Online Appendix for
A Static Capital Buffer is Hard To Beat

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This appendix consists of five sections. In Section A we complete the derivation of the equilibrium conditions and provide proofs for propositions 1 and 2. In Section B, we summarize the equilibrium conditions for the model. In Section C, we discuss the computation of the asset-weighted bank failure rate. In Section D, we provide further details on the mechanisms that trigger banks to take on excessive risk. Finally, in Section E, we derive a consumption equivalent variation that we use to interpret the welfare costs of simple rules for capital requirements.

A Derivation of Equilibrium Conditions

This section completes the derivation of the equilibrium conditions for the model. We tackle the optimization problem of households before considering banks and nonfinancial firms. We then consider the financing of the deposit insurance scheme provided by the government. We conclude by stating the resource constraints. Along the way, we prove propositions 1 and 2.

A.1 Households

In the main text, we consider a case in which we have only one representative household. The household problem involves saving through deposits and bank equity, with the capital requirement on banks determining the equilibrium split between these two assets, and providing labor services to firms inelastically. In our baseline specification, we leave the managerial labor needed to run banks unmodeled. This is not too different from abstracting from modeling the managerial services for other types of firms in the model. But there are other approaches to setting up the household problem in models that include financial intermediaries but that are in the standard RBC mold otherwise. Some of the alternatives include differentiating more explicitly between roles within households.

In this section of the appendix, we spell out how to recast the setup for the household problem described in the main text of the paper into an alternative with explicit differentiation between workers and bankers. This recasting follows the blueprint in the model of Gertler and Karadi (2011), which splits the household into workers and bankers. In that model, bankers receive an endowment untied to the utility maximization problem of the household, whereas we endogenize the endowment choice. We will show that, mutatis mutandis, there is no change in the equilibrium conditions relative to the baseline setup with a representative household.

We assume here that households are composed of workers and two bankers. The household seeks to maximize

$$\max_{C_t, D_t, E_t^s, E_t^r} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t - \kappa C_{t-1})^{1-\varsigma_c} - 1}{1-\varsigma_c} + \varsigma_0 \frac{D_t^{1-\varsigma_d} - 1}{1-\varsigma_d} \right],$$

where $0 < \beta < 1$ is the worker's subjective discount factor, assumed identical across workers, i.e. $\beta_w = \beta$ for all w.

This maximization problem is subject to the sequence of budget constraints

$$
\begin{aligned}
C_t + D_t + E_t^s + E_t^r &= W_t + R_{t-1}^d D_{t-1} + R_t^{e,s} E_{t-1}^s + R_t^{e,r} E_{t-1}^r - T_t, \\
E_t^s &\geq 0, \\
E_t^r &\geq 0.
\end{aligned}
$$

Households save through deposits, D_t, which pay a gross real rate R_t^d, and by endowing bankers with initial capital: E_t^s is the endowment for bankers operating a "safe" bank, which lends to a safe firm and pays $R_{t+1}^{e,s}$ next period; E_t^r is the endowment for bankers operating a "risky" bank, which lends to a risky firm and pays $R_{t+1}^{e,r}$. As for the baseline formulation of the household problem, the returns that will be shared by the bankers are not known when the household makes the endowment decisions. By contrast, the return on deposits is known, and deposits are protected by deposit insurance. Households pay lump sum taxes, T_t, to fund the government's deposit insurance program.

Bankers run banks that they do not own.[1] They are elected to run as bankers for two periods at the end of which they return to the household. They share consumption with the household as well as the proceeds from running safe and risky banks at the end of the two-period stint as managers.

As in the new-Keynesian literature that considers sticky wages, see for example Erceg et al. (2000), we assume that households can also access a complete set of state contingent claims traded across households but in zero net supply—we leave this set unspecified in the households' objective functions and constraints. These state-contingent claims ensure equalization of consumption and deposit decisions across households, counteracting the idiosyncratic risk borne by operating only one risky bank. In our baseline formulation of the household problem, we do not need this additional assumption as we think of bank equity in the same vein as a mutual fund that assigns aliquot shares of capital to different banks.

Using λ_{ct} as the Lagrangian multiplier on the budget constraint, and ζ_t^s and ζ_t^r on the non-negativity constraints for safer and risky equity, respectively, the FOCs of the representative household's problem are:

$$C_t: \quad (C_t - \kappa C_{t-1})^{-\varsigma_c} - \beta\kappa \mathbb{E}_t \left(C_{t+1} - \kappa C_t\right)^{-\varsigma_c} - \lambda_{ct} = 0, \tag{A.1}$$

$$D_t: \quad \varsigma_0 D_t^{-\varsigma_d} - \lambda_{ct} + \beta \mathbb{E}_t \left\{\lambda_{ct+1}\right\} R_t^d = 0, \tag{A.2}$$

$$E_t^s: \quad -\lambda_{ct} + \beta \mathbb{E}_t \left\{\lambda_{ct+1} R_{t+1}^{e,s}\right\} + \zeta_t^s = 0, \tag{A.3}$$

$$E_t^r: \quad -\lambda_{ct} + \beta \mathbb{E}_t \left\{\lambda_{ct+1} R_{t+1}^{e,r}\right\} + \zeta_t^r = 0 \tag{A.4}$$

[1] Bankers can be viewed as managers with expertise in operating firms.

together with the complementary slackness conditions:

$$\zeta_t^s E_t^s = 0, \tag{A.5}$$

$$\zeta_t^r E_t^r = 0. \tag{A.6}$$

These conditions are equivalent to the conditions that characterize the solution to the household's problem in the main text: equation (A.1) corresponds to (16) in the main text, (A.2) corresponds to (17) in the main text, (A.3) corresponds to (18) in the main text, (A.4) corresponds to (19) in the main text, (A.5) corresponds to (20) in the main text, and (A.6) corresponds to (21) in the main text.

A.2 The Bank's Problem

There is a unit measure of banks. As described in Section 2.3.2, banks maximize expected dividends

$$\max_{l_t, d_t, e_t, \sigma_t} \mathbb{E}_t \left\{ \psi_{t.t+1} \left[\int_{\varepsilon_{t+1}^*}^{\infty} nw_{t+1} \, \mathrm{d}G(\varepsilon_{t+1}) \right] \right\} - e_t,$$

subject to

$$\begin{aligned} l_t &= e_t + d_t, \\ e_t &\geq \gamma_t l_t, \\ l_t &\geq 0, \\ \underline{\sigma} &\leq \sigma_t \leq \overline{\sigma}, \end{aligned}$$

where $\psi_{t,t+1} = \beta \frac{\lambda_{ct+1}}{\lambda_{ct}}$ is the stochastic discount factor.

A reminder on notation is in order. We use lower-case letters to denote variables pertaining to single banks or firms, and upper-case letters to denote aggregate variables.

A.2.1 First-Order Conditions

Substituting $d_t = l_t - e_t$ into the maximization problem and writing $\mathrm{d}G(\varepsilon_{t+1})$ explicitly turn the objective into

$$\max_{l_t, e_t, \sigma_t} \mathbb{E}_t \left\{ \psi_{t,t+1} \left[\int_{\varepsilon_{t+1}^*}^{\infty} \left(\left(R_{t+1}^s + \sigma_t \frac{\varepsilon_{t+1}}{Q_t} \right) l_t - R_t^d (l_t - e_t) \right) \frac{1}{\sqrt{2\pi\tau_t^2}} \exp\left(-\frac{(\varepsilon_{t+1} + \xi)^2}{2\tau_t^2} \right) \mathrm{d}\varepsilon_{t+1} \right] - e_t \right\},$$

subject to

$$e_t \geq \gamma_t l_t,$$
$$l_t \geq 0,$$
$$\underline{\sigma} \leq \sigma_t \leq \overline{\sigma},$$

where $\psi_{t,t+1} = \beta \frac{\lambda_{ct+1}}{\lambda_{ct}}$ is the stochastic discount factor and $\varepsilon^*_{t+1} = \left(\frac{R^d_t - R^s_{t+1}}{\sigma_t} - \frac{R^d_t e_t}{\sigma_t l_t} \right) Q_t$ is the shield of limited liability. Note that we expressed ε^*_{t+1} from $\left(R^s_{t+1} + \sigma_t \frac{\varepsilon^*_{t+1}}{Q_t} \right) l_t - R^d_t \left(l_t - e_t \right) = 0$ to get the lower limit of the integral.

Append the Lagrangian multiplier χ_{1t} to the constraint $e_t \geq \gamma l_t$ and χ_{2t} to the constraint $l_t \geq 0$. We will split the maximization problem into two parts. We will first condition on a choice of risk, σ_t, and then show how to pin down that choice (see Section A.2.6). Accordingly, conditional on the optimal choice of σ_t, the first-order conditions are:

$$\frac{\partial \mathcal{L}}{\partial l_t} = \mathbb{E}_t \left[\psi_{t,t+1} \overbrace{\left(\left(R^s_{t+1} + \sigma_t \left(\frac{R^d_t - R^s_{t+1}}{\sigma_t} - \frac{R^d_t e_t}{\sigma_t l_t} \right) \right) l_t - R^d_t \left(l_t - e_t \right) \right)}^{=0} \cdot \frac{\partial \varepsilon^*_{t+1}}{\partial l_t} \right] + \chi_{2t} +$$
$$\mathbb{E}_t \left[\int\limits_{\varepsilon^*_{t+1}}^{\infty} \psi_{t,t+1} \frac{\partial}{\partial l_t} \left(\left(R^s_{t+1} + \sigma_t \frac{\varepsilon_{t+1}}{Q_t} \right) l_t - R^d_t \left(l_t - e_t \right) \right) \frac{1}{\sqrt{2\pi\tau_t^2}} \exp\left(- \frac{\left(\varepsilon_{t+1} + \xi \right)^2}{2\tau_t^2} \right) \mathrm{d}\varepsilon_{t+1} \right] - \gamma \chi_{1t} = 0.$$

$$\frac{\partial \mathcal{L}}{\partial e_t} = -\mathbb{E}_t \left[\psi_{t,t+1} \overbrace{\left(\left(R^s_{t+1} + \sigma_t \left(\frac{R^d_t - R^s_{t+1}}{\sigma_t} - \frac{R^d_t e_t}{\sigma_t l_t} \right) \right) l_t - R^d_t \left(l_t - e_t \right) \right)}^{=0} \cdot \frac{\partial \varepsilon^*_{t+1}}{\partial e_t} \right] + \chi_{1t} +$$
$$\mathbb{E}_t \left[\int\limits_{\varepsilon^*_{t+1}}^{\infty} \psi_{t,t+1} \frac{\partial}{\partial e_t} \left(\left(R^s_{t+1} + \sigma_t \frac{\varepsilon_{t+1}}{Q_t} \right) l_t - R^d_t \left(l_t - e_t \right) \right) \frac{1}{\sqrt{2\pi\tau_t^2}} \exp\left(- \frac{\left(\varepsilon_{t+1} + \xi \right)^2}{2\tau_t^2} \right) \mathrm{d}\varepsilon_{t+1} \right] - 1 = 0,$$

together with the complementary slackness conditions:

$$\begin{aligned}
&\chi_{1t}\left(e_t - \gamma_t l_t\right) = 0,\\
&\chi_{2t} l_t = 0,\\
&e_t - \gamma_t l_t \geq 0,\\
&l_t \geq 0,\\
&\chi_{1t} \geq 0,\\
&\chi_{2t} \geq 0,
\end{aligned}$$

We are using the Leibniz integral rule above to find the partial derivatives of the profit function. Note that the first term is zero in the differentiation because the upper limit of the integral does not depend on any of the choice variables.

Next, express the integrals in the first-order conditions above using the erf function, wherever possible. Note that we omit the stochastic discount factor and the expectation operator in writing up the expressions of the next integrals. We include those terms in the final exposition.

Work on $\frac{\partial}{\partial l_t}$:

$$\int_{\left(\frac{R_t^d - R_{t+1}^s}{\sigma_t} - \frac{R_t^d e_t}{\sigma_t l_t}\right) Q_t}^{\infty} \frac{\partial}{\partial l_t}\left(\left(\left(R_{t+1}^s + \sigma_t \frac{\varepsilon_{t+1}}{Q_t}\right) l_t - R_t^d\left(l_t - e_t\right)\right) \frac{1}{\sqrt{2\pi\tau_t^2}} \exp\left(-\frac{\left(\varepsilon_{t+1}+\xi\right)^2}{2\tau_t^2}\right) \mathrm{d}\varepsilon_{t+1} =$$

$$\int_{\left(\frac{R_t^d - R_{t+1}^s}{\sigma_t} - \frac{R_t^d e_t}{\sigma_t l_t}\right) Q_t}^{\infty} \left(R_{t+1}^s + \sigma_t \frac{\varepsilon_{t+1}}{Q_t} - R_t^d\right) \frac{1}{\sqrt{2\pi\tau_t^2}} \exp\left(-\frac{\left(\varepsilon_{t+1}+\xi\right)^2}{2\tau_t^2}\right) \mathrm{d}\varepsilon_{t+1} =$$

Break the calculation of the integral into two parts.

$$\frac{\sigma_t}{Q_t} \int_{\left(\frac{R_t^d - R_{t+1}^s}{\sigma_t} - \frac{R_t^d e_t}{\sigma_t l_t}\right) Q_t}^{\infty} \varepsilon_{t+1} \frac{1}{\sqrt{2\pi\tau_t^2}} \exp\left(-\frac{\left(\varepsilon_{t+1}+\xi\right)^2}{2\tau_t^2}\right) \mathrm{d}\varepsilon_{t+1} +$$

$$\left(R_{t+1}^s - R_t^d\right) \int_{\left(\frac{R_t^d - R_{t+1}^s}{\sigma_t} - \frac{R_t^d e_t}{\sigma_t l_t}\right) Q_t}^{\infty} \frac{1}{\sqrt{2\pi\tau_t^2}} \exp\left(-\frac{\left(\varepsilon_{t+1}+\xi\right)^2}{2\tau_t^2}\right) \mathrm{d}\varepsilon_{t+1}.$$

Work on the first part

$$\int\limits_{\left(\frac{R_t^d - R_{t+1}^s}{\sigma_t} - \frac{R_t^d e_t}{\sigma_t l_t}\right) Q_t}^{\infty} \varepsilon_{t+1} \frac{1}{\sqrt{2\pi\tau_t^2}} \exp\left(-\frac{(\varepsilon_{t+1}+\xi)^2}{2\tau_t^2}\right) \mathrm{d}\varepsilon_{t+1},$$

by introducing a change in variables to recast the integral in terms of the Standard Normal distribution. Use $v = \frac{\varepsilon_{t+1}+\xi}{\sqrt{2}\tau_t}$, or equivalently $\varepsilon_{t+1} = v\sqrt{2}\tau_t - \xi$, and remember that for the change $x = \varphi(t)$, the integral $\int_{\varphi(a)}^{\varphi(b)} f(x)dx$ becomes $\int_a^b f(\varphi(t))\varphi'(t)dt$. Here, we use that $dv = \frac{d\varepsilon_{t+1}}{\sqrt{2}\tau_t}$, so we need to multiply dv by $\sqrt{2}\tau_t$ to express $d\varepsilon_{t+1}$ in terms of dv. Moreover, we need to transform the lower limit using v. So we need to add ξ to the lower limit of the integral and divide the result by $\sqrt{2}\tau_t$.

$$\int\limits_{\frac{\left(R_t^d(l_t-e_t)-R_{t+1}^s l_t\right)Q_t+\xi\sigma_t l_t}{\sigma_t l_t\sqrt{2}\tau_t}}^{\infty} \left(v\sqrt{2}\tau_t - \xi\right) \frac{\sqrt{2}\tau_t}{\sqrt{2\pi\tau_t^2}} \exp\left(-v^2\right) \mathrm{d}v =$$

$$\frac{\sqrt{2}\tau_t}{\sqrt{\pi}} \int\limits_{\frac{\left(R_t^d(l_t-e_t)-R_{t+1}^s l_t\right)Q_t+\xi\sigma_t l_t}{\sigma_t l_t\sqrt{2}\tau_t}}^{\infty} v\exp\left(-v^2\right) \mathrm{d}v - \frac{\xi}{\sqrt{\pi}} \int\limits_{\frac{\left(R_t^d(l_t-e_t)-R_{t+1}^s l_t\right)Q_t+\xi\sigma_t l_t}{\sigma_t l_t\sqrt{2}\tau_t}}^{\infty} \exp\left(-v^2\right) \mathrm{d}v =$$

$$-\frac{\sqrt{2}\tau_t}{2\sqrt{\pi}} \exp\left(-v^2\right)\Big|_{\frac{\left(R_t^d(l_t-e_t)-R_{t+1}^s l_t\right)Q_t+\xi\sigma_t l_t}{\sigma_t l_t\sqrt{2}\tau_t}}^{\infty} - \frac{\xi}{\sqrt{\pi}}\left[\int\limits_0^{\infty} \exp\left(-v^2\right)\mathrm{d}v - \int\limits_0^{\frac{\left(R_t^d(l_t-e_t)-R_{t+1}^s l_t\right)Q_t+\xi\sigma_t l_t}{\sigma_t l_t\sqrt{2}\tau_t}} \exp\left(-v^2\right)\mathrm{d}v\right] =$$

$$0 + l_t\frac{\tau_t}{\sqrt{2\pi}} \exp\left(-\left(\frac{\left(R_t^d\left(l_t-e_t\right)-R_{t+1}^s l_t\right)Q_t+\xi\sigma_t l_t}{\sigma_t l_t\sqrt{2}\tau_t}\right)^2\right) -$$

$$\frac{\xi}{\sqrt{\pi}}\left[\frac{\sqrt{\pi}}{2}\mathrm{erf}(\infty) - \frac{\sqrt{\pi}}{2}\mathrm{erf}\left(\frac{\left(R_t^d\left(l_t-e_t\right)-R_{t+1}^s l_t\right)Q_t+\xi\sigma_t l_t}{\sigma_t l_t\sqrt{2}\tau_t}\right)\right] =$$

$$\frac{\tau_t}{\sqrt{2\pi}} \exp\left(-\left(\frac{\left(R_t^d\left(l_t-e_t\right)-R_{t+1}^s l_t\right)Q_t+\xi\sigma_t l_t}{\sigma_t l_t\sqrt{2}\tau_t}\right)^2\right) - \frac{\xi}{2}\left[1-\mathrm{erf}\left(\frac{\left(R_t^d\left(l_t-e_t\right)-R_{t+1}^s l_t\right)Q_t+\xi\sigma_t l_t}{\sigma_t l_t\sqrt{2}\tau_t}\right)\right],$$

where we used that $\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x \exp\left(-v^2\right)$.

Therefore,

$$\int\limits_{\left(\frac{R_t^d - R_{t+1}^s}{\sigma_t} - \frac{R_t^d e_t}{\sigma_t l_t}\right)Q_t}^{\infty} \varepsilon_{t+1} \frac{1}{\sqrt{2\pi\tau_t^2}} \exp\left(-\frac{(\varepsilon_{t+1}+\xi)^2}{2\tau_t^2}\right) \mathrm{d}\varepsilon_{t+1} = \tag{A.7}$$
$$\frac{\tau_t}{\sqrt{2\pi}} \exp\left(-\left(\frac{\left(R_t^d\left(l_t - e_t\right) - R_{t+1}^s l_t\right) Q_t + \xi\sigma_t l_t}{\sigma_t l_t \sqrt{2}\tau_t}\right)^2\right) - \frac{\xi}{2}\left[1 - \operatorname{erf}\left(\frac{\left(R_t^d\left(l_t - e_t\right) - R_{t+1}^s l_t\right) Q_t + \xi\sigma_t l_t}{\sigma_t l_t \sqrt{2}\tau_t}\right)\right]$$

Going to the second part, we write $\int\limits_{\left(\frac{R_t^d - R_{t+1}^s}{\sigma_t} - \frac{R_t^d e_t}{\sigma_t l_t}\right)Q_t}^{\infty} \left(\frac{1}{\sqrt{2\pi\tau_t^2}} \exp\left(-\frac{(\varepsilon_{t+1}+\xi)^2}{2\tau_t^2}\right)\right) \mathrm{d}\varepsilon_{t+1}$ in terms of the error function. Again, use the transformation $v = \frac{\varepsilon_{t+1}+\xi}{\sqrt{2}\tau_t}$ or $\varepsilon_{t+1} = v\sqrt{2}\tau_t - \xi$

$$\int\limits_{\frac{\left(R_t^d(l_t - e_t) - R_{t+1}^s l_t\right)Q_t + \xi\sigma_t l_t}{\sigma_t l_t \sqrt{2}\tau_t}}^{\infty} \frac{\sqrt{2}\tau_t}{\sqrt{2\pi\tau_t^2}} \exp\left(-v^2\right) \mathrm{d}v = \frac{1}{\sqrt{\pi}} \int\limits_{\frac{\left(R_t^d(l_t - e_t) - R_{t+1}^s l_t\right)Q_t + \xi\sigma_t l_t}{\sigma_t l_t \sqrt{2}\tau_t}}^{\infty} \exp\left(-v^2\right) \mathrm{d}v =$$
$$\frac{1}{2}\left(1 - \operatorname{erf}\left(\frac{\left(R_t^d\left(l_t - e_t\right) - R_{t+1}^s l_t\right) Q_t + \xi\sigma_t l_t}{\sigma_t l_t \sqrt{2}\tau_t}\right)\right).$$

Therefore,

$$\int\limits_{\left(\frac{R_t^d - R_{t+1}^s}{\sigma_t} - \frac{R_t^d e_t}{\sigma_t l_t}\right)Q_t}^{\infty} \left(\frac{1}{\sqrt{2\pi\tau_t^2}} \exp\left(-\frac{(\varepsilon_{t+1}+\xi)^2}{2\tau_t^2}\right)\right) \mathrm{d}\varepsilon_{t+1} = \tag{A.8}$$
$$\frac{1}{2}\left(1 - \operatorname{erf}\left(\frac{\left(R_t^d\left(l_t - e_t\right) - R_{t+1}^s l_t\right) Q_t + \xi\sigma_t l_t}{\sigma_t l_t \sqrt{2}\tau_t}\right)\right).$$

Combining the results for the two parts and simplifying,

$$\mathbb{E}_t\left[\int\limits_{\left(\frac{R_t^d-R_{t+1}^s}{\sigma_t}-\frac{R_t^d e_t}{\sigma_t l_t}\right)Q_t}^{\infty}\frac{\partial}{\partial l_t}\left(\left(\left(R_{t+1}^s+\sigma_t\frac{\varepsilon_{t+1}}{Q_t}\right)l_t-R_t^d\left(l_t-e_t\right)\right)\frac{1}{\sqrt{2\pi\tau_t^2}}\exp\left(-\frac{(\varepsilon_{t+1}+\xi)^2}{2\tau_t^2}\right)\mathrm{d}\varepsilon_{t+1}\right]=$$

$$\mathbb{E}_t\left[\int\limits_{\left(\frac{R_t^d-R_{t+1}^s}{\sigma_t}-\frac{R_t^d e_t}{\sigma_t l_t}\right)Q_t}^{\infty}\left(R_{t+1}^s+\sigma_t\frac{\varepsilon_{t+1}}{Q_t}-R_t^d\right)\frac{1}{\sqrt{2\pi\tau_t^2}}\exp\left(-\frac{(\varepsilon_{t+1}+\xi)^2}{2\tau_t^2}\right)\mathrm{d}\varepsilon_{t+1}\right]=$$

$$\mathbb{E}_t\left[\frac{\sigma_t}{Q_t}\frac{\tau_t}{\sqrt{2\pi}}\exp\left(-\left(\frac{\left(R_t^d\left(l_t-e_t\right)-R_{t+1}^s l_t\right)Q_t+\xi\sigma_t l_t}{\sigma_t l_t\sqrt{2}\tau_t}\right)^2\right)-\frac{\sigma_t\xi}{2Q_t}\left[1-\operatorname{erf}\left(\frac{\left(R_t^d\left(l_t-e_t\right)-R_{t+1}^s l_t\right)Q_t+\xi\sigma_t l_t}{\sigma_t l_t\sqrt{2}\tau_t}\right)\right]\right]+$$

$$\mathbb{E}_t\left[\left(R_{t+1}^s-R_t^d\right)\frac{1}{2}\left(1-\operatorname{erf}\left(\frac{\left(R_t^d\left(l_t-e_t\right)-R_{t+1}^s l_t\right)Q_t+\xi\sigma_t l_t}{\sigma_t l_t\sqrt{2}\tau_t}\right)\right)\right]=$$

$$\mathbb{E}_t\left[\frac{\sigma_t}{Q_t}\frac{\tau_t}{\sqrt{2\pi}}\exp\left(-\left(\frac{\left(R_t^d\left(l_t-e_t\right)-R_{t+1}^s l_t\right)Q_t+\xi\sigma_t l_t}{\sigma_t l_t\sqrt{2}\tau_t}\right)^2\right)+\right.$$
$$\left.\left(\frac{R_{t+1}^s-\frac{\sigma_t\xi}{Q_t}-R_t^d}{2}\right)\left[1-\operatorname{erf}\left(\frac{\left(R_t^d\left(l_t-e_t\right)-R_{t+1}^s l_t\right)Q_t+\xi\sigma_t l_t}{\sigma_t l_t\sqrt{2}\tau_t}\right)\right]\right].$$

Similarly, work on $\frac{\partial}{\partial e_t}$

$$\int\limits_{\left(\frac{R_t^d-R_{t+1}^s}{\sigma_t}-\frac{R_t^d e_t}{\sigma_t l_t}\right)Q_t}^{\infty}\frac{\partial}{\partial e_t}\left(\left(\left(R_{t+1}^s+\sigma_t\frac{\varepsilon_{t+1}}{Q_t}\right)l_t-R_t^d\left(l_t-e_t\right)\right)\frac{1}{\sqrt{2\pi\tau_t^2}}\exp\left(-\frac{(\varepsilon_{t+1}+\xi)^2}{2\tau_t^2}\right)\mathrm{d}\varepsilon_{t+1}=$$

$$\int\limits_{\left(\frac{R_t^d-R_{t+1}^s}{\sigma_t}-\frac{R_t^d e_t}{\sigma_t l_t}\right)Q_t}^{\infty}R_t^d\frac{1}{\sqrt{2\pi\tau_t^2}}\exp\left(-\frac{(\varepsilon_{t+1}+\xi)^2}{2\tau_t^2}\right)\mathrm{d}\varepsilon_{t+1}=R_t^d\frac{1}{2}\left(1-\operatorname{erf}\left(\frac{R_t^d\left(l_t-e_t\right)-R_{t+1}^s l_t+\xi\sigma_t l_t}{\sigma_t l_t\sqrt{2}\tau_t}\right)\right).$$

In sum, the FOCs can be written as follows:

$$\mathbb{E}_t\left\{\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[\frac{\sigma_t}{Q_t}\frac{\tau_t}{\sqrt{2\pi}}\exp\left(-\left(\frac{\left(R_t^d\left(1-\frac{e_t}{l_t}\right)-R_{t+1}^s\right)Q_t+\xi\sigma_t}{\sigma_t\sqrt{2}\tau_t}\right)^2\right)+\right.\right.$$
$$\left.\left.\left(\frac{R_{t+1}^s-\frac{\sigma_t\xi}{Q_t}-R_t^d}{2}\right)\left[1-\operatorname{erf}\left(\frac{\left(R_t^d\left(1-\frac{e_t}{l_t}\right)-R_{t+1}^s\right)Q_t+\xi\sigma_t}{\sigma_t\sqrt{2}\tau_t}\right)\right]\right]\right\}+\chi_{2t}=\gamma\chi_{1t}, \tag{A.9}$$

$$\mathbb{E}_t\left\{\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[R_t^d\frac{1}{2}\left(1-\text{erf}\left(\frac{\left(R_t^d\left(1-\frac{e_t}{l_t}\right)-R_{t+1}^s\right)Q_t+\xi\sigma_t}{\sigma_t\sqrt{2}\tau_t}\right)\right)\right]\right\}-1+\chi_{1t}=0. \quad \text{(A.10)}$$

Finally, the complementary slackness conditions can be expressed as

$$(e_t-\gamma l_t)\,\chi_{1t} = 0, \quad \text{(A.11)}$$

$$l_t\chi_{2t} = 0. \quad \text{(A.12)}$$

A.2.2 Proof of Proposition 1

In equilibrium, capital requirements always bind; that is,

$$e_t=\gamma_t l_t.$$

We will show that the Lagrange multiplier associated with this constraint is always positive, hence the constraint must bind. To this end, we also employ the household problem's first-order conditions (FOCs) with respect to equity, described in Appendix A.1.

Equations (A.3) and (A.4) can be expressed as

$$\beta\mathbb{E}_t\left\{\frac{\lambda_{ct+1}}{\lambda_{ct}}R_{t+1}^{e,i}\right\}=1-\frac{\zeta_t^i}{\lambda_{ct}},$$

where $i\in\{s,r\}$ denotes the type of equity. In this expression, substitute equation (A.10) for 1. Therefore,

$$\mathbb{E}_t\left\{\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[R_t^d\frac{1}{2}\left(1-\text{erf}\left(\frac{\left(R_t^d\left(1-\frac{e_t^i}{l_t^i}\right)-R_{t+1}^s\right)Q_t+\xi\sigma_t^i}{\sigma_t^i\sqrt{2}\tau_t}\right)\right)\right]-R_{t+1}^{e,i}\right\}-\frac{\zeta_t^i}{\lambda_{ct}}+\chi_{1t}^i=0. \quad \text{(A.13)}$$

Since the range of the erf function is between -1 and 1, i.e.$-1\leq \text{erf}(x)\leq 1$, we know that the following expression is between Ψ_1^{i*} and Ψ_2^{i*}:

$$\Psi_1^{i*}\leq\mathbb{E}_t\left\{\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[R_t^d\frac{1}{2}\left(1-\text{erf}\left(\frac{\left(R_t^d\left(1-\frac{e_t^i}{l_t^i}\right)-R_{t+1}^s\right)Q_t+\xi\sigma_t^i}{\sigma_t^i\sqrt{2}\tau_t}\right)\right)-R_{t+1}^{e,i}\right]\right\}\leq\Psi_2^{i*},$$

where

$$\Psi_1^{i*} = \mathbb{E}_t \left\{ \beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \left[0 - R_{t+1}^{e,i} \right] \right\},$$
$$\Psi_2^{i*} = \mathbb{E}_t \left\{ \beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \left[R_t^d - R_{t+1}^{e,i} \right] \right\}.$$

Plugging λ_{ct} from equations (A.3) and (A.4) into equation (A.2), we get:

$$\mathbb{E}_t \left\{ \beta \lambda_{ct+1} \left[R_t^d - R_{t+1}^{e,i} \right] \right\} = -\varsigma_0 D_t^{-\varsigma_d} + \zeta_t^i.$$

Note that $\varsigma_0 D_t^{-\varsigma_d} > 0$ under the usual (and mild) assumptions on the preferences for liquidity. Moreover, the Lagrangian multiplier on the households budget constraint, λ_{ct}, is positive. It reflects the fact that the budget constraint always binds given the standard assumptions on the preferences (the Inada conditions). The latest expression is transformed into the following after dividing it by λ_{ct}:

$$\underbrace{\mathbb{E}_t \left\{ \beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \left[R_t^d - R_{t+1}^{e,i} \right] \right\}}_{=\Psi_2^{i*}} - \frac{\zeta_t^i}{\lambda_{ct}} = -\frac{\varsigma_0 D_t^{-\varsigma_d}}{\lambda_{ct}} < 0.$$

Thus, $\Psi_2^{i*} < \frac{\zeta_t^i}{\lambda_{ct}}$.

Rewriting Equation (A.13)

$$\mathbb{E}_t \left\{ \beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \left[R_t^d \frac{1}{2} \left(1 - \operatorname{erf} \left(\frac{\left(R_t^d \left(1 - \frac{e_t^i}{l_t^i} \right) - R_{t+1}^s \right) Q_t + \xi \sigma_t^i}{\sigma_t^i \sqrt{2} \tau_t} \right) \right) \right] - R_{t+1}^{e,i} \right\} = \frac{\zeta_t^i}{\lambda_{ct}} - \chi_{1t}^i,$$

combine it with $\Psi_2^{i*} < \frac{\zeta_t^i}{\lambda_{ct}}$ to find

$$\frac{\zeta_t^i}{\lambda_{ct}} - \chi_{1t} < \Psi_2^{i*} < \frac{\zeta_t^i}{\lambda_{ct}}.$$

Hence, $\chi_{1t}^i > 0$ for each $i \in \{s, r\}$. $\square$

A.2.3 Combined First-Order Conditions

$$\mathbb{E}_t \left\{ \beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \left[\frac{\sigma_t}{Q_t} \frac{\tau_t}{\sqrt{2\pi}} \exp \left(- \left(\frac{\left(R_t^d \left(1 - \frac{e_t}{l_t}\right) - R_{t+1}^s \right) Q_t + \xi \sigma_t}{\sigma_t \sqrt{2} \tau_t} \right)^2 \right) + \right.\right.$$

$$\left.\left. \left(\frac{R_{t+1}^s - \frac{\sigma_t \xi}{Q_t} - R_t^d}{2} \right) \left[1 - \mathrm{erf} \left(\frac{\left(R_t^d \left(1 - \frac{e_t}{l_t}\right) - R_{t+1}^s \right) Q_t + \xi \sigma_t}{\sigma_t \sqrt{2} \tau_t} \right) \right] \right] \right\} + \chi_{2t} = \gamma \chi_{1t},$$

$$\mathbb{E}_t \left\{ \beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \left[R_t^d \frac{1}{2} \left(1 - \mathrm{erf} \left(\frac{\left(R_t^d \left(1 - \frac{e_t}{l_t}\right) - R_{t+1}^s \right) Q_t + \xi \sigma_t}{\sigma_t \sqrt{2} \tau_t} \right) \right) \right] \right\} - 1 + \chi_{1t} = 0.$$

Since $\chi_{1t} > 0$, multiply the second equation by γ_t and add it to the first equation using $\frac{e_t}{l_t} = \gamma_t$. Therefore, the FOCs can be combined into:

$$\begin{aligned} &\mathbb{E}_t \left\{ \beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \left[\frac{\sigma_t}{Q_t} \frac{\tau_t}{\sqrt{2\pi}} \exp \left(- \left(\frac{\left(R_t^d (1 - \gamma_t) - R_{t+1}^s \right) Q_t + \xi \sigma_t}{\sigma_t \sqrt{2} \tau_t} \right)^2 \right) + \right.\right. \\ &\left.\left. \frac{1}{2} \left(R_{t+1}^s - \frac{\sigma_t \xi}{Q_t} - R_t^d (1 - \gamma_t) \right) \left[1 - \mathrm{erf} \left(\frac{\left(R_t^d (1 - \gamma_t) - R_{t+1}^s \right) Q_t + \xi \sigma_t}{\sigma_t \sqrt{2} \tau_t} \right) \right] \right] \right\} = \gamma_t - \chi_{2t}, \end{aligned} \tag{A.14}$$

$$\chi_{2t} l_t = 0. \tag{A.15}$$

A.2.4 Zero-Profit Condition

In step 1, we derive the expression of the zero-profit condition under all states of nature. In step 2, we show that this zero-profit condition implies the FOCs derived in Appendix A.2.3.

Step 1: Since there is no agency problem between banks and households, this condition captures the fact that all the profits (or losses) are distributed to equity holders after realization of shocks at the beginning of each period. In each aggregate state, banks whose investments in risky firms pan out will have returns that satisfy on average (over the realizations of the idiosyncratic shock) $\left[\left(R_{t+1}^s + \frac{\sigma_t}{Q_t} \right) l_t - R_t^d (l_t - e_t) \right] - \int R_{t+1,b}^e(b) \cdot e_t = 0$, where the bounds of the integral are chosen such that we integrate over banks for which the profit is non-negative, while banks whose risky investments earn low (negative) returns will have

$R^e_{t+1,b} = 0$. Therefore,

$$R^e_{t+1} = \int\limits_{\left(\frac{R^d_t(1-\gamma_t)-R^s_{t+1}}{\sigma_t}\right)Q_t}^{\infty} \frac{\left(\left(R^s_{t+1} + \sigma_t \frac{\varepsilon_{t+1}}{Q_t}\right) l_t - R^d_t d_t\right) \frac{1}{\sqrt{2\pi\tau_t^2}} \exp\left(-\frac{(\varepsilon_{t+1}+\xi)^2}{2\tau_t^2}\right) \mathrm{d}\varepsilon_{t+1}}{e_t} +$$

$$\int\limits_{-\infty}^{\left(\frac{R^d_t(1-\gamma_t)-R^s_{t+1}}{\sigma_t}\right)Q_t} 0 \cdot \frac{1}{\sqrt{2\pi\tau_\mathrm{t}^2}} \exp\left(-\frac{(\varepsilon_{\mathrm{t+1}}+\xi)^2}{2\tau_\mathrm{t}^2}\right) \mathrm{d}\varepsilon_{\mathrm{t+1}} =$$

$$\frac{1}{e_t} \int\limits_{\left(\frac{R^d_t(1-\gamma_t)-R^s_{t+1}}{\sigma_t}\right)Q_t}^{\infty} \left(R^s_{t+1} l_t - R^d_t d_t\right) \frac{1}{\sqrt{2\pi\tau_t^2}} \exp\left(-\frac{(\varepsilon_{t+1}+\xi)^2}{2\tau_t^2}\right) \mathrm{d}\varepsilon_{t+1} +$$
$$\frac{1}{e_t} \int\limits_{\left(\frac{R^d_t(1-\gamma_t)-R^s_{t+1}}{\sigma_t}\right)Q_t}^{\infty} \sigma_t \frac{\varepsilon_{t+1}}{Q_t} l_t \frac{1}{\sqrt{2\pi\tau_t^2}} \exp\left(-\frac{(\varepsilon_{t+1}+\xi)^2}{2\tau_t^2}\right) \mathrm{d}\varepsilon_{t+1} =$$

$$\frac{1}{e_t}\left[\left(R^s_{t+1} l_t - R^d_t d_t\right) \frac{1}{2}\left(1 - \operatorname{erf}\left(\frac{\left(R^d_t(1-\gamma_t)-R^s_{t+1}\right)Q_t+\xi\sigma_t}{\sigma_t\sqrt{2}\tau_t}\right)\right) + \right.$$
$$\left. \frac{\sigma_t l_t}{Q_t}\left(\frac{\tau_t}{\sqrt{2\pi}} \exp\left(-\left(\frac{\left(R^d_t(1-\gamma_t)-R^s_{t+1}\right)Q_t+\xi\sigma_t}{\sigma_t\sqrt{2}\tau_t}\right)^2\right) - \frac{\xi}{2}\left[1 - \operatorname{erf}\left(\frac{\left(R^d_t(1-\gamma_t)-R^s_{t+1}\right)Q_t+\xi\sigma_t}{\sigma_t\sqrt{2}\tau_t}\right)\right]\right)\right] =$$

$$\frac{l_t}{e_t}\left\{\frac{\sigma_t}{Q_t}\frac{\tau_t}{\sqrt{2\pi}} \exp\left(-\left(\frac{\left(R^d_t(1-\gamma_t)-R^s_{t+1}\right)Q_t+\xi\sigma_t}{\sigma_t\sqrt{2}\tau_t}\right)^2\right) + \right.$$
$$\left. \frac{1}{2}\left(R^s_{t+1} - \frac{\sigma_t\xi}{Q_t} - R^d_t(1-\gamma_t)\right)\left[1 - \operatorname{erf}\left(\frac{\left(R^d_t(1-\gamma_t)-R^s_{t+1}\right)Q_t+\xi\sigma_t}{\sigma_t\sqrt{2}\tau_t}\right)\right]\right\}.$$

Since $\frac{l_t}{e_t} = \frac{1}{\gamma_t}$, we can rewrite the latter condition as (using that it holds for each $i \in \{s, r\}$):

$$R^{e,i}_{t+1} = \frac{1}{\gamma_t}\frac{\sigma^i_t}{Q_t}\frac{\tau_t}{\sqrt{2\pi}} \exp\left(-\left(\frac{\left(R^d_t(1-\gamma_t)-R^s_{t+1}\right)Q_t+\xi\sigma^i_t}{\sigma^i_t\sqrt{2}\tau_t}\right)^2\right) +$$
$$\frac{1}{\gamma_t}\frac{1}{2}\left(R^s_{t+1} - \frac{\sigma^i_t\xi}{Q_t} - R^d_t(1-\gamma_t)\right)\left[1 - \operatorname{erf}\left(\frac{\left(R^d_t(1-\gamma_t)-R^s_{t+1}\right)Q_t+\xi\sigma^i_t}{\sigma^i_t\sqrt{2}\tau_t}\right)\right]. \tag{A.16}$$

Step 2: Note that the combined FOC from Appendix A.2.3 can be expressed as:

$$\begin{aligned}
&\mathbb{E}_t\left\{\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[\frac{\sigma_t^i}{Q_t}\frac{\tau_t}{\sqrt{2\pi}}\exp\left(-\left(\frac{\left(R_t^d\left(1-\gamma_t\right)-R_{t+1}^s\right)Q_t+\xi\sigma_t^i}{\sigma_t^i\sqrt{2}\tau_t}\right)^2\right)+\right.\right.\\
&\left.\left.\frac{1}{2}\left(R_{t+1}^s-\frac{\sigma_t^i\xi}{Q_t}-R_t^d(1-\gamma_t)\right)\left[1-\operatorname{erf}\left(\frac{\left(R_t^d\left(1-\gamma_t\right)-R_{t+1}^s\right)Q_t+\xi\sigma_t^i}{\sigma_t^i\sqrt{2}\tau_t}\right)\right]\right]\right\}= \qquad (A.17)\\
&\gamma_t-\chi_{2t}^i=\gamma_t\left(\mathbb{E}_t\left\{\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}R_{t+1}^{e,i}\right\}+\frac{\zeta_t^i}{\lambda_{ct}}\right)-\chi_{2t}^i,
\end{aligned}$$

where we substitute for 1 from Household's FOC with respect to the two types of equity, i.e.,

$$\beta\mathbb{E}_t\left\{\frac{\lambda_{ct+1}}{\lambda_{ct}}R_{t+1}^{e,i}\right\}=1-\frac{\zeta_t^i}{\lambda_{ct}}$$

for each $i\in\{s,r\}$.

When $l_t^i>0$, the complementary slackness conditions in equations (A.5), (A.6), and (A.15) imply both $\zeta_t^i=0$ and $\chi_{2t}^i=0$. Plugging these results into equation (A.17):

$$\begin{aligned}
&\mathbb{E}_t\left\{\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}R_{t+1}^{e,i}\right\}=\frac{1}{\gamma_t}\mathbb{E}_t\left\{\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[\frac{\sigma_t^i}{Q_t}\frac{\tau_t}{\sqrt{2\pi}}\exp\left(-\left(\frac{\left(R_t^d\left(1-\gamma_t\right)-R_{t+1}^s\right)Q_t+\xi\sigma_t^i}{\sigma_t^i\sqrt{2}\tau_t}\right)^2\right)+\right.\right. \qquad (A.18)\\
&\left.\left.\frac{1}{2}\left(R_{t+1}^s-\frac{\sigma_t^i\xi}{Q_t}-R_t^d(1-\gamma_t)\right)\left[1-\operatorname{erf}\left(\frac{\left(R_t^d\left(1-\gamma_t\right)-R_{t+1}^s\right)Q_t+\xi\sigma_t^i}{\sigma_t^i\sqrt{2}\tau_t}\right)\right]\right]\right\}.
\end{aligned}$$

Notice that equation (A.18) differs from the zero-profit condition derived in Step 1 above by only the stochastic discount factor attached to both sides of the equation. Therefore, the zero-profit condition implies the FOC.

A.2.5 Expression of Expected Dividends

Expected dividends (valued on date t) are defined as

$$\begin{aligned}
&\mathit{\Omega}_t\left(\sigma_t;\, l_t,\, d_t,\, e_t\right)=-e_t+\\
&\mathbb{E}_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\int\limits_{\left(\frac{R_t^d(l_t-e_t)}{\sigma_t l_t}-\frac{R_{t+1}^s}{\sigma_t}\right)Q_t}^{\infty}\left(\left(R_{t+1}^s+\sigma_t\frac{\varepsilon_{t+1}}{Q_t}\right)l_t-R_t^d\left(l_t-e_t\right)\right)\frac{1}{\sqrt{2\pi\tau_t^2}}\exp\left(-\frac{\left(\varepsilon_{t+1}+\xi\right)^2}{2\tau_t^2}\right)\mathrm{d}\varepsilon_{t+1}\right]
\end{aligned}$$

In Appendix A.2.1, we calculated all the necessary ingredients for the above integral. Multiply equation (A.7) by $\frac{\sigma_t l_t}{Q_t}$ and make a sum with equation (A.8) multiplied by $R_{t+1}^s l_t-$

$R_t^d(l_t - e_t)$. Use that the capital requirement always binds, i.e., $e_t = \gamma_t l_t$ Therefore,

$$\begin{aligned} &\Omega\left(\sigma_t;\, l_t,\, d_t,\, e_t\right) = -l_t\gamma_t + \\ &l_t\mathbb{E}_t\left\{\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[\frac{\sigma_t}{Q_t}\frac{\tau_t}{\sqrt{2\pi}}\exp\left(-\left(\frac{\left(R_t^d\left(1-\gamma_t\right)-R_{t+1}^s\right)Q_t+\xi\sigma_t}{\sigma_t\sqrt{2}\tau_t}\right)^2\right)+\right.\right. \\ &\left.\left.\frac{\left(R_{t+1}^s-R_t^d\left(1-\gamma_t\right)-\frac{\sigma_t\xi}{Q_t}\right)}{2}\left[1-\mathrm{erf}\left(\frac{\left(R_t^d\left(1-\gamma_t\right)-R_{t+1}^s\right)Q_t+\xi\sigma_t}{\sigma_t\sqrt{2}\tau_t}\right)\right]\right]\right\}. \end{aligned} \tag{A.19}$$

We define ε_{t+1}^* as the realization of the idiosyncratic shock below which the bank's net worth is negative; that is, $\left(R_{t+1}^s + \sigma_t\frac{\varepsilon_{t+1}^*}{Q_t}\right)l_t - R_t^d d_t = 0$. Expressing

$$\varepsilon_{t+1}^* = -\frac{Q_t}{\sigma_t}\left[R_{t+1}^s - R_t^d\left(1-\gamma_t\right)\right]$$

and plugging it into the cdf of the Normal distribution:

$$G(\varepsilon_{t+1}^*) = \frac{1}{2}\left[1+\mathrm{erf}\left(\frac{\varepsilon_{t+1}^*+\xi}{\tau_t\sqrt{2}}\right)\right] = \frac{1}{2}\left[1+\mathrm{erf}\left(\frac{-\frac{Q_t}{\sigma_t}\left[R_{t+1}^s-R_t^d\left(1-\gamma_t\right)\right]+\xi}{\tau_t\sqrt{2}}\right)\right].$$

Therefore, $1 - G(\varepsilon_{t+1}^*) =$

$$1-\frac{1}{2}\left[1+\mathrm{erf}\left(\frac{-\frac{Q_t}{\sigma_t}\left[R_{t+1}^s-R_t^d\left(1-\gamma_t\right)\right]+\xi}{\tau_t\sqrt{2}}\right)\right] = \frac{1}{2}-\frac{1}{2}\mathrm{erf}\left(\frac{\left(R_t^d\left(1-\gamma_t\right)-R_{t+1}^s\right)Q_t+\xi\sigma_t}{\sigma_t\sqrt{2}\tau_t}\right).$$

Using these results, we can re-write equation (A.19) as follows:

$$\begin{aligned} \Omega\left(\sigma_t;\, l_t,\, d_t,\, e_t\right) = -l_t\gamma_t + l_t\mathbb{E}_t&\left\{\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[\left(R_{t+1}^s-R_t^d\left(1-\gamma_t\right)-\frac{\xi\sigma_t}{Q_t}\right)\left(1-G(\varepsilon_{t+1}^*)\right)+\right.\right. \\ &\left.\left.\left(\frac{\sigma_t}{Q_t}\right)\frac{\tau_t}{\sqrt{2\pi}}\exp\left(-\left(\frac{\varepsilon_{t+1}^*+\xi}{\tau_t\sqrt{2}}\right)^2\right)\right]\right\}. \end{aligned} \tag{A.20}$$

By denoting

$$\begin{aligned} \omega_1 &\equiv \left(R_{t+1}^s-R_t^d\left(1-\gamma_t\right)-\frac{\xi\sigma_t}{Q_t}\right)\left(1-G(\varepsilon_{t+1}^*)\right), \\ \omega_2 &\equiv \left(\frac{\sigma_t}{Q_t}\right)\frac{\tau_t}{\sqrt{2\pi}}\exp\left(-\left(\frac{\varepsilon_{t+1}^*+\xi}{\tau_t\sqrt{2}}\right)^2\right), \end{aligned}$$

Equation (A.20) can be described as

$$\Omega(\sigma_t;\, l_t,\, d_t,\, e_t) = \left(\mathbb{E}_t\left[\psi_{t,t+1}\left(\omega_1 + \omega_2\right)\right] - \gamma_t\right) l_t.$$

A.2.6 Choice of Risk

In this section of the appendix, we prove that the expected dividend function of banks is convex in the risk parameter σ_t. This result guarantees that banks choose either the maximum risk, $\overline{\sigma}$, or the minimum risk, $\underline{\sigma}$, to maximize expected shareholder value, net of initial equity investment. So all the intermediate values of σ_t, which may result from the first-order conditions with respect to σ_t, are not optimal.

The proof provided here generalizes the proof of Van den Heuvel (2008) to the presence of aggregate uncertainty. Our proof applies to an arbitrary distribution of the idiosyncratic shock, ε_{t+1}, with non-positive mean, so our example of a Normal distribution considered in the analysis is not a special case that can drive our results.

Assumption. *ε has a cumulative distribution function G_ε with support $[\underline{\varepsilon},\, \overline{\varepsilon}]$, with $\underline{\varepsilon} < 0 < \overline{\varepsilon}$. The mean of ε is equal to $-\xi$ ($\xi > 0$). ε is independent of the aggregate shock. The aggregate shock does not depend on the choice of σ_t.*

Note that we do not restrict the analysis to the bounded support[2], so $\underline{\varepsilon}$ and $\overline{\varepsilon}$ can take $-\infty$ and $+\infty$, respectively. Note that G_ε need not be continuous.

Let $\hat{\varepsilon}(\sigma_t,\, R^s_{t+1}) \equiv \left(\frac{R^d_t d_t}{\sigma_t l_t} - \frac{R^s_{t+1}}{\sigma_t}\right) Q_t = \frac{R^d_t(1-\gamma_t) - R^s_{t+1}}{\sigma_t} Q_t$, where the latter equation uses the result that the capital requirement constraint always binds. It denotes the realization of the idiosyncratic shock below which the bank's net worth is negative. Let $\pi(\sigma_t,\, R^s_{t+1}) = E_\varepsilon\left[\left(\left(R^s_{t+1} + \frac{\sigma_t\varepsilon}{Q_t}\right) l_t - R^d_t d_t\right)^+\right]$ be a function of expected net worth (taken over the idiosyncratic shock only) under some realization of R^s_{t+1} which is considered to be fixed in this function. To account for the aggregate uncertainty, R^s_{t+1} needs to be a random variable. Therefore, expected net worth taken into account both idiosyncratic and aggregate uncertainty is

$$\Pi(\sigma_t) = \int_\Omega \psi_{t,t+1}\pi\left(\sigma_t,\, R^s_{t+1}(\omega)\right) P(d\omega) = \mathbb{E}_t\left[\psi_{t,t+1}\int_{\hat{\varepsilon}(\sigma_t,\, R^s_{t+1})}^{\overline{\varepsilon}} \left(\left(R^s_{t+1} + \frac{\sigma_t\varepsilon}{Q_t}\right) l_t - R^d_t d_t\right) dG_\varepsilon\right] =$$

$$\mathbb{E}_t\left[\psi_{t,t+1}\int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \left(\left(R^s_{t+1} + \frac{\sigma_t\varepsilon}{Q_t}\right) l_t - R^d_t d_t\right) dG_\varepsilon\right] - \mathbb{E}_t\left[\psi_{t,t+1}\int_{\underline{\varepsilon}}^{\hat{\varepsilon}(\sigma_t,\, R^s_{t+1})} \left(\left(R^s_{t+1} + \frac{\sigma_t\varepsilon}{Q_t}\right) l_t - R^d_t d_t\right) dG_\varepsilon\right] =$$

[2] Unbounded support is more relevant if we consider aggregate risk.

$$\mathbb{E}_t\left[\psi_{t,t+1}\left(R^s_{t+1}l_t - R^d_t d_t - \frac{\sigma_t \xi}{Q_t} l_t\right)\right] - \frac{\sigma_t l_t}{Q_t}\mathbb{E}_t\left[\psi_{t,t+1}\int\limits_{\underline{\varepsilon}}^{\hat{\varepsilon}(\sigma_t,\, R^s_{t+1})}\left(\varepsilon - \hat{\varepsilon}(\sigma_t,\, R^s_{t+1})\right) dG_\varepsilon\right] =$$

$$\mathbb{E}_t\left[\psi_{t,t+1}\left(R^s_{t+1}l_t - R^d_t d_t\right)\right] + \frac{l_t}{Q_t}\left(\sigma_t\mathbb{E}_t\left[\psi_{t,t+1}\int\limits_{\underline{\varepsilon}}^{\hat{\varepsilon}(\sigma_t,\, R^s_{t+1})}\left(\hat{\varepsilon}(\sigma_t,\, R^s_{t+1}) - \varepsilon\right) dG_\varepsilon\right] - \sigma_t \xi \mathbb{E}_t \psi_{t,t+1}\right).$$

Note that in the derivations above we express $\left(R^s_{t+1} + \frac{\sigma_t \varepsilon}{Q_t}\right) l_t - R^d_t d_t$ in terms of $\hat{\varepsilon}(\sigma_t,\, R^s_{t+1})$ and ε using the definition of $\hat{\varepsilon}(\sigma_t,\, R^s_{t+1})$.

The proof below shows that $\Pi(\sigma_t)$ is convex in σ_t. Note that the expression of $\Pi(\sigma_t)$ involves the term which is linear in σ_t and $\frac{l_t}{Q_t} \geq 0$. Moreover, $\psi_{t,t+1} > 0$. Therefore, the sufficient condition for $\Pi(\sigma_t)$ to be convex in σ_t is that

$$H(\sigma_t) \equiv \mathbb{E}_t\left[\int_{\underline{\varepsilon}}^{\hat{\varepsilon}(\sigma_t)} \left(\hat{\varepsilon}(\sigma_t) - \varepsilon\right) dG_\varepsilon\right]\sigma_t$$

is convex in σ_t.

Claim. $H(\sigma_t) \equiv \mathbb{E}_t\left[\int_{\underline{\varepsilon}}^{\hat{\varepsilon}(\sigma_t)}\left(\hat{\varepsilon}(\sigma_t,\, R^s_{t+1}) - \varepsilon\right) dG_\varepsilon\right]\sigma_t$ *is convex in* σ_t*:*

Proof. Steps of the proof: □

1. Define $h(\sigma_t,\, R^s_{t+1}) \equiv \sigma_t\left[\int_{\underline{\varepsilon}}^{\hat{\varepsilon}(\sigma_t,\, R^s_{t+1})}\left(\hat{\varepsilon}(\sigma_t,\, R^s_{t+1}) - \varepsilon\right) dG_\varepsilon\right]$ in which the aggregate uncertainty is taken off. Consider 3 cases:

   (a) The realization of R^s_{t+1} is such that $\hat{\varepsilon}(\sigma_t,\, R^s_{t+1}) = \frac{R^d_t(1-\gamma_t) - R^s_{t+1}}{\sigma_t} > 0$, so $R^s_{t+1} < R^d_t\left(1-\gamma_t\right),$

   (b) The realization of R^s_{t+1} is such that $\hat{\varepsilon}(\sigma_t,\, R^s_{t+1}) = \frac{R^d_t(1-\gamma_t) - R^s_{t+1}}{\sigma_t} < 0$, so $R^s_{t+1} > R^d_t\left(1-\gamma_t\right),$

   (c) The realization of R^s_{t+1} is such that $\hat{\varepsilon}(\sigma_t,\, R^s_{t+1}) = \frac{R^d_t(1-\gamma_t) - R^s_{t+1}}{\sigma_t} = 0$, so $R^s_{t+1} = R^d_t\left(1-\gamma_t\right),$

   Show that $h(\sigma_t,\, R^s_{t+1})$ is convex in σ_t in cases 1a and 1b and $h(\sigma_t,\, R^s_{t+1})$ is linear in σ_t in case 1c.

2. Employ the argument that convexity is preserved under non-negative scaling and addition (guaranteed by the expectation operator over the aggregate uncertainty) to find that $H(\sigma_t)$ is convex.

Let's show each step of the proof formally:

1. Let $\sigma_{1t} < \sigma_{2t}$ and, for $\lambda \in (0,\, 1)$, define $\sigma_{\lambda t} = \lambda\sigma_{1t} + (1-\lambda)\sigma_{2t}$. Let $\hat{\varepsilon}_i = \hat{\varepsilon}(\sigma_{it},\, R^s_{t+1}) \equiv \frac{R^d_t(1-\gamma_t)-R^s_{t+1}}{\sigma_{it}}Q_t$, for $i = 1,\, 2,\, \lambda$. Therefore,

$$\hat{\varepsilon}_1\sigma_{1t} = \hat{\varepsilon}_2\sigma_{2t} = \hat{\varepsilon}_\lambda(\lambda\sigma_{1t} + (1-\lambda)\sigma_{2t}) \tag{A.21}$$

Using the definition of the convex function, we need to show that

$$h(\sigma_{\lambda t}) \leq \lambda h(\sigma_{1t}) + (1-\lambda)h(\sigma_{2t}).$$

(a) $R^s_{t+1} < R^d_t(1-\gamma_t)$: it implies that $\hat{\varepsilon}_2 < \hat{\varepsilon}_\lambda < \hat{\varepsilon}_1$,

$$h(\sigma_{\lambda t}) = (\lambda\sigma_{1t} + (1-\lambda)\sigma_{2t})\left\{\int_{\underline{\varepsilon}}^{\hat{\varepsilon}(\sigma_{\lambda t})}\left(\hat{\varepsilon}(\sigma_{\lambda t}) - \varepsilon\right)dG_\varepsilon\right\} =$$

$$\begin{aligned}
\lambda\sigma_{1t}&\left\{\int_{\underline{\varepsilon}}^{\hat{\varepsilon}_1}\left(\hat{\varepsilon}_\lambda - \varepsilon\right)dG_\varepsilon - \int_{\hat{\varepsilon}_\lambda}^{\hat{\varepsilon}_1}\left(\hat{\varepsilon}_\lambda - \varepsilon\right)dG_\varepsilon\right\} + \\
&(1-\lambda)\sigma_{2t}\left\{\int_{\underline{\varepsilon}}^{\hat{\varepsilon}_2}\left(\hat{\varepsilon}_\lambda - \varepsilon\right)dG_\varepsilon + \int_{\hat{\varepsilon}_2}^{\hat{\varepsilon}_\lambda}\left(\hat{\varepsilon}_\lambda - \varepsilon\right)dG_\varepsilon\right\} = \\
&\lambda\sigma_{1t}\left\{\int_{\underline{\varepsilon}}^{\hat{\varepsilon}_1}\left(\hat{\varepsilon}_1 - \varepsilon\right)dG_\varepsilon + \left(\hat{\varepsilon}_\lambda - \hat{\varepsilon}_1\right)G_\varepsilon(\hat{\varepsilon}_1) + \int_{\hat{\varepsilon}_\lambda}^{\hat{\varepsilon}_1}\left(\varepsilon - \hat{\varepsilon}_\lambda\right)dG_\varepsilon\right\} + \\
&(1-\lambda)\sigma_{2t}\left\{\int_{\underline{\varepsilon}}^{\hat{\varepsilon}_2}\left(\hat{\varepsilon}_2 - \varepsilon\right)dG_\varepsilon + \left(\hat{\varepsilon}_\lambda - \hat{\varepsilon}_2\right)G_\varepsilon(\hat{\varepsilon}_2) + \int_{\hat{\varepsilon}_2}^{\hat{\varepsilon}_\lambda}\left(\hat{\varepsilon}_\lambda - \varepsilon\right)dG_\varepsilon\right\} \leq \\
&\lambda\sigma_{1t}\left\{\int_{\underline{\varepsilon}}^{\hat{\varepsilon}_1}\left(\hat{\varepsilon}_1 - \varepsilon\right)dG_\varepsilon + \left(\hat{\varepsilon}_\lambda - \hat{\varepsilon}_1\right)G_\varepsilon(\hat{\varepsilon}_1) + \int_{\hat{\varepsilon}_\lambda}^{\hat{\varepsilon}_1}\left(\hat{\varepsilon}_1 - \hat{\varepsilon}_\lambda\right)dG_\varepsilon\right\} + \\
&(1-\lambda)\sigma_{2t}\left\{\int_{\underline{\varepsilon}}^{\hat{\varepsilon}_2}\left(\hat{\varepsilon}_2 - \varepsilon\right)dG_\varepsilon + \left(\hat{\varepsilon}_\lambda - \hat{\varepsilon}_2\right)G_\varepsilon(\hat{\varepsilon}_2) + \int_{\hat{\varepsilon}_2}^{\hat{\varepsilon}_\lambda}\left(\hat{\varepsilon}_\lambda - \hat{\varepsilon}_2\right)dG_\varepsilon\right\},
\end{aligned}$$

where the inequality sign comes from $\int_{\hat{\varepsilon}_\lambda}^{\hat{\varepsilon}_1}\left(\varepsilon - \hat{\varepsilon}_\lambda\right)dG_\varepsilon \leq \int_{\hat{\varepsilon}_\lambda}^{\hat{\varepsilon}_1}\left(\hat{\varepsilon}_1 - \hat{\varepsilon}_\lambda\right)dG_\varepsilon$ and $\int_{\hat{\varepsilon}_2}^{\hat{\varepsilon}_\lambda}\left(\hat{\varepsilon}_\lambda - \varepsilon\right)dG_\varepsilon \leq \int_{\hat{\varepsilon}_2}^{\hat{\varepsilon}_\lambda}\left(\hat{\varepsilon}_\lambda - \hat{\varepsilon}_2\right)dG_\varepsilon$. Substituting for the definitions of $h(\sigma_{1t}) = \sigma_{1t}\int_{\underline{\varepsilon}}^{\hat{\varepsilon}_1}\left(\hat{\varepsilon}_1 - \varepsilon\right)dG_\varepsilon$ and $h(\sigma_{2t}) = \sigma_{2t}\int_{\underline{\varepsilon}}^{\hat{\varepsilon}_2}\left(\hat{\varepsilon}_2 - \varepsilon\right)dG_\varepsilon$, we get:

$$\begin{aligned}
&h(\sigma_{\lambda t}) \leq \lambda h(\sigma_{1t}) + (1-\lambda)h(\sigma_{2t}) + \lambda\sigma_{1t}\left\{\left(\hat{\varepsilon}_\lambda - \hat{\varepsilon}_1\right)G_\varepsilon(\hat{\varepsilon}_\lambda)\right\} + (1-\lambda)\sigma_{2t}\left\{\left(\hat{\varepsilon}_\lambda - \hat{\varepsilon}_2\right)G_\varepsilon(\hat{\varepsilon}_\lambda)\right\} = \\
&\lambda h(\sigma_{1t}) + (1-\lambda)h(\sigma_{2t}) + G_\varepsilon(\hat{\varepsilon}_\lambda)\left(\lambda\sigma_{1t}\left(\hat{\varepsilon}_\lambda - \hat{\varepsilon}_1\right) + (1-\lambda)\sigma_{2t}\left(\hat{\varepsilon}_\lambda - \hat{\varepsilon}_2\right)\right) = \lambda h(\sigma_{1t}) + (1-\lambda)h(\sigma_{2t}),
\end{aligned}$$

where for the last equality we use equation (A.21) to show that

$$\lambda\sigma_{1t}\left(\hat{\varepsilon}_\lambda - \hat{\varepsilon}_1\right) + (1-\lambda)\sigma_{2t}\left(\hat{\varepsilon}_\lambda - \hat{\varepsilon}_2\right) = \hat{\varepsilon}_\lambda\left(\lambda\sigma_{1t} + (1-\lambda)\sigma_{2t}\right) - \lambda\sigma_{1t}\hat{\varepsilon}_1 - (1-\lambda)\sigma_{2t}\hat{\varepsilon}_2 = 0.$$

(b) $R^s_{t+1} > R^d_t(1-\gamma_t)$: it implies that $\hat{\varepsilon}_1 < \hat{\varepsilon}_\lambda < \hat{\varepsilon}_2$

$$
\begin{aligned}
h(\sigma_{\lambda t}) = (\lambda\sigma_{1t} + (1-\lambda)\sigma_{2t}) \left\{ \int_{\underline{\varepsilon}}^{\hat{\varepsilon}(\sigma_{\lambda t})} (\hat{\varepsilon}(\sigma_{\lambda t}) - \varepsilon)\, dG_\varepsilon \right\} = \\
\lambda\sigma_{1t} \left\{ \int_{\underline{\varepsilon}}^{\hat{\varepsilon}_1} (\hat{\varepsilon}_\lambda - \varepsilon)\, dG_\varepsilon + \int_{\hat{\varepsilon}_1}^{\hat{\varepsilon}_\lambda} (\hat{\varepsilon}_\lambda - \varepsilon)\, dG_\varepsilon \right\} + \\
(1-\lambda)\sigma_{2t} \left\{ \int_{\underline{\varepsilon}}^{\hat{\varepsilon}_2} (\hat{\varepsilon}_\lambda - \varepsilon)\, dG_\varepsilon - \int_{\hat{\varepsilon}_\lambda}^{\hat{\varepsilon}_2} (\hat{\varepsilon}_\lambda - \varepsilon)\, dG_\varepsilon \right\} = \\
\lambda\sigma_{1t} \left\{ \int_{\underline{\varepsilon}}^{\hat{\varepsilon}_1} (\hat{\varepsilon}_1 - \varepsilon)\, dG_\varepsilon + (\hat{\varepsilon}_\lambda - \hat{\varepsilon}_1)\, G_\varepsilon(\hat{\varepsilon}_1) + \int_{\hat{\varepsilon}_1}^{\hat{\varepsilon}_\lambda} (\hat{\varepsilon}_\lambda - \varepsilon)\, dG_\varepsilon \right\} + \\
(1-\lambda)\sigma_{2t} \left\{ \int_{\underline{\varepsilon}}^{\hat{\varepsilon}_2} (\hat{\varepsilon}_2 - \varepsilon)\, dG_\varepsilon + (\hat{\varepsilon}_\lambda - \hat{\varepsilon}_2)\, G_\varepsilon(\hat{\varepsilon}_2) + \int_{\hat{\varepsilon}_\lambda}^{\hat{\varepsilon}_2} (\varepsilon - \hat{\varepsilon}_\lambda)\, dG_\varepsilon \right\} \leq \\
\lambda\sigma_{1t} \left\{ \int_{\underline{\varepsilon}}^{\hat{\varepsilon}_1} (\hat{\varepsilon}_1 - \varepsilon)\, dG_\varepsilon + (\hat{\varepsilon}_\lambda - \hat{\varepsilon}_1)\, G_\varepsilon(\hat{\varepsilon}_1) + \int_{\hat{\varepsilon}_1}^{\hat{\varepsilon}_\lambda} (\hat{\varepsilon}_\lambda - \hat{\varepsilon}_1)\, dG_\varepsilon \right\} + \\
(1-\lambda)\sigma_{2t} \left\{ \int_{\underline{\varepsilon}}^{\hat{\varepsilon}_2} (\hat{\varepsilon}_2 - \varepsilon)\, dG_\varepsilon + (\hat{\varepsilon}_\lambda - \hat{\varepsilon}_2)\, G_\varepsilon(\hat{\varepsilon}_2) + \int_{\hat{\varepsilon}_\lambda}^{\hat{\varepsilon}_2} (\hat{\varepsilon}_2 - \hat{\varepsilon}_\lambda)\, dG_\varepsilon \right\},
\end{aligned}
$$

where the inequality sign comes from $\int_{\hat{\varepsilon}_1}^{\hat{\varepsilon}_\lambda} (\hat{\varepsilon}_\lambda - \varepsilon)\, dG_\varepsilon \leq \int_{\hat{\varepsilon}_1}^{\hat{\varepsilon}_\lambda} (\hat{\varepsilon}_\lambda - \hat{\varepsilon}_1)\, dG_\varepsilon$ and $\int_{\hat{\varepsilon}_\lambda}^{\hat{\varepsilon}_2} (\varepsilon - \hat{\varepsilon}_\lambda)\, dG_\varepsilon \leq \int_{\hat{\varepsilon}_\lambda}^{\hat{\varepsilon}_2} (\hat{\varepsilon}_2 - \hat{\varepsilon}_\lambda)\, dG_\varepsilon$. Substituting for the definitions of $h(\sigma_{1t}) = \sigma_{1t} \int_{\underline{\varepsilon}}^{\hat{\varepsilon}_1} (\hat{\varepsilon}_1 - \varepsilon)\, dG_\varepsilon$ and $h(\sigma_{2t}) = \sigma_{2t} \int_{\underline{\varepsilon}}^{\hat{\varepsilon}_2} (\hat{\varepsilon}_2 - \varepsilon)\, dG_\varepsilon$, we get:

$$
\begin{aligned}
h(\sigma_{\lambda t}) \leq \lambda h(\sigma_{1t}) + (1-\lambda) h(\sigma_{2t}) + \lambda\sigma_{1t} \left\{ (\hat{\varepsilon}_\lambda - \hat{\varepsilon}_1)\, G_\varepsilon(\hat{\varepsilon}_\lambda) \right\} + \\
(1-\lambda)\sigma_{2t} \left\{ (\hat{\varepsilon}_\lambda - \hat{\varepsilon}_2)\, G_\varepsilon(\hat{\varepsilon}_\lambda) \right\} = \lambda h(\sigma_{1t}) + (1-\lambda) h(\sigma_{2t}) + \\
G_\varepsilon(\hat{\varepsilon}_\lambda) \left(\lambda\sigma_{1t} (\hat{\varepsilon}_\lambda - \hat{\varepsilon}_1) + (1-\lambda)\sigma_{2t} (\hat{\varepsilon}_\lambda - \hat{\varepsilon}_2) \right) = \lambda h(\sigma_{1t}) + (1-\lambda) h(\sigma_{2t}),
\end{aligned}
$$

where the last equality follows from the same reasoning employed in the previous case.

(c) $R^s_{t+1} = R^d_t(1-\gamma_t)$. Hence, $\hat{\varepsilon}(\sigma_t) = 0$ and

$$
h(\sigma_t) = \sigma_t \left[\int_{\underline{\varepsilon}}^{0} (0 - \varepsilon)\, dG_\varepsilon \right],
$$

which is linear in σ_t

2. We found in step 1 that $h(\sigma_t,\ R^s_{t+1})$ is convex in σ_t for each $R^s_{t+1} \in \mathbb{R}$. Consider

$P(\omega) \geq 0$ for each $R^s_{t+1}(\omega) \in \mathbb{R}$. Then the following function[3]:

$$\int_\Omega h\left(\sigma_t,\ R^s_{t+1}(\omega)\right) P(d\omega) = \mathbb{E}_t\, h(\sigma_t,\ R^s_{t+1}) \equiv H(\sigma_t)$$

is convex in σ_t. It follows directly from the linearity of the expectation operator which puts a non-negative weight on every realization of R^s_{t+1} and the fact that the sum of convex functions is a convex function. Therefore, $\Pi(\sigma_t)$ is convex in σ_t.

Since the expected dividend function is defined as the difference between expected net worth and the initial equity investment (i.e., it is an affine transformation of expected net worth) and does not depend on σ_t—that is, $\Omega(\sigma_t;\, l_t,\, d_t,\, e_t) \equiv \Pi(\sigma_t) - e_t$—the convexity of $\Pi(\sigma_t)$ guarantees that $\Omega(\sigma_t;\, l_t,\, d_t,\, e_t)$ is convex. $\square$

A.2.7 Safe and Risky Banks

The results established in the preceding section imply that at any one time, depending on the state of the economy and the realization of aggregate shocks, only one type of bank may exist, either the risky bank or the safe bank. However, we have not been able to rule out analytically that risky and safe banks may coexist. Accordingly, we allow for this possibility in the numerical solution of the model. Nevertheless, in our model simulations, we have not found any such case.

We let μ_t denote the fraction of banks with risky portfolios (banks that choose $\sigma_t = \overline{\sigma}$) at date t; the remaining fraction $1 - \mu_t$ are safe banks ($\sigma_t = \underline{\sigma}$).

The fraction μ_t is endogenously determined by equity positions of households: we have $\mu_t = \frac{E^r_t}{E^r_t + E^s_t}$. At any point in time, the economy may be in a safe equilibrium (with $\mu_t = 0$), a risky equilibrium (with $\mu_t = 1$), or a mixed equilibrium (with $0 < \mu_t < 1$).

Each bank within a group (safe or risky) is alike and solves the same maximization problem in which it chooses l^i_t, d^i_t, e^i_t according to its type $i \in \{s, r\}$.

A.3 The Non-Financial Firm's Problem

We consider the problem of safe firms before that of risky firms.

[3]Linearity in σ_t for one particular value of R^s_{t+1} can be considered as a weakly convex function, so it does not change the nature of the argument

A.3.1 Safe Firms

Let π^s_{t+1} denote the revenue of a safe firm in period $t+1$ net of expenses:

$$\pi^s_{t+1} = y^s_{t+1} + (1-\delta)Q_t k^s_{t+1} - W_{t+1}h^s_{t+1} - R^s_{t+1}l^{f,s}_t.$$

In this notation, the problem of the safe firm is to

$$\max_{l^{f,s}_t, k^s_{t+1}} \mathbb{E}_t \left\{ \max_{h^s_{t+1}} \pi^s_{t+1} \right\}.$$

The first-order condition for $\max_{h^s_{t+1}} \pi^s_{t+1}$ is $\frac{\partial \pi^s_{t+1}}{\partial h^s_{t+1}} = 0$. It implies that

$$W_{t+1} = \frac{\partial y^s_{t+1}}{\partial h^s_{t+1}} = (1-\alpha)\frac{y^s_{t+1}}{h^s_{t+1}} = (1-\alpha) A_{t+1} \left(\frac{k^s_{t+1}}{h^s_{t+1}} \right)^{\alpha}, \tag{A.22}$$

$$h^s_{t+1} = (1-\alpha)\frac{y^s_{t+1}}{W_{t+1}} = (1-\alpha)\frac{A_{t+1}\left(k^s_{t+1}\right)^{\alpha}\left(h^s_{t+1}\right)^{1-\alpha}}{W_{t+1}}. \tag{A.23}$$

Accordingly, the safe firm's Lagrangian is:

$$\begin{aligned}\mathcal{L}^{\text{safe}} =& \mathbb{E}_t \left\{ A_{t+1}\left(k^s_{t+1}\right)^{\alpha}\left(h^s_{t+1}\right)^{1-\alpha} + (1-\delta)Q_{t+1}k^s_{t+1} - W_{t+1}h^s_{t+1} - R^s_{t+1}l^{f,s}_t \right\} + \\ & \lambda^s_{ht}\mathbb{E}_t \left\{ (1-\alpha)\frac{A_{t+1}\left(k^s_{t+1}\right)^{\alpha}\left(h^s_{t+1}\right)^{1-\alpha}}{W_{t+1}} - h^s_{t+1} \right\} + \lambda^s_{lt}\left(l^{f,s}_t - Q_t k^s_{t+1} \right).\end{aligned}$$

Notice that there is no expectation operator on the Lagrangian multipliers because those constraints hold under every state of nature. The problem implies the following first-order conditions

$$\frac{\partial \mathcal{L}^{\text{safe}}}{\partial l^{f,s}_t} = -\mathbb{E}_t\left[R^s_{t+1}\right] + \lambda^s_{lt} = 0,$$

$$\frac{\partial \mathcal{L}^{\text{safe}}}{\partial k^s_{t+1}} = \mathbb{E}_t\left[\alpha\frac{y^s_{t+1}}{k^s_{t+1}} + (1-\delta)Q_{t+1}\right] + \lambda^s_{ht}(1-\alpha)\alpha\mathbb{E}_t\left[\frac{A_{t+1}}{W_{t+1}}\left(\frac{k^s_{t+1}}{h^s_{t+1}}\right)^{\alpha-1}\right] - \lambda^s_{lt}Q_t = 0,$$

$$\frac{\partial \mathcal{L}^{\text{safe}}}{\partial h^s_{t+1}} = (1-\alpha)\frac{A_{t+1}\left(k^s_{t+1}\right)^{\alpha}\left(h^s_{t+1}\right)^{1-\alpha}}{W_{t+1}} - W_{t+1} + \lambda^s_{ht}\left[(1-\alpha)^2\frac{A_{t+1}}{W_{t+1}}\left(\frac{k^s_{t+1}}{h^s_{t+1}}\right)^{\alpha} - 1\right] = 0.$$

Combining $\frac{\partial \mathcal{L}^{\text{safe}}}{\partial h^s_{t+1}} = 0$ with equation (A.23) yields $\lambda^s_{ht} = 0$. Then, plugging $\frac{\partial \mathcal{L}^{\text{safe}}}{\partial l^{f,s}_t} = 0$ into

$\frac{\partial \mathcal{L}^{\text{safe}}}{\partial k^s_{t+1}}$ for λ^s_{lt}, we get

$$\mathbb{E}_t\left[R^s_{t+1}\right]Q_t = \mathbb{E}_t\left[\alpha\frac{y^s_{t+1}}{k^s_{t+1}} + (1-\delta)Q_{t+1}\right].$$

Consider the zero-profit condition of the safe firm under all states of nature. Since the production function has constant returns to scale,

$$y^s_{t+1} = \frac{\partial y^s_{t+1}}{\partial k^s_{t+1}}k^s_{t+1} + \frac{\partial y^s_{t+1}}{\partial h^s_{t+1}}h^s_{t+1} = \alpha A_{t+1}\left(\frac{k^s_{t+1}}{h^s_{t+1}}\right)^{\alpha-1}k^s_{t+1} + W_{t+1}h^s_{t+1},$$

where we use equation (A.23) to substitute for W_{t+1} in the last equality. Plugging the expression of y^s_{t+1} into $\pi^s_{t+1} = 0$ and using $Q_t k^s_{t+1} = l^{f,s}_t$, we find that:

$$\alpha A_{t+1}\left(\frac{k^s_{t+1}}{h^s_{t+1}}\right)^{\alpha-1}k^s_{t+1} + (1-\delta)Q_{t+1}k^s_{t+1} - R^s_{t+1}Q_t k^s_{t+1} = 0.$$

Since $k^s_{t+1} > 0$, we can divide by k^s_{t+1} to get

$$R^s_{t+1}Q_t = \alpha A_{t+1}\left(\frac{k^s_{t+1}}{h^s_{t+1}}\right)^{\alpha-1} + (1-\delta)Q_{t+1} \tag{A.24}$$

under all states of nature. This condition implies

$$R^s_t Q_{t-1} = \alpha A_t\left(\frac{k^s_t}{h^s_t}\right)^{\alpha-1} + (1-\delta)Q_t.$$

A.3.2 Risky Firms

Let π^r_{t+1} denote the revenue of a risky firm in period $t+1$ net of expenses:

$$\pi^r_{t+1} = y^r_{t+1} + (1-\delta)Q_t k^r_{t+1} - W_{t+1}h^r_{t+1} - R^r_{t+1}l^{f,r}_t.$$

In this notation, the problem of the risky firm is to

$$\max_{l^{f,r}_t, k^r_{t+1}} \mathbb{E}_t\left\{\max_{h^r_{t+1}} \pi^r_{t+1}\right\}.$$

The first-order condition for $\max_{h^r_{t+1}} \pi^r_{t+1}$ is $\frac{\partial \pi^r_{t+1}}{\partial h^r_{t+1}} = 0$. It implies that

$$W_{t+1} = \frac{\partial y^r_{t+1}}{\partial h^r_{t+1}} = (1-\alpha) A_{t+1} \left(\frac{k^r_{t+1}}{h^r_{t+1}}\right)^{\alpha}, \tag{A.25}$$

$$h^r_{t+1} = (1-\alpha) \frac{A_{t+1} \left(k^r_{t+1}\right)^{\alpha} \left(h^r_{t+1}\right)^{1-\alpha}}{W_{t+1}}. \tag{A.26}$$

Accordingly, the risky firm's Lagrangian is:

$$\begin{aligned} \mathcal{L}^{\text{risky}} =& \mathbb{E}_t \left[A_{t+1} \left(k^r_{t+1}\right)^{\alpha} \left(h^r_{t+1}\right)^{1-\alpha} + \varepsilon_{t+1} k^r_{t+1} + (1-\delta) Q_{t+1} k^r_{t+1} - W_{t+1} h^r_{t+1} - R^r_{t+1} l^{f,r}_t\right] + \\ & \lambda^r_{ht} \mathbb{E}_t \left[(1-\alpha) \frac{A_{t+1} \left(k^r_{t+1}\right)^{\alpha} \left(h^r_{t+1}\right)^{1-\alpha}}{W_{t+1}} - h^r_{t+1}\right] + \lambda^r_{lt} \left(l^{f,r}_t - Q_t k^r_{t+1}\right). \end{aligned}$$

Notice that there is no expectation operator on the Lagrangian multipliers because those constraints hold under every state of nature. The problem implies the following first-order conditions

$$\begin{aligned} \frac{\partial \mathcal{L}^{\text{risky}}}{\partial l^{f,r}_t} &= -\mathbb{E}_t \left[R^r_{t+1}\right] + \lambda^r_{lt} = 0, \\ \frac{\partial \mathcal{L}^{\text{risky}}}{\partial k^r_{t+1}} &= \mathbb{E}_t \left[\alpha A_{t+1} \left(\frac{k^r_{t+1}}{h^r_{t+1}}\right)^{\alpha-1} + \varepsilon_{t+1} + (1-\delta) Q_{t+1}\right] + \\ & \quad \lambda^r_{ht} \mathbb{E}_t \left[\alpha (1-\alpha) \frac{A_{t+1}}{W_{t+1}} \left(\frac{k^r_{t+1}}{h^r_{t+1}}\right)^{\alpha-1}\right] - \lambda^r_{lt} Q_t = 0, \\ \frac{\partial \mathcal{L}^{\text{risky}}}{\partial h^r_{t+1}} &= (1-\alpha) A_{t+1} \left(\frac{k^r_{t+1}}{h^r_{t+1}}\right)^{\alpha} - W_{t+1} + \lambda^r_{ht} \left[(1-\alpha)^2 \frac{A_{t+1}}{W_{t+1}} \left(\frac{k^r_{t+1}}{h^r_{t+1}}\right)^{\alpha} - 1\right] = 0. \end{aligned}$$

Equation (A.25) together with $\frac{\partial \mathcal{L}^{\text{risky}}}{\partial h^r_{t+1}} = 0$ yield $\lambda^r_{ht} = 0$. Plugging $\frac{\partial \mathcal{L}^{\text{risky}}}{\partial l^{f,r}_t} = 0$ into $\frac{\partial \mathcal{L}^{\text{risky}}}{\partial k^r_{t+1}}$ for λ^r_{lt}, we get

$$\mathbb{E}_t \left[R^r_{t+1}\right] Q_t = \mathbb{E}_t \left[\alpha A_{t+1} \left(\frac{k^r_{t+1}}{h^r_{t+1}}\right)^{\alpha-1} + (1-\delta) Q_{t+1} + \varepsilon_{t+1}\right].$$

Combining equation (A.22) with equation (A.25):

$$\frac{k^s_{t+1}}{h^s_{t+1}} = \frac{k^r_{t+1}}{h^r_{t+1}} \tag{A.27}$$

under all states of nature. But remember that the first-order condition of the safe firm implies

$$\mathbb{E}_t\left[R^s_{t+1}\right]Q_t = \mathbb{E}_t\left[\alpha A_{t+1}\left(\frac{k^s_{t+1}}{h^s_{t+1}}\right)^{\alpha-1} + (1-\delta)Q_{t+1}\right].$$

Therefore

$$\mathbb{E}_t\left[R^r_{t+1}\right]Q_t = \mathbb{E}_t\left[R^s_{t+1}Q_t + \varepsilon_{t+1}\right].$$

Consider the zero-profit condition of the risky firm under all states of nature.

$$\begin{aligned}
\pi^r_{t+1} = y^r_{t+1} + (1-\delta)Q_t k^r_{t+1} - W_{t+1}h^r_{t+1} - R^r_{t+1}l^{f,r}_t = \\
y^r_{t+1} + (1-\delta)Q_t k^r_{t+1} - (1-\alpha)A_{t+1}\left(k^r_{t+1}\right)^{\alpha}\left(h^r_{t+1}\right)^{1-\alpha} - R^r_{t+1}l^{f,r}_t = \\
\alpha A_{t+1}\left(k^r_{t+1}\right)^{\alpha}\left(h^r_{t+1}\right)^{1-\alpha} + \varepsilon_{t+1}k^r_{t+1} + (1-\delta)Q_t k^r_{t+1} - R^r_{t+1}l^{f,r}_t = \\
\alpha A_{t+1}\left(\frac{k^r_{t+1}}{h^r_{t+1}}\right)^{\alpha-1} k^r_{t+1} + \varepsilon_{t+1}k^r_{t+1} + (1-\delta)Q_t k^r_{t+1} - R^r_{t+1}l^{f,r}_t = 0,
\end{aligned}$$

where we use equation (A.26) to substitute for $W_{t+1}h^r_{t+1}$. Using equation (A.24) together with equation (A.27), we can express

$$\alpha A_{t+1}\left(\frac{k^r_{t+1}}{h^r_{t+1}}\right)^{\alpha-1} = R^s_{t+1}Q_t - (1-\delta)Q_{t+1},$$

that holds under all states of nature. Plugging it into the zero-profit condition and using $Q_t k^r_{t+1} = l^{f,r}_t$, we find that:

$$R^s_{t+1}Q_t k^r_{t+1} - (1-\delta)Q_{t+1}k^r_{t+1} + \varepsilon_{t+1}k^r_{t+1} + (1-\delta)Q_t k^r_{t+1} - R^r_{t+1}Q_t k^r_{t+1} = 0.$$

Since $k^r_{t+1} > 0$, we can divide by k^r_{t+1} to get

$$R^r_{t+1}Q_t = R^s_{t+1}Q_t + \varepsilon_{t+1}$$

under all states of nature. This condition implies

$$R^r_t Q_{t-1} = R^s_t Q_{t-1} + \varepsilon_t.$$

A.3.3 Aggregating Across Firms

Here we show that we can aggregate individual firms into two representative firms. Let $k^i_{j,t}$ denote the capital chosen by firm i that is financed by borrowing from bank j. Both i and j lie within the continuum of measure 1 of banks and firms, respectively. In this notation, equation (A.27) is written as

$$\frac{k^i_{j,t+1}}{h^i_{j,t+1}} = \frac{k_{t+1}}{h_{t+1}}, \tag{A.28}$$

for all $j \in [0,1]$ and $i \in [0,1]$. Each firm chooses the same capital-to-labor ratio independently of the type of bank it borrows from.

Note that σ_t is the fraction of risky firms at date t; the remaining fraction $1-\sigma_t$ of firms are safe firms. Let's index firms as follows: firm j_1, with $j_1 \in [0, \sigma_t]$, can only access a risky technology subject to both aggregate and idiosyncratic shocks; firm j_2, with $j_2 \in [\sigma_t, 1]$ has access to a safe production technology subject to aggregate shocks only. Since there are no equilibria with $\underline{\sigma} < \sigma_t < \overline{\sigma}$, the fraction of risky firms is linked to the fraction of banks with risky portfolios as follows:

$$\sigma_t = (1-\mu_t)\,\underline{\sigma} + \mu_t \overline{\sigma}.$$

Define the following objects: Let $K^s_{s,t+1} = \int_{\sigma_t}^1 \int_{\mu_t}^1 k^i_{j,t+1} dj di$ be the total capital allocated to the safe technology and financed by borrowing from the banks that choose a fraction $\underline{\sigma}$ of risky projects. Let $K^s_{r,t+1} = \int_{\sigma_t}^1 \int_0^{\mu_t} k^i_{j,t+1} dj di$ be the total capital allocated to the safe technology and financed by borrowing from the banks that choose a fraction $\overline{\sigma}$ of risky projects. We let K^s_{t+1} denote the total capital allocated to the safe technology. Thus,

$$K^s_{t+1} = \int\limits_{\sigma_t}^{1} \int\limits_{0}^{1} k^i_{j,t+1} dj di = K^s_{s,t+1} + K^s_{r,t+1},$$

Let $K^r_{s,t+1} = \int_0^{\sigma_t} \int_{\mu_t}^1 k^i_{j,t+1} dj di$ be the total capital allocated to the risky technology and financed by borrowing from the banks that choose a fraction $\underline{\sigma}$ of risky projects. Let $K^r_{r,t+1} = \int_0^{\sigma_t} \int_0^{\mu_t} k^i_{j,t+1} dj di$ be the total capital allocated to the safe technology and financed by borrowing from the banks that choose a fraction $\overline{\sigma}$ of risky projects. We let K^r_{t+1} denote the total capital allocated to the risky technology. Thus,

$$K^r_{t+1} = \int\limits_{0}^{\sigma_t} \int\limits_{0}^{1} k^i_{j,t+1} dj di = K^r_{s,t+1} + K^r_{r,t+1},$$

The same upper and lower case notation applies to labor, i.e. $H^s_{s,t+1} = \int_{\sigma_t}^1 \int_{\mu_t}^1 h^i_{j,t+1} dj di$; $H^s_{r,t+1} = \int_{\sigma_t}^1 \int_0^{\mu_t} h^i_{j,t+1} dj di$; $H^r_{s,t+1} = \int_0^{\sigma_t} \int_{\mu_t}^1 h^i_{j,t+1} dj di$; $H^r_{r,t+1} = \int_0^{\sigma_t} \int_0^{\mu_t} h^i_{j,t+1} dj di$.

Safe representative firm produces:

$$Y^s_t = \int\limits_{\sigma_{t-1}}^{1} \int\limits_{0}^{1} A_t \left(k^i_{j,t}\right)^{\alpha} \left(h^i_{j,t}\right)^{1-\alpha} dj di = \int\limits_{\sigma_{t-1}}^{1} \int\limits_{0}^{1} F\left(k^i_{j,t}, h^i_{j,t}\right) dj di =$$

Using that the technology has Constant Returns to Scale:

$$= \int_{\sigma_{t-1}}^{1} \int_{0}^{1} \left[F_{k^i_{j,t}} \left(k^i_{j,t}, h^i_{j,t} \right) k^i_{j,t} + F_{h^i_{j,t}} \left(k^i_{j,t}, h^i_{j,t} \right) h^i_{j,t} \right] dj di =$$

where $F_{k^i_{j,t}} \left(k^i_{j,t}, h^i_{j,t} \right)$ and $F_{h^i_{j,t}} \left(k^i_{j,t}, h^i_{j,t} \right)$ denote the partial derivative of $F \left(k^i_{j,t}, h^i_{j,t} \right)$ with respect to $k^i_{j,t}$ and $h^i_{j,t}$, respectively. Since these partial derivatives are homogeneous of degree zero, we can express them in term of capital-labor ratio, i.e.

$$= \int_{\sigma_{t-1}}^{1} \int_{0}^{1} \left[f_{k^i_{j,t}} \left(\frac{k^i_{j,t}}{h^i_{j,t}} \right) k^i_{j,t} + f_{h^i_{j,t}} \left(\frac{k^i_{j,t}}{h^i_{j,t}} \right) h^i_{j,t} \right] dj di = \text{Plugging equation (A.28)} =$$

$$= \int_{\sigma_{t-1}}^{1} \int_{0}^{1} \left[f_{k_t} \left(\frac{k_t}{h_t} \right) k^i_{j,t} + f_{h_t} \left(\frac{k_t}{h_t} \right) h^i_{j,t} \right] dj di =$$

$$f_{k_t} \left(\frac{k_t}{h_t} \right) \left[\int_{\sigma_t}^{1} \int_{0}^{1} k^i_{j,t} dj di \right] + f_{h_t} \left(\frac{k_t}{h_t} \right) \left[\int_{\sigma_t}^{1} \int_{0}^{1} h^i_{j,t} dj di \right] = f_{k_t} \left(\frac{k_t}{h_t} \right) K^s_t + f_{h_t} \left(\frac{k_t}{h_t} \right) H^s_t =$$

Since $\frac{K^s_{s,t}}{H^s_{s,t}} = \frac{K^s_{r,t}}{H^s_{r,t}} = \frac{k_t}{h_t}$, then $\frac{K^s_t}{H^s_t} \frac{h_t}{k_t} = \left(\frac{K^s_{s,t} + K^s_{r,t}}{H^s_{s,t} + H^s_{r,t}} \right) \frac{H^s_{r,t}}{K^s_{r,t}} = 1$. Therefore $\frac{K^s_t}{H^s_t} = \frac{k_t}{h_t}$.

$$= f_{K^s_t} \left(\frac{K^s_t}{H^s_t} \right) K^s_t + f_{H^s_t} \left(\frac{K^s_t}{H^s_t} \right) H^s_t.$$

Hence, we obtain that

$$Y^s_t = A_t \left(K^s_t \right)^{\alpha} \left(H^s_t \right)^{1-\alpha}. \tag{A.29}$$

Moving to the risky representative firm,

$$Y^r_t = \int_{0}^{\sigma_{t-1}} \int_{0}^{1} \left[A_t \left(k^i_{j,t} \right)^{\alpha} \left(h^i_{j,t} \right)^{1-\alpha} + \varepsilon^i_{j,t} k^i_{j,t} \right] dj di = \int_{0}^{\sigma_{t-1}} \int_{0}^{1} F \left(k^i_{j,t}, h^i_{j,t} \right) dj di + \int_{0}^{\sigma_{t-1}} \int_{0}^{1} \varepsilon^i_{j,t} k^i_{j,t} dj di.$$

Note that similar steps described above apply to the first term in the summation, so that $\int_0^{\sigma_{t-1}} \int_0^1 F \left(k^i_{j,t}, h^i_{j,t} \right) dj di = A_t \left(K^r_t \right)^{\alpha} \left(H^r_t \right)^{1-\alpha}$. To express the second term, notice that since each risky firm solves the same maximization problem, it chooses the same amount of capital independently of the type of bank it borrows from. Therefore, $\int_0^{\sigma_{t-1}} \int_0^1 \varepsilon^i_{j,t} k^i_{j,t} dj di = -\xi K^r_t$. Hence,

$$Y^r_t = A_t \left(K^r_t \right)^{\alpha} \left(H^r_t \right)^{1-\alpha} - \xi K^r_t. \tag{A.30}$$

A.3.4 Capital Producing Firms

At the end of period t, goods producing firms sell their capital to competitive capital producing firms. Letting I_t^g denote gross investment, the evolution of capital follows

$$I_t = \eta_t \left[1 - \frac{\phi}{2}\left(\frac{I_t^g}{I_{t-1}^g} - 1\right)^2\right] I_t^g,$$

where η_t is a shock to investment-specific technology (ISP), and ϕ is a measure of the severity of investment adjustment costs. The aggregate capital stock evolves according to

$$K_{t+1}^s + K_{t+1}^r = I_t + (1-\delta)\left(K_t^s + K_t^r\right).$$

The capital producing firms are owned by households, and solve the problem

$$\max_{I_{t+i}^g} \mathbb{E}_t \sum_{i=0}^{\infty} \psi_{t,t+i} \left\{ \eta_{t+i} Q_{t+i} \left[1 - \frac{\phi}{2}\left(\frac{I_{t+i}^g}{I_{t+i-1}^g} - 1\right)^2\right] I_{t+i}^g - I_{t+i}^g \right\},$$

where $\psi_{t,t+i} = \beta \frac{\lambda_{ct+i}}{\lambda_{ct}}$ is the stochastic discount factor of the households.

$$\begin{aligned} \eta_t Q_t \left[1 - \frac{\phi}{2}\left(\frac{I_t^g}{I_{t-1}^g} - 1\right)^2\right] & - \eta_t Q_t \phi \left(\frac{I_t^g}{I_{t-1}^g} - 1\right) \frac{I_t^g}{I_{t-1}^g} - 1 + \\ & \beta \mathbb{E}_t \left[\eta_{t+1} \frac{\lambda_{ct+1}}{\lambda_{ct}} Q_{t+1} \phi \left(\frac{I_{t+1}^g}{I_t^g} - 1\right) \frac{I_{t+1}^g}{\left(I_t^g\right)^2} I_{t+1}^g \right] = 0. \end{aligned}$$

A.4 The Government

The government levies the tax to fully compensate for the loss to the deposit insurance fund due to rescue of defaulted banks.

$$T_t = -\int_{-\infty}^{\left(\frac{R^d_{t-1}D_{t-1}}{\sigma_{t-1}L_{t-1}} - \frac{R^s_t}{\sigma_{t-1}}\right)Q_{t-1}} \left(\left(R^s_t + \frac{\sigma_{t-1}\varepsilon_t}{Q_{t-1}}\right)L_{t-1} - R^d_{t-1}D_{t-1}\right) \mathrm{d}G(\varepsilon_t) =$$
$$-\left[\int_{-\infty}^{\infty} \left(\left(R^s_t + \frac{\sigma_{t-1}\varepsilon_t}{Q_{t-1}}\right)L_{t-1} - R^d_{t-1}D_{t-1}\right) \mathrm{d}G(\varepsilon_t) - \int_{\left(\frac{R^d_{t-1}D_{t-1}}{\sigma_{t-1}L_{t-1}} - \frac{R^s_t}{\sigma_{t-1}}\right)Q_{t-1}}^{\infty} \left(\left(R^s_t + \frac{\sigma_{t-1}\varepsilon_t}{Q_{t-1}}\right)L_{t-1} - R^d_{t-1}D_{t-1}\right) \mathrm{d}G(\varepsilon_t)\right] =$$

Note that in the square bracket the first term equals $\left(R^s_t - \frac{\sigma_{t-1}\xi}{Q_{t-1}}\right)L_{t-1} + R^d_{t-1}D_{t-1}$. We have already calculated the second term. Therefore,

$$T_t = \frac{\sigma_{t-1}L_{t-1}}{Q_{t-1}}\frac{\tau_{t-1}}{\sqrt{2\pi}}\exp\left(-\left(\frac{R^d_{t-1}\left(1-\gamma_{t-1}\right)Q_{t-1} - R^s_tQ_{t-1} + \xi\sigma_{t-1}}{\sigma_{t-1}\sqrt{2}\tau_{t-1}}\right)^2\right) - \left(R^s_t - \frac{\sigma_{t-1}\xi}{Q_{t-1}}\right)L_{t-1} + R^d_{t-1}D_{t-1} +$$
$$\frac{1}{2}L_{t-1}\left(R^s_t - \frac{\sigma_{t-1}\xi}{Q_{t-1}} - \left(1-\gamma_{t-1}\right)R^d_{t-1}\right)\left[1 - \mathrm{erf}\left(\frac{R^d_{t-1}\left(1-\gamma_{t-1}\right)Q_{t-1} - R^s_tQ_{t-1} + \xi\sigma_{t-1}}{\sigma_{t-1}\sqrt{2}\tau_{t-1}}\right)\right],$$

which can be rewritten as

$$\begin{aligned} T_t =& \frac{\sigma_{t-1}L_{t-1}}{Q_{t-1}}\frac{\tau_{t-1}}{\sqrt{2\pi}}\exp\left(-\left(\frac{R^d_{t-1}\left(1-\gamma_{t-1}\right)Q_{t-1} - R^s_tQ_{t-1} + \xi\sigma_{t-1}}{\sigma_{t-1}\sqrt{2}\tau_{t-1}}\right)^2\right) - \\ & \frac{1}{2}\left(R^s_tL_{t-1} - \frac{\sigma_{t-1}\xi}{Q_{t-1}}L_{t-1} - R^d_{t-1}D_{t-1}\right)\left[1 + \mathrm{erf}\left(\frac{R^d_{t-1}\left(1-\gamma_{t-1}\right)Q_{t-1} - R^s_tQ_{t-1} + \xi\sigma_{t-1}}{\sigma_{t-1}\sqrt{2}\tau_{t-1}}\right)\right]. \end{aligned} \tag{A.31}$$

A.5 Resource Constraints

The aggregate loans to the (representative) safe firm come from two sources: 1) from all safe banks (of measure $1-\mu_t$) that allocate $1-\underline{\sigma}$ share of their loan portfolio to safe projects and 2) from all risky banks (of measure μ_t) that allocate $1-\overline{\sigma}$ share of their loan portfolio to safe projects. Therefore, the equilibrium conditions linking our bank-level and firm-level variables representing loans are

$$Q_tK^s_{t+1} = \left(1-\underline{\sigma}\right)\left(1-\mu_t\right)l^s_t + \left(1-\overline{\sigma}\right)\mu_tl^r_t.$$

Similarly,

$$Q_t K^r_{t+1} = \underline{\sigma} \left(1 - \mu_t\right) l^s_t + \overline{\sigma} \mu_t l^r_t.$$

The aggregate bank loans are linked to the individual bank loans by: $L^r_t = \mu_t l^r_t$ and $L^s_t = (1 - \mu_t) l^s_t$. Therefore, we can describe the latter two equations by using aggregate loans

$$\begin{aligned} Q_t K^s_{t+1} &= \left(1 - \underline{\sigma}\right) L^s_t + \left(1 - \overline{\sigma}\right) L^r_t, \\ Q_t K^r_{t+1} &= \underline{\sigma} L^s_t + \overline{\sigma} L^r_t. \end{aligned}$$

The equity positions taken by households, in turn, determine the equity positions of individual banks: $E^r_t = \mu_t e^r_t$ and $E^s_t = (1 - \mu_t) e^s_t$. The returns on the equity positions taken by households at date t are linked to the dividends paid by banks at date $t+1$. We have:

$$\mathbb{E}_t R^{e,r}_{t+1} = \left(\omega^r_1 + \omega^r_2\right) L^r_t,$$
$$\mathbb{E}_t R^{e,s}_{t+1} = \left(\omega^s_1 + \omega^s_2\right) L^s_t,$$

where we use the fact that $\max \left[n w^r_{t+1}, 0 \right]$ is linear in loans; ω_1 and ω_2 were defined in equations (27) and (28). Deposits held by households are issued by (safe and risky) banks: $D_t = D^s_t + D^r_t$ where $D^s_t = L^s_t - E^s_t$ and $D^r_t = L^r_t - E^r_t$.

The equilibrium conditions linking our aggregate and individual firm-specific variables are straightforward, but cumbersome in terms of notation. We state the conditions in Appendix B. The market-clearing conditions for labor, capital, and goods are

$$H^s_t + H^r_t = 1,$$

$$K^s_t + K^r_t = K_t,$$

and

$$Y^s_t + Y^r_t = C_t + I^g_t.$$

B List of Equilibrium Conditions

From the first-order conditions for the household problem, derived in Section A.1

$$(C_t - \kappa C_{t-1})^{-\varsigma_c} - \beta\kappa \mathbb{E}_t \left(C_{t+1} - \kappa C_t\right)^{-\varsigma_c} - \lambda_{ct} = 0 \tag{B.1}$$

$$\varsigma_0 D_t^{-\varsigma_d} - \lambda_{ct} + \mathbb{E}_t \beta \lambda_{ct+1} R_t^d = 0, \tag{B.2}$$

$$-\lambda_{ct} + \mathbb{E}_t \beta \lambda_{ct+1} R_{t+1}^{e,s} + \zeta_t^s = 0, \tag{B.3}$$

$$-\lambda_{ct} + \mathbb{E}_t \beta \lambda_{ct+1} R_{t+1}^{e,r} + \zeta_t^r = 0. \tag{B.4}$$

From Section A.2.6, we know that we need to track two types of banks, safe and risky banks:

$$\sigma_t^s = \underline{\sigma}, \tag{B.5}$$

$$\sigma_t^r = \overline{\sigma}. \tag{B.6}$$

From the bank problem in Section A.2, for $\forall i \in \{s, r\}$

$$l_t^i = d_t^i + e_t^i. \tag{B.7}$$

From the proof that capital requirements always bind in Section A.2.2, for $\forall i \in \{s, r\}$

$$e_t^i = \gamma_t l_t^i. \tag{B.8}$$

From the combined first-order conditions for banks in Section A.2.3, for $\forall i \in \{s, r\}$

$$\gamma_t - \chi_{2t}^i = \mathbb{E}_t \left\{ \beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \left[\frac{\sigma_t^i}{Q_t} \frac{\tau_t}{\sqrt{2\pi}} \exp\left(- \left(\frac{\left(R_t^d(1-\gamma_t) - R_{t+1}^s\right)Q_t + \xi\sigma_t^i}{\sigma_t^i \sqrt{2}\tau_t} \right)^2 \right) + \right.\right. \tag{B.9}$$

$$\left.\left. \frac{1}{2}\left(R_{t+1}^s - \frac{\sigma_t^i \xi}{Q_t} - R_t^d(1-\gamma_t)\right) \left[1 - \operatorname{erf}\left(\frac{\left(R_t^d(1-\gamma_t) - R_{t+1}^s\right)Q_t + \xi\sigma_t^i}{\sigma_t^i \sqrt{2}\tau_t} \right)\right]\right]\right\},$$

$$\chi_{2t}^i l_t^i = 0. \tag{B.10}$$

From the zero-profit condition for banks derived in Section A.2.4, for $\forall i \in \{s, r\}$

$$R_{t+1}^{e,i} = \frac{1}{\gamma_t} \left\{ \frac{\sigma_t^i}{Q_t} \frac{\tau_t}{\sqrt{2\pi}} \exp\left(- \left(\frac{\left(R_t^d(1-\gamma_t) - R_{t+1}^s\right)Q_t + \xi\sigma_t^i}{\sigma_t^i \sqrt{2}\tau_t} \right)^2 \right) + \right. \tag{B.11}$$

$$\left. \frac{1}{2}\left(R_{t+1}^s - \frac{\sigma_t^i \xi}{Q_t} - R_t^d(1-\gamma_t)\right) \left[1 - \operatorname{erf}\left(\frac{\left(R_t^d(1-\gamma_t) - R_{t+1}^s\right)Q_t + \xi\sigma_t^i}{\sigma_t^i \sqrt{2}\tau_t} \right)\right]\right\}.$$

Definition of the share of risky banks, from the aggregation across banks in Section A.2.7

$$\mu_t = \frac{E_t^r}{E_t^s + E_t^r}. \tag{B.12}$$

From the problem of safe firms in Section A.3.1

$$R_t^s = \frac{\alpha A_t}{Q_{t-1}} \left(\frac{K_t^s}{H_t^s}\right)^{\alpha-1} + (1-\delta)\frac{Q_t}{Q_{t-1}}. \tag{B.13}$$

From the problem of risky firms in Section A.3.2

$$R_t^r = R_t^s + \frac{\varepsilon_t}{Q_{t-1}}. \tag{B.14}$$

From Equation A.22 in Section A.3.1 and Section A.3.3

$$W_t = (1-\alpha)\, \tfrac{Y_t^s}{H_t^s}. \tag{B.15}$$

From the aggregation across safe and risky firms in Section A.3.3

$$\tfrac{K_t^s}{H_t^s} = \tfrac{K_t^r}{H_t^r}, \tag{B.16}$$

$$Y_t^s = A_t \left(K_t^s\right)^{\alpha} \left(H_t^s\right)^{1-\alpha}, \tag{B.17}$$

$$Y_t^r = A_t \left(K_t^r\right)^{\alpha} \left(H_t^r\right)^{1-\alpha} - \xi K_t^r. \tag{B.18}$$

From the problem of capital producing firms in Section A.3.4

$$K_{t+1} = I_t + (1-\delta) K_t, \tag{B.19}$$

$$I_t = \eta_t \left[1 - \tfrac{\phi}{2}\left(\tfrac{I_t^g}{I_{t-1}^g} - 1\right)^2\right] I_t^g, \tag{B.20}$$

$$\eta_t Q_t \left[1 - \tfrac{\phi}{2}\left(\tfrac{I_t^g}{I_{t-1}^g} - 1\right)^2\right] - \eta_t Q_t \phi \left(\tfrac{I_t^g}{I_{t-1}^g} - 1\right) \tfrac{I_t^g}{I_{t-1}^g} - 1 + \tag{B.21}$$

$$\beta \mathbb{E}_t \left[\eta_{t+1} \tfrac{\lambda_{ct+1}}{\lambda_{ct}} Q_{t+1} \phi \left(\tfrac{I_{t+1}^g}{I_t^g} - 1\right) \tfrac{I_{t+1}^g}{\left(I_t^g\right)^2} I_{t+1}^g\right] = 0.$$

The tax for the deposit insurance scheme is derived in Section A.4

$$T_t = L_{t-1} \left\{ \tfrac{\sigma_{t-1}}{Q_{t-1}} \tfrac{\tau_{t-1}}{\sqrt{2\pi}} \exp\left(-\left(\tfrac{\left(R_{t-1}^d(1-\gamma_{t-1}) - R_t^s\right) Q_{t-1} + \xi\sigma_{t-1}}{\sigma_{t-1}\sqrt{2}\tau_{t-1}}\right)^2\right) - \right. \tag{B.22}$$

$$\left. \tfrac{1}{2}\left(R_t^s - R_{t-1}^d\left(1-\gamma_{t-1}\right) - \tfrac{\xi\sigma_{t-1}}{Q_{t-1}}\right)\left[1 + \operatorname{erf}\left(\tfrac{\left(R_{t-1}^d(1-\gamma_{t-1}) - R_t^s\right) Q_{t-1} + \xi\sigma_{t-1}}{\sigma_{t-1}\sqrt{2}\tau_{t-1}}\right)\right]\right\}.$$

Using the resource constraints from Section A.5

$$L_t^s = (1-\mu_t)\, l_t^s, \tag{B.23}$$
$$L_t^r = \mu_t l_t^r, \tag{B.24}$$
$$E_t^i = \gamma_t L_t^i \quad \forall i \in \{s, r\}\,, \tag{B.25}$$
$$L_t^i = D_t^i + E_t^i \quad \forall i \in \{s, r\}\,, \tag{B.26}$$
$$Q_t K_{t+1}^s = (1-\underline{\sigma})\, L_t^s + (1-\overline{\sigma})\, L_t^r, \tag{B.27}$$
$$Q_t K_{t+1}^r = \underline{\sigma} L_t^s + \overline{\sigma} L_t^r, \tag{B.28}$$
$$D_t = D_t^s + D_t^r, \tag{B.29}$$
$$K_t = K_t^s + K_t^r, \tag{B.30}$$
$$H_t^s + H_t^r = 1, \tag{B.31}$$
$$Y_t^s + Y_t^r = C_t + I_t^g. \tag{B.32}$$

From the first-order conditions of the bank problem in Section A.2.1, the Lagrange multipliers χ_{2t}^i on the loan constraints $l_t^s > 0$, where $i \in \{s, r\}$, govern the transition between different regimes, demarcated as follows:

1. safe regime: $\chi_{2t}^s = 0$, $l_t^s > 0$, $\chi_{2t}^r > 0$, and $l_t^r = 0$;
2. risky regime: $\chi_{2t}^s > 0$, $l_t^s = 0$, $\chi_{2t}^r = 0$, and $l_t^r > 0$;
3. mixed regime: $\chi_{2t}^s = 0$, $l_t^s > 0$, $\chi_{2t}^r = 0$, and $l_t^r > 0$.

These conditions, together with binding capital requirements, also imply whether the inequality constraints on the household bank equity holdings bind or not. Accordingly,

1. safe regime: $\zeta_t^s = 0$, $E_t^s > 0$, $\zeta_t^r > 0$, $E_t^r = 0 \implies \mu_t = 0$;
2. risky regime: $\zeta_t^s > 0$, $E_t^s = 0$, $\zeta_t^r = 0$, and $E_t^r > 0 \implies \mu_t = 1$;
3. mixed regime: $\zeta_t^s = 0$, $E_t^s > 0$, $\zeta_t^r = 0$, and $E_t^r > 0 \implies 0 < \mu_t < 1$;

and remember that μ_t is the share of risky bank equity in total bank equity. Notice that in the safe regime, $l_t^r = 0$ and the binding capital requirement make $E_t^r = 0$ redundant; in the risky regime, $l_t^s = 0$ and the binding capital requirement make $E_t^s = 0$ redundant; and in the mixed regime, the fact that $\chi_{2t}^r = 0$ and that $\zeta_t^r = 0$ together with Equation B.4 and Equation B.11, make Equation B.9 redundant.

Finally, in the numerical implementation of the model, we use a guess-and-verify approach to determine the regime sequence expected to occur after the realization of the aggregate shocks period by period (Guerrieri & Iacoviello, 2015). In the iterative procedure to determine the regime guesses, the following conditions prompt a new guess:

1. safe regime exit: $l_t^s < 0$,
2. risky regime exit: $l_t^r < 0$,
3. mixed regime exit: $l_t^s < 0$ or $l_t^r < 0$.

C Computing Bank Failure Rates

In this section we provide additional details on our computation of the asset-weighted average bank failure rate. In particular we reconcile the 1.52 percent annualized failure rate used in our calibration with the somewhat lower rate of 0.5 percent in Elenev et al. (2021).

We obtain the list of failed banks from Bank Failure and Assistance Data of the Federal Deposit Insurance Corporation (FDIC) and obtain the assets for those banks from the last report filed with the Federal Financial Institutions Examination Council. The total assets of the U.S. banking sector are from the Federal Reserve's H.8 Release, Assets and Liabilities of Commercial Banks in the United States. The H.8 Release does not include thrifts. This omission seems negligible late in the calibration sample but is more substantive early on, when thrifts were a more prominent part of the U.S. banking system. Nonetheless, we prefer using the H.8 Release as it extends further back, allowing us to start the sample in 1980:Q1.

The main difference between the failure rate we calculated and the one in Elenev et al. (2021) turns out not to be our use of the H.8 release. Rather, the difference stems from including in our list of "failed" banks two large banks that received FDIC assistance during the Global Financial Crisis—Citicorp and Bank of America. In our calculation, we treat the provision of "assistance" by the FDIC, as tantamount to preventing failure. The assistance cases are not numerous, and the ones that truly make a difference are those for Citicorp and Bank of America because of the enormous sizes of their assets. For these two banks, the FDIC invoked a Systemic Risk Exception (SRE). The SRE is a rarely used, post-1991 provision that allows regulators to waive the "least-cost" requirement for failing banks if it threatens financial stability.

We confirmed that the two entries pertaining to Citicorp and Bank of America in the FDIC dataset that we are using to construct failure rates coincide with the dates when the FDIC invoked the SRE. A white paper of the Federal Deposit Insurance Corporation (2018) offers a history of FDIC interventions; the SRE invocation for Citicorp is described on Pages 73, and the one for Bank of America on page 86. The FDIC account explains that the structure of the packages offered to the two banks was similar and included Treasury participation through the Troubled Asset Relief Program (TARP). Other banks received government funds through TARP, but the FDIC's Bank Failure and Assistance Data do not include them.

With our procedure, the average failure rate drops to 0.62 if we exclude all the entries in the FDIC dataset categorized as "assistance." And if we also construct asset weights using FDIC data that include thrifts, the failure rate drops further down to 0.52 for a sample running from 1984:Q1 through 2024:Q4, which seems like a close match for the 0.5 failure rate in the calibration of Elenev et al. (2021).

We certainly recognize that excluding banks that received “assistance” is a defensible choice, but we prefer including them in our calculation for a closer match to what we intended to capture as bank failures in our model.

D Discussion of the Excessive Risk-Taking Mechanism

Following our results derived in Appendix A.2.5, we can decompose expected dividends into the following components:

$$\Omega\left(\sigma_t;\ l_t, d_t, e_t\right) = \mathbb{E}_t\left\{\psi_{t,t+1} l_t\left[\omega_1 + \omega_2\right]\right\} - \gamma_t l_t,$$

where

$$\left[\omega_1 + \omega_2\right] = \left[\underbrace{\left(R^s_{t+1} - R^d_t\left(1-\gamma_t\right) - \frac{\xi\sigma_t}{Q_t}\right)\underbrace{\left(1 - G(\varepsilon^*_{t+1})\right)}_{\text{non-defaulted}}}_{\substack{\omega_1 \equiv \text{returns from a loan} \\ \text{portfolio with riskiness } \sigma_t}} + \underbrace{\left(\frac{\sigma_t}{Q_t}\right)\frac{\tau_t}{\sqrt{2\pi}}\exp\left(-\left(\frac{\varepsilon^*_{t+1}+\xi}{\tau_t\sqrt{2}}\right)^2\right)}_{\substack{\omega_2 \equiv \text{bonus from} \\ \text{projects volatility}}}\right],$$

and the cutoff point ε^*_{t+1} is defined by $R^d_t\left(1-\gamma_t\right)Q_t - R^s_{t+1}Q_t = \sigma_t\varepsilon^*_{t+1}$.

The first component, ω_1, represents loan returns of riskiness σ_t controlling for the variance of idiosyncratic shock (when τ is taken as given). The bank trades off the benefits from limited liability and deposit insurance with a smaller profitability of riskier projects. The term $\frac{\xi\sigma_t}{Q_t}$ reflects, in expectation, the reduction of loan returns for the bank holding σ_t share of risky projects. The bank receives net income on loans, $R^s_{t+1} - R^d_t\left(1-\gamma_t\right) - \frac{\xi\sigma_t}{Q_t}$, if it does not default on deposits which happens with probability $1 - G(\varepsilon^*_{t+1})$. If the bank defaults, it gets zero, i.e. $0 \cdot G(\varepsilon^*_{t+1})$ which is not shown in the expression explicitly.

The second component, ω_2, represents the extra effect of σ_t on expected net worth owing to more dispersed returns from projects. In fact, ω_2 is strictly increasing in τ: the bank views projects as a call option the value of which rises with volatility associated with higher upside. Limited liability bounds the payoff to zero in the worst case scenario.

Risk-taking incentives depend on the difference between returns on safe loans and returns on deposits. Table D1 illustrates the effects of greater risk-taking on the components of dividends for each realization of the aggregate returns. We map aggregate returns into states of nature and consider two cases depending on the sign of ε^*_{t+1}. The aggregate returns influence the value of the shield of limited liability. Risk amplifies the effect of the idiosyncratic shock. So, in every state of nature, the bank's choice of risk is determined by the expected effect of the idiosyncratic shock on the value of the shield of limited liability and returns on loans. The up-turn arrow, $\Uparrow$, indicates that greater risk-taking increases the corresponding component of bank's dividends. The down-turn arrow, $\Downarrow$, means that the corresponding component of bank's dividends decreases with greater risk-taking. Two arrows turned in the opposite directions, $\Uparrow\Downarrow$, signify that the effect of greater risk-taking is undetermined and depends on the parameterization.

Table D1: Illustrating the Effects of Higher Risk on Dividends.

<table>
<tr><th rowspan="2">States of nature where</th><th colspan="2">Effects on ω_1</th><th rowspan="2">Effects on ω_2</th></tr>
<tr><th>$R^s_{t+1} - R^d_t\,(1-\gamma_t) - \frac{\xi\sigma_t}{Q_t}$</th><th>$1 - G(\varepsilon^*_{t+1})$</th></tr>
<tr><td>$R^s_{t+1} < R^d_t\,(1-\gamma_t) \quad \Leftrightarrow \quad \varepsilon^*_{t+1} > 0$</td><td>$\Downarrow$</td><td>$\Uparrow$</td><td>$\Uparrow$</td></tr>
<tr><td rowspan="2">$R^s_{t+1} > R^d_t\,(1-\gamma_t) \quad \Leftrightarrow \quad \varepsilon^*_{t+1} < 0$</td><td rowspan="2">$\Downarrow$</td><td rowspan="2">$\Downarrow$</td><td>if $\varepsilon^*_{t+1} > -\xi$, then $\Uparrow\Downarrow$</td></tr>
<tr><td>if $\varepsilon^*_{t+1} \leqslant -\xi$, then $\Uparrow$</td></tr>
</table>

First, $\varepsilon^*_{t+1} > 0$ indicates that the bank makes losses on safe loans. It happens in those states of nature where the net income from the zero-risk portfolio is negative, so the bank is behind the shield of limited liability. By accepting more risk, the bank is more likely to get a positive net return under a favorable realization of the idiosyncratic shock as risk acts like a leverage on the size of the shock. Therefore, $1 - G(\varepsilon^*_{t+1})$ rises. This balances with smaller returns on a portfolio with more risky loans, i.e. $R^s_{t+1} - R^d_t\,(1-\gamma_t) - \frac{\xi\sigma_t}{Q_t}$ goes down. Similarly, gambling on more dispersed returns allows the bank to move away from a zero return that comes from the limited liability to some positive return that is accompanied by less frequent defaults. So, the effect of σ_t on expected dividends from ω_2 is positive.

Second, $\varepsilon^*_{t+1} < 0$ shows that the bank makes positive profits on safe loans. The bank is more likely to default when it takes on more risk because any negative idiosyncratic shock would be amplified by risk. The bank internalizes that riskier projects are less profitable. Therefore, the overall effect of greater risk on ω_1 is negative when $\varepsilon^*_{t+1} < 0$.

Then consider the bonus from projects volatility. If $-\xi < \varepsilon^*_{t+1} < 0$, there are two contrasting forces. On the one hand, the bank always benefits from limited liability that makes the variance of projects returns attractive. On the other hand, the bank is more concerned about (and more vulnerable to) the variability of returns in the situation when taking on more risk would result in zero payoff instead of some positive payoff achieved by smaller risk. It occurs when $-\xi < \varepsilon^*_{t+1} < 0$. In these states of nature, the bank requires greater than average realization of the idiosyncratic shock in order to get a positive return. Call this type of shock a good idiosyncratic shock. This shock happens with probability smaller than 0.5. Define a bad idiosyncratic shock as a complement to a good idiosyncratic shock. An increase in risk increases the profits under a good shock. It captures the benefits from greater upside. At the same time, an increase in risk makes it more likely to get a bad shock. The bank trades off marginal profits coming from a good shock with marginal losses coming from the reduction of profits due to more defaults. Since the probability of the latter is greater than the probability of the former, the losses from defaults can dominate the benefits from greater volatility. This force goes in the opposite direction when $\varepsilon^*_{t+1} \leqslant -\xi$. The difference is that here the bank is more likely to get a good shock than a bad shock. Therefore, the bank puts more weight on the benefits from risk-taking than on its costs. It is verified mathematically that the effects of σ_t on ω_2 are unambiguously positive when $\varepsilon^*_{t+1} \leqslant -\xi$.

In sum, we find that net returns on safe loans, $R^s_{t+1} - R^d_t\,(1-\gamma_t)$, is the main driver for the bank's choice of risk. In the partial-equilibrium setting, we differentiate between three cases that characterize incentives for risk-taking.

First, $R^s_{t+1} < R^d_t\,(1-\gamma_t)$ applies to the states of nature where a relatively large negative aggregate shock is realized. Two forces against the one that seems to be of lesser relevance make the bank benefit most from taking risk. Second, $-\xi < R^d_t\,(1-\gamma_t) - R^s_{t+1} < 0$ applies to the states of nature where intermediate values (not too large and not too small) of either negative or positive aggregate shock are realized. There are more forces that lower incentives for risk. Third, $R^d_t\,(1-\gamma_t) - R^s_{t+1} < -\xi$ applies to the states of nature where a positive aggregate shock of a larger size is realized. Interestingly, there is a force associated with the bonus from projects volatility that makes it possible for the bank to increase risk. The choice of risk depends on the strength of that force, ω_2, relative to the negative exposure of returns from a loan portfolio to risk, ω_1. It still remains a quantitative question to find out how risk-taking is determined in the general equilibrium set-up.

Capital requirements affect risk-taking through a change in ε^*_{t+1}. When γ_t increases, ε^*_{t+1} falls. It means that the bank will be more likely to find itself in the states of nature where ε^*_{t+1} is negative. It forces the bank to keep more skin in the game, make the shield of limited liability less attractive and prevent the switch into financing risky projects.

E Consumption Equivalent Variation

Let $Welf^{opt}$ be the welfare level attained under the optimal policy:

$$Welf^{opt} = E\sum_{t=0}^{\infty}\beta^t\left[\frac{\left(C_{\text{opt},t}-\kappa C_{\text{opt},t-1}\right)^{1-\varsigma_c}-1}{1-\varsigma_c}+\varsigma_0\frac{D_{\text{opt},t}^{1-\varsigma_d}-1}{1-\varsigma_d}\right].$$

And let $Welf^{rule}$ be the welfare level attained under a simple rule:

$$Welf^{rule} = E\sum_{t=0}^{\infty}\beta^t\left[\frac{\left(C_{\text{rule},t}-\kappa C_{\text{rule},t-1}\right)^{1-\varsigma_c}-1}{1-\varsigma_c}+\varsigma_0\frac{D_{\text{rule},t}^{1-\varsigma_d}-1}{1-\varsigma_d}\right].$$

For given paths of consumption and deposits, we are interested in sizing a permanent tax Δ applied to the consumption utility stream under the optimal policy such that the level of welfare under the optimal policy with the tax is equal to the level of welfare under the suboptimal rule. Thus,

$$E\sum_{t=0}^{\infty}\beta^t\left[\frac{\left((1-\Delta)\left(C_{\text{opt},t}-\kappa C_{\text{opt},t-1}\right)\right)^{1-\varsigma_c}-1}{1-\varsigma_c}+\varsigma_0\frac{D_{\text{opt},t}^{1-\varsigma_d}-1}{1-\varsigma_d}\right]=Welf^{rule},$$

which can be rewritten as

$$E\sum_{t=0}^{\infty}\beta^t\left[\frac{(1-\Delta)^{1-\varsigma_c}\left(C_{\text{opt},t}-\kappa C_{\text{opt},t-1}\right)^{1-\varsigma_c}-1}{1-\varsigma_c}+\varsigma_0\frac{D_{\text{opt},t}^{1-\varsigma_d}-1}{1-\varsigma_d}\right]=Welf^{rule}.$$

Taking out $(1-\Delta)^{1-\sigma_c}$, we get

$$E\sum_{t=0}^{\infty}\beta^t\left[(1-\Delta)^{1-\varsigma_c}\,\frac{\left(C_{\text{opt},t}-\kappa C_{\text{opt},t-1}\right)^{1-\varsigma_c}-1}{1-\varsigma_c}+\varsigma_0\frac{D_{\text{opt},t}^{1-\varsigma_d}-1}{1-\varsigma_d}+\frac{(1-\Delta)^{1-\varsigma_c}-1}{1-\varsigma_c}\right]=Welf^{rule}.$$

With some additional manipulations, we obtain

$$\begin{aligned}E\sum_{t=0}^{\infty}\beta^t\Bigg[\left((1-\Delta)^{1-\varsigma_c}-1\right)\frac{\left(C_{\text{opt},t}-\kappa C_{\text{opt},t-1}\right)^{1-\varsigma_c}-1}{1-\varsigma_c}+\frac{\left(C_{\text{opt},t}-\kappa C_{\text{opt},t-1}\right)^{1-\varsigma_c}-1}{1-\varsigma_c}+\\ \varsigma_0\frac{D_{\text{opt},t}^{1-\varsigma_d}-1}{1-\varsigma_d}+\frac{(1-\Delta)^{1-\varsigma_c}-1}{1-\varsigma_c}\Bigg]=Welf^{rule}.\end{aligned}$$

Let $Welf_C^{opt}$ be the welfare from the consumption utility stream attained under the optimal policy,

$$Welf_C^{opt} = E\sum_{t=0}^{\infty}\beta^t\left[\frac{\left(C_{\text{opt},t}-\kappa C_{\text{opt},t-1}\right)^{1-\varsigma_c}-1}{1-\varsigma_c}\right].$$

Hence,

$$\left((1-\Delta)^{1-\varsigma_c}-1\right)Welf_C^{opt}+Welf^{opt}+\frac{(1-\Delta)^{1-\varsigma_c}-1}{(1-\beta)(1-\varsigma_c)}=Welf^{rule}.$$

From the equation above, we can derive Δ, the consumption equivalent variation:

$$\left((1-\Delta)^{1-\varsigma_c}-1\right)\left(Welf_C^{opt}+\frac{1}{(1-\beta)(1-\varsigma_c)}\right)+Welf^{opt}=Welf^{rule},$$

$$(1-\Delta)^{1-\varsigma_c}-1=\frac{Welf^{rule}-Welf^{opt}}{\left(Welf_C^{opt}+\frac{1}{(1-\beta)(1-\varsigma_c)}\right)},$$

$$(1-\Delta)^{1-\varsigma_c}=1-\frac{Welf^{opt}-Welf^{rule}}{\left(Welf_C^{opt}+\frac{1}{(1-\beta)(1-\varsigma_c)}\right)},$$

$$\Delta=1-\left(1-\frac{Welf^{opt}-Welf^{rule}}{\left(Welf_C^{opt}+\frac{1}{(1-\beta)(1-\varsigma_c)}\right)}\right)^{\frac{1}{1-\varsigma_c}}.$$

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