Supplemental Appendix: Event Studies with Feedback

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LEMMA 1 (Elimination of Time Effects): *Suppose the outcome equation* (1) *is augmented with additive time effects,*

$$Y_{it} = \rho_Y Y_{i,t-1} + \alpha_i + X_{it}'\beta + \sum_{j\in\mathcal{J}} D_{it}^j \delta_{ij} + \gamma_t + U_{it}, \quad t = 1, \ldots, T.$$

Define the cross-sectional averages

$$\bar{Y}_t = \frac{1}{N}\sum_{i=1}^N Y_{it}, \quad \bar{X}_t = \frac{1}{N}\sum_{i=1}^N X_{it}, \quad \bar{\alpha} = \frac{1}{N}\sum_{i=1}^N \alpha_i, \quad \bar{U}_t = \frac{1}{N}\sum_{i=1}^N U_{it},$$

and

$$\bar{D}_t(\delta) = \frac{1}{N}\sum_{i=1}^N \sum_{j\in\mathcal{J}} D_{it}^j \delta_{ij}.$$

Let the demeaned variables be

$$\dot{Y}_{it} = Y_{it} - \bar{Y}_t, \quad \dot{X}_{it} = X_{it} - \bar{X}_t, \quad \dot{\alpha}_i = \alpha_i - \bar{\alpha}, \quad \dot{U}_{it} = U_{it} - \bar{U}_t.$$

Then the demeaned outcome satisfies

$$\dot{Y}_{it} = \rho_Y \dot{Y}_{i,t-1} + \dot{\alpha}_i + \left(\sum_{j\in\mathcal{J}} D_{it}^j \delta_{ij} - \bar{D}_t(\delta)\right) + \dot{X}_{it}'\beta + \dot{U}_{it}, \tag{A.1}$$

so that the time effects $\{\gamma_t\}$ *are eliminated from the dynamic equation for* $\dot{Y}_{it}$.

PROOF:

Averaging the augmented outcome equation over $i = 1, \ldots, N$ yields

$$\bar{Y}_t = \rho_Y \bar{Y}_{t-1} + \bar{\alpha} + \bar{D}_t(\delta) + \gamma_t + \bar{X}_t'\beta + \bar{U}_t.$$

Subtracting this average from the individual equation gives

$$\begin{aligned} Y_{it} - \bar{Y}_t = \rho_Y (Y_{i,t-1} - \bar{Y}_{t-1}) + (\alpha_i - \bar{\alpha}) \\ + \left(\sum_{j\in\mathcal{J}} D_{it}^j \delta_{ij} - \bar{D}_t(\delta)\right) + (\gamma_t - \gamma_t) + (X_{it} - \bar{X}_t)'\beta + (U_{it} - \bar{U}_t). \end{aligned}$$

The term $(\gamma_t - \gamma_t)$ is identically zero. Using the definitions of the demeaned variables, this simplifies to

$$\dot{Y}_{it} = \rho_Y \dot{Y}_{i,t-1} + \dot{\alpha}_i + \left(\sum_{j\in\mathcal{J}} D_{it}^j \delta_{ij} - \bar{D}_t(\delta)\right) + \dot{X}_{it}'\beta + \dot{U}_{it},$$

which is exactly (A.1). Hence the time effects $\{\gamma_t\}$ are eliminated from the dynamic equation for the demeaned outcome.