Climate Policy and the Long-run Interest Rate
Supplementary Appendix
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1 Value added production function

To determine TFP, we calculate the value-added production function, equal to the production function for gross output minus the optimized values of the intermediate energy inputs. Profit maximization for the representative producer of energy services yields the following relative demands for each type of energy:

$$p_t^i + \tau_t = \gamma_i p_t^E \left(\frac{E_t}{E_t^i}\right), \quad i = 1, 2, 3; \quad p_t^0 = \gamma_0 p_t^E \left(\frac{E_t}{E_t^0}\right), \tag{1}$$

and the price index for energy services

$$p_t^E = \tilde{\gamma}(p_t^0)^{\gamma_0} \prod_{i=1}^{3} (p_t^i + \tau_t)^{\gamma_i}. \tag{2}$$

Variable $\tilde{\gamma} = \prod_{i=0}^{3} \gamma_i^{-\gamma_i}$ is a constant. We use g_{p_E} to denote the growth rate of the price of energy services. The zero-profit condition for the energy-services producer implies that

$$p_t^E E_t = p_t^0 E_t^0 + \sum_{i=1}^{3} (p_t^i + \tau_t) E_t^i. \tag{3}$$

Profit maximization for the final good producer yields the relative demand for energy services and capital,

$$p_t^E = \nu K_t^{\alpha} E_t^{\nu-1} \left(A_t L_t\right)^{1-\alpha-\nu}, \tag{4}$$

$$R_t = \alpha K_t^{\alpha-1} E_t^{\nu} \left(A_t L_t\right)^{1-\alpha-\nu}, \tag{5}$$

where R_t is the rental rate and $r_t = R_t - \delta$. Combining (4) with (2), (3), and the production function for gross outputyields the value-added production function,

$$Y_t = \left(1 - \nu \left(\gamma_0 + \sum_{i=1}^{3} \gamma_i \frac{p_t^i}{\tau_t + p_t^i}\right)\right) \nu^{\frac{\nu}{1-\nu}} \left(p_t^E\right)^{\frac{-\nu}{1-\nu}} K_t^{\tilde{\alpha}} \left(A_t L_t\right)^{1-\tilde{\alpha}}. \tag{6}$$

Constant $\tilde{\alpha} \equiv \frac{\alpha}{1-\nu}$ is the share of value-added paid to capital.

2 Balanced Growth

Definition 1. *A* balanced growth path (BGP) *is a path along with C_t and K_t grow at constant rates, g_C^* and g_K^*. We use asterisks $(*)$ to denote BGP values. An* asymptotic balanced growth path *(ABGP) is a BGP that cannot be reached with finite prices and quantities.*

The ABGP is the steady state of the model. Rebating the carbon tax revenue implies that this steady state only occurs in the limit. As is often the case the macro-energy literature, we interpret the ABGP as describing the behavior of the economy at decadal time scales (e.g., Hassler et al., 2021).

Starting with the the value-added production function, we solve for the ABGP using the standard steps. We define the intensive form as

$$z_t = \frac{Z_t}{TFP_t^{\frac{1}{1-\tilde{\alpha}}} L_t}, \quad Z_t \in \{C_t, Y_t, K_t\}.$$

Let $g_{TFP,t}$ be the growth rate of TFP. Then, the dynamics of the economy in intensive form equal:

$$\begin{aligned}
y_t &= k_t^{\tilde{\alpha}}, \\
k_{t+1} &= \frac{y_t - c_t + (1-\delta)k_t}{(1+g_{TFP,t})^{\frac{1}{1-\tilde{\alpha}}}} \\
c_{t+1} &= \left(\frac{\beta(1+r_{t+1})}{(1+g_{TFP,t})^{\frac{\sigma}{1-\tilde{\alpha}}}} c_t^{\sigma} \right)^{\frac{1}{\sigma}} \\
r_t &= \tilde{\alpha} k_t^{\tilde{\alpha}-1} - \delta.
\end{aligned}$$

On the ABGP, g_{TFP}, y_t, k_t, c_t and r_t are constant. The interest rate on the ABGP, r^*, equals:

$$r^* = \beta^{-1}(1+g_{TFP}^*)^{\frac{\sigma}{1-\tilde{\alpha}}} - 1, \tag{7}$$

and the remaining variables are given by

$$
\begin{aligned}
k^* &= \left(\frac{r^* + \delta}{\tilde{\alpha}}\right)^{\frac{1}{\tilde{\alpha}-1}} \\
y^* &= (k^*)^{\tilde{\alpha}} \\
c^* &= y^* - (1 + g^*_{TFP})^{\frac{1}{1-\tilde{\alpha}}} k^* + (1-\delta)k^*.
\end{aligned}
$$

3 Derivation of Proposition 1

Any effect of climate policy on the long-run interest rate must occur though the effect of the policy on the long-run growth rate of TFP. We re-write the expression for the long-run real interest rate in (7) as:

$$\ln(1 + r^*) = \frac{\sigma}{1 - \tilde{\alpha}} \ln(1 + g^*_{TFP}) - \ln(\beta). \tag{8}$$

The growth rate of TFP, in turn, depends on the growth rate of technology and the growth rate of the price of energy services. From the TFP expression in the main text we have:

$$\ln(1 + g^*_{TFP}) = (1 - \tilde{\alpha}) \ln(1 + g_A) - \frac{\nu}{1 - \nu} \ln(1 + g^*_{P_E}). \tag{9}$$

The growth rate of technology, g_A, is exogenous to our model. However, the growth rate of the price of energy services, (2), depends on the growth rate of the price of clean energy and on the growth rates of the tax-inclusive price of each type of fossil energy. On the ABGP, the growth rate of the tax-inclusive fossil-energy prices will either equal the growth rate of the carbon tax or the growth rate of the underlying fossil-energy price, whichever is largest.

We use this intuition to derive the effect of a carbon tax on the long-run interest rate as a function of the growth rates of fossil-energy prices and the growth rate of the tax. First, the growth rate of the price of energy services on the ABGP equals:

$$\ln(1 + g^*_{P_E}) = \begin{cases} \gamma_0 \ln(1 + g_{p^0}) + \sum_{i=1}^{3} \gamma_i \ln(1 + g_{p^i}) & \text{if } g_{p^1} > g_\tau \\ \gamma_0 \ln(1 + g_{p^0}) + \gamma_1 \ln(1 + g_\tau) + \sum_{i=2}^{3} \gamma_i \ln(1 + g_{p^i}) & \text{if } g_{p^2} > g_\tau \geq g_{p^1} \\ \gamma_0 \ln(1 + g_{p^0}) + \sum_{i=1}^{2} \gamma_i \ln(1 + g_\tau) + \gamma_3 \ln(1 + g_{p^3}) & \text{if } g_{p^3} > g_\tau \geq g_{p^2} \\ \gamma_0 \ln(1 + g_{p^0}) + \sum_{i=1}^{3} \gamma_i \ln(1 + g_\tau) & \text{if } g_\tau \geq g_{p^3}. \end{cases} \tag{10}$$

We combine (8), (9) and (10) and apply small value approximations to derive the effect of a change in the growth rate of the carbon tax on the long-run interest rate given in Proposition 1.

4 Derivation of Proposition 2

Since the solution to this problem is well-known, we provide a relatively quick proof. The strategy is to derive the first-order conditions for the optimal allocation and then compare them to first-order conditions for the decentralized equilibrium. In the absence of policy, the decentralized equilibrium ignores the dirty energy use target, $E^{\max}$. This is the only market failure, and it can be corrected with a Pigouvian tax on on dirty energy. The marginal external cost of dirty energy is equal to the shadow value on the constraint imposed by the target.

The Lagrangian for the social planner's problem can be written as

$$\begin{aligned}\mathcal{L} = \sum_{t=0}^{\infty} \beta^t u \left(K_t^{\alpha} \left(A_t L_t\right)^{1-\alpha-\nu} \left(\prod_{i=0}^{3} (E_t^i)^{\gamma_i} \right)^{\nu} - \sum_{i=0}^{3} p_t^i E_t^i - K_{t+1} + (1-\delta) K_t \right) \\ -\Omega \left(\left(\sum_{t=0}^{\infty} \sum_{i=1}^{3} E_t^i \right) - E^{\max} \right). \end{aligned} \tag{11}$$

The first order condition for K_{t+1} is

$$u'(C_t) = \beta(1 + r_{t+1}) u'(C_{t+1}), \tag{12}$$

where we have used (5) to write the result in terms of the interest rate in the decentralized economy.

The first order condition for flow energy use is given by

$$p_t^0 = p_t^E \gamma_0 \left(\frac{E_t}{E_t^0} \right), \tag{13}$$

$$p_t^i + \beta^{-t} (u'(C_t))^{-1} \Omega = p_t^E \gamma_i \left(\frac{E_t}{E_t^i} \right), \quad i = 1, 2, 3, \tag{14}$$

where we have used the production function for energy and (4) to write the results in terms of E_t and p_t^E in the decentralized equilibrium.

Comparing these results to the first order conditions from the decentralized equilibrium,

$$C_t^{-\sigma} = \beta(1+r_{t+1})C_{t+1}^{-\sigma}. \tag{15}$$

and (1), we observe that the optimal allocation can be implemented with a single instrument:

$$\tau_t^{\text{opt}} = \beta^{-t}(u'(C_t))^{-1}\Omega. \tag{16}$$

Combining this result with (12) gives

$$\frac{\tau_{t+1}^{\text{opt}}}{\tau_t^{\text{opt}}} - 1 \equiv g_\tau^{\text{opt}} = r_{t+1}. \tag{17}$$

The net-zero tax, τ^{nz}, is a Pigouvian tax, where the social cost of fossil energy use is given by the shadow value of the carbon budget measured in units of discounted marginal utility. Discounting implies that this value rises over time at the rate of interest.

5 Least-Cost Net-Zero Policy

We use Proposition 2 to solve for r^*_{nz} under the net-zero tax. First, we combine (8) -(10) to derive r^*_{nz} as a function of g_τ,

$$\begin{aligned}\ln(1+r^*_{nz}) =& \psi - \frac{\sigma}{1-\tilde{\alpha}} \times \frac{\nu}{1-\nu} \\ & \times \begin{cases} \gamma_0 \ln(1+g_{p^0}) + \sum_{i=1}^{3}\gamma_i \ln(1+g_{p^i}) & \text{if } g_{p^1} > g_\tau \\ \gamma_0 \ln(1+g_{p^0}) + \gamma_1 \ln(1+g_\tau) + \sum_{i=2}^{3}\gamma_i \ln(1+g_{p^i}) & \text{if } g_{p^2} > g_\tau \geq g_{p^1} \\ \gamma_0 \ln(1+g_{p^0}) + \sum_{i=1}^{2}\gamma_i \ln(1+g_\tau) + \gamma_3 \ln(1+g_{p^3}) & \text{if } g_{p^3} > g_\tau \geq g_{p^2} \\ \gamma_0 \ln(1+g_{p^0}) + \sum_{i=1}^{3}\gamma_i \ln(1+g_\tau) & \text{if } g_\tau \geq g_{p^3}, \end{cases}\end{aligned} \tag{18}$$

where $\psi \equiv \frac{\sigma}{1-\tilde{\alpha}}(1-\tilde{\alpha})\ln(1+g_A) - \ln\beta$ is common to all cases. Then, we can use the result that $g_\tau^{nz} = r^*_{nz}$ to numerically solve (18) for the r^*_{nz} under the net-zero carbon tax. This differs from the exercise with exogenous carbon taxes, because the growth rate of the carbon tax is now endogenous to r^*.

References

Hassler, J., P. Krusell, and C. Olovsson (2021): "Directed technical change as a response to natural resource scarcity," *Journal of Political Economy*, 129, 3039–3072.