# Supplemental Appendix
# Learning in the Marriage Market: The Economics of Dating

Yair Antler, Daniel Bird, and Daniel Fershtman

## B Alternative Learning Technology

In this appendix we analyze sorting patterns in a version of our model with the following learning technology: while a couple are dating, a good, fully revealing, signal arrives at a rate of $\lambda_g$ if the couple are compatible, whereas a bad, fully revealing, signal arrives at a rate of $\lambda_b$ if the couple are incompatible.[25] In the absence of any signal, a couple's belief evolves according to $\dot{q}_t = -(\lambda_g - \lambda_b)q_t(1 - q_t)$.

Our baseline model is a special case of this model where $\lambda_b = 0$. That assumption is for the sake of exposition and all of our results would remain valid so long as beliefs drift down conditional on no signal arriving, i.e., if $\lambda_g > \lambda_b$. In this appendix we consider the case where $\lambda_g < \lambda_b$, so that the belief is increasing in the absence of a signal, i.e., no news is good news. To simplify exposition, we assume that the rate at which marriages dissolve does not depend on compatibility, i.e., we assume that $\delta_{nc} = \delta$.

Under a no-news-is-good-news learning technology, conditional on no signal arriving, an agent's expected payoff from marrying the partner whom they are dating increases over time. By contrast, the value of separating and returning to the singles pool does not change over time. It follows that the only time at which an agent will voluntarily break up with the partner whom they are dating (and return to the singles market) is when a bad signal arrives. Hence, conditional on beginning to date, a couple's only choice is when to get married in the absence of a signal (if a signal arrives while a couple is dating, they will marry immediately if it is good and separate immediately if it is bad).

The strategy of agent $x$ therefore consists of choosing a set of agents that they are willing to begin dating and, for each such agent $y$, the duration of the dating period (in the absence of a signal) after which agent $x$ agrees to marry agent $y$. We denote this duration by $T_x^{gn}(y)$. We assume that marriage requires mutual consent, and therefore the (maximal) dating time

[25] The implicit assumption that a signal does not arrive after a couple marries is immaterial since a married couple will not separate before the dissolution shock even if they learn that they are incompatible. Intuitively, not only does marriage provide the maximal flow payoff of 1, but postponing the termination of a bad marriage also postpones the cost of dissolving the bad marriage.

before the couple $\langle x, y\rangle$ marry is $\max\{T_x^{gn}(y), T_y^{gn}(x)\}$.[26]

Let $W_s^{gn}(x)$ denote the continuation value of a single agent of type $x$ in this variant of the model. The continuation value for agent $x$ from dating agent $y$ for at most $T$ units of time, conditional on being *compatible*, is

$$\begin{aligned} V_c^{gn}(x,T;y) = &\int_0^T \lambda_g e^{-\lambda_g s}\left(-\frac{(1-e^{-rs})c}{r} + e^{-rs}\left(\frac{1}{r+\delta} + \frac{\delta W_s^{gn}(x)}{r+\delta}\right)\right) ds \\ &+ e^{-\lambda_g T}\left(-\frac{(1-e^{-rT})c}{r} + e^{-rT}\left(\frac{1}{r+\delta} + \frac{\delta W_s^{gn}(x)}{r+\delta}\right)\right). \end{aligned}$$

The first term in $V_c^{gn}(x,T;y)$ corresponds to the event that a good signal arrives before the couple has dated for $T$ units of time. In this case, the couple marry upon arrival of the signal, and the agent incurs the cost of dating up to that time, enjoys a flow payoff of 1 until the marriage is dissolved (which occurs at rate $\delta$), and then returns to the singles pool and receives a continuation value of $W_s^{gn}(x)$. The second term is similar, reflecting the event that a good signal did not arrive for $T$ units of time, at which point the couple marry anyway.

The continuation value for agent $x$ from dating agent $y$ for at most $T$ units of time, conditional on being *incompatible*, is

$$\begin{aligned} V_{nc}^{gn}(x,T;y) = &\int_0^T \lambda_b e^{-\lambda_b s}\left(-\frac{(1-e^{-rs})c}{r} + e^{-rs} W_s^{gn}(x)\right) ds \\ &+ e^{-\lambda_b T}\left(-\frac{(1-e^{-rT})c}{r} + e^{-rT}\left(\frac{1}{r+\delta} + \frac{\delta}{r+\delta}(W_s^{gn}(x) - Z)\right)\right). \end{aligned}$$

The first term corresponds to the event where a bad signal arrives before the couple has dated for $T$ units of time, at which point the agent returns to the single pool (incurring the flow cost of dating until that time). The second term corresponds to the event where a bad signal does not arrive for $T$ units of time. In this case the agent incurs the cost of dating for $T$ units of time, marries after dating for $T$ units of time and enjoys the flow payoff from marriage until the marriage is dissolved, and then returns to the singles pool and incurs the cost of dissolving an incompatible marriage.

The expected continuation value for agent $x$ from dating agent $y$ for at most $T$ units of time is therefore

$$V_d^{gn}(x,T;y) = q_0(x,y)V_c^{gn}(x,T;y) + (1-q_0(x,y))V_{nc}^{gn}(x,T;y).$$

[26] We have implicitly assumed that agents do not bargain over the duration of dating upon meeting. This reflects the fact that agents cannot credibly commit to a cap on the duration of dating.

Agent $x$'s choices of whether or not to date/marry agent $y$, are relevant only if agent $y$ chooses to date/marry agent $x$. As in the baseline model, we assume that all agents make their choices as if they were pivotal, that is, $T_x(y) \in argmax\{V_d^{gn}(x,T;y)\}$ for all $x, y \in X$. On the other hand, the choice of whom to begin dating depends on the strategy of each potential partner, i.e., $x$ chooses to date any $y$ for which $V_d^{gn}(x, \max\{T_x^{gn}(y), T_y^{gn}(x)\}; y) \geq W_s^{gn}(x)$.

The expected value from marriage is increasing in the belief about compatibility. Therefore, agent $x$'s choice of when to marry agent $y$ is given by a threshold belief, $q_m^\star(x)$, such that agent $x$ chooses to marry agent $y$ once $q_t(x,y) \geq q_m^\star(x)$. The following lemma characterizes the marriage threshold.

**Lemma B.1** *Agent x's marriage threshold is*

$$q_m^\star(x) = 1 - \frac{c\delta + r(c + \delta W_s^{gn}(x) + 1)}{\lambda_b(rW_s^{gn}(x) + \delta Z - 1) + \delta r Z}. \tag{B.1}$$

*Moreover, this threshold is decreasing in x if*

$$(c+1)\lambda_b(\delta + r) < Z\delta^2\left(r + \lambda_b\right). \tag{B.2}$$

**Proof.** Taking the derivative of $V_d^{gn}(x,T;y)$ with respect to $T$ yields

$$\frac{(q_0 - 1)e^{-T(\lambda_b + r)}(c\delta + r(c + (\delta - \lambda_b)W_s^{gn}(x) - \delta Z + 1) + \lambda_b(1 - \delta Z))}{\delta + r} - \frac{q_0 e^{-T(\lambda_g + r)}(c\delta + r(c + \delta W_s^{gn}(x) + 1))}{\delta + r}.$$

The optimal marriage threshold is given by equating this derivative to zero. Due to the agent's dynamic consistency, this is equivalent to asking for what $q_0$ is the above derivative equal to zero when evaluated at $T = 0$. That is, the threshold is given implicitly by

$$\frac{c\delta + r(c + \lambda_b(q_m^\star(x) - 1)W_s^{gn}(x) + \delta((q_m^\star(x) - 1)Z + W_s^{gn}(x)) + 1) + \lambda_b(q_m^\star(x) - 1)(\delta Z - 1)}{\delta + r} = 0.$$

Equation (B.1) is derived by solving the above equation.

Next we analyze the monotonicity of $q_m^\star(\cdot)$. From (B.1)

$$\frac{\partial q_m^\star(x)}{\partial x} = r\frac{(c+1)\lambda_b(\delta + r) - Z\delta^2\left(r + \lambda_b\right)}{(\lambda_b(rW_s^{gn}(x) + \delta Z - 1) + \delta r Z)^2}\frac{\partial W_s^{gn}(x)}{\partial x}.$$

The fraction in the above expression is negative under Condition (B.2). Moreover, by a mimicking argument analogous to that in the baseline model, $W_s^{gn}(x)$ is increasing in $x$. This establishes the second part of this result. ■

In what follows, we assume that Condition (B.2) holds. This occurs if the cost of dissolving a bad marriage, $Z$, is large enough. Under this condition, higher types will have a lower marriage threshold. Intuitively, while the expected payoff from marriage does not depend on an agent's type, high-type agents have a higher outside option of returning to the singles pool. Thus, high types have a higher cost of learning, and so they are willing to marry at a lower belief.

To understand the implications of this alternative learning technology on equilibrium sorting, it is necessary to consider not only when agents marry, but also which agents will start dating one another. Since marriage thresholds are decreasing in type, if agent $x$ is willing to begin dating agent $y$, they are also willing to date all agents $y' > y$ as well. This follows from the fact that the probability that agent $x$ is compatible with agent $y'$ is higher than the probability that agent $x$ is compatible with agent $y$, and because agent $y'$ is willing to marry at a lower belief than agent $y$. That is, the mutual agreement constraint for marriage is less stringent. The following lemma is an immediate consequence.

**Lemma B.2** *There exists a function $\underline{y} : X \to X$, such that the set of partners that agent $x$ is willing to date is the interval $[\underline{y}(x), 1]$.*

Using the above characterization of agents' dating and marriage behavior, the following result establishes that equilibria exhibit PAM.

**Proposition B.1** *Every steady-state equilibrium conversion-rate function satisfies the single-crossing property.*

**Proof.** Consider the agents $x''$ and $x'$, where $x'' > x'$. Let $y^\dagger$ be the infimum of $y \in [\underline{y}(x''), 1]$ such that $y$ is willing to date $x''$. For any $y < y^\dagger$, $\alpha(x'', y) = 0$. Thus, for $y \in A_\alpha(x', x'')$ such that $y < y^\dagger$, we have that $\alpha(x'', y) < \alpha(x', y)$.

Next, consider $y \geq y^\dagger$, since the marriage threshold is decreasing all couples $\langle x'', y \rangle$ start dating. Compatible couples marry with probability one. On the other hand, an incompatible couple $\langle x, y \rangle$ marry unless the bad signal arrives during the time it takes for their belief to increase from $q_0(x, y)$ to $\max\{q_m^\star(x), q_m^\star(y)\}$. Since $q_0(x, y)$ is increasing in $x$, and $q_m^\star(x)$ is decreasing in $x$, the corresponding interval of beliefs for the couple $\langle x'', y \rangle$ is a subset of the one for $\langle x', y \rangle$. It follows, that the couple $\langle x'', y \rangle$ is not only more likely to be compatible than the couple $\langle x', y\rangle$– in which case they marry with probability one – but is also more likely to marry if they are not compatible. Hence, for any $y \in A_\alpha(x', x'')$ such that $y > y^\dagger$, we have that $\alpha(x'', y) > \alpha(x', y)$. ■

To gain intuition for this result, consider two agents $x'' > x'$ and a potential partner $y$ such that both agents are willing to date $y$ and $y$ is willing to date each of them. First, observe that the couple $\langle x'', y\rangle$ dates for less time than the couple $\langle x', y\rangle$ since (i) $q_0(x'', y) > q_0(x', y)$ and (ii) the marriage threshold is lower for $\langle x'', y\rangle$ than for $\langle x', y\rangle$ (Lemma B.1). This means that, conditional on being incompatible, $\langle x'', y\rangle$ are less likely than $\langle x', y\rangle$ to encounter a bad signal that leads them to break up while dating, and so they are more likely to marry. Second, if a couple is compatible, then they marry with probability 1. As $\langle x'', y\rangle$ are more likely to be compatible than $\langle x', y\rangle$, we have that, overall, $x''$ is more likely than $x'$ to marry $y$. To conclude the argument, recall that any $y$ who is willing to start dating $x'$ is also willing to date $x''$ (Lemma B.2)

In contrast to the baseline model, equilibrium sorting exhibits PAM regardless of whether $q_0(\cdot, \cdot)$ is supermodular or not. The reason for this difference is that in the baseline model increasing attractiveness has a potentially ambiguous effect on the conversion rate. Namely, since the breakup threshold is increasing in attractiveness, agents with high attractiveness need not be more likely to marry than agents with low attractiveness. The assumption that $q_0(\cdot, \cdot)$ is supermodular guaranteed that the increase in the agents' breakup thresholds is not the dominant force.

In the baseline model we showed that efficient sorting may require NAM. This finding remains valid in the no-news-is-good-news setting. Intuitively, if couples learn their compatibility instantaneously, the specifics of the learning technology are immaterial. Hence, the argument used in Proposition 3 remains valid.

Formally, consider the binary setting studied in Section 3.3.2 in which no-news-is-good-news. In this setting, assume that for every couple $\langle x, y\rangle$ a social planner chooses the (stationary) probability with which the couple starts dating, $p(x, y)$, and the amount of time after which a dating couple will marry even if no signal arrives, $T^{gn}(x, y)$. Note that setting $T^{gn}(x, y) = 0$ for a couple that starts dating with positive probability is strictly suboptimal from a social perspective: not only do the couple receive a negative expected utility, but removing them from the singles pool increases the search time for other agents. Hence, the social planner chooses $p(\cdot, \cdot) \in [0, 1]$, and $T^{gn}(\cdot, \cdot) \in (0, \infty)$.

At the limit of $\lambda_b \to \infty$, for any fixed policy chosen by the social planner, a couple that starts dating will marry if and only if they are compatible. Hence, as $\lambda_b \to \infty$, the social planner can take $T^{gn}(\cdot,\cdot)$ to zero in order to avoid the cost of dating. Formally, the rate of convergence of $T^{gn}(\cdot,\cdot)$ must be slow enough so that $q_{T^{gn}(\cdot,\cdot)}(\cdot,\cdot)$ converges to one. Note, that at this double limit, $p(x,y)$ represents the probability with which a couple $\langle x,y\rangle$ will marry if they are compatible. It follows that at the limit the social planner is choosing the conversion rate, where the choice is restricted to $\alpha(\cdot,\cdot)\in[0,q_0(\cdot,\cdot)]$. Moreover, since at the limit of $T^{gn}(\cdot,\cdot)\to 0$ the time of dating converges to zero, by the same argument used in step 1 of Proposition 3 welfare is given by $(1-u(\ell)-u(h))$. The arguments used in steps 2 and 3 of Proposition 3 are valid, and give the following result.

**Proposition B.2** *Negative assortative matching is optimal under the same conditions derived in Proposition 3.*

## B.1 Vanishing Search Frictions

A key difference between the two learning technologies is observed at the limit where search frictions vanish. In particular, in contrast to the case where no-news-is-bad-news, vanishing search frictions do not lead to a dating apocalypse when no-news-is-good-news. The intuitive reason for this difference is that with a no-news-is-good-news learning technology, a couple $\langle x,y\rangle$ that have dated for a short amount of time without having received a signal have a *higher* belief about their compatibility than when they started dating. Thus, the fact that both $x$ and $y$ found dating preferable to continued search at belief $q_0(x,y)$, implies that they will have the same preference at any higher belief. This basic intuition, which is embedded in the use of marriage thresholds rather than breakup thresholds, implies that agents will not break up without receiving a bad signal, regardless of the meeting rate. As the probability of receiving a bad signal is bounded from above by $1-q_0(0,0)$, the expected number of partners agents date before marriage is bounded from above by $1/q_0(0,0)$.

An alternative interpretation of Proposition 4 is that maximal dating times converge to zero as search frictions vanish. To show that this is not the case under a no-news-is-good-news technology, we argue that marriage after dating for a very short amount of time without receiving a signal is suboptimal. Intuitively, the rate at which beliefs change (conditional on no signal arriving) is independent of the meeting rate. Thus, a couple that marries after a very short period of dating obtains a payoff that is close to the payoff from marriage with no learning at all, $\frac{q_0(x,y)-z(1-q_0(x,y))}{r+\delta}$, which is negative by assumption. Formally,

**Proposition B.3** *The time agents date before marriage without receiving a signal is bounded away from zero uniformly over $\mu$.*

**Proof.** Let $q^\dagger < 1$ denote the belief at which the expected value of marriage is zero, and note that by assumption $q^\dagger > q_0(1,1)$. By an analogous argument to the one used in Lemma 3, the amount of time it takes $q_t \in [q_0(0,0), \frac{q^\dagger + q_0(1,1)}{2}]$ to increase by $\epsilon < \frac{q^\dagger - q_0(1,1)}{2}$ (conditional on no signal arriving) is bounded from below by $K^{gn}\epsilon$, where $K^{gn} = \min_{q\in[q_0(0,0),q^\dagger]}\{\frac{1}{(\lambda_b-\lambda_g)q(1-q)}\}$.

By definition, marrying while $q_t < q_\dagger$ is dominated by separating. Since $q_0(\cdot,\cdot)$ is increasing and $q_0(1,1) < q^\dagger$, it follows that no couple will marry without observing a signal unless they have dated for at least $K^{gn}\frac{q^\dagger - q_0(1,1)}{2}$ units of time. ■

# C No Search Frictions

In our model there are both search and learning frictions. To better understand the implications of these frictions, it is constructive to compare our model to models in which only one of the frictions is relevant. Models of two-sided matching with search frictions (and no learning frictions) are abundant. In fact, this strand of the literature has tackled the main questions (i.e., equilibrium existence, sorting, and limiting behavior as search frictions vanish) studied in our paper, as discussed in the literature review. By contrast, to the best of our knowledge, there are no models of decentralized matching with time-consuming learning and no search frictions. Thus, in this appendix, we develop and analyze such a benchmark model and compare its results to the limit results we derived in Section 4.1.

We consider a marriage market that is identical to the model in the main text, except for the following changes. The market operates in discrete time – with periods of length $dt$ – and agents discount the future at a rate $e^{-rdt} < 1$. In each period, single agents are matched according to a stationary dating assignment matrix $\chi_{M,W}$, where $W$ (resp., $M$) is the set of women (resp., men) who are single at the beginning of that period and $\chi_{M,W}(m,w)$ is the probability that a man of type $m$ is matched to a woman of type $w$. We assume that the dating matrix is symmetric for men and women, and that, if $\chi_{M,W}(m,w) > 0$, the specific agents of types $m$ and $w$ are matched uniformly at random.

A matching $\nu_t : M_t \cup W_t \to M_t \cup W_t$ is a one-to-one correspondence such that a man is matched to a woman if and only if that woman is matched to him. We write $\nu(x) = y$ to denote that agent $x$ is matched to agent $y$ under $\mu$. Similarly, we write $\nu(x) = \emptyset$ to say that $x$ is single under $\nu$.

At the beginning of period $t$, the dating matrix induces a matching $\nu_t$. Let $\tilde{\nu}_t$ be a matching that extends $\nu_t$ as follows. The matching $\tilde{\nu}_t$ consists of all the matches in $\nu_t$ as well as all of the dating couples in $\tilde{\nu}_{t-1}$ who did not separate at the end of period $t-1$. We denote the latter set by $D_t$.

After the extended matching $\tilde{\nu}$ is formed, couples that are matched under $\tilde{\nu}$ observe a signal about their compatibility. Specifically, compatible couples observe a signal with probability $\lambda dt > 0$, whereas incompatible ones do not observe a signal. Couples who observe a signal learn that they are compatible and marry, whereas couples who do not observe a signal update their belief about their compatibility.

At the end of each period $t$, couples who did not observe a signal have to decide whether to separate or not. Each of them makes that choice independently, and if both choose not to separate, then they remain together (i.e., the couple also belongs to $D_{t+1}$). Otherwise, both agents start period $t+1$ as singles. As for married couples, at the end of each period, each such couple separates exogenously with probability $\delta dt > 0$, in which case both partners join the singles pool at the beginning of the next period.

*Agents' strategies.* We focus on stationary equilibria. That is, agents choose for how long to date each partner to whom they have been assigned.

*Solution concept.* A steady-state equilibrium is a dating matrix $\chi_{M,W}$ and a profile of agents' strategies such that:

- for any couple $\langle x, y \rangle$ that has been together for $\tau \geq 0$ periods:
  - There exists no single agent $i$ such that $q_0(x,i) > q_\tau(x,y)$.
  - There exists no couple $(x', y')$ that has been together for $\tau' \geq 0$ periods such that $q_0(x,y') > q_\tau(x,y)$ and $q_0(x,y') > q_{\tau'}(x',y')$.
- There are no two single agents who prefer to start dating each other.
- The size of the singles pool, and the distribution of attractiveness therein, are constant over time.

*Discussion of solution concept.* The solution concept captures the most basic idea in the two-sided matching literature: in equilibrium, there are no two agents that are not dating one another who can benefit from forming a couple. Note that while this model is dynamic, the probability of being compatible with a partner at a specific point is a sufficient statistic for one's preferences over partners. This results from the fact that: i) an agent's dating assignment in period $t$ is independent of their choices in the past, and ii) if agent $x$ prefers to pair up with an agent with attractiveness $y$ in a given period, it will be profitable for them to switch to new partners of that level of attractiveness in each subsequent period. Thus, an unmatched couple can benefit from pairing up if they are more likely to be compatible with each other than with their actual partners.

We now characterize the steady-state equilibrium of the benchmark model, and establish that the matching outcomes in the equilibrium of the baseline model converge to those of the benchmark model as search frictions vanish.

**Proposition C.1** *In the unique steady-state equilibrium, all agents date (and eventually marry) agents of their own type; i.e., there is perfect positive assortative matching.*

**Proof.** Consider a candidate profile of strategies and a dating matrix $\chi$ that matches each agent to an agent of the same type on the other side of the market. Suppose further that couples who do not observe a signal break up immediately. Agent $x$ does not benefit from leaving their assigned partner for an agent $y < x$ as $q_0(x, y) < q_0(x, x)$. Moreover, agent $x$ does not benefit from leaving their current partner for a different partner of type $x$. Thus, there are no blocking pairs. The balanced flow is satisfied for the measure of singles $u(x)$ for which

$$u(x)q_0(x, x)\lambda dt = (g(x) - u(x))\delta dt.$$

Hence, the profile and dating matrix we described constitute a steady-state equilibrium.

The next step of the proof shows that, in equilibrium, agents always break up after dating for one period if they do not observe a signal. Consider a couple $(m, w)$ who continue dating after not observing a signal. Since $q_1(m, w) < q_0(m, w)$, it follows that agent $m$ will form a blocking pair with another agent $w$ from a different $(m, w)$ dating couple who have dated for more than one period, in contradiction to the profile being part of an equilibrium.

Now suppose by way of contradiction that, in equilibrium, agents are assigned to agents of different types on the other side of the market with positive probability. Consider a woman $y$ who is assigned to a man of attractiveness $x < y$. By symmetry of the dating matrix, there exists a man of attractiveness $y$ who is assigned to a woman of attractiveness $x$. Hence, under the assignment matrix $\chi$, there are two couples, $(x, y)$ and $(y, x)$. Since $q_0(y, y) > q_0(x, y)$, woman $y$ (from the former couple) and man $y$ (from the latter couple) will form a blocking pair. ■