Supplemental appendix to "R&D Uncertainty and Cycles"

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A Additional results on cyclical sensitivity

Table 1 reports measures of the cyclical sensitivity of $log(I/P)$, where I is a measure of investment and P is the GDP deflator, as in the main paper. Additionally, it reports measures of the cyclical sensitivity of I/K, where K is an estimate of the corresponding capital stock. Scope of data sources and frequencies of observations are reported in the caption to the table. Cyclical sensitivity is defined as the correlation of the measure with the HP-filtered component of a measure of value added for the corresponding set of firms.

A.1 Description

Let t index a year and let $\mathcal{C}_t$ denote the set of continuing firms, that is, firms active in the sample at both $t-1$ and t. Firms are indexed by j. (For our particular application, where we define the investment rate as I_t/K_t where K_t is measured as of the end of year $t-1$, this requires firms to be in sample at $t-2$, $t-1$ and t). Let $\alpha \in [0,1]$ be a fixed scalar. We write:

$$\begin{aligned}
\Delta \frac{I_t}{K_t} &\equiv \sum_{j\in\mathcal{C}_t} w_{j,t}\frac{I_{j,t}}{K_{j,t}} - \sum_{j\in\mathcal{C}_t} w_{j,t-1}\frac{I_{j,t-1}}{K_{j,t-1}} \\
&= \sum_{j\in\mathcal{C}_t} (\alpha w_{j,t} + (1-\alpha) w_{j,t-1}) \left(\frac{I_{j,t}}{K_{j,t}} - \frac{I_{j,t-1}}{K_{j,t-1}}\right) \quad \text{(within-firm)} \\
&+ \sum_{j\in\mathcal{C}_t} \left((1-\alpha)\frac{I_{j,t}}{K_{j,t}} + \alpha \frac{I_{j,t-1}}{K_{j,t-1}}\right)(w_{j,t} - w_{j,t-1}) \quad \text{(reallocation)}
\end{aligned}$$

To see why this decomposition holds, simplifying notation let $x_j \equiv I_{j,t}/K_{j,t}$, $y_j \equiv I_{j,t-1}/K_{j,t-1}$, $w_j \equiv w_{j,t}$, and $v_j \equiv w_{j,t-1}$. The decomposition is:

$$\sum_j w_j x_j - \sum_j v_j y_j = \sum_j (\alpha w_j + (1-\alpha)v_j)(x_j - y_j) + \sum_j ((1-\alpha)x_j + \alpha y_j)(w_j - v_j)$$

Expand the first term on the right-hand side:

$$\sum_j (\alpha w_j + (1-\alpha)v_j)(x_j - y_j) = \sum_j \left[\alpha w_j x_j - \alpha w_j y_j + (1-\alpha)v_j x_j - (1-\alpha)v_j y_j\right]$$

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Then expand the second term on the right-hand side

$$\sum_j ((1-\alpha)x_j + \alpha y_j)(w_j - v_j) = \sum_j [(1-\alpha)x_j w_j - (1-\alpha)x_j v_j + \alpha y_j w_j - \alpha y_j v_j]$$

Add the two expansions and collect terms by coefficient to obtain that the right-hand side is equal to:

$$\sum_j \Big[\underbrace{(\alpha + (1-\alpha))}_{=1} w_j x_j + \underbrace{(-\alpha + \alpha)}_{=0} w_j y_j + \underbrace{((1-\alpha) - (1-\alpha))}_{=0} v_j x_j + \underbrace{(-(1-\alpha) - \alpha)}_{=-1} v_j y_j \Big],$$

which establishes the result.

This decomposition follows the productivity decomposition literature that separates aggregate changes into within-firm and reallocation components, and in particular Melitz and Polanec (2015). The parameter α controls how the weights are averaged across periods: setting $\alpha = 1$ yields the decomposition of Baily et al. (1992), which uses end-period weights; setting $\alpha = 0$ yields the decomposition of Foster, Haltiwanger and Krizan (2001), which uses base-period weights; and setting $\alpha = 0.5$ yields the decomposition of Griliches and Regev (1995), which uses the average of base- and end-period weights. We follow Griliches and Regev (1995) and set $\alpha = 0.5$, so that the within-firm component captures changes in firm-level investment rates weighted by the average of initial and final capital shares, while the reallocation component captures shifts in capital shares weighted by the average of initial and final investment rates. Note that our decomposition is exact because we focus on the sample of continuing firms, and thus omit the further contributions of entry and exit to the actual aggregate investment rate (see Melitz and Polanec 2015 for a treatment of entry and exit). This would complicate exposition of the results without substantially altering them because the entry and exit components have little material impact at higher frequencies, as the annual entry and exit rates from the Compustat sample are small.

Finally, before plotting the cumulative change in each of the two components above, we remove a linear time trend from each of the components. The reallocation component of $\Delta I_t/K_t$ has a positive mean so that the series trends upward on average, likely reflecting the fact that firms that initially small firms that gain weight are those that experience persistently high investment rates, while the within-firm component trends down on average, likely reflecting mean-reversion in investment rates for larger firms. We use detrending the cumulative series instead of demeaning the series in changes because the latter would impose that the components are all equal to zero at the end of the sample. Either approach leaves the cyclicality of the different components (discussed below) unchanged.

A.2 Results

Figure 2 reports the results of the decomposition for R&D investment. Figures 3 and 4 in this Appendix show the decomposition for physical investment and total investment rates, respectively. In those cases the pattern is somewhat starker and all the cyclical variation in the aggregate investment rate is driven by the within-firm term. Finally, Table 2 reports the cyclical sensitivity, defined, as in the main text, as the correlation with annual HP filtered log value added, deflated by the GDP deflator, for the NFCB sector, for each type of investment and each of its components. The results confirm the visual intuition that the bulk of the cyclical variation in either type of investment is driven by within-firm effects, not by reallocation.

B Model

B.1 Description

Time t is continuous. A firm has an irreversible project with known future payoff V. The time of project completion, T, is on the other hand unknown, and depends on how quickly the firm can complete the project.

In order to complete the project, the firm must incur costs, which we interpret as investment. The project is completed once the remaining costs to pay reach zero. These costs, however, are subject to uncertainty. Specifically, the remaining cost to completion evolves as:

$$dC_t = -I_t dt + \sigma\left(\beta(I_t C_t)^{\frac{1}{2}} dz_t + (1-\beta) C_t dw_t\right), \tag{1}$$

where $I_t dt$ represent investment costs incurred in the time period $[t, t+dt]$, and C_t are the current costs to completion. Formally, the time of project completion is the first passage time of C_t through zero:

$$T = \inf_{s \geq t} \{ \quad s \quad \text{s.t.} \quad C_s \leq 0 \quad | \quad C_t \quad \}. \tag{2}$$

It can be shown that C_t, the current cost to completion, is also the *expected* cost to completion, that is, $C_t = \mathbb{E}_t\left[\int_t^T I dt\right]$, whenever the firm uses a constant intensity investment policy, $I_t = I$.

The investment rate I_t is controlled by the firm, so that the firm can, in particular, choose to interrupt investment in the project according to the evolution of C_t. We assume that it must satisfy:

$$I_t \in \left[0, \overline{I}\right], \tag{3}$$

where $\overline{I}$ is the maximum feasible investment rate.

Finally, dz_t and dw_t represent two sources of uncertainty. As explained in the main text, we associate the dz_t shocks with R&D uncertainty, as investment in this case helps the firm accrue information about true project costs (not just lower remaining costs to completion). We associate instead the dw_t shocks with cost uncertainty, more prominent for traditional investment projects, and which can change the net value of the project regardless of whether the firm is actively investing.

Assume that investors are risk-neutral and discount the future at rate r. At any point in time, for a firm with remaining cost to completion C_t, the value of continuing the project is:

$$\begin{aligned} F(C_t) \quad &= \quad \max_{I_t} \quad \mathbb{E}_t\left[e^{-r(T-t)} V - \int_t^T e^{-r(s-t)} I_s ds\right] \\ \text{s.t.} \quad & C_T = 0 \\ & dC_s = -I_s ds + \sigma\left(\beta(I_s C_s)^{\frac{1}{2}} dz_s + (1-\beta) C_t dw_s\right) \\ & I_s \in \left[0, \overline{I}\right] \end{aligned}$$

Note that the firm can always pause investment ($I_t = 0$). As we will see below, in the case $\beta = 1$, pausing investment freezes costs permanently, so that the project is effectively abandoned. When $\beta < 1$, completion costs can continue to evolve even if investment is paused, so that the project is effectively still active.

B.2 No uncertainty

When there is no uncertainty ($\sigma = 0$), the model reduces to:

$$
\begin{aligned}
F(C_t) &= \max_{I_t} \quad e^{-r(T-t)}V - \int_t^T e^{-r(s-t)} I_s ds \\
\text{s.t.} \quad & C_T = 0 \\
& dC_t = -I_t dt \\
& I_t \in \left[0, \bar{I}\right]
\end{aligned}
$$

Assuming that the solution is bang-bang, under the policy $I_s = \bar{I}$ for all $s \geq t$, we have:

$$T = t + \frac{C_t}{\bar{I}}. \tag{4}$$

Thus:

$$\int_t^T e^{-r(s-t)} I_s ds = \bar{I} \int_t^{t+\frac{C_t}{\bar{I}}} e^{-r(s-t)} ds = \frac{\bar{I}}{r}\left(1 - e^{-r\frac{C_t}{\bar{I}}}\right). \tag{5}$$

Therefore,

$$F(C_t) = \left(V + \frac{\bar{I}}{r}\right) e^{-r\frac{C_t}{\bar{I}}} - \frac{\bar{I}}{r}. \tag{6}$$

Under the policy $I_t = 0$, $F(C_t) = 0$. Thus the firm undertakes the project if and only if $C_t \leq C^*$, where:

$$C^* = \frac{\bar{I}}{r} \log\left(1 + r\frac{V}{\bar{I}}\right). \tag{7}$$

As $r \to 0$, the value of investing is:

$$F(C_t) = V - C_t. \tag{8}$$

The investment threshold converges to $C^* = V$ as $r \to 0$. When $r > 0$, $F(C_t) < V - C_t$ and $C^* < V$, because the payoff is further into the future than the completion costs.

B.3 Uncertainty

When there is uncertainty, standard arguments show that the value of the project can be written, in recursive form, as:

$$rF(C_t) = \max_{I_t \in [0,\bar{I}]} \quad -I_t \quad + \quad \frac{1}{dt}\mathbb{E}_t\left[dF(C_t)\right]. \tag{9}$$

Assuming differentiability, and applying Ito's Lemma, we have, up to terms of order $o(dt)$:

$$
\begin{aligned}
\mathbb{E}_t\left[dF(C_t)\right] &= F_C \mathbb{E}_t\left[dC_t\right] + \frac{1}{2} F_{CC} \mathbb{E}_t\left[(dC_t)^2\right] \\
&= -F_C I_t dt \\
&+ \; F_{CC}\sigma^2\beta^2 I_t C_t dt \\
&+ \; F_{CC}\sigma^2(1-\beta)^2 C_t^2 dt
\end{aligned}
$$

$$+ \quad F_{CC}\sigma^2\beta(1-\beta)I_t^{\frac{1}{2}}C_t^{\frac{3}{2}}\mathbb{E}_t\left[dw_t dz_t\right]$$

We focus on the case where the two sources of uncertainty are uncorrelated, so that $\mathbb{E}_t\left[dw_t dz_t\right] = 0$.

In this case the recursive representation of the problem is:

$$rF(C_t) = \max_{I_t \in [0,\bar{I}]} -(1+F_C)I_t + \frac{1}{2}\left(\beta^2 I_t C_t + (1-\beta)^2 C_t^2\right) F_{CC}\sigma^2 \tag{10}$$

subject to the following boundary conditions:

$$F(0) = V \tag{11}$$

$$\lim_{C_t \to +\infty} F(C_t) = 0 \tag{12}$$

The maximand is linear in I_t. Suppose that there is an interior ($C^* > 0$) threshold such that:

$$I_t = \begin{cases} \bar{I} & \text{if} \quad C_t \leq C^* \\ 0 & \text{if} \quad C_t \geq C^*, \end{cases} \tag{13}$$

then C^* would need to satisfy the smooth-pasting condition:

$$\frac{1}{2}\sigma^2\beta^2 F_{CC}(C^*)C^* = 1 + F_C(C^*). \tag{14}$$

Thus the value function satisfies:

$$rF(C_t) = \begin{cases} -(1+F_C)\bar{I} \quad + \quad \frac{1}{2}\left(\beta^2\bar{I}C_t + (1-\beta)^2C_t^2\right)F_{CC}\sigma^2 & \text{if} \quad C_t \leq C^* \\ (1-\beta)^2 C_t^2 F_{CC}\sigma^2 & \text{if} \quad C_t \geq C^* \end{cases} \tag{15}$$

In the region where $C_t \geq C^*$, if $r > 0$, this equation has the solution:

$$F(C_t) = AC_t^{-\zeta}, \quad \zeta \equiv \frac{1}{2}\left(\sqrt{1 + 8\frac{r}{\sigma(1-\beta)}} - 1\right) > 0. \tag{16}$$

The value of the parameter A must be obtained from the smooth-pasting condition and the solution on $C_t < C_t^*$. From here on we can study only either special cases or numerical solutions.

B.4 Special analytical case: $r = 0$, $\beta = 1$

When $r = 0$, in the region $C_t \geq C^*$, $F(C_t) = 0$ (using the boundary condition $\lim_{C_t \to +\infty} F(C_t) = 0$). In the region $C_t \leq C^*$, the value function follows:

$$0 = -(1+F_C) + \frac{1}{2}\bar{I}C_t F_{CC}\sigma^2. \tag{17}$$

The unique solution is:

$$\begin{aligned} C^* &= \left(1 + \frac{1}{2}\sigma^2\right)V > V \\ F(C_t) &= \begin{cases} V - C_t + \sigma^2\left(\frac{V}{2}\right)^{-\frac{2}{\sigma^2}}\left(\frac{C_t}{\sigma^2+2}\right)^{\frac{\sigma^2+2}{\sigma^2}} & \text{if} \quad C_t \leq C^* \\ 0 & \text{if} \quad C_t \geq C^* \end{cases} \end{aligned} \tag{18}$$

In particular, the cost completion threshold is increasing in the degree of uncertainty, σ^2, so that when uncertainty rises, certain projects that were put on hold are accelerated.

B.5 Special analytical case: $r = 0$, $\beta = 0$

Suppose that there is a solution with an interior threshold $C^* > 0$. In this case the smooth-pasting condition becomes:

$$1 + F_C(C^*) = 0. \tag{19}$$

Recall that for $C_t \geq C^*$, $F(C_t) = 0$. For $C_t \leq C^*$, the HJB becomes:

$$0 = -(1 + F_C)\bar{I} + \frac{1}{2}C_t^2 F_{CC}\sigma^2 \tag{20}$$

A general solution to this equation must satisfy:

$$1 + F_C(C_t) = A \exp\left(-\frac{2\bar{I}}{\sigma^2 C_t}\right). \tag{21}$$

With an interior threshold, the smooth-pasting condition imposes $A = 0$. Thus $F(C_t) = B - C_t$. By $F(C^*) = 0$, $B = C^*$. By $F(0) = V$, $C^* = V$. Thus it must be that $F(C_t) = V - C_t$ for $C_t \leq V$.

However, this is not the globally optimal strategy. Indeed the threshold strategy $C^* = 0$ (i.e. waiting infinitely long for the cost to be zero) yields a value of V regardless of current costs. So the optimum is $F(C_t) = V$ and infinite waiting times in that case.

This is the purest case of the waiting effect. Costs will hit 0 almost surely as $t \to +\infty$ (as the GBM drifts downwards in logs). If there is no discounting, then the best thing to do is to wait for that to happen and reap the reward, V.

B.6 General case

Figure 5, 6 and 7 show examples of solutions of the model computed numerically for three particular cases, $\beta = 0$ (pure external uncertainty), $\beta = 1$ (pure technical uncertainty), and $\beta = 0.5$. The numerical method is standard and described in Section B.7 of this document. Except for the values of β, the structural parameter values in Figures 5 through 7 are the same.

B.7 Numerical solution method for general $\beta \in [0, 1]$

The general case combines both investable and external uncertainty. The HJB equation is:

$$rF(C_t) = \max_{I_t \in [0, \bar{I}]} \left\{ -(1 + F_C)I_t + \frac{1}{2}\sigma^2 \left[\beta^2 I_t C_t + (1 - \beta)^2 C_t^2\right] F_{CC} \right\} \tag{22}$$

The maximand is linear in I_t. The coefficient on I_t is:

$$-(1 + F_C) + \frac{1}{2}\sigma^2 \beta^2 C_t F_{CC} \tag{23}$$

Thus the firm invests ($I_t = \bar{I}$) when:

$$\frac{1}{2}\sigma^2 \beta^2 C_t F_{CC} \geq 1 + F_C \tag{24}$$

Suppose there is an interior threshold $C^* > 0$ such that the firm invests for $C_t \leq C^*$ and waits for $C_t > C^*$. Then the value function satisfies:

Investment region ($C_t \le C^*$):

$$rF = -(1 + F_C)\bar{I} + \frac{1}{2}\sigma^2 \left[\beta^2 \bar{I} C_t + (1-\beta)^2 C_t^2\right] F_{CC} \tag{25}$$

Waiting region ($C_t > C^*$):

$$rF = \frac{1}{2}\sigma^2 (1-\beta)^2 C_t^2 F_{CC} \tag{26}$$

Note that unlike the $\beta = 1$ case, the waiting region has a nontrivial ODE to solve (costs still evolve due to external uncertainty). Unlike the $\beta = 0$ case, the investment criterion depends on F_{CC}.

Boundary conditions:

- $F(0) = V$ (project completion payoff)
- $\lim_{C_t \to \infty} F(C_t) = 0$
- Smooth pasting at C^*:

$$\frac{1}{2}\sigma^2 \beta^2 C^* F_{CC}(C^*) = 1 + F_C(C^*) \tag{27}$$

B.7.1 Discretization

Discretize $[0, C_{\max}]$ with $N + 1$ points:

$$C_i = i \cdot \Delta C, \quad i = 0, 1, \dots, N, \quad \Delta C = \frac{C_{\max}}{N} \tag{28}$$

Let $F_i \approx F(C_i)$. Note that $C_0 = 0$.

Using upwind (forward) difference for advection and central difference for diffusion:

$$F_C(C_i) \approx \frac{F_{i+1} - F_i}{\Delta C} \tag{29}$$

$$F_{CC}(C_i) \approx \frac{F_{i+1} - 2F_i + F_{i-1}}{(\Delta C)^2} \tag{30}$$

Define the diffusion coefficients for each region:

$$a_i^{\text{inv}} = \frac{\sigma^2 \left[\beta^2 \bar{I} C_i + (1-\beta)^2 C_i^2\right]}{2(\Delta C)^2} \tag{31}$$

$$a_i^{\text{wait}} = \frac{\sigma^2 (1-\beta)^2 C_i^2}{2(\Delta C)^2} \tag{32}$$

and the advection coefficient $c = \bar{I}/\Delta C$.

Investment region ($I_i = \bar{I}$): The HJB becomes:

$$rF_i = -\bar{I} - \bar{I}\frac{F_{i+1} - F_i}{\Delta C} + a_i^{\text{inv}}(F_{i+1} - 2F_i + F_{i-1}) \tag{33}$$

Rearranging:

$$a_i^{\text{inv}} F_{i-1} - (2a_i^{\text{inv}} + r - c)F_i + (a_i^{\text{inv}} - c)F_{i+1} = \bar{I} \tag{34}$$

Waiting region ($I_i = 0$): The HJB becomes:

$$rF_i = a_i^{\text{wait}}(F_{i+1} - 2F_i + F_{i-1}) \tag{35}$$

Rearranging:

$$a_i^{\text{wait}}F_{i-1} - (2a_i^{\text{wait}} + r)F_i + a_i^{\text{wait}}F_{i+1} = 0 \tag{36}$$

Boundary conditions:

- $F_0 = V$
- $F_N = 0$

B.7.2 Matrix formulation

In matrix form, $\mathbf{AF} = \mathbf{d}$, where $\mathbf{F} = (F_1, \ldots, F_{N-1})^T$. The matrix $\mathbf{A}$ is tridiagonal. For row i:

- If $I_i = \bar{I}$: coefficients are $(a_i^{\text{inv}}, -(2a_i^{\text{inv}} + r - c), a_i^{\text{inv}} - c)$ and $d_i = \bar{I}$
- If $I_i = 0$: coefficients are $(a_i^{\text{wait}}, -(2a_i^{\text{wait}} + r), a_i^{\text{wait}})$ and $d_i = 0$

The boundary conditions enter the right-hand side:

- If $I_1 = \bar{I}$: $d_1 = \bar{I} - a_1^{\text{inv}}V$
- If $I_1 = 0$: $d_1 = -a_1^{\text{wait}}V$

B.7.3 Policy iteration algorithm

- **Initialize:** Set $F_i^{(0)} = \max(V - C_i, 0)$
- **Policy update:** Compute F_C and F_{CC} at each grid point using:

$$F_C(C_i) \approx \frac{F_{i+1} - F_i}{\Delta C} \tag{37}$$

$$F_{CC}(C_i) \approx \frac{F_{i+1} - 2F_i + F_{i-1}}{(\Delta C)^2} \tag{38}$$

  Evaluate the investment criterion:

$$\frac{1}{2}\sigma^2\beta^2 C_i F_{CC}^{(n)}(C_i) - \left(1 + F_C^{(n)}(C_i)\right) \tag{39}$$

  Enforce monotonicity: find the largest index j^* such that the criterion is ≥ 0, and set:

$$I_i^{(n)} = \begin{cases} \bar{I} & \text{if } i \leq j^* \\ 0 & \text{if } i > j^* \end{cases} \tag{40}$$

- **Value update:** Assemble $\mathbf{A}^{(n)}$ and $\mathbf{d}^{(n)}$ using the policy $I_i^{(n)}$, then solve:

$$\mathbf{A}^{(n)}\mathbf{F}^{(n+1)} = \mathbf{d}^{(n)} \tag{41}$$

- **Convergence check:** Stop if $\|F^{(n+1)} - F^{(n)}\|_\infty < \varepsilon$
- **Extract threshold:** $C^* \approx C_{j^*}$

B.7.4 Implementation notes

Handling $\beta = 1$: When $\beta = 1$, the waiting region has $a_i^{\text{wait}} = 0$, which would give a singular system if we try to solve $-rF_i = 0$. Instead, we impose $F_i = 0$ directly for $i > j^*$ by setting the matrix row to $(0, 1, 0)$ with $d_i = 0$.

Handling β close to 1: For β close to but not equal to 1, the waiting region diffusion $(1-\beta)^2 C_i^2$ becomes very small, potentially causing numerical issues. For the calibration used in this paper, these numerical issues do not appear to be prominent.

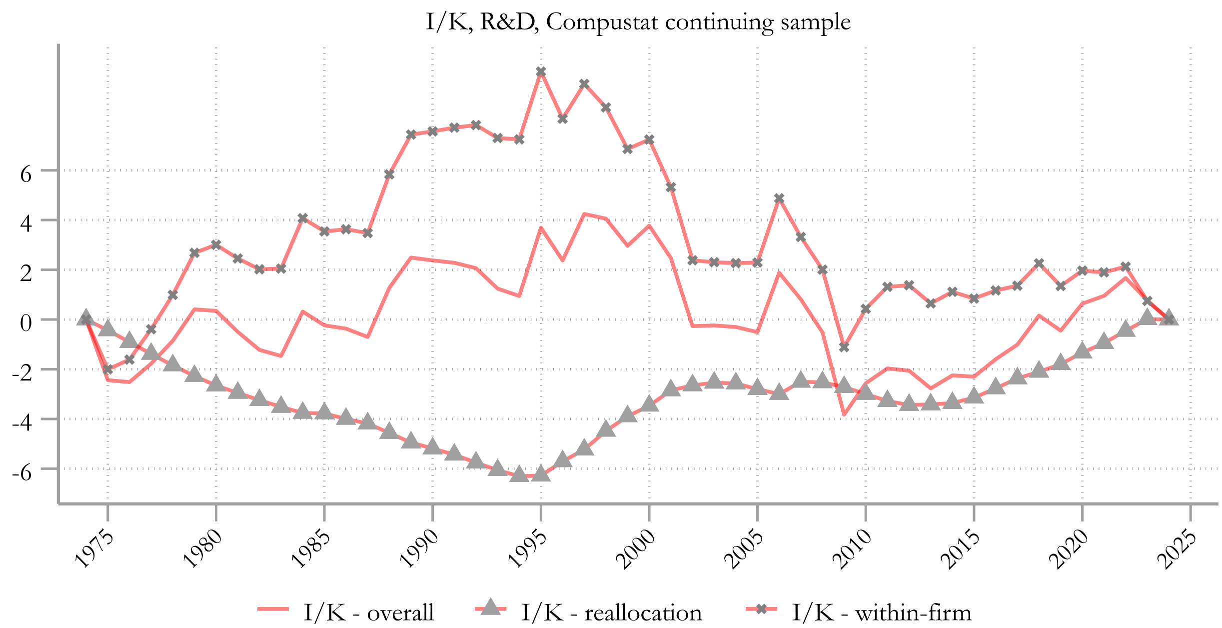

Figure 1: Within-firm and reallocation components of R&D investment.

Figure 2: This graph decomposes changes in the aggregate ratio of R&D investment to the stock of R&D capital in the sample of continuing Compustat firms into a within-firm component, which reflects changes in investment rates keeping the distribution of R&D capital across firms constant, and a reallocation component. The underlying sample are all US publicly traded non-financial firms. Details of the decomposition are reported in Appendix.

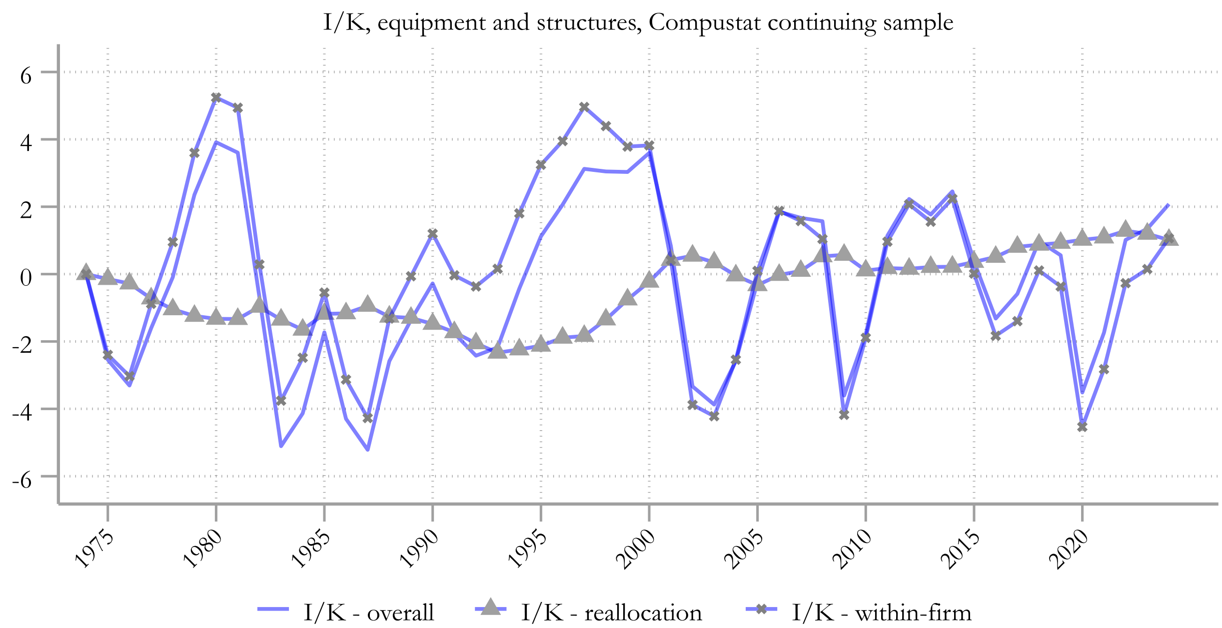

Figure 3: This graph decomposes changes in the aggregate ratio of investment in equipment and structures to the stock of equipment and structures in the sample of continuing Compustat firm into a within-firm component, which reflects changes in investment rates keeping the distribution of capital across firms constant, and a reallocation component. The underlying sample are all US publicly traded non-financial firms. Details of the decomposition are reported in Section A.

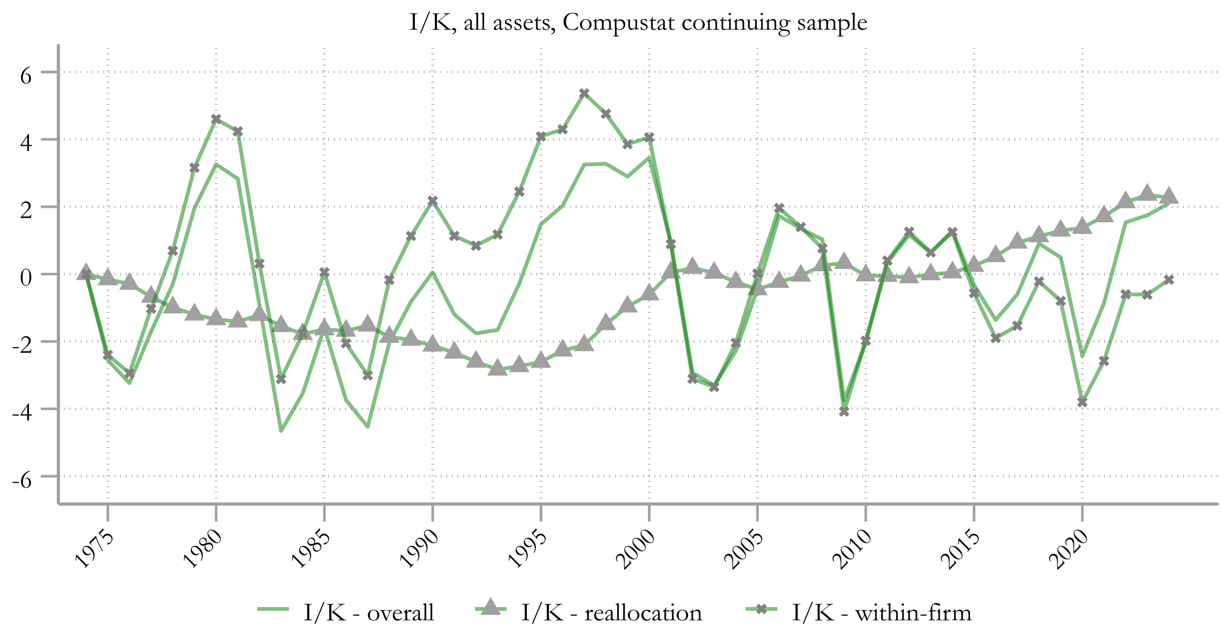

Figure 4: This graph decomposes changes in the aggregate ratio of total investment to the total stock of capital in the sample of continuing Compustat firm into a within-firm component, which reflects changes in investment rates keeping the distribution of capital across firms constant, and a reallocation component. The underlying sample are all US publicly traded non-financial firms. Details of the decomposition are reported in Section A.

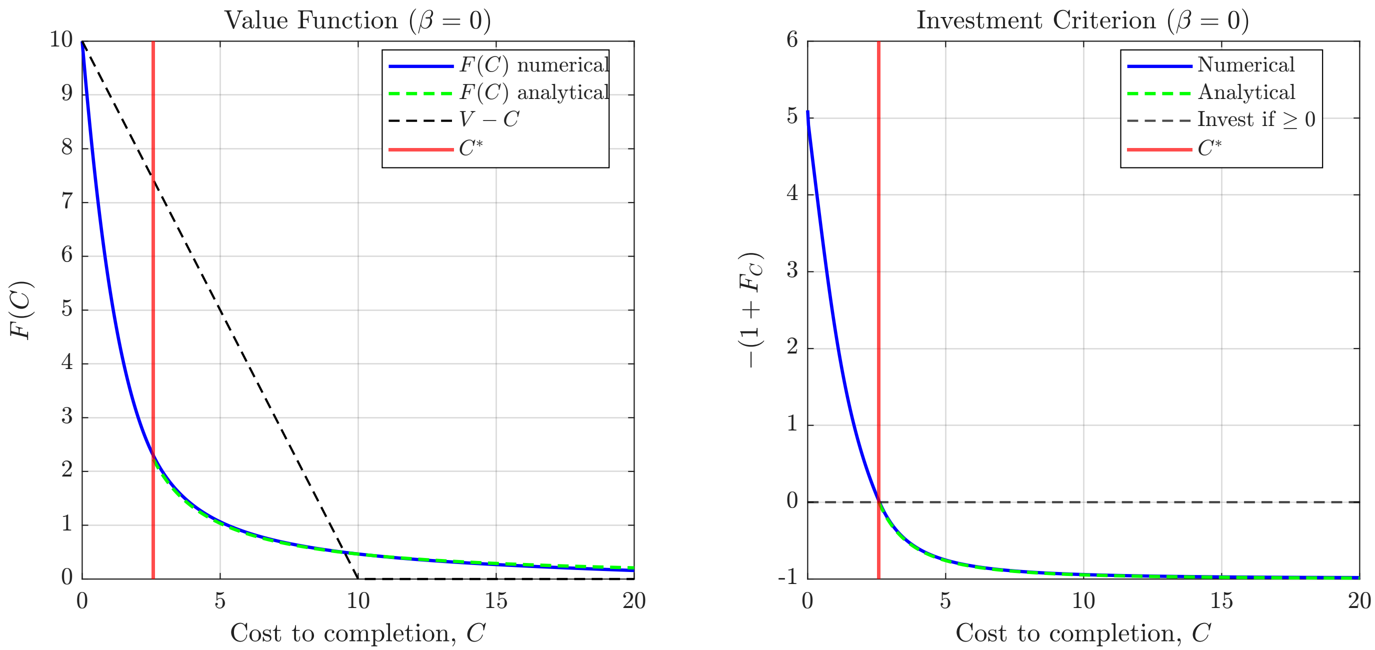

Figure 5: Value function and derivative for $\beta = 0$ case. Parameter values are $V = 10$, $\bar{I} = 0.1$, $r = 0.05$, $\sigma = 0.2$.

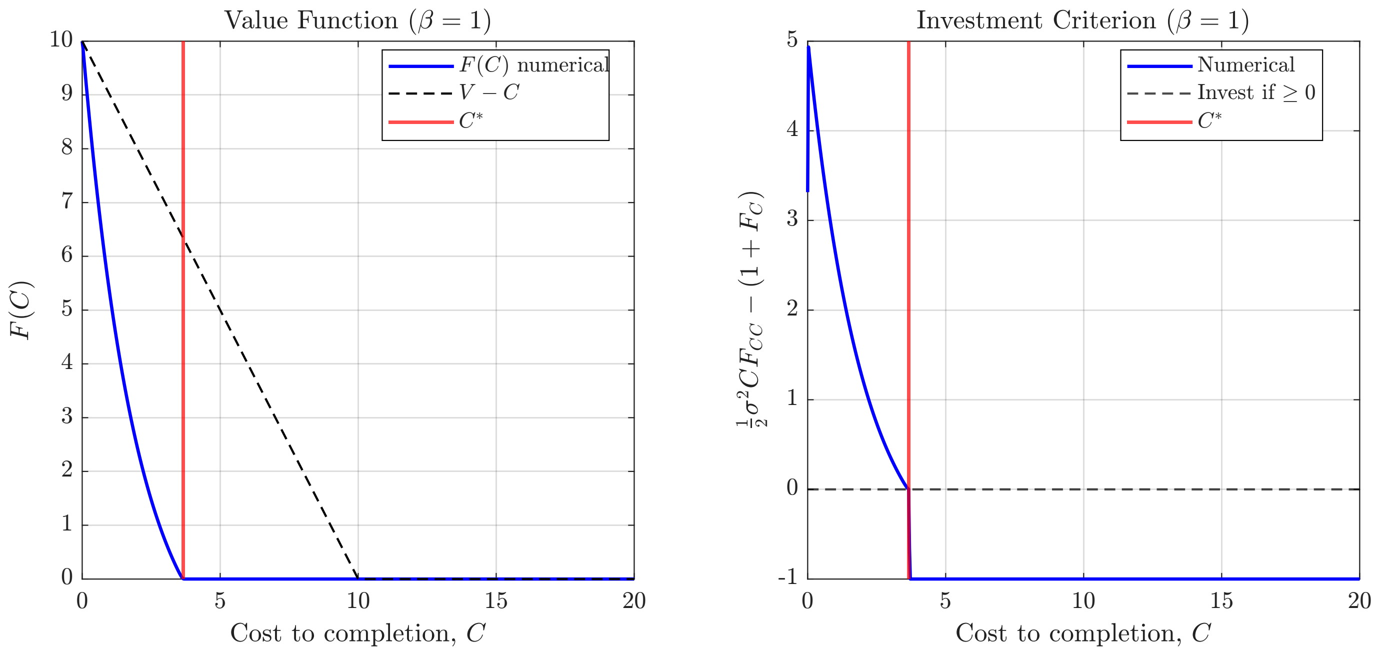

Figure 6: Value function and derivative for $\beta = 1$ case. Parameter values are $V = 10$, $\bar{I} = 0.1$, $r = 0.05$, $\sigma = 0.2$.

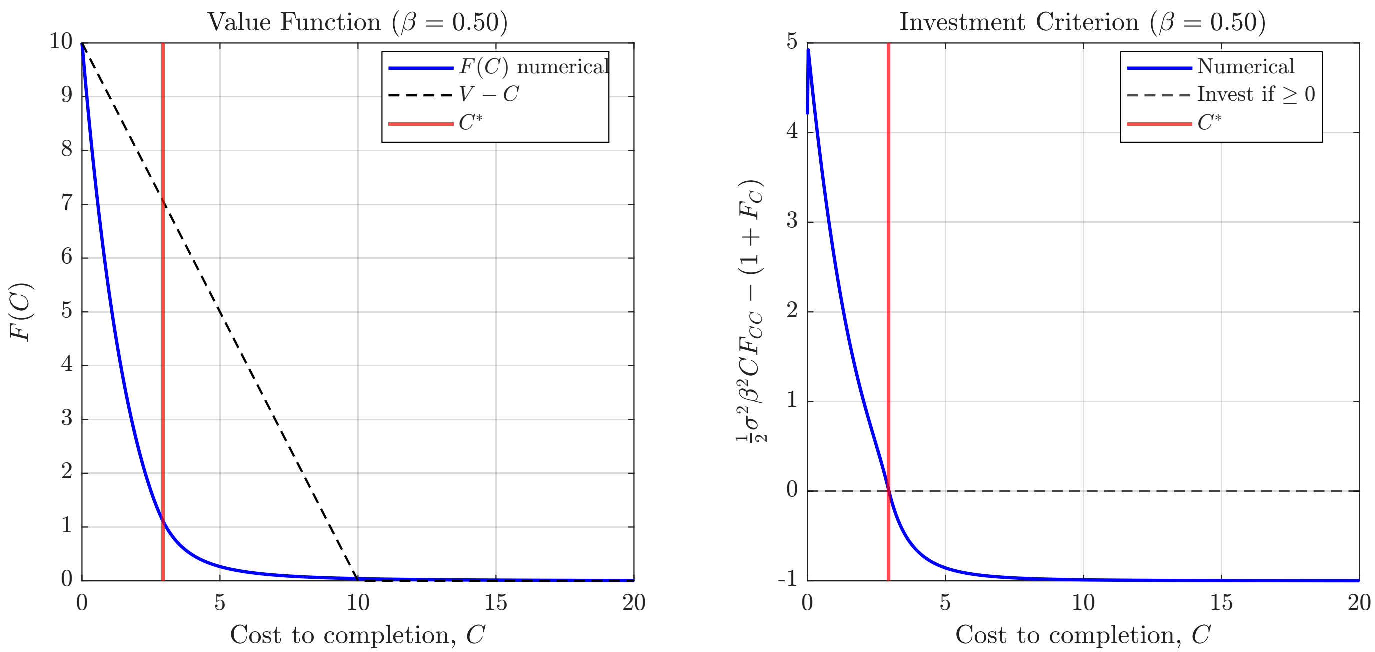

Figure 7: Value function and derivative for $\beta = 0.5$ case. Parameter values are $V = 10$, $\bar{I} = 0.1$, $r = 0.05$, $\sigma = 0.2$.

Panel A: Cyclical sensitivity of $\log(I/P)$, P = GDP deflator

	Private sector		**NFCB sector**		**Compustat**
	Quarterly	Annual	Quarterly	Annual	Annual
Total investment	0.74	0.78	0.72	0.78	0.66
	(<0.01)	*(<0.01)*	*(<0.01)*	*(<0.01)*	*(<0.01)*
Equipment & structures	0.73	0.77	0.73	0.78	0.68
	(<0.01)	*(<0.01)*	*(<0.01)*	*(<0.01)*	*(<0.01)*
R&D	0.45	0.35	0.26	0.44	0.44
	(<0.01)	*(<0.01)*	*(<0.01)*	*(<0.01)*	*(<0.01)*
p-value for difference in cyclical sensitivity	<0.01	<0.01	<0.01	<0.01	0.01
# obs	289	74	289	74	49

Panel B: Cyclical sensitivity of I/K

	Private sector		**NFCB sector**		**Compustat**
	Quarterly	Annual	Quarterly	Annual	Annual
Total investment	0.38	0.46	0.26	0.46	0.46
	(<0.01)	*(<0.01)*	*(<0.01)*	*(<0.01)*	*(<0.01)*
Equipment & structures	0.38	0.42	0.28	0.46	0.37
	(<0.01)	*(<0.01)*	*(<0.01)*	*(<0.01)*	*(0.01)*
R&D	0.09	0.13	0.14	0.28	0.26
	(0.12)	*(0.25)*	*(0.01)*	*(0.02)*	*(0.07)*
p-value for difference in cyclical sensitivity	<0.01	0.08	<0.01	0.02	0.26
# obs	289	74	289	74	49

Table 1: Measures of cyclical sensitivity. In Panel A, cyclical sensitivity is defined as the correlation between the HP-filtered component of the log of investment deflated by the GDP deflator, and the HP-filtered component of a measure of value added deflated by the GDP deflator for the corresponding sector. In Panel B, cyclical sensitivity is defined as the correlation between the I/K ratio and the HP-filtered component of a measure of value added for the corresponding sector. For Compustat, the measure of value added is NFCB sector value added. For quarterly time series the smoothing parameter is 1600; for annual time series the smoothing parameter is 6.25. Numbers in parentheses are p-values for statistical significance of the correlation. The test at the bottom of each panel reports the p-value for the null that physical investment has a lower cyclical sensitivity than R&D, $\text{corr}(I_{\text{Physical}}, \text{GDP}) \leq \text{corr}(I_{\text{RD}}, \text{GDP})$.

	Overall	**Reallocation**	**Within-firm**
Total investment	0.46	−0.03	0.46
	(<0.01)	*(0.84)*	*(<0.01)*
Equipment and structures	0.37	−0.06	0.48
	(0.01)	*(0.69)*	*(<0.01)*
R&D	0.26	−0.01	0.18
	(0.07)	*(0.98)*	*(0.23)*
# obs	49	49	49

Table 2: Cyclical sensitivity of I/K: reallocation vs. within-firm components. Cyclical sensitivity is defined as the correlation to the HP-filtered component of value added for the non-financial corporate business sector; the smoothing parameter is 6.25. Cyclicality is reported separately for the overall aggregate I/K ratio and for its within-firm and reallocation components, and is broken down by investment in equipment and structures and investment in R&D. Numbers in parentheses refer to p-values for the null that the correlation is different from 0.

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