Supplemental Appendix

An Experimental Implementation of the Core

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A.1. **New Experiment with an Empty Core.** This subsection reports the efficiency, duration, and average payoff when the core is empty. It is included at the request of an anonymous reviewer. Two new treatments are added with the following parameters. In Game (a), $v(AB) = 140$, $v(AC) = 100$, $v(BC) = 90$; in Game (b), $v(AB) = 140$, $v(AC) = 120$, $v(BC) = 70$. The value of the grand coalition is still 150, and an individual player gets nothing. In other words, compared to Game 1(a) and Game 1(b) in the main experiment, the value of each two-player coalition is increased by 10 points. The two new games are named E(a) and E(b), because the core is Empty. In E(a), the nucleolus and Shapley value are equal to Games 1(a) and 3(a); In E(b), the nucleolus and Shapley value are equal to Games 1(b) and 3(b). The new experimental sessions are also conducted at Nanjing Audit University. A session has 27 or 18 participants, and each new treatment has eight matching groups.[30]

The efficiency is plotted in Figure A.1. It is calculated by the sum of the three players' obtained payoffs divided by 150 points. Scenario 1 is indicated by two blue lines; Scenario 2 is indicated by two red lines; Scenario 3 is indicated by two green lines; the new games with empty cores are indicated by two khaki lines. The two khaki lines of E(a) and E(b) remain at the bottom of the graph. Their average efficiency levels are 74.38% and 76.18%, respectively. However, the average efficiency levels of 1(a) and 1 (b) are 83.71%, 83.89%, and those of 3(a) and 3 (b) are 93.38% and 92.42%. The efficiency level in 2(a) is 92.82%, closer to Scenario 3, whereas that in 2(b) is 86.10%, closer to Scenario 1. Overall, the efficiency appears to increase with the area of the core, but its change is not as large as, e.g. the frequency of the grand coalition displayed in Table A.2.[31]

This part also presents the time when the whole coalition formation process ends. It complements Figure 6, which plots the total number of submitted offers. Figure A.2 plots the finishing time in each treatment.[32] As several rounds of negotiation occur, the game does not end within a few seconds after starting. The relative ranking is generally consistent with the total number of submitted offers in Figure 6. The finishing times for E(a) and E(b) are at their lowest levels. Particularly during the last 15 periods, the durations are 19.62 and 18.67 seconds, while the bargaining process in the other treatments lasts over 20 seconds. The two blue lines of Scenario 1 remain around 30 seconds. The other four lines fluctuate between 30 and 90 seconds. As the area of the core expands and the grand coalition is formed more often, participants appear to need more time to negotiate. In addition, the process ends more quickly in the last five periods, as the durations of these four treatments are mostly below 60

[30]The new experiment does not have nine groups because I have exhausted the subject pool at Nanjing Audit University, as the experiment already involves 630 students. Eight matching groups are sufficient for the behaviour in the Appendix.

[31]Not all of the pairwise non-parametric tests are statistically significant for the efficiency reported in this paragraph and the finishing time reported in the next paragraph.

[32]Games terminated after 200 seconds are not taken into account.

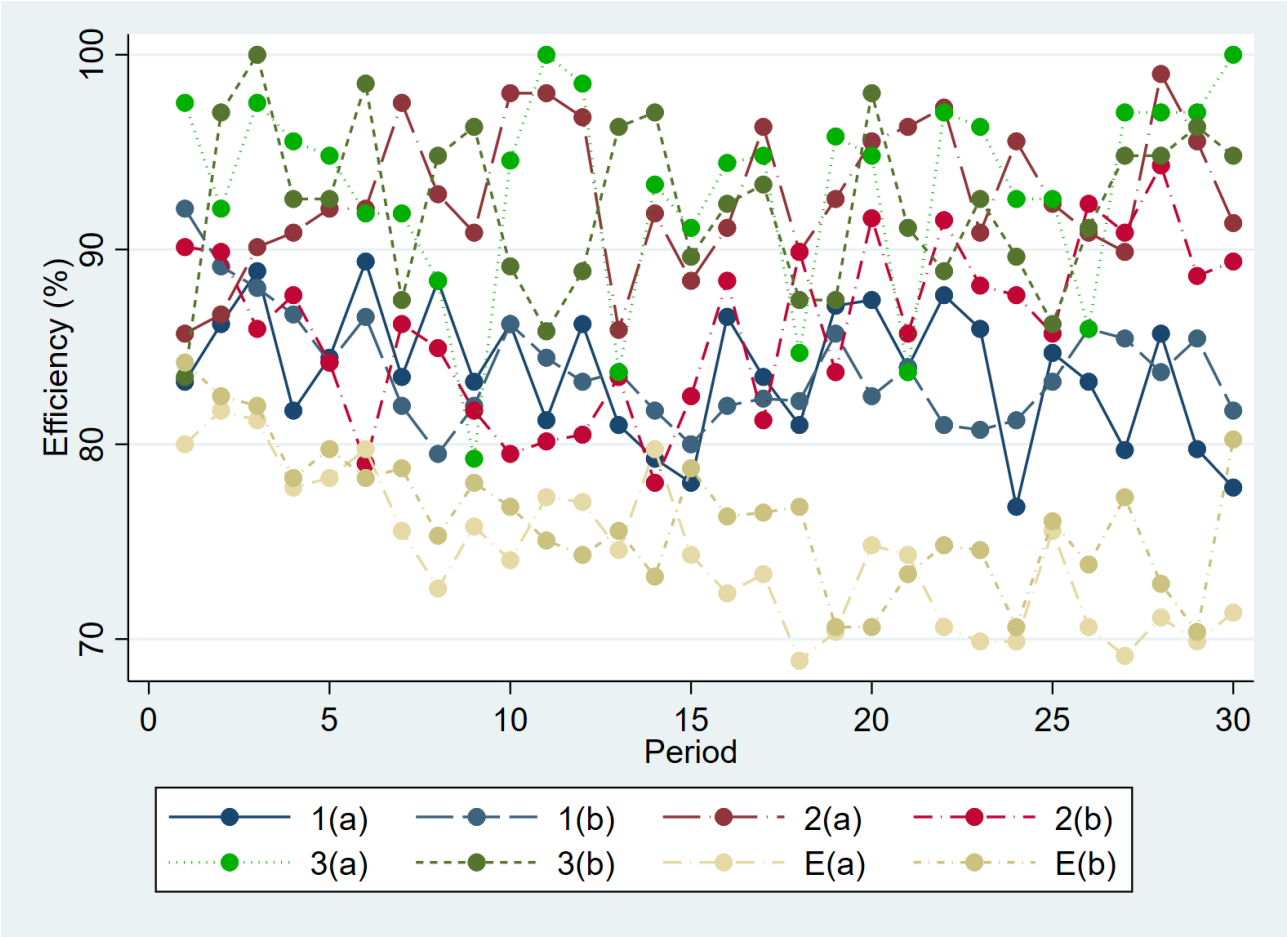


FIGURE A.1. Efficiency over Periods

seconds. Perhaps participants have gradually learned the acceptable amount over previous periods.

Finally, I have added the average payoffs in coalitions in Table A.1 as supplementary information. As the core is empty, the results do not support any of the solution concepts. Because the two-player coalitions now have high values, players frequently walk away from the grand coalition. The sum of the payoffs obtained by the included players exceeds the core's prediction.

A.2. **Average Payoffs.** Table A.2 lists players' average payoffs in each formed coalition in all periods. It is included in the Appendix to conserve space. Table A.2 supplements Section 5.1 and Section 5.2 by providing specific information on the average payoffs and formed coalitions. The numbers in parentheses are the standard deviation. The last row of each segment displays the average payoffs of only the included players.

A.3. **Results of All Players.** Table A.3 lists the average payoffs obtained for Players A, B, and C, in each treatment. The payoffs are taken into consideration whether the players are included in a coalition, excluded from any coalition, or unable to reach an agreement within the time limit. Because zero payoffs lower players' average payoffs, the numbers in Table A.3 differ from any of the solution concepts. As explained in the main text, the core, nucleolus and Shapley value all assume the sum of the three players' payoffs is 150 points.

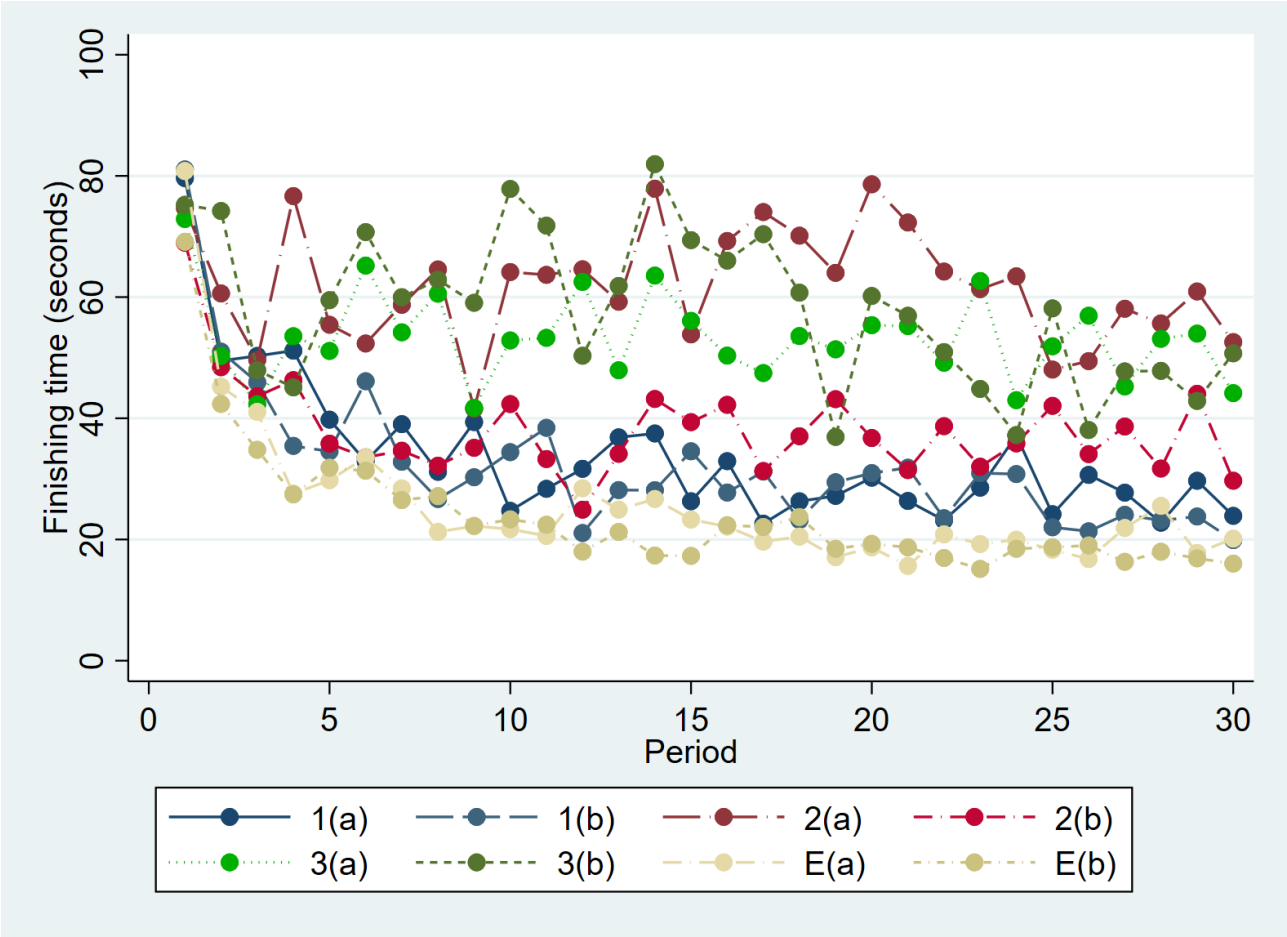


FIGURE A.2. Finishing Time

TABLE A.1. Average Payoffs with an Empty Core in All Periods

E(a)	**Freq.**	**A**	**B**	**C**	**E(b)**	**Freq.**	**A**	**B**	**C**
AB	48.89%	74.55	65.36		AB	47.78%	84.67	55.33	
		(7.19)	(7.23)				(13.00)	(13.00)	
AC	26.67%	78.44		21.41	AC	39.17%	89.10		30.88
		(7.79)		(7.78)			(10.03)		(10.06)
BC	9.86%		63.30	26.70	BC	6.11%		40.34	29.66
			(12.57)	(12.57)				(7.73)	(7.73)
ABC	14.44%	57.00	53.93	38.88	ABC	6.94%	64.80	49.20	35.80
		(11.43)	(9.91)	(16.30)			(15.18)	(9.86)	(16.08)
None	0.14%				None	0.00%			
Avg.	–	72.88	62.82	27.38	Avg.	–	85.05	53.13	31.47
(inclusion)		(10.86)	(9.75)	(13.89)	(inclusion)		(13.48)	(13.09)	(10.83)
Avg.	–	65.69	46.05	13.98	Avg.	–	79.85	32.32	16.39
(all)		(24.08)	(16.91)	(10.96)	(all)		(24.21)	(27.89)	(17.57)

Note. The numbers in the parentheses are the standard deviation. A blank space means the player is not included in the coalition. A "–" means the number cannot be calculated.

Section 5.3 has reported the distance to each solution concept using payoffs in grand coalitions. This section provides the distance using payoffs in both two-player and three-player coalitions as supplementary information.[33] The excluded player gets nothing, and her payoff is far from any of the solution concepts. The points chosen to calculate the

[33]The data does not include games where no agreement is reached within the time limit.

TABLE A.2. Average Payoffs in Formed Coalitions in All Periods

1(a)	**Freq.**	**A**	**B**	**C**	**1(b)**	**Freq.**	**A**	**B**	**C**
AB	27.65%	68.97	60.85		AB	42.22%	79.98	49.65	
		(5.77)	(6.34)				(11.02)	(11.01)	
AC	21.36%	70.05		19.89	AC	27.28%	83.78		26.15
		(7.20)		(7.16)			(8.71)		(8.75)
BC	8.02%		57.51	22.48	BC	5.06%		35.61	24.39
			(10.81)	(10.81)				(9.89)	(9.89)
ABC	42.72%	65.55	60.59	23.77	ABC	25.31%	77.16	43.37	29.40
		(7.68)	(7.32)	(12.28)			(19.07)	(10.59)	(13.49)
None	0.25%				None	0.12%			
Avg.	–	67.63	60.37	22.48	Avg.	–	80.32	46.48	27.42
(inclusion)		(7.31)	(7.48)	(10.96)	(inclusion)		(13.37)	(11.54)	(11.29)
2(a)	**Freq.**	**A**	**B**	**C**	**2(b)**	**Freq.**	**A**	**B**	**C**
AB	16.79%	67.48	62.45		AB	33.33%	73.79	56.21	
		(4.12)	(4.26)				(10.77)	(10.77)	
AC	4.44%	67.36		12.64	AC	23.46%	77.25		22.73
		(8.70)		(8.70)			(8.33)		(8.33)
BC	0.37%		40.00	20.00	BC	1.60%		20.85	19.15
			(10.00)	(10.00)				(8.72)	(8.72)
ABC	75.68%	69.01	65.66	14.49	ABC	41.11%	79.84	50.58	22.94
		(5.74)	(5.91)	(7.96)			(14.36)	(12.39)	(10.66)
None	2.72%				None	0.49%			
Avg.	–	68.67	65.94	14.41	Avg.	–	77.16	49.71	22.77
(inclusion)		(5.70)	(6.56)	(8.02)	(inclusion)		(12.17)	(12.62)	(9.76)
3(a)	**Freq.**	**A**	**B**	**C**	**3(b)**	**Freq.**	**A**	**B**	**C**
AB	4.81%	46.26	43.74		AB	3.82%	44.19	45.87	
		(4.26)	(4.26)				(2.98)	(2.98)	
AC	0.62%	27.20		22.80	AC	0.37%	40.00		30.00
		(16.02)		(16.02)			(8.66)		(8.66)
BC	0.37%		26.67	13.33	BC	0.12%		10.00	10.00
			(11.55)	(11.55)				–	–
ABC	89.88%	50.94	50.58	48.48	ABC	89.75%	51.54	50.25	48.20
		(3.33)	(2.82)	(4.59)			(4.10)	(2.06)	(3.88)
None	4.32%				None	5.93%			
Avg.	–	50.55	50.14	48.13	Avg.	–	51.20	50.01	48.08
(inclusion)		(4.16)	(3.62)	(5.69)	(inclusion)		(4.38)	(2.70)	(4.30)

Note. The numbers in the parentheses are the standard deviation. A blank space means the player is not included in the coalition. A "–" means the number cannot be calculated.

TABLE A.3. Players' All Payoffs During All Periods

	Freq.	**A**	**B**	**C**		**Freq.**	**A**	**B**	**C**
1(a)	–	62.04	47.32	16.21	**1(b)**	–	76.16	33.74	15.81
		(19.91)	(25.73)	(13.73)			(22.07)	(22.96)	(16.04)
2(a)	–	66.55	60.96	11.60	**2(b)**	–	75.54	38.46	15.07
		(13.14)	(17.87)	(9.19)			(16.36)	(24.16)	(13.39)
3(a)	–	48.18	47.66	43.76	**3(b)**	–	48.10	46.86	43.39
		(11.44)	(11.43)	(14.89)			(12.93)	(12.43)	(14.84)

distance to the core in Scenarios 2 and 3 are set to be the same as in Section 5.3. Table A.4 presents the distance from the obtained payoff to the core, nucleolus, Shapley value, and the

equal split. The number in each cell is larger than Table 3. Nevertheless, the distance to the core remains the smallest in each treatment.

TABLE A.4. Distance to Each Solution Concept Using Data in All Coalitions During All Periods

	Core	**Nucleolus**	**Shapley value**	**Equal Split**
1(a)	29.80	29.80	36.83	47.56
1(b)	33.46	33.46	40.60	55.65
2(a)	11.09	18.01	25.47	46.91
2(b)	28.00	36.05	32.75	52.88
3(a)	5.28	36.15	19.12	5.28
3(b)	5.27	49.28	24.94	5.27

A.4. **Behaviour During Periods 16-30.** Table A.5 displays the payoffs obtained during Periods 16-30. Compared to Table A.2, which lists the payoffs in all periods, the payoffs in Periods 16-30 are further away from the equal split in Scenarios 1 and 2. In particular, in 1(b), 2(a), and 2(b), the payoffs in Periods 16-30 coincide better with the core or fall closer to its edge than in Periods 1-30. In Scenarios 3, the payoffs are slightly more different from the equal split compared to Table A.2. Nevertheless, they remain in the core, and 50-50-50 remains the most frequently accepted offer. Overall, the data in Periods 16-30 are even closer to the core's prediction.

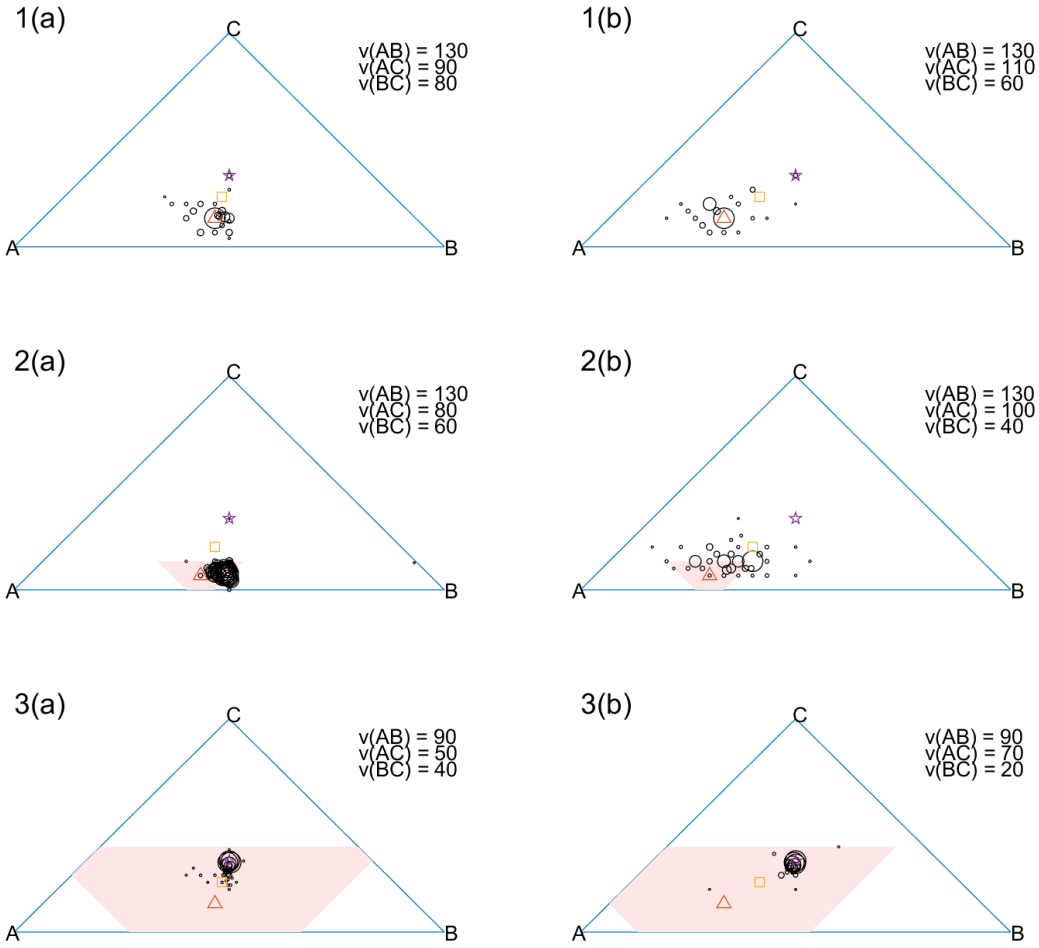
Figure A.3 plots the obtained payoffs in grand coalitions in Periods 16-30. The general pattern is consistent with Figure 5, which plots payoffs in all periods. In fact, the behaviour during the second half of the experiment gives stronger support for the core. In each treatment, the most frequent points coincide with the core or fall within its range. Also, the payoff allocations in Periods 16-30 become more concentrated as participants are still learning and converging at the start of the experiment. Compared with Figure 5, results in Periods 16-30 better prove that the core correctly predicts the average payoffs in grand coalitions.

A.5. **Other Coalition Formation Process.** This section presents the payoffs proposed in the first offer averaged across all coalitions. In Table A.6, the first proposed payoffs are similar to the accepted payoffs displayed in Table A.2. The payoff difference for each player in each treatment is less than 4.56 points. Perhaps the accepted payoffs in the previous period have an anchoring effect for the first offer in the present period. It is worth noting that the acceptance rate for the first offer is similar to the acceptance rate for the overall offer in Table 4.

TABLE A.5. Average Payoffs in Formed Coalitions in Periods 16-30

1(a)	**Freq.**	**A**	**B**	**C**	**1(b)**	**Freq.**	**A**	**B**	**C**
AB	18.77%	71.58	57.89		AB	34.57%	87.64	42.21	
		(4.83)	(6.37)				(8.45)	(8.61)	
AC	23.21%	70.94		19.06	AC	25.93%	87.94		21.91
		(4.55)		(4.55)			(6.92)		(6.95)
BC	10.12%		57.24	22.76	BC	8.89%		37.08	22.92
			(9.91)	(9.91)				(8.89)	(8.89)
ABC	47.90%	67.40	60.74	21.72	ABC	30.62%	84.60	38.95	26.37
		(6.69)	(7.02)	(10.00)			(14.93)	(7.16)	(11.03)
None	0.00%				None	0.00%			
Avg.	–	69.18	59.58	21.09	Avg.	–	86.70	40.25	24.14
(inclusion)		(6.13)	(7.45)	(8.86)	(inclusion)		(10.83)	(8.27)	(9.52)
2(a)	**Freq.**	**A**	**B**	**C**	**2(b)**	**Freq.**	**A**	**B**	**C**
AB	6.91%	67.54	62.46		AB	23.46%	79.20	50.80	
		(2.86)	(2.86)				(10.44)	(10.44)	
AC	3.95%	65.94		14.06	AC	17.53%	81.06		18.94
		(10.15)		(10.15)			(7.17)		(7.17)
BC	0.25%		40.00	20.00	BC	2.72%		21.00	19.00
			-	-				(8.43)	(8.43)
ABC	85.43%	69.94	67.66	12.41	ABC	55.80%	85.50	44.05	20.39
		(4.64)	(4.51)	(3.70)			(9.93)	(9.22)	(5.58)
None	3.46%				None	0.49%			
Avg.	–	69.60	67.19	12.50	Avg.	–	83.17	45.23	20.01
(inclusion)		(4.96)	(4.82)	(4.20)	(inclusion)		(10.00)	(10.95)	(6.11)
3(a)	**Freq.**	**A**	**B**	**C**	**3(b)**	**Freq.**	**A**	**B**	**C**
AB	4.44%	45.28	44.72		AB	3.70%	45.07	44.93	
		(1.18)	(1.18)				(0.26)	(0.26)	
AC	0.49%	27.50		22.50	AC	0.49%	35.00		35.00
		(3.54)		(3.54)			(0.00)		(0.00)
BC	0.25%		40.00	0.00	BC	0.00%		–	–
			–	–				–	–
ABC	90.62%	51.24	50.61	48.14	ABC	89.38%	51.38	50.27	48.36
		(3.11)	(2.26)	(4.18)			(3.55)	(1.69)	(3.40)
None	4.20%				None	6.42%			
Avg.	–	50.84	50.31	47.87	Avg.	–	51.04	50.06	48.29
(inclusion)		(3.70)	(2.59)	(5.21)	(inclusion)		(3.87)	(1.95)	(3.52)

Note. The numbers in the parentheses are the standard deviation. A blank space means the player is not included in the coalition. A "–" means the number cannot be calculated.



Note. The frequency of payoff allocations is indicated by the size of circles.

FIGURE A.3. Payoff Allocations in Grand Coalitions During Periods 16-30

TABLE A.6. Average Payoffs of Included Players in the First Proposed Offer During All Periods

	A	**B**	**C**
1(a)	66.27	59.45	25.65
	(8.68)	(8.10)	(12.96)
1(b)	77.23	48.04	30.01
	(13.88)	(11.94)	(11.87)
2(a)	67.85	65.20	15.89
	(7.85)	(7.11)	(10.85)
2(b)	73.99	52.63	25.35
	(13.96)	(11.91)	(13.60)
3(a)	51.48	50.56	47.64
	(5.77)	(4.47)	(6.32)
3(b)	52.63	50.07	47.06
	(6.34)	(4.48)	(6.28)