(s,a,1) > k(s,a,0)$, makes the firm less constrained, that is, $\lambda(s,a,1) < \lambda(s,a,0)$, and generates a positive private sector spillover, that is, $y_p(s,a,1) > y_p(s,a,0)$. Instead, if $\phi_g < \phi_p$, a procurement shock makes the firm more constrained, generates a negative private sector spillover, and might reduce firm size if ϕ_g is low enough. Whenever $\phi_g = \phi_p$, a procurement shock changes neither λ nor y_p and allows the firm to increase its size.*

Proof: Let's define $R_p(s,a,d) \equiv p_p(s,a,d)\, y_p(s,a,d)$ and $R_g(s,a,d) \equiv p_g(s,a,d)\, y_g(s,a,d)$. We can take the difference of the borrowing constraint for $d = 1$ and $d = 0$ firms to write,

$$\tilde{\phi}_p \frac{R_p(s,a,0)}{k_p(s,a,0)} = u(s,a,1)\, \tilde{\phi}_p \frac{R_p(s,a,1)}{k_p(s,a,1)} + (1 - u(s,a,1))\, \tilde{\phi}_g \frac{R_g(s,a,1)}{k_g(s,a,1)} \tag{E.35}$$

where $u \equiv k_p/k$, and we have used equations (E.6) and (E.7) to replace revenues minus labor costs in the financial constraint and have defined $\tilde{\phi}_p \equiv \phi_p[1-(1-\alpha)(\sigma-1)/\sigma]$ and $\tilde{\phi}_g \equiv \phi_g[1-(1-\alpha)(\sigma-1)/\sigma]$. If $\phi_g = \phi_p$ (and hence $\tilde{\phi}_g = \tilde{\phi}_p$), firms with $d = 1$ equalize the average product in the public and private sectors, see equations (E.4) and (E.5), and hence equation (E.35) becomes $R_p(s,a,0)/k_p(s,a,0) = R_p(s,a,1)/k_p(s,a,1)$, which implies $k_p(s,a,1) = k_p(s,a,0)$ and hence there is no private sector spillover. Because the average product in the public sector goes to infinity when $k_g = 0$, it must be that $k_g(s,a,1) > 0$ and hence $k(s,a,1) \equiv k_p(s,a,1) + k_g(s,a,1) > k(s,a,0)$, that is, a procurement shock makes the firm grow. If $\phi_g > \phi_p$, then the second term in the weighted average of the right-hand-side of equation (E.35) is larger than the first one. This can be easily seen by multiplying both sides of equation (E.4) by $\tilde{\phi}_p$ and both sides of equation (E.5) by $\tilde{\phi}_g$. Then equation (E.35) implies that $R_p(s,a,1)/k_p(s,a,1) < R_p(s,a,0)/k_p(s,a,0)$ and hence $k_p(s,a,1) > k_p(s,a,0)$, that is, there is a positive private sector spillover, and because $k_g(s,a,1) > 0$ it must be that $k(s,a,1) > k(s,a,0)$. If $\phi_g < \phi_p$, then the second term in the weighted average of the right-hand side of equation (E.35) is smaller than the first one. Then $R_p(s,a,1)/k_p(s,a,1) > R_p(s,a,0)/k_p(s,a,0)$ and $k_p(s,a,1) < k_p(s,a,0)$, that is, there is a negative private sector spillover. The sign of $k(s,a,1) - k(s,a,0) = k_p(s,a,1) - k_p(s,a,0) + k_g(s,a,1)$ is undetermined. However, note that when $\phi_g = 0$, the whole second term in the right-hand-side of equation (E.35) vanishes and we have that $R_p(s,a,0)/k_p(s,a,0) = R_p(s,a,1)/k(s,a,1)$ or $k(s,a,1)/k(s,a,0) = R_p(s,a,1)/R_p(s,a,0)$. Because of the negative private sector spillover, this ratio is less than one and hence $k(s,a,1) < k(s,a,0)$. That is, with a small enough ϕ_g a procurement shock may make the firms shrink in size. Finally, the results for λ follow from Lemma 7. ∎

That is, when firms have no collateral and the only financing comes from revenues, if $\phi_g = \phi_p$ a procurement shock has a similar effect as for unconstrained firms: it leaves private sector activity unchanged and the firm increases in size to serve the public sector demand. If $\phi_g > \phi_p$, a procurement shock increases the supply of credit more than its demand, it allows the firm to grow, it makes it less constrained, and it generates a positive private sector spillover. This is not in contradiction with part (b) in Proposition 4, which states that whenever $\phi_g > \phi_p$ the firm tilts capital towards the public sector. To understand why, imagine that a procurement shock generated no private sector spillover such that $k_p(s,a,1) = \tilde{\phi}_p R_p(s,a,1)$ and $k_g(s,a,1) = \tilde{\phi}_g R_g(s,a,1)$. With $\phi_g > \phi_p$ this would allow a lower marginal (and average) product of capital in the public sector, that is $R_g(s,a,1)/k_g(s,a,1) < R_p(s,a,1)/k_p(s,a,1)$. A positive private sector spillover happens because part of the extra financing in the public sector is used in the private sector, which allows $k_p(s,a,1) > k_p(s,a,0)$ while still having $R_g(s,a,1)/k_g(s,a,1) < R_p(s,a,1)/k_p(s,a,1)$.

Finally, putting the last two propositions together, we can state that,

Proposition 11 *Take the case with $\phi_a > 0$.*

(a) Whenever $\phi_g = \phi_p$, a procurement shock increases firm size, makes the firm more constrained, and generates a negative private sector spillover.

(b) Whenever $\phi_g < \phi_p$, a procurement shock has the same effects as the case $\phi_g = \phi_p$ unless ϕ_g is small enough, which may lead to a decline in firm size.

(c) Whenever $\phi_g > \phi_p$, a procurement shock has the same effects as the case $\phi_g = \phi_p$ unless a is small enough, which may relax the firm financial constraint and generate a positive private sector spillover.

Proof: Proposition 9 states that whenever $\phi_a > 0$ and $\phi_g = \phi_p = 0$, because of scarce collateral a, a procurement shock does not let the firm borrow more and grow, makes the firm more financially constrained because of the extra demand, and generates a negative private sector spillover to allow production for the public sector. When $\phi_a = 0$ and $\phi_g = \phi_p > 0$ Proposition 10 shows that a procurement shock allows the firm to grow, does not change the severity of the financial constraint, and does not generate any private sector spillover. Hence, putting together both propositions proves (a). For the case (b), $\phi_a > 0$ and $\phi_g < \phi_p$, Proposition 9 and Proposition 10 coincide in stating an increase in the severity of the financial constraint and a negative private sector spillover. Regarding the change in firm size, the collateral channel in Proposition 9 does not allow the firm to grow with a procurement shock, while the revenue channel in Proposition 10 states that the firm will generally grow in size, but it may shrink if ϕ_g is small enough. So this property should be inherited when both channels are active. To show it formally, we can take the difference between the borrowing constraint of any firm under the $d = 1$ and $d = 0$ cases to get,

$$k(s,a,1) - k(s,a,0) = \tilde{\phi}_p \big[R_p(s,a,1) + R_g(s,a,1) - R_p(s,a,0)\big] + \left(\tilde{\phi}_g - \tilde{\phi}_p\right) R_g(s,a,1) \tag{E.36}$$

where we have defined $R_p(s,a,d)$, $R_g(s,a,d)$, $\tilde{\phi}_p$, and $\tilde{\phi}_g$ as in the proof of Proposition 10. Now, if $\phi_g = \phi_p$, the second term in the right-hand-side of equation (E.36) disappears. If the firm keeps capital unchanged after a procurement shock, $k(s,a,1) = k(s,a,0)$, and reallocates some capital to the public sector such that $k_g(s,a,1) > 0$ revenues increase due to the concavity of the revenue functions, that is, $R_p(s,a,1) + R_g(s,a,1) > R_p(s,a,0)$ which increases borrowing capacity and hence $k(s,a,1)$. However, whenever $\phi_p > \phi_g$, the extra borrowing through higher revenues of shifting capital towards the public sector is offset by the lower pledgeability of public sector revenues, as shown in the 2nd term of equation (E.36). In net, it is not clear whether total borrowing capacity and hence $k(s,a,1)$ will go up or down, as it will depend on the size of $\tilde{\phi}_g$. For example, with $\phi_p > \phi_g = 0$ we will have $k(s,a,1) < k(s,a,0)$. To see why, note that with $\phi_g = 0$ equation (E.36) becomes

$$k(s,a,1) - k(s,a,0) = \tilde{\phi}_p\big[R_p(s,a,1) - R_p(s,a,0)\big]$$

Because of the negative private sector spillover whenever $\phi_p > \phi_g$, $R_p(s,a,1) < R_p(s,a,0)$ and thus $k(s,a,1) < k(s,a,0)$. Finally, for the case (c), $\phi_a > 0$ and $\phi_g > \phi_p$, Proposition 9 and Proposition 10 together imply that a procurement shock makes the firm grow because of the extra credit that comes with procurement. Regarding the private sector spillover, let's do as in proof of Proposition 10 and rewrite the difference of the borrowing constraint for $d = 1$ and $d = 0$ firms as,

$$\begin{aligned} \tilde{\phi}_p \frac{R_p(s,a,0)}{k_p(s,a,0)} &= \tilde{\phi}_p \frac{R_p(s,a,1)}{k_p(s,a,1)} + (1 - u(s,a,1))\left[\tilde{\phi}_g \frac{R_g(s,a,1)}{k_g(s,a,1)} - \tilde{\phi}_p \frac{R_p(s,a,1)}{k_p(s,a,1)}\right] \\ &+ \phi_a a\left[\frac{1}{k(s,a,1)} - \frac{1}{k(s,a,0)}\right] \end{aligned} \tag{E.37}$$

The second term in equation (E.37) is positive whenever $\phi_g > \phi_p$ (see proof of Proposition 10). Hence, if $a = 0$, equation (E.37) implies $R_p(s,a,1)/k_p(s,a,1) < R_p(s,a,0)/k_p(s,a,0)$, which requires $k_p(s,a,1) > k_p(s,a,0)$, a positive private sector spillover. Now, if $a > 0$, the third term in the right-hand side of equation (E.37) reappears. Because $k(s,a,1) > k(s,a,0)$ whenever $\phi_g > \phi_p$ this term is negative. With low enough a, the sum of the second and third term in the right-hand side of equation (E.37) is positive and hence we still have $R_p(s,a,1)/k_p(s,a,1) < R_p(s,a,0)/k_p(s,a,0)$ and $k_p(s,a,1) > k_p(s,a,0)$. However, with larger a the sum of the second and third term in the right-hand side of equation (E.37) is negative and hence we will have $R_p(s,a,1)/k_p(s,a,1) > R_p(s,a,0)/k_p(s,a,0)$ and $k_p(s,a,1) < k_p(s,a,0)$, that is, a negative private sector spillover. The result for $\lambda(s,a,1) - \lambda(s,a,0)$ follows from Lemma 7. ■

Hence, whenever $\phi_g > \phi_p$ (our empirically relevant case), a negative private sector spillover happens due to the collateral constraints and despite the earnings-based financial constraint, which pushes in the opposite direction.

Proposition 12 *The value of procurement is strictly positive, that is,* $\pi(s,a,1) - \pi(s,a,0) > 0\ \forall s,a$. *For* $\phi_g \geq \phi_p$, *the value of procurement is increasing in net worth* a *and in productivity* s, *except for the case of firms with very low net worth whenever* $\phi_g > \phi_p$.

Proof: The first part is trivial. A firm with $d = 1$ can always replicate the profits of a firm with $d = 0$ by choosing $k_p(s,a,1) = k_p(s,a,0)$ and $k_g(s,a,1) = 0$. Because of the concavity of the revenue functions, it is optimal for any firm with $d = 1$ to reallocate capital across sectors, choose $k_g(s,a,1) > 0$, and increase profits. For the effect of net worth a on the value of procurement, we want to show that $\partial[\pi(s,a,1) - \pi(s,a,0)]/\partial a > 0$. Equation (E.33) implies that

$$\frac{\partial[\pi(s,a,1) - \pi(s,a,0)]}{\partial a} = \phi_a\big[\lambda(s,a,1) - \lambda(s,a,0)\big]$$

and the sign of $\lambda(s,a,1)-\lambda(s,a,0)$ is given by Proposition 11. Finally, for the effect of productivity s on the value of procurement we want to show that $\partial[\pi(s,a,1)-\pi(s,a,0)]\,\partial s>0$ whenever $\phi_g\geq\phi_p$. Equation (E.34) implies

$$\begin{aligned}\frac{\partial[\pi(s,a,1)-\pi(s,a,0)]}{\partial s} &= (1+\phi_p\lambda(s,a,1))\frac{\partial p_p y_p}{\partial s}+(1+\phi_g\lambda(s,a,1))\frac{\partial p_g y_g}{\partial s}\\ &- (1+\phi_p\lambda(s,a,0))\frac{\partial p_p y_p}{\partial s}\end{aligned}$$

Note that $\frac{\partial p_p y_p}{\partial s}=\frac{1}{\alpha}\frac{k_p}{s}\frac{\partial p_p y_p}{\partial k_p}=\frac{1}{\alpha}\frac{k_p}{s}\frac{r+\delta+\lambda}{1+\lambda\phi_p}$ and an analogous expression holds for $\frac{\partial p_g y_g}{\partial s}$. Substituting these expressions in the above equation gives

$$\frac{\partial[\pi(s,a,1)-\pi(s,a,0)]}{\partial s}=\frac{r+\delta+\lambda(s,a,1)}{\alpha s}[k_p(s,a,1)+k_g(s,a,1)]-\frac{r+\delta+\lambda(s,a,0)}{\alpha s}k_p(s,a,0)$$

With $\phi_g\geq\phi_p$, Proposition 11 states that $k_p(s,a,1)+k_g(s,a,1)>k_p(s,a,0)$. Therefore, whenever $\lambda(s,a,1)>\lambda(s,a,0)$ we can guarantee that $\frac{\partial[\pi(s,a,1)-\pi(s,a,0)]}{\partial s}>0$. Proposition 11 shows this will always happen when $\phi_g=\phi_p$, and also for $\phi_g>\phi_p$ except for very low a. ■

F Equilibrium definition

Let $\mathbf{X}\equiv\mathbf{S}\times\mathbf{A}\times\{0,1\}$ be the state space of the entrepreneur problem, $\mathbf{X}_1\equiv\mathbf{S}\times\mathbf{A}\times\{1\}$ the subset of the state space for firms with a procurement project, $\mathcal{X}$ a σ-algebra generated by $\mathbf{X}$, and Γ a probability measure over $\mathcal{X}$. Given government policy parameters (Y_g,m_g) and a distribution of entrant entrepreneurs Γ_0 over $\mathcal{X}$, we define the steady state equilibrium of the model as (i) a triplet of prices (w,r,P_g), (ii) a level constant $\bar{p}$ and a tax τ, (iii) firms' policy functions, (iv) macroeconomic aggregates (Y_p,C,A,T), (v) and a probability measure Γ, such that:

1. The household solves its optimization problem, which delivers the modified golden rule condition $\beta(1+r)=1$ and the budget constraint $C=Ar+w-\tau+T$

2. Entrepreneurs solve their optimization problem, which characterizes the firms' policy functions

3. The final good producers solve their optimization problems and the markets for each intermediate variety clear. This means that,

$$\int_{\mathbf{X}_1}p_g(s,a,1)\,y_g(s,a,1)\,\mathrm{d}\Gamma=P_gY_g\qquad\text{and}\qquad\int_{\mathbf{X}}p_p(s,a,d)\,y_p(s,a,d)\,\mathrm{d}\Gamma=Y_p$$

4. The factor markets clear

$$\int_{\mathbf{X}}k(s,a,d)\,\mathrm{d}\Gamma=\int_{\mathbf{X}}a\,\mathrm{d}\Gamma+A\qquad\text{and}\qquad\int_{\mathbf{X}}n(s,a,d)\,\mathrm{d}\Gamma=1$$

5. The probability of obtaining procurement projects is consistent with the measure of goods bought by the public sector:

$$\int_{\mathbf{X}}\Pr(d'=1\mid b(s,a,d),d,s)\,\mathrm{d}\Gamma=m_g\tag{F.1}$$

6. The budget constraint of the government holds: $P_g Y_g = \tau$

7. The transfers to the household are given by the net accidental bequests,

$$T = (1-\theta)\left[\int_{\mathbf{X}} a'(s,a,d)\mathrm{d}\Gamma - \int_{\mathbf{X}} a\,\mathrm{d}\Gamma_0\right]$$

8. Γ is the stationary distribution of entrepreneurs consistent with the law of motion of the distribution properly obtained from the policy functions of the entrepeneurs, the law of motion of the productivity shocks s, and the probability function of procurement.

9. By Walras law, the market for the private good clears:

$$Y_p = \int_{\mathbf{X}} \left[b\left(s,a,d\right) + c\left(s,a,d\right) + \delta k\left(s,a,d\right)\right]\mathrm{d}\Gamma + C$$

G Aggregate output, productivity, and prices

GDP and aggregate TFP. We can define GDP in the model as

$$Y \;\equiv\; Y_p + P_g Y_g \;=\; \mathrm{TFP}_p K_p^{\alpha} N_p^{1-\alpha} + P_g \mathrm{TFP}_g K_g^{\alpha} N_g^{1-\alpha} \;=\; \mathrm{TFP} K^{\alpha} N^{1-\alpha} \tag{G.1}$$

where $K \equiv K_p + K_g$, $N \equiv N_p + N_g$, and aggregate TFP in units of the private goods is defined by,

$$\mathrm{TFP} \equiv \left(\frac{K_p}{K}\right)^{\alpha}\left(\frac{N_p}{N}\right)^{1-\alpha}\mathrm{TFP}_p + P_g\left(\frac{K_g}{K}\right)^{\alpha}\left(\frac{N_g}{N}\right)^{1-\alpha}\mathrm{TFP}_g \tag{G.2}$$

sectoral TFP. The TFP for the private and public sectors are given by,

$$\mathrm{TFP}_p \;\equiv\; \frac{Y_p}{K_p^{\alpha} N_p^{1-\alpha}} = \left\{\int_{[0,1]} \left[s_i\left(\frac{\overline{\mathrm{MRPK}}_p}{\mathrm{MRPK}_{ip}}\right)^{\alpha}\left(\frac{\overline{\mathrm{MRPN}}_p}{\mathrm{MRPN}_{ip}}\right)^{1-\alpha}\right]^{\sigma_p-1}\mathrm{d}i\right\}^{\frac{1}{\sigma_p-1}} \tag{G.3}$$

$$\mathrm{TFP}_g \;\equiv\; \frac{Y_g}{K_g^{\alpha} N_g^{1-\alpha}} = \left\{\int_{I_g} \frac{1}{m_g}\left[s_i\left(\frac{\overline{\mathrm{MRPK}}_g}{\mathrm{MRPK}_{ig}}\right)^{\alpha}\left(\frac{\overline{\mathrm{MRPN}}_g}{\mathrm{MRPN}_{ig}}\right)^{1-\alpha}\right]^{\sigma_g-1}\mathrm{d}i\right\}^{\frac{1}{\sigma_g-1}} \tag{G.4}$$

where
$$\frac{1}{\overline{\mathrm{MRPK}}_p} \equiv \int_{[0,1]} \frac{p_{ip}y_{ip}}{P_p Y_p}\frac{1}{\mathrm{MRPK}_{ip}}\mathrm{d}i, \qquad \frac{1}{\overline{\mathrm{MRPK}}_g} \equiv \int_{I_g} \frac{p_{ig}y_{ig}}{P_g Y_g}\frac{1}{\mathrm{MRPK}_{ig}}\mathrm{d}i$$
$$\frac{1}{\overline{\mathrm{MRPN}}_p} \equiv \int_{[0,1]} \frac{p_{ip}y_{ip}}{P_p Y_p}\frac{1}{\mathrm{MRPN}_{ip}}\mathrm{d}i, \qquad \frac{1}{\overline{\mathrm{MRPN}}_g} \equiv \int_{I_g} \frac{p_{ig}y_{ig}}{P_g Y_g}\frac{1}{\mathrm{MRPN}_{ig}}\mathrm{d}i$$

As shown in Section III.C, there is no heterogeneity in MRPN_{ig} nor MRPN_{ip} in the model. Therefore, sectoral productivities become:

$$\mathrm{TFP}_p \equiv \frac{Y_p}{K_p^{\alpha} N_p^{1-\alpha}} = \left\{ \int_{[0,1]} \left[s_i \left(\frac{\overline{\mathrm{MRPK}}_p}{\mathrm{MRPK}_{ip}} \right)^{\alpha} \right]^{\sigma_p - 1} \mathrm{d}i \right\}^{\frac{1}{\sigma_p - 1}} \tag{G.5}$$

$$\mathrm{TFP}_g \equiv \frac{Y_g}{K_g^{\alpha} N_g^{1-\alpha}} = \left\{ \int_{I_g} \frac{1}{m_g} \left[s_i \left(\frac{\overline{\mathrm{MRPK}}_g}{\mathrm{MRPK}_{ig}} \right)^{\alpha} \right]^{\sigma_g - 1} \mathrm{d}i \right\}^{\frac{1}{\sigma_g - 1}} \tag{G.6}$$

Absent financial frictions, there would be no heterogeneity in MRPK_{ip} and MRPK_{ig} either and optimal TFP in the private and public sectors (conditional on selection) would be,

$$\mathrm{TFP}_p^* = \left[\int_{[0,1]} s_i^{\sigma_p - 1} \mathrm{d}i \right]^{\frac{1}{\sigma_p - 1}} \quad \text{and} \quad \mathrm{TFP}_g^* = \left[\int_{I_g} \frac{1}{m_g} s_i^{\sigma_g - 1} \mathrm{d}i \right]^{\frac{1}{\sigma_g - 1}} \tag{G.7}$$

Relative price of public sector good. Using the definitions of P_g and P_p in equation 3, the relative price can be written as,

$$\frac{P_g}{P_p} = \frac{\left[\int_{I_g} \frac{1}{m_g} p_{ig}^{1-\sigma_g} \mathrm{d}i \right]^{\frac{1}{1-\sigma_g}}}{\left[\int_{[0,1]} p_{ip}^{1-\sigma_p} \mathrm{d}i \right]^{\frac{1}{1-\sigma_p}}} = \left(\frac{\sigma_g/(\sigma_g - 1)}{\sigma_p/(\sigma_p - 1)} \right) \frac{\left[\int_{I_g} \frac{1}{m_g} \left(\frac{1}{s_i} \mathrm{MRPK}_{ig}^{\alpha} \, \mathrm{MRPN}_{ig}^{1-\alpha} \right)^{1-\sigma_g} \mathrm{d}i \right]^{\frac{1}{1-\sigma_g}}}{\left[\int_{[0,1]} \left(\frac{1}{s_i} \mathrm{MRPK}_{ip}^{\alpha} \, \mathrm{MRPN}_{ip}^{1-\alpha} \right)^{1-\sigma_p} \mathrm{d}i \right]^{\frac{1}{1-\sigma_p}}}$$

which follows from the definitions of MRPK_{ip} and MRPN_{ip}, and the production function as,

$$\begin{aligned} \mathrm{MRPK}_{ip} &\equiv \frac{\partial p_{ip} y_{ip}}{\partial k_{ip}} = \alpha \frac{\sigma_p - 1}{\sigma_p} \frac{p_{ip} y_{ip}}{k_{ip}}, \quad \mathrm{MRPN}_{ip} \equiv \frac{\partial p_{ip} y_{ip}}{\partial n_{ip}} = (1-\alpha) \frac{\sigma_p - 1}{\sigma_p} \frac{p_{ip} y_{ip}}{n_{ip}} \\ &\Rightarrow \quad p_{ip} = \frac{\sigma_p}{\sigma_p - 1} \frac{1}{s_i} \left(\frac{\mathrm{MRPK}_{ip}}{\alpha} \right)^{\alpha} \left(\frac{\mathrm{MRPN}_{ip}}{1-\alpha} \right)^{1-\alpha} \end{aligned}$$

and the same applies for MRPK_{ig} and MRPN_{ig}. Next multiplying and dividing by $\overline{\mathrm{MRPK}}_g^{\alpha}$ and $\overline{\mathrm{MRPN}}_g^{1-\alpha}$ in the numerator, by $\overline{\mathrm{MRPK}}_p^{\alpha}$ and $\overline{\mathrm{MRPN}}_p^{1-\alpha}$ in the denominator, and using the fact that our model generates no dispersion in MRPN_{ig}, we obtain,

$$\frac{P_g}{P_p} = \left(\frac{\sigma_g/(\sigma_g - 1)}{\sigma_p/(\sigma_p - 1)} \right) \left(\frac{\overline{\mathrm{MRPK}}_g}{\overline{\mathrm{MRPK}}_p} \right)^{\alpha} \frac{\left[\int_{I_g} \frac{1}{m_g} \left[\frac{1}{s_i} \left(\frac{\mathrm{MRPK}_{ig}}{\overline{\mathrm{MRPK}}_g} \right)^{\alpha} \right]^{1-\sigma_g} \mathrm{d}i \right]^{\frac{1}{1-\sigma_g}}}{\left[\int_{[0,1]} \left[\frac{1}{s_i} \left(\frac{\mathrm{MRPK}_{ip}}{\overline{\mathrm{MRPK}}_p} \right)^{\alpha} \right]^{1-\sigma_p} \mathrm{d}i \right]^{\frac{1}{1-\sigma_p}}} = \left(\frac{\sigma_g/(\sigma_g - 1)}{\sigma_p/(\sigma_p - 1)} \right) \left(\frac{\overline{\mathrm{MRPK}}_g}{\overline{\mathrm{MRPK}}_p} \right)^{\alpha} \frac{\mathrm{TF}}{\mathrm{TF}}$$

Relative sectoral TFP. Given the definition of TFP_p in equation (G.3), we can write

$$\begin{aligned} \mathrm{TFP}_p &= \left[m_g \int_{I_g} \frac{1}{m_g} \left(s_i \left(\frac{\overline{\mathrm{MRPK}}_p}{\mathrm{MRPK}_{ip}} \right)^{\alpha} \right)^{\sigma_p - 1} \mathrm{d}i + (1 - m_g) \int_{I_g^c} \frac{1}{1 - m_g} \left(s_i \left(\frac{\overline{\mathrm{MRPK}}_p}{\mathrm{MRPK}_{ip}} \right)^{\alpha} \right)^{\sigma_p - 1} \mathrm{d}i \right]^{\frac{1}{\sigma_p - 1}} \\ &= \left[m_g \mathrm{TFP}_{p,I_g}^{\sigma_p - 1} + (1 - m_g) \, \mathrm{TFP}_{p,I_g^c}^{\sigma_p - 1} \right]^{\frac{1}{\sigma_p - 1}} \end{aligned}$$

where we have defined TFP_{p,I_g} and TFP_{p,I_g^c} as the average TFP in the private sector within the set of procurement (I_g) and non-procurement (I_g^c) firms respectively. Then, dividing by TFP_g in both sides we get the expression for $\text{TFP}_p/\text{TFP}_g$:

$$\frac{\text{TFP}_p}{\text{TFP}_g} = \left[m_g \left(\frac{\text{TFP}_{p,I_g}}{\text{TFP}_g} \right)^{\sigma_p - 1} + (1 - m_g) \left(\frac{\text{TFP}_{p,I_g^c}}{\text{TFP}_g} \right)^{\sigma_p - 1} \right]^{\frac{1}{\sigma_p - 1}} \tag{G.8}$$

The first term in equation (G.8) reflects the within-firm misallocation. With $\sigma_g = \sigma_p$ this term would be equal to 1 if $\phi_g = \phi_p$ or if there were no financial frictions ($\lambda_i = 0\ \forall i$). Instead, if $\phi_g > \phi_p$ procurement firms switch their capital relatively towards the public sector and the dispersion of MRPK_{ig} declines, which makes $\text{TFP}_{p,I_g}/\text{TFP}_g$ fall. The second term in equation (G.8) reflects both between-firm misallocation and selection into procurement. If firms with higher s self-select into procurement, then $\text{TFP}_{p,I_g^c}/\text{TFP}_g$ declines. If there is more dispersion in MRPK_{ip} between non-procurement firms than in MRPK_{ig} between procurement firms, then $\text{TFP}_{p,I_g^c}/\text{TFP}_g$ is lower. In short, absent financial frictions the only reason for $\text{TFP}_p/\text{TFP}_g \neq 1$ would be the selection of firms into procurement. In the first best (no financial frictions and the government selects the firms with highest s) we would have $\text{TFP}_p/\text{TFP}_g < 1$.

Relative sectoral $\overline{\textbf{MRPK}}$. Given the definition of $\overline{\text{MRPK}}_p$ in (G.5), we can write

$$\begin{aligned}
\overline{\text{MRPK}}_p &= \left[\frac{R_{p,I_g}}{P_p Y_p} \int_{I_g} \frac{p_{ip} y_{ip}}{R_{p,I_g}} \text{MRPK}_{ip}^{-1} \text{d}i + \frac{R_{p,I_g^c}}{P_p Y_p} \int_{I_g^c} \frac{p_{ip} y_{ip}}{R_{p,I_g^c}} \text{MRPK}_{ip}^{-1} \text{d}i \right]^{-1} \\
&= \left[\frac{R_{p,I_g}}{P_p Y_p} \overline{\text{MRPK}}_{p,I_g}^{-1} + \frac{R_{p,I_g^c}}{P_p Y_p} \overline{\text{MRPK}}_{p,I_g^c}^{-1} \right]^{-1}
\end{aligned}$$

where R_{p,I_g} and R_{p,I_g^c} denote total revenues in the private sector by procurement firms and non-procurement firms respectively. Then, dividing by $\overline{\text{MRPK}}_g$ in both sides we obtain the expression for $\overline{\text{MRPK}}_p/\overline{\text{MRPK}}_g$

$$\frac{\overline{\text{MRPK}}_p}{\overline{\text{MRPK}}_g} = \left[\frac{R_{p,I_g}}{P_p Y_p} \left(\frac{\overline{\text{MRPK}}_{p,I_g}}{\overline{\text{MRPK}}_g} \right)^{-1} + \frac{R_{p,I_g^c}}{P_p Y_p} \left(\frac{\overline{\text{MRPK}}_{p,I_g^c}}{\overline{\text{MRPK}}_g} \right)^{-1} \right]^{-1} \tag{G.9}$$

Whenever $\overline{\text{MRPK}}_p \neq \overline{\text{MRPK}}_g$ there is misallocation of capital across sectors. The term $\overline{\text{MRPK}}_{p,I_g}/\overline{\text{MRPK}}_g$ in the right-hand side of equation (G.9) reflects the effects of within-firm misallocation on this between-sector misallocation. With $\sigma_g = \sigma_p$, this term would be equal to 1 if $\phi_g = \phi_p$ or if there were no financial frictions ($\lambda_i = 0\ \forall i$). Instead, if $\phi_g > \phi_p$, firms switch their capital relatively towards the public sector and hence $\overline{\text{MRPK}}_{p,I_g}/\overline{\text{MRPK}}_g > 1$. The term $\overline{\text{MRPK}}_{p,I_g^c}/\overline{\text{MRPK}}_g$ in the right-hand side of equation (G.9) reflects both between-firm misallocation and selection into procurement.

H Additional moments

In this section, Table H.1 provides information on additional moments, both in the data and in the model. Most of them are untargeted moments in our calibration.

Table H.1. Additional moments

(A) Selection	Data	Model
Employment*	1.18	1.14
MRPK*	0.13	0.15
TFPQ	0.56	0.43
Fixed assets	1.44	0.98
Value added	1.56	1.14
Age	0.26	0.07
Leverage	0.05	0.05
$\Pr(d_{t+1} = 1 \mid d_t = 1)^*$	0.60	0.59
$\Pr(\sum_{j=1}^{2} d_{t+j} \geq 1 \mid d_t = 1)$	0.71	0.64
$\Pr(\sum_{j=1}^{3} d_{t+j} \geq 1 \mid d_t = 1)$	0.76	0.69

(B) Treatment (LPs)	Data	Model
$p_p y_p$ $(h = 0)$	-0.147	-0.086
$p_p y_p$ $(h = 4)$	0.034	0.020
credit $(h = 0)$	0.051	0.244
credit $(h = 4)$	0.050	0.088

(C) AR1 process	Data	Model
corr(log y_t,log y_{t-1})	0.85	0.90
std(Δ log y_t)	0.67	0.67

(D) Procurement sales (share)	Data	Model
Mean	0.325	0.298
pct 25	0.031	0.288
pct 50	0.105	0.294
pct 75	0.304	0.304

(E) Firm growth

	Δlog $p_{it}y_{it}$		Δlog n_{it}		Δlog k_{it}	
	Data	Model	Data	Model	Data	Model
Mean	0.15	0.13	0.07	0.13	0.11	0.22
Age 1	0.30	0.29	0.18	0.29	0.18	0.50
Age 2	0.07	0.09	0.01	0.09	0.04	0.14
Age 3	0.05	0.03	0.01	0.03	0.03	0.05

Notes: This table shows a number of moments as measured in our calibrated economy versus the data. The symbol * indicates that the moment has been explicitly targeted in the calibration. The rest are untargeted moments. The top part of **Panel A** reports the *ex-ante* procurement premia for employment, MRPK, fixed assets, age, value added, and TFPQ, as explained in Block #3 of Section IV.A. The bottom part of **Panel A** reports the (cumulative) probabilities of obtaining procurement contracts in $t+1$, $t+2$, and $t+3$, respectively, conditional on being active in procurement in period t (period t=2006 in the data). **Panel B** shows the point estimates of the local projection regressions shown in Figure 7 for $h = 0$ and $h = 4$. For the case of leverage (defined as the ratio of credit to fixed assets), the premium is not expressed in log points but in percentage points. **Panel C** shows the one-year autocorrelation of firms' log sales and the standard deviation of firms' sales growth, both in the data (year 2005-2006) and in the model. **Panel D** shows $p_{ig}y_{ig}/p_iy_i$ for firms with $p_{ig}y_{ig} > 0$, both in the data (year 2006) and model. **Panel E** shows the distribution of firm growth in terms of output, employment, and capital, both in the data (year 2005-2006) and model.

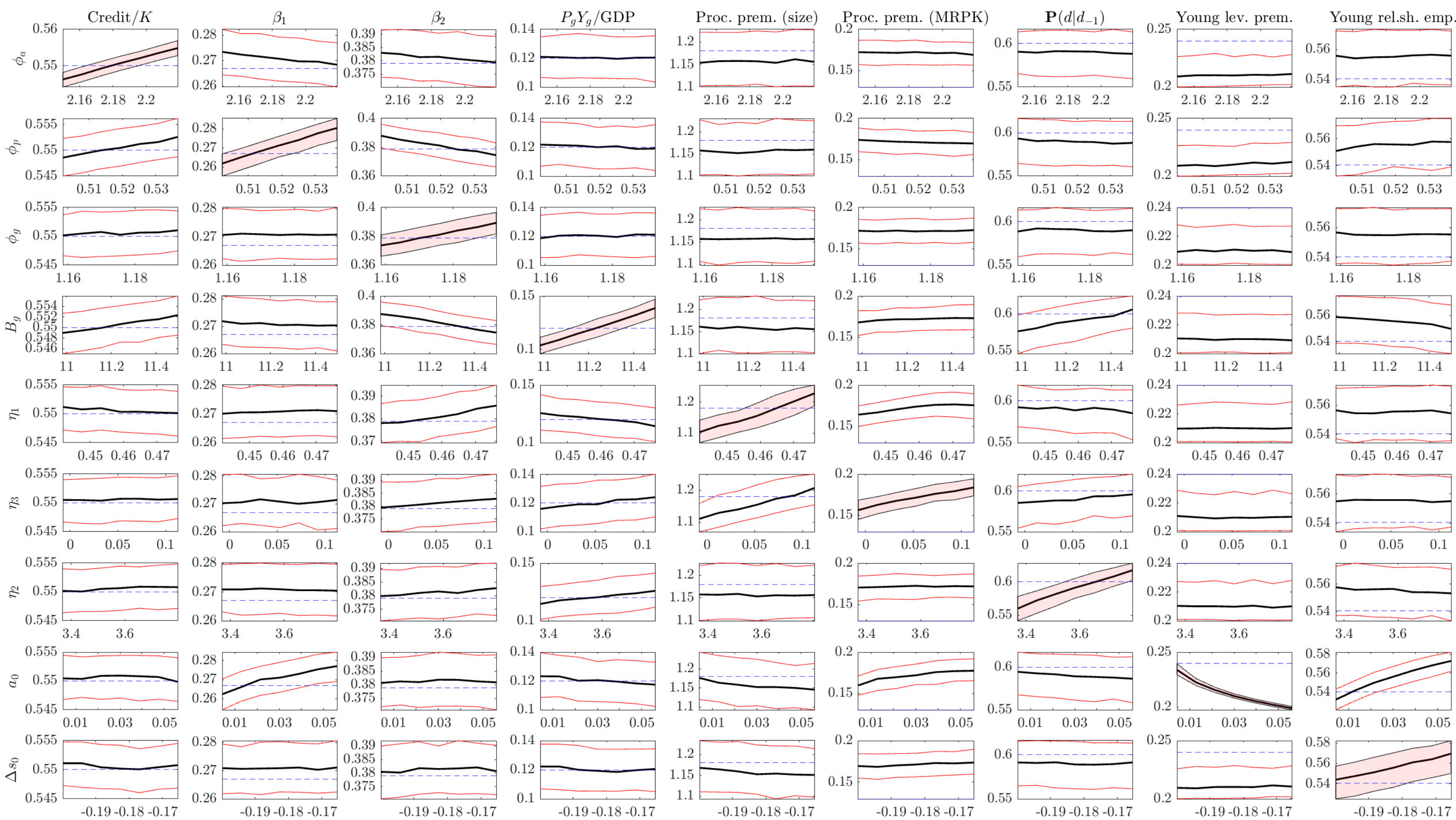
I Identification of model parameters

Our calibration strategy consists of choosing the values of 9 parameters so that the model matches 9 moments. In this appendix, we show that our choice of targets is justified by the fact that each of these targets is "particularly informative" of a particular parameter.

To do so, we proceed as follows. First, we draw a large random sample of parameter combinations based on a 9-dimensional hypercube. Second, for each vector of parameters, we solve for the model's steady-state equilibrium and calculate the relevant moments.[7] And third, we plot how the 15th percentile, the 50th, and the 85th percentile of the distribution of each moment as we move along the values of each parameter. We report all this in the 81 panels of Figure I.1. Intuitively, each panel in Figure I.1 documents how a particular moment changes with a specific parameter, while the other parameters are allowed to vary randomly. The steeper the slope of the relationship between the parameter values and percentiles of the moment, the stronger the identification. The diagonal panels in Figure I.1 shows that each moment is especially informative of one parameter.

[7] We note that, for each vector of parameters, we need to iterate over w, $\bar{p}$, and Y_p to ensure that the equilibrium conditions 4, 5, and 9 are satisfied. We do not need to iterate over τ or T because the equilibrium conditions 6 and 7 are immediate.

Figure I.1. Identification of the model parameters



Notes: This figure plots the relationship between percentiles of a given moment and a specific parameter. The thick black line refers to the median, and the thinner black lines above and below refer to the 85th and 15th percentiles, respectively. The dashed blue line represents the targeted value of the moment. These lines have been generated by solving the steady-state of the model using combinations of parameters drawn from a 9-dimensional hypercube.

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