Supplemental Appendix: Global Imbalances, Tariffs, and Industrial Policy*

July, 2026

1 Introduction

This appendix outlines a two-country New Keynesian model for the purposes of assessing current account impacts of various shocks and policies. The model contains a range of frictions above and beyond the standard representative-agent, labor-only production New Keynesian model of Galí and Monacelli (2005), such as those featured in Smets and Wouters (2007). It also has a rich set of open economy features allowing international trade and financial transactions to occur, with the possibility of government intervention for both. Some of the key features it contains are:

- Two economies that produce both a traded good and a nontraded good. The presence of these sectors that are poor substitutes in demand allows the model to incorporate inter-sectoral spillovers from policies and shocks targeted at the export-focused tradable sector.
- Producer currency pricing (PCP) with a single non-contingent financial asset that can be held by either country in net positive or negative quantities (that define the NIIP and allows the existence of current account surpluses/deficits).
- Adjustment costs for holdings of the international financial asset that can be increased to simulate the effects of capital controls. The government can directly purchase net foreign assets by issuing domestic debt or raising taxes, replicating reserve accumulation policies.
- A production framework that includes capital so that the effects of investment dynamics on current accounts can be assessed.
- Perpetual youth (Blanchard, 1985) and hand-to-mouth agents to break Ricardian Equivalence. This ensures that fiscal policy choices related to subsidies and tariffs generate realistic responses from households.
- A government sector that can issue debt, raise revenue through tariffs, labor income taxes, consumption taxes, and taxes on profits. The government can spend through general transfers and subsidize investment.
- Frictions for price and wage setting to replicate delayed relative price adjustment.

*This is an appendix to the paper: Global Imbalances, Industrial Policy and Tariffs by Pierre-Olivier Gourinchas, Gene Kindberg-Hanlon, Manasa Patnam, Lorenzo Rotunno and Michele Ruta. Contact: gkindberg-hanlon@imf.org

The appendix first outlines the details of the model and its calibration. It then simulates the cases described in *Global Imbalances, Industrial Policy and Tariffs*. The appendix shows all cases in the working paper version, a subset of which are shown in the published *Journal of Economic Perspectives* version of the paper.

2 Model

2.1 Households and consumption

Households in the home (H) and foreign (F) economy consume tradable goods produced in both countries, and nontradable goods produced domestically.

Consumption bundles and price indices The aggregate consumption bundle C_t is defined as a CES aggregate of tradable and non-tradable goods:

$$C_t = \left[\gamma^{\frac{1}{\chi}}(C_{T,t})^{\frac{\chi-1}{\chi}} + (1-\gamma)^{\frac{1}{\chi}}(C_{N,t})^{\frac{\chi-1}{\chi}}\right]^{\frac{\chi}{\chi-1}}$$

where $\gamma \in [0,1]$ is the share of tradable goods in the consumption basket, and $\chi > 0$ represents the elasticity of substitution between tradable and non-tradable goods. The demand functions for the sectoral bundles are:

$$C_{T,t} = \gamma\left(\frac{P_{T,t}}{P_t}\right)^{-\chi} C_t, \quad C_{N,t} = (1-\gamma)\left(\frac{P_{N,t}}{P_t}\right)^{-\chi} C_t$$

The Consumer Price Index (CPI) is consequently defined as $P_t = \left[\gamma P_{T,t}^{1-\chi} + (1-\gamma)P_{N,t}^{1-\chi}\right]^{\frac{1}{1-\chi}}$.

The tradable goods bundle is defined over home (H) and imported (F) varieties, given by:

$$C_{T,t} = \left[(1-\alpha)^{\frac{1}{\eta}} C_{Ht}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{Ft}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$$

where $\alpha \in [0,1]$ is a measure of international linkages in demand, or $1-\alpha$ is a measure of home bias, and $\eta > 0$ is a measure of the substitutability of home and foreign consumption bundles. Each consumption subindex in turn is given by a CES aggregate of home- and foreign-produced tradable varieties and home produced nontradable varieties:

$$C_{Ht} = \left[\int_0^1 C_{Ht}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad C_{Ft} = \left[\int_0^1 C_{Ft}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad C_{N,t} = \left[\int_0^1 C_{N,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where $\varepsilon > 1$ measures the degree of substitutability between varieties.

Finally, the demand functions for domestic and imported tradable goods are:

$$C_{Ht} = (1-\alpha)\left(\frac{P_{Ht}}{P_{T,t}}\right)^{-\eta} C_{T,t}, \quad C_{Ft} = \alpha\left(\frac{P_{Ft}}{P_{T,t}}\right)^{-\eta} C_{T,t},$$

where $P_{T,t} = \left[(1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}$ is the consumer price index (CPI) for tradable goods.

Foreign households Foreign households, denoted with '$*$' for each relevant variable, have the same preferences for domestically-produced relative to goods produced overseas, so that their consumption can be written:

$$C_t^* = \left[\gamma^{\frac{1}{\chi}} (C_{T,t}^*)^{\frac{\chi-1}{\chi}} + (1-\gamma)^{\frac{1}{\chi}} (C_{N,t}^*)^{\frac{\chi-1}{\chi}}\right]^{\frac{\chi}{\chi-1}}$$

They also have an analogous degree of home bias, so that:

$$C_{T,t}^* = \left[(1-\alpha)^{\frac{1}{\eta}} C_{Ft}^{*\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{Ht}^{*\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$$

The equations for establishing $C_{F,t}^*$, $C_{H,t}^*$ as a function of relative prices follow accordingly.

Hand to mouth and perpetual youth households There are two types of household in each economy. A hand-to-mouth household that is credit constrained consumes all income each period. Second, a perpetual youth household can hold positive or negative net assets and consumption can deviate from income. All households face a sales tax of τ_C on their consumption.

The parameter ω reflects the share of HTM households in the economy, such that:

$$C_t = \omega C_t^H + (1-\omega) C_t^P$$

The hand-to-mouth agent is credit constrained to consume all income from wages or government transfers each period, and has no other sources of income:

$$(1+\tau_{C,t}) C_t^H = (1-\tau_t^l) w_t N_t + T_t$$

The perpetual youth household maximizes utility of the form:

$$E_0 \sum_{t=0}^{\infty} (\beta\theta)^t \left(\frac{1}{1-\sigma} C_t^{p1-\sigma} - \frac{1}{1+\nu} N_t^{p1+\nu}\right),$$

where C is a consumption index and N is hours worked. Utility is discounted by both β and the probability of the survival of the household (θ). Note that in the simulations shown in the paper, the perpetual youth household has sticky consumption habits, a standard feature of many medium-scale New Keynesian models. This feature is excluded in the derivations below to reduce the complexity of the notation. This feature is elaborated on in Section 6.1.

The perpetual youth household can both borrow and hold net positive assets in the form of domestic government debt B_H and an international bond (denominated in domestic currency), B_I. Both types of bond are denoted in real terms. The perpetual youth household earns income from both wages and profits from the production of the domestic good (Π).

$$(1+\tau_{C,t})C_t^P + \frac{B_{Ht}}{(1-\omega)} + \frac{B_{It}}{(1-\omega)} + \frac{\varphi}{(1-\omega)2}(B_{I,t} - B_{I,ss})^2$$
$$= \frac{R_{H,t-1}}{\theta(1-\omega)\pi_t}B_{Ht-1} + \frac{R_{I,t-1}}{\theta(1-\omega)\pi_t}B_{I,t-1} + (1-\tau_{l,t})w_t N_t + \frac{\Pi_t}{(1-\omega)} + T_t$$

In the perpetual youth framework of Blanchard (1985), households earn an additional return, $\frac{1}{\theta}$, effectively inheritance from households that have not survived that period, (newly formed households that replace them have no assets initially).

Households face an adjustment cost φ of changing their holdings of the international bond. This reflects the potential for capital controls, with their strictness increasing in φ. The parameter is also set at very low levels to ensure the stationarity of net bond holdings in the baseline calibration.[1]

Labor supply Labor from both types of households is supplied to a union. The union collects undifferentiated labor from the households and maximizes their weighted utility by selling differentiated labor to firms. The union faces adjustment costs in changing the wages of the differentiated labor it supplies.

The union maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{1}{1+\nu}\left[\left(\frac{W_t(i)}{W_t}\right)^{-\epsilon_w} N_t \right]^{1+\nu} \right)$$

noting that individual i's labor demand will be a function of average labor demand, the elasticity of subsitution ϵ_w and the relative wage. Note that the aggregates C and N are simply a weighted (ω) average of the HTM and perpetual youth households consumption and labor supply (which is what the union seeks to maximize the utility of).

This maximization is subject to the budget constraint including wage adjustment costs:

$$\lambda_t \left[(1-\tau_{l,t})\frac{W_t(i)}{P_t}\left(\frac{W_t(i)}{W_t}\right)^{-\epsilon_w} N_t - \frac{\phi_w}{2}\left(\frac{W_t(i)}{W_{t-1}(i)} - 1\right)^2 Y_t + ... \right]$$

$\partial W(i)$:

$$\epsilon_w \frac{1}{W_t(i)} N_t^{1+\nu} + \lambda_t(1-\tau_{l,t})(1-\epsilon_w)\frac{N_t}{P_t} - \lambda_t\phi_w \frac{1}{W_{t-1}}(\pi_w - 1)Y_t + \beta\lambda_{t+1}\phi_w \frac{W_{t+1}}{W_t^2}(\pi_{w,t+1} - 1)Y_{t+1} = 0$$

Now multiply by W

$$\epsilon_w N_t^{1+\nu} + \lambda_t(1-\tau_{l,t})(1-\epsilon_w)w_t N_t + \beta\lambda_{t+1}\phi_w \pi_{w,t+1}(\pi_{w,t+1} - 1)Y_{t+1} = \lambda_t \phi_w \pi_{w,t}(\pi_w - 1)Y_t$$

With flexible wages ($\phi_w = 0$) we can see the equation collapses to the standard form, where post-tax wages are a markup over the MRS between labor and consumption:

[1] Note that this formulation for the adjustment cost is the same as that used by Schmitt-Grohe and Uribe (2003) to maintain stationarity of international bond holdings. However, because of the perpetual youth household, the model's dynamics for international bond holdings are already stationary.

$$\frac{\epsilon_w}{\epsilon_w - 1} C_t^{\sigma} N_t^{\nu} = (1 - \tau_{I,t}) w_t$$

Intertemporal optimality and international financial linkages By taking the FOCs of the perpetual youth household with respect to C_t^P and B_t^H , the standard Euler equation is obtained.

$$1 = \beta E_t \left[\left(\frac{C_{t+1}^P}{C_t^P} \right)^{-\sigma} \frac{R_{H,t}}{\pi_{t+1}} \right] \tag{1}$$

The FOC for the international bond, and therefore the link between consumption and interest rates in both economies, also accounts for the capital control friction generated by parameter φ.

$$\partial B_{I,t} : \frac{\beta \lambda_{t+1}}{\lambda_t} \frac{R_{I,t}}{\pi_{t+1}} = 1 + \varphi(B_{I,t} - B_{I,ss})$$

We can therefore describe a 'wedge' between the international bond interest rate and the domestic government bond. The higher the adjustment costs and deviation from steady state bond holdings are, the higher are international bond interest rates compared to domestic rates.

$$\frac{R_{I,t}}{R_{H,t}} = 1 + \varphi(B_{I,t} - B_{I,ss})$$

For simplicity, we assume that the foreign economy has no adjustment costs for the international bond, but follows a similar structure (although must account for the home-currency denomination of the international bond). Foreign economy perpetual youth households face the household budget constraint:

$$C_t^{*,P} + \frac{B_{Ft}}{(1-\omega)} + \frac{B_{It}^*}{\varepsilon_t(1-\omega)} = \frac{R_{t-1}^F}{\theta(1-\omega)\pi_t^*} B_{Ft-1} - \frac{R_{t-1}^I}{\varepsilon_t \theta(1-\omega)\pi_t^*} B_{I,t-1}^* + w_t^* N_t^* + \frac{\Pi_t^*}{(1-\omega)} + T_t^*,$$

Where ε is the currency rate of exchange from the foreign to domestic economy. For the foreign economy:

$$\frac{\varepsilon_t}{\varepsilon_{t+1}} \frac{R_{I,t}}{\pi_{t+1}^*} = \frac{R_{F,t}}{\pi_{t+1}^*} = \frac{\lambda_t^*}{\beta \lambda_{t+1}^*}$$

This equation has the simple intuition that the exchange rate of the foreign economy must be expected to appreciate (depreciate) if the international bond yields are higher (lower) than domestic bond yields (the standard UIP condition).

Finally, the real exchange rate is

$$\mathcal{Q}_t = \frac{\varepsilon_t P_t^*}{P_t}$$

Using the FOCs for the international bond relative to the domestic bond, the link between real interest rates and the real exchange rate in each economy can be written :

$$\frac{\mathcal{Q}_{t+1}}{\mathcal{Q}_t} \frac{R_{F,t}}{\pi_{t+1}^*} = \frac{R_{H,t}}{\pi_{t+1}} \left(1 + \varphi(B_{I,t} - B_{I,ss})\right)$$

This collapses to the standard UIP condition when capital restrictions, φ, fall to zero.

Consumption of the perpetual youth household The perpetual youth household's consumption can be written as a function of its asset endowment (of government and international bonds), and also its discounted future income. The latter, (written Y^P), reflects, wage income, profits and transfers net of taxes.

In addition, because the return on the international bond (when accounting for adjustment costs) is equivalent to the domestic rate, we can write consumption of the perpetual youth household as:

$$C_t^P \sum_{\tau=0}^{\infty} (\theta\beta^{1/\sigma})^{\tau}(Q_{t+\tau})^{1-1/\sigma} = \frac{R_{t-1}}{(1-\omega)\pi_t} B_{H,t-1} + \frac{R_{I,t-1}}{(1-\omega)\pi_t} B_{I,t-1} + \sum_{\tau=0}^{\infty} \theta^{\tau} Q_{t+\tau}(Y_{t+\tau}^P)$$

Where $Q_{t+\tau} = \frac{\pi_{t+1}}{R_t}\frac{\pi_{t+2}}{R_{t+1}}...\frac{\pi_{t+\tau}}{R_{t+\tau-1}}$. A complete derivation also including the effects of consumption taxes and consumption habits is included in section 6.1.

2.2 Tariffs, prices, and exchange rates

Both the Home and Foreign economy can impose time-varying tariffs on one another. This modifies some of the earlier defined price index equations and has implications for the relative price and RER.

Tradable price indices and tariffs With the introduction of tariffs, the price index for the tradable bundle in the Home economy must account for the ad-valorem tariff τ_t levied on the foreign good. The tradable price index is given by:

$$P_{T,t} = \left[(1-\alpha)P_{Ht}^{1-\eta} + \alpha((1+\tau_t)P_{Ft})^{1-\eta}\right]^{\frac{1}{1-\eta}}$$

Analogously, the Foreign tradable price index faces a tariff τ_t^* on goods imported from the Home country:

$$P_{T,t}^* = \left[(1-\alpha)P_{F,t}^{*,1-\eta} + \alpha((1+\tau_t^*)P_{H,t}^*)^{1-\eta}\right]^{\frac{1}{1-\eta}}$$

We can now express tradable inflation as a function of domestic inflation and the terms of trade. By dividing the Home tradable price index by $P_{H,t}$, and defining the terms of trade $S_t = \frac{P_{F,t}}{P_{Ht}}$, we obtain:

$$\frac{P_{T,t}}{P_{Ht}} = \left[(1-\alpha) + \alpha((1+\tau_t)S_t)^{1-\eta}\right]^{\frac{1}{1-\eta}}$$

Taking the ratio of this expression to its value in period $t-1$, and noting that $\pi_{T,t} = P_{T,t}/P_{T,t-1}$ and $\pi_{H,t} = P_{H,t}/P_{H,t-1}$, tradable inflation can be written as:

$$\pi_{T,t} = \pi_{H,t}\left(\frac{(1-\alpha) + \alpha((1+\tau_t)S_t)^{1-\eta}}{(1-\alpha) + \alpha((1+\tau_{t-1})S_{t-1})^{1-\eta}}\right)^{\frac{1}{1-\eta}}$$

Similarly, Foreign tradable inflation can be linked to the Foreign producer price inflation ($\pi_{F,t}^*$) and the terms of trade. Recalling that from the foreign perspective, the relative price of imports is the inverse of the terms of trade (S_t^{-1}), we derive:

$$\pi^*_{T,t} = \pi^*_{F,t}\left(\frac{(1-\alpha)+\alpha(1+\tau^*_t)^{1-\eta}S_t^{\eta-1}}{(1-\alpha)+\alpha(1+\tau^*_{t-1})^{1-\eta}S_{t-1}^{\eta-1}}\right)^{\frac{1}{1-\eta}}$$

A tariff shock generates an immediate wedge between domestic price inflation (π_H) and tradable consumer price inflation (π_T), which subsequently feeds into aggregate CPI inflation according to the share of tradables γ.

However, the tariff does not directly enter the household budget constraint other than through its effects on relative prices (see Lemma A.1).

Non-tradable and Aggregate Inflation Unlike the tradable sector, the non-tradable sector is shielded from direct external price pressures such as tariffs or exchange rate fluctuations. Inflation in the non-tradable sector, $\pi_{N,t}$, is driven by domestic real marginal costs, $MC_{N,t}$.

Aggregate inflation, $\pi_t = P_t/P_{t-1}$, is a composite of inflation in both sectors. Given the price index $P_t = [\gamma P_{T,t}^{1-\chi} + (1-\gamma)P_{N,t}^{1-\chi}]^{\frac{1}{1-\chi}}$, aggregate inflation evolves according to:

$$\pi_t = \left[\gamma\left(\pi_{T,t}\frac{P_{T,t-1}}{P_{t-1}}\right)^{1-\chi} + (1-\gamma)\left(\pi_{N,t}\frac{P_{N,t-1}}{P_{t-1}}\right)^{1-\chi}\right]^{\frac{1}{1-\chi}}$$

Real Exchange Rate Decomposition The real exchange rate, $\mathcal{Q}_t = \frac{\varepsilon_t P_t^*}{P_t}$, is driven by the relative price of tradable goods, but it also depends on the relative price of non-tradables in each country. This can be seen by decomposing the RER:

$$\mathcal{Q}_t = \underbrace{\frac{\varepsilon_t P^*_{T,t}}{P_{T,t}}}_{\text{Relative tradables prices}} \times \underbrace{\left[\frac{\gamma + (1-\gamma)(P^*_{N,t}/P^*_{T,t})^{1-\chi}}{\gamma + (1-\gamma)(P_{N,t}/P_{T,t})^{1-\chi}}\right]^{\frac{1}{1-\chi}}}_{\text{Relative tradable/nontradable wedge}}$$

Changes in productivity or demand in the non-tradable sector will alter the internal price ratio $P_{N,t}/P_{T,t}$, driving movements in the aggregate RER even if tradable prices satisfy the law of one price.

2.3 Firms and production

Goods producers Firms producing both (domestic) tradable and non-tradable goods ($k = H, N$) have a production technology that combines capital and labor. Firms can receive a subsidy on the amount they invest ($\tau_{sub,t}$), but also pay corporate taxes on their profits of amount τ_K.

Individual firms use the production technology:

$$Y_{k,t}(i) = A_{k,t}(i)K^{\mu}_{k,t}(i)N^{1-\mu}_{k,t}(i)$$

Investment goods are produced from consumption goods of the sector in question (nontradable output is produced using capital from nontradable output). Firms face adjustment costs when investing. Profits in standard tax codes deduct the value of capital depreciation ($P_k\delta K_{k,t}$) from revenues rather than the cost of investment itself, and this is also the approach taken in this model.

$$\sum_{i=0}^{T}\beta^i\frac{\lambda_{t+i}}{\lambda_t}\left(\frac{1}{P_{t+i}}\left((1-\tau_{K,t})\left(P_{k,t+i}Y_{k,t+i}-P_{t+i}w_{t+i}N_{t+i}\right)+\tau_{K,t+i}P_{T/N,t+i}\delta K_{k,t+i}-(1-\tau^k_{sub,t+i})P_{T/N,t+i}I_{k,t+i}\right)\right.$$

$$\left.-q_t\left(K_{k,t+1+i}-I_{t+i}+\frac{\kappa}{2}\left(\frac{I_{k,t+i}}{I_{k,t+i-1}}-1\right)^2 I_{k,t+i}-(1-\delta)K_{k,t+i}\right)-v_{t+i}\left(Y_{k,t+i}-A_{k,t+i}K^{\mu}_{k,t+i}N^{1-\mu}_{k,t+i}\right)\right)$$

$\partial N_{k,t}$

$$v_t(1-\mu)A_{k,t}K^{\mu}_{k,t}N^{-\mu}_{k,t}=(1-\tau_{K,t})w_t$$

The lagrangian multiplier v can be determined via the $\partial Y_{k,t}=\frac{P_{k,t}}{P_t}(1-\tau_{K,t})\frac{1}{\mathcal{M}_t}=v_t$ condition. The multiplier v is also interpretable as the post-tax marginal cost. In other words, firms produce up to the point where price equals marginal costs plus a markup $\frac{P_{k,t}}{P_t}=\mathcal{M}_t\frac{w_t/P_t}{MPL_t}$.[2] Since the corporate tax loss (from revenues) and gain (from higher costs being tax-deductible) cancel out:

$$v_t=(1-\tau_{k,t})mc_{k,t}=\frac{(1-\tau_{k,t})w_t}{(1-\mu)A_{k,t}K^{\mu}_{k,t}N^{-\mu}_{k,t}}$$

The remaining equations for optimal investment are largely standard, and define 'Tobin's Q' and its evolution. Although to note that the expected return on capital also includes future expected relative prices (i.e. q is higher if future relative prices of the product produced domestically is higher).

∂I_t:

$$(1-\tau^k_{sub,t})\frac{P_{T/N,t}}{P_t}=q_{k,t}\left(1-\frac{\kappa}{2}(\frac{I_{k,t}}{I_{k,t-1}}-1)^2-\kappa(\frac{I_{k,t}}{I_{k,t-1}}-1)(\frac{I_{k,t}}{I_{k,t-1}})\right)+\beta\frac{\lambda_{t+1}}{\lambda_t}q_{k,t+1}\kappa\left((\frac{I_{k,t+1}}{I_{k,t}}-1)(\frac{I_{k,t+1}}{I_{k,t}})^2\right)$$

$\partial K_{k,t+1}$

$$q_{k,t}=\beta\frac{\lambda_{t+1}}{\lambda_t}\left(mc_{k,t+1}(1-\tau_{k,t+1})\mu A_{k,t+1}K^{\mu-1}_{k,t+1}N^{1-\mu}_{k,t+1}+\tau_{k,t+1}\frac{P_{k,t+1}}{P_{t+1}}\delta+q_{k,t+1}(1-\delta)\right)$$

Note that mc here is in real terms, deflated by the aggregate price level, $mc=\frac{MC^n}{P_k}\frac{P_k}{P}$. In a New Keynesian framework, firms with market power restrict output to maintain prices, creating a wedge between price and marginal cost (the markup). Consequently, the firm values additional productive capacity only at the cost

[2]For convenience, we do not show the pricing adjustment terms and price elasticity of demand in this lagrangian, which would be required to show the existence of the markup term. Note that the firm faces declining prices from higher output, such that $P_{k,t}(i)Y_{k,t}(i)=P_{k,t}\left(\frac{Y_{k,t}(i)}{Y_{k,t}}\right)^{-\frac{1}{\epsilon}}$ and $\partial Y_{k,t}(i)=P_{k,t}(\frac{\epsilon-1}{\epsilon})$ Although these are shown in the pricing determination section.

savings it generates (lowering mc). It also increases incentives to invest when relative prices of the sector are high ($\frac{P_k}{P}$).

Note also in the above that the *cost* of investment is P_T or P_N, since for tradable investment, imported goods can be used, not just domestically produced tradables (P_H).

This is a key transmission mechanism for nonmarket policies - a directive to produce at a lower markups (higher mc) *increases* q and desired investment. It also draws more labor into the sector to increase output.

Market clearing Each sector must produce enough to satisfy demand for consumption and investment. It is assumed that investment in the tradable sector is made up of output in the tradable sector, while investment in the nontradable sector requires the output of the nontradable sector.

The market clearing condition therefore becomes:

$$Y_{H,t} = I_{H,t} + C_{H,t} + C^*_{H,t} + I^*_{H,t}$$

We assume that investment I_t is produced using the same tradable aggregator as consumption. This implies that foreign investment decisions directly generate demand for home tradable goods (exports).

The market clearing condition for the Home Tradable good ($Y_{H,t}$) must account for both domestic and foreign demand for consumption and investment:

$$Y_{H,t} = \underbrace{(1-\alpha)\left(\frac{P_{H,t}}{P_{T,t}}\right)^{-\eta}(C_{T,t} + I_{T,t})}_{\text{Domestic Use}} + \underbrace{\alpha\left(\frac{P_{H,t}}{\varepsilon_t P^*_{T,t}}\right)^{-\eta}(C^*_{T,t} + I^*_{T,t})}_{\text{Exports}}$$

For the non-tradable sector, the market must clear domestically. Assuming non-tradable goods are used for consumption and a portion of investment services:

$$Y_{N,t} = C_{N,t} + I_{N,t}$$

Pricing and tradable inflation Now we examine the optimal pricing decision under PCP for the individual firm making home tradable goods, accounting for demand in both economies using the demand function:

$$Y_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_{T,t}}\right)^{-\eta}(C_{T,t} + I_{T,t}) + \alpha\left(\frac{P_{H,t}}{\varepsilon_t P^*_{T,t}}\right)^{-\eta}(C^*_{T,t} + I^*_{T,t})$$

The retailer faces adjustment costs when it changes prices of the form: $\frac{\phi_\pi}{2}\left(\frac{P_{H,t+j}(i)}{P_{H,t+j-1}(i)} - 1\right)^2 Y_{H,t+j}$. The maximization problem becomes:

$$\max_{\{P_{H,t+j}(i)\}_{j=0}^{\infty}} E_t \sum_{j=0}^{\infty} \Lambda_{t,t+j} \frac{1}{P_t} \Bigg\{ (1-\tau_{K,t}) \left(P_{H,t+j}(i) - MC_{t+j}\right) \left(\frac{P_{H,t+j}(i)}{P_{H,t+j}}\right)^{-\epsilon}$$

$$\underbrace{\left[(1-\alpha) \left(\frac{P_{H,t}}{P_{T,t}}\right)^{-\eta} (C_{T,t} + I_{T,t}) + \alpha \left(\frac{P_{H,t}}{\varepsilon_t P^*_{T,t}}\right)^{-\eta} (C^*_{T,t} + I^*_{T,t}) \right]}_{Y_{H,t}}$$

$$- (1-\tau_{K,t}) \frac{\phi_\pi}{2} \left(\frac{P_{H,t+j}(i)}{P_{H,t+j-1}(i)} - 1\right)^2 Y_{H,t+j} \Bigg\}$$

In this case the FOC wrt $P_{H,t}(i)$ yields (assuming all firms face the same FOC and $P_{H,t}(i) = P_{H,t}$, and noting that the corporate tax term drops out)

$$\frac{Y_{H,t}}{P_t} \left[(1-\epsilon) + \epsilon \frac{MC_t}{P_{H,t}} \right] - \phi_\pi \pi_{H,t} \left(\pi_{H,t} - 1\right) \frac{Y_{H,t}}{P_{H,t}} + \beta \phi_\pi Y_{H,t+1} \frac{P_{H,t+1}}{P^2_{H,t}} \left(\pi_{H,t+1} - 1\right) = 0$$

Multiplying through by $P_{H,t}$ and dividing by $Y_{H,t}$ yields:

$$\frac{1}{P_t} \left[(1-\epsilon) P_{H,t} + \epsilon MC_t \right] - \phi_\pi \pi_{H,t} \left(\pi_{H,t} - 1\right) + \beta \phi_\pi \frac{Y_{H,t+1}}{Y_{H,t}} \pi_{H,t+1} \left(\pi_{H,t+1} - 1\right) = 0$$

$$\left[(1-\epsilon) \frac{P_{H,t}}{P_t} + \epsilon \frac{MC_t}{P_t} \right] - \phi_\pi \pi_{H,t} \left(\pi_{H,t} - 1\right) + \beta \phi_\pi \frac{Y_{H,t+1}}{Y_{H,t}} \pi_{H,t+1} \left(\pi_{H,t+1} - 1\right) = 0$$

In the absence of adjustment costs, the equation reduces to the standard markup over marginal costs pricing:

$$\frac{P_{H,t}}{P_t} = \frac{\epsilon}{\epsilon - 1} \frac{MC_t}{P_t}$$

The equations for nontradable and foreign counterparts take an analogous form.

The potential for a shock to markups $\mathcal{M}^\epsilon$ is applied in the simulated version of the model.

Aggregate profits Aggregate profits to distribute to the perpetual youth household are equivalent to the post-tax markup over the marginal product of labor and the return on capital, offset by the depreciation deduction for corporate taxes. Since the profits to the household are in "cashflow" form, the cost of investment is also deducted from the distributed profits. Profits of both the tradable and non tradable sector are distributed to the perpetual youth household only.

$$\Pi_{k,t} = \frac{P_{k,t}}{P_t} \left((1-\tau_{K,t}) \left((1 - \frac{MC_t}{P_{k,t}})(1-\mu) Y_{k,t} + (\mu) Y_{k,t} \right) + \delta \tau_{k,t} K_t - I_{k,t} \right)$$

$$\Pi_t = \Pi_{H,t} + \Pi_{N,t}$$

Tax revenues associated with corporate profits are denoted as:

$$\Pi_t^{\tau_K} = \Pi_{H,t}^{\tau_K} + \Pi_{N,t}^{\tau_K}$$

2.4 Accounting identities and policy rules

Fiscal and monetary policy The government collects revenues from tariffs, the consumption tax, labor tax, and corporate tax each period. On the expenditure side, the government can use these revenues to finance lump-sum transfers and investment subsidies. The government can issue debt or pay down debt in the case of revenue and spending mismatches.

Revenues, Rev_t are:

$$Rev_t = \tau_t \frac{P_{F,t}}{P_t} C_{F,t} + \tau_{C,t} C_t + \tau_{l,t} w_t N_t + \Pi_t^{\tau_K}$$

Expenditure, E_t:

$$E_t = T_t + \tau_{sub,t} I_t$$

Government debt evolves as:

$$B_{H,t} + Rev_t = E_t + \frac{R_{t-1}}{\pi_t} B_{H,t-1} + B_I^{\epsilon}$$

The term B_I^{ϵ} reflects changes in the government's holdings of the international bond (B_{It}^G). It evolves as:

$$B_{I,t}^{\epsilon} = B_{I,t}^G - \frac{R_{I,t-1}}{\pi_t} B_{I,t-1}^G + \epsilon_{G,t}$$

The government can demand to hold more of the international bond (reserve accumulation) via the exogenous shock ϵ_G. This has consequences for the balance sheet, requiring additional domestic bond issuance, lower expenditure, or higher taxes to fund it.

Total net holdings of the international bond by the government and private households defines the NIIP:

$$B_{it}^T = B_{It} + B_{It}^G$$

The government pursues a fiscal rule of the form:

$$\frac{E_t}{Y_t} = \rho_t^T \frac{Rev_t}{Y_t} - \rho^B \frac{B_{H,t}}{Y_t}$$

The roles of expenditure and revenue can be switched - the component that serves to stabilize debt must be an individual component of expenditure or revenue for identification.

A lower value for ρ^T will result in the stock of debt being the primary absorber of shocks to expenditure or revenue, while a value of 1 will make shocks to any component deficit neutral. ρ^B ensures debt stability in the long run (taking a very small value).

Finally, the monetary policy rule is as follows:

$$R_{H,t} = R_{ss} \left(\frac{\pi_t}{\pi_{ss}} \right)^{\psi_\pi}$$

Aggregates Aggregate output is

$$Y_t = GDP_t = \frac{P_{H,t}}{P_t} Y_H + \frac{P_{N,t}}{P_t} Y_N$$

Aggregate investment is:

$$I_t = \frac{P_{H,t}}{P_t} I_{H,t} + \frac{P_{N,t}}{P_t} I_{N,t}$$

Trade and the current account The current account balance is:

$$CAB_t = \left(\underbrace{Y_t - C_t - I_t}_{\textbf{Trade balance}} + \underbrace{(R_{I,t-1} - 1) \frac{B^T_{I,t-1}}{\pi_t}}_{\textbf{Income balance}} \right),$$

Net holdings of the international debt security evolves based on the CAB surplus or deficit of the economy. It must be equivalent to the negative of the change in the foreign households holdings of the international bond for markets to clear (the foreign government does not hold the international bond).

$$B^T_{It} - B^T_{It-1} = CAB_t = -(B^*_{It} - B^*_{It-1})$$

3 Parameters

Table 1: Calibrated Parameters

Parameter	Symbol	Value	Description
Preferences & Households			
Discount Factor	β	0.995	Subjective discount factor
Inter-temporal substitution (inverse)	σ	2.0	
Inverse Frisch Elasticity	ν	0.33	Inverse elasticity of labor supply
Habit Persistence	ρ	0.7	Internal habit formation in consumption
Hand-to-Mouth Share	ω	0.3	Share of liquidity-constrained households
Technology & Capital			
Capital Share	μ_k	0.33	Share of capital in production function
Depreciation Rate	δ	0.025	Quarterly capital depreciation rate
Inv. Adjustment Cost	κ	3.0	Capital adjustment cost parameter
Openness & Sectors			
Tradable Share	γ	0.5	Share of tradables in consumption basket
T/NT Elasticity	χ	0.5	Subst. elasticity between Tradables/Non-Tradables
Trade Elasticity	η	2.0	Elasticity of substitution (Home vs. Foreign)
Openness	α	0.3	Share of imports in tradable bundle
Nominal Frictions & Policy			
Price Elasticity	ϵ	6.0	Elasticity of substitution between varieties
Rotemberg Parameter (prices)	ϕ_π	70	Adjustment cost for prices
Rotemberg Parameter (wages)	ϕ_w	70	Adjustment cost for wages
Taylor Rule (Inf)	ψ_π	1.5	Taylor rule parameter on inflation

4 Shock processes

The model incorporates varying exogenous shock processes that drive fluctuations in productivity, fiscal instruments, monetary policy, and preferences. These are modeled as autoregressive processes of order one (AR(1)).

Sector-specific productivity follow AR(1) processes in logs.

$$\ln(A_{T,t}) = \rho_A \ln(A_{T,t-1}) + (1-\rho_A)\ln(\bar{A}_T) + \varepsilon_{A,T,t}$$

$$\ln(A_{N,t}) = \rho_A \ln(A_{N,t-1}) + (1-\rho_A)\ln(\bar{A}_N) - \varepsilon_{A,N,t}$$

$$\ln(A^*_{T,t}) = \rho_A \ln(A^*_{T,t-1}) + (1-\rho_A)\ln(\bar{A}^*_T) + \varepsilon^*_{A,T,t}$$
$$\ln(A^*_{N,t}) = \rho_A \ln(A^*_{N,t-1}) + (1-\rho_A)\ln(\bar{A}^*_N) + \varepsilon^*_{A,N,t}$$

Where A_T and A_N represent productivity in the tradable and non-tradable sectors respectively, and variables with an asterisk ($*$) denote foreign equivalents. The shock to nontradable home productivity enters with a minus sign as we simulate a contraction.

The model also includes shocks to investment subsidies, tariffs, and factor taxes. The subsidy on tradable investment follows a persistent process.

$$\tau^T_{sub,T,t} = \rho_{\tau,sub}\tau^T_{sub,t-1} + (1-\rho_{\tau,sub})\bar{\tau}^T + \varepsilon^T_{\tau,sub,t}$$

The model includes the potential to shock markups on tradable sector output.

$$\mathcal{M}^\epsilon_t = \rho_{\mathcal{M}}\mathcal{M}^\epsilon_{t-1} + \epsilon^{\mathcal{M}}_t$$

Tariffs (τ) and taxes on labor (τ_L) and capital (τ_K) follow AR(1) processes. The shocks to labor and capital taxes enter with a negative sign, representing a tax cut.

$$\tau_t = \rho_\tau \tau_{t-1} + \varepsilon_{\tau,t}$$
$$\tau_{L,t} = \rho_{\tau,L}\tau_{L,t-1} - \varepsilon_{\tau,L,t}$$
$$\tau_{K,t} = \rho_{\tau,K}\tau_{K,t-1} - \varepsilon_{\tau,K,t}$$

5 Simulations

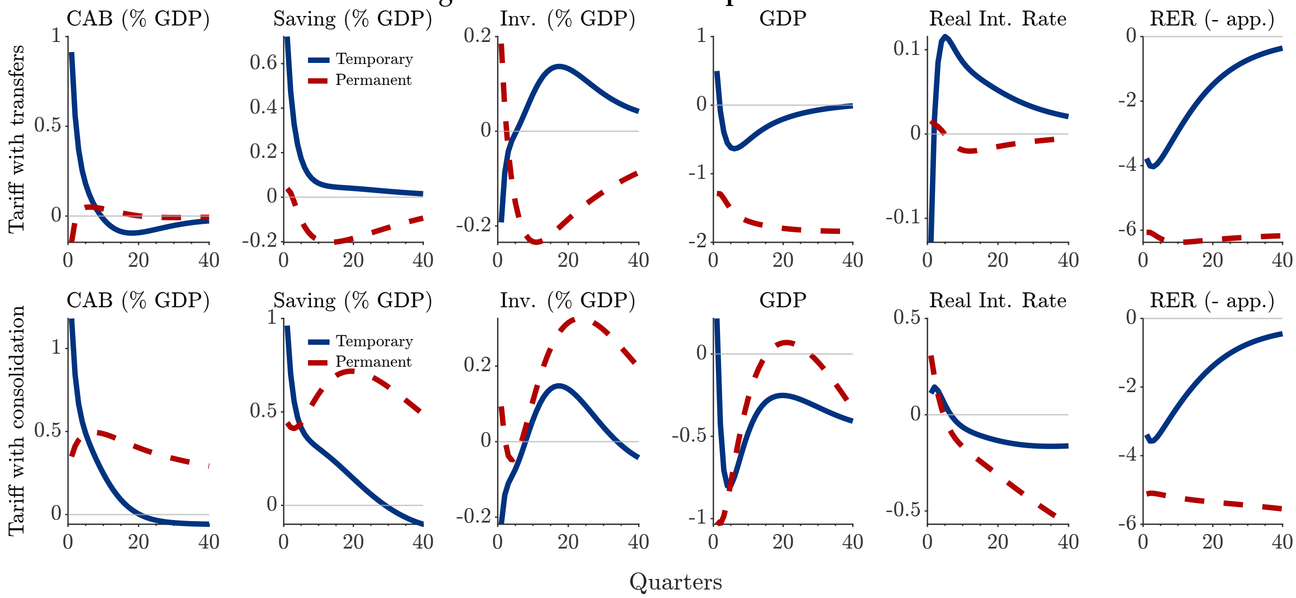
In this section we provide a description of the scenarios shown in the working paper. The published version of the paper shows a subset of the results on tariffs and combined macro- and micro-IP.

5.1 Tariffs

In this section, we show the results of unilateral temporary and permanent tariffs by the Home economy in the model. The tariff is simulated as a 10 percentage point increase in τ. In the temporary case, the half life of the tariff is 10 quarters. We further show results where the tariff revenues are immediately redistributed to households as lump-sum transfers and a second case where tariff revenues are used to lower the stock of outstanding debt. In the model, a very low value of ρ^T is used in the fiscal consolidation case so that there is initially only a small increase in lump-sum transfers relative to the size of tariff revenues. However, transfers rise slowly over time to stabilize debt (due to the small ρ^B parameter).

As explained in the paper, the permanent tariff has only a modest effect on saving in the case where revenues are fully paid out as transfers. Households have no incentive to save more or less in response to the tariff given it will remain constant over time, leading to a broadly neutral current account effect. In the temporary case, saving rises sharply in anticipation of lower prices in the future as the tariff is withdrawn, increasing the current account balance in the short-term. Saving dynamics change in the fiscal consolidation case. Because the presence of hand-to-mouth and perpetual youth households break Ricardian equivalence, higher government saving due to the tariff revenues leads to higher national saving. The current account balance is persistently higher in response.

Figure 1: **Model-based impact of tariffs**

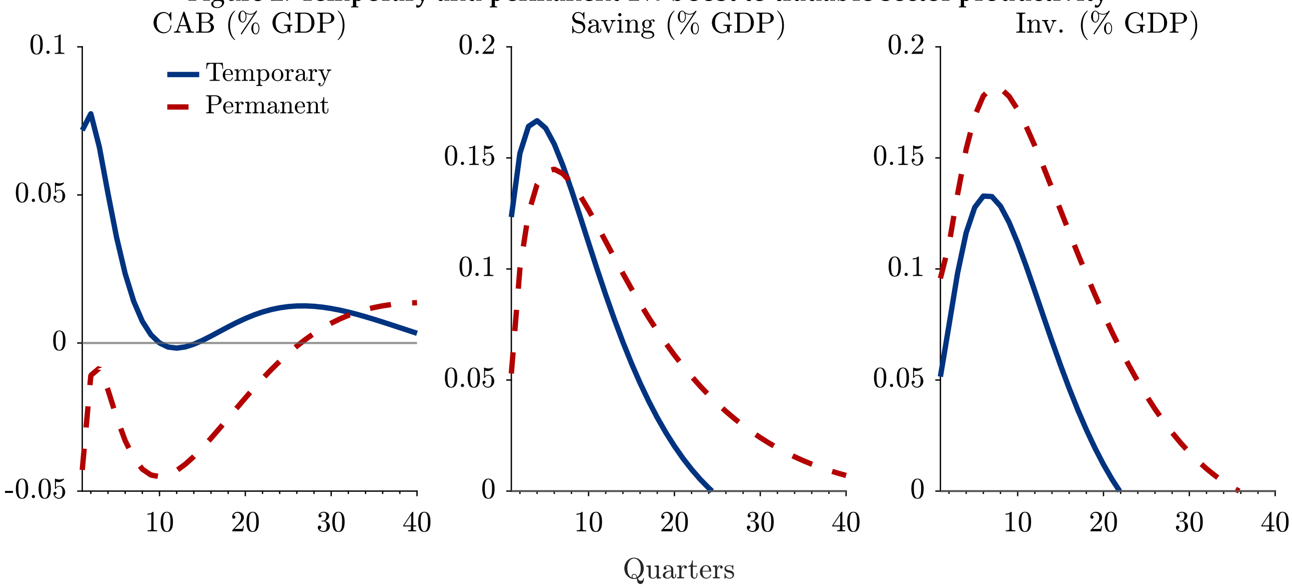


5.2 Micro-industrial policy: productivity related developments

Productivity-enhancing micro-industrial policy We first show the effects of a micro-industrial policy that succeeds in raising tradable sector productivity. This is simulated as a 1% boost to tradable sector productivity. In the permanent case, the persistence of the shock (ρ_A) is set to 1. In the temporary case, ρ_A is set to 0.9. Perpetual youth households initially save part of the windfall created by the temporary improvement in productivity, generating a current account surplus. This exceeds the initial increase in investment, partly due to investment adjustment frictions and the limited persistence of the productivity rise (reducing the incentive to invest). In the permanent case, saving initially rises but by less than in the temporary case to reflect consumption smoothing in anticipation of higher future income. That said, the presence of hand-to-mouth agents in the model and consumption habits limit the increase in consumption relative to output so that saving still rises overall as a share of GDP. The investment response is larger in the permanent case to reflect the more persistent increase in productivity, and hence discounted returns on capital.

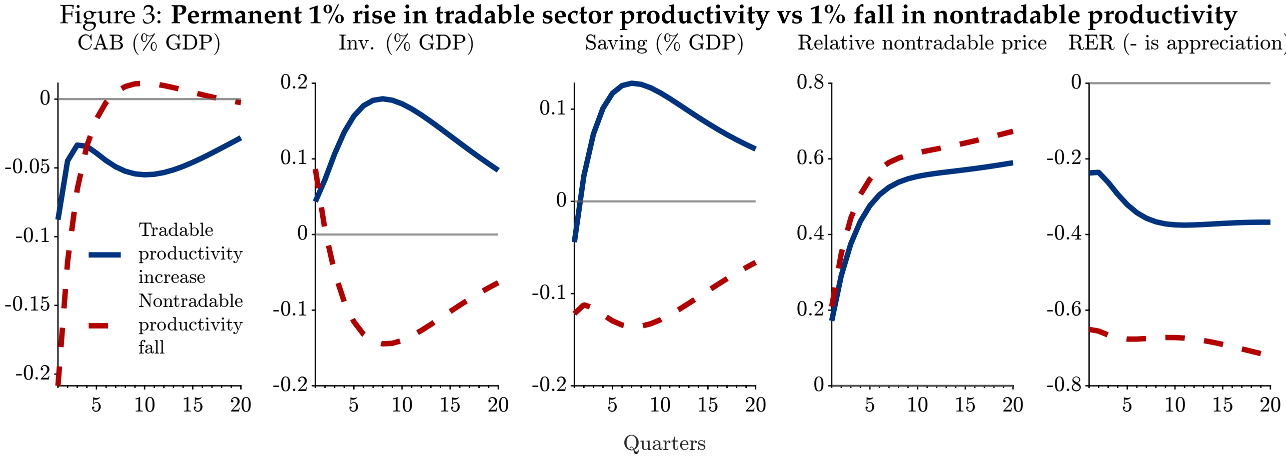
Falling nontradable productivity and its equivalence with rising tradable sector productivity Distortionary industrial policies that reallocate resources to the export-focused tradable sector could harm productivity and output in the nontradable sector. Paradoxically, a permanent negative shock to non-tradable productivity (A_N) produces qualitatively similar impacts on the current account as a positive tradable productivity shock (A_T). This is driven by the low elasticity of substitution between sectors ($\chi < 1$). When A_N falls, the relative price of nontradables surges. Because households view tradables and nontradables as complements, their substitution away from the now-expensive nontradables is limited. Consequently, the RER appreciates as the price of nontradables becomes relatively more expensive than internationally-priced tradables. The real appreciation grows over time given the slow adjustment of prices and reallocation of capital between sectors, so

Figure 2: **Temporary and permanent 1% boost to tradable sector productivity**



that the relative cost of the home consumption basket increasingly becomes more expensive. This incentivizes dissaving, while aggregate investment also contracts to reflect falling productivity. The net result is a decline in the current account balance.

Figure 3: **Permanent 1% rise in tradable sector productivity vs 1% fall in nontradable productivity**



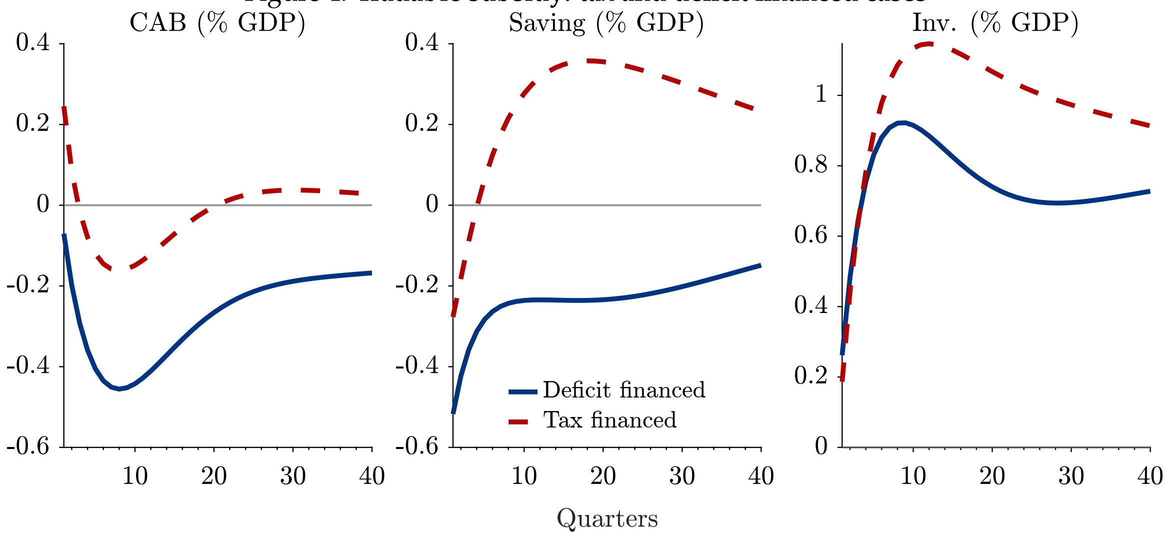
5.3 Micro-industrial policy: Subsidies

We then consider a simple subsidy to the cost of investment (τ_{sub}). This subsidy encourages capital deepening, and therefore has similar impacts to the productivity simulations considered above (given it influences productivity via capital deepening). In contrast to the earlier case, we now consider the fiscal implications of so-called 'micro' industrial policy and how this affects the current account.

We consider two cases, one in which the subsidy is fully funded via a tax on consumption (τ_c). Second, we consider a case where the subsidy is initially heavily deficit financed (via a very low value of ρ^T, reducing the increase in taxes to a small fraction of the increase in subsidy spending initially). In the case of the debt-financed subsidy, future tax liabilities from the cost of the subsidy are partially discounted by the perpetual youth households due to the probability of survival θ. Households therefore save less than the cost of the subsidy, lowering aggregate saving. In contrast, in the tax-financed case, the cost of the subsidy immediately lowers consumption, as perpetual youth households bear the cost immediately and HTM households lower consumption given income constraints from lower post-tax income. In both cases, investment rises in response to the subsidy.

Lower national saving in the deficit-financed case leads to a more negative current account impact than in the tax-financed case.

Figure 4: **Tradable subsidy: tax and deficit financed cases**

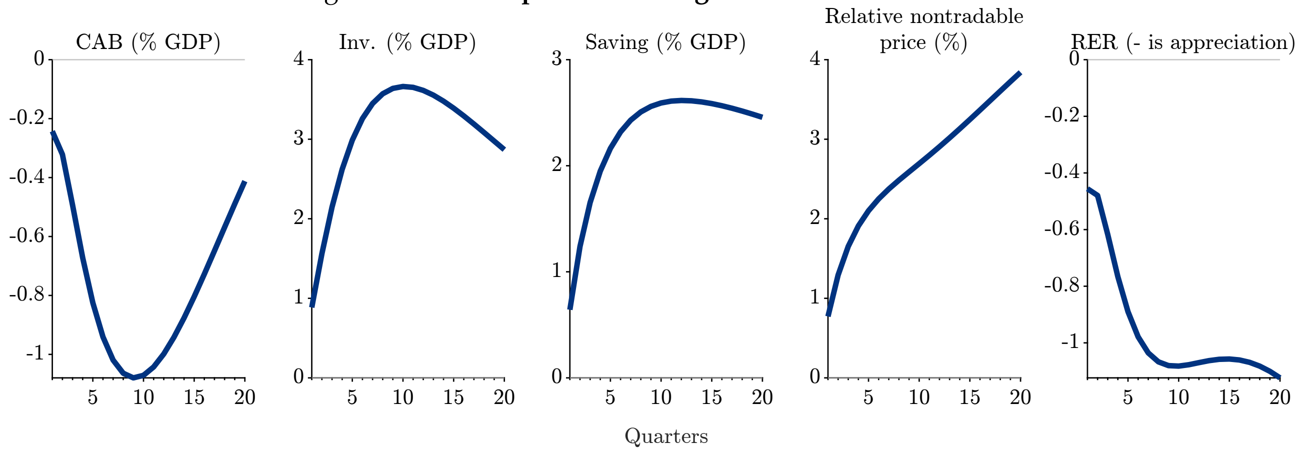


5.4 Micro-industrial policy: Production targets

Industrial policy can also be performed by implementing strategic targets for production. For example, a mandated increase in output may eventually drive marginal costs lower if scale economies are achieved, or if they encourage learning by doing. Here, we only consider the effects of the mandated increase in output and not the hoped-for second round effect on productivity.

We simulate this as a permanent fall in the markup in the tradable sector. This has two direct effects: lowering the price of output, and increasing production and factor inputs as firms are forced to increase production and bear the consequence of lower profits. This shock is close to an approximation of meeting output targets while sacrificing commercial interests (profit) in the short run. The shock has many similar effects to the productivity improvement case considered above — the increased output for tradable products at lower prices creates a relative shortage of nontradable products. Inelastic demand for nontradables results in a sharp rise in relative prices, increasing investment and labor demand to alleviate shortages. The overall price level rises in response to rising demand, resulting in a RER appreciation. Higher expected future income as capital deepening progresses reduces the increase in saving relative to investment, leaving the current account balance lower.

Figure 5: **Effect of production targets in the tradable sector**



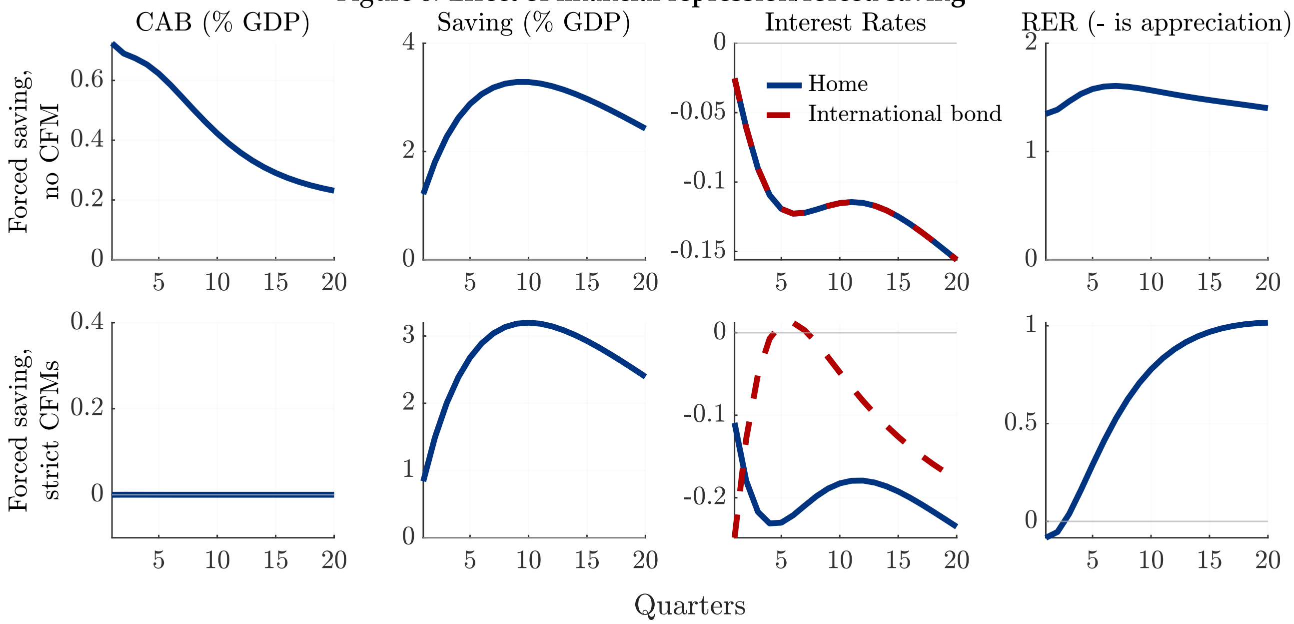
5.5 Macro-industrial policy: Forced saving and capital controls

As discussed in the paper, forced saving and financial repression policies may be used to lower domestic interest rates and increase credit availability, supporting domestic industrial policy objectives. Often this policy can be combined with capital controls that create a 'wedge' with global interest rates (allowing them to be lower).

Policy to increase household saving and lower interest rates is simulated as a persistent preference shock to the household discount rate β (raising it). The simulation is conducted for two cases to approximate the impact of capital controls. When φ (the adjustment cost term on international bond holdings) is 0, this is considered to policy of free financial flows with no capital flow restrictions. We set φ to a very high level to proxy the implementation of strict capital flow restrictions that make international bond transactions too costly to undertake. In the free capital mobility case, the persistent increase in saving drives the current account balance higher and domestic and global (reflecting the international bond yield) interest rates lower. The exchange rate initially depreciates before investment begins to rise in response to lower interest rates.

In the strict 'CFM' case (capital flow management), households cannot accumulate foreign assets to match desired saving. A reduction in import demand from this consumption suppression instead leads to a gradual appreciation of the RER. Domestic interest rates fall below the international bond rate and domestic investment rises sufficiently to leave the current account balance little changed.

Figure 6: **Effect of financial repression/forced saving**

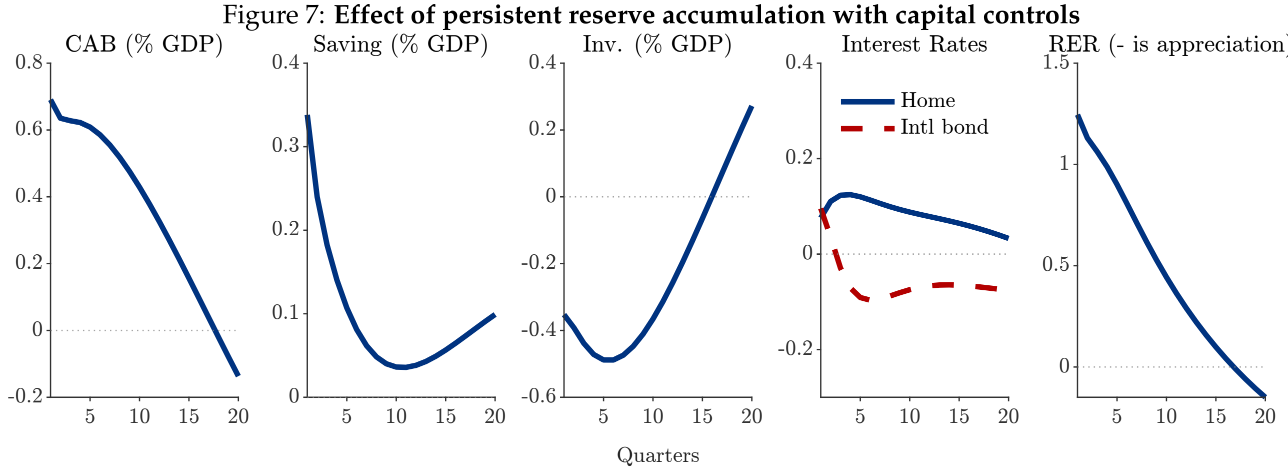


5.6 Macro-industrial policy: reserve accumulation

We now consider government reserve accumulation of international bonds with the aim of increasing the current account balance and depreciating the real exchange rate.

To simulate the presence of capital controls for the private sector, φ is set to 1, introducing high costs for households to engage in international bond transactions. ϵ^G is shocked to generate a persistent accumulation of the international bond by the government (higher B_I^G). The accumulation is funded primarily through domestic bond issuance, requiring higher domestic interest rates to encourage the private sector to purchase the new net supply of bonds (they are unable to sell down holdings of the international bond given the high transaction costs). The result is a rise in domestic interest rates above the global bond rate and a persistent depreciation given a fall in domestic absorption from lower consumption and investment.

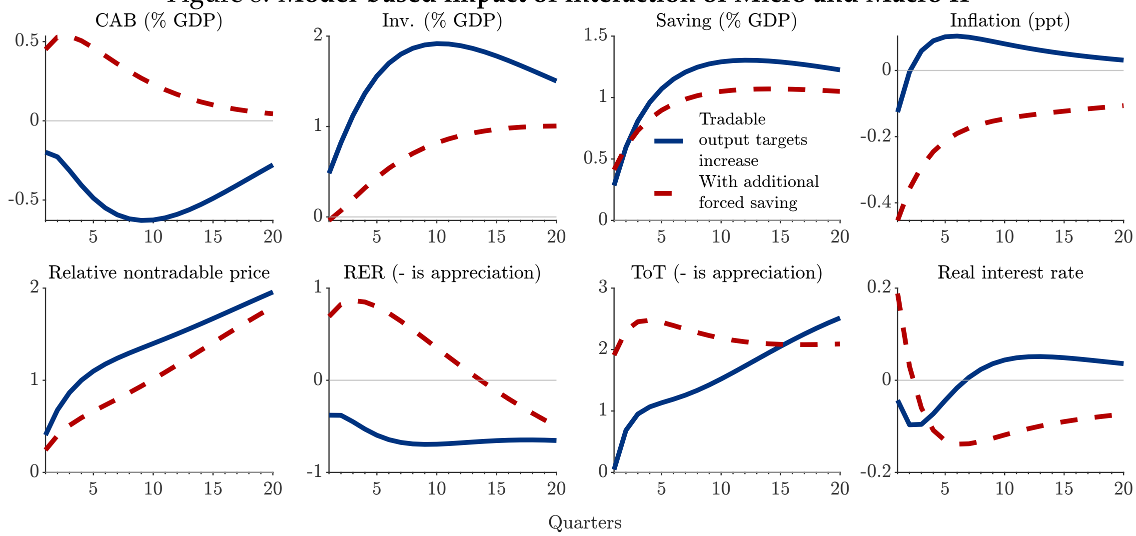
Figure 7: **Effect of persistent reserve accumulation with capital controls**



5.7 Combined Micro and Macro industrial policy

In this section, we show that the combination of micro and macro industrial policy in our framework can offset some of the 'undesirable' side-effects of micro-IP that result in a lower current account balance. As in our previous example of production targets in the tradable sector, we lower desired markups to replicate higher output targets at the cost of near term profitability. Expectations of higher future income from capital deepening in response to higher demand and lower prices also lower saving relative to investment. In addition, the relative shortage of nontradable goods leads to a large relative price increase and an RER appreciation. Now we combine this production targets policy with consumption repression. Using an intertemporal preference shock (to β), we lower consumption persistently, as in the financial repression/forced saving case described above (with no capital controls in this case). This naturally *increases* saving relative to investment, but also counteracts the real exchange rate appreciation that occurs in the standalone micro-IP case by reducing demand and limiting the higher relative price adjustment of nontradable goods.

Figure 8: **Model-based impact of interaction of Micro and Macro IP**



Note: Simulated as a highly persistent (but declining) fall in the desired markup for tradable sector firms. In the 'with additional forced saving' scenario, an additional shock to intertemporal preferences lowers consumption.

6 Model extensions

6.1 Consumption smoothing

We now maximize household utility for:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{1}{1-\sigma} (C_t(i) - \rho C_{t-1})^{1-\sigma} - \frac{1}{1+\nu} N_t^{1+\nu} \right),$$

This implies that household i has habits referencing aggregate (not individual household) consumption in the previous period. It avoids the FOCs yielding forward looking and backward looking components to the consumption smoothing parameter which make the perpetual youth household framework have no analytically tractable solution.

Replacing $\tilde{C}_t = C_t - \rho C_{t-1}$ We also can modify the budget constraint so that it is written in terms of $\tilde{C}$

$$(1+\tau_{C,t})C_t^P + \frac{B_{Ht}}{(1-\omega)} + \frac{B_{It}}{(1-\omega)} + \frac{\varphi}{(1-\omega)2}(B_{I,t} - B_{I,ss})^2$$
$$= \frac{R_{H,t-1}}{\theta(1-\omega)\pi_t}B_{Ht-1} + \frac{R_{I,t-1}}{\theta(1-\omega)\pi_t}B_{I,t-1} + (1-\tau_t^l)w_t N_t + \frac{\Pi_t}{(1-\omega)} + T_t$$

First, we know from the first order conditions that the return on the international bond, inclusive of adjustment costs, must be equal to the domestic government bond, so we can consolidate all bonds into a single aggregate $B_{T,t}$ with return $\frac{R_t}{\pi_{t+1}}$.

$$(1+\tau_{C,t})\tilde{C}_t + B_{T,t} = \frac{R_{t-1}}{\theta\pi_t}\frac{B_{T,t-1}}{1-\omega} + (1-\tau_t^l)w_t N_t + \frac{\Pi_t}{1-\omega} + T_t - (1+\tau_{C,t})\rho C_{t-1}$$

And the FOC for $\tilde{C}$ becomes:

$$(\tilde{C}_t)^{-\sigma} = (1+\tau_{C,t})\lambda_t$$

While the standard bond condition yields:

$$\lambda_t = \beta\frac{R_t}{\pi_{t+1}}\lambda_{t+1}$$

How does the forward iteration of the budget constraint change (for simplicity, Y is income net of investment)?:

$$Y_t - (1+\tau_{C,t})(\tilde{C}_t + \rho C_{t-1}) + \frac{R_{t-1}}{\theta\pi_t}\frac{B_{T,t-1}}{1-\omega} = \frac{B_{T,t}}{1-\omega}$$

$$Y_{t+1} - (1+\tau_{C,t+1})(\tilde{C}_{t+1} + \rho C_t) + \frac{R_t}{\theta\pi_{t+1}}(Y_t - (1+\tau_{C,t})(\tilde{C}_t + \rho C_{t-1})) + \frac{R_t}{\pi_{t+1}}\frac{R_{t-1}}{\theta^2\pi_t}\frac{B_{T,t-1}}{1-\omega} = \frac{B_{T,t+1}}{1-\omega}$$

Where $Q_t = \frac{\pi_t}{R_{t-1}}$, $Q_{t+1} = \frac{\pi_{t+1}}{R_t}\frac{\pi_t}{R_{t-1}}$. This leads to the following lifetime budget constraint:

$$\sum_{\tau=0}^{\infty}\theta^\tau Q_{t+\tau}(Y_{t+\tau} - (1+\tau_{C,t+\tau})(\tilde{C}_{t+\tau} + \rho C_{t-1+\tau})) + \frac{R_{t-1}}{(1-\omega)\theta\pi_t}B_{t-1} = \frac{B_T}{1-\omega}\theta^T Q_{t+T}$$

$$\sum_{\tau=0}^{\infty}\theta^\tau Q_{t+\tau}((1+\tau_{C,t+\tau})\tilde{C}_{t+\tau}) = \sum_{\tau=0}^{\infty}\theta^\tau Q_{t+\tau}(Y_{t+\tau} - (1+\tau_{C,t+\tau})\rho C_{t-1+\tau}) + \frac{R_{t-1}}{(1-\omega)\theta\pi_t}B_{T,t-1}$$

We can re-write the budget constraint by substituting in the consumption Euler.:

$$(1+\tau_{C,t})\tilde{C}_t = (1+\tau_{C,t})(\frac{Q_{t+1}(1+\tau_{C,t+1})}{\beta(1+\tau_{C,t})})^{1/\sigma}\tilde{C}_{t+1}$$

$$\tilde{C}_t\theta(Q_{t+1}(1+\tau_{C,t+1}))^{1-1/\sigma}(1+\tau_{C,t})^{1/\sigma-1}\beta^{1/\sigma} = Q_{t+1}(1+\tau_{C,t+1})\theta\tilde{C}_{t+1}$$

$$(1+\tau_{C,t})\tilde{C}_t\theta(Q_{t+1}\frac{(1+\tau_{C,t+1})}{(1+\tau_{C,t})})^{1-1/\sigma}\beta^{1/\sigma} = Q_{t+1}(1+\tau_{C,t+1})\theta\tilde{C}_{t+1}$$

By substituting for the first period of the budget constraint $\tilde{C}_t(1+\tau_{C,t})$

$$(1+\tau_{C,t})\tilde{C}_t\sum_{\tau=0}^{\infty}(\theta\beta^{1/\sigma})^{\tau}(Q_{t+\tau}\frac{(1+\tau_{C,t+\tau})}{(1+\tau_{C,t})})^{1-1/\sigma} = \frac{R_{t-1}}{\theta(1-\omega)\pi_t}B_{T,t-1} + \sum_{\tau=0}^{\infty}\theta^{\tau}Q_{t+\tau}(Y_{t+\tau}-(1+\tau_{C,t+\tau})\rho C_{t-1+\tau})$$

This equation for consumption only applies to those households who have survived the previous period and inherited assets. We now need to adjust the aggregate consumption function to account for those agents who have just entered the world with no assets but who do have income from wages and profits. This is a simple weighting of θ for the surviving households and $(1-\theta)$ times zero for the new households. The end result is a simple elimination of the θ term on the return to $B_{T,t-1}$

$$(1+\tau_{C,t})\tilde{C}_t\sum_{\tau=0}^{\infty}(\theta\beta^{1/\sigma})^{\tau}(Q_{t+\tau}\frac{(1+\tau_{C,t+\tau})}{(1+\tau_{C,t})})^{1-1/\sigma} = \frac{R_{t-1}}{(1-\omega)\pi_t}B_{T,t-1} + \sum_{\tau=0}^{\infty}\theta^{\tau}Q_{t+\tau}(Y_{t+\tau}-(1+\tau_{C,t+\tau})\rho C_{t-1+\tau})$$

A Proofs and Technical Results

Lemma A.1 (Tariffs and the budget constraint) *The imposition of tariffs does not alter the standard form of the budget constraint, with the effects generated instead through relative and aggregate price adjustment.*

The real cost of consumption of tradables can be written as:

$$\frac{P_{H,t}}{P_t}C_{H,t} + (1+\tau_t)\frac{P_{F,t}}{P_t}C_{F,t}$$

Substituting for the relative price terms as functions of the terms of trade results in:

$$\left[(1-\alpha) + \alpha(1+\tau_t)^{1-\eta}S_t^{1-\eta}\right]^{\frac{-1}{1-\eta}} C_{Ht} + \left[(1-\alpha)S^{\eta-1} + \alpha(1+\tau_t)^{1-\eta}\right]^{\frac{-1}{1-\eta}} (1+\tau_t)C_{Ft}$$

This can further be re-written as a function of C_T by substituting for C_H and C_F given their dependence on total consumption and relative prices:

$$(1-\alpha)\left[(1-\alpha) + \alpha(1+\tau_t)^{1-\eta}S_t^{1-\eta}\right]^{\frac{\eta-1}{1-\eta}} C_T + \alpha\left[(1-\alpha)S^{\eta-1} + \alpha(1+\tau_t)^{1-\eta}\right]^{\frac{\eta-1}{1-\eta}} (1+\tau)^{1-\eta}C_T \tag{2}$$

Extracting the home and foreign price from each term respectively:

$$(1-\alpha)\left[(1-\alpha) + \alpha(1+\tau_t)^{1-\eta}S_t^{1-\eta}\right]^{\frac{\eta-1}{1-\eta}} C_T + \alpha(1+\tau_t)^{1-\eta}\left[(1-\alpha)S^{\eta-1} + \alpha(1+\tau_t)^{1-\eta}\right]^{\frac{\eta-1}{1-\eta}} C_T$$
$$(1-\alpha)P_H^{1-\eta}\left[(1-\alpha)P_H^{1-\eta} + \alpha(1+\tau_t)^{1-\eta}P_F^{1-\eta}\right]^{-1} C_T + \alpha(1+\tau)^{1-\eta}P_F^{1-\eta}\left[(1-\alpha)P_H^{1-\eta} + \alpha(1+\tau_t)^{1-\eta}P_F^{1-\eta}\right]^{-1} C_T$$

This trivially reduces to C_T. So the inclusion of tariffs does not change the budget constraint from the household's perspective beyond its effects on the aggregate and relative price effects.

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