Online Appendix: Production and Financial Networks in Interplay

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Appendix A: Additional results

A.1. Additional Tables

TABLE A1

SUMMARY STATISTICS

			Mean	S.D.	P25	Median	P75
Link Level							
Upstream propagation							
Δlog(sales from supplier to customer)	The log of change of firm's sales to its customer between 2008 and 2009	%	-12.062	61.407	-53.513	-13.894	17.599
Direct (Bank) Shock	Bank (supply) Shock is a variable capturing whether the firm was borrowing before the global financial crisis from banks which significantly reduced credit supply during the global financial crisis. To construct this variable, we use the firm level shock estimated following Amiti & Weinstein (2018) as the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values implies a credit reduction), where the firm-level shock is aggregated using the lagged December 2007 credit between the firm and each bank as weights		0.151	0.050	0.125	0.152	0.176
Customer (Bank) Shock (1st Order Effect)	Direct (bank credit supply) shock of the customer of a firm		0.151	0.047	0.125	0.151	0.176
Dummy Direct (Bank) Shock	A binary variable that takes the vaule of one when the Direct Shock is above its median and zero otherwise	0/1	0.544	0.498	0.000	1.000	1.000
Dummy Customer (Bank) Shock (1st Order Effect)	A binary variable that takes the value of one when the Customer (Bank) Shock (1st Order Effect) is above its median and zero otherwise	0/1	0.497	0.500	0.000	0.000	1.000
Higher Order Customer (Bank) Shock	A network aggregate of shocks hitting suppliers of the customer of any order (see equations (8) and (8R) and Section 5 and 6)		0.120	0.035	0.096	0.117	0.140
Customer Reduction of Bank Debt	The (negative) change in overall bank credit of the customer between 2008 and 2009	%	8.563	61.479	-7.496	7.449	27.897
Customer (Bank) Net Interbank Borrowing	The net interbank position (interbank deposits minus interbank loans) of the customer's weighted average banks, where weights are based on the lagged credit	0.0x	0.025	0.019	0.011	0.023	0.036
Δlog(sales from supplier to customer/sales of customer)	The log of change of firm's sales to its customer between 2008 and 2009 minus the log of change of the customer's sales	%	5.214	63.099	-33.604	0.973	35.927
Downstream propagation							
Δlog(sales from supplier to customer)	The log of change of a supplier's sales to the firm between 2008 and 2009	%	-11.932	60.414	-52.008	-12.730	16.381
Direct (Bank) Shock	Bank (supply) Shock is a variable capturing whether the firm was borrowing before the global financial crisis from banks which significantly reduced credit supply during the global financial crisis. To construct this variable, we use the firm level shock estimated following Amiti & Weinstein (2018) as the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values implies a credit reduction), where the firm-level shock is aggregated using the lagged December 2007 credit between the firm and each bank as weights		0.150	0.050	0.121	0.152	0.180
Supplier (Bank) Shock (1st Order Effect)	Direct (bank credit supply) shock of the supplier of a firm		0.151	0.044	0.129	0.151	0.173
Dummy Direct (Bank) Shock	A binary variable that takes the vaule of one when the Direct Shock is above its median and zero otherwise	0/1	0.541	0.498	0.000	1.000	1.000
Dummy Supplier (Bank) Shock (1st Order Effect)	A binary variable that takes the vaule of one when the Supplier (Bank) Shock (1st Order Effect) is above its median and zero otherwise	0/1	0.505	0.500	0.000	1.000	1.000
Higher Order Supplier (Bank) Effect	A network aggregate of shocks hitting suppliers of the supplier of any order (see equations (8) and (8R) and Section 5 and 6)		0.108	0.041	0.081	0.106	0.133
Supplier Reduction of Bank Debt	The (negative) change in overall bank credit of the supplier between 2008 and 2009	%	10.071	58.433	-6.275	8.696	28.814
Supplier (Bank) Net Interbank Borrowing	The net interbank position (interbank deposits minus interbank loans) of the supplier's weighted average banks, where weights are based on the lagged credit	0.0x	0.027	0.019	0.015	0.024	0.036
Δlog(sales from supplier to customer/sales of customer)	The log of change of a supplier's sales to the firm between 2008 and 2009 minus the log of change of the firm (customer)'s sales	%	6.280	62.476	-31.823	2.272	36.550
Node Level							
Δlog(sales)	The log of change of firm' sales to all its customers between 2008 and 2009	%	-19.970	40.779	-40.314	-16.800	-0.585
Δlog(employment)	The log of change of firm' employment between 2008 and 2009	%	-8.967	30.296	-20.743	-0.358	0.000
Direct (Bank) Shock	Bank (supply) Shock is a variable capturing whether the firm was borrowing before the global financial crisis from banks which significantly reduced credit supply during the global financial crisis. To construct this variable, we use the firm level shock estimated following Amiti & Weinstein (2018) as the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values implies a credit reduction), where the firm-level shock is aggregated using the lagged December 2007 credit between the firm and each bank as weights		0.148	0.061	0.109	0.149	0.185
Supplier (Bank) Shock (1st Order Effect)	Aggregate of all the firm's suppliers' Direct (Bank) Shock, weighted by the lagged sales from each supplier to the firm		0.063	0.043	0.030	0.058	0.088
Customer (Bank) Shock (1st Order Effect)	Aggregate of all the firm's customers' Direct (Bank) Shock, weighted by the lagged sales from the firm to each customer		0.026	0.032	0.002	0.015	0.039
Upstream (1st & Higher Order Effects)	A network aggregate of shocks hitting customers of any order (see equations (10) and (10R) and Section 5 and 6)		0.034	0.049	0.000	0.008	0.054
Bidirectional (Up & Down 1st & Higher Order Effects)	A network aggregate of shocks hitting suppliers and customers of any order (see equations (10) and (10R) and Section 5 and 6)		0.041	0.046	0.004	0.026	0.064

Notes: This table reports the definition, mean, standard deviation and first, second and third quartiles of the variables used in the analysis. See Section 3 to 6 of the paper for a more in depth explanation.

TABLE A2

DIFFERENCE IN MEAN TESTS DEPENDING ON EX-ANTE LINKS WITH BANKS WITH STRONG NEGATIVE CREDIT SUPPLY

	Firms Exposed to Unconstrained Banks		Firms Exposed to Constrained Banks		Difference in Means	Normalized Differences	Dependent Variable: Bank Credit Supply Shock	
	Mean	S.D.	Mean	S.D.	t test	test	Coefficient	S.E.
Firm Characteristics								
Short Term Debt	49.57	(15.36)	49.90	(15.36)	4.39	0.02	0.001	(0.001)
Log(Age)	2.63	(0.34)	2.63	(0.34)	-2.57	-0.01	-0.000	(0.001)
Own Funds/Total Assets	31.56	(14.93)	31.36	(14.93)	-2.63	-0.01	0.001	(0.001)
Log(Total Assets)	7.57	(0.97)	7.58	(0.97)	1.21	0.00	0.003	(0.003)
Liquidity Ratio	16.25	(13.72)	16.11	(13.72)	-2.08	-0.01	-0.000	(0.001)
Average Bank Characteristics								
Log(Total Assets)	18.32	(0.77)	17.82	(0.69)	-138.55	-0.48	-0.090**	(0.037)
Own Funds/Total Assets	0.05	(0.01)	0.05	(0.01)	29.97	0.10	0.006	(0.054)
Net Interbank Borrowing	0.02	(0.01)	0.03	(0.01)	82.81	0.29	0.055**	(0.026)
ROA	0.01	(0.00)	0.01	(0.00)	40.09	0.14	0.020	(0.031)
NPL	0.03	(0.01)	0.03	(0.01)	28.73	0.10	-0.007	(0.035)
Loans/Deposits	0.62	(0.09)	0.63	(0.09)	7.70	0.03	0.012	(0.035)
% Construction & Real Estate	0.47	(0.05)	0.48	(0.06)	36.33	0.13	0.046	(0.040)
Savings Bank	0.53	(0.50)	0.41	(0.49)	-48.33	-0.17	-0.061	(0.055)
Network variables								
Bonacich centrality	0.64	(0.43)	0.66	(0.44)	10.28	0.04	-0.001	(0.002)
Bonacich centrality (γ=1)	1.47	(0.38)	1.49	(0.38)	13.02	0.05	-0.003	(0.002)
Upstreamness	1.68	(0.42)	1.70	(0.42)	11.09	0.04	0.005	(0.003)
In degree	18.61	(8.97)	19.04	(9.09)	9.72	0.03	0.007	(0.004)
Out degree	12.36	(9.31)	12.97	(9.44)	13.25	0.05	0.002	(0.002)
Expenditure share (γ_i)	0.00	(0.00)	0.00	(0.00)	5.15	0.02	-0.001	(0.002)
R-squared							0.212	
No. of Observations	80,884		85,999				166,883	

Notes: This table (in the first four columns) reports means and standard deviations of firm characteristics as of December 2007. Firms are classified in two groups. The first two columns refer to firms that ex-ante worked with unconstrained banks (its bank credit supply is below the median of the bank supply factor estimated following Amiti & Weinstein (2018), see below and Section 3), while the third and fourth columns refer to firms that worked with constrained banks (above the median). Column (5) reports the t-statistic of the differences in mean and column (6) shows the normalized difference test (a scale-and-sample-size-free estimator) proposed by Imbens and Wooldridge (2009), for which Imbens and Rubin (2015) suggested a heuristic threshold of 0.25 in absolute value for significant differences. Bank characteristics at the firm level are computed as a weighted average of the bank variables at the firm-bank level, using as weights the credit amount of each relationship. Columns (7) and (8) shows the results of a OLS regressions where the dependent variable is the firm level shock estimated following Amiti & Weinstein (2018) as the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values implies a credit reduction), where the firm-level shock is aggregated using the lagged credit between the firm and each bank as weights, and the rest of the variables are standardized. Industry*province dummies are included. Vector of Bonacich centralities is defined with $\mathbf{v} = \kappa(\mathbf{I} - \alpha\mathbf{G})^{-1}\boldsymbol{\gamma}$ where $\kappa > 0$ is a normalizing constant, upstreamness is the measure defined in Antras et al. (2012), In degree is the number of suppliers, and Out degree is the number of customers. Coefficients are listed in the first row, robust standard errors are reported in the adjacent column which are corrected for clustering at the four-digit NACE, province and main bank. *** Significant at 1%, ** significant at 5%, * significant at 10%.

TABLE A3

FIRM-LEVEL EFFECTS OF BANK SUPPLY SHOCKS

Dependent Variable:	ΔCredit		
	(1)	(2)	(3)
	2009	2008	2007
Direct (Bank) Shock	-0.604**	-0.214	0.297
	(0.285)	(0.612)	(0.353)
Firm Controls	Yes	Yes	Yes
Spatial & Industry Fixed Effects	Yes	Yes	Yes
R-squared	0.057	0.086	0.102
Observations	196,171	99,257	87,085

Notes: This table reports estimates from OLS. See Section 4. Observations are at the level of the firm (node-level). The dependent variable is the change in bank credit. Bank shock is a variable capturing whether the firm was borrowing before the global financial crisis from banks which significantly reduced credit supply during the global financial crisis. To construct this variable, we first use the firm level shock estimated following Amiti & Weinstein (2018) as the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values implies a credit reduction) and then we aggregate it using the lagged credit between the firm and each bank as weights. As we cannot control for firm fixed effects, we control for industry, zip code fixed effects and main bank fixed effects. All shocks are standardized. For the list of firm controls, see Section 4. Coefficients for each regressor are listed in the first row, while robust standard errors are reported in the row below (corrected for clustering at the level of the main bank). In each column, the word Yes indicates that the set of characteristics or fixed effects is included. *** Significant at 1%, ** significant at 5%, * significant at 10%.

TABLE A4

LINK-LEVEL: PROPAGATION OF BANK CREDIT SUPPLY SHOCKS THROUGH THE NETWORK OF CUSTOMERS/SUPPLIERS. REDUCED FORM. DUMMY SHOCK

Panel A. Upstream propagation (indirect shocks via bank credit supply shocks to first-order customers)

			IV. Instrument: Bank Net Interbank Borrowing		IV. Instrument: Bank Shock	
			1st Stage	2º Stage	1st Stage	2º Stage
	(1)	(2)	(3)	(4)	(5)	(6)
Dummy Direct (Bank) Shock	-0.914*					
	(0.493)					
Dummy Customer (Bank) Shock (1st Order Effect)		-2.620**		-4.803**	3.366***	
		(1.173)		(1.984)	(0.602)	
Customer (Bank) Net Interbank Borrowing			0.413***			
			(0.017)			
Customer Reduction of Bank Debt						-0.558***
						(0.265)
Customer:						
Controls	-	Yes	Yes	Yes	Yes	Yes
Spatial*Industry Fixed Effects	-	Yes	Yes	Yes	Yes	Yes
Fixed Effects	Yes	No	No	No	No	No
Firm:						
Controls	Yes	-	-	-	-	-
Spatial*Industry Fixed Effects	Yes	-	-	-	-	-
Fixed Effects	No	Yes	Yes	Yes	Yes	Yes
Firm*Supplier Spatial & Industry Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Effective First Stage F statistic	-	-	91.77	-	31.88	-
R-squared	0.404	0.488	-	-	-	-
Observations	1,119,169	1,119,169	1,119,169	1,119,169	1,119,169	1,119,169

Panel B. Downstream propagation (indirect shocks via bank credit supply shocks to first-order suppliers)

			IV. Instrument: Bank Net Interbank Borrowing		IV. Instrument: Bank Shock	
			1st Stage	2º Stage	1st Stage	2º Stage
	(1)	(2)	(3)	(4)	(5)	(6)
Dummy Direct (Bank) Shock	-2.902**					
	(1.452)					
Dummy Supplier (Bank) Shock (1st Order Effect)		-1.180**		-3.335***	2.173***	
		(0.516)		(1.225)	(0.434)	
Supplier (Bank) Net Interbank Borrowing			0.848***			
			(0.037)			
Supplier Reduction of Bank Debt						-0.453**
						(0.094)
Supplier:						
Controls	-	Yes	Yes	Yes	Yes	Yes
Spatial*Industry Fixed Effects	-	Yes	Yes	Yes	Yes	Yes
Fixed Effects	Yes	No	No	No	No	No
Firm:						
Controls	Yes	-	-	-	-	-
Spatial*Industry Fixed Effects	Yes	-	-	-	-	-
Fixed Effects	No	Yes	Yes	Yes	Yes	Yes
Firm*Supplier Spatial & Industry Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Effective First Stage F statistic	-	-	1,121.63	-	11.07	-
R-squared	0.358	0.483	-	-	-	-
Observations	1,114,421	1,114,421	1,114,421	1,114,421	1,114,421	1,114,421

Notes: This table reports estimates from WLS results. See Section 4. Observations are at the level of the firm-customer (Panel A) or firm-supplier (Panel B), i.e. link-level. The dependent variable is the change in the log of sales from supplier to customer between 2008 and 2009 for all columns but (3) and (5). In column (4) the firm bank shock is instrumented with the firm financial shock derived from the (weighted) average net interbank borrowing of the firm across all its banks before the crisis (column (3)) and corrected for the standardization of the instrumental variable. In column (6) the reduction in bank debt between 2008 and 2009 is instrumented with the firm financial shock (column (5)). The continuous bank shock is a variable capturing whether the firm was borrowing before the global financial crisis from banks which significantly reduced credit supply during the global financial crisis. To construct this variable, we use the firm level shock estimated following Amiti & Weinstein (2018) as the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values implies a credit reduction). Here we discretize this variable based on the median of the distribution. All shocks are standardized. For the list of controls, see Section 4. First stage effective F statistic is based on Montial Olea and Pfleuger (2013) and it is robust to heteroskedasticity, serial correlation, and clustering. Coefficients for each regressor are listed in the first row, while robust standard errors are reported in the row below (corrected for clustering at the firm, main bank, and supplier or customer levels but for columns (4) and (6) that correct for firm and main bank). In each column, the word Yes indicates that the corresponding set of characteristics or fixed effects (FE) is included, No that it is not included, and - that it is comprised by the set of fixed effects. *** Significant at 1%, ** significant at 5%, * significant at 10%.

TABLE A5

LINK-LEVEL: PROPAGATION OF BANK CREDIT SUPPLY SHOCKS THROUGH THE NETWORK. REDUCED FORM.
BANK SHOCKS COMPUTED USING A FIRM VARYING DEMAND BY LOAN TYPE

Dependent Variable: Δlog(sales from supplier to customer)	Upstream propagation			Downstream propagation		
	Continuous Shock		Discrete Shock	Continuous Shock		Discrete Shock
	(1)	(2)	(3)	(4)	(5)	(6)
Direct (Bank) Shock	-1.094**			-3.072**		
	(0.442)			(1.470)		
Customer (Bank) Shock (1st Order Effect)		-2.445**	-2.923**			
		(1.233)	(0.098)			
Supplier (Bank) Shock (1st Order Effect)					-0.990***	-1.377***
					(0.372)	(0.508)
Customer/Supplier:						
Controls	-	Yes	Yes	-	Yes	Yes
Spatial*Industry Fixed Effects	-	Yes	Yes	-	Yes	Yes
Fixed Effects	Yes	No	No	Yes	No	No
Firm:						
Controls	Yes	-	-	Yes	-	-
Spatial*Industry Fixed Effects	Yes	-	-	Yes	-	-
Fixed Effects	No	Yes	Yes	No	Yes	Yes
Firm*Customer/Supplier Spatial & Industry Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
R-squared	0.405	0.474	0.474	0.358	0.484	0.483
Observations	1,119,169	1,119,169	1,119,169	1,114,421	1,114,421	1,114,421

Notes: This table reports estimates from WLS results. See Section 4. Observations are at the level of firm-customer/supplier, i.e. link-level. The dependent variable is the change in the log of sales from supplier to customer between 2008 and 2009. Bank shock is a variable capturing whether the firm was borrowing before the global financial crisis from banks which significantly reduced credit supply during the global financial crisis. To construct this variable, we use the firm level shock estimated following Amiti & Weinstein (2018), but allowing firm-loan type fixed effects (where loan types are asset-based loans, cash flow loans, trade finance agreements, and leases following Ivashina, Laeven, and Moral-Benito (2022)), as the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values implies a credit reduction). In column (3) and (6) the discrete bank supply shock is used based on the median of the distribution. All shocks are standardized. Coefficients for each regressor are listed in the first row, while robust standard errors are reported in the row below (corrected for clustering at the firm, main bank, and supplier or customer levels). In each column, the word Yes indicates that the corresponding set of characteristics or fixed effects (FE) is included, No that it is not included, and - that it is comprised by the set of fixed effects. *** Significant at 1%, ** significant at 5%, * significant at 10%.

TABLE A6

LINK-LEVEL: PROPAGATION OF BANK CREDIT SUPPLY SHOCKS THROUGH THE NETWORK. REDUCED FORM.
BANK SHOCKS COMPUTED USING A FIRM VARYING DEMAND BY INDUSTRY AND PROVINCE OF THE FIRM VS BANK'S SPECIALIZATION

Dependent Variable: Δlog(sales from supplier to customer)	Upstream propagation			Downstream propagation		
	Continuous Shock		Discrete Shock	Continuous Shock		Discrete Shock
	(1)	(2)	(3)	(4)	(5)	(6)
Direct (Bank) Shock	-1.209**			-2.623**		
	(0.570)			(1.191)		
Customer (Bank) Shock (1st Order Effect)		-2.665**	-1.680*			
		(1.261)	(0.957)			
Supplier (Bank) Shock (1st Order Effect)					-0.896**	-0.937***
					(0.409)	(0.459)
Customer/Supplier:						
Controls	-	Yes	Yes	-	Yes	Yes
Spatial*Industry Fixed Effects	-	Yes	Yes	-	Yes	Yes
Fixed Effects	Yes	No	No	Yes	No	No
Firm:						
Controls	Yes	-	-	Yes	-	-
Spatial*Industry Fixed Effects	Yes	-	-	Yes	-	-
Fixed Effects	No	Yes	Yes	No	Yes	Yes
Firm*Customer/Supplier Spatial & Industry Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
R-squared	0.404	0.474	0.474	0.358	0.483	0.484
Observations	1,119,169	1,119,169	1,119,169	1,114,421	1,114,421	1,114,421

Notes: This table reports estimates from WLS results. See Section 4. Observations are at the level of firm-customer/supplier, i.e. link-level. The dependent variable is the change in the log of sales from supplier to customer between 2008 and 2009. In column (3) and (6) the discrete bank supply shock is used based on the median of the distribution. Bank shock is a variable capturing whether the firm was borrowing before the global financial crisis from banks which significantly reduced credit supply during the global financial crisis. To construct this variable, we use the firm level shock estimated following Amiti & Weinstein (2018), but allowing firm fixed effects to vary depending on whether the firm and the bank match in their industry and/or province (where the province or industry, NACE two digits, of the bank relates to its main province or industry computed in terms of total credit at December of 2007), as the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values implies a credit reduction). In column (3) and (6) the discrete bank supply shock is used based on the median of the distribution. All shocks are standardized. Coefficients for each regressor are listed in the first row, while robust standard errors are reported in the row below (corrected for clustering at the firm, main bank, and supplier or customer levels). In each column, the word Yes indicates that the corresponding set of characteristics or fixed effects (FE) is included, No that it is not included, and - that it is comprised by the set of fixed effects. *** Significant at 1%, ** significant at 5%, * significant at 10%.

TABLE A7

LINK-LEVEL: PROPAGATION OF BANK CREDIT SUPPLY SHOCKS THROUGH THE NETWORK. REDUCED FORM.
BANK SHOCKS COMPUTED USING A FIRM VARYING DEMAND DEPENDING ON BANK'S SPECIALIZATION IN REAL ESTATE

Dependent Variable: Δlog(sales from supplier to customer)	Upstream propagation			Downstream propagation		
	Continuous Shock		Discrete Shock	Continuous Shock		Discrete Shock
	(1)	(2)	(3)	(4)	(5)	(6)
Direct (Bank) Shock	-0.992**			-3.017**		
	(0.471)			(1.417)		
Customer (Bank) Shock (1st Order Effect)		-2.146*	-2.453**			
		(1.196)	(1.104)			
Supplier (Bank) Shock (1st Order Effect)					-1.103**	-1.394***
					(0.556)	(0.497)
Customer/Supplier:						
Controls	-	Yes	Yes	-	Yes	Yes
Spatial*Industry Fixed Effects	-	Yes	Yes	-	Yes	Yes
Fixed Effects	Yes	No	No	Yes	No	No
Firm:						
Controls	Yes	-	-	Yes	-	-
Spatial*Industry Fixed Effects	Yes	-	-	Yes	-	-
Fixed Effects	No	Yes	Yes	No	Yes	Yes
Firm*Customer/Supplier Spatial & Industry Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
R-squared	0.404	0.474	0.474	0.358	0.483	0.484
Observations	1,119,169	1,119,169	1,119,169	1,114,421	1,114,421	1,114,421

Notes: This table reports estimates from WLS results. See Section 4. Observations are at the level of firm-customer/supplier, i.e. link-level. The dependent variable is the change in the log of sales from supplier to customer between 2008 and 2009. In column (3) and (6) the discrete bank supply shock is used based on the median of the distribution. Bank shock is a variable capturing whether the firm was borrowing before the global financial crisis from banks which significantly reduced credit supply during the global financial crisis. To construct this variable, we use firm level shock estimated following Amiti & Weinstein (2018), but allowing firm fixed effects to vary depending on whether the bank is specialized in the real estate sector or not (in terms of total credit at December 2007), as the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values implies a credit reduction). In column (3) and (6) the discrete bank supply shock is used based on the median of the distribution. All shocks are standardized. Coefficients for each regressor are listed in the first row, while robust standard errors are reported in the row below (corrected for clustering at the firm, main bank, and supplier or customer levels). In each column, the word Yes indicates that the corresponding set of characteristics or fixed effects (FE) is included, No that it is not included, and - that it is comprised by the set of fixed effects. *** Significant at 1%, ** significant at 5%, * significant at 10%.

TABLE A8. PANEL A

LINK-LEVEL: PROPAGATION OF BANK CREDIT SUPPLY SHOCKS THROUGH THE NETWORK. REDUCED FORM. BANK SHOCKS COMPUTED USING A FIRM VARYING DEMAND BY THE WEIGHT OF WEAK BANKS IN THE PROVINCE

Dependent Variable: Δlog(sales from supplier to customer)	Upstream propagation			Downstream propagation		
	Continuous Shock		Discrete Shock	Continuous Shock		Discrete Shock
	(1)	(2)	(3)	(4)	(5)	(6)
Direct (Bank) Shock	-0.913***			-2.738***		
	(0.497)			(1.041)		
Customer (Bank) Shock (1st Order Effect)		-2.464**	-2.654**			
		(1.202)	(1.118)			
Supplier (Bank) Shock (1st Order Effect)					-1.028*	-1.310***
					(0.524)	(0.426)
Customer/Supplier:						
Controls	-	Yes	Yes	-	Yes	Yes
Spatial*Industry Fixed Effects	-	Yes	Yes	-	Yes	Yes
Fixed Effects	Yes	No	No	Yes	No	No
Firm:						
Controls	Yes	-	-	Yes	-	-
Spatial*Industry Fixed Effects	Yes	-	-	Yes	-	-
Fixed Effects	No	Yes	Yes	No	Yes	Yes
Firm*Customer/Supplier Spatial & Industry Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
R-squared	0.404	0.474	0.474	0.358	0.483	0.484
Observations	1,119,169	1,119,169	1,119,169	1,114,421	1,114,421	1,114,421

Notes: This table reports estimates from WLS results. Observations are at the level of firm-customer/supplier, i.e. link-level. The dependent variable is the change in the log of sales from supplier to customer between 2008 and 2009. In column (3) and (6) the discrete bank supply shock is used based on the median of the distribution. Bank shock is a variable capturing whether the firm borrowing before the global financial crisis from banks which significantly reduced credit supply during the global financial crisis. To construct this variable, we use the firm level shock estimated following Amiti & Weinstein (2018), but allowing firm-province fixed effects, where provinces are classified by quintiles based on the market share of the weak banks (following Bentolila et al., 2018), as the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values implies a credit reduction). All shocks are standardized. Coefficients for each regressor are listed in the first row, while robust standard errors are reported in the row below (corrected for clustering at the firm, main bank, and supplier or customer levels). In each column, the word Yes indicates that the corresponding set of characteristics or fixed effects (FE) is included, No that it is not included, and - that it is comprised by the set of fixed effects. *** Significant at 1%, ** significant at 5%, * significant at 10%.

TABLE A8. PANEL B

LINK-LEVEL: PROPAGATION OF BANK CREDIT SUPPLY SHOCKS THROUGH THE NETWORK. REDUCED FORM. BANK SHOCKS COMPUTED USING A FIRM VARYING DEMAND BY WEAK BANKS

Dependent Variable: Δlog(sales from supplier to customer)	Upstream propagation			Downstream propagation		
	Continuous Shock		Discrete Shock	Continuous Shock		Discrete Shock
	(1)	(2)	(3)	(4)	(5)	(6)
Direct (Bank) Shock	-1.056**			-2.657**		
	(0.520)			(1.046)		
Customer (Bank) Shock (1st Order Effect)		-2.510**	-2.843**			
		(1.245)	(1.229)			
Supplier (Bank) Shock (1st Order Effect)					-0.949*	-1.288***
					(0.490)	(0.442)
Customer/Supplier:						
Controls	-	Yes	Yes	-	Yes	Yes
Spatial*Industry Fixed Effects	-	Yes	Yes	-	Yes	Yes
Fixed Effects	Yes	No	No	Yes	No	No
Firm:						
Controls	Yes	-	-	Yes	-	-
Spatial*Industry Fixed Effects	Yes	-	-	Yes	-	-
Fixed Effects	No	Yes	Yes	No	Yes	Yes
Firm*Customer/Supplier Spatial & Industry Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
R-squared	0.404	0.474	0.474	0.358	0.483	0.484
Observations	1,119,169	1,119,169	1,119,169	1,114,421	1,114,421	1,114,421

Notes: This table reports estimates from WLS results. Observations are at the level of firm-customer/supplier, i.e. link-level. The dependent variable is the change in the log of sales from supplier to customer between 2008 and 2009. In column (3) and (6) the discrete bank supply shock is used based on the median of the distribution. Bank shock is a variable capturing whether the firm borrowing before the global financial crisis from banks which significantly reduced credit supply during the global financial crisis. To construct this variable, we use the firm level shock estimated following Amiti & Weinstein (2018), but allowing firm-weak bank fixed effects, where weak banks are defined following Bentolila et al. (2018), as the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values implies a credit reduction). All shocks are standardized. Coefficients for each regressor are listed in the first row, while robust standard errors are reported in the row below (corrected for clustering at the firm, main bank, and supplier or customer levels). In each column, the word Yes indicates that the corresponding set of characteristics or fixed effects (FE) is included, No that it is not included, and - that it is comprised by the set of fixed effects. *** Significant at 1%, ** significant at 5%, * significant at 10%.

TABLE A9

LINK-LEVEL: PROPAGATION OF A BANK SUPPLY SHOCK THROUGH THE NETWORK OF CUSTOMERS/SUPPLIERS: STRUCTURAL FORM. HETEROGENEITY WITH CUSTOMER/SUPPLIER

Dependent Variable: Δlog(sales from supplier to customer/sales of customer)	Upstream propagation			Downstream propagation		
	(1)	(2)	(3)	(4)	(5)	(6)
Customer (Bank) Shock (1st Order Effect)	-2.054**	-2.484**	-2.623**			
	(0.932)	(1.096)	(1.148)			
Customer (Bank) Shock (1st Order Effect)*Debt/Assets of the Customer	-0.740**	-0.945**	-1.445**			
	(0.354)	(0.399)	(0.583)			
Customer (Bank) Shock (1st Order Effect)*SME Customer		-0.971**	-0.967**			
		(0.438)	(0.439)			
Customer (Bank) Shock (1st Order Effect)*Debt/Assets of the Customer*SME Customer			-0.646**			
			(0.253)			
Supplier (Bank) Shock (1st Order Effect)				-1.217**	-1.316**	-1.335**
				(0.576)	(0.656)	(0.661)
Supplier (Bank) Shock (1st Order Effect)*Debt/Assets of the Supplier				-1.603	-1.608	-1.957
				(1.085)	(1.074)	(1.271)
Supplier (Bank) Shock (1st Order Effect)*SME Supplier					-0.248	-0.323
					(0.249)	(0.293)
Supplier (Bank) Shock (1st Order Effect)*Debt/Assets of the Supplier*SME Supplier						-0.703
						(0.558)
Debt/Assets of the Customer	0.155	0.121	0.213			
	(0.751)	(0.810)	(1.044)			
SME Customer		-0.317	-0.312			
		(0.357)	(0.373)			
Debt/Assets of the Customer*SME Customer			0.072			
			(0.530)			
Debt/Assets of the Supplier				0.065	0.067	-0.019
				(0.583)	(0.579)	(0.574)
SME Supplier					-1.075***	-1.132***
					(0.370)	(0.364)
Debt/Assets of the Supplier*SME Supplier						-0.321
						(0.321)
Higher Order Customer (Bank) Effect	1.770***	1.710***	1.709***			
	(0.631)	(0.621)	(0.626)			
Higher Order Supplier (Bank) Effect				-2.162*	-2.171*	-2.250*
				(1.168)	(1.162)	(1.179)
Supplier/Customer:						
Controls	Yes	Yes	Yes	Yes	Yes	Yes
Spatial*Industry Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Firm Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Firm*Supplier/Customer Spatial & Industry Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
R-squared	0.478	0.478	0.478	0.493	0.493	0.493
Observations	1,119,169	1,119,169	1,119,169	1,114,420	1,114,420	1,114,420

This table reports estimates from WLS results. See Section 6. Observations are at the level of the firm-customer (columns (1) to (3)) or firm-supplier (columns (4) to (6)), i.e. link-level. The dependent variable is the change in the sales from supplier to customer, minus the change in the log of total sales of the customer between 2008 and 2009. Bank shock is a variable capturing whether the firm was borrowing before the global financial crisis from banks which significantly reduced credit supply during the global financial crisis. To construct this variable, we use the firm level shock estimated following Amiti & Weinstein (2018) as the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values implies a credit reduction). For the definition of higher order bank shock effects, see Section 5 and 6 of the paper. SME is a dummy variable that takes the value 1 if the customer or the supplier of the firm is not a large firm, and 0 otherwise. Debt/Assets is the ratio between the debt with cost over total assets. All variables are standardized. For the list of controls, see Section 6. Coefficients for each regressor are listed in the first row, while robust standard errors are reported in the row below (corrected for clustering at the firm, main bank, and supplier or customer levels). In each column, the word Yes indicates that the corresponding set of characteristics or fixed effects is included. *** Significant at 1%, ** significant at 5%, * significant at 10%.

TABLE A10

NODE-LEVEL: THE EFFECT OF BANK CREDIT SUPPLY SHOCK ON TRADE CREDIT

Dependent Variable:	Change commercial debtors: Customers		Δ(Commercial debtors: Customers/Sales)	
	(1)	(2)	(3)	(4)
Direct (Bank) Shock	0.609	0.626	0.181	0.171
	(0.595)	(0.598)	(0.135)	(0.133)
Customer (Bank) Shock (1st Order Effect)		-0.583**		0.353***
		(0.228)		(0.043)
Supplier (Bank) Shock (1st Order Effect)		0.102		-0.065**
		(0.264)		(0.026)
Firm Controls	Yes	Yes	Yes	Yes
Spatial & Industry Fixed Effects	Yes	Yes	Yes	Yes
R-squared	0.071	0.071	0.073	0.074
Observations	165,455	165,455	181,393	181,393

This table reports estimates from OLS. For columns (1) and (2), the dependent variable is the symmetric change between 2008 and 2009 for commercial debtors (customers) – trade credit is from suppliers to customers – in percentage. For columns (3) and (4), the dependent variable is the change between 2008 and 2009 for commercial debtors (customers) over total assets, in percentage. As we cannot control for firm fixed effects, we control for industry (NACE at two digits) times province, spatial (zip code), and main bank fixed effects. All shocks are standardized. Coefficients for each regressor are listed in the first row, while robust standard errors are reported in the row below (corrected for clustering at main bank). In each column, the word Yes indicates that the corresponding set of characteristics or fixed effects is included. *** Significant at 1%, ** significant at 5%, * significant at 10%.

TABLE A11

LINK-LEVEL: PROPAGATION OF A BANK SUPPLY SHOCK THROUGH THE NETWORK OF CUSTOMERS/SUPPLIERS: STRUCTURAL FORM

Panel A: Aggregating suppliers at 2 or 3-digit NACE

Dependent Variable: Δlog(sales from supplier to customer/sales of customer)	2-digit NACE		3-digit NACE	
	(1)	(2)	(3)	(4)
Supplier (Bank) Shock (1st Order Effect)	-0.560**	-0.423*	-0.601***	-0.495**
	(0.256)	(0.250)	(0.221)	(0.221)
Higher Order Supplier (Bank) Shock		-1.122**		-0.960**
		(0.533)		(0.486)
Supplier/Customer:				
Controls	Yes	Yes	Yes	Yes
Customer Spatial*Industry Fixed Effects	Yes	Yes	Yes	Yes
Firm:				
Fixed Effects	Yes	Yes	Yes	Yes
Firm*Customer Spatial & Industry Fixed Effects	Yes	Yes	Yes	Yes
R-squared	0.481	0.483	0.410	0.412
Observations	526,429	526,429	697,019	697,019

Panel B: Firm fixed effects*supplier industry (2 or 3-digit NACE) fixed effects

Dependent Variable: Δlog(sales from supplier to customer/sales of customer)	2-digit NACE		3-digit NACE	
	(1)	(2)	(3)	(4)
Supplier (Bank) Shock (1st Order Effect)	-1.787*	-1.768*	-2.733**	-2.556**
	(0.950)	(0.920)	(1.340)	(1.165)
Higher Order Supplier (Bank) Shock		-1.574		-0.884
		(1.446)		(1.916)
Supplier/Customer:				
Controls	Yes	Yes	Yes	Yes
Spatial*Industry Fixed Effects	Yes	Yes	Yes	Yes
Firm:				
Firm Fixed Effects*Supplier Industry Fixed Effects	Yes	Yes	Yes	Yes
Firm*Supplier/Customer Spatial & Industry Fixed Effects	Yes	Yes	Yes	Yes
R-squared	0.595	0.595	0.595	0.646
Observations	757,067	757,067	577,859	577,859

Panel C: Estimated sigma from panel A and panel B

	Aggregating suppliers at:		Firm f.e.*supplier industry at:	
	2-digit NACE	3-digit NACE	2-digit NACE	3-digit NACE
Sigma	1.25***	1.28***	1.95**	2.37**
	(0.15)	(0.16)	(0.77)	(1.00)

This table reports estimates from WLS results. See Section 6. Observations are at the level of the firm-supplier, i.e. link-level. The dependent variable is the change in the sales from supplier to customer, minus the change in the log of total sales of the customer between 2008 and 2009. Bank shock is a variable capturing whether the firm was borrowing before the global financial crisis from banks which significantly reduced credit supply during the global financial crisis. To construct this variable, we use the firm level shock estimated following Amiti & Weinstein (2018) as the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values implies a credit reduction). For the definition of higher order bank shock effects, see Section 5 and 6 of the paper. All variables are standardized. For the list of controls, see Section 6. Coefficients for each regressor are listed in the first row, while robust standard errors are reported in the row below (corrected for clustering at the firm, main bank, and supplier or customer levels). In each column, the word Yes indicates that the corresponding set of characteristics or fixed effects is included. *** Significant at 1%, ** significant at 5%, * significant at 10%.

TABLE A12

LINK-LEVEL: PROPAGATION OF A BANK SUPPLY SHOCK THROUGH THE NETWORK OF CUSTOMERS/SUPPLIERS: STRUCTURAL FORM. LONG-RUN EFFECTS

Dependent Variable: Δlog(sales from supplier to customer/sales of customer)	Upstream	Downstream	Joint estimation Non-standardized
	(1)	(2)	(3)
Customer (Bank) Shock (1st Order Effect)	-2.008**		-42.541**
	(0.896)		(20.376)
Higher Order Customer (Bank) Shock	1.979*		56.372*
	(1.022)		(29.128)
Supplier (Bank) Shock (1st Order Effect)		-2.158**	-49.308**
		(0.903)	(20.709)
Higher Order Supplier (Bank) Shock		1.730	44.453
		(1.098)	(28.755)
Supplier/Customer:			
Controls	Yes	Yes	Yes
Spatial*Industry Fixed Effects	Yes	Yes	Yes
Firm:			
Fixed Effects	Yes	Yes	Yes
Firm*Supplier/Customer Spatial & Industry Fixed Effects	Yes	Yes	Yes
R-squared	0.525	0.538	0.531
Estimated Sigma			2.17** (0.88)
Observations	1,115,199	1,107,968	2,223,167

This table reports estimates from WLS results. See Section 6. Observations are at the level of the firm-customer (columns (1), (4) and (7)) or firm-supplier (columns (2), (5) and (8)), i.e. link-level. The dependent variable is the change in the sales from supplier to customer, minus the change in the log of total sales of the customer between 2008 and 2010. Bank shock is a variable capturing whether the firm was borrowing before the global financial crisis from banks which significantly reduced credit supply during the global financial crisis. To construct this variable, we use the firm level shock estimated following Amiti & Weinstein (2018) as the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values implies a credit reduction). For the definition of higher order bank shock effects, see Section 5 and 6 of the paper. All variables are standardized but those of column (3), (6) and (9). For the list of controls, see Section 6. Coefficients for each regressor are listed in the first row, while robust standard errors are reported in the row below (corrected for clustering at the firm, main bank, and supplier or customer levels). In each column, the word Yes indicates that the corresponding set of characteristics or fixed effects is included. *** Significant at 1%, ** significant at 5%, * significant at 10%.

TABLE A13

LINK-LEVEL: PROPAGATION OF A BANK SUPPLY SHOCK THROUGH THE NETWORK OF CUSTOMERS/SUPPLIERS: HETEROGENEOUS INPUT ELASTICITIES. STRUCTURAL FORM

Dependent Variable: Δlog(sales from supplier to customer/sales of customer)	Upstream propagation		Downstream propagation	
	(1)	(2)	(3)	(4)
Customer (Bank) Shock (1st Order Effect)	-2.006**	-1.960**		
	(0.921)	(0.886)		
Higher Order Customer (Bank) Shock	2.409**	1.598*		
	(1.000)	(0.857)		
Customer Capital Input Effect		2.713*		
		(1.555)		
Customer Labor Input Effect		-1.378**		
		(0.692)		
Supplier (Bank) Shock (1st Order Effect)			-1.077**	- 1.059**
			(0.547)	(0.515)
Higher Order Supplier (Bank) Shock			-3.326**	-2.713*
			(1.595)	(1.527)
Supplier Capital Input Effect				-4.062***
				(1.316
Supplier Labor Input Effect				-0.985
				(1.722)
Supplier/Customer:				
Controls	Yes	Yes	Yes	Yes
Spatial*Industry Fixed Effects	Yes	Yes	Yes	Yes
Firm:				
Fixed Effects	Yes	Yes	Yes	Yes
Firm*Supplier/Customer Spatial & Industry Fixed Effects	Yes	Yes	Yes	Yes
R-squared	0.478	0.478	0.493	0.494
No. of Observations	1,119,169	1,119,169	1,114,421	1,114,421

Notes: This table reports estimates from WLS. See online Appendix. Observations are at the level of the firm (node-level). The dependent variables is the change, between 2008 and 2009, in the log of firm-level aggregate sales to all customers over E, the (nominal) GDP. Bank shock is a variable capturing whether the firm was borrowing before the global financial crisis from banks which significantly reduced credit supply during the global financial crisis. To construct this variable, we use the firm level shock estimated following Amiti & Weinstein (2018) as the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values implies a credit reduction). As we cannot control for firm fixed effects, we control for spatial and industry, and main bank fixed effects. All variables are standardized. Coefficients for each regressor are listed in the first row, while robust standard errors are reported in the row below (corrected for clustering at the firm, main bank, and supplier or customer levels). In each column, the word Yes indicates that the corresponding set of characteristics or fixed effects is included. *** Significant at 1%, ** significant at 5%, * significant at 10%.

TABLE A14

NODE-LEVEL: FIRM-LEVEL EFFECTS OF BANK SUPPLY SHOCKS THROUGH THE PRODUCTION NETWORK: HETEROGENEOUS INPUT ELASTICITIES. STRUCTURAL FORM.

Dependent Variable: Δlog(sales/E)	(1)	(2)	(3)	(4)	(5)
Upstream (1st & Higher Order Effects)	-1.810***		-1.534***	-1.531***	-1.506***
	(0.326)		(0.321)	(0.351)	(0.321)
Bidirectional (Up & Down 1st & Higher Order Effects)		-1.542***	-0.715**	-0.725*	-0.498*
		(0.344)	(0.304)	(0.402)	(0.292)
Direct (Bank) Shock				0.033	
				(0.599)	
Bidirectional Capital Input Effect					-2.274***
					(0.283)
Bidirectional Labor Input Effect					-0.443
					(0.475)
Firm Controls	Yes	Yes	Yes	Yes	Yes
Spatial & Industry Fixed Effects	Yes	Yes	Yes	Yes	Yes
R-squared	0.358	0.357	0.358	0.358	0.359
Observations	196,171	196,171	196,171	196,171	196,171

Notes: This table reports estimates from WLS. See online Appendix. Observations are at the level of the firm (node-level). The dependent variables are the change, between 2008 and 2009, in the log of firm-level aggregate sales to all customers over E, the (nominal) GDP. Bank shock is a variable capturing whether the firm was borrowing before the global financial crisis from banks which significantly reduced credit supply during the global financial crisis. To construct this variable, we use the firm level shock estimated following Amiti & Weinstein (2018) as the sum of the common shock and the firm-level bank shock (multiplied by -1, so higher values implies a credit reduction). See also Section 6 of the paper. As we cannot control for firm fixed effects, we control for spatial and industry, and main bank fixed effects. All variables are standardized. For the list of firm controls, see Section 4. Coefficients for each regressor are listed in the first row, while robust standard errors are reported in the row below (corrected for clustering at the level of the main bank). In each column, the word Yes indicates that the set of characteristics or fixed effects is included. *** Significant at 1%, ** significant at 5%, * significant at 10%.

TABLE A15

SUMMARY STATISTICS: SUPPLIER-CUSTOMER DATASET

	Num. Obs.	Mean	S.D.	P25	Median	P75
			Links			
Links by Year:						
2008	13,810,158	78,039	5,281,888	4,922	9,453	25,396
2009	11,988,607	71,641	4,989,742	4,791	8,950	23,361
Links Appearing in Both Years:						
2008	7,655,815	115,145	7,035,309	6,353	13,424	38,043
2009	7,655,815	92,495	5,089,617	5,487	10,942	30,073
			Nodes			
2008						
Number of Customers	696,152	20	336	2	4	13
Number of Suppliers	808,973	17	55	3	7	17
2009						
Number of Customers	691,792	17	309	1	4	11
Number of Suppliers	789,118	15	52	2	6	15

Notes: This table reports means, standard deviations and first/second/third quartiles of annual bilateral transactions for 2008 and 2009 (Links), as well as the number of suppliers/customers for years 2008 and 2009 (Nodes). A firm is a supplier (customer) if it has at least one customer (supplier) in the network in a given year. Link *ji* between two firms appears in both years if supplier *j* reports a sale to customer *i* (or *i* reports a purchase from *j*) in both 2008 and 2009.

A.2. Heterogeneous input elasticities

Equation (8) does not feature network propagation of effects coming from labor and capital. The intuition behind this is that $\mathbf{G}$ is a column stochastic matrix, ensuring that changes in wages and the price of physical capital affect all firms equally, given that they have identical labor and capital input shares in the production function.

We now explore the implications of relaxing the assumption that α, β and ρ are common to all firms, both for the link-level analysis from Section IV.D and node-level analysis from Section IV.E.

As we show in Proposition 1, when input shares are firm specific (8) becomes:

$$\begin{aligned}\text{dlog}\left(\frac{s_{ji}}{s_i}\right) =& -\theta_i - (\sigma-1)\theta_j \\ & -(\sigma-1)\boldsymbol{e}_j'\mathbf{AG}'(\mathbf{I}-\mathbf{AG}')^{-1}\boldsymbol{\theta} + (\sigma-1)\boldsymbol{e}_i'\mathbf{G}'(\mathbf{I}-\mathbf{AG}')^{-1}\boldsymbol{\theta} \\ & -(\sigma-1)\boldsymbol{e}_j'(\mathbf{I}-\mathbf{AG}')^{-1}(\boldsymbol{\beta}\text{d}w + \boldsymbol{\rho}\text{d}r) + (\sigma-1)\boldsymbol{e}_i'\mathbf{G}'(\mathbf{I}-\mathbf{AG}')^{-1}(\boldsymbol{\beta}\text{d}w + \boldsymbol{\rho}\text{d}r).\end{aligned} \tag{1}$$

The first two lines of (1) are analogous to (8). The only difference is in that the scalar α is replaced with the diagonal matrix $\mathbf{A}\equiv diag(\alpha_i)$ in which i-th element is equal to α_i. The third line of the expression is new, and it captures how the change of the change in wage (dw) and the change in the price of capital (dr) affect the production costs of supplier j and customer i, respectively. Intuitively, the effects of a change in the wage and the capital price are proportional to how much a firm relies, directly and indirectly, on labor and capital in production, respectively.

In the empirical implementation, we estimate sector-specific parameters α, β and ρ using standard approach Wooldridge (2009). We assign these parameters to firms according to their respective sectors. While we do not observe dw, dr, we are able to calculate network measures $(\mathbf{I}-\mathbf{AG}')^{-1}\boldsymbol{\beta}$ and $(\mathbf{I}-\mathbf{AG}')^{-1}\boldsymbol{\rho}$. Therefore, when estimating equation (1) the theory implied effect of $\boldsymbol{e}_j'(\mathbf{I}-\mathbf{AG}')^{-1}\boldsymbol{\beta}$ is equal to $-(\sigma-1)\text{d}w$, and analogously $(\mathbf{I}-\mathbf{AG}')^{-1}\boldsymbol{\rho}$. We report the estimation results in Table A13. We label estimated parameters of $\boldsymbol{e}_j'(\mathbf{I}-\mathbf{AG}')^{-1}\boldsymbol{\beta}$ and $\boldsymbol{e}_j'(\mathbf{I}-\mathbf{AG}')^{-1}\boldsymbol{\rho}$ as the *Supplier Labor Input Effect* and the *Supplier Capital Input Effect*, respectively. Similarly, the parameters of $\boldsymbol{e}_i'(\mathbf{I}-\mathbf{AG}')^{-1}\boldsymbol{\beta}$ and $\boldsymbol{e}_i'(\mathbf{I}-\mathbf{AG}')^{-1}\boldsymbol{\rho}$ are labeled as the *Customer Labor Input Effect* and the *Customer Capital Input Effect*, respectively. We find that our estimates of parameters reported in panel A of Table 3 are not affected.

By allowing firm specific values of parameters α, β and ρ equation (10) becomes:

$$\text{dlog}\left(\frac{s_i}{E}\right) = -\boldsymbol{e}_i'(\mathbf{I}-\mathbf{H})^{-1}\mathbf{H}\boldsymbol{\theta} + (1-\sigma)\boldsymbol{e}_i'\boldsymbol{\Lambda}\boldsymbol{\theta} + (1-\sigma)\boldsymbol{e}_i'\boldsymbol{\Lambda}(\boldsymbol{\beta}\text{d}w + \boldsymbol{\rho}\text{d}r), \tag{2}$$

where now, $\mathbf{H}=\mathbf{V}^{-1}\mathbf{GAMV}$ and $\boldsymbol{\Lambda}=(\mathbf{I}-\mathbf{H})(diag(\mathbf{H1})-\mathbf{HG}')(\mathbf{I}-\mathbf{AG}')^{-1}$.

Turning to the node-level analysis, relative to (10), equation (2) features bidirectional propagation coming from the changes in labor and capital prices due to bank shocks. Since, in this case, firms are different with respect to the intensity in which they use inputs, changes in labor and capital price affect firms differently. Note, also, that both upstream and bidirectional propagation of bank shocks in (2) depend on the firm-specific values of parameter α_i, which is captured by diagonal matrix $\mathbf{A}$. The intuition behind these two types of propagation remains the same, as explained Section IV.E.

We estimate equation (9) and report the results in Table A14 in online Appendix A. We label estimated parameters of $\boldsymbol{e}_i'\boldsymbol{\Lambda\beta}$ and $\boldsymbol{e}_i'\boldsymbol{\Lambda\rho}$ as the *Bidirectional Capital Input Effect* and the *Bidirectional Labor Input Effect*, respectively.

A.3. A more complete literature review

The fast-growing literature studying the phenomenon of shock propagation in large economies has mostly evolved by studying separately the real and the financial networks. In the first case, the main focus has been on the supply chains that underlie the production of the non-financial firms of the economy and the role of the network structure in the propagation and aggregation of, for the most part, productivity shocks.[1] In the second case, the analysis has mainly centered on the banks alone as the main actors, the links among them typically conceived as reflecting some form of financial flows.[2] In comparison with these two largely unconnected branches of the literature, our contribution considers both the real and financial sides of the economy and focuses the analysis on the interaction between them.

There is a rich literature that has explored whether credit-supply shocks may lead to significant real effects on the production side of the economy, but its analysis of the problem abstracts from the role played by the production network as a propagation structure of those shocks. As a representative sample of its more theoretical branch, we can refer to e.g. Holmstrom and Tirole (1997); Stein (1998); Gertler and Kiyotaki (2010), while for its empirical branch we can mention e.g. Khwaja and Mian (2008), Chodorow-Reich (2014), Greenstone, Mas and Nguyen (2014), Jiménez et al. (2012, 2017), Amiti and Weinstein (2018), and Galaasen et al. (2020). There are only a few papers that are close to ours in that they also aim to understand the process by which bank shocks propagate through the real production network. To the best of our knowledge, the following two papers are the most related.[3]

The first paper is by Costello (2020), who studies the downstream propagation of shocks through their influence on the trade credit that firms extend to their customers. Relying on data obtained from a third-party trade credit information platform, this paper documents that firms with greater exposure to a large decline in finance reduce their trade credit to customers,

[1] See for instance Acemoglu et al. (2012); Barrot and Sauvagnat (2016); Baqaee (2018); Carvalho et al. (2020).

[2] See for instance Allen and Gale (2000); Freixas, Parigi and Rochet (2000); Iyer and Peydro (2011); Niepmann and Schmidt-Eisenlohr (2013); Elliott, Golub and Jackson (2014); Cabrales, Gottardi and Vega-Redondo (2017).

[3] Another more distantly related paper is Alfaro, García-Santana and Moral-Benito (2021), which investigates the propagation bank shocks through *industry-level input-output data.* We outline here three important differences. First, they analyze reduced-form estimates while we show that such reduced-form estimation may miss about half of the overall propagation effects, hence substantially underestimating the extent of shock propagation. Second, and relatedly, they do not investigate higher-order propagation effects, but our findings suggest that these high-order effects are as crucial as first-order effects.

Third, their reliance on industry-aggregated data raises identification concerns that our transaction-level data at the firm and supplier-customer level can handle in a significantly more effective manner.

In addition, we also refer to the paper by Dewachter, Tielens and Hove (2020), which complements our research by studying a dynamic Keynesian model that also displays an interplay of financial and production networks and is applied to Belgium data similar to ours. Their concern is quite different from ours in that their primary focus is on how bank concentration and its effect on bank competition bears on macroeconomic volatility.

and consequently induce negative effects on employment.[4] In contrast with this paper, we use administrative registers and focus on the effects of bank shocks on sales at the firm-to-firm (link) and firm (node) levels. The advantage of our approach, relative to the one taken in Costello (2020), is that it enables us to account for the general equilibrium and higher-order network effects of bank shocks and interpret the estimates as structural parameters of the model. In contrast with Costello (2020), we show that: (a) besides downstream propagation, upstream propagation is also important, with even larger economic effects; (b) in addition to first-order effects, also higher-order effects matter; (c) complex bidirectional propagation matters as well. This type of propagation of bank shocks has not yet been studied in the literature.

The second paper is by Cortes, Silva and Doornik (2019), who uses firm-to-firm payment data across different banks from Brazil to approximate transaction data across firms and estimate the indirect effects of state-owned bank shocks. Methodologically, however, this paper differs from ours in several key respects. First, it only considers first-order propagation, while we also analyze the transmission of shocks through higher-order linkages. Second, it considers bank shocks by state-owned banks, while we consider bank shocks from all banks. Note that there is extensive literature showing that state-owned banks generate large inefficiencies (see e.g. La Porta, Lopez-de Silanes and Shleifer (2002)), and hence, changes in credit through such banks may not identify bank shocks appropriately. Third, due to data limitations, Cortes, Silva and Doornik (2019) only exploit transactions between firms working with different banks while we exploit all transactions. And fourth, in contrast to their paper, our approach is theory-based in that we propose and study a general equilibrium model of the problem and then use it for the estimation.

Our paper is also connected to studies that estimate the elasticity of substitution across intermediate inputs using various methods and sources of variation. Several papers use import data to estimate the elasticity of substitution between imported and domestic intermediate inputs. Halpern, Koren and Szeidl (2015) uses annual Hungarian import data and estimates the elasticity of substitution between imported and domestic intermediate inputs to be between 4 and 7. In their estimation, they fix the elasticity of substitution between intermediate inputs and factors of production to be 1. Using monthly import data from Japan to USA in combination with the Tohoku earthquake shock, Boehm, Flaaen and Pandalai-Nayar (2019) estimates the elasticity of substitution between intermediate goods imported from Japan and other domestic (US) intermediate inputs to be between 0.201 and 0.624.

Another approach to estimating the elasticity is using sector-level data and plausible variation in input prices. Using government spending demand shocks and annual US sector level input-output tables Atalay (2017) estimates the elasticity of substitution between intermediate inputs between -0.13 and - 0.07. Using trade liberalization shock in India, Peter, Ruane et al. (2022) estimates the long-run elasticity of substitution between material input categories to be 3.1.

The papers closest to ours use firm-level production network data to recover the elasticity of substitution across intermediate inputs. Even though they do not estimate the elasticity of substitution directly, Barrot and Sauvagnat (2016) argue that their results on the effects of

[4] Related to this, Demir et al. (2024) show that a negative shock to the cost of import financing gets propagated from liquidity-constrained firms to their customers (see also Jacobson and von Schedvin (2015)).

supply chain disruption on firm-level outcomes are consistent with Leontief production function in the short run (quarterly data). Using partial annual production network data from Japan and the shock caused by the Tohoku earthquake, Carvalho et al. (2020) estimate the elasticity of substitution across intermediate inputs to be 1.18. Finally, Fujiy, Ghose and Khanna (2024), using monthly firm-level transaction data from India and disruptions caused by governmental responses to Covid-19 pandemic, find that the elasticity of substitution across intermediate inputs lies within a range of 0.50 to 0.66.

We conclude by reiterating the important point made in Ruhl et al. (2008) and Boehm, Flaaen and Pandalai-Nayar (2019): caution is required when comparing results across different studies, as the elasticity of substitution is inherently linked to the time horizon and the nature of the shocks considered.

Appendix B: Proofs

Lemma 1. *The marginal cost of firm i is given by*

$$mc_i = (1+\theta_i)\frac{1}{\kappa_i} r^{\rho_i} w^{\beta_i} P_i^{\alpha_i}, \tag{3}$$

where $\kappa_i \equiv \zeta_i \rho_i^{\rho_i} \alpha_i^{\alpha_i} \beta_i^{\beta_i}$ *and* $P_i \equiv \left[\sum_{k\in N_i^+} g_{ki} p_k^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$.

Proof of Lemma 1. Given any feasible production plan $[\ell_i, k_i, (z_{ij})_{j=1}^n]$ and shock θ_i, firm i minimizes:

$$(1+\theta_i)\left(w\ell_i + rk_i + \sum_{j\in N_i^+} p_j z_{ji}\right), \tag{4}$$

subject to the technological constraint:

$$y_i \leq \zeta_i k_i^{\rho_i} \ell_i^{\beta_i} \left[\left(\sum_{j\in N_i^+} g_{ji}^{\frac{1}{\sigma}} z_{ji}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}\right]^{\alpha_i}.$$

The above constraint holds with equality. Hence the Lagrangian of this problem is:

$$\mathscr{L} = (1+\theta_i)\left(w\ell_i + rk_i + \sum_{j\in N_i^+} p_j z_{ji}\right) - \varphi_i\left[\zeta_i k_i^{\rho_i} \ell_i^{\beta_i} M_i^{\alpha_i} - y_i\right].$$

The first-order necessary conditions (FONC) which are also sufficient, given the postulated convexity conditions, read:

$$\begin{aligned}
&(1+\theta_i)p_j = \varphi_i \zeta_i k_i^{\rho_i} \ell_i^{\beta_i} \alpha_i M_i^{\alpha_i - 1}\frac{\partial M_i}{\partial z_{ji}} = \varphi_i \alpha_i y_i \frac{1}{M_i}\frac{\partial M_i}{\partial z_{ji}} = \varphi_i \alpha_i y_i \frac{1}{M_i}\left[\sum_k g_{ki}^{\frac{1}{\sigma}} z_{ki}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1}{\sigma-1}} g_{ji}^{\frac{1}{\sigma}} z_{ji}^{-\frac{1}{\sigma}},\\
&(1+\theta_i)w = \varphi_i \beta_i \frac{1}{\ell_i} y_i,\\
&(1+\theta_i)r = \varphi_i \rho_i \frac{1}{k_i} y_i.
\end{aligned} \tag{5}$$

From (5) follows directly that for any two interemediate inputs j and k used by firm i, we have:

$$\frac{p_j}{p_k} = \left[\frac{g_{ji}}{g_{ki}}\right]^{\frac{1}{\sigma}} \left[\frac{z_{ji}}{z_{ki}}\right]^{-\frac{1}{\sigma}} \Rightarrow z_{ki} = \left[\frac{p_j}{p_k}\right]^{\sigma} \frac{g_{ki}}{g_{ji}} z_{ji}.$$

Substituting z_{ji} from above in (5) we get:

$$M_i = \left[\sum_{k\in N_i^+, k\neq j} g_{ki}^{\frac{1}{\sigma}}\left[\left[\frac{p_j}{p_k}\right]^{\sigma}\frac{g_{ki}}{g_{ji}} z_{ji}\right]^{\frac{\sigma-1}{\sigma}} + g_{ji}^{\frac{1}{\sigma}} z_{ji}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \Rightarrow z_{ji} = g_{ji} p_j^{-\sigma} P_i^{\sigma} M_i, \tag{6}$$

where

$$P_i \equiv \left[\sum_{k\in N_i^+} g_{ki} p_k^{1-\sigma}\right]^{\frac{1}{1-\sigma}},$$

is the price index of intermediate inputs firm i uses in the production. Using (5), the definition of P_i and (6) we can write the conditional demand for intermediate inputs, labor and capital:

$$\begin{aligned} \ell_i(y_i;w,r,\boldsymbol{p},\theta_i) &= \varphi_i \frac{1}{1+\theta_i}\beta_i \frac{y_i}{w}, \\ k_i(y_i;w,r,\boldsymbol{p},\theta_i) &= \varphi_i \frac{1}{1+\theta_i}\rho_i \frac{y_i}{r}, \\ M_i(y_i;w,r,\boldsymbol{p},\theta_i) &= \varphi_i \frac{1}{1+\theta_i}\alpha_i \frac{y_i}{P_i}, \\ z_{ji}(y_i;w,r,\boldsymbol{p},\theta_i) &= \varphi_i \frac{1}{1+\theta_i}\alpha_i g_{ji} p_j^{-\sigma} P_i^{\sigma-1} y_i. \end{aligned} \tag{7}$$

Substituting (7) in (4) we get that $mc_i = \varphi_i$. Then, to derive the expression for φ_i, substitute (7) in (4) to obtain:

$$y_i = \zeta_i \left(\frac{\varphi_i \rho_i y_i}{(1+\theta_i)r}\right)^{\rho_i} \left(\frac{\varphi_i \beta_i y_i}{(1+\theta_i)w}\right)^{\beta_i} \left(\frac{\varphi_i \alpha_i y_i}{(1+\theta_i)P_i}\right)^{\alpha_i} = \frac{\zeta_i}{1+\theta_i}\varphi_i y_i \left(\frac{\rho_i}{r}\right)^{\rho_i} \left(\frac{\beta_i}{w}\right)^{\beta_i} \left(\frac{\alpha_i}{P_i}\right)^{\alpha_i},$$

which gives:

$$mc_i = \varphi_i = \frac{1+\theta_i}{\zeta_i}\rho_i^{-\rho_i}\beta_i^{-\beta_i}\alpha_i^{-\alpha_i} r^{\rho_i} w^{\beta_i} P_i^{\alpha_i} = \frac{1+\theta_i}{\kappa_i} r^{\rho_i} w^{\beta_i} P_i^{\alpha_i},$$

as desired. □

Steady state and normalization

To facilitate the calibration, following Baqaee (2018); Baqaee and Farhi (2019), we define the steady state as a contingency in which there are no financial shocks $\theta_i = 0, \forall i$, and $\kappa_i = \mu_i, \forall i$.

From the Lemma 1 and the pricing rule, we can write:

$$\log p_i = \log\left[\frac{1+\theta_i}{\kappa_i}\mu_i\right] + \rho_i \log r + \beta_i \log w + \frac{\alpha_i}{1-\sigma}\log\left[\sum_k g_{ki} p_k^{1-\sigma}\right],$$

which in the steady state reduces to:

$$\log p_i = \rho_i \log r + \beta_i \log w + \frac{\alpha_i}{1-\sigma}\log\left[\sum_k g_{ki} p_k^{1-\sigma}\right].$$

Clearly, $p_i = 1 \forall (i \in N)$, $w=1$ and $r=1$ satisfies this equation for every i.

The consumer's problem

Lemma 2. *Let $\bar{p} = \prod_i \left(\frac{p_i}{\gamma_i}\right)^{\gamma_i}$ denote the price index of consumption goods. The consumer chooses consumption plan such that*

$$\frac{p_i c_i}{\bar{p} c} = \gamma_j \text{ and } \frac{c^{-\gamma}}{L^\eta} = \frac{\bar{p}}{w}.$$

Proof of Lemma 2. The consumer solves the following problem:

$$\max_{\boldsymbol{c},L} \frac{1}{1-\delta}\left[\overbrace{\prod_i c_i^{\gamma_i}}^{c}\right]^{1-\delta} - \frac{L^{1+\eta}}{1+\eta}$$
$$s.t. \sum_i p_i c_i \leq E.$$

The monotonicity of preferences implies $\bar{p}c=E$. The fact that $\frac{p_i c_i}{\bar{p}c}=\gamma_j$ is directly obtained by solving for the expenditure minimization problem, where $\bar{p}c$ is the resulting expenditure. $\frac{c^{-\gamma}}{L^\eta}=\frac{\bar{p}}{w}$ follows directly from the utility maximization problem with respect to c and L. □

Cost share, revenue share, sales share

We now introduce some additional notation. Let $s_{ji}\equiv p_j z_{ji}$ and $s_i=p_i y_i$. Furthermore, let $\tilde{\omega}_{ji}$ denote cost share on intermediate input j among intermediate inputs firm i uses in production, that is $\tilde{\omega}_{ji}\equiv\frac{s_{ji}}{\sum_\ell s_{\ell i}}$. Let $\omega_{ji}\equiv\frac{s_{ji}}{s_i}$ denote the expenditure share in sales on input j (we'll refer to it as *revenue share*). Finally, let $h_{ji}\equiv\frac{s_{ji}}{s_j}$ denote the sales share.

Let us now relate the technological parameters $(g_{ji})_{ij}$ with cost shares $\tilde{\omega}_{ji}$, revenue shares ω_{ji}, and sales shares h_{ji}.

Lemma 3. *The following holds in the equilibrium:*

$$\tilde{\omega}_{ji}=g_{ji}p_j^{1-\sigma}P_i^{\sigma-1}=\frac{g_{ji}p_j^{1-\sigma}}{\sum_k g_{ki}p_k^{1-\sigma}}, \text{ or in terms of quantities, } \tilde{\omega}_{ji}=g_{ji}^{\frac{1}{\sigma}}\left[\frac{z_{ji}}{M_i}\right]^{\frac{\sigma-1}{\sigma}}, \tag{8}$$

$$\omega_{ji}=\frac{\alpha_i}{(1+\theta_i)\mu_i}\tilde{\omega}_{ji}, \tag{9}$$

and

$$h_{ji}=\omega_{ji}\frac{s_i}{s_j}=\frac{\alpha_i}{(1+\theta_i)\mu_i}\tilde{\omega}_{ji}\frac{s_i}{s_j}. \tag{10}$$

Proof of Lemma 3. The first equation in (8) follows directly from the fact that $s_{ji}=g_{ji}p_j^{1-\sigma}P_i^{\sigma-1}P_iM_i$. To prove that the second equation in (8) we note that the FONC of the cost minimization problem with respect to z_{ji} in (5) gives:

$$(1+\theta_i)p_j=mc_i\alpha_i y_i M_i^{\frac{1-\sigma}{\sigma}} g_{ji}^{\frac{1}{\sigma}} z_{ji}^{-\frac{1}{\sigma}} \Rightarrow \frac{(1+\theta_i)p_j z_{ji}}{mc_i\alpha_i y_i}=M_i^{\frac{1-\sigma}{\sigma}} g_{ji}^{\frac{1}{\sigma}} z_{ji}^{\frac{\sigma-1}{\sigma}} \overbrace{\Rightarrow}^{\text{by (7)}} \tilde{\omega}_{ji}=g_{ji}^{\frac{1}{\sigma}} M_i^{\frac{1-\sigma}{\sigma}} z_{ji}^{\frac{\sigma-1}{\sigma}}.$$

The equation (9) follows directly from (7) and the pricing rule $p_i=\mu_i mc_i$. Finally, (10) follows from equality $\frac{s_{ji}}{s_j}=\frac{s_{ji}}{s_i}\frac{s_i}{s_j}$, which concludes the proof. □

Let $\mathbf{\Omega}$, $\tilde{\mathbf{\Omega}}$ and $\mathbf{H}$ be matrices with elements ω_{ij}, $\tilde{\omega}_{ij}$ and h_{ij} respectively. Recall that, $\mathbf{A}$, $\mathbf{M}$ and $\mathbf{T}$ stand for diagonal matrices with elements α_i, $\frac{1}{\mu_i}$ and $\frac{1}{1+\theta_i}$ on the main diagonal, respectively. Lemma 3 implies that in steady state:

$$\tilde{\mathbf{\Omega}} \overbrace{=}^{\text{steady state}} \mathbf{G},$$

$$\mathbf{\Omega}{=}\tilde{\mathbf{\Omega}}\mathbf{AMT} \overbrace{=}^{\text{steady state}} \mathbf{GAM},$$

$$\mathbf{H}{=}\mathbf{V}^{-1}\mathbf{\Omega}\mathbf{V} \overbrace{=}^{\text{steady state}} \mathbf{V}^{-1}\mathbf{GAMV},$$

where $\mathbf{V}$ are diagonal matrices with elements diagonal elements equal to entries of the vector $\boldsymbol{v}(\boldsymbol{\theta}){=}(v_i(\boldsymbol{\theta}))_{i=1}^n{=}(\mathbf{I}-\mathbf{GAMT})^{-1}\boldsymbol{\gamma}$, which, in steady state, is equal to $\boldsymbol{v}(\mathbf{0}){=}(\mathbf{I}-\mathbf{GAM})^{-1}\boldsymbol{\gamma}$.

Effect of shocks on prices

Lemma 4. *In the steady state:*

$$\frac{\partial \log \boldsymbol{p}}{\partial \theta_k}{=}\left[\mathbf{I}-\mathbf{AG}'\right]^{-1}\left[\frac{\partial \log w}{\partial \theta_k}\boldsymbol{\beta}+\frac{\partial \log r}{\partial \theta_k}\boldsymbol{\rho}+\boldsymbol{e}_k\right].$$

In the special case when $\alpha_i{=}\alpha$, $\beta_i{=}\beta$:

$$\frac{\partial \log \boldsymbol{p}}{\partial \theta_k}{=}\left[\mathbf{I}-\alpha\mathbf{G}'\right]^{-1}\boldsymbol{e}_k+\frac{\beta}{1-\alpha}\frac{\partial \log w}{\partial \theta_k}\mathbf{1}+\frac{\rho}{1-\alpha}\frac{\partial \log r}{\partial \theta_k}\mathbf{1}.$$

Proof. Consider $\frac{\partial \log p_i}{\partial \theta_k}$. From the expression for marginal cost of firm i (Lemma 1), we have:

$$\log p_i{=}\log(1+\theta_i)+\log\mu_i-\log\kappa_i+\beta_i\log w+\rho_i\log r_i+\alpha_i\log P_i.$$

Differentiating with respect to θ_k we get:

$$\frac{\partial \log p_i}{\partial \theta_k}{=}\frac{\partial \log(1+\theta_i)}{\partial \theta_k}+\beta_i\frac{\partial \log w}{\partial \theta}+\rho_i\frac{\partial \log r}{\partial \theta_k}+\alpha_i\frac{\partial \log P_i}{\partial \theta_k}.$$

To get the expression for $\frac{\partial \log P_i}{\partial \theta_k}$ we note that from the definition of P_i it follows that:

$$\frac{\partial \log P_i}{\partial \theta_k}{=}\frac{1}{\sum_\ell g_{\ell i}p_\ell^{1-\sigma}}\sum_j g_{ji}p_j^{1-\sigma}\frac{\partial \log p_j}{\partial \theta_k}.$$

At the steady state:

$$\left.\frac{\partial \log P_i}{\partial \theta_k}\right|_{\boldsymbol{\theta}=0}{=}\sum_j g_{ji}\left.\frac{\partial \log p_j}{\partial \theta_k}\right|_{\boldsymbol{\theta}=0}{=}\boldsymbol{e}_i'\mathbf{G}'\frac{\partial \log \boldsymbol{p}}{\partial \theta_k}.$$

In what follows we omit $|_{\boldsymbol{\theta}=0}$ whenever it is clear that the derivatives are evaluated at the steady state. Finally, evaluating at the steady state:

$$\frac{\partial \log p_i}{\partial \theta_k}{=}-\delta_{ki}+\beta_i\frac{\partial \log w}{\partial \theta_k}+\rho_i\frac{\partial \log r}{\partial \theta_k}+\alpha_i\sum_j g_{ji}\frac{\partial \log p_j}{\partial \theta_k},$$

where δ_{ki} denotes the Kroenecker's delta. Writing this expression for each price in vector notation gives:

$$\frac{\partial \log \boldsymbol{p}}{\partial \theta_k}{=}\left[\mathbf{I}-\mathbf{AG}'\right]^{-1}\left[\frac{\partial \log w}{\partial \theta_k}\boldsymbol{\beta}+\frac{\partial \log r}{\partial \theta_k}\boldsymbol{\rho}+\boldsymbol{e}_k\right].$$

In the special case when $\alpha_i = \alpha$, $\beta_i = \beta$ for all i the above expression becomes:

$$\frac{\partial \log \boldsymbol{p}}{\partial \theta_k} = \left[\mathbf{I} - \alpha \mathbf{G}'\right]^{-1} \boldsymbol{e}_k + \frac{\beta}{1-\alpha} \frac{\partial \log w}{\partial \theta_k} \mathbf{1} + \frac{\rho}{1-\alpha} \frac{\partial \log r}{\partial \theta_k} \mathbf{1}.$$

□

The following corollary is implied by Lemma 4.

Corollary 1. *At the steady state:*

$$\frac{\partial \log \boldsymbol{P}}{\partial \theta_k} = \mathbf{G}' \left[\mathbf{I} - \mathbf{A}\mathbf{G}'\right]^{-1} \left[\frac{\partial \log w}{\partial \theta_k} \boldsymbol{\beta} + \frac{\partial \log r}{\partial \theta_k} \boldsymbol{\rho} + \boldsymbol{e}_k\right],$$

and when $\alpha_i = \alpha$, $\beta_i = \beta$ for all i:

$$\frac{\partial \log \boldsymbol{P}}{\partial \theta_k} = \frac{\beta}{1-\alpha} \frac{\partial \log w}{\partial \theta_k} \mathbf{1} + \frac{\rho}{1-\alpha} \frac{\partial \log r}{\partial \theta_k} \mathbf{1} + \mathbf{G}' \left[\mathbf{I} - \alpha \mathbf{G}'\right]^{-1} \boldsymbol{e}_k.$$

Link-level outcomes

Proposition 1. *The first order approximation of the change* $\log \frac{s_{ji}}{s_i} = \log w_{ji}$ *at the steady state is given with* (1)*. In the special case when all firms have equal input shares* α, β *and* ρ (1) *becomes* (8).

Proof of Proposition 1. We use the following approximation:

$$d \log s_{ji} = \sum_{k \in N} \frac{\partial \log s_{ji}}{\partial \theta_k} \theta_k, \tag{11}$$

where derivatives are evaluated at point $\boldsymbol{\theta} = \mathbf{0}$.

From firms' pricing rule ($p_i = \mu_i mc_i$), Lemma 1, and (6) it directly follows that:

$$p_j z_{ji} = g_{ji} (\mu_j mc_j)^{1-\sigma} P_i^{\sigma} M_i \Rightarrow p_j z_{ji} = g_{ji} (\mu_j (1+\theta_j) \kappa_j^{-1} r_j^{\rho_j} w^{\beta_j} P_j^{\alpha_j})^{1-\sigma} P_i^{\sigma} M_i,$$

which together with (7) implies:

$$p_j z_{ji} = g_{ji} (\mu_j (1+\theta_j) \kappa_j^{-1} r_j^{\rho_j} w^{\beta_j} P_j^{\alpha_j})^{1-\sigma} P_i^{\sigma-1} \mu_i^{-1} (1+\theta_i)^{-1} \alpha_i s_i.$$

Taking logs, and using $s_{ji} \equiv p_j z_{ji}$ we get:

$$\log s_{ji} = (1-\sigma) \log(1+\theta_j) - \log(1+\theta_i) + \log g_{ji} + (1-\sigma)(\log \mu_j - \log \kappa_j + \beta_j \log w + \rho_j \log r) + \log \alpha_i + \log s_i - \log \mu_i + (1-\sigma) \alpha_j \log P_j + (\sigma - 1) \log P_i.$$

Differentiating with respect to θ_k we get:

$$\begin{aligned} \frac{\partial \log s_{ji}}{\partial \theta_k} = & -\frac{1}{1+\theta_i} \delta_{ki} - (\sigma-1) \frac{1}{1+\theta_j} \delta_{kj} + \frac{\partial \log s_i}{\theta_k} - \\ & (\sigma - 1) \left[\beta_j \frac{\partial \log w}{\partial \theta_k} + \rho_j \frac{\partial \log r}{\partial \theta_k} + \alpha_j \frac{\partial \log P_j}{\partial \theta_k}\right] + (\sigma - 1) \frac{\partial \log P_i}{\partial \theta_k}, \end{aligned} \tag{12}$$

where δ_{jk} is Kroeneker's delta.

Corollary 1 implies that we can write (12) as:

$$\begin{aligned}\frac{\partial \log s_{ji}}{\partial\theta_k}=&-\frac{1}{1+\theta_i}\delta_{ki}-(\sigma-1)\frac{1}{1+\theta_j}\delta_{kj}+\frac{\partial\log s_i}{\theta_k}-\\ &(\sigma-1)\boldsymbol{e}_j'\left[\frac{\partial \log w}{\partial\theta_k}\boldsymbol{\beta}+\frac{\partial\log r}{\partial\theta_k}\boldsymbol{\rho}+\mathbf{AG}'\left(\mathbf{I}-\mathbf{AG}'\right)^{-1}\left(\frac{\partial\log w}{\partial\theta_k}\boldsymbol{\beta}+\frac{\partial\log r}{\partial\theta_k}\boldsymbol{\rho}+\boldsymbol{e}_k\right)\right]+\\ &(\sigma-1)\boldsymbol{e}_i'\left[\mathbf{G}'\left(\mathbf{I}-\mathbf{AG}'\right)^{-1}\left(\frac{\partial\log w}{\partial\theta_k}\boldsymbol{\beta}+\frac{\partial\log r}{\partial\theta_k}\boldsymbol{\rho}+\boldsymbol{e}_k\right)\right],\end{aligned}\tag{13}$$

which simplifies to:

$$\begin{aligned}\frac{\partial \log s_{ji}}{\partial\theta_k}=&-\frac{1}{1+\theta_i}\delta_{ki}-(\sigma-1)\frac{1}{1+\theta_j}\delta_{kj}+\frac{\partial\log s_i}{\partial\theta_k}-\\ &(\sigma-1)\boldsymbol{e}_j'\left[\left(\mathbf{I}-\mathbf{AG}'\right)^{-1}\left(\frac{\partial\log w}{\partial\theta_k}\boldsymbol{\beta}+\frac{\partial\log r}{\partial\theta_k}\boldsymbol{\rho}\right)+\mathbf{AG}'\left(\mathbf{I}-\mathbf{AG}'\right)^{-1}\boldsymbol{e}_k\right]+\\ &(\sigma-1)\boldsymbol{e}_i'\left[\mathbf{G}'\left(\mathbf{I}-\mathbf{AG}'\right)^{-1}\left(\frac{\partial\log w}{\partial\theta_k}\boldsymbol{\beta}+\frac{\partial\log r}{\partial\theta_k}\boldsymbol{\rho}\right)+\mathbf{G}'(\mathbf{I}-\mathbf{AG}')^{-1}\boldsymbol{e}_k\right].\end{aligned}\tag{14}$$

Using (14) in (11) and evaluating derivatives at $\boldsymbol{\theta}=\mathbf{0}$ gives (1), where we use the fact that $\mathbf{I}+\mathbf{AG}'(\mathbf{I}-\mathbf{AG}')^{-1}=(\mathbf{I}-\mathbf{AG}')^{-1}$.

In the special case when $\alpha_i=\alpha$ and $\beta_i=\beta$ for all firms i (14) simplifies to:

$$\begin{aligned}\frac{\partial \log s_{ji}}{\partial\theta_k}=&-\frac{1}{1+\theta_i}\delta_{ki}-(\sigma-1)\frac{1}{1+\theta_j}\delta_{kj}+\frac{\partial\log s_i}{\partial\theta_k}-\\ &(\sigma-1)\left[\frac{\beta}{1-\alpha}\frac{\partial\log w}{\partial\theta_k}+\frac{\rho}{1-\alpha}\frac{\partial\log r}{\partial\theta_k}+\alpha\boldsymbol{e}_j'\mathbf{G}'(\mathbf{I}-\alpha\mathbf{G})^{-1}\boldsymbol{e}_k\right]+\\ &(\sigma-1)\left[\frac{\beta}{1-\alpha}\frac{\partial\log w}{\partial\theta_k}+\frac{\rho}{1-\alpha}\frac{\partial\log r}{\partial\theta_k}+\boldsymbol{e}_i'\mathbf{G}'\left[\mathbf{I}-\alpha\mathbf{G}'\right]^{-1}\boldsymbol{e}_k\right]\\ =&-\frac{1}{1+\theta_i}\delta_{ki}-(\sigma-1)\frac{1}{1+\theta_j}\delta_{kj}+\frac{\partial\log s_i}{\theta_k}-(\sigma-1)\left[\alpha\boldsymbol{e}_j'\mathbf{G}'(\mathbf{I}-\alpha\mathbf{G})^{-1}\boldsymbol{e}_k-\boldsymbol{e}_i'\mathbf{G}'\left(\mathbf{I}-\alpha\mathbf{G}'\right)^{-1}\boldsymbol{e}_k\right].\end{aligned}$$

This concludes the proof. □

Node-level outcomes

Proposition 2. *A first-order approximation of the change in* $\log\left(\frac{s_i}{E}\right)$ *at the steady state is given with* (2)*. In the special case when all firms have equal input shares* α*,* β *and* ρ (2) *becomes* (9)*.*

Proof of Proposition 2. We first provide expression for $\frac{\partial\log \boldsymbol{s}}{\partial\theta_k}-\frac{\partial\log E}{\partial\theta_k}$. Equations (2) and (9) then follow directly.

Market clearing condition for intermediate inputs read:

$$y_i=c_i+\sum_j z_{ij},$$

which be written as:

$$p_iy_i=p_ic_i+\sum_j p_iz_{ij}\Rightarrow s_i=\gamma_iE+\sum_j\omega_{ij}s_j,$$

and therefore:

$$s{=}E(\mathbf{I}-\mathbf{\Omega})^{-1}\boldsymbol{\gamma}{=}E\boldsymbol{v}.$$

Taking derivatives, we get:

$$\begin{aligned}\frac{\partial \boldsymbol{s}}{\partial\theta_k}&{=}\frac{\partial E}{\partial\theta_k}(\mathbf{I}-\mathbf{\Omega})^{-1}\boldsymbol{\gamma}-(\mathbf{I}-\mathbf{\Omega})^{-1}\frac{\partial(\mathbf{I}-\mathbf{\Omega})}{\partial\theta_k}(\mathbf{I}-\mathbf{\Omega})^{-1}E\boldsymbol{\gamma}\\ &{=}\frac{\partial \log E}{\partial\theta_k}\boldsymbol{s}+(\mathbf{I}-\mathbf{\Omega})^{-1}\frac{\partial\mathbf{\Omega}}{\partial\theta_k}\boldsymbol{s}{=}\left[\frac{\partial \log E}{\partial\theta_k}\mathbf{I}+\overbrace{(\mathbf{I}-\mathbf{\Omega})^{-1}}^{\mathbf{\Psi}}\frac{\partial\mathbf{\Omega}}{\partial\theta_k}\right]\boldsymbol{s}.\end{aligned}$$

For a given firm i, we have:

$$\frac{\partial s_i}{\partial\theta_k}{=}\frac{\partial \log E}{\partial\theta_k}s_i+\sum_\ell\sum_j\psi_{i\ell}\frac{\partial\omega_{\ell j}}{\partial\theta_k}s_j,$$

and consequently

$$\frac{\partial \log s_i}{\partial\theta_k}{=}\frac{\partial \log E}{\partial\theta_k}+\frac{1}{s_i}\sum_\ell\sum_j\psi_{i\ell}\frac{\partial\omega_{\ell j}}{\partial\theta_k}s_j{=}\frac{\partial \log E}{\partial\theta_k}+\frac{1}{s_i}\sum_\ell\sum_j\psi_{i\ell}\omega_{\ell j}\frac{\partial \log\omega_{\ell j}}{\partial\theta_k}s_j.$$

Substituting the expression for $\frac{\partial \log\omega_{\ell j}}{\partial\theta_k}$ (from Proposition 1), the previous expression becomes:

$$\frac{\partial \log s_i}{\partial \log\theta_k}{=}\frac{\partial \log E}{\partial\theta_k}+\frac{1}{s_i}\left[\sum_\ell\sum_j\psi_{i\ell}\omega_{\ell j}s_j\left(-\frac{\partial \log(1+\theta_j)}{\partial\theta_k}+(1-\sigma)\left(\frac{\partial \log p_\ell}{\partial\theta_k}-\sum_r\tilde{\omega}_{rj}\frac{\partial \log p_r}{\partial\theta_k}\right)\right)\right].$$

To write the expression for $\frac{\partial \log \boldsymbol{s}}{\partial\theta_k}$, we consider the parts of the right-hand side of the previous expression in the brackets separately.

First, we note that:

$$-\frac{1}{s_i}\left[\sum_\ell\sum_j\psi_{i\ell}\omega_{\ell j}s_j\frac{\partial \log(1+\theta_j)}{\partial\theta_k}\right]{=}-\frac{1}{v_i}\left[\sum_\ell\sum_j\psi_{i\ell}\omega_{\ell j}v_j\frac{\partial \log(1+\theta_j)}{\partial\theta_k}\right],$$

where we used $s_i{=}Ev_i$. For each i this can be written in the matrix notation as:

$$-\frac{1}{1+\theta_k}\mathbf{V}^{-1}[\mathbf{I}-\mathbf{\Omega}]^{-1}\mathbf{\Omega}\mathbf{V}\boldsymbol{e}_k{=}-\frac{1}{1+\theta_k}(\mathbf{I}-\mathbf{H})^{-1}\mathbf{H}\boldsymbol{e}_k,$$

since:

$$\mathbf{V}^{-1}(\mathbf{I}-\mathbf{\Omega})^{-1}\mathbf{\Omega}\mathbf{V}\boldsymbol{e}_k{=}\left(\mathbf{V}^{-1}\mathbf{\Omega}\mathbf{V}+\mathbf{V}^{-1}\mathbf{\Omega}\mathbf{V}\mathbf{V}^{-1}\mathbf{\Omega}\mathbf{V}+...\right)\boldsymbol{e}_k{=}\left(\sum_{i=1}^{\infty}\mathbf{H}^i\right)\boldsymbol{e}_k{=}(\mathbf{I}-\mathbf{H})^{-1}\mathbf{H}\boldsymbol{e}_k.$$

The expression:

$$(1-\sigma)\frac{1}{s_i}\left[\sum_\ell\sum_j\psi_{i\ell}\omega_{\ell j}s_j\frac{\partial \log p_\ell}{\partial\theta_k}\right],$$

for each $i{\in}N$ can be written as:

$$(1-\sigma)\mathbf{V}^{-1}[\mathbf{I}-\mathbf{\Omega}]^{-1}diag(\mathbf{\Omega}\mathbf{V}\mathbf{1})\frac{\partial \log\boldsymbol{p}}{\partial\theta_k}.$$

Finally, consider:

$$-(1-\sigma)\frac{1}{s_i}\left[\sum_{\ell}\sum_{j}\psi_{i\ell}\omega_{\ell j}s_j\sum_{r}\tilde{\omega}_{rj}\frac{\partial \log p_r}{\partial\theta_k}\right],$$

and note that we can write this expression (for each i) in matrix notation as:

$$-(1-\sigma)\mathbf{V}^{-1}[\mathbf{I}-\mathbf{\Omega}]^{-1}\mathbf{\Omega}\mathbf{V}\tilde{\mathbf{\Omega}}'\frac{\partial \log \boldsymbol{p}}{\partial\theta_k}.$$

Putting everything together, we get:

$$\frac{\partial \log \boldsymbol{s}}{\partial\theta_k}=\frac{\partial \log E}{\partial\theta_k}-\frac{1}{1+\theta_k}[\mathbf{I}-\mathbf{H}]^{-1}\mathbf{H}\boldsymbol{e}_k+(1-\sigma)\mathbf{V}^{-1}[\mathbf{I}-\mathbf{\Omega}]^{-1}\left[diag(\mathbf{\Omega}\mathbf{V}\mathbf{1})-\mathbf{\Omega}\mathbf{V}\tilde{\mathbf{\Omega}}'\right]\frac{\partial \log \boldsymbol{p}}{\partial\theta_k}. \quad (15)$$

Plugging in the expression for $\frac{\partial \log \boldsymbol{p}}{\partial\theta_k}$ (from Lemma 4) in (15) gives:

$$\frac{\partial \log \boldsymbol{s}}{\partial\theta_k}=\frac{\partial \log E}{\partial\theta_k}-\frac{1}{1+\theta_k}[\mathbf{I}-\mathbf{H}]^{-1}\mathbf{H}\boldsymbol{e}_k+$$
$$(1-\sigma)\mathbf{V}^{-1}[\mathbf{I}-\mathbf{\Omega}]^{-1}\left[diag(\mathbf{\Omega}\mathbf{V}\mathbf{1})-\mathbf{\Omega}\mathbf{V}\tilde{\mathbf{\Omega}}'\right]\left[\mathbf{I}-\mathbf{A}\tilde{\mathbf{\Omega}}'\right]^{-1}\left[\frac{\partial \log w}{\partial\theta_k}\boldsymbol{\beta}+\frac{\partial \log r}{\partial\theta_k}\boldsymbol{\rho}+\frac{1}{1+\theta_k}\boldsymbol{e}_k\right]. \quad (16)$$

Evaluating in the steady state ($\tilde{\omega}_{ij}=g_{ij}$, $\omega_{ij}=\frac{\alpha_j}{\mu_j}g_{ij}$, and $\boldsymbol{\theta}=\mathbf{0}$) the previous expression becomes:

$$\frac{\partial \log \boldsymbol{s}}{\partial\theta_k}-\frac{\partial \log E}{\partial\theta_k}=-[\mathbf{I}-\mathbf{V}^{-1}\mathbf{GAMV}]^{-1}\mathbf{V}^{-1}\mathbf{GAMV}\boldsymbol{e}_k+$$
$$(1-\sigma)\mathbf{V}^{-1}[\mathbf{I}-\mathbf{GAM}]^{-1}\left[diag(\mathbf{GAMV1})-\mathbf{GAMVG}'\right][\mathbf{I}-\mathbf{AG}']^{-1}\left[\frac{\partial \log w}{\partial\theta_k}\boldsymbol{\beta}+\frac{\partial \log r}{\partial\theta_k}\boldsymbol{\rho}+\boldsymbol{e}_k\right].$$

Let us now define $\mathbf{\Lambda}\equiv\mathbf{V}^{-1}[\mathbf{I}-\mathbf{GAM}]^{-1}[diag(\mathbf{GAMV1})-\mathbf{GAMVG}'][\mathbf{I}-\mathbf{AG}']^{-1}$.

We can write (evaluating at the steady state):

$$\frac{\partial \log \boldsymbol{s}}{\partial\theta_k}-\frac{\partial \log E}{\partial\theta_k}=-[\mathbf{I}-\mathbf{V}^{-1}\mathbf{GAMV}]^{-1}\mathbf{V}^{-1}\mathbf{GAMV}\boldsymbol{e}_k+(1-\sigma)\mathbf{\Lambda}\left[\frac{\partial \log w}{\partial\theta_k}\boldsymbol{\beta}+\frac{\partial \log r}{\partial\theta_k}+\boldsymbol{e}_k\right].$$

In the special case $\alpha_i=\alpha$ and $\beta_i=\beta$ for all $i\in N$ we get:

$$\frac{\partial \log \boldsymbol{s}}{\partial\theta_k}-\frac{\partial \log E}{\partial\theta_k}=-\alpha[\mathbf{I}-\alpha\mathbf{V}^{-1}\mathbf{GMV}]^{-1}\mathbf{V}^{-1}\mathbf{GMV}\boldsymbol{e}_k+(1-\sigma)\mathbf{\Lambda}\boldsymbol{e}_k,$$

where we used the fact that in the symmetric case $(\mathbf{I}-\alpha\mathbf{G}')^{-1}\mathbf{1}=\frac{1}{1-\alpha}\mathbf{1}$, and $[diag(\mathbf{GMV1})-\mathbf{GMVG}']\mathbf{1}=\mathbf{0}$.

□

Auxiliary result

To obtain (10) from (9) we use the following result.

Lemma 5. *In steady state:*

$$\mathbf{V}^{-1}[\mathbf{I}-\mathbf{GAM}]^{-1}\left[diag(\mathbf{GAMV1})-\mathbf{GAMVG}'\right][\mathbf{I}-\mathbf{AG}']^{-1}=[\mathbf{I}-\mathbf{H}]^{-1}\left[diag(\mathbf{H1})-\mathbf{HG}'\right][\mathbf{I}-\mathbf{AG}']^{-1}.$$

Proof.

$$\mathbf{V}^{-1}[\mathbf{I}-\mathbf{GAM}]^{-1}\left[diag(\mathbf{GAMV1})-\mathbf{GAMVG}'\right]\left[\mathbf{I}-\mathbf{AG}'\right]^{-1}=$$
$$\mathbf{V}^{-1}[\mathbf{I}-\mathbf{GAM}]^{-1}\mathbf{VV}^{-1}\left[\mathbf{V}diag(\mathbf{V}^{-1}\mathbf{GAMV1})-\mathbf{VV}^{-1}\mathbf{GAMVG}'\right]\left[\mathbf{I}-\mathbf{AG}'\right]^{-1}=$$
$$[\mathbf{I}-\mathbf{H}]^{-1}\left[diag(\mathbf{H1})-\mathbf{HG}'\right]\left[\mathbf{I}-\mathbf{AG}'\right]^{-1}.$$

□

Aggregation

We now examine the effect of the shocks on the real GDP. We consider the case with homogeneous α, β and ρ. We follow Baqaee and Farhi (2019) and choose as a numeraire the nominal GDP, therefore normalizing $E=\bar{p}c=\sum p_i c_i=1$, where $\bar{p}\equiv\sum_{i\in N}\left(\frac{p_i}{\gamma_i}\right)^{\gamma_i}$ is the consumer price index, and c is the aggregate production of the consumption good as defined in Section IV.C, which is equal to the real GDP in our model. Given the chosen normalization, we can write:

$$\frac{\partial \log c}{\partial \theta_k}=-\frac{\partial \log \bar{p}}{\partial \theta_k}=-\sum_{i\in N}\gamma_i\frac{\partial \log p_i}{\partial \theta_k}, \tag{17}$$

and consequently:

$$d\log c=-\sum_{i=1}^{n}\gamma_i d\log p_i. \tag{18}$$

Proposition 3. *The first-order approximation of the effects of financial shocks on GDP is given by:*

$$d\log c=-\boldsymbol{\gamma}'\left[\mathbf{I}-\alpha\mathbf{G}'\right]^{-1}\boldsymbol{\theta}-\frac{\beta}{1-\alpha}d\log w-\frac{\rho}{1-\alpha}d\log r, \tag{19}$$

where

$$d\log w=\frac{\eta}{1+\eta}\frac{\beta}{wL}\boldsymbol{s}'\mathbf{M}d\log\boldsymbol{s}-\frac{1-\delta}{1+\eta}d\log c, \tag{20}$$

$$d\log r=\frac{\rho}{rK}\boldsymbol{s}'\mathbf{M}d\log\boldsymbol{s}, \tag{21}$$

and

$$d\log\boldsymbol{s}=-[\mathbf{I}-\mathbf{H}]^{-1}\mathbf{H}\boldsymbol{\theta}+(1-\sigma)\boldsymbol{\Lambda}\boldsymbol{\theta}.$$

Proof of Proposition 3. We first find the expression for $\frac{\partial \log c}{\partial \theta_k}$. From Lemma 4:

$$\frac{\partial \log \boldsymbol{p}}{\partial \theta_k}=\left[\mathbf{I}-\alpha\mathbf{G}'\right]^{-1}\boldsymbol{e}_k+\frac{\beta}{1-\alpha}\frac{\partial \log w}{\partial \theta_k}\mathbf{1}+\frac{\rho}{1-\alpha}\frac{\partial \log r}{\partial \theta_k}\mathbf{1},$$

from where we can write:

$$\frac{\partial \log c}{\partial \theta_k}=-\boldsymbol{\gamma}'\left[\mathbf{I}-\alpha\mathbf{G}'\right]^{-1}\boldsymbol{e}_k-\left(\frac{\beta}{1-\alpha}\frac{\partial \log w}{\partial \theta_k}+\frac{\rho}{1-\alpha}\frac{\partial \log r}{\partial \theta_k}\right). \tag{22}$$

Therefore,

$$d\log c = -\boldsymbol{\gamma}'\left[\mathbf{I}-\alpha\mathbf{G}'\right]^{-1}\boldsymbol{\theta}-\frac{\beta}{1-\alpha}\sum_{k\in N}\frac{\partial\log w}{\partial\theta_k}\theta_k-\frac{\rho}{1-\alpha}\sum_{k\in N}\frac{\partial\log r}{\partial\theta_k}\theta_k=$$
$$-\boldsymbol{\gamma}'\left[\mathbf{I}-\alpha\mathbf{G}'\right]^{-1}\boldsymbol{\theta}-\frac{\beta}{1-\alpha}\mathrm{d}\log w-\frac{\rho}{1-\alpha}\mathrm{d}\log r,$$

which delivers equation (19).[5]

We now show that equations (21) and (20) hold. Combining the firm's optimal demand for capital (7) and the market clearing condition for capital, we get:

$$rK=\rho\sum_{i=1}^{n}\frac{s_i}{\mu_i}=\rho\boldsymbol{s}'\mathbf{M}\mathbf{1},$$

where we recall that K is the aggregate (inelastic) supply of the physical capital. Similarly, for labor, we get:

$$wL=\beta\sum_{i=1}^{n}\frac{s_i}{\mu_i}=\beta\boldsymbol{s}'\mathbf{M}\mathbf{1}.$$

Therefore, we can write:

$$\frac{\partial\log r}{\partial\theta_k}=\frac{\partial\log(rK)}{\partial\theta_k}=\frac{\partial\log(rK)}{\partial(rK)}\frac{\partial(rK)}{\partial\theta_k}=\frac{\rho}{rK}\left[\sum_{i=1}^{n}\frac{s_i}{\mu_i}\frac{\partial\log s_i}{\partial\theta_k}\right]=\frac{\rho}{rK}\boldsymbol{s}'\mathbf{M}\frac{\partial\log\boldsymbol{s}}{\partial\theta_k},$$

from where we get (21). We recall that from Proposition 2 $\mathrm{d}\log\boldsymbol{s}=-\left[\mathbf{I}-\mathbf{H}\right]^{-1}\mathbf{H}\boldsymbol{\theta}+(1-\sigma)\boldsymbol{\Lambda}\boldsymbol{\theta}$.

As for $\frac{\partial\log w}{\partial\theta_k}$, from the market clearing condition for labor we have:

$$\frac{\partial\log w}{\partial\theta_k}=\frac{\beta}{wL}\boldsymbol{s}'\mathbf{M}\frac{\partial\log\boldsymbol{s}}{\partial\theta_k}-\frac{\partial\log L}{\partial\theta_k}.$$

To calculate $\frac{\partial\log L}{\partial\theta_k}$, we note that from the representative household's problem, we have:

$$\log L=\frac{1-\delta}{\eta}\log c+\frac{1}{\eta}\log w,$$

and therefore:

$$\frac{\partial\log L}{\partial\theta_k}=\frac{1-\delta}{\eta}\frac{\partial\log c}{\partial\log\theta_k}+\frac{1}{\eta}\frac{\partial\log w}{\partial\theta_k},$$

which in turn implies:

$$\frac{\partial\log w}{\partial\theta_k}=\frac{\eta}{1+\eta}\frac{\beta}{wL}\boldsymbol{s}'\mathbf{M}\frac{\partial\log\boldsymbol{s}}{\partial\theta_k}-\frac{1-\delta}{1+\eta}\frac{\partial\log c}{\partial\theta_k},$$

and therefore:

$$\mathrm{d}\log w=\frac{\eta}{1+\eta}\frac{\beta}{wL}\boldsymbol{s}'\mathbf{M}\mathrm{d}\log\boldsymbol{s}-\frac{1-\delta}{1+\eta}\mathrm{d}\log c,$$

which is exactly (20). This completes the proof.

□

[5] Equation (19) corresponds to a more general expression derived in equation (4) in Baqaee and Farhi (2019), labeled as *ex-ante effect of distortions*. While Baqaee and Farhi (2019) do not provide an explicit expression for the ex-ante effect of distortions on the GDP, we are able to do it (see Corollary 2) thanks to our parametric assumptions.

Corollary 2.

$$\begin{aligned}\mathrm{dlog}c= & -\left(1-\frac{1-\delta}{1+\eta}\frac{\beta}{1-\alpha}\right)^{-1}\boldsymbol{\gamma}'\left[\mathbf{I}-\alpha\mathbf{G}'\right]^{-1}\boldsymbol{\theta} \\ & -\frac{1}{1-\alpha}\left(1-\frac{1-\delta}{1+\eta}\frac{\beta}{1-\alpha}\right)^{-1}\left(\frac{\eta\beta^2}{(1+\eta)wL}+\frac{\rho^2}{rK}\right)\boldsymbol{s}'\mathbf{M}\Big(-[\mathbf{I}-\mathbf{H}]^{-1}\mathbf{H}\boldsymbol{\theta}+(1-\sigma)\boldsymbol{\Lambda}\boldsymbol{\theta}\Big)\end{aligned} \tag{23}$$

Proof. Follows directly from Proposition 3.

□

Corollary 3. *In the absence of the production network*

$$\mathrm{dlog}c=-\left(1-\frac{1-\delta}{1+\eta}\beta\right)^{-1}\boldsymbol{\gamma}'\boldsymbol{\theta} \tag{24}$$

Proof. In counterfactual without network, we set $\alpha=0$, which implies $[\mathbf{I}-\alpha\mathbf{G}']^{-1}=\mathbf{I}$. Moreover, $\alpha=0$ implies $\mathbf{H}=0$, which basically captures the idea that firms are selling only to the final consumer, and therefore $\mathrm{dlog}\,\boldsymbol{s}=0$. Hence, (23) becomes (24). □

Recovering markups

From Lemma 3, it follows that at the steady state: $\mu_i=\alpha_i\frac{\tilde{\omega}_{ji}}{\omega_{ji}}$. We observe $\tilde{\omega}_{ji}$ and ω_{ji} in our VAT data. For firms that have more than one supplier, we calculate μ_i as the average across all suppliers, as follows:

$$\mu_i=\alpha_i\frac{1}{d_i^-}\sum_j\frac{\tilde{\omega}_{ji}}{\omega_{ji}},$$

where d_i^- is the in-degree of firm i.

References

Acemoglu, Daron, Vasco M Carvalho, Asuman Ozdaglar, and Alireza Tahbaz-Salehi. 2012. "The Network Origins of Aggregate Fluctuations." *Econometrica*, 80(5).

Alfaro, Laura, Manuel García-Santana, and Enrique Moral-Benito. 2021. "On the direct and indirect real effects of credit supply shocks." *Journal of Financial Economics*, 139(3).

Allen, Franklin, and Douglas Gale. 2000. "Financial Contagion." *Journal of Political Economy*, 108(1).

Amiti, Mary, and David E Weinstein. 2018. "How much do idiosyncratic bank shocks affect investment? Evidence from matched bank-firm loan data." *Journal of Political Economy*, 126(2).

Atalay, Enghin. 2017. "How important are sectoral shocks?" *American Economic Journal: Macroeconomics*, 9(4).

Baqaee, David Rezza. 2018. "Cascading failures in production networks." *Econometrica*, 86(5).

Baqaee, David Rezza, and Emmanuel Farhi. 2019. "Productivity and misallocation in general equilibrium." *The Quarterly Journal of Economics*, 135(1).

Barrot, Jean-Noël, and Julien Sauvagnat. 2016. "Input Specificity and the Propagation of Idiosyncratic Shocks in Production Networks." *The Quarterly Journal of Economics*, 131(3).

Boehm, Christoph E, Aaron Flaaen, and Nitya Pandalai-Nayar. 2019. "Input linkages and the transmission of shocks: Firm-level evidence from the 2011 Tōhoku earthquake." *Review of Economics and Statistics*, 101(1).

Cabrales, Antonio, Piero Gottardi, and Fernando Vega-Redondo. 2017. "Risk Sharing and Contagion in Networks." *The Review of Financial Studies*, 30(9).

Carvalho, Vasco M, Makoto Nirei, Yukiko U Saito, and Alireza Tahbaz-Salehi. 2020. "Supply Chain Disruptions: Evidence from the Great East Japan Earthquake." *The Quarterly Journal of Economics.*

Chodorow-Reich, Gabriel. 2014. "The employment effects of credit market disruptions: Firm-level evidence from the 2008–9 financial crisis." *The Quarterly Journal of Economics*, 129(1).

Cortes, Gustavo, Thiago Silva, and Bernardus Doornik. 2019. "Credit Shock Propagation in Firm Networks: evidence from government bank credit expansions." *Working paper, Central Bank of Brazil, Research Department.*

Costello, Anna M. 2020. "Credit market disruptions and liquidity spillover effects in the supply chain." *Journal of Political Economy*, 128(9).

Demir, Banu, Beata Javorcik, Tomasz K Michalski, and Evren Ors. 2024. "Financial constraints and propagation of shocks in production networks." *Review of Economics and Statistics*, 106(2).

Dewachter, H, J Tielens, and JV Hove. 2020. "The Production Network Sources of Bank Influence." Working Paper.

Elliott, Matthew, Benjamin Golub, and Matthew O Jackson. 2014. "Financial Networks and Contagion." *American Economic Review*, 104(10).

Freixas, Xavier, Bruno M. Parigi, and Jean-Charles Rochet. 2000. "Systemic risk, interbank relations, and liquidity provision by the central bank." *Journal of Money, Credit and Banking*, 32(3).

Fujiy, Brian C, Devaki Ghose, and Gaurav Khanna. 2024. "Production Networks and Firm-level Elasticities of Substitution." The World Bank.

Galaasen, Sigurd, Rustam Jamilov, Ragnar Juelsrud, and Hélène Rey. 2020. "Granular Credit Risk." *Working paper.*

Gertler, Mark, and Nobuhiro Kiyotaki. 2010. "Financial intermediation and credit policy in business cycle analysis." In *Handbook of Monetary Economics.* Vol. 3. Elsevier.

Greenstone, Michael, Alexandre Mas, and Hoai-Luu Nguyen. 2014. "Do credit market shocks affect the real economy? Quasi-experimental evidence from the Great Recession and 'normal' economic times." *Working paper.*

Halpern, László, Miklós Koren, and Adam Szeidl. 2015. "Imported inputs and productivity." *American economic review*, 105(12).

Holmstrom, Bengt, and Jean Tirole. 1997. "Financial intermediation, loanable funds, and the real sector." *the Quarterly Journal of Economics*, 112(3).

Iyer, Rajkamal, and Jose-Luis Peydro. 2011. "Interbank Contagion at Work: Evidence from a Natural Experiment." *The Review of Financial Studies*, 24(4).

Jacobson, Tor, and Erik von Schedvin. 2015. "Trade Credit and the Propagation of Corporate Failure: an Empirical Analysis." *Econometrica*, 83(4).

Jiménez, Gabriel, Steven Ongena, José-Luis Peydró, and Jesús Saurina. 2012. "Credit Supply and Monetary Policy: Identifying the Bank Balance-sheet Channel with Loan Applications." *American Economic Review*, 102(5).

Jiménez, Gabriel, Steven Ongena, José-Luis Peydró, and Jesús Saurina. 2017. "Macroprudential policy, countercyclical bank capital buffers, and credit supply: evidence from the Spanish dynamic provisioning experiments." *Journal of Political Economy*, 125(6).

Khwaja, Asim Ijaz, and Atif Mian. 2008. "Tracing the Impact of Bank Liquidity Shocks: Evidence from an Emerging Market." *American Economic Review*, 98(4).

La Porta, Rafael, Florencio Lopez-de Silanes, and Andrei Shleifer. 2002. "Government Ownership of Banks." *The Journal of Finance*, 57(1).

Niepmann, Friederike, and Tim Schmidt-Eisenlohr. 2013. "Bank bailouts, international linkages, and cooperation." *American Economic Journal: Economic Policy*, 5(4).

Peter, Alessandra, Cian Ruane, et al. 2022. "The aggregate importance of intermediate input substitutability." *Princeton University, memo.*

Ruhl, Kim J, et al. 2008. "The international elasticity puzzle." *NYU Mimeo.*

Stein, Jeremy C. 1998. "An adverse-selection model of bank asset and liability management with implications for the transmission of monetary policy." *RAND Journal of Economics*, 29(3).

Wooldridge, Jeffrey M. 2009. "On estimating firm-level production functions using proxy variables to control for unobservables." *Economics letters*, 104(3).