

Supplemental Appendix

for “Real Credit Cycles”

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July 2025

A Model

A.1 Solving the Model

The computational algorithm involves iteration on an outer loop (related to debt pricing) and an inner loop (related to firm policies). Before solving the model, we discretize the state space $(s, k, b) = (z, \varepsilon_z, A, \varepsilon_A, k, b)$ into $n_z \times n_z \times n_A \times n_A \times n_k \times n_b$ grid points. We then discretize the rational and perceived diagnostic transitions of the exogenous states according to [Tauchen \(1986\)](#). The computational algorithm - following [Strebulaev and Whited \(2012\)](#) - proceeds as follows:

Start outer loop.

1. Guess a default policy $df^\theta(s, k, b)$, and compute the implied debt prices $q^\theta(s, k, b)$ according to the lenders diagnostic zero-profit condition Equation (17).

Start inner loop.

- (a) Given the debt prices $q^\theta(s, k, b)$ and default policy $df^\theta(s, k, b)$, solve the diagnostic firm’s Bellman Equations (14), (15), and (16) for $V^\theta(s, k, b)$, $V_{ND}^\theta(s, k, b)$, and $V_D^\theta(s)$ as well as the implied optimal policies for investment and debt issuance $k'^\theta(s, k, b)$, $b'^\theta(s, k, b)$. Use standard discrete-state, discrete-policy dynamic programming policy iteration to do so.
2. Compute updated default policies $df^\theta(s, k, b)$ according to the default choice defining V^θ in Equation (14), i.e., $V_{ND}^\theta(s, k, b) < V_D^\theta(s)$.
3. Compute the ergodic distribution $\mu(s, k, b)$ implied by the firm policies for default, capital, and debt $df^\theta(s, k, b)$, $k'^\theta(s, k, b)$, and $b'^\theta(s, k, b)$.

4. Compute the mass of states in which the guessed default policy differs from the updated default policy. If this set of states has mass lower than some tolerance, exit. If not, then go to top and restart with the updated set of default states as your new guess.

Table A.1 reports the value of several dimensions used for the baseline solution of the model. Figure A.1 plots the perceived value function V^θ in our estimated DE model for a range of different capital and productivity news realizations, with overreaction generating more perceived value after goods news even conditional upon today’s productivity state. The value function in the RE model, also plotted, displays no such overreaction in the direction of recent news.

Table A.1: Computational Choices

Quantity	Description	Value
T^{sim}	Simulated periods	6
T^{erg}	Initially discarded periods	50
N^{firm}	Number of firms	1000
N^{IRF}	Number of IRF economies	10000
T^{IRF}	Length of IRF economies	75
T^{decomp}	Length of historical decomposition	2
n_z	Micro productivity grid size	5
n_A	Macro productivity grid size	5
n_k	Capital grid size	30
n_b	Debt grid size	30

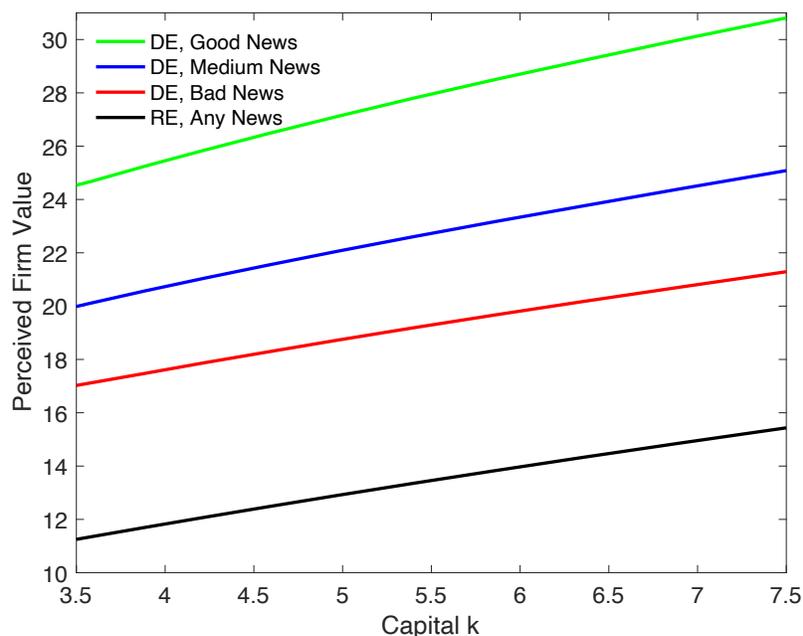
Notes: The table reports various computational values used in discretizing and solving the model.

A.2 Simulating the Model

After the model is solved, we unconditionally simulate the model by drawing exogenous uniform random shocks and combining this information with the transition matrix for macro TFP to simulate the macro process for A_t for some periods $t = 1, \dots, T^{sim} + T^{erg}$. At the micro level, we simulate the model “non-stochastically” according to the method of Young (2010), i.e., we store the dynamics of the weight of the cross-sectional distribution at each discretized point in the state space (s, k, b) rather than simulating a large number of firms. Note that when simulating the model, all macro shocks and distributional dynamics are determined according to the *rational* or true representations of the driving process, even though debt pricing and firm policies may involve diagnostic expectations.

With the simulated distribution in hand for each period, macro series of interest are simply weighted sums of micro-level outcomes across this distribution, discarding the first T^{erg} periods to remove the influence of initial conditions. Note that we do in fact simulate

Figure A.1: Firm Value and Diagnosticity



Notes: The figure plots the perceived value function V^θ as a function of firm capital k for the estimated DE model, together with the firm's value function V in the estimated RE model. All lines hold fixed micro TFP z , macro TFP A , macro news ε_A , and the firm's debt b at identical representative values. The four lines reflect different realizations of micro TFP news ε_z , with positive news in the DE model (green line), medium news in the DE model (blue line), bad news in the DE model (red line), and any news in the RE model (black line).

a number of individual firms N^{firm} for the purpose of computing moments within our SMM estimation algorithm, but this is not a step required for the purpose of solving the model or simulating within-period business cycle aggregates.

A.3 Computing Impulse Responses

Our approach to impulse response calculation in this nonlinear context follows [Koop et al. \(1996\)](#), i.e., we compute nonlinear generalized impulse responses. To understand the impact of a given sequence of shocks, we perform the following:

1. For a large number N^{irf} of economies of length T^{irf} , simulate two different versions of the simulation, the “shock” and “no shock” versions. For each economy and each version, we simulate the macro TFP process by first drawing T^{irf} uniform shocks for comparison with the macro TFP transition matrix. Then, simulate both versions unconditionally using identical macro TFP shocks until period $T^{shock} < T^{irf}$.
2. From period T^{shock} and continuing as long as the desired sequence of exogenous innovations you wish to impose lasts, impose a number of periods of certain pre-determined innovations in productivity for the “shock” case, while continuing to simulate the “no shock” economy unconditionally.
3. After the imposed shocks sequence is complete, simulate macro TFP in both economies as normal.
4. After the macro TFP process is determined for each pair of economies, compute the business cycle aggregates of interest in each economy, period, and version by using the simulation approach outlined above.
5. If business cycle aggregate $X_{i,t}^{shock}$ is series X in economy i in period t in the shock case, and $X_{i,t}^{noshock}$ is series X in economy i in period t in the no shock case, then define the impulse response to the predetermined sequence of innovations as

$$IRF_t^X = \frac{1}{N^{irf}} \sum_{i=1}^{N^{irf}} \frac{X_{i,t}^{shock} - X_{i,t}^{noshock}}{X_{i,t}^{noshock}}.$$

The main text’s set of impulse response figures reports the series IRF^X for the indicated macro-financial aggregates. Note, however, that the impulse responses presented in the text are scaled to equal an exact shock size, while the productivity grid in the model varies discretely. We achieve this by imposing movements up or down by a single grid point, imposing [Step 2](#) above only with a certain probability chosen in each period to deliver the desired average shock size.

A.4 Performing the Spread Matching Exercise

In a classic linear setting, performing historical decompositions such as the one used in [Section 6](#) for the Great Financial Crisis is typically a trivial matter of inverting a data

path using simple linear algebra. However, our nonlinear model with heterogeneity and a discretized productivity process poses some additional computational challenges. Given the empirical path to match across T_{decomp} periods for macro credit spreads $(S_1, S_2, \dots, S_T^{decomp})$ we proceed as follows.

First, we pick an initial period drawn from a representative location in the unconditional simulation of the model, fixing the associated simulated cross-sectional distribution of firm-level states μ_0 drawn from the simulation of the model. Call this period $t = 0$, and note that at the end of period 0 a cross-sectional distribution μ_1 is pre-determined. Then for each period $t = 1, \dots, T^{decomp}$, do the following:

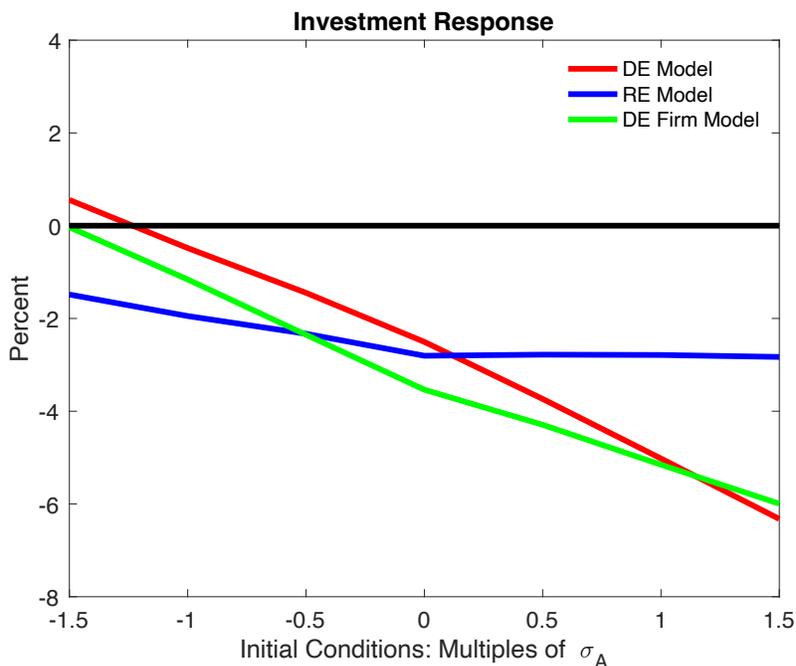
1. Guess a value for macro TFP A_t , and find the bracketing interval $[A_{i-1}, A_i]$ together with linear interpolation weights $\omega(A_t, i) = \frac{A_t - A_{i-1}}{A_i - A_{i-1}}$ for the guessed value of productivity.
2. Compute the implied policies of all firms in the cross-sectional distribution μ_t given a macro TFP level equal to A_i , together with the implied macro spread level $S(A_i)$. Repeat the process for macro TFP equal to A_{i-1} to obtain $S(A_{i-1})$.
3. Assume that firms play a “mixed strategy” over the two macro TFP grid points, in which case the resulting macro spread level is $(1 - \omega(A_t, i))S(A_{i-1}) + \omega(A_t, i)S(A_i)$.
4. If the implied macro spread level is not equal to the desired spread value S_t to within some tolerance, then update your guess for macro TFP A_t and return to Step 1. Otherwise proceed.
5. Given a productivity guess which delivers exactly the correct interpolated value of macro productivity in period t , compute the beginning-of-period distribution μ_{t+1} of firm-level states by pushing forward a fraction $\omega(A_t, i)$ of the distribution μ_t using firm policies associated with A_i and a fraction $1 - \omega(A_t, i)$ of the distribution μ_t using firm policies associated with A_{i-1} .

At the end of this process, you have determined a smooth value of productivity A_t which gives you an implied macro spread series exactly consistent with the target value in period t , and you have updated the cross-sectional distribution in an internally consistent fashion given the smooth value of productivity between grid points. Repeating this process for each period $t = 1, \dots, T^{decomp}$ yields a productivity path A_t , as well as a set of cross-sectional distributions μ_t , which exactly match the target data path for spread. All other macro aggregates of interest can then be computed from the distributional and macro TFP path. Note that for the spread matching exercise for the Great Recession and financial crisis in Section 6, we set $T^{decomp} = 2$, with $t = 1$ being the “Pre-Crisis” period and $t = 2$ being the “Crisis” period.

A.5 DE Firms with RE Lenders

Figure A.2 plots the equivalent of Figure 2, plotting investment responses to a negative TFP shock for a range of initial conditions, in a re-estimated model in which firms exhibit DE with $\theta > 0$ but lenders exhibit RE with $\theta = 0$. Figure

Figure A.2: Investment Nonlinearity with Rational Pricing



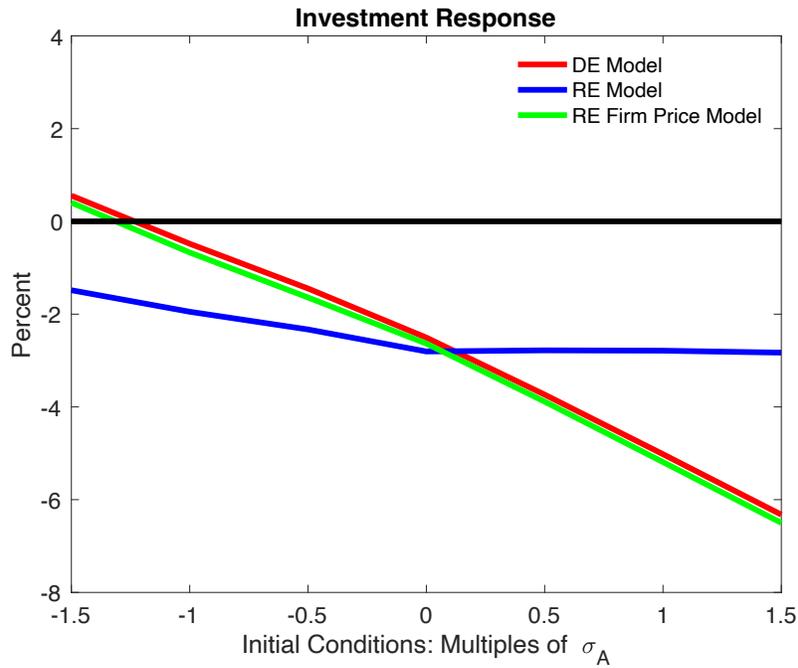
Notes: The vertical axis in the figure reports the simulated first-period impulse response of macro investment to a one-standard deviation negative shock to macro TFP, and the horizontal axis reports the initial conditions, i.e., the magnitude of the shock to macro TFP in the previous period. The DE model (red line), RE model (blue line), and DE model with diagnostic firms and rational lenders (green line) are reported on the figure.

Figure A.3 also plots investment responses to a negative aggregate TFP shock in a model in which lenders exhibit DE with respect to aggregate shocks but RE with respect to firm-level shocks.

A.6 Belief Updating versus Endogenous Sensitivity

The IRF nonlinearity revealed in the main text in Figure 2 can in principle stem from one of two not mutually exclusive sources: i) direct reversals of overoptimistic diagnostic beliefs after good news, or ii) higher riskiness in the distribution of overoptimistic firms' endogenous states b and k after a good news. To piece apart the direct contribution of beliefs versus the indirect contribution of firm states, we compute the response of the diagnostic economy to a one-standard deviation negative shock to TFP after a range of different initial shocks, in

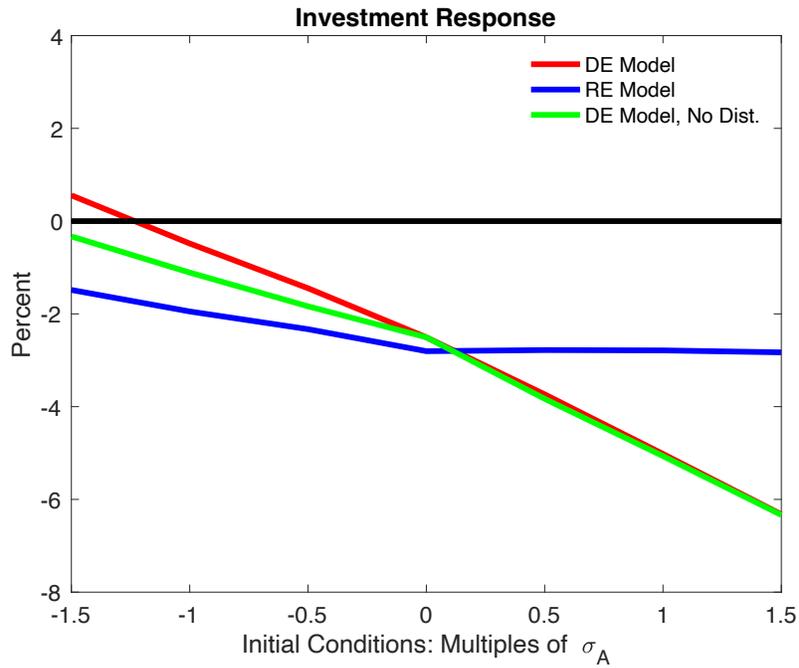
Figure A.3: Investment Nonlinearity with Rational Pricing at Firm Level



Notes: The vertical axis in the figure reports the simulated first-period impulse response of macro investment to a one-standard deviation negative shock to macro TFP, and the horizontal axis reports the initial conditions, i.e., the magnitude of the shock to macro TFP in the previous period. The DE model (red line), RE model (blue line), and model with DE firms, lender RE on firm-level shocks, and lender DE on aggregate shocks (green line) are reported on the figure.

each case holding the distribution of endogenous states equal to the one which would have obtained without the negative TFP shock. The green line in Figure A.4 reveals the resulting investment responses as a function of the initial conditions. Without the endogenously higher riskiness in capital and debt positions generated by firms, the green line continues to slope down but is not quite as strongly sloped as the red line in some regions. Therefore, both the direct belief reversal and endogenous distributional shifts due to diagnosticity matter for generating the overall nonlinearity in Figure 2.

Figure A.4: Investment Nonlinearity with No Distributional Dynamics



Notes: The vertical axis in the figure reports the simulated first-period impulse response of macro investment to a one-standard deviation negative shock to macro TFP, and the horizontal axis reports the initial conditions, i.e., the magnitude of the shock to macro TFP in the previous period. The DE model (red line), RE model (blue line), and DE model with no shift in the distribution of endogenous states (light blue line) are reported on the figure.

A.7 Investment Wedges

Consider a benchmark representative firm version of our PE model without investment frictions, i.e., without adjustment costs or financial frictions. In this case, the representative firm solves

$$\max_{K'} \dots - K' + \frac{1}{R} \mathbb{E}^{firm} [\Pi(A', K') + (1 - \delta)K',] + \dots$$

where the profit function is defined as

$$\Pi(A, K) = \max_N AK^\alpha N^\nu - WN$$

and the expectations operator \mathbb{E}^{firm} obeys whatever belief system the firm uses (RE, DE, etc...). Static labor optimization implies that

$$\Pi(A, K) = (1 - \nu) \left(\frac{\nu}{W} \right)^{\frac{\nu}{1-\nu}} A^{\frac{1}{1-\nu}} K^{\frac{\alpha}{1-\nu}},$$

so the capital FOC for the frictionless representative economy is given by

$$R - 1 + \delta = \mathbb{E}^{firm} \frac{\partial}{\partial K'} \Pi(A', K'), \quad (20)$$

where again the expectations operator can be either DE or RE depending on the firm's belief system. FOC (20) together with the profit function expression lead to the capital choice

$$K' = \left(\frac{\alpha}{R - 1 + \delta} \right)^{\frac{1-\nu}{1-(\alpha+\nu)}} \left(\frac{\nu}{W} \right)^{\frac{\nu}{1-(\alpha+\nu)}} \left(\mathbb{E}^{firm} \left[A'^{\frac{1}{1-\nu}} \right] \right)^{\frac{1-\nu}{1-(\alpha+\nu)}}. \quad (21)$$

Ok, so now consider an investment wedge representation of a similar economy. In this case, the firm is assumed to solve the problem

$$\max_{K'} \dots - \tau_I K' + \frac{1}{R} \mathbb{E} [\Pi(A', K') + (1 - \delta)K' \tau_I',] + \dots \quad (22)$$

where τ_I is typically assumed to be an investment wedge that follows an AR(1) with autocorrelation ρ_I and \mathbb{E} is the RE operator. In this case, the FOC with the investment wedge is given by

$$\begin{aligned} \tau_I &= \frac{1}{R} \mathbb{E} \left[\frac{\partial}{\partial K'} \Pi(A', K') + (1 - \delta) \tau_I' \right] \\ \tau_I [R - \rho_I (1 - \delta)] &= \mathbb{E} \left[\frac{\partial}{\partial K'} \Pi(A', K') \right]. \end{aligned} \quad (23)$$

Here, the expectation \mathbb{E} on the RHS is the rational expectation, and K' is whatever capital choice is made by the firm. Two results are immediate.

First, if capital K' is chosen by a RE decisionmaker, then $\mathbb{E}^{firm} = \mathbb{E}$. And inspecting

Equations (20) and (23) reveals that only a constant $\tau_I = 1$ (and associated $\rho_I = 1$) are consistent with the model. In other words, the investment wedge does not move if there are no frictions and the representative decisionmaker has RE beliefs. This is not an interesting result, it's simply stating that in a RE model with no frictions, the investment wedge is trivially constant and equal to 1.

Second, if the decisionmaker chooses capital K' in equation (21) subject to DE beliefs, then the RE version of FOC (20) will not be satisfied and in general you must have movement in the investment wedge τ_I in Equation (23) which must be satisfied under RE. In particular, the investment wedge equation becomes

$$\tau_I [R - \rho_I(1 - \delta)] = \mathbb{E} \left[\frac{\partial}{\partial K'} \Pi(A', K'^{\theta}) \right], \quad (24)$$

where the expectation \mathbb{E} is rational but the DE value K'^{θ} is given from (21) in this specialized case as

$$K'^{\theta} = \left(\frac{\alpha}{R - 1 + \delta} \right)^{\frac{1-\nu}{1-(\alpha+\nu)}} \left(\frac{\nu}{W} \right)^{\frac{\nu}{1-(\alpha+\nu)}} \left(\mathbb{E}^{\theta} \left[A'^{\frac{1}{1-\nu}} \right] \right)^{\frac{1-\nu}{1-(\alpha+\nu)}}. \quad (25)$$

Straightforward calculations, substituting (25) into (24), reveal that

$$\tau_I \propto \frac{\mathbb{E} A'^{\frac{1}{1-\nu}}}{\mathbb{E}^{\theta} A'^{\frac{1}{1-\nu}}}.$$

The numerator is RE, the denominator is DE. But these are both analytically tractable lognormal expectations under a Gaussian lognormal AR(1) assumption for A :

$$\begin{aligned} \mathbb{E} A'^{\frac{1}{1-\nu}} &= e^{\frac{\rho_A}{1-\nu} \log A + \frac{\sigma_A^2}{2(1-\nu)^2}} \\ \mathbb{E}^{\theta} A'^{\frac{1}{1-\nu}} &= e^{\frac{\rho_A}{1-\nu} (\log A + \theta \varepsilon_A) + \frac{\sigma_A^2}{2(1-\nu)^2}}. \end{aligned}$$

We therefore obtain the formula

$$\tau_I \propto e^{-\frac{\rho_A}{1-\nu} \theta \varepsilon_A}, \quad (26)$$

This reveals that the investment wedge is indeed an AR(1) with a lognormal innovation and no persistence. In logs, as this is usually presented, we obtain that

$$\log \tau_I = \Omega - \frac{\rho_A}{1-\nu} \theta \varepsilon_A$$

up to some constant Ω . So, we have uncovered that in a PE frictionless representative agent version of our economy, DE does indeed generate investment wedge fluctuations. In this economy DE **only** creates investment wedge fluctuations, with efficiency, labor, and government consumption wedges constant trivially due to the representative firm's lack of financial

frictions, static labor optimization, and lack of a government respectively. Intuitively, with good news, an economist doing business cycle accounting under RE perceives that the DE firm acts like its price of capital investment declines. With bad news, the RE economist doing business cycle accounting perceives that the DE firm acts like its price of capital investment increases.

A.8 General Equilibrium Definition

In Section 7.2 we consider an extended model with a representative household and endogenously flexible wages. Here, we outline the household, lender, and firm problems, define the equilibrium, and derive the two household first-order conditions used in the main text.

A.8.1 Household

A diagnostic representative household supplies labor N and saves in a risk-free bond B' in zero-net supply according to

$$H^\theta(\mu, A, \varepsilon_A, B) = \max_{B', N} \left[C - \frac{\omega}{1 + \frac{1}{\lambda}} N^{1 + \frac{1}{\lambda}} + \beta \mathbb{E}^\theta (H^\theta(\mu', A', \varepsilon'_A, B') | \mu, A, \varepsilon_A) \right]. \quad (27)$$

The household states include the distribution $\mu(s, k, b)$ across firm states $s = (z, \epsilon_z)$, k , and b , macro TFP A , macro news ε_A , and holdings of the bond B . The household's budget constraint is given by

$$C + B' = RB + WN + D^F + D^L + T.$$

Above D^F reflects net aggregate payouts from the firm sector, and D^L reflects aggregate payouts from lenders, both of whom are owned by the household. T reflects lump-sum transfers of corporate taxes from firms and lenders to the household. The household takes as given both the real interest rate R and the wage W .

A.8.2 Lenders

A continuum of diagnostic risk-neutral lenders funds itself via risk-free debt, taking as given the real interest rate R . Each lender participates in a range of competitive credit markets for firms with state s , future capital k' , and a desired loan amount b' . If a borrower firm defaults on its debt, the lender receives the after-tax recovery rate

$$\mathcal{R}(k', b') = \gamma(1 - \tau)(1 - \delta) \frac{k'}{b'},$$

where $1 - \gamma$ reflects a deadweight loss upon default. A lender's expected present discounted value of participating in the market for lending to firm (s, k', b') given the aggregate state

(μ, A, ε_A) is given by

$$-q(s, k', b', \mu, A, \varepsilon_A)b' + \frac{1}{R}\mathbb{E}^\theta [(1 - df(s', k', b', \mu', A', \varepsilon'_A))b' + df(s', k', b', \mu', A', \varepsilon'_A)b'\mathcal{R}(k', b')|s, \mu, A, \varepsilon_A]. \quad (28)$$

A lender must choose to enter, or not to enter, each individual credit market, and any net profits are paid to the household owner.

A.8.3 Firms

A continuum of competitive diagnostic firms with idiosyncratic states (s, k, b) owned by the household takes as given the real interest rate R , the wage W , and a debt price schedule $q(s, k', b', \mu, A, \varepsilon_A)$, choosing investment for future capital k' , debt issuance b' , static labor demand n , and whether to default in order to maximize the expected present discounted value of its payouts according to

$$V^\theta(s, k, b, \mu, A, \varepsilon_A) = \max_{df \in \{0,1\}} [dfV_D^\theta(s, \mu, A, \varepsilon_A) + (1 - df)V_{ND}^\theta(s, k, b, \mu, A, \varepsilon_A)] \quad (29)$$

$$V_{ND}^\theta(s, k, b, \mu, A, \varepsilon_A) = \max_{k', b', n} \left\{ d - IC(d) + \frac{1}{R}\mathbb{E}^\theta [V^\theta(s', k', b', \mu', A', \varepsilon'_A)|s, \mu, A, \varepsilon_A] \right\} \quad (30)$$

$$d = \frac{(1 - \tau)[y - Wn - AC(i, k) - \phi]}{+q(s, k', b', \mu, A, \varepsilon_A)b' - i - b + \tau(R - 1 + \delta k)}$$

$$V_D^\theta(s, \mu, A, \varepsilon_A) = \left\{ 0 + \frac{1}{R}\mathbb{E}^\theta [V(s', 0, 0, \mu', A', \varepsilon'_A)|s, \mu, A, \varepsilon_A] \right\} \quad (31)$$

A.8.4 Definition

An equilibrium in this economy under diagnostic expectations is a collection including

- a wage function $W^\theta(\mu, A, \varepsilon_A)$,
- a fixed real interest rate R ,
- a debt pricing function $q^\theta(s, k', b', \mu, A, \varepsilon_A)$,
- household value H^θ and policy functions N^θ, B'^θ (each functions of $(\mu, A, \varepsilon_A, B)$),
- lender participation decisions for each credit market (s, k', b') ,

- firm value and policy functions V^θ , V_{ND}^θ , V_D^θ , k'^θ , b'^θ , n^θ and df^θ (each functions of $(s, k, b, \mu, A, \varepsilon_A)$), and
- a transition mapping $\mu' = \Gamma^\theta(\mu, A, \varepsilon_A)$ for the distribution $\mu(s, k, b)$ across periods

such that

- taking as given wages and the interest rate, household values and policies satisfy their dynamic problem (27) under DE beliefs from (12),
- taking as given debt prices and firm policies, lenders optimally whether to participate or not in each credit market given their payoffs (28) under DE beliefs from (12) and (11),
- taking as given wages, the interest rate, and debt prices, firm values and policies satisfy their dynamic problem defined by (29), (30), and (31) under DE beliefs from (12) and (11),
- labor markets clear with labor demand from firms equal to labor supply from households

$$\int n^\theta(s, k, b, \mu, A, \varepsilon_A) d\mu(s, k, b) = N^\theta(\mu, A, \varepsilon_A, B),$$

- individual firm credit markets clear with nonpositive expected payouts in (28) for each market (s, k', b') and strictly zero expected payouts when the market is active,
- the risk-free debt market clears with risk-free debt supply from the households equal to the total funding needs of the lenders as required by firm borrowing

$$B'^\theta(\mu, A, \varepsilon_A, B) = \int b'^\theta(s, k, b, \mu, A, \varepsilon_A) d\mu,$$

and

- the transition mapping Γ^θ accurately reflects transitions of states given firm policies and diagnostic expectations, i.e.,

$$\int_{\{(s', k', b') \in \mathcal{A}\}} d\mu'(s', k', b') = \int_{\mathcal{B}} f^\theta(s'|s) d\mu(s, k, b)$$

$$\mathcal{B}(\mathcal{A}, \mu, A, \varepsilon_A) = \{(s, k, b) | (s', k'^\theta, b'^\theta) \in \mathcal{A} \text{ for some } s'\}$$

for any well behaved set \mathcal{A} whenever $\mu' = \Gamma^\theta(\mu, A, \varepsilon_A)$.

A.8.5 Characterization of Prices

Note that the linearity of household utility over consumption, together with optimal savings in risk-free debt, implies that a constant risk-free rate $R = \frac{1}{\beta}$ is the only one consistent with equilibrium. Discounted payoffs of firms and lenders at this interest rate are therefore equivalent to expected present discounted value maximization at the household's stochastic discount factor (in this case degenerate to β). Furthermore, a household's labor supply optimality condition is given by

$$\omega N^{\frac{1}{\lambda}} = W,$$

and since this expression doesn't depend upon consumption C it must also not depend upon household debt B . Therefore, the labor market clearing condition above can be rewritten

$$\left(\frac{W^\theta(\mu, A, \varepsilon_A)}{\omega} \right)^\lambda = \int n^\theta(s, k, b, \mu, A, \varepsilon_A) d\mu(s, k, b),$$

which is equivalent to the Equation (19) given in the main text. Also note that linearity of the household's preferences allows us to avoid specifying the nature of aggregate resource constraints or goods market clearing, since these details do not impact firm, lender, or household policies at the margin, although it is natural and harmless to assume that the structure of taxation clears a fiscal budget constraint each period and that the household absorbs any unexpectedly high or low aggregate payouts from diagnostic firms and lenders. Finally, note that for any market in which lending is active, the competitive nature of lending together yields zero diagnostically expected discounted profits in Equation (28) so that

$$q^\theta(s, k', b', \mu, A, \varepsilon_A) = \frac{1}{R} \mathbb{E}^\theta \left[(1 - df^\theta(s', k', b', \mu', A', \varepsilon'_A)) + df^\theta(s', k', b', \mu', A', \varepsilon'_A) \mathcal{R}(k', b') \mid s, \mu, A, \varepsilon_A \right]. \quad (32)$$

A.8.6 General Equilibrium Solution Algorithm

One might suspect that this problem poses more difficulty than the usual rational expectations model because of diagnostic expectations. But the characterization of prices above, dependent upon the careful assumption of linear consumption utility, actually implies that we face only the usual numerical challenges that arise in any model with a "Krusell Smith problem." In other words, there are two familiar computational challenges to numerically solving the general equilibrium model laid out above. First, the distributional state μ entering into the pricing function W^θ is intractable. Second, the transition mapping Γ^θ (implicitly appearing in the optimal debt pricing condition and firm value functions) and the wage mapping W^θ appearing in firm and household problems are also intractable and unavailable in closed form.

We implement a tractable approach to addressing both problems by replacing the distributional state μ with a truncated history of length K of the aggregate TFP shocks $(A_{t-K}, A_{t-K+1}, \dots, A_{t-1}, A_t)$. For moderate history lengths K , the approximate state is more tractable than the distribution μ , and the mapping Γ^θ collapses to the simple diagnostic expectations of the evolution of the exogenous process which we have already exploited. With this simplification in hand, we then follow an outer loop/inner loop approach to solving the model.

1. Guess a mapping from a truncated history of macro states

$$(A_{t-K}, A_{t-K+1}, \dots, A_{t-1}, A_t) \rightarrow W_t.$$

2. Solve the model conditional upon this tractable truncated history, where $(A_t, A_{t-1}, \dots, A_{t-K})$ enters the firm's state vector and hence the Bellman equations determining investment, default, and debt issuance policies.
3. Simulate the model for a large number of periods $t = 1, \dots, T$, clearing markets with W_t in each period t by numerically solving the nonlinear equation

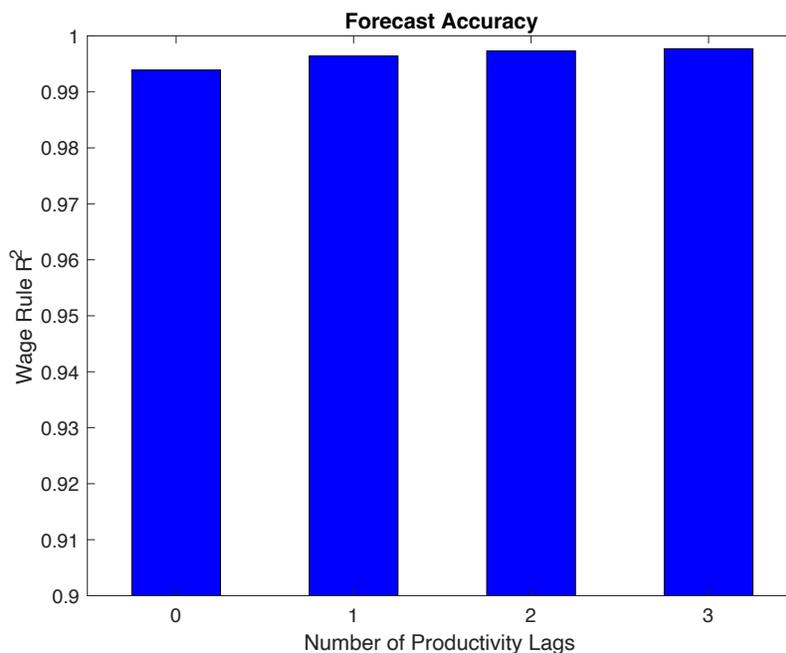
$$\left(\frac{W_t}{\omega}\right)^\lambda = \int n(s, k, b|W_t) d\mu_t(s, k, b)$$

for each period t in the simulation. Note that this is a well behaved nonlinear equation in one variable. The analytically computable static labor policies $n(s, k, b|W_t)$ are strictly declining in W_t on the RHS and the function on the LHS is strictly increasing in W_t . In practice, markets can be cleared robustly using bisection or another similar algorithm.

4. Based on the simulated wage data, update your wage prediction mapping from Step 1. If the mapping has converged to within some tolerance, exit. If not update the mapping and return to Step 1.

A few practical comments are in order about this approach to solving the Krusell Smith problem, which to our knowledge is a novel, although conceptually straightforward, solution method. First, given the discretized macro TFP state space, we store the wage mapping nonparametrically as a matrix of mean wages conditional upon each combination of truncated macro TFP histories. After simulation, the wage mapping update step simply involves repeated calculations of mean wages within the appropriate subsamples of the simulated data. Second, because the macro state is replaced with macro TFP shock histories rather than with an augmented endogenous macro moment, there is no need to create an approximate anticipated default rule used to price debt. Lenders simply price debt according to the usual

Figure A.5: Wage Predictions and TFP Lags

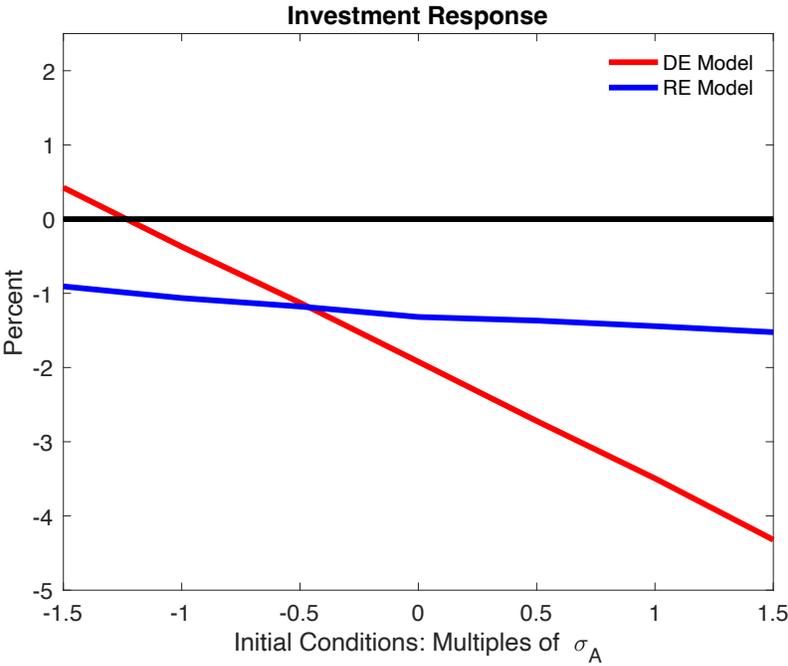


Notes: The figure plots the R^2 of nonparametric regressions of log wages on discrete histories of macro TFP of increasing length.

no-arbitrage condition in Equation (32) above. Third, because no endogenous moments are forecasted in our solution method, there is no Den Haan (2010)-style distinction between static and dynamic forecasts of the wage. In other words, there is no room for forecast errors about endogenous macro moments to accumulate over time, since only exogenous shock histories are used for forecasts. So, unlike in typical adaptations of the Krusell and Smith (1998) method, the R^2 of the implicit wage forecast rule is in this case an appropriate metric of accuracy. With this in mind, Figure A.5 plots the estimated R^2 of regressions of the log wage on fully populated sets of dummies for macro TFP histories of up to a given lag length. Once a single lag is taken into account, incorporating information from yesterday's TFP level about the current distribution of capital and hence labor demand in the cross section, the R^2 measures mostly stabilize. Our baseline case, which uses a single lag with $K = 1$ in the wage prediction rule, is therefore a parsimonious but apparently accurate choice.

We use this solution to re-compute investment nonlinearities in Figure 2 for the GE case, with the results contained in Figure A.6.

Figure A.6: Investment Nonlinearity with General Equilibrium

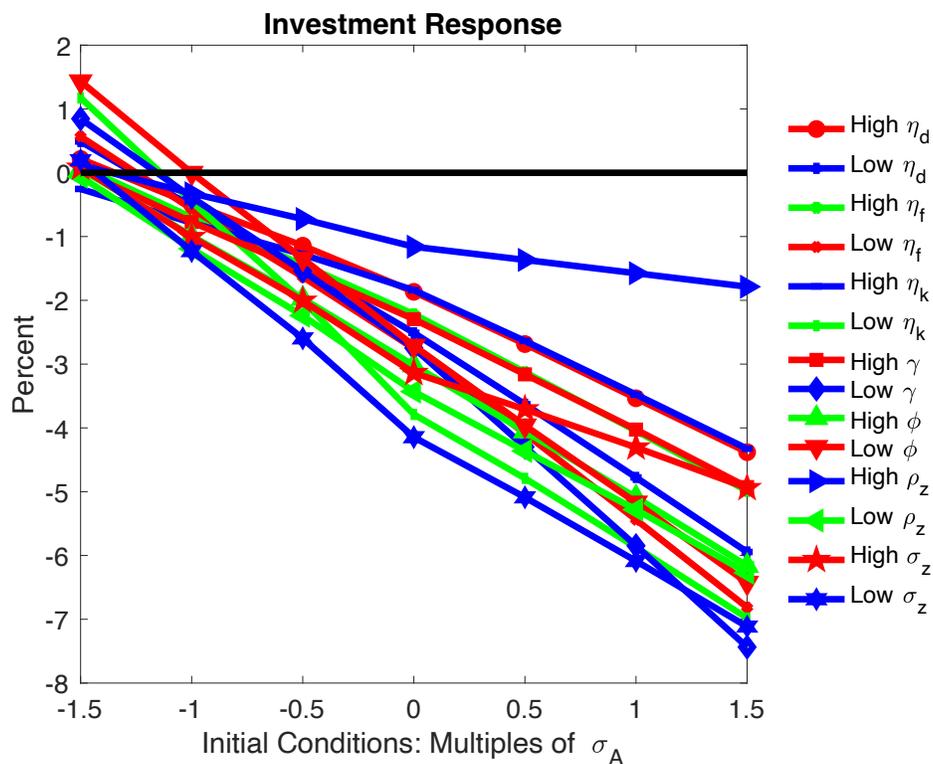


Notes: The vertical axis in the figure reports the simulated first-period impulse response of macro investment to a one-standard deviation negative shock to macro TFP, and the horizontal axis reports the initial conditions, i.e., the magnitude of the shock to macro TFP in the previous period. Both the DE model (red line) and RE model (blue line) are reported on the figure. These results are computed in the general equilibrium model with labor market clearing.

A.9 Parameter Robustness Checks for Investment Nonlinearity

In the baseline estimated DE model in partial equilibrium we recomputed investment the nonlinearity plot in Figure 2 for a range of parameter robustness checks, with the resulting responses of investment plotted in Figure A.7.

Figure A.7: Investment Nonlinearity with Alternative Parameters



Notes: The vertical axis in the figure reports the simulated impulse response of macro investment to a one-standard deviation negative shock to macro TFP, and the horizontal axis reports the initial conditions, i.e., the magnitude of the shock to macro TFP in the previous period. All models are perturbations of the estimated Baseline DE model, with a single parameter shifted to round higher or lower values bracketing the parameter point estimate. The autocorrelation of micro TFP ρ_z is set to 0.75 (high) and 0.6 (low). The volatility of micro TFP σ_z is set to 0.3 (high) and 0.2 (low). The capital adjustment cost η_k is set to 4.3 (high) and 3.7 (low). The fixed operating cost ϕ is set to 0.25 (high) and 0.05 (low). The recovery rate γ is set to 0.30 (high) and 0.15 (low). The equity fixed cost η_f is set to 0.025 (high) and 0.005 (low). The equity linear cost η_d is set to 0.15 (high) and 0.05 (low). Macro TFP volatility σ_A , not reported, is held constant in order to keep the shock experiment constant. The magnitude of diagnosticity θ , also held constant, is already varied in Figure 2 to extreme values. The noise in earnings σ_π , also not reported, affects only simulated micro moments.

B Data

B.1 Microdata on Firm Beliefs from Compustat and IBES-Guidance

In our analysis of firm financial and profit forecasts we use a combination of the Compustat Fundamentals Annual and IBES manager guidance databases (S&P Global and CRSP, 2020; Thomson Reuters, 2017, 2019; WRDS, 2020b). The combined sample for the Compustat-IBES data spans 1999-2017 for 9666 firm-fiscal years spanning 1391 firms. To construct our sample, we discard utilities and financials as well as any firm-years with negative values for assets, capital, employment or sales. Descriptive statistics for each variable from this sample used in our analysis, as well as firm revenues and capital, are reported in Table B.1.

Table B.1: Sample Descriptive Statistics

Quantity	Mean	Standard Deviation
Sales	6549.4	22231.5
Capital	1492.1	6185.2
Assets	6689.6	27242.3
Profit	0.4362	1.0310
Investment	0.3104	0.2331
Debt	0.2708	0.2377
Forecast Error	-0.2609	0.7842
Forecast	0.7994	0.9256

Notes: The table reports descriptive statistics for the sample of 1391 firms from 1999-2017 and 9666 firm-years in the combined Compustat-IBES database. The first three rows represent revenues and the book value of the capital stock and total assets, in \$ millions. The remaining rows reflect the ratio of realized earnings to the book value of the capital stock, the capital expenditures investment rate, the ratio of end of period total liabilities to the asset stock, the next-period forecast error defined as realized future profits minus manager guidance scaled by firm capital, and the next period forecast defined as manager profit guidance scaled by firm capital. The sample was winsorized before computing the descriptive statistics above.

The variable definitions are given as follows, with both empirical and model information attached:

- **Earnings or profits** are equal to GAAP net income, Compustat `ib`. The model equivalent is $\pi = (1 - \tau)(y - Wn - AC(i, k) - \phi) + \tau((R - 1)b + \delta k) - \delta k$.
- **Capital** k is equal to the book value of plants, property, and equipment, Compustat `ppent`. The model equivalent is the state variable k .
- **Investment** i is equal to the total value of capital expenditures, Compustat `capxv`. The model equivalent is the policy variable $i = k' - (1 - \delta)k$.
- **Debt** b is equal to the total net value of liabilities, Compustat `dltt + dlc - che`. The model equivalent is the state variable b .

- **Forecast error** fe is equal to the realized value of earnings π minus the forecast level of earnings π^f made from the previous fiscal year, where realized earnings are Compustat `ib` and forecast earnings are equal to manager guidance from the IBES database. The model equivalent is the earnings value π above, minus the forecast level implied by firm-level diagnostic expectations, the definition of π , and firm policies predetermined in the previous period.

We also use the merged Compustat-IBES guidance sample to run various robustness checks to the firm forecast error predictability regressions reported in the main text. Table B.3 shows similar forecast error predictability maintains after the Great Recession, in a sample of firms present for five or more years in the data, and after discarding all firms with high-yield debt as classified by Moody’s ratings.

B.2 Microdata on Bonds from FISD-TRACE

We use the WRDS US Corporate Bond Return database ([WRDS, Mergent, and FINRA, 2020](#); [WRDS, 2020a](#)), which merges the Mergent FISD and FINRA TRACE datasets with issuance and secondary market information on corporate bond issues, respectively. We consider only unsecured, unconvertible debentures and convert secondary market yields to spreads based on comparable Treasury rates, with a resulting dataset of around 80,000 issues from mid-2002 to late 2019. We link the bond return database to Compustat firm financials through the WRDS CRSP link, and we aggregate from the issue to firm level by computing average yields and bond returns for a firm in Q4 of a given year. The resulting dataset spans around 1,500 large US public firms. Linking this panel to the IBES-manager guidance data yields the sample used in Table 2 in the main text. Table B.5 replicates Table 2 conditioning only on investment grade bonds. Table B.6 replicates Table 2 but does not include current profit controls for the investment regressions nor lagged spread controls for the spread regressions.

B.3 Macro Data

At the macro level, we use a combination of information from the NIPA accounts, the US Treasury, and John Fernald’s website ([BEA, 2020b](#); [US Treasury, 2020](#); [BEA, 2020a](#); [Fernald, 2020](#)). The following variables are relevant, all at annual frequency or converted to annual frequency by averaging.

- **Output** Y is real GDP from the NIPA accounts.
- **Investment** I is real nonresidential private investment from the NIPA accounts in the data.

- **Spreads** are the mean spread across firms in a given year in our FISD-TRACE or WRDS US Corporate Bond Return Database. The spreads are computed as yields relative to a matched-maturity Treasury yield in the same period.
- **TFP** is the annualized value of the series `dtfp` from John Fernald’s website.
- **Profit Fcst** is the sum of earnings guidance across firms in a given year from our Compustat-IBES merged database.

In the spread matching/Great Recession exercise in Figure 4, we report the average spread growth and the average growth of investment, TFP, and profit forecasts in each subperiod.

B.4 SMM Estimation

Our SMM estimation exercise in Section 5.1 involves three steps: 1) moment and covariance matrix calculations, 2) model estimation, and 3) standard error calculation. We detail each of these steps in turn.

B.4.1 Moment and Covariance Matrix Calculation

Table 5 reports a set of target moments at the micro and macro levels for our SMM estimation exercise. The micro moments are a covariance matrix of the vector

$$X_{it} = (\text{Profit}_{it}, \text{Investment}_{it}, \text{Debt}_{it}, \text{Spread}_{it})'$$

for firm i in fiscal year t from our merged Compustat-IBES-FISD-TRACE sample. The micro moments also include the covariance of future forecast error growth with lagged investment and debt issuance growth at a two-period horizon, together with the variance of future forecast errors. The merged sample with all of the required variables available spans 493 firms and 4697 total observations. To compute the micro moments, we use the following procedure:

- Demean X_{it} by firm and year to obtain \hat{X}_{it}
- Compute the covariance matrix as the mean of $\hat{X}_{it}\hat{X}'_{it}$.
- Apply the standard formula for the clustered covariance of a mean vector to obtain the moment covariance matrix Ω_{Micro} , clustering across firms.

With the estimated micro moments and the estimated moment covariance matrix for the micro moments in hand, we then turn to the calculation of the macro moments and their covariance matrix. Note that the macro moments are the mean default rate, the mean

spread, and the standard deviation of real GDP growth. We compute the mean default and spread series from our merged FISD-TRACE data on corporate debt, and we compute real GDP growth from the NIPA data. The point estimates of these macro moments are trivial to compute. We then compute an estimate of the covariance matrix of these macro moments Ω_{Macro} using a stationary block bootstrap.

Note that for our later inference based on clustering at the firm level, we will rely upon an assumption that the macro sample length T^{macro} and the total number of observations in the microdata sample T behave proportionally with $T^{macro}/T \rightarrow \gamma$ for some constant γ as $N \rightarrow \infty$. This allows us to rely on asymptotics of the basic form

$$\sqrt{T^{macro}}(\hat{\mu} - \mu) \rightarrow_d N(0, \Omega), \quad (33)$$

where $\hat{\mu}$ is the estimated moment vector (with micro and macro moments) and Ω is the joint moment covariance matrix. Table 5 reports $\hat{\mu}$.

B.4.2 Point Estimate Calculation

We compute the point estimates $\hat{\beta}$ for the vector of estimated parameters β in Table 4 by solving the following standard SMM optimization problem

$$\min_{\beta} (\mu^S(\beta) - \hat{\mu})' \hat{\Omega}^{-1} (\mu^S(\beta) - \hat{\mu})$$

where $\mu^S(\beta)$ is the model value of the moments given β computed from simulated data, $\hat{\Omega}^{-1}$ is the asymptotically efficient weighting matrix given by the inverse of the estimated moment covariance matrix, and $\hat{\mu}$ is the empirical moment vector. We employ particle swarm optimization to solve this optimization problem, a stochastic global optimization routine that bears substantial similarity to simulated annealing and genetic algorithms.

B.4.3 Standard Error Calculation

Given the ratio between the number of observations T^{sim} in the model simulation used to compute $\mu^S(\beta)$ and the empirical number of observations T , the SMM estimator's asymptotic covariance matrix Σ follows

$$\sqrt{T}(\hat{\beta} - \beta) \rightarrow_d N(0, \Sigma) \quad (34)$$

where

$$\Sigma = \left(1 + \frac{T}{T^{sim}}\right) \left(\frac{\partial \mu^S(\beta)}{\partial \beta'} \Omega^{-1} \frac{\partial \mu^S(\beta)}{\partial \beta}\right)^{-1}. \quad (35)$$

Equation (35) yields a feasible formula for Σ after substitution of the estimated covariance

matrix $\hat{\Omega}$ and numerical calculation of the moment Jacobian matrix $\frac{\partial \mu^S(\beta)}{\partial \beta'}$ within the model using forward differentiation from the point estimates $\hat{\beta}$. With these elements in hand, Tables 4 reports standard errors based on the approximating variance from (34).

B.5 GMM Regression Coefficients with Firm Fixed Effects

In this appendix section we provide a statistical framework for the analysis of forecast over-reaction allowing for firm fixed effects through the use of within-firm differences and GMM estimators. We use these results directly to provide reduced-form regression coefficients with firm fixed effects in Appendix Table B.2 via GMM, and they also prove useful for generating intuition for the identification of the beliefs parameter θ in our quantitative SMM estimation of the neoclassical model in Section 5.1.

B.5.1 Data Generating Process

Let x_{it} be the observed value of the forecasted variable \tilde{x}_{it} (e.g., profits) for firm i in period t , following an AR(1) with firm fixed effects μ_i^x and subject to measurement error ν_{it}^x with MA(1) dynamics:

$$\begin{aligned}\tilde{x}_{it} &= \mu_i^x + \rho \tilde{x}_{it-1} + \epsilon_{it}, & \epsilon_{it} &\sim N(0, \sigma_\epsilon^2) \\ x_{it} &= \tilde{x}_{it} + \nu_{it}^x + \gamma_x \nu_{it-1}^x, & \nu_{it}^x &\sim N(0, \sigma_{x,\nu}^2)\end{aligned}$$

Let y_{it} be the observed value of a variable \tilde{y}_{it} (e.g., investment) linked linearly to the forecasted variable, subject to its own fixed effects μ_i^y and its own MA(1) measurement error ν_{it}^y :

$$\begin{aligned}\tilde{y}_{it} &= \mu_i^y + \alpha \tilde{x}_{it} \\ y_{it} &= \tilde{y}_{it} + \nu_{it}^y + \gamma_y \nu_{it-1}^y, & \nu_{it}^y &\sim N(0, \sigma_{y,\nu}^2).\end{aligned}$$

Allow forecasts $\tilde{x}_{it+1|t}^f$ to follow the DE formula, but also allow for forecast bias in the form of fixed effects μ_i^f and for MA(1) measurement noise ν_{it}^f in the observed value $x_{it+1|t}^f$

$$\begin{aligned}\tilde{x}_{it+1|t}^f &= \mu_i^f + \rho \tilde{x}_{it} + \rho \theta \epsilon_{it} \\ x_{it+1|t}^f &= \tilde{x}_{it+1|t}^f + \nu_{it}^f + \gamma_f \nu_{it-1}^f, & \nu_{it}^f &\sim N(0, \sigma_{f,\nu}^2).\end{aligned}$$

The implied observed forecast errors $f e_{it+1} = x_{it+1} - x_{it+1|t}^f$ are then given by

$$f e_{it+1} = (\mu_i^x + \rho \tilde{x}_{it} + \epsilon_{it+1} + \nu_{it+1}^x + \gamma_x \nu_{it}^x) - (\mu_i^f + \rho \tilde{x}_{it} + \rho \theta \epsilon_{it} + \nu_{it}^f + \gamma_f \nu_{it-1}^f)$$

$$fe_{it+1} = \left(\mu_i^x - \mu_i^f \right) + \epsilon_{it+1} + \nu_{it+1}^x + \gamma_x \nu_{it}^x - \rho \theta \epsilon_{it} - \nu_{it}^f - \gamma_f \nu_{it-1}^f$$

B.5.2 Nonoverlapping Difference Covariances as Useful Moments

Choose any difference horizons $\bar{s}, \underline{s} \geq 1$. Then the data generating process laid out above implies that the covariance of future forecast error growth with current profit growth in the firm is given by

$$\begin{aligned} & cov(fe_{it+\bar{s}} - fe_{it}, x_{it-1} - x_{it-\underline{s}}) = \\ & cov \left(\begin{array}{cc} \epsilon_{it+\bar{s}} + \nu_{it+\bar{s}}^x + \gamma_x \nu_{it+\bar{s}-1}^x - \rho \theta \epsilon_{it+\bar{s}-1} - \nu_{it+\bar{s}-1}^f - \gamma_f \nu_{it+\bar{s}-2}^f & \epsilon_{it-1} + \nu_{it-1}^x + \gamma_x \nu_{it-2}^x + \rho \tilde{\alpha}_{t-2} \\ -\epsilon_{it} - \nu_{it}^x - \gamma_x \nu_{it-1}^x + \rho \theta \epsilon_{it-1} + \nu_{it-1}^f + \gamma_f \nu_{it-2}^f & -\rho \tilde{\alpha}_{t-2-\underline{s}} - \epsilon_{it-1-\underline{s}} - \nu_{it-1-\underline{s}}^x - \gamma_x \nu_{it-1-\underline{s}-1}^x \end{array} \right) \\ & = \rho \theta cov(\epsilon_{it-1}, \epsilon_{it-1}) - \gamma_x cov(\nu_{it-1}^x, \nu_{it-1}^x) = \rho \theta \sigma_\epsilon^2 - \gamma_x \sigma_{\nu,x}^2. \end{aligned}$$

We immediately obtain our first result, that the covariance of future forecast growth with profit growth can offer evidence of diagnosticity in beliefs but may be inconclusive in the presence of MA(1) measurement error in profits.

Result 1: If $cov(fe_{it+\bar{s}} - fe_{it}, x_{it-1} - x_{it-\underline{s}}) = \rho \theta \sigma_\epsilon^2 - \gamma_x \sigma_{\nu,x}^2 > 0$ for some $\bar{s}, \underline{s} \geq 1$ and $\gamma_x > 0$, then $\theta > 0$.

To exploit similar intuition but avoid cross-contamination with profit measurement error, we can also compute the covariance of future forecast error growth with the growth of outcomes y_{it} in the firm such as investment or debt.

$$cov(fe_{it+\bar{s}} - fe_{it}, y_{it-1} - y_{it-\underline{s}}) = \alpha \rho \theta cov(\epsilon_{it-1}, \epsilon_{it-1}) = \alpha \rho \theta \sigma_\epsilon^2$$

We then immediately obtain our second result, which is that covariances of future forecast error growth with investment or debt growth in the firm should be zero unless expectations are diagnostic with $\theta = 0$.

Result 2: If $cov(fe_{it+\bar{s}} - fe_{it}, y_{it-1} - y_{it-\underline{s}}) = \alpha \rho \theta \sigma_\epsilon^2 > 0$ for some $\bar{s}, \underline{s} \geq 1$ and $\alpha > 0$, then $\theta > 0$.

Result 2, which holds analytically in closed form and still holds approximately in our nonlinear neoclassical model, provides direct motivation for our use of the forecast error growth covariance moments in Section 5.1.

B.5.3 Obtaining Regression Coefficients

In this section, we derive formulas for the asymptotically normal and consistent GMM estimation of regression coefficients of future forecast errors on investment in the firm, allowing for both measurement error and fixed effects in all series. In particular, using the notation of our data generating process, our goal is to estimate regression coefficients linking the following ‘‘cleansed’’ variables to one another:

- Forecast errors without measurement error or fixed effects $fe_{it+1}^* = \epsilon_{it+1} - \rho\theta\epsilon_{it}$
- The linked variable y without measurement error or fixed effects $y_{it}^* = \tilde{y}_{it} - \mu_i^y$

Now, note a few simple results based on our data generating process above.

$$\begin{aligned} cov(fe_{it+1}^*, y_{it}^*) &= cov(\epsilon_{it+1} - \rho\theta\epsilon_{it}, \alpha_y\epsilon_{it} + \dots) = -\alpha_y\rho\theta\sigma_\epsilon^2 \\ var(y_{it}^*) &= \alpha^2 \frac{\sigma_\epsilon^2}{1 - \rho^2}. \end{aligned}$$

Therefore, the probability limit of a univariate regression of forecast errors fe_{it+1}^* on the linked variable y_{it}^* is given by

$$\beta_y = \frac{cov(fe_{it+1}^*, y_{it}^*)}{var(y_{it}^*)} = \frac{-\alpha\rho\theta\sigma_\epsilon^2}{\alpha^2 \frac{\sigma_\epsilon^2}{1-\rho^2}} = \frac{-\rho\theta\sigma_\epsilon^2}{\alpha \frac{\sigma_\epsilon^2}{1-\rho^2}}.$$

Now, this is true for each linked variable y , and in particular for $y = i$ (investment) and $y = b'$ (debt issuance). The statistical framework above implies the following moment relationships.

$$\begin{aligned} m_1 &= cov(fe_{it+1} - fe_{it}, i_{it-1} - i_{it-1}) = \alpha_i\rho\theta\sigma_\epsilon^2 \\ m_2 &= cov(fe_{it+1} - fe_{it}, b'_{it-1} - b'_{it-1}) = \alpha_b\rho\theta\sigma_\epsilon^2 \\ m_3 &= cov(b'^*_{it-1}, i^*_{it}) = \alpha_i\alpha_b \frac{\sigma_\epsilon^2}{1 - \rho^2} \end{aligned}$$

But all of these are estimable consistently. m_1 and m_2 are probability limits of covariances of differences. m_3 is the probability limit of the covariance of residualized investment and debt. Consider a general central limit theorem

$$\sqrt{N}(m(X) - m) \rightarrow_d N(0, \Sigma)$$

where $m(X) = (m_1, m_2, m_3)'$ is the moment estimate from the data X , N is the observation count and Σ is the asymptotic variance of $m(X)$ allowing for firm-level clustering. Note that Σ can be estimated via off-the-shelf closed-form form econometric formulas. Then consider the desired regression coefficient $\beta_i = -\frac{m_2}{m_3}$, which is a function of the underlying vector m . Via the Delta method we have

$$\sqrt{N}(\hat{\beta}_i - \beta_i) \rightarrow_d N(0, \Omega),$$

where the asymptotic variance of the regression coefficient vector is given by

$$\Omega = \frac{\partial\beta_i}{\partial m'} \Sigma \frac{\partial\beta_i}{\partial m}.$$

All of these objects have feasible estimators, since the Jacobian has a simple form given by

$$\frac{\partial \beta_i}{\partial m'} = \begin{bmatrix} 0 & \frac{-1}{m_3} & \frac{m_2}{m_3^2} \end{bmatrix}.$$

With this framework in hand, we note that Appendix Table B.2 in column (3) reports the regression coefficient point estimate $\hat{\beta}_i = \beta_i(\hat{m})$ as well as the standard error $\sqrt{\frac{\text{diag}\hat{\Omega}}{N}}$.

B.6 AR(1) with Misperceived Persistence

There are multiple departures from full information rational expectations which in principle can generate overreaction of beliefs, including both DE as well as models in which agents systematically inflate the perceived persistence of an underlying AR(1) model. To compare the two belief systems, consider data X_t which follows an underlying true AR(1)

$$\log X_{t+1} = \rho \log X_t + \epsilon_{t+1}, \quad \epsilon_{t+1} \sim N(0, \sigma^2).$$

A agent with diagnostic beliefs governed by parameter θ makes forecast

$$\mathbb{E}_t^\theta \log X_{t+1} = \rho \log X_t + \theta \rho \epsilon_t,$$

leading to forecast error

$$\begin{aligned} FE_{t+1}^\theta &= \log X_{t+1} - \mathbb{E}_t^\theta \log X_{t+1} \\ &= \rho \log X_{t+1} + \epsilon_{t+1} - (\rho \log X_t + \theta \rho \epsilon_t) \\ &= \epsilon_{t+1} - \theta \rho \epsilon_t, \end{aligned}$$

implying a negative covariance, i.e., overreaction, given by

$$\begin{aligned} \text{cov} [FE_{t+1}^\theta, \log X_t] &= \text{cov} [\epsilon_{t+1} - \theta \rho \epsilon_t, \rho \log X_{t-1} + \epsilon_t] \\ &= -\theta \rho \sigma^2. \end{aligned}$$

Now, as an alternative, consider an agent who in the same context perceives an AR(1) process with persistence $\hat{\rho}$. The agent's forecast is

$$\mathbb{E}_t^{\hat{\rho}} \log X_{t+1} = \hat{\rho} \log X_t,$$

leading to forecast error

$$\begin{aligned} FE_{t+1}^{\hat{\rho}} &= \log X_{t+1} - \hat{\rho} \log X_t \\ &= \rho \log X_t + \epsilon_{t+1} - \hat{\rho} \log X_t \end{aligned}$$

$$= (\rho - \hat{\rho}) \log X_t + \epsilon_{t+1},$$

implying a covariance

$$\begin{aligned} \text{cov} [FE_{t+1}^{\hat{\rho}}, \log X_t] &= \text{cov} [(\rho - \hat{\rho}) \log X_t + \epsilon_{t+1}, \log X_t] \\ &= (\rho - \hat{\rho}) \text{cov} [\log X_t, \log X_t] \\ &= (\rho - \hat{\rho}) \frac{\sigma^2}{1 - \rho^2}. \end{aligned}$$

This covariance is negative, i.e., overreaction occurs on the part of the agent, whenever $\hat{\rho} > \rho$. So overestimated persistence maps to overreaction.

To compare the two belief systems, it is useful to consider the degree of perceived persistence $\hat{\rho}$ required to equalize the degree of overreaction across the diagnostic and misperceived AR(1) versions of beliefs. In this case $\hat{\rho}$ must satisfy

$$\begin{aligned} \text{cov} [FE_{t+1}^{\hat{\rho}}, \log X_t] &= \text{cov} [FE_{t+1}^{\theta}, \log X_t] \\ (\rho - \hat{\rho}) \frac{\sigma^2}{1 - \rho^2} &= -\theta \rho \sigma^2 \\ \hat{\rho} &= \rho (1 + \theta(1 - \rho^2)). \end{aligned}$$

As expected, to match diagnostic expectations with overreaction and $\theta > 0$ requires $\hat{\rho} > \rho$.

Operationalizing this formula using our baseline micro and macro TFP process estimates, together with the estimated degree of diagnosticity, from Table 4 implies perceived persistence parameters given by

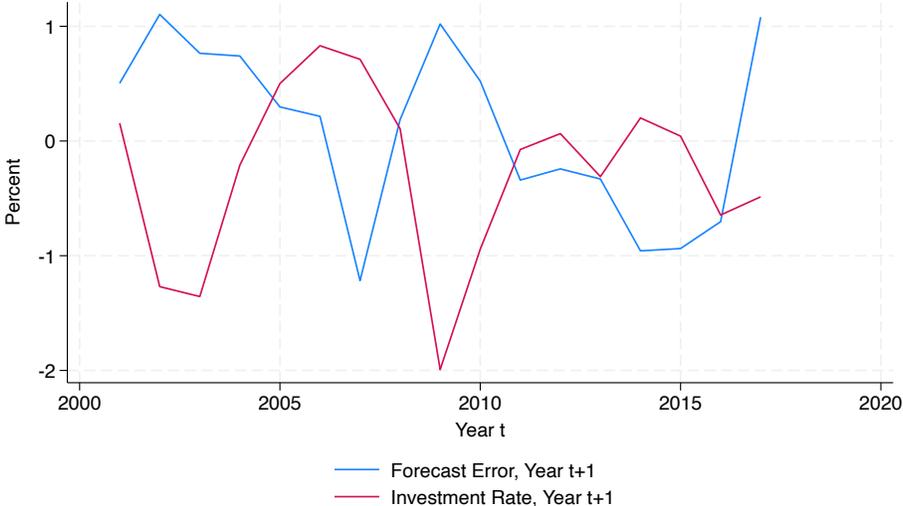
$$\text{Micro TFP: } \hat{\rho}_z = 1.0071$$

$$\text{Macro TFP: } \hat{\rho}_A = 1.0346.$$

Forecast overreaction to the degree we estimate in our baseline requires nonstationary AR(1) beliefs with $\hat{\rho} > 1$. Nonstationary beliefs are incompatible with the stationary Bellman Equations (15) and (16) characterizing firm decisionmaking. Intuitively, the degree of overreaction and forecast error volatility we see in the data are high, requiring a perceived persistence incompatible with stationarity because of the high fundamental persistence common in macroeconomic and firm-level analysis. An advantage of diagnostic beliefs in this context is therefore the maintained stationarity of beliefs which nevertheless match observed overreaction patterns.

B.7 Robustness Tables and Figures

Figure B.1: Aggregate Forecast Errors



Notes: Horizontal axis is year. Red line is mean investment rate. Blue line is mean forecast error in the following year. Variables standardized. Correlation in the two series is -0.61.

Table B.2: Predictable Forecast Errors, OLS vs GMM

	(1)	(2)	(3)
Dependent Variable:	Forecast Error _{t+1}		
Estimation Method:	OLS	OLS	GMM
Investment _t	-1.037*** (0.069)	-0.457*** (0.065)	-3.647* (2.080)
Firm Effects		✓	✓
Year Effects	✓	✓	✓
Years	1999-2018	1999-2018	1999-2018
Firm-Years	9666	9666	7095

Notes: The table reports panel estimates from the merged Compustat-IBES sample of the coefficients of a regression of forecast errors on the indicated variable. Columns 1 and 2 reflect OLS panel estimators, while column 3 reflects the GMM estimator outlined in Data Appendix B. The standard errors are clustered at the firm level. All variables are scaled by the firm's tangible capital stock and measured at the firm-fiscal year level. Forecast errors in $t + 1$ are realized earnings in $t + 1$ minus firm forecasts in t . Investment in t is capital expenditures. * = 10% level, ** = 5% level, and *** = 1% level. Note that 0.01 = 1% relative to the firm's assets.

Table B.3: Predictable Forecast Errors, Robustness

	(1)	(2)	(3)	(4)	(5)	(6)
Dep. Variable:	Forecast Error _{t+1}					
Sample:	Baseline	Inv. Grade	≥ 5 Yrs.	Post-G.R.	High Inv.	Low Inv.
Investment _t	-0.457*** (0.065)	-0.448*** (0.067)	-0.365*** (0.073)	-0.468*** (0.107)	-0.372*** (0.087)	-0.569*** (0.199)
Firm Effects	✓	✓	✓	✓	✓	✓
Year Effects	✓	✓	✓	✓	✓	✓
Years	1999-2018	1999-2018	1999-2018	2009-2018	1999-2018	1999-2018
Firm-Years	9666	8639	7973	4388	4625	4599

Notes: The table reports panel OLS estimates from the merged Compustat-IBES sample of the coefficients of a regression of forecast errors on the indicated variable. Each column, across rows, reports coefficients for a particular sample of interest. The standard errors are clustered at the firm level. All variables are scaled by the firm's tangible capital stock and measured at the firm-fiscal year level. Forecast errors in $t + 1$ are realized earnings in $t + 1$ minus firm forecasts in t . Investment in t is capital expenditures. * = 10% level, ** = 5% level, and *** = 1% level. Note that 0.01 = 1% relative to the firm's assets.

Table B.4: Predictable Forecast Errors by Horizon

	(1)	(2)	(3)
Dependent Variable:		Forecast Error _{t+1}	
Estimation Method:	OLS	OLS	OLS
Δ Investment _t	-0.179*** (0.058)		
Δ Investment _{t-1}		-0.153*** (0.055)	
Δ Investment _{t-2}			-0.029 (0.049)
Year Effects	✓	✓	✓
Years	1999-2018	2000-2018	2001-2018
Firm-Years	9747	9342	8937

Notes: The table reports panel OLS estimates from the merged Compustat-IBES sample of the coefficients of a regression of forecast errors on the indicated variable. The standard errors are clustered at the firm level. All variables are scaled by the firm's tangible capital stock and measured at the firm-fiscal year level. Forecast errors in $t + 1$ are realized earnings in $t + 1$ minus firm forecasts in t . Investment in t is capital expenditures. * = 10% level, ** = 5% level, and *** = 1% level. Note that 0.01 = 1% relative to the firm's assets.

Table B.5: Linking Forecast Errors and Firm Reversals: Investment Grade Firms

Panel A: Second Stage	(1)	(2)	(3)	(4)
Dep. Variable:	Return _t	Return _t	Δ Investment _t	Δ Investment _t
Estimation Method:	OLS	IV	OLS	IV
Forecast Error _t	0.000 (0.000)	0.006* (0.004)	0.022** (0.009)	0.436*** (0.078)
Panel B: First Stage	Dep. Variable: Forecast Error _t			
Investment _{t-1}		-0.523*** (0.118)		-0.523*** (0.118)
Year Effects	✓	✓	✓	✓
Years	2002-2018	2002-2018	2002-2018	2002-2018
Firm-Years	2000	2000	2000	2000
First Stage F		20		20

Notes: The table reports estimates of specifications on the merged Compustat - IBES - FISD/TRACE sample at the firm-fiscal year level, restricting to firms with Moody's rated investment grade debt. The top panel plots OLS and IV second-stage estimates. The bottom panel, where relevant, reports IV first-stage estimates. Columns (3)-(4) control for current profits in the second stage. Standard errors are clustered at the firm level. * = 10% level, ** = 5% level, and *** = 1% level. Note that 0.01 = 1% relative to the firm's assets.

Table B.6: Linking Forecast Errors and Firm Reversals: No Controls

Panel A: Second Stage	(1)	(2)	(3)	(4)
Dep. Variable:	Return _t	Return _t	Δ Investment _t	Δ Investment _t
Estimation Method:	OLS	IV	OLS	IV
Forecast Error _t	0.001 (0.001)	0.007* (0.004)	0.008 (0.007)	0.452*** (0.095)
Panel B: First Stage	Dep. Variable: Forecast Error _t			
Investment _{t-1}		-0.565*** (0.104)		-0.565*** (0.104)
Year Effects	✓	✓	✓	✓
Years	2002-2018	2002-2018	2002-2018	2002-2018
Firm-Years	2852	2852	2852	2852
First Stage F		29		29

Notes: The table reports estimates of specifications on the merged Compustat - IBES - FISD/TRACE sample at the firm-fiscal year level. The top panel plots OLS and IV second-stage estimates. The bottom panel, where relevant, reports IV first-stage estimates. Standard errors are clustered at the firm level. * = 10% level, ** = 5% level, and *** = 1% level. Note that 0.01 = 1% relative to the firm's assets.

Table B.7: Linking Forecast Errors and Industry Reversals in the Data: SIC2 Sectors

	(1)	(2)	(3)
Dependent Variable:	Forecast Error _t	Return _t	ΔInvestment _t
IV Stage:	First	Second	Second
Forecast Error _t		0.048*** (0.014)	0.567*** (0.046)
Investment _{t-1}	-0.873*** (0.246)		
Industry Effects	✓	✓	✓
Industries	35	35	35
Industry-Years	453	453	453
First Stage F	13		

Notes: The table reports first- and second-stage IV estimates based on industry aggregated data from the Compustat- IBES-FISD/TRACE sample at the SIC2 × fiscal year level. Column (1) reports the first stage, and columns (2)-(3) report second-stage regressions. Standard errors are clustered at the industry level. * = 10% level, ** = 5% level, and *** = 1% level. Forecast error is average realized minus expected profits normalized by capital stocks. Investment is the average investment rate, i.e., capital expenditures normalized by the capital stock. The return is the average realized bond return. Columns (2)-(3) control for the lagged spread and current profits relative to the capital stock. Note that 0.01 = 1% relative to the firm's assets.

Table B.8: Linking Forecast Errors and Industry Reversals in the Data: No Controls

	(1)	(2)	(3)
Dependent Variable:	Forecast Error _t	Return _t	ΔInvestment _t
IV Stage:	First	Second	Second
Forecast Error _t		0.047*** (0.016)	1.266*** (0.408)
Investment _{t-1}	-0.476*** (0.153)		
Industry Effects	✓	✓	✓
Industries	111	111	111
Industry-Years	1291	1291	1291
First Stage F	10		

Notes: The table reports first- and second-stage IV estimates based on industry aggregated data from the Compustat- IBES-FISD/TRACE sample at the SIC3 × fiscal year level. Column (1) reports the first stage, and columns (2)-(3) report second-stage regressions. Standard errors are clustered at the industry level. * = 10% level, ** = 5% level, and *** = 1% level. Forecast error is average realized minus expected profits normalized by capital stocks. Investment is the average investment rate, i.e., capital expenditures normalized by the capital stock. The return is the average realized bond return. Note that 0.01 = 1% relative to the firm's assets.

Table B.9: Linking Forecast Errors and Industry Reversals in the Data: No Industry Effects

	(1)	(2)	(3)
Dependent Variable:	Forecast Error _t	Return _t	ΔInvestment _t
IV Stage:	First	Second	Second
Forecast Error _t		0.009*** (0.003)	0.242*** (0.025)
Investment _{t-1}	-0.956*** (0.142)		
Industries	111	111	111
Industry-Years	1291	1291	1291
First Stage F	46		

Notes: The table reports first- and second-stage IV estimates based on industry aggregated data from the Compustat- IBES-FISD/TRACE sample at the SIC3 × fiscal year level. Column (1) reports the first stage, and columns (2)-(3) report second-stage regressions. Standard errors are clustered at the industry level. * = 10% level, ** = 5% level, and *** = 1% level. Forecast error is average realized minus expected profits normalized by capital stocks. Investment is the average investment rate, i.e., capital expenditures normalized by the capital stock. The return is the average realized bond return. Columns (2)-(3) control for the lagged spread and current profits relative to the capital stock. Note that 0.01 = 1% relative to the firm's assets.

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