E-comment on "Naïve Herding in Rich-Information Settings"

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Abstract

This note identifies a gap in the proof of Proposition 4 in Eyster and Rabin (2010) and provides a correction that leaves the proposition's conclusion unchanged.

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Eyster and Rabin (2010) (hereafter ER) study naïve herding in a sequential social learning model with binary state $\omega \in \{0, 1\}$. Each player $t \in \mathbb{N}$ receives a private signal $S_t \in [0, 1]$, i.i.d. conditional on ω with a continuous density, and chooses an action $a_t \in [0, 1]$. In their Proposition 4, they show that under boundedly rational ("BRTNI") behavior, actions converge almost surely (a.s.) to 0 or 1. The proof defines[1]

$$X_t := 2^{-t} \ln\left(\frac{S_t}{1 - S_t}\right)$$

and shows that $2^{1-t} P_t = \sum_{\tau=1}^{t-1} X_\tau$ converges a.s. to a nonzero limit V_ω, where

$$P_t := \sum_{\tau=1}^{t-1} 2^{t-1-\tau} \ln\left(\frac{S_\tau}{1 - S_\tau}\right)$$

is the log-odds ratio of public beliefs at time t. This note points out a gap in that proof and provides a correction.[2] The conclusion of ER's Proposition 4, which we paraphrase below, is unchanged.

Proposition 4. *As $t \to \infty$, BRTNI actions $\{a_t\}_{t \in \mathbb{N}}$ converge a.s. to either 0 or 1.*

Player t's action satisfies $\ln\left(\frac{a_t}{1-a_t}\right) = P_t + \ln\left(\frac{S_t}{1-S_t}\right)$, so convergence of actions to 0 or 1 is equivalent to convergence of beliefs to 0 or 1.

The issue.

After establishing that $V_\omega := \sum_{\tau=1}^{\infty} X_\tau$ is a.s. well-defined in $\mathbb{R}$ for each state ω, ER's proof must show that $\mathbb{P}(V_\omega = 0 \mid \omega) = 0$. ER argue (p. 241): "Since $2^{1-t} P_t = 0$ if and only if $\ln\left(\frac{S_t}{1-S_t}\right) = -2P_{t-1}$ and $\ln\left(\frac{S_t}{1-S_t}\right)$ is atomless with negative mean when $\omega = 0$, this can happen for only finitely many t; hence, $2^{1-t} P_t$ converges a.s. to something other than zero." This argument has a gap: a sequence can be strictly nonzero for every t and still converge to 0.

[1] This is well-defined a.s. since $\mathbb{P}(S_t \in \{0, 1\}) = 0$ under the assumed continuous density.

[2] For completeness, I note a second, minor issue. ER verify one of the conditions of Kolmogorov's Three-Series Theorem by applying Chebyshev's inequality to show that $\sum_{\tau=1}^{\infty} \mathbb{P}(|X_\tau| \geq 1 \mid \omega) < \infty$. However, $\mathbb{E}\left[\ln\left(\frac{S}{1-S}\right) \mid \omega = 0\right] \neq 0$, so Chebyshev's inequality cannot be directly applied in that form. One can instead apply Markov's inequality to X_τ^2, obtaining the same summability conclusion using the second moment rather than the variance. Since ER already establish finiteness of the second moment in their proof of Proposition 3, the argument goes through unchanged.

THE CORRECTION.

Write $V_\omega = X_1 + W_\omega$, where $W_\omega := \sum_{t=2}^{\infty} X_t$. Conditional on ω, X_1 depends only on S_1 while W_ω depends only on $(S_t)_{t\geq 2}$, so X_1 and W_ω are independent. Since S_1 has a continuous density on $(0,1)$ and $x \mapsto \frac{1}{2}\ln(x/(1-x))$ is a continuous strictly monotone map, X_1 has an absolutely continuous distribution. Because V_ω is the sum of two independent random variables, one of which is absolutely absolutely continuous, its distribution conditional on ω is absolutely continuous. Consequently, $\mathbb{P}(V_\omega = 0 \mid \omega) = 0$.

To conclude that a_t converges a.s., note that

$$2^{1-t}\ln\left(\frac{a_t}{1-a_t}\right) = 2^{1-t}P_t + 2^{1-t}\ln\left(\frac{S_t}{1-S_t}\right) = 2^{1-t}P_t + 2X_t.$$

The term $2^{1-t}P_t$ converges to $V_\omega \neq 0$ a.s. Since $\sum_{t=1}^{\infty} X_t$ converges a.s., we have $X_t \to 0$ a.s., and hence $2X_t \to 0$ a.s. Therefore,

$$2^{1-t}\ln\left(\frac{a_t}{1-a_t}\right) \to V_\omega \neq 0$$

a.s. Since $2^{1-t} \to 0$, it follows that $|\ln(a_t/(1-a_t))| \to \infty$, and hence $a_t \to 0$ or $a_t \to 1$ a.s.

References

Eyster, Erik, and Matthew Rabin. 2010. "Naïve Herding in Rich-Information Settings." *American Economic Journal: Microeconomics*, 2(4): 221–243.