Supplemental Appendix

Firm Heterogeneity, Market Power and Macroeconomic Fragility

Alessandro Ferrari
UPF, CREi, BSE & CEPR

Francisco Queirós
ISEG Research in Economics and Management, CSEF & CEF.UP

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B.1 The Quantitative Model

Calibration

Steady State We perform three different calibrations of our model – to match the average level of markups and its dispersion in 1975, 1990 and in 2007. We need to calibrate five technology parameters: the elasticity of substitution σ_I and σ_G (which are time-invariant), the log-normal standard deviation λ, the fixed production cost c, the fraction of markets in the uncompetitive sector f_u and the fraction of uncompetitive markets with fixed costs x_c.

We start by specifying a grid for these six different parameters $(\sigma_I, \sigma_G, \lambda, c, f_u, x_c)$, and then construct a vector with different values for the capital stock K. We then compute the aggregate equilibrium for each parameter combination and for each value K.[1] We start by assuming that all firms are active, so that there are N firms in each of the I industries. We compute the aggregate equilibrium using equations (21) and (22). We then compute the profits net of the fixed cost that each firm makes

$$\left(p_{ijt} - \frac{\Theta_t}{\tau_{ijt}}\right) y_{ijt} - c_i$$

and identify the firm with the largest negative value. We exclude this firm and recompute the aggregate equilibrium. We repeat this iterative procedure until all firms have non-negative profits (net of the fixed production cost). If equilibrium multiplicity arises, this algorithm allows us to consistently select the equilibrium that features the largest number of firms.

For each triplet (λ, c, f_{comp}), we then have the general equilibrium computed for all possible capital values. The steady states of our economy correspond to the values of K for with the rental

[1] Aggregate TFP e^{z_t} is assumed to be constant and equal to one.

rate R_t is equal to $\frac{1}{\beta} - (1-\delta)$.

When multiple steady states arise (as in the 1990 and 2007 economies), we compute model moments in the highest steady state.

Solution Algorithm for the Dynamic Optimization Problem

We now explain the algorithm we use for the dynamic optimization problem of the representative household. We take the calibrated parameters (λ, c) and form a grid for aggregate capital with $n_K = 75$ points. This grid is centered around the highest steady state K_H^{ss}, with a lower-bound $0.25 \times K_H^{\text{ss}}$ and upper bound $2.25 \times K_H^{\text{ss}}$. We also form a grid for aggregate TFP, A. We use Tauchen's algorithm with $n_A = 11$ points, autocorrelation parameter ϕ_A and standard deviation for the innovations σ_ε (the last two parameters are calibrated, as explained in the main text). We compute the aggregate equilibrium for each value of K and A.

We next compute a numerical approximation for the household policy function, by iterating on the Euler equation. We start by making a guess about the savings rate

$$s\left(X_t\right) \coloneqq \frac{C\left(X_t\right)}{Y\left(X_t\right)}$$

for every combination of the vector of state-variables $X_t \coloneqq (K_t, A_t)$. Given a guess $s^{(n)}\left(X_t\right)\ \forall X_t$ for the savings rate, we use the Euler equation to obtain a new guess $s^{(n+1)}\left(X_t\right)$ as follows

$$\frac{1}{\left(1-s^{(n+1)}\left(X_t\right)\right)Y\left(X_t\right)-\frac{W\left(X_t\right)^{(1+\nu)/\nu}}{1+\nu}} = \mathbb{E}_t\left\{\frac{\beta\left[R\left(X_{t+1}\right)+(1-\delta)\right]}{\left(1-s^{(n)}\left(X_{t+1}\right)\right)Y\left(X_{t+1}\right)-\frac{W\left(X_{t+1}\right)^{(1+\nu)/\nu}}{1+\nu}}\right\}$$

$$\Leftrightarrow \quad s^{(n+1)}\left(X_t\right) = 1-\frac{1}{Y\left(X_t\right)}\left\{\frac{W\left(X_t\right)^{(1+\nu)/\nu}}{1+\nu}+\left[\mathbb{E}_t\left\{\frac{\beta\left[R\left(X_{t+1}\right)+(1-\delta)\right]}{\left(1-s^{(n)}\left(X_{t+1}\right)\right)Y\left(X_{t+1}\right)-\frac{W\left(X_{t+1}\right)^{(1+\nu)/\nu}}{1+\nu}}\right\}\right]^{-1}\right\}.$$

We iterate on this procedure until

$$\left|s^{(n+1)}\left(X_t\right)-s^{(n)}\left(X_t\right)\right| \ < \ \epsilon \quad \forall X_t.$$

Steady States

$K^*_{H,75}$	$K^*_{H,90}$	$K^*_{L,07}$	$K_{U,07}$	$K^*_{H,07}$
0	-0.008	-0.21	-0.15	-0.033

Table B.1: Steady-state location

Note: K^*_L and K^*_H denote the two stable steady states (low and high), while K_U denotes the unstable steady state.

Steady-state location Table B.1 reports the values of the capital stock in all (non-stochastic) steady states of our calibrated economies. Since the levels are themselves uninformative, we report them in log deviations from the steady-state capital of the 1975 economy. This allows for a better comparison across calibrations.

We normalize the log steady-state capital in 1975 to zero, so that the values in the other steady states can be interpreted as growth rates. These numbers indicate that the level of K^*_H (that is, the largest steady state of each economy) was lowest in 2007. These results are consistent with our theory (see, for example, the discussion of the two comparative statics exercises in Figure 5 of the paper). We also highlight that our theory does not feature any source of growth, which could make K^*_H larger in 2007.

Log-linearization of the Euler Equation We report here the derivation of the dynamic system, log-linearized around the steady-state values. We show that the system is saddle-path stable in the steady states we analyze.

We start with the Euler equation

$$U_c\left(C_t, L_t\right) \;=\; \beta\,\mathbb{E}_t\left\{\left[R_{t+1} + (1-\delta)\right] U_c\left(C_{t+1}, L_{t+1}\right)\right\}$$

$$\Leftrightarrow \quad \frac{1}{C_t - \dfrac{L_t^{1+\nu}}{1+\nu}} \;=\; \beta\,\mathbb{E}_t\left\{\frac{\alpha\Theta_{t+1}K_{t+1}^{\alpha-1}L_{t+1}^{1-\alpha} + (1-\delta)}{C_{t+1} - \dfrac{L_{t+1}^{1+\nu}}{1+\nu}}\right\}.$$

Using

$$L_t = \left[(1-\alpha)\;\Theta_t\,K_t^{\alpha}\right]^{1/(\alpha+\nu)},$$

and $\Theta_t = \Theta^*$, $\Phi_t = \Phi^*$, the Euler equation becomes (and assuming no uncertainty)

$$\frac{(1+\nu)\,C_{t+1} - L_{t+1}^{1+\nu}}{(1+\nu)\,C_t - L_t^{1+\nu}} = \beta\left[\alpha\Theta^* K_{t+1}^{\alpha-1} L_{t+1}^{1-\alpha} + (1-\delta)\right]$$

$$\Leftrightarrow \frac{(1+\nu)\,C_{t+1} - \left[(1-\alpha)\ \Theta^*\ K_{t+1}^{\alpha}\right]^{(1+\nu)/(\alpha+\nu)}}{(1+\nu)\,C_t - \left[(1-\alpha)\ \Theta^*\ K_t^{\alpha}\right]^{(1+\nu)/(\alpha+\nu)}} = \beta\left[\alpha\Theta^* K_{t+1}^{\alpha-1}\left[(1-\alpha)\ \Theta^*\ K_{t+1}^{\alpha}\right]^{(1-\alpha)/(\alpha+\nu)} + (1-\delta)\right]$$

$$\Leftrightarrow \beta^{-1}(1+\nu)\,C_{t+1} - \beta^{-1}\omega_1 K_{t+1}^{\sigma_1} = (1+\nu)\,\omega_2 C_t K_{t+1}^{-\sigma_2} - \omega_1\omega_2 K_t^{\sigma_1} K_{t+1}^{-\sigma_2} + (1-\delta)\left[(1+\nu)\,C_t - \omega_1 K_t^{\sigma_1}\right].$$

Taking a first order Taylor expansion around $k_0 = k^* := \log(K^*)$ and $c_0 = c^* := \log(C^*)$

$$\begin{aligned} K_{t+1}^{\sigma_1} &\approx \exp(\sigma_1 k^*) + \sigma_1 \exp(\sigma_1 k^*)(k_{t+1} - k^*) \\ &= \exp(\sigma_1 k^*)\left[1 + \sigma_1 (k_{t+1} - k^*)\right] \end{aligned}$$

$$\begin{aligned} K_{t+1}^{-\sigma_2} &\approx \exp(-\sigma_2 k^*) - \sigma_2 \exp(-\sigma_2 k^*)(k_{t+1} - k^*) \\ &= \exp(-\sigma_2 k^*)\left[1 - \sigma_2 (k_{t+1} - k^*)\right] \end{aligned}$$

$$\begin{aligned} K_t^{\sigma_1} K_{t+1}^{-\sigma_2} &\approx \exp(\sigma_1 k^* - \sigma_2 k^*) + \sigma_1 \exp(\sigma_1 k^* - \sigma_2 k^*)(k_t - k^*) - \sigma_2 \exp(\sigma_1 k^* - \sigma_2 k^*)(k_{t+1} - k^*) \\ &= \exp(\sigma_1 k^* - \sigma_2 k^*)\left[1 + \sigma_1 (k_t - k^*) - \sigma_2 (k_{t+1} - k^*)\right] \end{aligned}$$

$$\begin{aligned} C_t K_{t+1}^{-\sigma_2} &\approx \exp(c^* - \sigma_2 k^*) + \exp(c^* - \sigma_2 k^*)(c_t - c^*) - \sigma_2 \exp(c^* - \sigma_2 k^*)(k_{t+1} - k^*) \\ &= \exp(c^* - \sigma_2 k^*)\left[1 + (c_t - c^*) - \sigma_2 (k_{t+1} - k^*)\right] \end{aligned}$$

$$C_t \approx \exp(c^*)(1 + c_t - c^*)$$

$$C_{t+1} \approx \exp(c^*)(1 + c_{t+1} - c^*).$$

The log-linearized version of the Euler equation is then

$$\begin{aligned} &\beta^{-1}(1+\nu)\exp(c^*)(1 + c_{t+1} - c^*) - \beta^{-1}\omega_1 \exp(\sigma_1 k^*)\left[1 + \sigma_1 (k_{t+1} - k^*)\right] \\ = \ &(1+\nu)\,\omega_2 \exp(c^* - \sigma_2 k^*)\left[1 + c_t - c^* - \sigma_2 (k_{t+1} - k^*)\right] - \\ &-\omega_1\omega_2 \exp(\sigma_1 k^* - \sigma_2 k^*)\left[1 + \sigma_1 (k_t - k^*) - \sigma_2 (k_{t+1} - k^*)\right] + \\ &+ (1-\delta)\left\{(1+\nu)\exp(c^*)(1 + c_t - c^*) - \omega_1 \exp(\sigma_1 k^*)\left[1 + \sigma_1 (k_t - k^*)\right]\right\}. \end{aligned}$$

We can put all the constants together and define

$$\begin{aligned}
& -\beta^{-1}(1+\nu)\exp(c^*)(1-c^*)+\beta^{-1}\omega_1\exp(\sigma_1 k^*)(1-\sigma_1 k^*) \\
+ & \ (1+\nu)\omega_2\exp(c^*-\sigma_2 k^*)(1-c^*+\sigma_2 k^*)-\omega_1\omega_2\exp(\sigma_1 k^*-\sigma_2 k^*)(1-\sigma_1 k^*+\sigma_2 k^*) \\
+ & \ (1-\delta)\left[(1+\nu)\exp(c^*)(1-c^*)-\omega_1\exp(\sigma_1 k^*)(1-\sigma_1 k^*)\right] \\
:= & \ a_1.
\end{aligned}$$

We can put all time t variables together and define

$$\begin{aligned}
& (1+\nu)\omega_2\exp(c^*-\sigma_2 k^*)c_t-\omega_1\omega_2\exp(\sigma_1 k^*-\sigma_2 k^*)\sigma_1 k_t+ \\
+ & \ (1-\delta)\left[(1+\nu)\exp(c^*)c_t-\omega_1\exp(\sigma_1 k^*)\sigma_1 k_t\right] \\
= & \ \left[(1+\nu)\omega_2\exp(c^*-\sigma_2 k^*)+(1-\delta)(1+\nu)\exp(c^*)\right]c_t+ \\
+ & \ \left[-\omega_1\omega_2\exp(\sigma_1 k^*-\sigma_2 k^*)\sigma_1-(1-\delta)\omega_1\exp(\sigma_1 k^*)\sigma_1\right]k_t \\
:= & \ b_{1c}c_t+b_{1k}k_t.
\end{aligned}$$

Finally, we can also put all time $t+1$ variables together and define

$$\begin{aligned}
& \beta^{-1}(1+\nu)\exp(c^*)c_{t+1}-\beta^{-1}\omega_1\exp(\sigma_1 k^*)\sigma_1 k_{t+1} \\
& +(1+\nu)\omega_2\exp(c^*-\sigma_2 k^*)\sigma_2 k_{t+1}-\omega_1\omega_2\exp(\sigma_1 k^*-\sigma_2 k^*)\sigma_2 k_{t+1} \\
= & \ \beta^{-1}(1+\nu)\exp(c^*)c_{t+1}+\left[-\beta^{-1}\omega_1\exp(\sigma_1 k^*)\sigma_1+\right. \\
+ & \ \left.(1+\nu)\omega_2\exp(c^*-\sigma_2 k^*)\sigma_2-\omega_1\omega_2\exp(\sigma_1 k^*-\sigma_2 k^*)\sigma_2\right]k_{t+1} \\
:= & \ d_{1c}c_{t+1}+d_{1k}k_{t+1}
\end{aligned} .$$

Then, the log-linearized version of the Euler equation can be written as

$$d_{1c}c_{t+1}+d_{1k}k_{t+1}=a_1+b_{1c}c_t+b_{1k}k_t.$$

Log-linearization of the aggregate resource constraint The aggregate resource constraint is

$$K_{t+1}=(1-\delta)K_t+\Phi_t K_t^{\alpha}L_t^{1-\alpha}-N_t^f f-C_t.$$

Using

$$L_t=\left[(1-\alpha)\ \Theta_t K_t^{\alpha}\right]^{1/(\alpha+\nu)}.$$

The aggregate resource constraint becomes

$$\begin{aligned} & K_{t+1} = (1-\delta) K_t + \Phi^* \left[(1-\alpha) \ \Theta^*\right]^{(1-\alpha)/(\alpha+\nu)} K_t^{\alpha(1+\nu)/(\alpha+\nu)} - N^{f*} f - C_t \\ \Leftrightarrow \quad & K_{t+1} = (1-\delta) K_t + \gamma K_t^{\sigma_1} - N^{f*} f - C_t \end{aligned}.$$

We have

$$K_{t+1} \approx \exp(k^*) (1 + k_{t+1} - k^*)$$

$$K_t \approx \exp(k^*) (1 + k_t - k^*)$$

$$C_t \approx \exp(c^*) (1 + c_t - c^*)$$

$$K_t^{\sigma_1} \approx \exp(\sigma_1 k^*) \left[1 + \sigma_1 (k_t - k^*)\right],$$

and so

$$\begin{aligned} & \exp(k^*) (1 + k_{t+1} - k^*) = (1-\delta) \exp(k^*) (1 + k_t - k^*) + \\ + \quad & \gamma \exp(\sigma_1 k^*) \left[1 + \sigma_1 (k_t - k^*)\right] - N^{f*} f - \exp(c^*) (1 + c_t - c^*) \\ \Leftrightarrow \quad & \exp(k^*) k_{t+1} = -\exp(k^*) (1 - k^*) + (1-\delta) \exp(k^*) (1 - k^*) + \\ + \quad & \gamma \exp(\sigma_1 k^*) (1 - \sigma_1 k^*) - \exp(c^*) (1 - c^*) - N^{f*} f \\ & + (1-\delta) \exp(k^*) k_t + \gamma \exp(\sigma_1 k^*) \sigma_1 k_t - \exp(c^*) c_t \\ \Leftrightarrow \quad & d_{2k} k_{t+1} = a_2 + b_{2k} k_t + b_{2c} c_t \end{aligned}.$$

Log-linear system In matrix form, we have

$$\begin{bmatrix} d_{1k} & d_{1c} \\ d_{2k} & 0 \end{bmatrix} \begin{bmatrix} k_{t+1} \\ c_{t+1} \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \begin{bmatrix} b_{1k} & b_{1c} \\ b_{2k} & b_{2c} \end{bmatrix} \begin{bmatrix} k_t \\ c_t \end{bmatrix},$$

which is equivalent to

$$\begin{bmatrix} k_{t+1} \\ c_{t+1} \end{bmatrix} = \begin{bmatrix} d_{1k} & d_{1c} \\ d_{2k} & 0 \end{bmatrix}^{-1} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \begin{bmatrix} d_{1k} & d_{1c} \\ d_{2k} & 0 \end{bmatrix}^{-1} \begin{bmatrix} b_{1k} & b_{1c} \\ b_{2k} & b_{2c} \end{bmatrix} \begin{bmatrix} k_t \\ c_t \end{bmatrix}.$$

Eigenvalues We study numerically the properties of the matrix $\begin{bmatrix} d_{1k} & d_{1c} \\ d_{2k} & 0 \end{bmatrix}^{-1} \begin{bmatrix} b_{1k} & b_{1c} \\ b_{2k} & b_{2c} \end{bmatrix}$, around the stable steady states of our economy. Table B.2 reports the eigenvalues of the system.

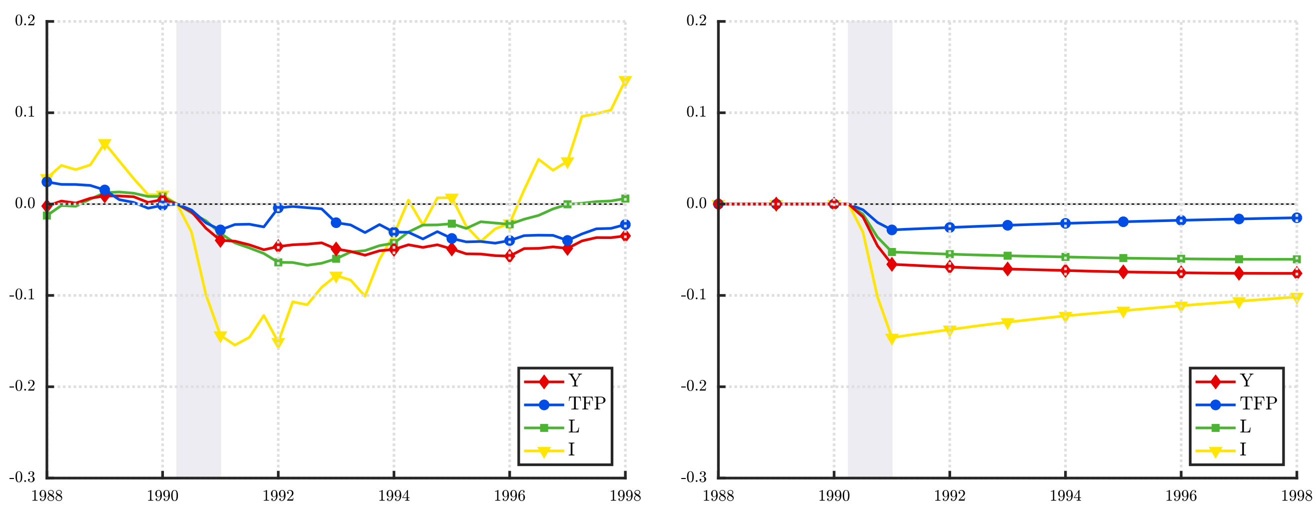
We find that for all the steady states we study in the paper for the three economies of 1975, 1990, and 2007, the system has one eigenvalue that is inside the unit circle and one that is outside. We conclude that the steady states we refer to as *stable* in the paper are all saddle-path stable: starting in a neighborhood of these steady states, there is only one consumption-savings path that leads the economy to that steady state.

	$\text{eig}_{H,75}$	$\text{eig}_{H,90}$	$\text{eig}_{L,07}$	$\text{eig}_{H,07}$
Low	0.98	0.98	0.99	0.98
High	1.04	1.04	1.05	1.04

Table B.2: Roots of the dynamic system (eigenvalues)

B.2 The 1990 Recession

The response in the 1990 economy



(a) 1990-1991 recession (data)

(b) The 1990-1991 shock in the 1990 model

Figure B.1: The 1990-1991 recession

The response in the 2007 economy

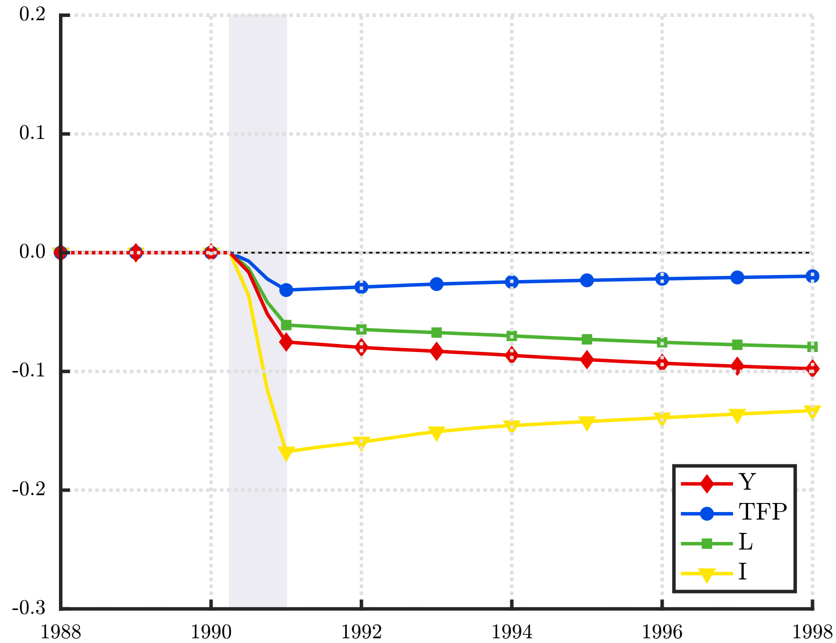


Figure B.2: The 1990-1991 shock in the 2007 model

B.3 Number of Firms

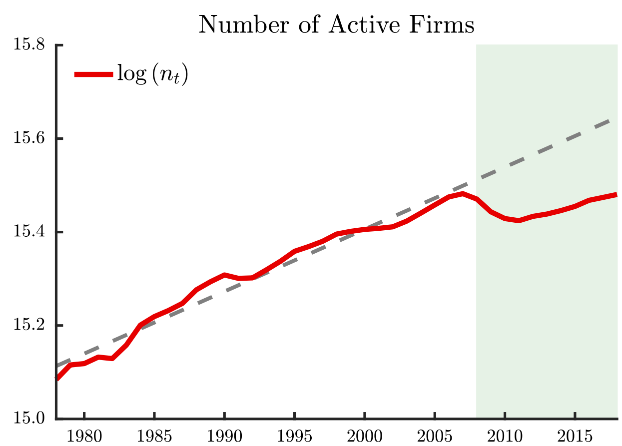


Figure B.3: **Number of Firms: 1978-2018**

Note: The red line shows the number of firms with at least one employee (in logs). The dashed grey line shows a linear trend computed for the period 1978-2007. Data is from the US Business Dynamics Statistics.

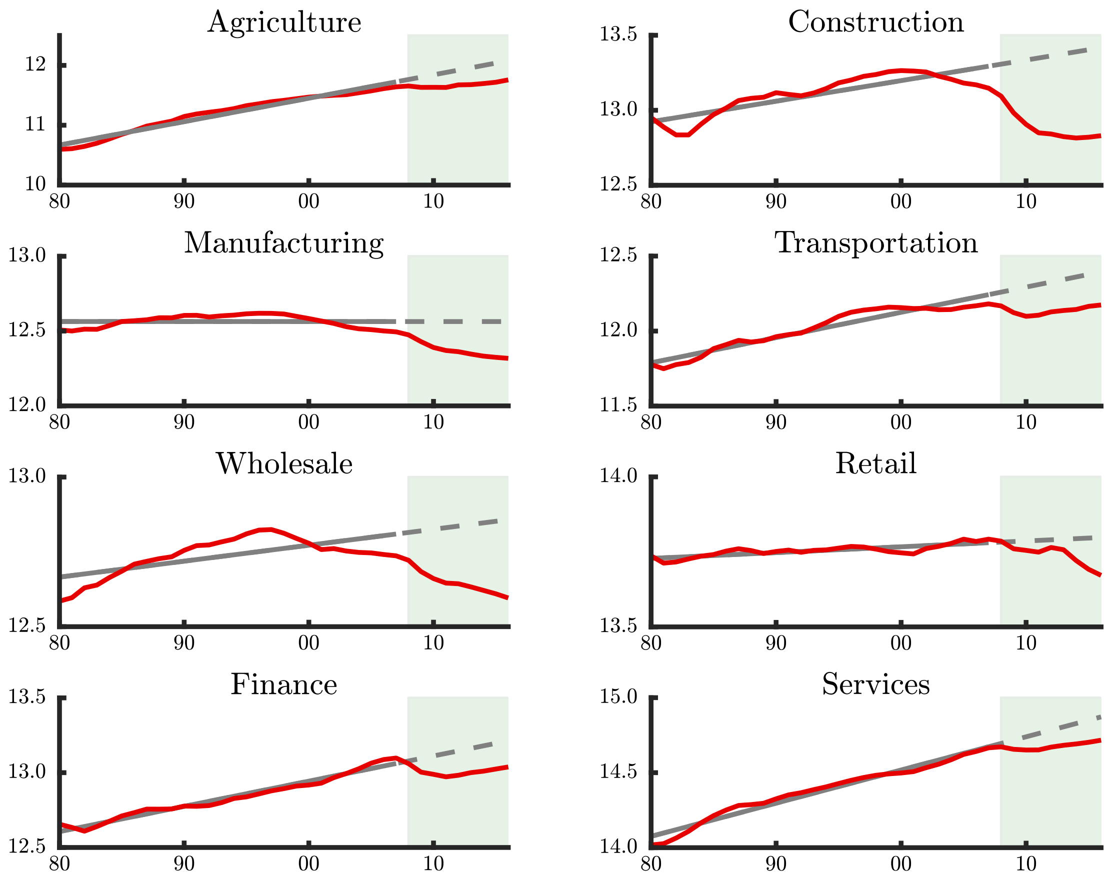


Figure B.4: **Number of Firms per Sector: 1980-2018**

Each panel shows the number of firms with at least one employee in each sector (in logs). For each series, the dashed grey line shows a linear trend computed over the 1980-2007 period. Data is from the US Business Dynamics Statistics

B.4 Alternative Source of Fluctuations

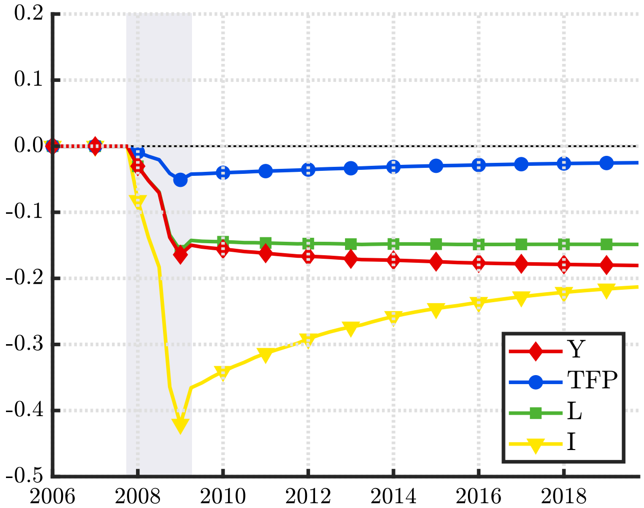


Figure B.5: The *great recession* in the 2007 model

	2007 Model			
	2009Q4	2015Q1	2019Q1	2040Q1
Output	-0.154	-0.174	-0.18	-0.182
TFP	-0.041	-0.030	-0.025	-0.022
Hours	-0.144	-0.148	-0.149	-0.147
Investment	-0.350	-0.250	-0.212	-0.191

Table B.3: Deviation from the high steady state

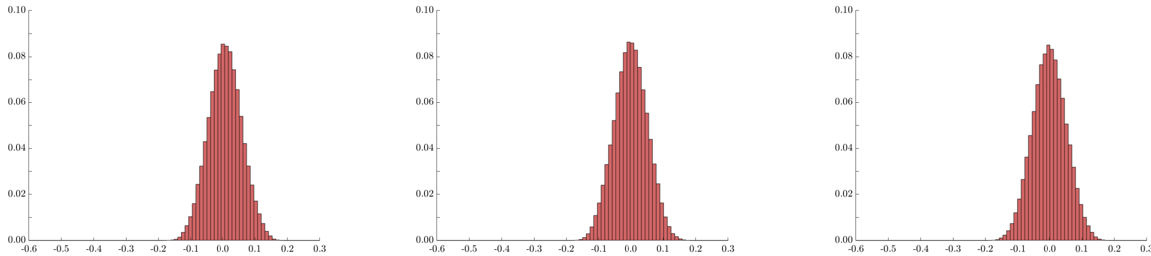
Note: This figure shows the response of the 2007 model to a sequence of shocks to the number of markets with positive fixed costs (see the main text for details).

B.5 Evaluating Different Channels

Fixed Number of Firms

First, we consider a version of the model without the extensive margin of firms. We do so by taking our calibrated model with the extensive margin and i) setting fixed costs to zero[2] and ii) fixing the set of active firms N_j. All the model parameter values are the same as our baseline models (with the exception of fixed costs, which are set to zero). We also use the same values for the exogenous TFP process. In this way, by comparing the volatility of this model to our baseline, we can quantify the role of endogenous entry and exit in contributing to aggregate fluctuations. We then subject the three economies (1975, 1990 and 2007) to a large history of draws and plot the ergodic distributions in Figure B.6.

[2] Note that without setting the fixed cost to zero, the economy would look identical other than that some units of the final good being subtracted from profits.



(a) 1975 (b) 1990 (c) 2007

Figure B.6: Ergodic distribution of output (fixed set of firms)

Note: This figure shows the distribution of log output for the 1975, 1990, and 2007 economies. We simulate each economy for 1,000,000 periods and plot log output in deviation from the steady state.

All three economies have an unimodal distribution of output, which indicates a unique (stochastic) steady state. To better understand this result, note that aggregate TFP is effectively fixed in this economy, up to the changes in the exogenous component.

Note that without entry and exit, the markup distribution is fixed since market shares do not move over the business cycle, and, in our model, they uniquely pin down markups. Since the markup distribution is constant over the cycle, the only remaining endogenous component of TFP is love for variety but this is driven by N, which is naturally kept fixed in this special case. As a consequence, the only endogenous response to changes in the exogenous component of TFP is coming from variable factor supply, as in a standard RBC model. This force alone cannot generate multiplicity since, absent the endogenous market structure, the rental rate is monotonically decreasing in the capital. There is just one value of capital such that $R_t = R^* := \beta^{-1} - (1-\delta)$. The economy moves around the unique steady state.

The three different economies (1975, 1990, and 2007) have very similar business cycle patterns. This is not surprising since they only differ in the endogenous (but fixed) component of TFP. Consequently, output changes are only driven by the exogenous TFP shocks, and by the factor supply responses (which are identically parametrized across economies). Hence, the three differ in levels but not in changes. From equation (15), we know that

$$Y_t \;= A_t \, \Phi\left(\mathbf{\Gamma}, \mathbf{N}_t\right) \, \left[(1-\alpha)\, \Theta\left(\mathbf{\Gamma}, \mathbf{N}_t\right)\right]^{\frac{1-\alpha}{\nu+\alpha}} \; K_t^{\alpha \frac{1+\nu}{\nu+\alpha}}.$$

We can simplify this expression by taking logs, and defining ζ and ξ appropriately:

$$\log Y_t = (1+\zeta) \log A_t + (1+\zeta) \log \Phi_t + \zeta \log\left[(1-\alpha)\Omega_t\right] + \xi \log K_t,$$

As a consequence, log deviations from the steady state are

$$\log Y_t - \log \bar{Y} = (1+\zeta)\,(\log A_t - \log \bar{A}) + (1+\zeta)\,(\log \Phi_t - \log \bar{\Phi}) \\ + \zeta\,(\log \Omega_t - \log \bar{\Omega}) + \xi\,(\log K_t - \log \bar{K}),$$

where upper bars denote steady-state variables. Under a constant market structure, $\log \Phi_t = \log \bar{\Phi}$, $\forall t$ and $\log \Omega_t = \log \bar{\Omega}$, $\forall t$. Consequently, differences in the dynamic response to a shock are entirely driven by the last term in the expression above (i.e., by capital accumulation).

To further compare these alternative models we compute the degree of amplification and persistence in each configuration. We use the stochastic process calibrated to our baseline economy and simulate a large time series for each model. In each time series, we then compute moments of log GDP deviations. These are reported in Table (7).

Monopolistic Competition

We consider a version of the model where firms operate under monopolistic competition, as in Dixit and Stiglitz (1977). Specifically, we look at a special case of our model in which there is at most one firm in each market. Aggregate output is

$$Y_t = \left(\sum_{i=1}^{I} y_{it}^{\rho}\right)^{1/\rho},$$

and each firm produces with the production function

$$y_{it} = A_t\,\gamma_i\,(k_{it})^{\alpha}\,(l_{it})^{1-\alpha}.$$

Each firm charges the monopoly markup $1/\rho$. This version of the model implies that the markup distribution is time-invariant and degenerate at $1/\rho$. We then have a free entry condition across markets.

In this special case, there is no heterogeneity in markups, and the factor share is equal to ρ (the inverse of the aggregate markup). We can write the aggregate production function as

$$Y = \frac{\left(\sum_{i=1}^{N} \gamma_i^{\rho/(1-\rho)}\right)^{1/\rho}}{\sum_{i=1}^{N} \gamma_i^{\rho/(1-\rho)}} A K^{\alpha} L^{1-\alpha},$$

where $\frac{\left(\sum_{i=1}^{N}\gamma_i^{\rho/(1-\rho)}\right)^{1/\rho}}{\sum_{i=1}^{N}\gamma_i^{\rho/(1-\rho)}}A$ is measured TFP. Absent heterogeneity, this boils down to the familiar love-of-variety in production term equal to $N^{(1-\rho)/\rho}$.

To calibrate the model, we use the average sales-weighted markup in the data in 2007 to obtain $\rho = 1/\hat{\mu}$. Hence, we set $\rho = 1/1.46 = 0.68$, so that the elasticity $\sigma_I = 1/(1-\rho) = 3.17$. As in our baseline model, we calibrate λ to match the standard deviation in log revenues. Under monopolistic competition, the market share of firm i is given by

$$s_i = \frac{p_i q_i}{PQ} = \frac{\gamma_i^{\epsilon-1}}{\sum_j \gamma_j^{\epsilon-1}}.$$

Therefore, the standard deviation of log market shares is simply given by

$$\mathrm{SD}\log(s) = (\sigma_I - 1)\mathrm{SD}\log(\gamma),$$

where $\mathrm{SD}\log(\gamma)$ is the standard deviation of log productivity. To make this model more directly comparable to our baseline model, we assume that only a fraction f_u of firms is subject to productivity differences. Among these, a fraction x_c is subject to positive fixed costs. These parameters are again calibrated to match the employment share in COMPUSTAT and the employment share in highly concentrated markets. This economy has a free entry condition (across industries), which holds exactly. To calibrate the fixed cost, we follow the steps used in the oligopoly model and target the fixed-to-total cost ratio in the steady state. The drawback of this approach is that the elasticity of substitution puts an upperbound $\overline{c/(vc+c)} = 1-\rho$ on the value of the fixed cost as a fraction of total costs.[3] This upperbound is binding for our model. Given our value of $\rho = 1/1.46 = 0.685$ the bound is 0.315. This is below the fixed to total cost ratio that we find in the data for 2007, which is equal to 0.369. Therefore, if the fixed-to-total cost ratio would take its data value, no firm would be active. To overcome this, we target a ratio of fixed to total costs equal to $0.8 \times \overline{c/(vc+c)}$.

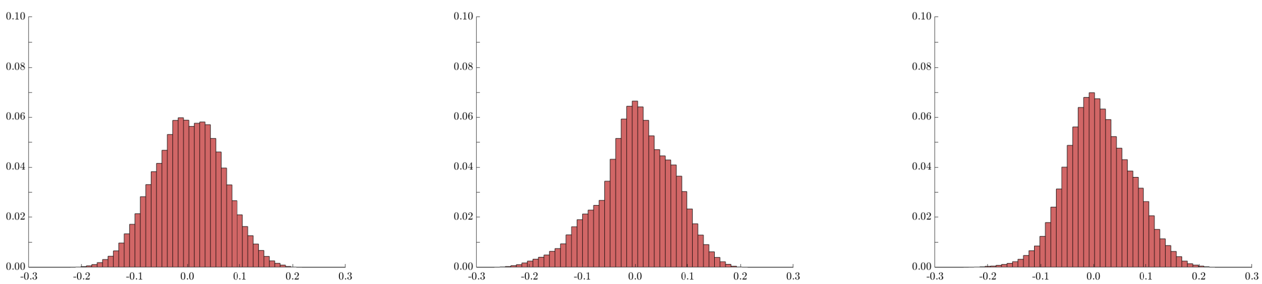
Once again, we use the same values for the exogenous TFP process. By comparing this model's volatility to our baseline, we can get a sense of how variable markups contribute to aggregate fluctuations.

[3]Net profits are equal to $vc_i\left(1/\rho - 1 - c/vc_i\right)$. This is positive provided that $c/vc_i < 1/\rho - 1$, which is equivalent to

$$\frac{c}{vc_i + c} < 1 - \rho.$$

Description	Parameter	Value			Source/Target
[B.1] Calibrated Parameters: Fixed					
Between product markets ES	σ_I	3.17			Sales-weighted average markup
Share of *uncompetitive* sector	f_u	0.31			Emp share COMPUSTAT
Share of *uncompetitive* markets with $c_i > 0$	x_c	0.60			Emp share concentrated industries
[B.2] Calibrated Parameters: Variable		1975	1990	2007	
Standard deviation of γ_{ij}	λ	0.66	0.85	0.93	Std log revenues
Fixed cost ($\times 10^{-3}$)	c	1.80	50.46	55.83	Average ratio fixed/total costs

Table B.4: Parameter Values



(a) 1975 (b) 1990 (c) 2007

Figure B.7: Ergodic distribution of output (monopolistic competition)

Note: This figure shows the distribution of log output for the 1975, 1990, and 2007 economies. We simulate each economy for 1,000,000 periods and plot log output in deviation from the high steady state.

We then subject the three economies to a large sequence of draws and plot the ergodic distributions in Figure B.7. All three economies have an unimodal distribution of output, which indicates a unique (stochastic) steady state.

Fixed Labor Supply

The special case where all factors are in fixed supply is of somewhat limited interest – if capital cannot be accumulated, transitions across steady states are not possible. For this reason, we consider the case where labor is in fixed supply and equal to $L_t = 1$, while capital is supplied elastically.

Households have an intertemporal utility

$$U_0 = \mathbb{E} \sum_{t=0}^{\infty} \beta^t \log (C_t),$$

and are subject to the budget constraint

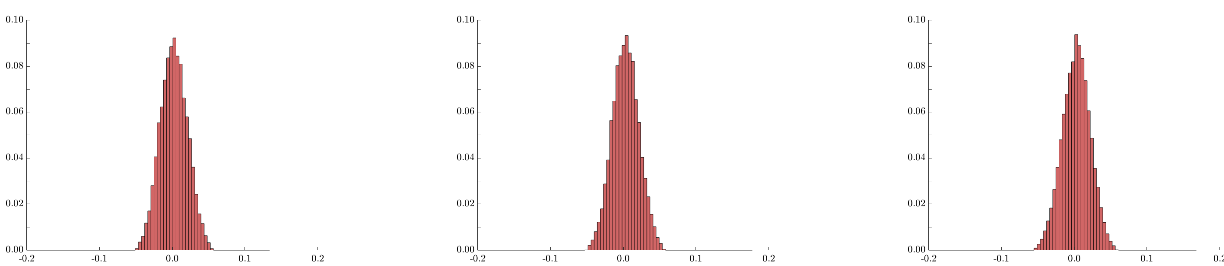
$$K_{t+1} = [R_t + (1 - \delta)] K_t + W_t + \Pi_t^N - C_t.$$

The aggregate production function becomes

$$Y_t = A_t \Phi (\mathbf{\Gamma}, \mathbf{N}_t) K_t^{\alpha},$$

with the understanding that endogenous productivity $\Phi(\cdot)$ depends on the number of active firms and, hence, on the aggregate capital stock K_t.

We follow the same steps as in the main model to calibrate our parameters of interest and report the model fit and the key experiment. Regarding the process of exogenous TFP, we use the same values for $\phi_A = 0.967$ and $\sigma_\varepsilon = 0.003$ as in the baseline model. In this way, by comparing the volatility of this model to our baseline, we can isolate the role of endogenous labor supply in contributing to aggregate fluctuations.



(a) 1975 (b) 1990 (c) 2007

Figure B.8: Ergodic distribution of output (fixed labor supply)

Note: This figure shows the distribution of log output for the 1975, 1990, and 2007 economies. We simulate each economy for 1,000,000 periods and plot log output in deviation from the high steady state.

We simulate each economy 1,000,000 times and plot the distribution of log output (in deviation from its mode). The three distributions are unimodal, which indicates that the three economies feature a unique (stochastic) steady state.

Description	Parameter	Value			Source/Target
[B.1] Calibrated Parameters: Fixed					
Between product markets ES	σ_I		1.42		Sales-weighted average markup
Within product market ES	σ_G		12.6		Sales-weighted average markup
Share of *uncompetitive* sector	f_u		0.436		Emp share COMPUSTAT
Share of *uncompetitive* markets with $c_i > 0$	x_c		0.324		Emp share concentrated industries
[B.2] Calibrated Parameters: Variable		1975	1990	2007	
Standard deviation of γ_{ij}	λ	0.168	0.197	0.220	Std log revenues
Fixed cost ($\times 10^{-4}$)	c	0.276	0.419	0.501	Average ratio fixed/total costs

Table B.5: Parameter Values

B.6 Aggregate Productivity

Average Firm Level TFP

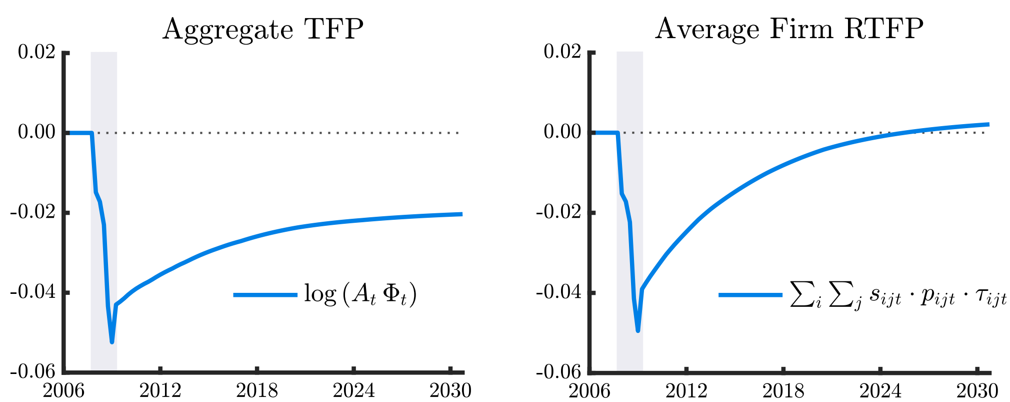


Figure B.9: Aggregate TFP versus Average Firm Level TFP

Note: The left panel shows aggregate TFP. The right panel shows a sales-weighted average of firm-level revenue TFP $p_{ijt} \cdot \tau_{ijt}$.

Figure B.9 reports a sales-weighted average of firm-level revenue TFP. A similar pattern emerges if one uses physical TFP instead.

Dispersion in Industry Output

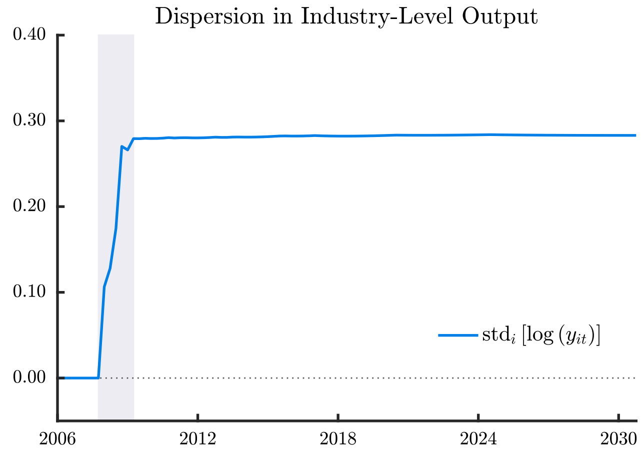


Figure B.10: Dispersion in $\log(y_{it})$

B.7 Regression Tables

	(1)	(2)	(3)	(4)
VARIABLES	$\Delta \log \text{emp}_{07-16}$	$\Delta \log \text{emp}_{07-16}$	$\Delta \log \text{emp}_{07-16}$	$\Delta \log \text{emp}_{07-16}$
$\text{concentration}_{07}$	-0.0223***	-0.0160**	-0.0177***	-0.0178**
	(0.00667)	(0.00688)	(0.00682)	(0.00732)
$\log \text{firms}_{07}$		0.00239***	0.00193***	0.00151
		(0.000705)	(0.000706)	(0.000983)
$\Delta \log \text{emp}_{03-07}$			0.0984***	0.0901***
			(0.0241)	(0.0247)
Observations	770	770	769	761
R-squared	0.014	0.029	0.050	0.064
Sector FE	NO	NO	NO	YES

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table B.6: Change in Employment: 2007-2016

Note: the table shows the results of regressing the growth rate of sectoral employment between 2007 and 2016 on the measure of concentration in 2007. The table presents the results of progressively adding controls and, in the last column, sector fixed effects.

VARIABLES	(1) $\Delta \log \text{payroll}_{07-16}$	(2) $\Delta \log \text{payroll}_{07-16}$	(3) $\Delta \log \text{payroll}_{07-16}$	(4) $\Delta \log \text{payroll}_{07-16}$
$\text{concentration}_{07}$	-0.0231***	-0.0177**	-0.0189***	-0.0194***
	(0.00679)	(0.00702)	(0.00697)	(0.00749)
$\log \text{firms}_{07}$		0.00203***	0.00164**	0.000991
		(0.000724)	(0.000725)	(0.00101)
$\Delta \log \text{payroll}_{03-07}$			0.0823***	0.0697***
			(0.0219)	(0.0225)
Observations	774	774	773	765
R-squared	0.015	0.025	0.043	0.054
Sector FE	NO	NO	NO	YES

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table B.7: Change in Total Payroll: 2007-2016

Note: The table shows the results of regressing the growth rate of sectoral total payroll between 2007 and 2016 on the measure of concentration in 2007. The table presents the results of progressively adding controls and, in the last column, sector fixed effects.

	(1)	(2)	(3)	(4)
VARIABLES	$\Delta \log \text{firms}_{07-16}$	$\Delta \log \text{firms}_{07-16}$	$\Delta \log \text{firms}_{07-16}$	$\Delta \log \text{firms}_{07-16}$
$\text{concentration}_{07}$	-0.0432***	-0.0391***	-0.0406***	-0.0231***
	(0.00608)	(0.00637)	(0.00635)	(0.00666)
$\log \text{firms}_{07}$		0.00137**	0.00119*	0.00449***
		(0.000663)	(0.000661)	(0.000897)
$\Delta \log \text{firms}_{03-07}$			0.0881***	0.0808***
			(0.0270)	(0.0273)
Observations	791	791	791	782
R-squared	0.060	0.065	0.078	0.151
Sector FE	NO	NO	NO	YES

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table B.8: Change in Number of Firms: 2007-2016

Note: the table shows the results of regressing the growth rate of the industry number of firms between 2007 and 2016 on the measure of concentration in 2007. The table presents the results of progressively adding controls and, in the last column, sector fixed effects.

	(1)	(2)	(3)	(4)
VARIABLES	Δlabor share$_{07-16}$	Δlabor share$_{07-16}$	Δlabor share$_{07-16}$	Δlabor share$_{07-16}$
concentration$_{07}$	-0.0314*	-0.0319*	-0.0314*	-0.0301
	(0.0167)	(0.0168)	(0.0167)	(0.0196)
log firms$_{07}$		-0.00111	-0.00120	-0.00255
		(0.00240)	(0.00240)	(0.00335)
Δlabor share$_{03-07}$			0.169*	0.146*
			(0.0867)	(0.0871)
Observations	99	99	98	97
R-squared	0.035	0.037	0.075	0.111
Sector FE	NO	NO	NO	YES

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table B.9: Change in Labor Share: 2007-2016

Note: The table shows the results of regressing the change of the sectoral labor share between 2007 and 2016 on the measure of concentration in 2007. The table presents the results of progressively adding controls and, in the last column, sector fixed effects.

References

Dixit, Avinash K and Joseph E Stiglitz, "Monopolistic Competition and Optimum Product Diversity," *American Economic Review*, June 1977, *67* (3), 297–308.