Supplemental Appendix for
"Sustaining International Rules in a Multipolar World"

By CECILIA CARVALHO, NICOLAS GOULART, DANIEL MONTE, AND EMANUEL ORNELAS

I. Mathematical Appendix

1) **Hegemons choose rules (Lemma A1).** In any symmetric Markov equilibrium, countries weakly prefer regime R in all states, and any hegemon optimally chooses R regardless of the inherited status quo.

2) **No rules under power (Lemma A2).** When the status quo is power-based, rules are not rebuilt in the multipolar state due to profitable unilateral deviations.

3) **Cyclic equilibria exist (Proposition 1).** A symmetric Markov equilibrium always exists in which countries choose P in the multipolar state and R under hegemony.

4) **Rules-based equilibria exist (Proposition 2).** A rules-based equilibrium exists if and only if the incentive condition in (A1) holds for all countries.

5) **Comparative statics (Proposition 3).** Greater multipolar persistence facilitates sustaining rules if and only if the number of countries exceeds the threshold in (A3).

A. *Lemma A1*

LEMMA A1 (Hegemons Choose *Rules*): *In any symmetric Markov equilibrium, countries weakly prefer entering any state under regime R: $V_i(\omega,R) \geq V_i(\omega,P)$ for all $\omega \in \Omega$ and all $i \in \mathscr{I}$. Moreover, $\sigma_i(i,\rho) = R$ for all $i \in \mathscr{I}$ and $\rho \in \{P,R\}$.*

PROOF:

Lemma A1 follows from Carvalho, Monte and Ornelas (2025). We first show that $V_i(i,R) \geq V_i(i,P)$ for all $i \in \mathscr{I}$ and $\omega \in \{0,i\}$. Since the only difference between inheriting status quo R versus P at Bellman equation is the indicator cost $\mathbf{1}\{\rho = P, a_i = R\}c \geq 0$ that must be paid when switching from P to R, we have $V_i(i,R) \geq V_i(i,P)$. The same argument applies to the multipolar state, yielding $V_i(0,R) \geq V_i(0,P)$ for all $i \in \mathscr{I}$. Thus, $V_i(\omega,R) \geq V_i(\omega,P)$ for all $i \in \mathscr{I}$ and $\omega \in \{0,i\}$.

Consider now country i's incentive compatibility constraint as hegemon in state i with inherited status quo ρ:

$$u_i(i,R) - \mathbf{1}\{\rho = P\}c + \delta \sum_{\omega' \in \Omega} q_{i,\omega'} V_i(\omega',R) \geq u_i(i,P) + \delta \sum_{\omega' \in \Omega} q_{i,\omega'} V_i(\omega',P).$$

But $V_i(\omega',R) \geq V_i(\omega',P)$ for all $\omega' \in \{0,i\}$, and by assumption, states transitions are smooth and hegemons myopically prefer *rules* net of switching costs: $u_i(i,R) - u_i(i,P) \geq c$. Therefore, the incentive compatibility constraint is satisfied, and $\sigma_i(i,\rho) = R$ for all $i \in \mathscr{I}$ and $\rho \in \{P,R\}$.

Consider now country i as a subordinate in state $j \neq i$. The hegemon j makes the regime choice, and country i's continuation value depends on tomorrow's state:

$$\begin{aligned} V_i(j,R) - V_i(j,P) &= u_i\big(j,\sigma_j(j,R)\big) - u_i\big(j,\sigma_j(j,P)\big) + \delta \sum_{\omega' \in \Omega} q_{j,\omega'} \Big[V_i\big(\omega',\sigma_j(j,R)\big) - V_i\big(\omega',\sigma_j(j,P)\big)\Big] \\ &= u_i(j,R) - u_i(j,P) > 0 \end{aligned}$$

where the last equality follows from the fact that, by the previous result, $\sigma_j(j,R) = \sigma_j(j,P) = R$, and the inequality follows from the assumption that subordinates myopically prefer *rules*. Therefore, we also have that $V_i(j,R) \geq V_i(j,P)$ for all $j \neq i$, completing the proof.

Intuitively, switching from R to P is costless, whereas switching from P to R is costly, so countries prefer a status quo regime R. Since by assumption hegemons also prefer R to P in the short-run, net of switching costs, the result follows.

B. Lemma A2

LEMMA A2 (Multipolar State Keeps *Power*): *In any symmetric Markov equilibrium, $\sigma_i(0,P) = P$ for all $i \in \mathscr{I}$.*

PROOF:

Suppose by contradiction that there exists an equilibrium with $\sigma_i(0,P) = R$ for all $i \in \mathscr{I}$.

Consider country i's incentive to deviate to P when all others choose R. The deviation gain equals:

$$\text{Deviation gain} = u_{i0}(P,\mathbf{a}_{-i}) - u_{i0}(R,\mathbf{a}_{-i}) + c + \delta \sum_{\omega' \in \Omega} q_{0,\omega'}\big(V_i(\omega',P) - V_i(\omega',R)\big).$$

We evaluate the continuation value differences. If country i becomes hegemon ($\omega' = i$), it chooses R by Lemma A1 regardless of status quo. Under status quo P, this costs c:

$$V_i(i,P) - V_i(i,R) = -c.$$

If another country becomes hegemon ($\omega' = j \neq i$), it also chooses R, so subordinate i receives the same payoff:

$$V_i(j,P) - V_i(j,R) = 0.$$

If the state remains multipolar ($\omega' = 0$), all countries choose R by hypothesis, but must pay c under status quo P:

$$V_i(0,P) - V_i(0,R) = -c.$$

Substituting these into the deviation gain:

$$\begin{aligned}\text{Deviation gain} &= u_{i0}(P,\mathbf{a}_{-i}) - u_{i0}(R,\mathbf{a}_{-i}) + c - \delta c(q_{0,i} + q_{0,0}) \\ &= u_{i0}(P,\mathbf{a}_{-i}) - u_{i0}(R,\mathbf{a}_{-i}) + c\big(1 - \delta(q_{0,i} + q_{0,0})\big).\end{aligned}$$

By the unilateral temptation assumption, $u_{i0}(P,\mathbf{a}_{-i}) > u_{i0}(R,\mathbf{a}_{-i})$. Since $\delta < 1$ and $q_{0,i} + q_{0,0} < 1$, we have $1 - \delta(q_{0,i} + q_{0,0}) > 0$. Therefore, the deviation gain is strictly positive, contradicting the equilibrium hypothesis.

C. Proof of Proposition 1

PROPOSITION 1 (Existence of Cyclic Equilibria): *A cyclic symmetric Markov equilibrium always exists.*

PROOF:

We construct a symmetric Markov equilibrium in which all countries choose P in the multipolar state regardless of the inherited status quo. By Lemma A1, hegemons optimally choose R. By Lemma A2, no country deviates from P in the multipolar state when the status quo is P.

Now consider the multipolar state with inherited status quo R. Suppose all other countries choose P. If country i deviates to R while all others choose P, the status quo next period becomes P since not all countries choose R. Thus, the continuation value is identical regardless of i's action:

$$\delta \sum_{\omega' \in \Omega} q_{0,\omega'} V_i(\omega', P) = \delta \sum_{\omega' \in \Omega} q_{0,\omega'} V_i(\omega', P).$$

The difference comes only from current payoffs. By the unilateral temptation assumption, $u_{i0}(P, \mathbf{a}_{-i}^P) > u_{i0}(R, \mathbf{a}_{-i}^P)$. Therefore, no country deviates from P in the multipolar state, establishing that this equilibrium always exists.

D. *Proof of Proposition 2*

PROPOSITION 2 (Existence of Rules-Based Equilibria): *A rules-based symmetric Markov equilibrium exists if and only if, for all $i \in \mathscr{I}$,*

$$\text{(A1)} \qquad u_{i0}(P, \mathbf{a}_{-i}^R) - u_{i0}(R, \mathbf{a}_{-i}^R) \;\leq\; \frac{\delta q_{0,0}}{1 - \delta q_{0,0}} \Big(u_{i0}(R, \mathbf{a}_{-i}^R) - u_{i0}(P, \mathbf{a}_{-i}^P) \Big) \;+\; \frac{c\,\delta q_{0,i}}{1 - \delta q_{0,0}}.$$

PROOF:

In a rules-based equilibrium, all countries choose R in the multipolar state when the status quo is R. Country i has no incentive to deviate if:

$$\text{(A2)} \qquad u_{i0}(R, \mathbf{a}_{-i}^R) - u_{i0}(P, \mathbf{a}_{-i}^R) + \delta \sum_{\omega' \in \Omega} q_{0,\omega'} \big(V_i(\omega', R) - V_i(\omega', P) \big) \geq 0.$$

We evaluate the continuation value differences. By Lemma A1, when country i becomes hegemon it chooses R regardless of status quo, paying c under status quo P. Thus $V_i(i,R) - V_i(i,P) = c$. When another country $j \neq i$ becomes hegemon, it also chooses R, so subordinate i receives the same payoff: $V_i(j,R) - V_i(j,P) = 0$. Substituting into (A2):

$$0 \leq u_{i0}(R, \mathbf{a}_{-i}^R) - u_{i0}(P, \mathbf{a}_{-i}^R) + c\delta q_{0,i} + \delta q_{0,0} \big(V_i(0,R) - V_i(0,P) \big).$$

To characterize $V_i(0,R) - V_i(0,P)$, note that countries choose R when status quo is R and P when status quo is P (Lemma A2). Using the Bellman equations:

$$V_i(0,R) - V_i(0,P) = u_{i0}(R, \mathbf{a}_{-i}^R) - u_{i0}(P, \mathbf{a}_{-i}^P) + \delta \Big[c q_{0,i} + q_{0,0} \big(V_i(0,R) - V_i(0,P) \big) \Big].$$

Rearranging yields:

$$V_i(0,R) - V_i(0,P) = \frac{u_{i0}(R, \mathbf{a}_{-i}^R) - u_{i0}(P, \mathbf{a}_{-i}^P) + c\delta q_{0,i}}{1 - \delta q_{0,0}}.$$

Substituting this back into the incentive compatibility condition and simplifying gives condition (A1).

E. *Proof of Proposition 3*

PROPOSITION 3 (Comparative Statics): *Assume symmetric transitions with $q_{0,i} = (1 - q_{0,0})/N$ for all $i \in \mathscr{I}$. Greater multipolar persistence ($q_{0,0}$) facilitates sustaining rules if and only if*

$$\text{(A3)} \qquad N \geq c(1 - \delta) / [u_{i0}(R, \mathbf{a}_{-i}^R) - u_{i0}(P, \mathbf{a}_{-i}^P)].$$

PROOF:

Under symmetric transitions with $q_{0,i} = (1-q_{0,0})/N$ for all $i \in \mathscr{I}$, condition (A1) becomes:

$$\text{(A4)} \qquad u_{i0}(P,\mathbf{a}^R_{-i}) - u_{i0}(R,\mathbf{a}^R_{-i}) \;\le\; \frac{\delta q_{0,0}}{1-\delta q_{0,0}}\Big(u_{i0}(R,\mathbf{a}^R_{-i}) - u_{i0}(P,\mathbf{a}^P_{-i})\Big) \;+\; \frac{c\,\delta(1-q_{0,0})}{N(1-\delta q_{0,0})}.$$

To determine when greater multipolar persistence ($q_{0,0}$) facilitates sustaining *rules*, we compute the derivative of the right-hand side of (A4) with respect to $q_{0,0}$.

The right-hand side of (A4) is:

$$\text{(A5)} \qquad \text{RHS}(q_{0,0}) = \frac{\delta q_{0,0}}{1-\delta q_{0,0}}(u_{i0}(R,\mathbf{a}^R_{-i}) - u_{i0}(P,\mathbf{a}^P_{-i})) + \frac{c\,\delta(1-q_{0,0})}{N(1-\delta q_{0,0})}.$$

$$\begin{aligned}
\frac{\partial}{\partial q_{0,0}}\left[\frac{\delta q_{0,0}}{1-\delta q_{0,0}}(u_{i0}(R,\mathbf{a}^R_{-i}) - u_{i0}(P,\mathbf{a}^P_{-i}))\right] &= \delta(u_{i0}(R,\mathbf{a}^R_{-i}) - u_{i0}(P,\mathbf{a}^P_{-i}))\cdot\frac{\partial}{\partial q_{0,0}}\left[\frac{q_{0,0}}{1-\delta q_{0,0}}\right] \\
&= \delta(u_{i0}(R,\mathbf{a}^R_{-i}) - u_{i0}(P,\mathbf{a}^P_{-i}))\cdot\frac{(1-\delta q_{0,0}) - q_{0,0}\cdot(-\delta)}{(1-\delta q_{0,0})^2} \\
&= \delta(u_{i0}(R,\mathbf{a}^R_{-i}) - u_{i0}(P,\mathbf{a}^P_{-i}))\cdot\frac{1-\delta q_{0,0}+\delta q_{0,0}}{(1-\delta q_{0,0})^2} \\
&= \frac{\delta(u_{i0}(R,\mathbf{a}^R_{-i}) - u_{i0}(P,\mathbf{a}^P_{-i}))}{(1-\delta q_{0,0})^2}.
\end{aligned}$$

$$\begin{aligned}
\frac{\partial}{\partial q_{0,0}}\left[\frac{c\,\delta(1-q_{0,0})}{N(1-\delta q_{0,0})}\right] &= \frac{c\,\delta}{N}\cdot\frac{\partial}{\partial q_{0,0}}\left[\frac{1-q_{0,0}}{1-\delta q_{0,0}}\right] \\
&= \frac{c\,\delta}{N}\cdot\frac{(-1)(1-\delta q_{0,0}) - (1-q_{0,0})\cdot(-\delta)}{(1-\delta q_{0,0})^2} \\
&= \frac{c\,\delta}{N}\cdot\frac{-1+\delta q_{0,0}+\delta-\delta q_{0,0}}{(1-\delta q_{0,0})^2} \\
&= \frac{c\,\delta}{N}\cdot\frac{\delta-1}{(1-\delta q_{0,0})^2} \\
&= -\frac{c\,\delta(1-\delta)}{N(1-\delta q_{0,0})^2}.
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \text{RHS}}{\partial q_{0,0}} &= \frac{\delta(u_{i0}(R,\mathbf{a}^R_{-i}) - u_{i0}(P,\mathbf{a}^P_{-i}))}{(1-\delta q_{0,0})^2} - \frac{c\,\delta(1-\delta)}{N(1-\delta q_{0,0})^2} \\
&= \frac{\delta}{(1-\delta q_{0,0})^2}\left[u_{i0}(R,\mathbf{a}^R_{-i}) - u_{i0}(P,\mathbf{a}^P_{-i}) - \frac{c(1-\delta)}{N}\right].
\end{aligned}$$

Since $\delta/[(1-\delta q_{0,0})^2] > 0$, the derivative $\partial\text{RHS}/\partial q_{0,0}$ has the same sign as the bracketed term:

$$u_{i0}(R,\mathbf{a}^R_{-i}) - u_{i0}(P,\mathbf{a}^P_{-i}) - c(1-\delta)/N.$$

Therefore $\partial\text{RHS}/\partial q_{0,0} \ge 0$ if and only if $u_{i0}(R,\mathbf{a}^R_{-i}) - u_{i0}(P,\mathbf{a}^P_{-i}) \ge c(1-\delta)/N$. Rearranging yields $N \ge c(1-\delta)/[u_{i0}(R,\mathbf{a}^R_{-i}) - u_{i0}(P,\mathbf{a}^P_{-i})]$.

Since the left-hand side of (A4) is independent of $q_{0,0}$, an increase in $q_{0,0}$ makes the inequality easier to satisfy (facilitates sustaining *rules*) if and only if the right-hand side increases with $q_{0,0}$.

This occurs precisely when:

$$N \geq c(1-\delta)/[u_{i0}(R, \mathbf{a}_{-i}^{R}) - u_{i0}(P, \mathbf{a}_{-i}^{P})],$$

REFERENCES

Carvalho, Cecilia, Daniel Monte, and Emanuel Ornelas. 2025. "Equilibrium Trade Regimes: Power- vs. Rules-Based." *CEPR Discussion Paper DP20751*.