(Not-For-Publication) Online Appendix

Games on Multiplex Networks

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A Empirical evidence

A.1 Multiplexing in social comparisons

We provide evidence that social comparisons and conspicuous consumption operate across different layers, with individuals comparing themselves to distinct reference groups in each.

(i) Layer 1: Friends. Friendships and peer relationships significantly shape conspicuous consumption. Individuals adjust their consumption to signal status within their social circles. For instance, Liang et al. (2018) found that socially excluded individuals, especially those who are relationship oriented, engage in conspicuous consumption to regain social acceptance among friends. Similarly, Oh (2021) showed that subjective social class moderates the link between social self-esteem and conspicuous consumption.

(ii) Layer 2: Neighbors. Neighbors also influence conspicuous consumption. Agarwal et al. (2021) examine peer effects of bankruptcies, showing that individuals living in the same building as a bankrupt neighbor reduce their monthly debit and credit card spending by 3.4%. Agarwal et al. (2020) provide causal evidence that "keeping up with the Joneses" contributes to financial distress, finding that a 1% increase in lottery winnings raises the bankruptcy rate of close neighbors by 0.04%. Their findings suggest that lottery winners spend more on visible assets (e.g., houses, cars) than invisible ones (e.g., cash, financial assets), influencing neighbors' financial behavior. Similarly, Grinblatt et al. (2008) found that automobile purchases are influenced by recent neighbor purchases, particularly among geographically proximate individuals. Other studies confirm similar trends in household spending on visible goods when people are exposed to wealthier neighbors (Bertrand and Morse, 2016; Kuhn et al., 2011).

(iii) Layer 3: Family. Conspicuous consumption extends to family networks, including in low-income settings. Studies highlight large expenditures on weddings and festivals, even among impoverished households, emphasizing social signaling within extended families (Rao, 2001; Bloch et al., 2004).

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(iv) Layer 4: Coworkers. Workplace environments also drive conspicuous consumption. Muggleton et al. (2022) found that income inequality within firms leads employees to increase status-signaling expenditures. Additionally, De Giorgi et al. (2020) show that coworkers' spending habits shape individual consumption choices. Even in academia, social comparisons play a role—Merton (1968) describes the "Matthew effect," where scholars signal intellectual prestige through publications, citations, and affiliations.

(v) Layer 5: Same ethnicity or race. Research on conspicuous consumption and race reveals significant differences in spending patterns. Charles et al. (2009) found that Black and Hispanic households devote a larger share of their expenditures to visible goods—such as clothing, jewelry, and cars—than White households with similar incomes, consistent with status-signaling behavior. Specifically, Black and Hispanic households spend up to 30% more, or about $2,300 extra per year, on publicly observable items. These findings suggest that social and economic factors shape consumption choices across racial and ethnic groups. Further analysis (Ryabov, 2016) show that Hispanics residing in more affluent neighborhoods were prone to allocate greater shares of their expenditure to conspicuous goods.

A.2 Multiplexing in public goods

We provide here some real-world examples of situations where efforts are strategic substitutes and agents exert positive externalities on each others. A natural application is the provision of public goods. In such settings, individual efforts act as substitutes because the benefits of the good are shared among all contributors. This means that if one individual increases their contribution, others may reduce their own effort without significantly diminishing the total amount of the public good that is available. At the same time, contributing to a public good generates positive externality, as others in the network continue to benefit from it without having to increase their own contributions.

Consider a neighborhood park project. If one resident donates a large sum, others may feel less compelled to contribute but will still enjoy the park. Similarly, in a group project, if one team member takes the lead and completes most of the work, others may reduce their effort while still benefiting from the final outcome.[3] A similar pattern emerges in neighborhood watch programs. When a few residents take the lead in organizing themselves and patrolling the area, others may reduce their participation, trusting that safety is being maintained. Despite the uneven participation, overall neighborhood security improves, benefiting all residents. Environmental conservation efforts follow a similar pattern. When certain community members actively engage in clean-up drives and tree planting, others might participate less,

[3] We explicitly explore this example in Section 5.4 in a multilayer framework.

assuming that the environment is already being cared for. Nevertheless, the entire community benefits from a cleaner and greener environment. Finally, consider public libraries as local public goods. If a group of volunteers dedicates significant time to running literacy programs or maintaining the library space, other potential volunteers may feel less inclined to contribute, assuming that the existing efforts are sufficient. Yet, the entire community continues to enjoy the educational and informational resources provided by the library.

B Interiority of the equilibrium effort

Under Assumption 1, there exists a unique equilibrium. If, for each $s \in \mathcal{S}$, $\mathbf{x}^{s*}$ defined in (14) is interior, then $\mathbf{X}^* = (\mathbf{x}^{s*}, s \in \mathcal{S})$ must be a unique equilibrium by Proposition 2. To this end, we derive first-order approximations of the equilibrium objectives when the ϕ^ss are not too large.

For simplicity, assume $\mathbf{v}^s = \mathbf{0}$. In this case, we obtain the following Taylor expansions:

$$\begin{aligned}
\mathbf{M}^s &\approx \mathbf{I} - \phi^s \mathbf{G}^s, \\
\boldsymbol{\mu} &\approx \frac{1}{\bar{\alpha}} \left(\mathbf{I} + \sum_{t \in \mathcal{S}} \frac{\alpha^t}{\bar{\alpha}} \phi^t \mathbf{G}^t \right) \mathbf{T}, \\
\mathbf{x}^{s*} &\approx \frac{\alpha^s}{\bar{\alpha}} \left(\mathbf{I} - \phi^s \mathbf{G}^s + \sum_{t \in \mathcal{S}} \frac{\alpha^t}{\bar{\alpha}} \phi^t \mathbf{G}^t \right) \mathbf{T},
\end{aligned}$$

where $\bar{\alpha} = \sum_{s \in \mathcal{S}} \alpha^s > 0$. These approximations are correct up to quadratic or higher-order terms of ϕ^ss. When $\mathbf{T} = \mathbf{1}$, we can further simplify these expressions. For each $i \in \mathcal{N}$, we obtain

$$\mu_i \approx \frac{1}{\bar{\alpha}} \left(1 + \sum_{t \in \mathcal{S}} \frac{\alpha^t}{\bar{\alpha}} \phi^t d_i^t \right), \tag{B.1}$$

$$x_i^{s*} \approx \frac{\alpha^s}{\bar{\alpha}} \left(1 - \phi^s d_i^s + \sum_{t \in \mathcal{S}} \frac{\alpha^t}{\bar{\alpha}} \phi^t d_i^t \right), \tag{B.2}$$

where d_i^t is the degree of agent $i \in \mathcal{N}$ in layer $t \in \mathcal{S}$. These expressions are useful to see the effects of spillovers ϕ^s on the equilibrium payoff U_i^*, which has a one-to-one relationship to μ_i by Corollary 1, and equilibrium efforts.

In particular, when $\phi^s \approx 0$, for all s, from the above approximations, we see that $x_i^{s^*}$ and μ_i must all be positive. By continuity, this shows that the equilibrium efforts must be interior when the spillovers are not too large. In all the simulations, we verify that, indeed, the equilibrium efforts are always strictly positive (see Section 4.2).

C Inefficiency of equilibrium allocations

Consider the welfare function given by (22). An allocation $\hat{\mathbf{X}} \in K = \prod_{i\in\mathcal{N}} K_i$ is called efficient if $W(\hat{\mathbf{X}}) \geq W(\mathbf{X})$ for any $\mathbf{X} \in \prod_{i\in\mathcal{N}} K_i$. The efficient allocation exists and it must be unique given the strict concavity of $W(\cdot)$ and the convexity and compactness of the choice set K. Determining the first-order conditions with respect to $x_i^s, i \in \mathcal{N}, s \in \mathcal{S}$ leads to the next proposition:

Proposition C1. *An interior allocation $\hat{\mathbf{X}}$ is efficient if and only if it satisfies the following equality:*[4]

$$\frac{\alpha^s}{(v_i^s + x_i^s + \phi^s \sum_{j\in\mathcal{N}} g_{ij}^s x_j^s)} + \sum_{j\neq i} \frac{\alpha^s \phi^s g_{ji}^s}{(v_j^s + x_j^s + \phi^s \sum_{k\in\mathcal{N}} g_{jk}^s x_k^s)} = \hat{\lambda}_i, \quad \forall i \in \mathcal{N}, s \in \mathcal{S} \tag{C.1}$$

where $\hat{\lambda}_i$ is the multiplier of agent i's budget constraint.

Proof of Proposition C1: An interior efficient allocation $\mathbf{X}^*$ must satisfy the corresponding FOCs with equalities, which lead to the system of equations in (C.1). Furthermore, W is strictly concave in $\mathbf{X}$. Thus, a solution to FOCs must be globally optimal. □

Comparing the conditions for an equilibrium allocation and those for the efficient allocation gives the discrepancy between these two allocations as each agent i in equilibrium does not take into account the effects of x_i^s on other agents' payoff. Basically, the first-order condition for each agent i at the Nash equilibrium corresponds to (C.1) without the second term on the left-hand side of this equation (see (16) with $\mu_i := 1/\hat{\lambda}_i$). When ϕ^s is positive (negative), agent i underestimates (overestimates) the marginal welfare effects of x_i^s. Since the aggregate budget is fixed for each player i, the discrepancy boils down to the relative allocations across different layers. Given the wedge between different private returns of effort and the social return, $\mathbf{X}^*$ is unlikely to be efficient in general.

To obtain some intuition, we consider again regular networks (Section H.1).

Example C1. *Consider regular networks with $v_i^s = 0$. Then, the unique efficient allocation satisfies $\hat{x}_i^s = \hat{x}^s, \forall i \in \mathcal{N}$, where $\hat{x}^s = \frac{\alpha^s}{\sum_{t\in\mathcal{S}} \alpha^t} T$.*

In particular, for two layers $s \neq t$,

$$\frac{\hat{x}^s}{\hat{x}^t} = \frac{\alpha^s}{\alpha^t}. \tag{C.2}$$

The equilibrium allocation $\mathbf{X}^*$ (72) satisfies $x_i^s = x^{*s}, \forall i \in N, s \in \mathcal{S}$, where

$$\frac{x^{*s}}{x^{*t}} = \frac{\alpha^s/[1+\phi^s d^s]}{\alpha^t/[1+\phi^t d^t]}. \tag{C.3}$$

[4] The Lagrange multiplier λ_i associated with the Nash equilibrium differs from that of the efficient outcome, which we denote by $\hat{\lambda}_i$.

By comparing (C.2) and (C.3), we observe a discrepancy between x^{s*} and $\hat{x}^s$. Interestingly, the efficient allocation $\hat{\mathbf{X}}^s$ does not depend on the degrees d^s and the network effect parameters ϕ^s while the equilibrium allocation $\mathbf{X}^*$ depends on these two parameters.

D Conditions for uniqueness: Spectral condition vs constrained positive definiteness

We can use the bordered Hessian test to check the positive definiteness of quadratic forms subject to linear constraints. However, this method is challenging to apply due to the numerous principal minors that must be computed. Alternatively, we can simplify the process by eliminating the last layer using the budget constraints. By rewriting the quadratic form without any constraints, the required condition becomes the following ($\bar{s}$ is the total number of layers):

$$diag(\frac{1}{\alpha^1}(\mathbf{I}+\phi^1\mathbf{G}^1),\cdots,\frac{1}{\alpha^{\bar{s}-1}}(\mathbf{I}+\phi^{\bar{s}-1}\mathbf{G}^{\bar{s}-1}))+\mathbf{1}_{(\bar{s}-1)\times(\bar{s}-1)}\otimes\frac{1}{\alpha^{\bar{s}}}(\mathbf{I}+\phi^{\bar{s}}\mathbf{G}^{\bar{s}}) \text{ is positive definite.} \tag{D.1}$$

Here $\mathbf{1}_{(\bar{s}-1)\times(\bar{s}-1)}$ denotes a $(\bar{s}-1)\times(\bar{s}-1)$ matrix where every entry is 1, and $\otimes$ denotes the Kronecker product of two matrices.

With two layers, $\bar{s}=2$, and the above condition is equivalent to positive definiteness of

$$\frac{1}{\alpha^1}(\mathbf{I}+\phi^1\mathbf{G}^1)+\frac{1}{\alpha^2}(\mathbf{I}+\phi^2\mathbf{G}^2),$$

which is equivalent to

$$\left(\frac{1}{\alpha^1}+\frac{1}{\alpha^2}\right)+\lambda_{\min}(\frac{\phi^1}{\alpha^1}\mathbf{G}^1+\frac{\phi^2}{\alpha^2}\mathbf{G}^2)>0.$$

This condition is implied by the following:[5]

$$\frac{1}{\alpha^1}(1+\lambda_{\min}(\phi^1\mathbf{G}^1))+\frac{1}{\alpha^2}(1+\lambda_{\min}(\phi^2\mathbf{G}^2))>0,$$

which is further implied by

$$1+\lambda_{\min}(\phi^1\mathbf{G}^1)>0, \text{ and } 1+\lambda_{\min}(\phi^2\mathbf{G}^2)>0.$$

When there are more than two layers, $\bar{s}>2$, we do not have such a simple equivalent characterization of the condition in (D.1) for the two-layer case. Instead, we impose a sufficient condition: for instance, if for every $s\in\mathcal{S}$, $\frac{1}{\alpha^1}(\mathbf{I}+\phi^1\mathbf{G}^1)$ is positive definite, i.e., $1+\lambda_{\min}(\phi^s\mathbf{G}^s)>0$, then condition in (D.1) must hold. On the other hand, the following condition is necessary for (D.1):

$$\left(\frac{1}{\alpha^s}+\frac{1}{\alpha^t}\right)+\lambda_{\min}(\frac{\phi^s}{\alpha^s}\mathbf{G}^s+\frac{\phi^t}{\alpha^t}\mathbf{G}^t)>0,\quad\forall s\neq t\in\ S.$$

Unless there are only two layers, the above condition is in general not sufficient.

[5]For two square symmetric matrices $\mathbf{A}$ and $\mathbf{B}$: $\lambda_{\min}(\mathbf{A}+\mathbf{B})\geq\lambda_{\min}(\mathbf{A})+\lambda_{\min}(\mathbf{B})$, where $\lambda_{\min}(\cdot)$ denotes the smallest eigenvalue of the respective matrix. For any scalar b, $\lambda_{\min}(b\mathbf{I}+\mathbf{A})=b+\lambda_{\min}(\mathbf{A})$.

Remark D1. *A sufficient condition for Assumption 1 (or equivalently condition in (D.1)) is*

$$\kappa_1 > 0, \cdots, \kappa_{\bar{s}-1} > 0, \text{ and } \kappa_{\bar{s}} > -\frac{1}{\sum_{j=1}^{\bar{s}-1} \frac{1}{\kappa_j}},$$

where $\kappa_s := \frac{1}{\alpha^s}(1 + \lambda_{\min}(\phi^s \mathbf{G}^s))$ *for each* $s \in \mathcal{S}$. *In other words, the spectral condition holds for the first* $\bar{s} - 1$ *layers as* $\kappa_1 > 0, \cdots, \kappa_{\bar{s}-1} > 0$*, for the last layer* $\kappa_{\bar{s}}$ *can be negative, but should exceed a certain threshold depending on the other layers,* $\kappa_1, \cdots, \kappa_{\bar{s}-1}$*. We will apply this condition in the multiple public goods application.*

On the other hand, suppose that we require that Assumption 1 holds for all preference weights $\{\alpha^1 > 0, \cdots, \alpha^{\bar{s}} > 0\}$*, then it must be the case that for each* $s \in \mathcal{S}$, $1 + \lambda_{\min}(\phi^s \mathbf{G}^s) \geq 0$*. In other words, the spectral condition in (11) is* almost *necessary in some sense.*

Let us now illustrate this remark by providing applications for which Assumption 1 holds but not Proposition 1.

D.1 The case of Bergstrom et al. (1986) (BBV)

In the BBV's model, there are two layers. The first layer is the empty network,i.e., $\mathbf{G}^1 = \mathbf{0}$, and the second layer is a complete network with $\phi^2 = 1$. The spectral condition does not hold as $1 + \lambda_{\min}(\phi^2 \mathbf{G}^2) = 0$ as $\phi^2 = 1$, $\lambda_{min}(\mathbf{G}^2) = -1$. Let us use Remark D1 to obtain a weaker condition. $\kappa_1 = \frac{1}{\alpha^1}(1 + \lambda_{\min}(\phi^1 \mathbf{G}^1)) = \frac{1}{\alpha^1} > 0$, and $\kappa_2 = \frac{1}{\alpha^2}(1 + \lambda_{\min}(\phi^2 \mathbf{G}^2)) = 0$. However, the following holds:

$$\underbrace{\kappa_2}_{=0} > -\underbrace{\frac{1}{\frac{1}{\kappa_1}}}_{>0}.$$

Thus, by Remark D1, Assumption 1 holds in the BBV setting. This implies that, in BBV, Assumption 1 holds but not Proposition 1(ii).

D.2 The case of multiplexing in public goods

Take the example in Figure 2 in Section 4.2.2 of multiplexing in public goods with $\phi^1 = 0.2, \phi^2 = 0, \phi^3 = 1$. We claim that Assumption 1 is satisfied, but the spectral condition in Proposition 1(ii) is not met. Let us illustrate it using Remark D1. In that example, the first layer is a star network with $\lambda_{min}(\mathbf{G}^1) = -2$, so $\kappa_1 = \frac{1}{\alpha^1}(1 + \lambda_{\min}(\phi^1 \mathbf{G}^1)) = \frac{(1+0.2(-2))}{\alpha^1} > 0$. The second layer is an empty network, so $\lambda_{min}(\mathbf{G}^2) = 0$, and $\kappa_2 = \frac{1}{\alpha^2}(1 + \lambda_{\min}(\phi^2 \mathbf{G}^2)) = \frac{1}{\alpha^2} > 0$, For layer three, it is a complete graph so $\lambda_{min}(\mathbf{G}^3) = -1$, so $\kappa_3 = \frac{1}{\alpha^3}(1 + \lambda_{\min}(\phi^3 \mathbf{G}^3)) = 0$. However, the following holds:

$$\underbrace{\kappa_3}_{=0} > -\underbrace{\frac{1}{\frac{1}{\kappa_1} + \frac{1}{\kappa_2}}}_{>0}.$$

Thus, by Remark D1, Assumption 1 holds in this example but not Proposition 1(ii).

E Examples of equilibria with interior and corner solutions

E.1 Example of equilibrium with strictly positive efforts

We consider a multilayer network with $n = 3$ agents and $\bar{s} = 2$ layers. Each agent $i = 1, 2, 3$ has a time constraint $T_i = 1$, and allocates effort across the two network layers. The utility weights on each layer are $\alpha^1 = \alpha^2 = 0.5$. The externalities from neighbors' efforts are assumed to be $\phi^1 = 0.5$ (strategic substitutes and positive externality, thus a public good) and $\phi^2 = -0.3$ (strategic complements and negative externality, thus a conspicuous good). The endowments in each layer are equal to $v_i^s = 0.1$, for all i, s. The network in each layer is displayed in the following figure:

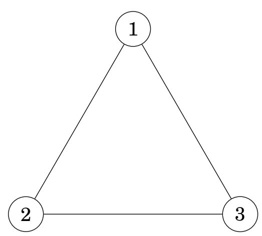

Figure E1: Layer 1: Fully Connected Triangle

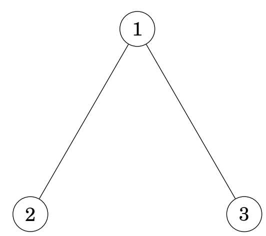

Figure E2: Layer 2: Star Network

Equilibrium Efforts. Since $1 + \lambda_{\min}(\phi^1 \mathbf{G}^1) = 0.5 > 0$ and $1 + \lambda_{\min}(\phi^2 \mathbf{G}^2) = 0.576 > 0$, there exists a unique equilibrium given by[6]

Agent	μ_i	$x_i^{1^*}$	$x_i^{2^*}$	$q_i^{1^*}$	$q_i^{2^*}$	T_i
1	1.0293	0.14638	0.85366	0.51463	0.51463	1
2	1.1512	0.26828	0.73172	0.57561	0.57561	1
3	1.1512	0.26828	0.73172	0.57561	0.57561	1

In this case, as well as in the one discussed below, the equilibrium profile is obtained by computing the (unique) global maximizer of the best-reply potential function θ; see Theorem 1 and Proposition 1.

All efforts are strictly positive, and each agent allocates their entire time endowment, normalized to $T_i = 1$. Since the spillover intensity is higher in layer 1 ($\phi^1 = 0.5$) than in layer 2 ($\phi^2 = -0.3$), free riding is more pronounced in layer 1, where efforts contribute to a public good. Consequently, agent 1 exerts the least effort in layer 1 and the most in layer 2, where she occupies a central position in this network.

[6]Remember that $q_i^s := v_i^s + x_i^s + \phi^s \sum_{j \in \mathcal{N}} g_{ij}^s x_j^s$.

E.2 Example of equilibrium with corner solutions

We consider the same 3-agent, 2-layer network setup as before, with the aim of illustrating a corner solution in which one agent exerts zero effort in the provision of the public good in layer 1. The parameters are identical to those in the previous section, except that we now assume $\phi^2 = -0.6$, instead of $\phi^2 = -0.3$, while maintaining $\phi^1 = 0.5$.

Equilibrium Efforts. Now, since $1+\lambda_{\min}(\phi^1 \mathbf{G}^1) = 0.5 > 0$ and $1+\lambda_{\min}(\phi^2 \mathbf{G}^2) = 0.152 > 0$, there exists a unique equilibrium given by[7]

Agent	μ_i	$x_i^{1^*}$	$x_i^{2^*}$	$q_i^{1^*}$	$q_i^{2^*}$	T_i
1	0.184	0	1	0.260	0.092	1
2	0.680	0.160	0.840	0.340	0.340	1
3	0.680	0.160	0.840	0.340	0.340	1

Changing only one parameter from $\phi^2 = -0.3$ to $\phi^2 = -0.6$—that is, increasing the strength of strategic complements in the star-shaped network (layer 2)—has a significant impact on equilibrium behavior. In particular, agent 1, who occupies the central position in the star network of layer 2, is induced to exert zero effort in layer 1 while substantially increasing their effort in layer 2. Although all agents respond similarly to the stronger incentives in layer 2, agent 1's central role in that layer amplifies their response. The intensified complementarity, combined with their centrality, results in a disproportionately large shift of effort toward layer 2. Consequently, the marginal return to effort in layer 1 for agent 1 drops below that of layer 2, leading to complete withdrawal from layer 1.

This result highlights how the structure of the network and the nature of strategic interactions (complements versus substitutes) jointly determine individual incentives. Moreover, this phenomenon arises due to the multilayer structure. Indeed, in a monolayer public good game, corner solutions naturally emerge (Bramoullé and Kranton, 2007), and increasing ϕ naturally leads to more corner solutions. In our two-layer framework, where layer 1 corresponds to a public good game with strategic substitutes in efforts and layer 2 to a conspicuous good game with strategic complements, increasing the strength of spillover effects in the non-public-good layer (layer 2) can cause a player to exert zero effort in the public good layer (layer 1). This occurs because of the effort arbitrage between the two layers and the fact that this player is highly central in layer 2 only.

Finally, observe that if we did not impose the constraint that negative solutions are not

[7] Negative efforts are not permitted here.

allowed, the unique equilibrium would exhibit very similar values and would be given by:

Agent	μ_i	$x_i^{1^*}$	$x_i^{2^*}$	$q_i^{1^*}$	$q_i^{2^*}$	T_i
1	0.3446	−0.08434	1.08434	0.17229	0.17229	1
2	0.5855	0.15663	0.84337	0.29277	0.29277	1
3	0.5855	0.15663	0.84337	0.29277	0.29277	1

Observe that even though the real effort $x_1^{1^*} = -0.08434$ is negative, the effective effort $q_1^{1^*}$ remains positive.

F Additional results on comparative statics exercises

We report comparative statics results with respect to all other parameters of the model, such as $\alpha^s, \phi^s, \mathbf{G}^s$ in the Theorem F1 below.

Theorem F1. *At an interior equilibrium $\mathbf{X}^*$, the following relationships hold*

(i) Effects of α^s:

$$\frac{\partial \boldsymbol{\mu}}{\partial \alpha^s} = -\left(\sum_{t\in\mathcal{S}} \alpha^t \mathbf{M}^t\right)^{-1} \mathbf{M}^s \boldsymbol{\mu}.$$

$$\frac{\partial \mathbf{x}^{s*}}{\partial \alpha^s} = \left[\mathbf{I}_n - \alpha^s \mathbf{M}^s \left(\sum_{t\in\mathcal{S}} \alpha^t \mathbf{M}^t\right)^{-1}\right] \mathbf{M}^s \boldsymbol{\mu}.$$

$$\frac{\partial \mathbf{x}^{s'*}}{\partial \alpha^s} = -\alpha^{s'} \mathbf{M}^{s'} \left(\sum_{t\in\mathcal{S}} \alpha^t \mathbf{M}^t\right)^{-1} \mathbf{M}^s \boldsymbol{\mu}, \text{ for } s' \neq s.$$

$$\frac{\partial \mathbf{U}^*}{\partial \alpha^s} = -\left(\sum_{t\in\mathcal{S}} \alpha^t\right) diag(1/\mu_1, \cdots, 1/\mu_n) \left(\sum_{t\in\mathcal{S}} \alpha^t \mathbf{M}^t\right)^{-1} \mathbf{M}^s \boldsymbol{\mu} + \ln(\boldsymbol{\mu}) + (\ln(\alpha^s) + 1)\mathbf{1}.$$

$$\frac{\partial W^*}{\partial \alpha^s} = -\left(\sum_{t\in\mathcal{S}} \alpha^t\right) (1/\mu_1, \cdots, 1/\mu_n) \left(\sum_{t\in\mathcal{S}} \alpha^t \mathbf{M}^t\right)^{-1} \mathbf{M}^s \boldsymbol{\mu} + \sum_{i\in\mathcal{N}} \ln(\mu_i) + n(\ln(\alpha^s) + 1).$$

(ii) *Effects of ϕ^s:*

$$\frac{\partial \boldsymbol{\mu}}{\partial \phi^s} = \left(\sum_{t\in\mathcal{S}} \alpha^t \mathbf{M}^t\right)^{-1} \mathbf{M}^s\mathbf{G}^s\mathbf{M}^s(\alpha^s\boldsymbol{\mu} - \mathbf{v}^s).$$

$$\frac{\partial \mathbf{x}^{s*}}{\partial \phi^s} = -\left[\mathbf{I}_n - \alpha^s\mathbf{M}^s\left(\sum_{t\in\mathcal{S}} \alpha^t \mathbf{M}^t\right)^{-1}\right] \mathbf{M}^s\mathbf{G}^s\mathbf{M}^s(\alpha^s\boldsymbol{\mu} - \mathbf{v}^s).$$

$$\frac{\partial \mathbf{x}^{s'*}}{\partial \phi^s} = \alpha^{s'}\mathbf{M}^{s'}\left(\sum_{t\in\mathcal{S}} \alpha^t \mathbf{M}^t\right)^{-1} \mathbf{M}^s\mathbf{G}^s\mathbf{M}^s(\alpha^s\boldsymbol{\mu} - \mathbf{v}^s), \text{ for } s' \neq s.$$

$$\frac{\partial \mathbf{U}^*}{\partial \phi^s} = \left(\sum_{t\in\mathcal{S}} \alpha^t\right) diag(1/\mu_1, \cdots, 1/\mu_n)\left(\sum_{t\in\mathcal{S}} \alpha^t \mathbf{M}^t\right)^{-1} \mathbf{M}^s\mathbf{G}^s\mathbf{M}^s(\alpha^s\boldsymbol{\mu} - \mathbf{v}^s).$$

$$\frac{\partial W^*}{\partial \phi^s} = \left(\sum_{t\in\mathcal{S}} \alpha^t\right) (1/\mu_1, \cdots, 1/\mu_n)\left(\sum_{t\in\mathcal{S}} \alpha^t \mathbf{M}^t\right)^{-1} \mathbf{M}^s\mathbf{G}^s\mathbf{M}^s(\alpha^s\boldsymbol{\mu} - \mathbf{v}^s).$$

(iii) *Effects of $\mathbf{G}^s$: For a given matrix $\boldsymbol{\eta}^s$, define the ϵ-perturbation of $\mathbf{G}^s$ with respect to $\boldsymbol{\eta}^s$ as $\mathbf{G}^s + \epsilon\boldsymbol{\eta}^s$.*[8] *Then:*

$$\left.\frac{\partial \boldsymbol{\mu}}{\partial \epsilon}\right|_{\epsilon=0} = \phi^s\left(\sum_{t\in\mathcal{S}} \alpha^t \mathbf{M}^t\right)^{-1} \mathbf{M}^s\boldsymbol{\eta}^s\mathbf{M}^s(\alpha^s\boldsymbol{\mu} - \mathbf{v}^s).$$

$$\left.\frac{\partial \mathbf{x}^{s*}}{\partial \epsilon}\right|_{\epsilon=0} = -\phi^s\left[\mathbf{I}_n - \alpha^s\mathbf{M}^s\left(\sum_{t\in\mathcal{S}} \alpha^t \mathbf{M}^t\right)^{-1}\right] \mathbf{M}^s\boldsymbol{\eta}^s\mathbf{M}^s(\alpha^s\boldsymbol{\mu} - \mathbf{v}^s).$$

$$\left.\frac{\partial \mathbf{x}^{s'*}}{\partial \epsilon}\right|_{\epsilon=0} = \phi^s\alpha^{s'}\mathbf{M}^{s'}\left(\sum_{t\in\mathcal{S}} \alpha^t \mathbf{M}^t\right)^{-1} \mathbf{M}^s\boldsymbol{\eta}^s\mathbf{M}^s(\alpha^s\boldsymbol{\mu} - \mathbf{v}^s), \text{ for } s' \neq s.$$

$$\left.\frac{\partial \mathbf{U}^*}{\partial \epsilon}\right|_{\epsilon=0} = \phi^s\left(\sum_{t\in\mathcal{S}} \alpha^t\right) diag(1/\mu_1, \cdots, 1/\mu_n)\left(\sum_{t\in\mathcal{S}} \alpha^t \mathbf{M}^t\right)^{-1} \mathbf{M}^s\boldsymbol{\eta}^s\mathbf{M}^s(\alpha^s\boldsymbol{\mu} - \mathbf{v}^s).$$

$$\left.\frac{\partial W^*}{\partial \epsilon}\right|_{\epsilon=0} = \phi^s\left(\sum_{t\in\mathcal{S}} \alpha^t\right) (1/\mu_1, \cdots, 1/\mu_n)\left(\sum_{t\in\mathcal{S}} \alpha^t \mathbf{M}^t\right)^{-1} \mathbf{M}^s\boldsymbol{\eta}^s\mathbf{M}^s(\alpha^s\boldsymbol{\mu} - \mathbf{v}^s).$$

This theorem parallels Theorem 2 in the main text, and its proof follows a similar approach. For each parameter of interest, we first obtain the comparative statics of $\boldsymbol{\mu}$ using (13). Next, we derive the equilibrium efforts using (14) and the equilibrium payoff using (15). Additionally, we frequently utilize the following identity for the differential of an inverse matrix: $d\mathbf{A}^{-1} = -\mathbf{A}^{-1}(d\mathbf{A})\mathbf{A}^{-1}$.

Proof of Theorem F1:

[8] Here, $\boldsymbol{\eta}^s$ denotes a nonnegative, symmetric $n \times n$ matrix with zeros on the diagonal. By continuity, the equilibrium profile is unique and interior when ϵ is sufficiently close to zero. Moreover, all equilibrium objectives are continuously differentiable in ϵ within an open neighborhood of zero.

Part (i):

$$\frac{\partial \boldsymbol{\mu}}{\partial \alpha^s} = -\left(\sum_{t\in\mathcal{S}} \alpha^t \mathbf{M}^t\right)^{-1} \mathbf{M}^s \left(\sum_{t\in\mathcal{S}} \alpha^t \mathbf{M}^t\right)^{-1} \left(\mathbf{T} + \sum_{t\in\mathcal{S}} \mathbf{M}^t \mathbf{v}^t\right) = -\left(\sum_{t\in\mathcal{S}} \alpha^t \mathbf{M}^t\right)^{-1} \mathbf{M}^s \boldsymbol{\mu}.$$

$$\frac{\partial \mathbf{x}^{s*}}{\partial \alpha^s} = \mathbf{M}^s \boldsymbol{\mu} + \alpha^s \mathbf{M}^s \frac{\partial \boldsymbol{\mu}}{\partial \alpha^s} = \left[\mathbf{I}_n - \alpha^s \mathbf{M}^s \left(\sum_{t\in\mathcal{S}} \alpha^t \mathbf{M}^t\right)^{-1}\right] \mathbf{M}^s \boldsymbol{\mu}.$$

$$\frac{\partial \mathbf{x}^{s'*}}{\partial \alpha^s} = \alpha^{s'} \mathbf{M}^{s'} \frac{\partial \boldsymbol{\mu}}{\partial \alpha^s} = -\alpha^{s'} \mathbf{M}^{s'} \left(\sum_{t\in\mathcal{S}} \alpha^t \mathbf{M}^t\right)^{-1} \mathbf{M}^s \boldsymbol{\mu}, \text{ for } s' \neq s.$$

$$\frac{\partial U_i^*}{\partial \alpha^s} = \ln(\mu_i) + \left(\sum_{t\in\mathcal{S}} \alpha^t\right) \frac{1}{\mu_i} \frac{\partial \mu_i}{\partial \alpha^s} + \ln(\alpha^s) + 1.$$

$$\frac{\partial \mathbf{U}^*}{\partial \alpha^s} = -\left(\sum_{t\in\mathcal{S}} \alpha^t\right) diag(1/\mu_1, \cdots, 1/\mu_n) \left(\sum_{t\in\mathcal{S}} \alpha^t \mathbf{M}^t\right)^{-1} \mathbf{M}^s \boldsymbol{\mu} + \ln(\boldsymbol{\mu}) + (\ln(\alpha^s) + 1)\mathbf{1}.$$

$$\frac{\partial W^*}{\partial \alpha^s} = -\left(\sum_{t\in\mathcal{S}} \alpha^t\right) (1/\mu_1, \cdots, 1/\mu_n) \left(\sum_{t\in\mathcal{S}} \alpha^t \mathbf{M}^t\right)^{-1} \mathbf{M}^s \boldsymbol{\mu} + \sum_{i\in\mathcal{N}} \ln(\mu_i) + n(\ln(\alpha^s) + 1).$$

Part (ii). First, we have

$$\frac{\partial \mathbf{M}^s}{\partial \phi^s} = -\mathbf{M}^s \mathbf{G}^s \mathbf{M}^s.$$

then

$$
\begin{aligned}
\frac{\partial \boldsymbol{\mu}}{\partial \phi^s} &= -\left(\sum_{t\in\mathcal{S}} \alpha^t \mathbf{M}^t\right)^{-1} (-\alpha^s \mathbf{M}^s \mathbf{G}^s \mathbf{M}^s) \left(\sum_{t\in\mathcal{S}} \alpha^t \mathbf{M}^t\right)^{-1} \left(\mathbf{T} + \sum_{t\in\mathcal{S}} \mathbf{M}^t \mathbf{v}^t\right) \\
&+ \left(\sum_{t\in\mathcal{S}} \alpha^t \mathbf{M}^t\right)^{-1} (-\mathbf{M}^s \mathbf{G}^s \mathbf{M}^s)\mathbf{v}^s \\
&= \left(\sum_{t\in\mathcal{S}} \alpha^t \mathbf{M}^t\right)^{-1} \mathbf{M}^s \mathbf{G}^s \mathbf{M}^s (\alpha^s \boldsymbol{\mu} - \mathbf{v}^s) \\
\frac{\partial \mathbf{x}^{s*}}{\partial \phi^s} &= (-\mathbf{M}^s \mathbf{G}^s \mathbf{M}^s)(\alpha^s \boldsymbol{\mu} - \mathbf{v}^s) + \alpha^s \mathbf{M}^s \frac{\partial \boldsymbol{\mu}}{\partial \phi^s} \\
&= -\left[\mathbf{I}_n - \alpha^s \mathbf{M}^s \left(\sum_{t\in\mathcal{S}} \alpha^t \mathbf{M}^t\right)^{-1}\right] \mathbf{M}^s \mathbf{G}^s \mathbf{M}^s (\alpha^s \boldsymbol{\mu} - \mathbf{v}^s) \\
\frac{\partial \mathbf{x}^{s'*}}{\partial \phi^s} &= \alpha^{s'} \mathbf{M}^{s'} \frac{\partial \boldsymbol{\mu}}{\partial \phi^s} = \alpha^{s'} \mathbf{M}^{s'} \left(\sum_{t\in\mathcal{S}} \alpha^t \mathbf{M}^t\right)^{-1} \mathbf{M}^s \mathbf{G}^s \mathbf{M}^s (\alpha^s \boldsymbol{\mu} - \mathbf{v}^s), \text{ for } s' \neq s \\
\frac{\partial \mathbf{U}^*}{\partial \phi^s} &= \left(\sum_{t\in\mathcal{S}} \alpha^t\right) diag(1/\mu_1, \cdots, 1/\mu_n) \frac{\partial \boldsymbol{\mu}}{\partial \phi^s} \\
&= \left(\sum_{t\in\mathcal{S}} \alpha^t\right) diag(1/\mu_1, \cdots, 1/\mu_n) \left(\sum_{t\in\mathcal{S}} \alpha^t \mathbf{M}^t\right)^{-1} \mathbf{M}^s \mathbf{G}^s \mathbf{M}^s (\alpha^s \boldsymbol{\mu} - \mathbf{v}^s) \\
\frac{\partial W^*}{\partial \phi^s} &= \left(\sum_{t\in\mathcal{S}} \alpha^t\right) (1/\mu_1, \cdots, 1/\mu_n) \left(\sum_{t\in\mathcal{S}} \alpha^t \mathbf{M}^t\right)^{-1} \mathbf{M}^s \mathbf{G}^s \mathbf{M}^s (\alpha^s \boldsymbol{\mu} - \mathbf{v}^s)
\end{aligned}
$$

Part (iii). First, we have:

$$
\left.\frac{\partial \mathbf{M}^s}{\partial \epsilon}\right|_{\epsilon=0} = -\phi^s \mathbf{M}^s \boldsymbol{\eta}^s \mathbf{M}^s.
$$

then

$$
\begin{aligned}
\left.\frac{\partial \boldsymbol{\mu}}{\partial \epsilon}\right|_{\epsilon=0} &= -\left(\sum_{t\in\mathcal{S}} \alpha^t \mathbf{M}^t\right)^{-1} (-\alpha^s \phi^s \mathbf{M}^s \boldsymbol{\eta}^s \mathbf{M}^s) \left(\sum_{t\in\mathcal{S}} \alpha^t \mathbf{M}^t\right)^{-1} \left(\mathbf{T} + \sum_{t\in\mathcal{S}} \mathbf{M}^t \mathbf{v}^t\right) \\
&+ \left(\sum_{t\in\mathcal{S}} \alpha^t \mathbf{M}^t\right)^{-1} (-\phi^s \mathbf{M}^s \boldsymbol{\eta}^s \mathbf{M}^s)\mathbf{v}^s \\
&= \phi^s \left(\sum_{t\in\mathcal{S}} \alpha^t \mathbf{M}^t\right)^{-1} \mathbf{M}^s \boldsymbol{\eta}^s \mathbf{M}^s (\alpha^s \boldsymbol{\mu} - \mathbf{v}^s). \\
\left.\frac{\partial \mathbf{x}^{s*}}{\partial \epsilon}\right|_{\epsilon=0} &= (-\phi^s \mathbf{M}^s \boldsymbol{\eta}^s \mathbf{M}^s)(\alpha^s \boldsymbol{\mu} - \mathbf{v}^s) + \alpha^s \mathbf{M}^s \frac{\partial \boldsymbol{\mu}}{\partial \phi^s} \\
&= -\phi^s \left[\mathbf{I}_n - \alpha^s \mathbf{M}^s \left(\sum_{t\in\mathcal{S}} \alpha^t \mathbf{M}^t\right)^{-1}\right] \mathbf{M}^s \boldsymbol{\eta}^s \mathbf{M}^s (\alpha^s \boldsymbol{\mu} - \mathbf{v}^s).
\end{aligned}
$$

$$\left.\frac{\partial \mathbf{x}^{s'*}}{\partial \epsilon}\right|_{\epsilon=0} = \alpha^{s'}\mathbf{M}^{s'}\frac{\partial \boldsymbol{\mu}}{\partial \phi^s} = \phi^s\alpha^{s'}\mathbf{M}^{s'}\left(\sum_{t\in\mathcal{S}}\alpha^t\mathbf{M}^t\right)^{-1}\mathbf{M}^s\boldsymbol{\eta}^s\mathbf{M}^s(\alpha^s\boldsymbol{\mu}-\mathbf{v}^s),\ \text{for } s'\neq s.$$

$$\begin{aligned}\left.\frac{\partial \mathbf{U}^*}{\partial \epsilon}\right|_{\epsilon=0} &= \left(\sum_{t\in\mathcal{S}}\alpha^t\right) diag(1/\mu_1,\cdots,1/\mu_n)\frac{\partial \boldsymbol{\mu}}{\partial \phi^s} \\ &= \phi^s\left(\sum_{t\in\mathcal{S}}\alpha^t\right) diag(1/\mu_1,\cdots,1/\mu_n)\left(\sum_{t\in\mathcal{S}}\alpha^t\mathbf{M}^t\right)^{-1}\mathbf{M}^s\boldsymbol{\eta}^s\mathbf{M}^s(\alpha^s\boldsymbol{\mu}-\mathbf{v}^s). \\ \left.\frac{\partial W^*}{\partial \epsilon}\right|_{\epsilon=0} &= \phi^s\left(\sum_{t\in\mathcal{S}}\alpha^t\right)(1/\mu_1,\cdots,1/\mu_n)\left(\sum_{t\in\mathcal{S}}\alpha^t\mathbf{M}^t\right)^{-1}\mathbf{M}^s\boldsymbol{\eta}^s\mathbf{M}^s(\alpha^s\boldsymbol{\mu}-\mathbf{v}^s).\end{aligned}$$

This completes the proof. □

G Extensions: Analytical details

In this section, we provide the analytical details of the four extensions presented in Section 5.

G.1 CES utility function

We generalize the Cobb–Douglas utility function in equation (1) to allow for a Constant Elasticity of Substitution (CES) aggregation of sub-utilities across layers, which is given by (36). Let us show that our main results are qualitatively the same under this more general specification. With the budget constraint, we have: $\sum_{s\in\mathcal{S}} x_i^s = T_i$. The Lagrangian is given by:

$$\mathcal{L} = \sum_{s\in\mathcal{S}}\alpha^s\frac{[(q_i^s)^\rho - 1]}{\rho} - \lambda_i\left(\sum_{s\in\mathcal{S}}x_i^s - T_i\right).$$

The first-order conditions are equal to

$$\frac{\partial U_i(\mathbf{x}_i,\mathbf{x}_{-i})}{\partial x_i^s} = \frac{\alpha^s}{(q_i^s)^{1-\rho}} - \lambda_i \leq 0\ \ (= 0 \text{ if } x_i^s > 0).$$

Using the SET transformation, we rewrite the above condition as

$$-\frac{q_i^s}{(\alpha^s)^{\frac{1}{1-\rho}}} + \mu_i \geq 0\ \ (= 0 \text{ if } x_i^s > 0).$$

where

$$\mu_i := \left(\frac{1}{\lambda_i}\right)^{\frac{1}{1-\rho}}.$$

Observe that μ_i decreases with λ_i. Define

$$\widetilde{\alpha}^s := (\alpha^s)^{\frac{1}{1-\rho}}.$$

Then, for an interior solution, we obtain

$$q_i^s = \widetilde{\alpha}^s \mu_i,$$

or equivalently

$$v_i^s + x_i^s + \phi^s \sum_{j \in \mathcal{N}} g_{ij}^s x_j^s = \widetilde{\alpha}^s \mu_i.$$

We can now obtain the same result as in the Cobb-Douglas utility case. Define the function θ on $K = \prod_i K_i$ as the best-reply potential of the multiplexity network game, with

$$-\theta(\mathbf{X}) = \sum_{s \in \mathcal{S}} \frac{1}{2\widetilde{\alpha}^s} \left\{ \sum_{i \in \mathcal{N}} (x_i^s)(2v_i^s + x_i^s + \phi^s \sum_{j \in \mathcal{N}} g_{ij}^s x_j^s) \right\} = \sum_{s \in \mathcal{S}} (\frac{1}{2\widetilde{\alpha}^s})(\mathbf{x}^s)'(2\mathbf{v}^s + (\mathbf{I}_n + \phi^s \mathbf{G}^s)\mathbf{x}^s).$$

Then, if $1 + \lambda_{\min}(\phi^s \mathbf{G}^s) > 0$, for all $s \in \mathcal{S}$, there exists a unique equilibrium. We can also characterize the equilibrium. We obtain

$$\mathbf{x}^s = (\mathbf{I} + \phi^s \mathbf{G})^{-1} (\widetilde{\alpha}^s \mu - \mathbf{v}^s),$$

where μ_i is determined by $\sum_{s \in \mathcal{S}} x_i^s = T_i$. This corresponds exactly to equation (14) where we replace $\widetilde{\alpha}^s$ by α^s.

Instead of the CES utility function (36), we can consider the CES utility function given by (38). The Lagrangian can be written as

$$\mathcal{L} = \left[\sum_{s \in \mathcal{S}} \alpha^s (q_i^s)^\rho \right]^{1/\rho} - \lambda_i \left(\sum_{s \in \mathcal{S}} x_i^s - T_i \right).$$

The first-order conditions are equal to

$$\frac{\partial U_i(\mathbf{x}_i, \mathbf{x}_{-i})}{\partial x_i^s} = \frac{1}{\rho} \left[\sum_{s \in \mathcal{S}} \alpha^s (q_i^s)^\rho \right]^{\frac{1}{\rho} - 1} \alpha^s \rho (q_i^s)^{\rho - 1} - \lambda_i \leq 0 \ \ (= 0 \text{ if } x_i^s > 0).$$

Using the SET transformation, we obtain

$$\frac{q_i^s}{(\alpha^s)^{\frac{1}{1-\rho}}} \geq \frac{\left[\sum_{s \in \mathcal{S}} \alpha^s (q_i^s)^\rho \right]^{\frac{1}{\rho}}}{(\lambda_i)^{\frac{1}{1-\rho}}} = \frac{U_i}{(\lambda_i)^{\frac{1}{1-\rho}}}.$$

Denote

$$\mu_i := U_i \left(\frac{1}{\lambda_i} \right)^{\frac{1}{1-\rho}}.$$

This leads to

$$\frac{q_i^s}{(\alpha^s)^{\frac{1}{1-\rho}}} - \mu_i \geq 0.$$

Define

$$\widetilde{\alpha}^s := (\alpha^s)^{\frac{1}{1-\rho}}.$$

Then, we obtain for interior solution

$$q_i^s = \widetilde{\alpha}^s \mu_i.$$

This corresponds exactly to equation (14) where we replace $\widetilde{\alpha}^s$ by α^s. The analysis is thus exactly the same as for the utility (36)

G.2 Convex costs

Consider the model for which each player's utility is given by (39), where the cost function is equal to (40).

Equilibrium uniqueness and best-reply potential Let us show that there exists a best-reply potential and that the equilibrium is unique. First, note that under the convex cost specification, each player's strategy space is a rectangular subset of $\mathbb{R}^{|\mathcal{S}|}$. This allows us to directly apply SET. In contrast, under a budget-type cost constraint, the strategy space becomes a simplex rather than a rectangle, except in the special case where there are only two layers.

Define

$$\mathcal{P}^C(\mathbf{X}) := \theta(\mathbf{X}) + \Omega^C(\mathbf{X}),$$

where the first term $\theta(\mathbf{X})$ is given by

$$\begin{aligned} -\theta(\mathbf{X}) &\equiv \sum_{s\in\mathcal{S}} \frac{1}{2\alpha^s} \left\{ \sum_{i\in\mathcal{N}} (x_i^s)(2v_i^s + x_i^s + \phi^s \sum_{j\in\mathcal{N}} g_{ij}^s x_j^s) \right\}. \\ &= \sum_{s\in\mathcal{S}} \frac{1}{\alpha^s} \left\{ (\mathbf{x}^s)'\mathbf{v}^s + \frac{1}{2}(\mathbf{x}^s)'(\mathbf{I}_n + \phi^s \mathbf{G}^s)\mathbf{x}^s \right\}. \end{aligned} \tag{G.1}$$

$\theta(\mathbf{X})$ is exactly the same as in the budget-constraint case and defined in the main text in (8). However, the second term

$$\Omega^C(\mathbf{X}) := \begin{cases} \sum_i \frac{1}{\gamma(1-\beta)} \left(\sum_{s\in\mathcal{S}} x_i^s\right)^{1-\beta}, & \text{if } \beta \neq 1 \text{ and } \beta \geq 0, \\ \sum_i \frac{1}{\gamma} \ln\left(\sum_{s\in\mathcal{S}} x_i^s\right), & \text{if } \beta = 1, \end{cases}$$

depends on the cost elasticity parameter β.

Proposition G2. *$\mathcal{P}^C(\cdot)$ is a best-reply potential of the game.*

Proof: Each player i's payoff U_i is strictly concave in $\mathbf{x}_i$. The first-order conditions at an equilibrium profile are given by

$$\frac{\partial U_i(\mathbf{X})}{\partial x_i^s} = \frac{\alpha^s}{q_i^s} - \gamma \left(\sum_{s\in\mathcal{S}} x_i^s\right)^{\beta} \leq 0 \quad \text{(with equality if } x_i^s > 0) \tag{G.2}$$

for each $s \in \mathcal{S}$.

Rewriting this system of first-order conditions as a variational inequality (VI) and applying SET using Zenou and Zhou (2025) yields the following equivalent system:

$$-\frac{q_i^s}{\alpha^s} + \frac{1}{\gamma\left(\sum_{s\in\mathcal{S}} x_i^s\right)^{\beta}} \leq 0 \quad \text{(with equality if } x_i^s > 0). \tag{G.3}$$

Therefore, the new system simplifies to

$$\frac{\partial \mathcal{P}^C(\mathbf{X})}{\partial x_i^s} \leq 0 \quad \text{(with equality if } x_i^s > 0\text{)}, \quad \forall s \in \mathcal{S}, \tag{G.4}$$

which is precisely the first-order condition system for maximizing this potential function in the first quadrant. In the final step, we use the observation that, for any $s \in \mathcal{S}$,

$$\frac{\partial \theta(\mathbf{X})}{\partial x_i^s} = -\frac{q_i^s}{\alpha^s} \quad \text{and} \quad \frac{\partial \Omega^C(\mathbf{X})}{\partial x_i^s} = \frac{1}{\gamma\left(\sum_{s\in\mathcal{S}} x_i^s\right)^\beta}. \tag{G.5}$$

In other words, fixing each $\mathbf{x}_{-i}$, we have

$$\arg\max_{\mathbf{x}_i \in \mathbb{R}_+^S} U_i(\mathbf{x}_i, \mathbf{x}_{-i}) = \arg\max_{\mathbf{x}_i \in \mathbb{R}_+^S} \mathcal{P}^C(\mathbf{x}_i, \mathbf{x}_{-i}).$$

This leads to the result in Theorem G2, and thus completes the proof. □

Now, we need to show that this best-reply potential is concave. We already know that the first term, $\theta(\mathbf{X})$, is concave under the spectral condition on each layer, i.e.,

$$1 + \lambda_{\min}(\phi^s \mathbf{G}^s) > 0, \quad \forall s \in \mathcal{S}. \tag{G.6}$$

For the second term, $\Omega^C(\mathbf{X})$ is always concave since each of its components is concave.[9]

Lemma 4. *$\mathcal{P}^C$ is strictly concave if, for each $s \in \mathcal{S}$, the spectral condition holds:*

$$1 + \lambda_{\min}(\phi^s \mathbf{G}^s) > 0. \tag{G.7}$$

This spectral condition is standard in the network literature, and we impose it on each layer. Furthermore, this sufficient condition does not depend on the cost parameter β or the cost shifter γ. In particular, for a fixed β, it may be possible to slightly relax the spectral condition by exploiting the concavity of the second term, $\Omega^C(\mathbf{X})$.

Remark G2. *The same techniques used to establish conditions for a unique equilibrium also apply to a more general convex cost function given by*

$$C_i = c_i\left(\sum_{s\in\mathcal{S}} x_i^s\right),$$

where, for each $i \in \mathcal{N}$, the function c_i satisfies $c_i'(t) > 0$ and $c_i''(t) \geq 0$ for all $t \geq 0$. Proposition G2 continues to hold if we redefine $\Omega^C(\mathbf{X})$ as follows:

$$\Omega^C(\mathbf{X}) := \sum_{i\in\mathcal{N}} \int_0^{T_i} \frac{dz}{c_i'(z)}, \quad \textit{where } T_i = \sum_{s\in\mathcal{S}} x_i^s. \tag{G.8}$$

It is straightforward to verify that $\Omega^C(\mathbf{X})$ is (weakly) concave in $\mathbf{X}$. This formulation accommodates both a broad class of convex cost functions and heterogeneity in costs across players.

[9] It suffices to show that each term of $\Omega^C(\mathbf{X})$ is concave. If $\beta > 1$, then $(1-\beta)z^{1-\beta}$ is concave in z because $z^{1-\beta}$ is convex in z and $1-\beta < 0$. If $0 \leq \beta < 1$, then $(1-\beta)z^{1-\beta}$ is also concave in z since $1-\beta > 0$ and $z^{1-\beta}$ is concave in z. If $\beta = 1$, then the corresponding term is $\ln z$, which is concave in z.

Properties/characterization of the equilibrium Consider the utility function:

$$U_i(\mathbf{x}_i, \mathbf{x}_{-i}) = \sum_{s\in S} \alpha^s \ln \left(v_i^s + x_i^s + \phi^s \sum_{j\in\mathcal{N}} g_{ij}^s x_j^s \right) - \frac{\gamma}{1+\beta} \left(\sum_{s\in S} x_i^s \right)^{1+\beta}.$$

Focusing on interior solutions, the first-order conditions yield

$$\frac{\alpha^s}{q_i^s} - \gamma \left(\sum_{s\in S} x_i^s \right)^{\beta} = 0,$$

where

$$\frac{\alpha^s}{q_i^s} := \frac{\alpha^s}{v_i^s + x_i^s + \phi^s \sum_{j\in\mathcal{N}} g_{ij}^s x_j^s}$$

is the *marginal benefit* of making effort x_i^s and

$$\lambda_i := \gamma \left(\sum_{s\in S} x_i^s \right)^{\beta} = \gamma (T_i)^{\beta} \tag{G.9}$$

is the *marginal cost* of making effort x_i^s. Define $\mu_i := 1/\lambda_i$ as the *inverse of the marginal cost*. Then, the first-order condition can be written as

$$\frac{\alpha^s}{q_i^s} = \frac{1}{\mu_i}.$$

That is, in equilibrium, we have

$$x_i^{s*} + \phi^s \sum_{j\in\mathcal{N}} g_{ij}^s x_j^{s*} = \alpha^s \mu_i^* - v_i^s.$$

In vector-matrix form:

$$\mathbf{x}^{s*} = (\mathbf{I} + \phi^s \mathbf{G})^{-1} (\alpha^s \boldsymbol{\mu}^* - \mathbf{v}^s) = \mathbf{M}^s (\alpha^s \boldsymbol{\mu}^* - \mathbf{v}^s).$$

Summing over s and using the fact that $\mathbf{T}$ denotes the vector of endogenous total efforts, with $T_i = \sum_s x_i^{s^*}$, yields

$$\sum_{s\in S} \mathbf{M}^s (\alpha^s \boldsymbol{\mu}^* - \mathbf{v}^s) = \sum_{s\in\mathcal{S}} \mathbf{x}^{s*} = \mathbf{T} = (\gamma)^{-\frac{1}{\beta}} (\boldsymbol{\mu}^*)^{-\frac{1}{\beta}},$$

where $\boldsymbol{\mu}^* = (\mu_1^*, ..., \mu_n^*)'$ is an is an $n-$dimensional column vector, and the exponentiation is elementwise. In the last equality we use the fact

$$\frac{1}{\mu_i^*} = \lambda_i = \gamma (T_i)^{\beta} \iff T_i = \left(\frac{1}{\gamma \mu_i^*} \right)^{\frac{1}{\beta}}.$$

Unlike the baseline model, now $\boldsymbol{\mu}^*$ is endogenous and implicitly given by the solution to the following system:

$$\sum_{s\in\mathcal{S}} \mathbf{M}^s (\alpha^s \boldsymbol{\mu}^* - \mathbf{v}^s) = \mathbf{T} = \begin{bmatrix} \left(\frac{1}{\gamma \mu_1^*} \right)^{1/\beta} \\ \vdots \\ \left(\frac{1}{\gamma \mu_n^*} \right)^{1/\beta} \end{bmatrix} \tag{G.10}$$

To identify a condition for the uniqueness of the solution to this μ-system, we construct another potential:

$$\phi(\boldsymbol{\mu}) := \sum_{s\in\mathcal{S}} (\boldsymbol{\mu})'\mathbf{M}^s(\frac{\alpha^s}{2}\boldsymbol{\mu} - \mathbf{v}^s) - \sum_{i\in\mathcal{N}} \left(\frac{1}{\gamma}\right)^{1/\beta} \left(\int_{\mu_i^0}^{\mu_i} z_i^{-1/\beta} dz_i\right).$$

This $\phi(\boldsymbol{\mu})$ has the required property that at an interior $\boldsymbol{\mu}$, $\nabla\phi = 0$ if and only if $\boldsymbol{\mu}$ solves the system (G.10).

Next, we verify that, under the spectral condition (G.7), $\phi(\boldsymbol{\mu})$ is strictly convex in $\boldsymbol{\mu}$. To see this, observe that the first term in $\phi(\boldsymbol{\mu})$ is strictly convex in $\boldsymbol{\mu}$, since its Hessian matrix, $\sum_s \alpha^s \mathbf{M}^s$, is positive definite under condition (G.7). The second term (without the minus sign) in $\phi(\boldsymbol{\mu})$ is clearly concave. Therefore, any interior solution to the system, if it exists, must be unique.

Assume that $\beta > 0$.[10] At any interior equilibrium, the payoff of agent i is given by

$$\begin{aligned} U_i^* &= \sum_{s\in\mathcal{S}} \alpha^s \ln(\alpha^s) + \left(\sum_{s\in\mathcal{S}} \alpha^s\right) \ln \mu_i^* - \frac{\gamma}{1+\beta} T_i^{1+\beta} \\ &= \sum_{s\in\mathcal{S}} \alpha^s \ln(\alpha^s) + \left(\sum_{s\in\mathcal{S}} \alpha^s\right) \ln \mu_i^* - \frac{1}{1+\beta} \cdot \frac{1}{\gamma^{1/\beta}(\mu_i^*)^{(1+\beta)/\beta}}. \end{aligned}$$

Observe that U_i^* is monotonic in μ_i^*, implying that for any two agents, $U_i^* \geq U_j^*$ if and only if $\mu_i^* \geq \mu_j^*$ in equilibrium, as in the case with a budget constraint.

G.3 The budget constraint model as a limiting case of the convex cost model

We can approximate the budget case as a limiting case of the convex cost model. Formally, define, for each agent i,

$$c_i(\mathbf{x}_i;\beta) := \frac{\gamma}{1+\beta}\left(\frac{\sum_{s\in\mathcal{S}} x_i^s}{T_i}\right)^{1+\beta},$$

where $\gamma > 0$ is a constant. For any $\beta > 0$, this cost is increasing and convex. It is easy to verify that in the limit the cost function exactly represents that in the budget case:

$$\lim_{\beta\to+\infty} c_i(\mathbf{x}_i;\beta) = \begin{cases} 0 & \text{if } \sum_{s\in\mathcal{S}} x_i^s \leq T_i \\ +\infty & \text{if } \sum_{s\in\mathcal{S}} x_i^s > T_i, \end{cases}$$

For each β, we obtain the best-reply potential function (see Section G.2) as

$$\mathcal{P}^C(\mathbf{X};\beta) := \theta(\mathbf{X}) + \Omega^C(\mathbf{X};\beta),$$

[10] In the case of $\beta = 0$, there is no linkage between layers, and we recover the setting of Bramoullé and Kranton (2007) in each layer independently.

where the first term $\theta(\mathbf{X})$ is defined in (G.1) and independent of β, and the second term (assuming $\beta > 1$) is

$$\Omega^C(\mathbf{X};\beta) = \sum_{i\in\mathcal{N}} \frac{1}{\gamma(1-\beta)} \left(\sum_{s\in\mathcal{S}} x_i^s\right)^{1-\beta} T_i^{1+\beta},$$

which depends on the cost elasticity parameter β. By definition, $\Omega^C(\mathbf{X};\beta) \leq 0$. Observe that, for each i,

$$\lim_{\beta\to+\infty} \frac{1}{\gamma(1-\beta)} \left(\sum_{s\in\mathcal{S}} x_i^s\right)^{1-\beta} T_i^{1+\beta} = \begin{cases} -\infty & \text{if } \sum_{s\in\mathcal{S}} x_i^s < T_i, \\ 0 & \text{if } \sum_{s\in\mathcal{S}} x_i^s \geq T_i. \end{cases}$$

In particular, if $\mathbf{X} \in K$, we have $\sum_{s\in\mathcal{S}} x_i^s = T_i$ for each i, and

$$\lim_{\beta\to+\infty} \Omega^C(\mathbf{X};\beta) = 0, \tag{G.11}$$

Assume the spectral conditions in (G.6) hold, so that for each β, $\mathcal{P}^C(\mathbf{X};\beta)$ is strictly concave, and, thus, the global maximizer of $\mathcal{P}^C(\mathbf{X};\beta)$ is unique, which we denote by $\mathbf{X}^*(\beta)$, i.e.,

$$\max_{\mathbf{X}\geq\mathbf{0}} \mathcal{P}^C(\mathbf{X};\beta) = \mathcal{P}^C(\mathbf{X}^*(\beta);\beta).$$

We can show that $\{\mathbf{X}^*(\beta), \beta > 1\}$ is uniformly bounded. Define $\lim_{\beta\to+\infty} \mathbf{X}^*(\beta) = \bar{\mathbf{X}}^*$.

Similarly, $\theta(\cdot)$ is strictly concave in $K = \prod_i K_i$ (recalling that K_i is the budget of agent i) and let $\mathbf{X}^* = (\mathbf{x}_1^*, \cdots, \mathbf{x}_n^*)$ denote its unique maximizer over K, i.e.,

$$\max_{\mathbf{X}\in K} \theta(\mathbf{X}) = \theta(\mathbf{X}^*).$$

Note that for each β, the feasible set of $\mathcal{P}^C(\mathbf{X};\beta)$ is larger than K, the feasible set of $\theta(\cdot)$.

Lemma 5.

$$\bar{\mathbf{X}}^* = \mathbf{X}^*.$$

This lemma follows directly from the following two results:

(i) $\bar{\mathbf{X}}^*$ is in K;

(ii) $\bar{\mathbf{X}}^*$ is the unique maximizer of $\theta(\cdot)$ in K, i.e.,

$$\theta(\bar{\mathbf{X}}^*) \geq \theta(\mathbf{X}), \quad \forall \mathbf{X} \in K \tag{G.12}$$

Proof of Lemma 5. Part (i). For each agent i, we need to rule out case (a) $\sum_{s\in\mathcal{S}} \bar{x}_i^{s^*} > T_i$ and (b) $\sum_{s\in\mathcal{S}} \bar{x}_i^{s^*} < T_i$.

In the former case, for sufficiently small $\epsilon > 0$, $\exists\hat{\beta} > 0$ such that, for any $\beta > \hat{\beta}$

$$\sum_{s\in\mathcal{S}} x_i^{s*}(\beta) \geq \sum_{s\in\mathcal{S}} \bar{x}_i^{s*} - \epsilon > T_i$$

$$q_i^{s*}(\beta) \geq \bar{q}_i^{s*} - \epsilon > 0, \quad \text{for any } s \in \mathcal{S}.$$

Then we have

$$\left.\frac{\partial \mathcal{P}^C}{\partial x_i^s}\right|_{(\mathbf{X}^*(\beta),\beta)} = -\frac{q_i^{s*}(\beta)}{\alpha^s} + \frac{T_i^{1+\beta}}{\gamma(\sum_{s\in\mathcal{S}} x_i^{s*}(\beta))^\beta} \leq -\frac{\bar{q}_i^{s*} - \epsilon}{\alpha^s} + \frac{T_i^{1+\beta}}{\gamma(\sum_{s\in\mathcal{S}} \bar{x}_i^{s*} - \epsilon)^\beta} < 0 \qquad \text{(G.13)}$$

for $\beta > \max\{\hat{\beta}, \beta^+\}$ and any $s \in \mathcal{S}$, where

$$\beta^+ = \max_{s\in\mathcal{S}} \frac{\ln\left(\frac{\gamma(\bar{q}_i^{s*}-\epsilon)}{\alpha^s T_i}\right)}{\ln\left(\frac{T_i}{(\sum_{s\in\mathcal{S}} \bar{x}_i^{s*}-\epsilon)}\right)}.$$

Note that $\mathbf{X}^*(\beta)$ is the global maximizer of $\mathcal{P}^C(\mathbf{X};\beta)$, which is strictly concave and equation (G.13) implies $x_i^{s*}(\beta) = 0$ for any $s \in \mathcal{S}$, which contradicts $\sum_{s\in\mathcal{S}} x_i^{s*}(\beta) > T_i > 0$.

In the latter case, for sufficiently small $\epsilon > 0$, $\exists \tilde{\beta} > 0$ such that, for any $\beta > \tilde{\beta}$

$$\sum_{s\in\mathcal{S}} x_i^{s*}(\beta) \leq \sum_{s\in\mathcal{S}} \bar{x}_i^{s*} + \epsilon < T_i$$

$$q_i^{s*}(\beta) \leq \bar{q}_i^{s*} + \epsilon, \quad \text{for any } s \in \mathcal{S}.$$

Then we have

$$\left.\frac{\partial \mathcal{P}^C}{\partial x_i^s}\right|_{(\mathbf{X}^*(\beta),\beta)} = -\frac{q_i^{s*}(\beta)}{\alpha^s} + \frac{T_i^{1+\beta}}{\gamma(\sum_{s\in\mathcal{S}} x_i^{s*}(\beta))^\beta} \geq -\frac{\bar{q}_i^{s*} + \epsilon}{\alpha^s} + \frac{T_i^{1+\beta}}{\gamma(\sum_{s\in\mathcal{S}} \bar{x}_i^{s*} + \epsilon)^\beta} > 0 \qquad \text{(G.14)}$$

for $\beta > \max\{\tilde{\beta}, \beta^{++}\}$ and any $s \in \mathcal{S}$, where

$$\beta^{++} = \max_{s\in\mathcal{S}} \frac{\ln\left(\frac{\gamma(\bar{q}_i^{s*}+\epsilon)}{\alpha^s T_i}\right)}{\ln\left(\frac{T_i}{(\sum_{s\in\mathcal{S}} \bar{x}_i^{s*}+\epsilon)}\right)}.$$

However, Equation (G.14) contradicts that $\mathbf{X}^*(\beta)$ is the global maximizer of $\mathcal{P}^C(\mathbf{X};\beta)$.

Therefore, both cases are ruled out.

Part (ii). Take any $\mathbf{X} \in K$. For each β, we have

$$\theta(\mathbf{X}^*(\beta)) \geq \theta(\mathbf{X}^*(\beta)) + \Omega^C(\mathbf{X}^*(\beta);\beta) = \mathcal{P}^C(\mathbf{X}^*(\beta);\beta) \geq \mathcal{P}^C(\mathbf{X};\beta) = \theta(\mathbf{X}) + \Omega^C(\mathbf{X};\beta).$$

The first inequality follows from the non-positivity of Ω^C, and the second inequality follows from the optimality of $\mathbf{X}^*(\beta)$. Therefore, for any β, we have

$$\theta(\mathbf{X}^*(\beta)) \geq \theta(\mathbf{X}) + \Omega^C(\mathbf{X};\beta).$$

Taking the limit as $\beta \to +\infty$ and using $\lim_{\beta\to+\infty} \Omega^C(\mathbf{X};\beta) = 0$ by (G.11), we obtain (G.12). □

G.4 Hypergraphs/multilateral interactions within each layer with an application to teams

As stated in Section 5.4, we consider a model of teams in organizations. There are $t \in \mathcal{T}$ teams in an organization, where each team t consists of a subset $\mathcal{N}^t \subseteq \mathcal{N}$ of agents. Each team forms a clique. Preferences are team-specific rather than layer-specific and are denoted by α^t.

Consider the utility function given by (44). Each agent i maximizes subject to the budget constraint $\sum_{t\in\mathcal{T}_i} x_i^t = B_i$. We have the following result:

Proposition G3.

(i) $\mathbf{X}$ *is a Nash equilibrium if and only if*

$$\mathbf{X} \in \arg\min_{\mathbf{X}\in\mathbf{F}} \theta(\mathbf{X}),$$

where

$$\theta(\mathbf{X}) \equiv \sum_{t\in\mathcal{T}} \frac{(y^t)^2}{\alpha^t} = \sum_{t\in\mathcal{T}} \frac{1}{\alpha^t} \left(\sum_{j\in\mathcal{N}^t} x_j^t \right)^2, \tag{G.15}$$

and $\mathbf{F}$ *is the set of* $\mathbf{X}$ *that satisfies the following feasibility conditions:*

$$\begin{cases} \mathbf{X} \geq \mathbf{0}, \\ \sum_{t\in\mathcal{T}_i} x_i^t = B_i, \quad \forall i \in \mathcal{N}. \end{cases} \tag{G.16}$$

(ii) Multiple equilibria may arise. However, the equilibrium team effort y^{t*} *is unique for every* $t \in \mathcal{T}$*, and consequently, equilibrium payoffs are also unique.*

This characterization of the equilibrium using a best-reply potential reveals several interesting observations. First, the set of equilibria may not be unique, but it forms a convex subset of the feasible set $\mathbf{F}$. Second, equilibrium team efforts are unique, which facilitates subsequent welfare analysis—such as examining the effects of changes in team structure (e.g., adding or removing members), the creation of new projects, or the inclusion of additional agents.

Remark G3. *The uniqueness of equilibrium fails in this case. The smallest eigenvalue of a clique is* -1*, and we have assumed* $\phi = 1$*. Consequently, the spectral condition* $1 + \lambda_{\min}(\phi^s \mathbf{G}^s) = 1 - 1 = 0$ *is exactly binding and therefore not strict.*

Assume an interior equilibrium and denote by λ_i the Lagrange multiplier. The first-order conditions are given by:

$$\frac{\partial u_i}{\partial x_i^t} = \frac{\alpha^t}{y^t} = \lambda_i, \quad \forall t \in \mathcal{T}_i.$$

As before, define $\mu_i = 1/\lambda_i$. Then, for any $t \in \mathcal{T}_i$, this implies:

$$y^t = \mu_i \alpha^t.$$

This further implies that if i and j are directly connected (i.e., belong to the same team, $i, j \in \mathcal{N}^t$), then $\mu_i = \mu_j$. If every pair of agents is directly or indirectly connected within a layer, then $\mu_i = \mu_j = \mu$. As mentioned above, since there may be multiple disconnected teams in each layer, the network $\mathbf{G}^s$ may not be connected for some s. In that case, we have:

$$y^t = \mu \alpha^t,$$

where $\mu_i = \mu_j = \mu$ for all i, j connected in the same component.

On the other hand, by a simple counting argument, we obtain:

$$\sum_{i \in \mathcal{N}} B_i = \sum_{i \in \mathcal{N}} \sum_{t \in \mathcal{T}_i} x_i^t = \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{N}^t} x_i^t = \sum_{t \in \mathcal{T}} y^t = \mu \left(\sum_{t \in \mathcal{T}} \alpha^t \right).$$

In other words,

$$\mu = \frac{\sum_{i \in \mathcal{N}} B_i}{\sum_{t \in \mathcal{T}} \alpha^t} = \frac{B}{\alpha}, \tag{G.17}$$

where $B \equiv \sum_{i \in \mathcal{N}} B_i$ and $\alpha \equiv \sum_{t \in \mathcal{T}} \alpha^t$.

Each player's equilibrium payoff is thus fully determined and given by:

$$U_i^* = \sum_{t \in \mathcal{T}_i} \alpha^t \ln y^{t*} = \sum_{t \in \mathcal{T}_i} \alpha^t \ln(\mu \alpha^t) = \left(\sum_{t \in \mathcal{T}_i} \alpha^t \ln \alpha^t \right) + \left(\sum_{t \in \mathcal{T}_i} \alpha^t \right) \ln \mu. \tag{G.18}$$

Unlike the baseline case, having a common μ does not imply that all players receive the same equilibrium payoff.

To provide further intuition, consider the following example:

Example G2. *Consider the networks displayed below in Figure 3 for layers A and B. We assume the following α values for each agent:*

$$\boldsymbol{\alpha}^A = \begin{bmatrix} \alpha^1 \\ \alpha^1 \\ \alpha^1 \\ \alpha^2 \\ \alpha^2 \end{bmatrix}, \quad \boldsymbol{\alpha}^B = \begin{bmatrix} \alpha^4 \\ \alpha^3 \\ \alpha^3 \\ \alpha^4 \\ \alpha^4 \end{bmatrix}, \tag{G.19}$$

where $\alpha^1, \alpha^2, \alpha^3, \alpha^4$ are all positive. Note that we allow for heterogeneous values of α_i^s, but these still satisfy the integrability condition in (43).

We observe that there are two distinct teams in each layer, each forming a clique, and that agents participate in different teams across layers. For example, agent 1 works with agents 2 *and 3 in layer 1, but with agents 4 and 5 in layer 2. Moreover, the α values are heterogeneous within each layer but identical within a team.*[11]

Note that we can give an alternative representation of this two-layer network using a hypergraph. It is displayed in Figure G3.

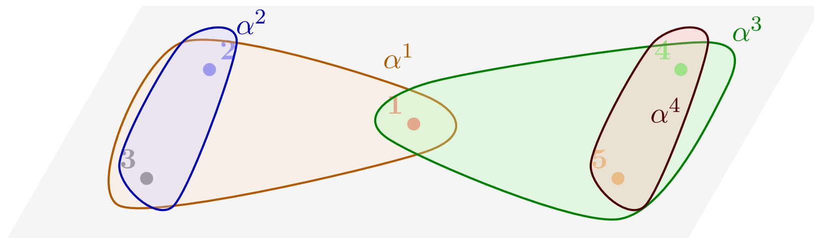

Figure G3: Hyper-graph

The question we address is how each agent allocates their effort across teams, given their time or budget constraint and the fact that, within any given team, all members occupy symmetric positions because each team network is complete; heterogeneity arises instead from team sizes, memberships, and the α^ts.

Let us calculate the equilibrium in this two-layer network team example. The adjacency matrices are given by:

$$\mathbf{G}^A = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad \mathbf{G}^B = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{G.20}$$

In this example, the equilibrium action profile may not be unique, but the equilibrium payoffs are unique. Indeed, we have:

$$\begin{cases} y^1 = x_1^1 + x_2^1 + x_3^1 = \alpha^1 \mu, \\ y^2 = x_4^1 + x_5^1 = \alpha^2 \mu, \\ y^3 = x_2^2 + x_3^2 = \alpha^3 \mu, \\ y^4 = x_1^2 + x_4^2 + x_5^2 = \alpha^4 \mu, \end{cases}$$

[11]We could interpret Figure 3 with four layers instead of two, which would be equivalent from a pure technical viewpoint (integrability is preserved). However, the economic interpretation would be different. In our Figure 3 with two layers, all five agents are active in both layers and are involved in different projects. In this case, the team or co-author example works well. In this new interpretation with four layers, each agent is only present in two layers out of four and thus it illustrates something very different.

where

$$\mu = \frac{B_1 + \cdots + B_5}{\alpha^1 + \alpha^2 + \alpha^3 + \alpha^4}.$$

In equilibrium, the payoffs are:

$$\begin{cases} U_1^* = \alpha^1 \ln(\alpha^1 \mu) + \alpha^4 \ln(\alpha^4 \mu), \\ U_2^* = U_3^* = \alpha^1 \ln(\alpha^1 \mu) + \alpha^3 \ln(\alpha^3 \mu), \\ U_4^* = U_5^* = \alpha^2 \ln(\alpha^2 \mu) + \alpha^4 \ln(\alpha^4 \mu). \end{cases}$$

When $\alpha^1 = \alpha^2$ and $\alpha^3 = \alpha^4$ (as in the baseline model where agents in the same layer have the same α), all players receive the same payoff. However, when $\alpha^1 \neq \alpha^2$, the payoffs of agents 1 and 4 can differ. Similarly, when $\alpha^3 \neq \alpha^4$, the payoffs of agents 1 and 2 can differ.

The equilibrium action profile is not unique because the rank of the following vectors:

$$\begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

is 3, which is less than the dimension of the space (5). However, we can uniquely determine the allocation of agent 1:

$$x_1^1 = y^1 + y^3 - (B_2 + B_3), \quad x_1^2 = y^2 + y^4 - (B_4 + B_5).$$

We cannot exactly pin down the allocation of agents 2 or 3, but the sum of their efforts in each project is uniquely determined. For example,

$$x_2^2 + x_3^2 = \alpha^3 \mu = y^3 \quad \textit{and} \quad x_2^1 + x_3^1 = (B_2 + B_3) - y^3 = y^1 - x_1^1.$$

To examine the effect of α more explicitly, let us take $B_i = 2$ for each i and define:

$$\alpha^1 = \alpha^4 = w, \quad \alpha^2 = \alpha^3 = 1 - w, \quad \textit{where } w \in (0, 1).$$

Then, we have:

- *If $0.2 < w < 1$, then $\mu = \mu_i = 5$, and* [12]

  $$y^1 = y^4 = 5w, \quad y^2 = y^3 = 5(1 - w).$$

  The equilibrium payoffs are:

  $$U_1^* = 2w \ln(5w), \tag{G.21}$$
  $$U_2^* = w \ln(5w) + (1 - w) \ln(5(1 - w)). \tag{G.22}$$

[12]Note that y^3 cannot exceed the total budget of agents 2 and 3, i.e., $5(1 - w) \leq 2 + 2 = 4$.

Thus,

$$U_1^* - U_2^* = w \ln(5w) - (1-w)\ln(5(1-w)),$$

which has the same sign as $w - 1/2$*. In particular, when* $w > 1/2$*, we have* $U_1^* > U_2^*$*; when* $0.2 < w < 0.5$*, we have* $U_1^* < U_2^*$*. In addition, multiple equilibria arise within this range of* w*.*

- *If* $0 < w \leq 0.2$*, we obtain a corner equilibrium in which agents 2 and 3 allocate all effort to the team with weight* α^3*, and agents 4 and 5 allocate all effort to the team with weight* α^2*. In this case:*

  $$y^1 = y^4 = 1, \quad y^2 = y^3 = 4.$$

  The multipliers μ_i *are not equal and are given by:*

  $$\mu_1 = \frac{1}{w} > \mu_2 = \cdots = \mu_5 = \frac{4}{1-w}.$$

  The payoffs are:

  $$U_1^* = 2w \ln(1) = 0, \tag{G.23}$$

  $$U_2^* = w \ln(1) + (1-w)\ln(4) = (1-w)\ln(4). \tag{G.24}$$

  Hence, $U_1^* < U_2^*$*. In this case, the equilibrium is unique.*

H Departures from monolayer predictions: Theoretical considerations

H.1 Regular networks with convex costs: Monolayer versus multilayer cases

To compare the predictions of the monolayer and multilayer cases, we now consider regular networks with convex costs.

Regular Networks: Monolayer versus multilayer predictions. Let us begin with the *multilayer network* with convex costs. As in Section 5.2, and assuming for simplicity that $\beta = 1$, the utility function is given by

$$U_i(\mathbf{x}_i, \mathbf{x}_{-i}) = \sum_{s \in S} \alpha^s \ln\left(v_i^s + x_i^s + \phi^s \sum_{j \in \mathcal{N}} g_{ij}^s x_j^s\right) - \frac{1}{2}\left(\sum_{s \in S} x_i^s\right)^2.$$

We easily obtain the equilibrium effort in each layer s as follows:

$$x_{\text{multi}}^{s*} = \frac{\alpha^s}{(1+\phi^s d^s)\sqrt{\sum_{s' \in S} \frac{\alpha^{s'}}{(1+\phi^{s'} d^{s'})}}}, \tag{H.1}$$

and the total effort:

$$X^*_{\text{multi}} := \sum_{s\in S} x^{s*}_{\text{multi}} = \sqrt{\sum_{s\in S} \frac{\alpha^s}{(1+\phi^s d^s)}}. \tag{H.2}$$

For the *monolayer* case with convex costs, the utility function becomes:

$$U_i(\mathbf{x}_i, \mathbf{x}_{-i}) = \alpha \ln \left(v_i + x_i + \phi \sum_{j\in\mathcal{N}} g_{ij} x_j \right) - \frac{1}{2} x_i^2.$$

Solving the model yields:

$$x^*_{\text{mono}} = \sqrt{\frac{\alpha}{1+\phi d}}. \tag{H.3}$$

We observe that the main difference between the two models is the additional term $\sqrt{\sum_{s\in S} \frac{\alpha^s}{(1+\phi^s d^s)}} := X^*_{\text{multi}}$ (total effort), which appears in the denominator of (H.1). This term links the different layers together.

To clarify the intuition behind this result, consider the two regular layers depicted in Figure H4, where $\mathbf{G}^A$ is a circle network and $\mathbf{G}^B$ is the complement network of $\mathbf{G}^A$, each consisting of four agents. The sum of these two networks yields the complete network, i.e., $\mathbf{G}^A + \mathbf{G}^B = K_4$.

To gain intuition, consider again the two networks displayed in Figure H4. Assume for simplicity $v_i^s = 1$ for all i and all s.

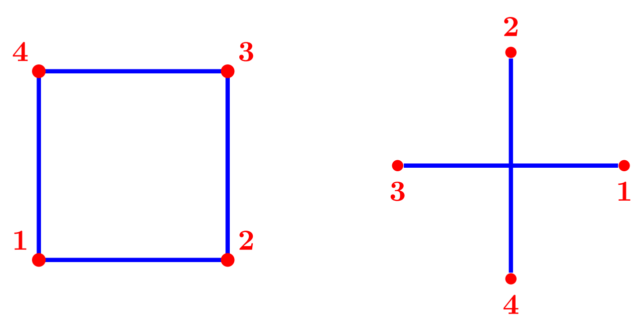

Figure H4: Two opposite regular networks

In the *monolayer case*, we treat $\mathbf{G}^A$ and $\mathbf{G}^B$ as *independent* and analyze them separately. Using equation (H.3), we obtain:

$$x^{A*}_{\text{mono}} = \sqrt{\frac{\alpha^A}{1+2\phi^A}}, \quad x^{B*}_{\text{mono}} = \sqrt{\frac{\alpha^B}{1+\phi^B}},$$

so that the total effort is given by:

$$X^*_{\text{mono}} = \sqrt{\frac{\alpha^A}{1+2\phi^A}} + \sqrt{\frac{\alpha^B}{1+\phi^B}}.$$

In the *multilayer case*, we treat $\mathbf{G}^A$ and $\mathbf{G}^B$ as *dependent* and analyze them jointly. Using equations (H.1) and (H.2), we obtain:

$$x_{\text{multi}}^{A*} = \frac{\alpha^A}{(1+2\phi^A)\sqrt{\frac{\alpha^A}{1+2\phi^A} + \frac{\alpha^B}{1+\phi^B}}}, \quad x_{\text{multi}}^{B*} = \frac{\alpha^B}{(1+\phi^B)\sqrt{\frac{\alpha^A}{1+2\phi^A} + \frac{\alpha^B}{1+\phi^B}}},$$

and the total effort is:

$$X_{\text{multi}}^{*} = \sqrt{\frac{\alpha^A}{1+2\phi^A} + \frac{\alpha^B}{1+\phi^B}}.$$

Clearly, the outcomes in the monolayer and multilayer cases differ significantly. In particular, the multiplex setting introduces an additional term, $\sqrt{\frac{\alpha^A}{1+2\phi^A} + \frac{\alpha^B}{1+\phi^B}}$, which links the two layers due to the constraint imposed by the total effort cost. In particular, in the multilayer case, the effort x_{multi}^{A*} in layer A depends on the parameters α^B and ϕ^B while this is not true in the monolayer case. Assume that $\phi^A < 0$ and $\phi^B < 0$, implying that efforts are complements and agents exert negative spillovers on each other. In the context of social comparisons (as discussed in Section 2.2), this corresponds well with real-world examples: layer A might represent "neighbors" and layer B "extended family." The efforts, in this case, could be interpreted as the fraction of income spent on cars (layer A) versus weddings and festivals (layer B). In the monolayer framework, these two types of expenditures are treated independently. Each is influenced solely by α^A or α^B (how much the individual values their neighbors or family) and by ϕ^A or ϕ^B (the strength of peer effects in each layer). However, this independence breaks down in the multilayer case. Because of the convex cost associated with total effort, the two activities become interconnected. In fact, the first-order condition in the multilayer case, for each layer $s = A, B$, is given by:

$$\frac{\alpha^s}{(1+\phi^s d^s)\, x_{multi}^s} = x_{multi}^A + x_{multi}^B,$$

where the left-hand side represents the marginal benefit of individual effort and the right-hand side captures its marginal cost. Notably, the two efforts x_{multi}^A and x_{multi}^B are perfect substitutes in the cost function—an increase in one must be offset by a decrease in the other to maintain optimality. In contrast, in the monolayer case, the first-order condition for each layer $s = A, B$ simplifies to:

$$\frac{\alpha^s}{(1+\phi^s d^s)\, x_{mono}^s} = x_{mono}^s.$$

While the marginal benefit remains the same, the marginal cost is now independent of effort in the other layer. This fundamental difference implies that monolayer predictions overlook the interaction between activities and fail to account for the constraint imposed by the overall cost of effort across layers.

H.2 The linear-quadratic utility model of Ballester et al. (2006): Mono versus multiplexing

H.2.1 Multiplexing: The linear-quadratic utility model of Ballester et al. (2006) with a budget cost

We extend the model of Ballester et al. (2006)—a linear-quadratic utility function with strategic complements in efforts and positive spillovers—to our multilayer framework. The utility function of each agent i is given by:

$$U_i = \sum_{s\in\mathcal{S}} \alpha^s \left(v_i^s x_i^s - \frac{1}{2}(x_i^s)^2 + \phi^s \sum_{j\in\mathcal{N}} g_{ij}^s x_i^s x_j^s \right),$$

where $\phi^s > 0$ and the constraint is $\sum_{s\in\mathcal{S}} x_i^s = T_i$.

Note that there is no need to use SET, as this utility function is linear-quadratic and thus constitutes an *exact potential*. In other words, we do not need to apply the SET transformation to the first-order conditions to obtain a best-response potential. Therefore, as in Ballester et al. (2006), the spectral condition for uniqueness is given by $1 - \lambda_{\max}(\phi^s \mathbf{G}^s) > 0$, for all $s \in \mathcal{S}$, where $\lambda_{\max}(\mathbf{H})$ denotes the largest eigenvalue of matrix $\mathbf{H}$.

The Lagrangian can be written as

$$\mathcal{L} = \sum_{s\in\mathcal{S}} \alpha^s \left(v_i^s x_i^s - \frac{1}{2}\left(x_i^s\right)^2 + \phi^s \sum_{j\in\mathcal{N}} g_{ij}^s x_i^s x_j^s \right) - \lambda_i \left(\sum_{s\in\mathcal{S}} x_i^s - T_i \right),$$

where λ_i is the Lagrangian multiplier. The first-order condition, at an interior equilibrium, is equal to

$$\frac{\partial U_i}{\partial x_i^s} = \alpha^s \left(v_i^s - x_i^s + \phi^s \sum_{j\in\mathcal{N}} g_{ij}^s x_j^s \right) - \lambda_i = 0.$$

By denoting $\mu_i = \lambda_i$ and solving this equation, we obtain:[13]

$$x_i^s = v_i^s + \phi^s \sum_{j\in\mathcal{N}} g_{ij}^s x_j^s - \frac{\mu_i}{\alpha^s}.$$

Thus, in the multilayer version of Ballester et al. (2006), the equilibrium is equal to

$$\mathbf{x}_{multi}^{s*} = [\mathbf{I}_n - \phi^s \mathbf{G}^s]^{-1} \left(\mathbf{v}^s - \frac{1}{\alpha^s}\boldsymbol{\mu} \right) = \widehat{\mathbf{M}}^s \left(\mathbf{v}^s - \frac{1}{\alpha^s}\boldsymbol{\mu} \right), \tag{H.4}$$

where $\boldsymbol{\mu}$ can be determined by the constraint $\sum_{s\in\mathcal{S}} \mathbf{x}^s = \mathbf{T}$, $\phi^s > 0$, and $\widehat{\mathbf{M}}^s = [\mathbf{I}_n - \phi^s \mathbf{G}^s]^{-1}$. Note that $\widehat{\mathbf{M}}^s$ differs from $\mathbf{M}^s$, as defined in (12), which is why we use a different notation. We obtain

$$\boldsymbol{\mu} = \left(\sum_{s\in\mathcal{S}} \frac{1}{\alpha^s} \widehat{\mathbf{M}}^s \right)^{-1} \left(\sum_{s\in\mathcal{S}} \widehat{\mathbf{M}}^s \mathbf{v}^s - \mathbf{T} \right). \tag{H.5}$$

[13] Observe that in our baseline model, μ_i is the inverse of the corresponding Lagrange multiplier associated with agent i's constraint, whereas here μ_i coincides with the multiplier λ_i.

We observe that the equilibrium effort in layer s increases in $\mathbf{v}^s$ and decreases in $\boldsymbol{\mu}$. In our benchmark model, the equilibrium is given by (14), which instead decreases in $\mathbf{v}^s$ and increases in $\boldsymbol{\mu}$ (under the conditions that ensure $\mathbf{M}^s \geq 0$). Thus, the two models would be equivalent if $\alpha^s = 1$ for all s, and if we replaced $(\mathbf{v}^s - \boldsymbol{\mu})$ with $(\boldsymbol{\mu} - \mathbf{v}^s)$. However, $\boldsymbol{\mu}$ differs between the two models.

H.2.2 Monolayer version of Ballester et al. (2006)

The utility is given by

$$U_i = v_i x_i - \frac{1}{2}(x_i)^2 + \phi \sum_{j \in \mathcal{N}} g_{ij} x_i x_j$$

As it is well-known, if $\phi \lambda_{\max}(\mathbf{G}) < 1$, the equilibrium is given by

$$x_i = v_i + \phi \sum_{j \in \mathcal{N}} g_{ij} x_j$$

That is,

$$\mathbf{x}_{mono} = [\mathbf{I}_n - \phi \mathbf{G}]^{-1} \mathbf{v} = \widehat{\mathbf{M}} \mathbf{v}. \tag{H.6}$$

To gain intuition, let us now compare the monolayer and multilayer versions of Ballester et al. (2006) for regular networks.

H.2.3 Monolayer versus Multilayer Version of Ballester et al. (2006): Regular Networks

Consider the two networks displayed in Figure H4 with four agents. The left network (a circle network) corresponds to layer A, while the right network corresponds to layer B, such that $\mathbf{G}^A + \mathbf{G}^B = K_4$. For simplicity, assume that $v_i^s = 1$ for all i and all s.

The Ballester et al. (2006)'s model with a budget constraint In the *monolayer* case, each layer can be solved independently. If $\phi^A < 0.5$ and $\phi^B < 1$, we obtain:

$$\mathbf{x}^A_{\text{mono}} = \left(\frac{1}{1 - 2\phi^A} \right) \mathbf{1}, \quad \mathbf{x}^B_{\text{mono}} = \left(\frac{1}{1 - \phi^B} \right) \mathbf{1}.$$

In the *multilayer* case, we solve the two layers jointly. Assume $\alpha^s = 1$ for all s and $T_i = T = 1$ for all i. Then, if $\phi^A < 0.5$ and $\phi^B < 1$, we have:

$$\mathbf{x}^A_{\text{multi}} = \frac{1 - \phi^B}{2(1 - \phi^A) - \phi^B} \mathbf{1}, \quad \mathbf{x}^B_{\text{multi}} = \frac{1 - 2\phi^A}{2(1 - \phi^A) - \phi^B} \mathbf{1},$$

and

$$\boldsymbol{\mu}_{\text{multi}} = \frac{(1 - 2\phi^A)\phi^B + (1 - \phi^B)}{2(1 - \phi^A) - \phi^B} \mathbf{1}. \tag{H.7}$$

It is straightforward to see that $\mathbf{x}^A_{\text{multi}}$ is increasing in ϕ^A (as in the monolayer case) but decreasing in ϕ^B. Similarly, $\mathbf{x}^B_{\text{multi}}$ is increasing in ϕ^B (as in the monolayer case) but decreasing in ϕ^A.

The Ballester et al. (2006)'s model with convex costs In the *monolayer* case, we can solve each layer separately. Assume $\alpha = 1$ and $v_i = 1$ for all i. If $\phi^A < 0.5$ and $\phi^B < 1$, we obtain

$$\mathbf{x}^{A*}_{mono} = \frac{1}{1 - 2\phi^A + c}\mathbf{1}, \quad \mathbf{x}^{B*}_{mono} = \frac{1}{1 - \phi^B + c}\mathbf{1}, \tag{H.8}$$

and $\mu_i^* := cx_i^*$, the marginal cost, is given by

$$\boldsymbol{\mu}^{A*}_{mono} = \frac{c}{1 - 2\phi^A + c}\mathbf{1}, \quad \boldsymbol{\mu}^{B*}_{mono} = \frac{c}{1 - \phi^B + c}\mathbf{1}.$$

In the *multilayer* case, we solve the two layers together. Assume $\alpha^s = 1$ for all s for all i and $v_i = 1$ for all i. Then, if $\phi^A < 0.5$ and $\phi^B < 1$, we have

$$\mathbf{x}^{A*}_{multi} = \frac{1 - \mu^*}{1 - 2\phi^A}\mathbf{1}, \quad \mathbf{x}^{B*}_{multi} = \frac{1 - \mu^*}{1 - \phi^B}\mathbf{1}.$$

However, $\boldsymbol{\mu}^*$ is now endogenous and determined by the *linear* systems of equations (48). We obtain:

$$\boldsymbol{\mu}^* = c\sum_{s\in\mathcal{S}} \frac{1}{(1 - \phi^s d^s)}\left(\mathbf{1} - \boldsymbol{\mu}^*\right) = c\left(\frac{1}{1 - 2\phi^A} + \frac{1}{1 - \phi^B}\right)\left(\mathbf{1} - \boldsymbol{\mu}^*\right).$$

Solving this yields

$$\boldsymbol{\mu}^* = \frac{c\left(\frac{1}{1-2\phi^A} + \frac{1}{1-\phi^B}\right)}{1 + c\left(\frac{1}{1-2\phi^A} + \frac{1}{1-\phi^B}\right)}\mathbf{1}. \tag{H.9}$$

References

Agarwal, Sumit, Vyacheslav Mikhed, and Barry Scholnick. 2020. "Peers' Income and Financial Distress: Evidence from Lottery Winners and Neighboring Bankruptcies." *Review of Financial Studies* 33 (1): 433–72.

Agarwal, Sumit, Wenlan Qian, and Xin Zou. 2021. "Thy Neighbor's Misfortune: Peer Effect on Consumption." *American Economic Journal: Economic Policy* 13 (2): 1–25.

Aldasoro, Inaki, and Ivan Alves. 2018. "Multiplex Interbank Networks and Systemic Importance: An Application to European Data." *Journal of Financial Stability* 35: 17–37.

Allouch, Nizar. 2015. "On the Private Provision of Public Goods on Networks." *Journal of Economic Theory* 157: 527–52.

Ballester, Coralio, Antoni Calvó-Armengol, and Yves Zenou. 2006. "Who's Who in Networks: Wanted: The Key Player." *Econometrica* 74 (5): 1403–17.

Banerjee, Abhijit, Emily Breza, Arun G. Chandrasekhar, Esther Duflo, Matthew O. Jackson, and Cynthia Kinnan. 2024. "Changes in Social Network Structure in Response to Exposure to Formal Credit Markets." *Review of Economic Studies* 91 (3): 1331–72.

Banerjee, Abhijit, Arun G. Chandrasekhar, Esther Duflo, and Matthew Jackson. 2013. "The Diffusion of Microfinance." *Science* 341 (6144): 363–63.

Bardoscia, Marco, Ginestra Bianconi, and Gerardo Ferrara. 2019. "Multiplex Network Analysis of the UK over the Counter Derivatives Market." *International Journal of Finance and Economics* 24 (4): 1520–44.

Belhaj, Mohamed, Yann Bramoulle, and Frederic Deroian. 2014. "Network Games under Strategic Complementarities." *Games and Economic Behavior* 88: 310–19.

Belhaj, Mohamed, and Frederic Deroian. 2014. "Competing Activities in Social Networks." *B.E. Journal of Economic Analysis & Policy* 14 (4): 1431–66.

Bergstrom, Theodore, Lawrence Blume, and Hal Varian. 1986. "On the Private Provision of Public Goods." *Journal of Public Economics* 29 (1): 25–49.

Bertrand, Marianne, and Adair Morse. 2016. "Trickle-down Consumption." *Review of Economics and Statistics* 98 (5): 863–79.

Billand, Pascal, Christophe Bravard, Subhadip Chakrabarti, and Sudipta Sarangi. 2014. "A Note on Networks of Collaboration in Multi-Market Oligopolies." *Annals of Regional Science* 53 (2): 325–35.

Billand, Pascal, Christophe Bravard, Subhadip Chakrabarti, and Sudipta Sarangi. 2016. "Business Intelligence and Multimarket Competition." *Journal of Public Economic Theory* 18 (2): 248–67.

Billand, Pascal, Christophe Bravard, Sumit Joshi, Ahmed Saber Mahmud, and Sudipta Sarangi. 2023. "A Model of the Formation of Multilayer Networks." *Journal of Economic Theory* 213: 105718.

Bloch, Francis, Vijayendra Rao, and Sonalde Desai. 2004. "Wedding Celebrations as Conspicuous Consumption: Signaling Social Status in Rural India." *Journal of Human Resources* 39 (3): 675–95.

Boccaletti, S., G. Bianconi, R. Criado, C. Del Genio, J. Gómez-Gardeñes, M. Romance, I. Sendiña Nadal, Z. Wang, and M. Zanin. 2014. "The Structure and Dynamics of Multilayer Networks." *Physics Reports* 544 (1): 1–122.

Bramoullé, Yann, and Christian Ghiglino. 2024. "Status Consumption in Networks: A Reference Dependent Approach." Unpublished.

Bramoullé, Yann, and Rachel Kranton. 2007. "Public Goods in Networks." *Journal of Economic Theory* 135 (1): 478–94.

Bramoullé, Yann, and Rachel Kranton. 2016. "Games Played on Networks." In *Oxford Handbook of the Economics of Networks*, edited by Yann Bramoullé, Andrea Galeotti and Brian W. Rogers, 83–112. Oxford University Press.

Bramoullé, Yann, Rachel Kranton, and Martin D'Amours. 2014. "Strategic Interaction and Networks." *American Economic Review* 104 (3): 898–930.

Chakrabarti, Subhadip, and Robert P. Gilles. 2007. "Network Potentials." *Review of Economic Design* 11 (1): 13–52.

Chandrasekhar, Arun, Vasu Chaudhary, Benjamin Golub, and Matthew O. Jackson. 2024. "Multiplexing and Diffusion." Unpublished.

Charles, Kerwin Kofi, Erik Hurst, and Nikolai Roussanov. 2009. "Conspicuous Consumption and Race." *Quarterly Journal of Economics* 124 (2): 425–67.

Chen, Ying-Ju, Yves Zenou, and Junjie Zhou. 2018. "Multiple Activities in Networks." *American Economic Journal: Microeconomics* 10 (3): 34–85.

Cheng, Chen, Wei Huang, and Yiqing Xing. 2021. "A Theory of Multiplexity: Sustaining Cooperation with Multiple Relations." Unpublished.

De Giorgi, Giacomo, Anders Frederiksen, and Luigi Pistaferri. 2020. "Consumption Network Effects." *Review of Economic Studies* 87 (1): 130–63.

Galeotti, Andrea, Benjamin Golub, and Sanjeev Goyal. 2020. "Targeting Interventions in Networks." *Econometrica* 88 (6): 2445–71.

Ghiglino, Christian, and Sanjeev Goyal. 2010. "Keeping up with the Neighbors: Social Interaction in a Market Economy." *Journal of the European Economic Association* 8 (1): 90–119.

Ghiglino, Christian, and Alastair Langtry. 2024. "Status Substitution and Conspicuous Consumption." Unpublished.

Golub, Benjamin, and Stephen Morris. 2017. "Higher-Order Expectations." Unpublished.

Golub, Benjamin, and Stephen Morris. 2020. "Expectations, Networks, and Conventions." Unpublished.

Grinblatt, Mark, Matti Keloharju, and Seppo Ikäheimo. 2008. "Social Influence and Consumption: Evidence from the Automobile Purchases of Neighbors." *Review of Economics and Statistics* 90 (4): 735–53.

Immorlica, Nicole, Rachel Kranton, Mihai Manea, and Greg Stoddard. 2017. "Social Status in Networks." *American Economic Journal: Microeconomics* 9 (1): 1–30.

Jackson, Matthew O., and Yves Zenou. 2015. "Games on Networks." In *Handbook of Game Theory*, Vol. 4, edited by H. Peyton Young and Shmuel Zamir, 95–163. Elsevier.

Joshi, Sumit, Ahmed Saber Mahmud, and Sudipta Sarangi. 2020. "Network Formation with Multigraphs and Strategic Complementarities." *Journal of Economic Theory* 188: 105033.

Joshi, Sumit, Ahmed Saber Mahmud, and Hector Tzavellas. 2025. "Endogenous Multiplex Networks." *Journal of Economic Behavior and Organization* 237: 107192.

Kivelä, Mikko, Alex Arenas, Marc Barthelemy, James P. Gleeson, Yamir Moreno, and Mason A. Porter. 2014. "Multilayer Networks." *Journal of Complex Networks* 2 (3): 203–71.

Kor, Ryan, and Junjie Zhou. 2022. "Multi-Activity Influence and Intervention." *Games and Economic Behavior* 137: 91–115.

Kuhn, Peter, Peter Kooreman, Adriaan Soetevent, and Arie Kapteyn. 2011. "The Effects of Lottery Prizes on Winners and their Neighbors: Evidence from the Dutch Postcode Lottery." *American Economic Review* 101 (5): 2226–47.

Lambotte, Mathieu. 2025. "Peer Effects in Binary Outcomes: Strategic Complementarity and Taste for Conformity with Endogenous Networks." *Journal of Applied Econometrics* 40 (6): 608–26.

Langtry, Alastair. 2023. "Keeping up with "the Joneses": Reference-Dependent Choice with Social Comparisons." *American Economic Journal: Microeconomics* 15 (3): 474–500.

Leister, Matthew C., Yves Zenou, and Junjie Zhou. 2022. "Social Connectedness and Local Contagion." *Review of Economic Studies* 89 (1): 372–410.

Liang, Shichang, Yun He, Yaping Chang, Xuebing Dong, and Donghong Zhu. 2018. "Showing to Friends or Strangers? Relationship Orientation Influences the Effect of Social Exclusion on Conspicuous Consumption." *Journal of Consumer Behaviour* 17 (4): 355–65.

Merton, Robert K. 1968. "The Matthew Effect in Science: The Reward and Communication Systems of Science are Considered." *Science* 159 (3810): 56–63.

Morris, Stephen, and Takashi Ui. 2005. "Generalized Potentials and Robust Sets of Equilibria." *Journal of Economic Theory* 124 (1): 45–78.

Muggleton, Naomi, Anna Trendl, Lukasz Walasek, David Leake, John Gathergood, and Neil Stewart. 2022. "Workplace Inequality is Associated with Status-Signaling Expenditure." *Proceedings of the National Academy of Sciences* 119 (15): 1–5.

Nagurney, Anna. 1999. *Network Economics: A Variational Inequality Approach*, Vol. 10. Springer.

Oh, Ga-Eun Grace. 2021. "Social Class, Social Self-Esteem, and Conspicuous Consumption." *Heliyon* 7 (2): e06318.

Parise, Francesca, and Asuman Ozdaglar. 2023. "Graphon Games: A Statistical Framework for Network Games and Interventions." *Econometrica* 91 (1): 191–225.

Poledna, Sebastian, Jose Luis Molina-Borboa, Serafin Martínez-Jaramillo, Marco Van Der Leij, and Stefan Thurner. 2015. "The Multi-Layer Network Nature of Systemic Risk and its Implications for the Costs of Financial Crises." *Journal of Financial Stability* 20: 70–81.

Rao, Vijayendra. 2001. "Poverty and Public Celebrations in Rural India." *Annals of the American Academy of Political and Social Science* 573 (1): 85–104.

Ryabov, Igor. 2016. "Conspicuous Consumption among Hispanics: Evidence from the Consumer Expenditure Survey." *Research in Social Stratification and Mobility* 44: 68–76.

Sadler, Evan, and Benjamin Golub. 2023. "Games on Endogenous Networks." Unpublished.

Tzavellas, Hector. 2023. "A Multilayer View of Systemic Importance and Aggregate Fluctuations." *International Economic Review* 64 (3): 1023–46.

Veblen, Thorstein. 1899. *The Theory of the Leisure Class: An Economic Study of Institutions*. Macmillan.

Walsh, Alan M. 2017. "Games on Multi-Layer Networks." Unpublished.

Zenou, Yves, and Junjie Zhou. 2025. "Sign-Equivalent Transformations and Equilibrium Systems: Theory and Applications." Unpublished.