Nonlinear Difference-in-Differences with Repeated Cross Sections

AEA Session: **Recent Advances in Difference-in-Differences**

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1. Introduction

- Much of the work on difference-in-differences for staggered interventions has focused on
  - Linear outcome models.
  - Panel data.
- Some papers allow repeated cross sections with linear outcome models:
  - Callaway and Sant'Anna (2021, Journal of Econometrics) [CS (2021)]
  - Borusyak, Jaravel, Spiess (2024, REStud) [BJS (2024)]
  - Deb, Norton, Wooldridge, Zabel (2025, WP) [DNWZ (2025)].

- What if we have repeated cross-sections and a binary response?
  - Almost certainly start with a linear probability model.
  - The (conditional) parallel trends assumption is sensitive to functional form; Roth and Sant'Anna (2023, Econometrica).
- As a robustness check, try a simple nonlinear model.
  - Extend Wooldridge (2023, Econometrics Journal) to repeated cross sections.
  - In some cases, a nonlinear model will pass pre-trends tests when a linear model does not.

- With Y_t the outcome, common situations that might call for a nonlinear model:
  - ▸ Y_t is binary.
  - ▸ Y_t is fractional.
  - ▸ Y_t is a count.
  - ▸ $Y_t \geq 0$ is a corner solution.
- Want estimation methods that, like OLS, impose very few assmptions for consistency.
  - ▸ Pooled quasi-maximum likelihood estimation (QMLE) in the linear exponential family (LEF).
  - ▸ Additional benefits from using the canonical link version of the mean function.

2. Notation, Assumptions, and Identification

- Identification assumes a stable population over time periods $t = 1$, ..., T.
- First intervention is at time q; more units treated through T.
- Potential outcomes for each treated state:

$$Y_t(g), g \in \{q, \ldots, T, \infty\}, t \in \{1, 2, \ldots, T\},$$

  - $Y_t(\infty)$ is the outcome in the never treated state.
- Cohort dummies (exhaustive and mutually exclusive):

$$D_g, g \in \{q, \ldots, T, \infty\}$$

  - $D_\infty = 1$ means a never treated unit.

• Following the literature, we want to (at least) estimate the average treatment effects on the treated (ATTs):

$$\begin{aligned}\tau_{gt} &= E[Y_t(g) - Y_t(\infty)|D_g = 1] \\ &= E(Y_t|D_g = 1) - E[Y_t(\infty)|D_g = 1], t = g, \ldots, T;\ g = q, \ldots, T,\end{aligned}$$

▸ $E(Y_t|D_g = 1)$ is easy to estimate given a random sample at time t: $\bar{Y}_{gt} = N_{gt}^{-1} \sum_{i=1}^{N_t} D_{itg} \cdot Y_{it}$.

▸ Hard part is the counterfactual mean:

$$E[Y_t(\infty)|D_g = 1]$$

• Allow covariates $\mathbf{X}_t$ (assume no bad controls).

• Rule out spillovers (SUTVA).

Assumption CNA (Conditional No Anticipation): For $g \in \{q, \ldots, T\}$ and $t \in \{1, \ldots, g-1\}$,

$$E[Y_t(g)|\mathbf{D}, \mathbf{X}_t] = E[Y_t(\infty)|\mathbf{D}, \mathbf{X}_t]. \ \square$$

- Rules out anticipatory changes in the potential outcomes.
- Difficult to relax, can skip over pre-treatment periods just before the intervention.

Assumption CPT (Conditional Parallel Trends): For $t = 1, 2, \ldots, T$ and $G(\cdot)$ a twice-continuously differentiable, strictly increasing function,

$$E[Y_t(\infty)|\mathbf{D}, \mathbf{X}_t] = G\Bigg(\alpha + \sum_{g=q}^{T} \beta_g D_g + \mathbf{X}_t \boldsymbol{\kappa} + \sum_{g=q}^{T} (D_g \cdot \dot{\mathbf{X}}_{tg}) \boldsymbol{\eta}_g + \gamma_t + \mathbf{X}_t \boldsymbol{\pi}_t\Bigg),$$

- $\dot{\mathbf{X}}_{tg} \equiv \mathbf{X}_t - E(\mathbf{X}_t | D_g = 1)$
- As normalizations, $\gamma_t = 0$, $\boldsymbol{\pi}_t = \mathbf{0}$. □

• Except when $G(z) = z$, Assumption CPT imposes parallel trends on the linear index, not $E[Y_t(\infty)|\mathbf{D}, \mathbf{X}_t]$.

- Define a time-varying treatment indicator:

$$W_t = D_q \cdot pq_t + D_{q+1} \cdot p(q+1)_t + \cdots + D_T \cdot pT_t,$$

$$pg_t = fg_t + \cdots + fT_t$$

fs_t are time period dummies

- Imposing Assumptions CNA and CPTgives

$$E(Y_t|\mathbf{D}, \mathbf{X}_t, W_t = 0) = G\left(\alpha + \sum_{g=q}^{T} \beta_j D_g + \mathbf{X}_t\boldsymbol{\kappa} + \sum_{g=q}^{T} (D_g \cdot \dot{\mathbf{X}}_{tg})\boldsymbol{\eta}_g + \sum_{s=2}^{T} \gamma_s fs_t + \sum_{s=2}^{T} (fs_t \cdot \mathbf{X}_t)\boldsymbol{\pi}_s\right)$$

3. Estimation by Imputation and Pooled QMLE

- Identification suggests estimating $E[Y_t(\infty)|D_g = 1]$ by imputation.
  - BJS (2024) in linear panel data case.
  - Wooldridge (2023) in nonlinear panel data case.
  - DNWZ (2025) in linear repeated cross sections case.
- Imputation makes inference more difficult, does not provide all information.
- Pooling across all observations makes inference easier.

- In sampling the data, let $t(i)$ denote the observed time period for unit i.
- Observed data are

$$\{Y_{i,t(i)}, \mathbf{X}_{i,t(i)}, D_{i,t(i),g},\ g = q, \ldots, T, i = 1, \ldots, N\}$$

- With independence across i, inference need only be made robust to distributional misspecification (heteroskedasticity in linear case).
- With cluster sampling or clustered assignment, need to cluster at a suitable level.
  - Common is cohort assignment is at, say, the county level.
- Estimates and valid inference can be based on a single conditional expectation.

$$
\begin{aligned}
E[Y_{it(i)}|\mathbf{D}_{i,t(i)},\mathbf{X}_{i,t(i)}] = G\Bigg[\ \alpha &+ \sum_{g=q}^{T}\beta_g D_{i,t(i),g} \\
&+ \mathbf{X}_{i,t(i)}\boldsymbol{\kappa} + \sum_{g=q}^{T}(D_{i,t(i),g}\cdot\dot{\mathbf{X}}_{i,t(i),g})\boldsymbol{\eta}_g \\
&+ \sum_{s=2}^{T}\gamma_s fs_t + \sum_{s=2}^{T}(fs_t\cdot\mathbf{X}_{i,t(i)})\boldsymbol{\pi}_s \\
&+ \sum_{g=q}^{T}\sum_{s=g}^{T}\delta_{gs}(W_{i,t(i)}\cdot D_{i,t(i),g}\cdot fs_t) \\
&+ \sum_{g=q}^{T}\sum_{s=g}^{T}(W_{i,t(i)}\cdot D_{i,t(i),g}\cdot fs_t\cdot\dot{\mathbf{X}}_{i,t(i),g})\boldsymbol{\xi}_{gs}\ \Bigg]
\end{aligned}
$$

- Obtain the average partial effects (APEs) with respect to $W_{i,t(i)}$.
  - Average across subsamples (g, t) to obtain $\hat{\tau}_{gt}$.
  - Average by exposure times.
  - Can obtain a single overall weighted effect.
  - In Stata, `margins` with `subpop() and vce(uncond)` does all the work.
- When $G^{-1}(\cdot)$ is the canonical link function, imputation = pooled QMLE.
  - Handy for checking calculations.
- The $\hat{\delta}_{gt}$, and aggregated versions, are also of some interest.
  - Proportionate effects or log odds.

Leads and Lags (Event Study) Estimation

• To the earlier conditional mean function, add the pre-treatment indicators and interactions:

$$NW_{i,t(i)} \cdot D_{i,t(i),g} \cdot fs_t,\ NW_{i,t(i)} \cdot D_{i,t(i),g} \cdot fs_t \cdot \dot{\mathbf{X}}_{i,t(i),g},\ s = 1,\ldots,g-2$$

▸ Coefficients for $s = g - 1$ normalized to zero.

▸ $NW_{i,t(i)} = 1 - W_{i,t(i)}$.

▸ Aggregate time until treatment using APEs with respect to $NW_{i,t(i)}$; `margins`.

▸ Produces, along with the ATTs, an aggregated event study graph.

• Imputation and pooled estimation are still the same with the canonical link.

▸ Linear + OLS, logit, flogit, exponential + Poisson.

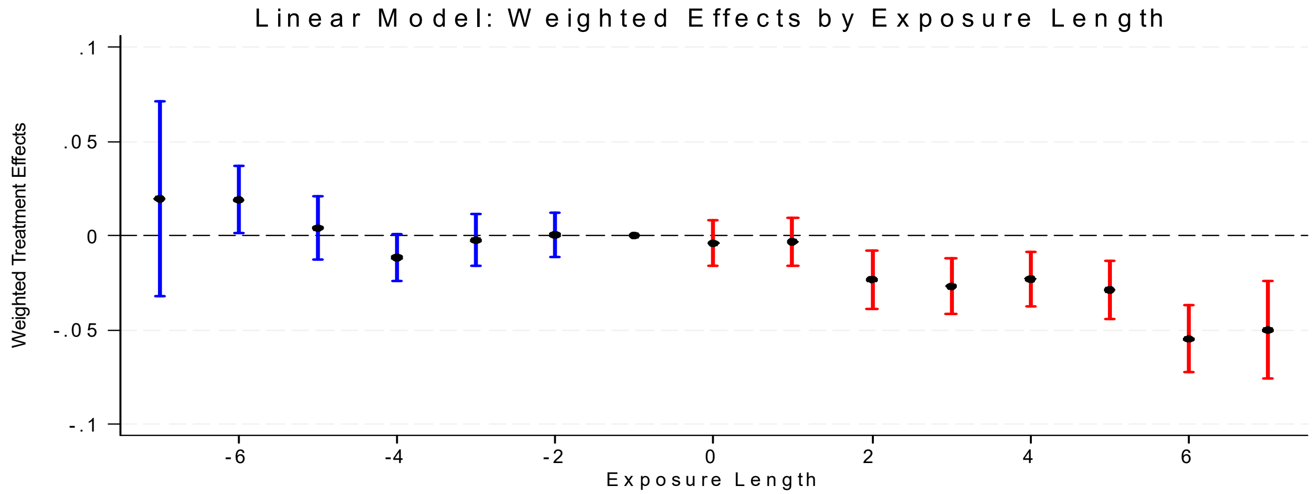
4. Application

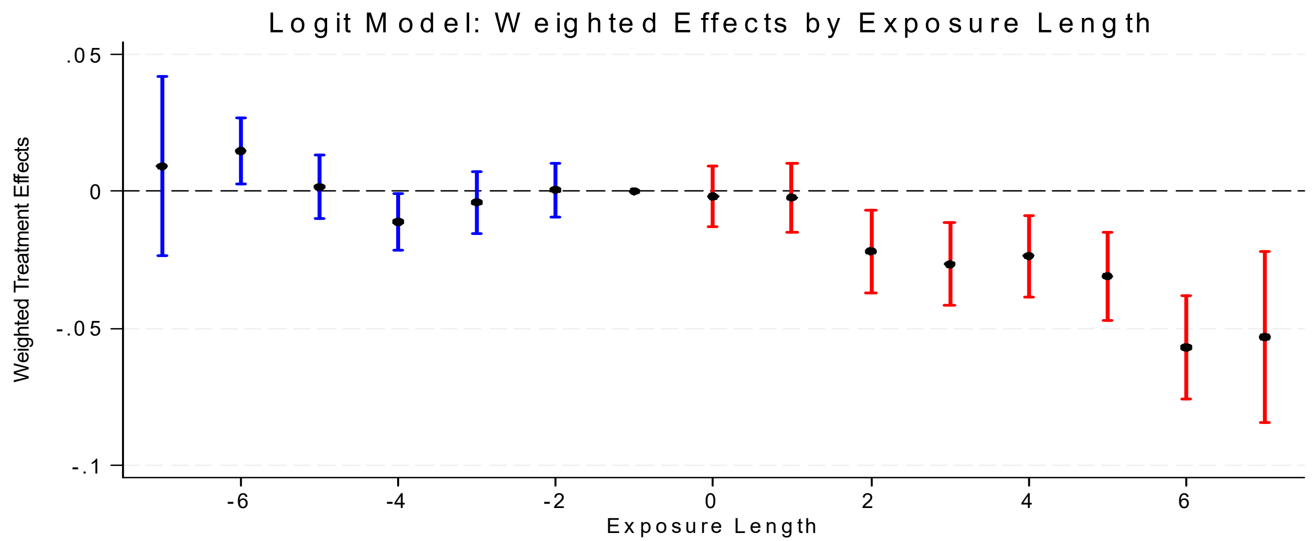
- Effects of Secure Communities immigration enforcement on SNAP enrollment (binary).
  - Individual survey data from the ACS, 2005-2016.
  - Hispanic U.S. citizens who reported not moving states.
  - Treatment at the county level; need to aggregate to PUMA.
  - Four treated cohorts: 2009, 2010, 2011, 2012.
  - More than 500,000 observations; 682 PUMAs across 32 states.
  - $\mathbf{X}_t$ includes three census region dummies; gender; age; education; poverty status.
  - Used in DNWZ (2025) in linear analysis.

Weighted Aggregated ATTs

	Lags Only	Leads and Lags	Heterogeneous Trends
	(1)	(2)	(3)
Linear	–0.0188 (0.0046)	–0.0220 (0.0058)	–0.0371 (0.0106)
Logit	–0.0172 (0.0047)	–0.0218 (0.0062)	–0.0455 (0.0148)

- Stata code available on request.
  - Standard errors clustered at the PUMA level.
  - Estimating hundreds of parameters (12 time periods, four cohorts, $\mathbf{X}_t$ is 1×7).


Linear Model: Weighted Effects by Exposure Length
Weighted Treatment Effects
.1
.05
0
-.05
-.1
-6
-4
-2
0
2
4
6
Exposure Length


Logit Model: Weighted Effects by Exposure Length
Weighted Treatment Effects
.05
0
-.05
-.1
-6
-4
-2
0
2
4
6
Exposure Length