Supplemental Appendix: Non-Compete Agreements and Bargaining Power

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I. Empirical Appendix

We draw on data from the National Longitudinal Survey of Youth 1997 (NLSY97). Our analysis uses survey waves from 2013 through 2021 and is limited to employed individuals with real hourly wages between $3 and $200. For respondents holding more than one job in a given survey year, we focus on their main job, defined as the current or most recent job at the time of the interview. If multiple jobs are ongoing, we designate the main job as the one with the longest tenure. All estimates apply the NLSY-provided sampling weights to ensure national representativeness.

II. Theory Appendix

A. *Proof of Proposition 1*

A1. INVESTMENT AND QUIT PROBABILITY UNDER NC AND WITHOUT NC

Proof: We first show that investment is weakly higher under NC for all θ.
When $\theta \geq \bar{\theta}_0$, $i_1^* \geq 0, i_0^* = 0$, which implies that $i_1^* \geq i_0^*$
Thus we only need to consider the case of $\theta \in [0, \bar{\theta}_0)$, when both i_1^*, i_0^* are nonzero.
Let

$$H_\delta(i) = (1-\theta)K_\delta \cdot (1 - e^{-\lambda K_\delta i}) - i$$

We find i_0^*, i_1^* by solving $H_0(i_0^*) = 0, H_1(i_1^*) = 0$.
Given the optimal investment without NC, i_0^*

$$H_1(i_0^*) = (1-\theta)r \cdot (1 - e^{-\lambda r i_0^*}) - i_0^* > (1-\theta)(r-\rho) \cdot (1 - e^{-\lambda (r-\rho) i_0^*}) - i_0^* = H_0(i_0^*) = 0$$

By Intermediate Value Theorem, since $H_1(i_0^*) > 0$, $\lim_{i\to\infty} H_1(i) < 0$, there must be $i_1^* > i_0^*$.
Now we show quit probability is weakly lower under NC. The quit probability under NC and No-NC are given by

$$q_1^* = P(v > r i_1^*) = e^{-\lambda r i_1^*} \; ; \; q_0^* = P(v > (r-\rho) i_0^*) = e^{-\lambda (r-\rho) i_0^*}$$

Since $i_1^* \geq i_0^*$, $r i_1^* \geq (r-\rho) i_0^*$. Thus $q_1^* = e^{-\lambda r i_1^*} \leq e^{-\lambda (r-\rho) i_0^*} = q_0^*$

A2. INEFFICIENTLY LOW INVESTMENT WITHOUT NC

Proof: We show that i_0^* is lower than the socially efficient level of investment i^*.
Let

$$H_p(i) = \rho + (r-\rho)(1 - e^{-\lambda (r-\rho) i}) - i$$

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The socially efficient investment, i^* satisfies $H_p(i^*) = 0$. Given the optimal investment without NC, i_0^*

$$H_p(i_0^*) = \rho + (r-\rho)(1-e^{-\lambda(r-\rho)i_0^*}) - i_0^* > (1-\theta)(r-\rho)(1-e^{-\lambda(r-\rho)i_0^*}) - i_0^* = H_0(i_0^*) = 0$$

Additionally, $\lim_{i\to\infty} H_p(i) = -\infty < 0$. By Intermediate Value Theorem, we have $i^* > i_0^*$

A3. Investment vs Bargaining Power

Proof: We show that i_1^*, i_0^* are non-increasing with worker's bargaining power θ.
When $\theta \geq \bar{\theta}_\delta, i_\delta^* = 0$. Thus $\frac{di_\delta^*}{d\theta} = 0$.
When $\theta < \bar{\theta_\delta}$, the second order condition for i_δ^* is given by $-1 + (1-\theta)\lambda K_\delta^2 e^{-\lambda K_\delta i_\delta^*} < 0$.
Using the implicit function theorem,

$$\frac{di_1^*}{d\theta} = -\frac{r(1-e^{-\lambda r i_1^*})}{1-(1-\theta)\lambda r^2 e^{-\lambda r i_1^*}} < 0$$

$$\frac{di_0^*}{d\theta} = -\frac{(r-\rho)(1-e^{-\lambda(r-\rho)i_0^*})}{1-(1-\theta)\lambda(r-\rho)^2 e^{-\lambda(r-\rho)i_0^*}} < 0$$

A4. Condition for nonzero investment

Proof: We prove that $i_\delta^* > 0$ if $\theta < \bar{\theta_\delta} = 1 - \frac{1}{\lambda K_\delta^2}$ and $i_\delta^* = 0$ otherwise.
From II.A we know i_δ^* is the solution to $H_\delta(i) = 0$. For an arbitrary i, we can derive

$$H_\delta'(i) = \lambda(1-\theta)K_\delta^2 e^{-\lambda K_\delta i} - 1\ ;\ H_\delta''(i) = -\lambda^2(1-\theta)K_\delta^3 e^{-\lambda K_\delta i} < 0$$

This shows that $H_\delta(i)$ is strictly concave.
$i = 0$ is always a solution to $H_\delta(i) = 0$. We need to show whether there exists a nonzero solution.

$$H_\delta(0) = 0\ ;\ H_\delta'(0) = \lambda(1-\theta)K_\delta^2 - 1$$

If $\theta > \bar{\theta_\delta}$

$$1-\theta < \frac{1}{\lambda K_\delta^2} \Rightarrow H_\delta'(0) < 0$$

Since $H_\delta(i)$ is strictly concave, for all $i \geq 0, H_\delta'(i) < 0$. Then since $H_\delta(0) = 0, H_\delta(i) < 0\ \forall i > 0$.
Thus $i_\delta^* = 0$ is the only solution to $H_\delta(i) = 0$.
If $\theta < \bar{\theta_\delta}$

$$1-\theta > \frac{1}{\lambda K_\delta^2} \Rightarrow H_\delta'(0) > 0$$

Given $H_\delta(0) = 0$, there exists an $i > 0$ such that $H_\delta(i) > 0$. Further we know that $\lim_{i\to\infty} H_\delta(i) = (1-\theta)K_\delta - i < 0$.
By the Intermediate Value Theorem, there must exist some $i_\delta^* > 0$ such that $H_\delta(i_\delta^*) = 0$. Since $H_\delta(i)$ is strictly concave and continuous, the nonzero solution i_δ^* is unique.

B. Proof of Proposition 2

B1. Derivation of Expected Joint Surplus and Contract Choice

The expected joint surplus with NC ($\Sigma_1(i_1^*)$) and without NC can be derived as ($\Sigma_0(i_0^*)$)

$$\Sigma_1(i_1^*) = -0.5 i_1^{*2} + r i_1^* + \frac{e^{-\lambda r i_1^*}}{\lambda}$$
$$\Sigma_0(i_0^*) = -0.5 i_0^{*2} + r i_0^* + \frac{e^{-\lambda (r-\rho) i_0^*}}{\lambda}$$

NC is chosen if and only if $\Sigma_1(i_1^*) > \Sigma_0(i_0^*)$.
We can decompose the differences in expected surplus as

$$\Delta_\Sigma = [r i_1^* - \frac{1}{2} i_1^{*2} - (r i_0^* - \frac{1}{2} i_0^{*2})] + \frac{1}{\lambda}(e^{-\lambda r i_1^*} - e^{-\lambda(r-\rho) i_0^*}) = (i_1^* - i_0^*)[r - \frac{1}{2}(i_0^* + i_1^*)] + \frac{1}{\lambda}(q_1^* - q_0^*)$$

The first term is larger or equal to 0 since $i_1^* < r, i_0^* < r$ and $i_1^* \geq i_0^*$. The second term is smaller or equal to 0 since $q_1^* \leq q_0^*$

B2. Comparison of w^* and B^*

First we derive B_1^* and B_0^* using $E(W_\delta^*) + B_\delta^* = \mu_0$.

$$B_0^* = \mu_0 - E(W_0^*) = \mu_0 - \left(\frac{1}{\lambda} + \rho i_0^* + \theta\left[(r-\rho) i_0^* - \frac{1}{\lambda} + \frac{e^{-\lambda(r-\rho) i_0^*}}{\lambda}\right]\right)$$
$$B_1^* = \mu_0 - E(W_1^*) = \mu_0 - \left(\frac{1}{\lambda} + \theta\left[r i_1^* - \frac{1}{\lambda} + \frac{e^{-\lambda r i_1^*}}{\lambda}\right]\right)$$
$$\Rightarrow \Delta_B = B_1^* - B_0^* = -\theta r(i_1^* - i_0^*) + (1-\theta)\rho i_0^* - \theta \cdot \frac{1}{\lambda}(q_1^* - q_0^*)$$

The first term is less than or equal to 0 since $i_1^* \geq i_0^*$. The second term is nonnegative. The third term is greater than or equal to 0 since $q_1^* \leq q_0^*$. Lastly, the comparison for second period wage is derived below

$$\Delta_w = w_1^* - w_0^* = \theta r(i_1^* - i_0^*) - (1-\theta)\rho i_0^*$$

B3. Wages Marked-to-Market ($\theta = 0$)

$\Delta_w = -\rho i_0^* \leq 0$; $\Delta_B = \rho i_0^* \geq 0$. Thus first period wage is higher NC and second period wage is higher without NC.
In a competitive market the firm's expected profit is set to 0.
The expected profit functions can be derived as

$$E\Pi_F(i_1^*, B_1) = -0.5 i_1^{*2} - B_1 + (1-\theta)[r i_1^* - \frac{1}{\lambda} + \frac{e^{-\lambda r i_1^*}}{\lambda}]$$
$$E\Pi_F(i_0^*, B_0) = -0.5 i_0^{*2} - B_0 + (1-\theta)[(r-\rho) i_0^* - \frac{1}{\lambda} + \frac{e^{-\lambda(r-\rho) i_0^*}}{\lambda}]$$

Therefore we can derive

$$B_1^* = -0.5 i_1^{*2} + (1-\theta)[r i_1^* - \frac{1}{\lambda} + \frac{e^{-\lambda r i_1^*}}{\lambda}]$$
$$B_0^* = -0.5 i_0^{*2} + (1-\theta)[(r-\rho) i_0^* - \frac{1}{\lambda} + \frac{e^{-\lambda (r-\rho) i_0^*}}{\lambda}]$$

The expected second period wage is

$$E(w_0) = \frac{1}{\lambda} + \rho i_0^* + \theta\left[(r-\rho) i_0^* - \frac{1}{\lambda} + \frac{e^{-\lambda (r-\rho) i_0^*}}{\lambda}\right]$$
$$E(w_1) = \frac{1}{\lambda} + \theta\left[r i_1^* - \frac{1}{\lambda} + \frac{e^{-\lambda r i_1^*}}{\lambda}\right]$$

Thus total compensation $T = B + E(w)$ can be derived as

$$T_1 = -0.5 i_1^{*2} + r i_1^* + \frac{e^{-\lambda r i_1^*}}{\lambda}$$
$$T_0 = -0.5 i_0^{*2} + r i_0^* + \frac{e^{-\lambda (r-\rho) i_0^*}}{\lambda}$$

The comparison of total compensation is

$$\Delta_T = [r i_1^* - \frac{1}{2} i_1^{*2} - (r i_0^* - \frac{1}{2} i_0^{*2})] + \frac{1}{\lambda}(e^{-\lambda r i_1^*} - e^{-\lambda (r-\rho) i_0^*}) = (i_1^* - i_0^*)[r - \frac{1}{2}(i_0^* + i_1^*)] + \frac{1}{\lambda}(q_1^* - q_0^*)$$

We observe that $\Delta_T = \Delta_\Sigma$. So when NC is used, $\Delta_T > 0$ and total compensation is higher under perfect competition.

C. Proof for Proposition 3

C1. $\theta \in [\bar{\theta}_0, \bar{\theta}_1)$

Proof: When $\theta \in [\bar{\theta}_0, \bar{\theta}_1)$, $\Delta_w = \theta r i_1^* > 0$; Thus the second period wage is higher with NC. $\Delta_B = -\theta r i_1^* + \theta \frac{1 - e^{-\lambda r i_1^*}}{\lambda}$. Plug in $i_1^* = r(1-\theta)(1 - e^{-\lambda r i_1^*})$ to Δ_B we get

$$\Delta_B = \theta(\frac{i_1^*}{\lambda(1-\theta)r} - r i_1^*) = \theta(\frac{r i_1^*}{\lambda(1-\theta)r^2} - r i_1^*) \leq 0$$

$\lambda(1-\theta)r^2 > 1$ when $\theta \in [\bar{\theta}_0, \bar{\theta}_1)$. Thus the first period wage is lower under NCs.

C2. $\theta \in [\bar{\theta}_1, 1)$

Proof: When $\theta \in [\bar{\theta}_1, 1), i_1^* = i_0^* = 0$. Thus the two contracts are equivalent.