Supplemental Appendix (Not for Publication)

Similarity of Information and Collective Action

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B1. On state-dependent changes in similarity

In our main model, a regime change in state 0 is costless. That is, $u(0) = v(0) = 0$. Suppose, instead, that a regime change in state 0 is socially costly: $u(0) = v(0) = -1$. We keep the model unchanged in all respects otherwise. Now a regime change when $\boldsymbol{\theta} = 0$ is welfare-reducing. We demonstrate how increasing similarity in state $\boldsymbol{\theta} = 0$ has the opposite effects from what we showed in the main paper. To this end, we specialize to symmetric strategies. Then, given any strategy σ, we have the associated participation and nonparticipation sets given by $P(\sigma)$ and $NP(\sigma)$ respectively.

For σ to be an equilibrium, the IC constraints for protesting and not-protesting are:

$$\begin{aligned}&\mu(s)\left[\mathcal{P}_s^1(P)\Lambda_{\mathrm{b}} + (1-\mathcal{P}_s^1(P))\Lambda_{\mathrm{o}}\right]\\ &-(1-\mu(s))\left[\mathcal{P}_s^0(P)\Lambda_{\mathrm{b}} + (1-\mathcal{P}_s^0(P))\Lambda_{\mathrm{o}}\right] \geqslant c \qquad \text{if } s \in P \qquad \text{(IC:P-S)}\end{aligned}$$

$$\mu(s)\mathcal{P}_s^1(P)\Lambda_{-\mathrm{o}} - (1-\mu(s))\mathcal{P}_s^0(P)\Lambda_{-\mathrm{o}} \leqslant c \qquad \text{if } s \notin P \qquad \text{(IC:NP-S)}$$

The only difference from our benchmark setup is the second term in the incentive constraints. This captures the probability of being pivotal in state $\boldsymbol{\theta} = 0$. It is straightforward to see that an increase in similarity in state 1 (i.e., CAD increases of $\mathcal{P}^1$) has the same impact as in the main model (for the natural modification of Condition M for this environment). But now consider the effects of increases in similarity in $\mathcal{P}^0$. We can interpret

$$\mathcal{P}_s^0(P)\Lambda_{\mathrm{b}} + (1-\mathcal{P}_s^0(P))\Lambda_{\mathrm{o}}$$

as the cost of making a difference in state $\theta = 0$ for a participant. In encouragement (discouragement) environments, an increase in similarity increases (decreases) this cost, thus reducing (increasing) the incentive for participation among participants. In other words, CAD increase of $\mathcal{P}^0$ has the opposite impact, compared to CAD increases of $\mathcal{P}^1$, on the incentive of the participants ((IC:P-S)).

For nonparticipants ($s \notin P$), higher similarity in state $\boldsymbol{\theta} = 1$ reduces the LHS in (IC:NP-S) while higher similarity in state $\boldsymbol{\theta} = 0$ increases it. Under the following assumption, the incentive constraint of the nonparticipants is always satisfied regardless of $\mathcal{P}^0$.

Assumption 2: *For any σ with $\mathscr{V}(\sigma) > \mathscr{V}(\sigma^*)$, and any $s \notin P(\sigma)$,*

$$\mu(s)\mathcal{P}_s^1(P(\sigma))\Lambda_{-o} < c.$$

We can again use CAD to characterize the effect of information similarity.

Proposition 7: *Suppose $\widehat{\mathcal{P}}^1$ satisfies assumption 2. Let $\mathcal{P} := (\widehat{\mathcal{P}}^1, \mathcal{P}^0)$ be an information structure such that $\mathcal{P}^0 \succcurlyeq_{CAD} \widehat{\mathcal{P}}^0$, then*

1. *$\mathscr{V}^*(\mathcal{P}) \geqslant \mathscr{V}^*(\widehat{\mathcal{P}})$ if $\Lambda_o > \Lambda_b$. And,*
2. *$\mathscr{V}^*(\mathcal{P}) \leqslant \mathscr{V}^*(\widehat{\mathcal{P}})$ if $\Lambda_b > \Lambda_o$.*

Proof. Let σ^* be a maximal equilibrium under $\mathcal{P}$ with associated participation and nonparticipation sets P^* and NP^* respectively.

Suppose that $\Lambda_{\mathrm{o}} > \Lambda_{\mathrm{b}}$. Then, for all $s \in P^*$

$$\mathcal{P}_s^0(P^*)\Lambda_{\mathrm{b}} + (1 - \mathcal{P}_s^0(P^*))\Lambda_{\mathrm{o}} \leqslant \widehat{\mathcal{P}}_s^0(P^*)\Lambda_{\mathrm{b}} + (1 - \widehat{\mathcal{P}}_s^0(P^*))\Lambda_{\mathrm{o}}$$

And, for all $s \notin P^*$, $\mu(s)\mathcal{P}_s^1(P^*) < c$ by Assumption 2. Therefore, $\sigma^* \in \mathcal{E}(\mathcal{P})$, proving part (1).

For (2), for all $s \in P^*$,

$$\mathcal{P}_s^0(P^*)\Lambda_{\mathrm{b}} + (1 - \mathcal{P}_s^0(P^*))\Lambda_{\mathrm{o}} \geqslant \widehat{\mathcal{P}}_s^0(P^*)\Lambda_{\mathrm{b}} + (1 - \widehat{\mathcal{P}}_s^0(P^*))\Lambda_{\mathrm{o}}.$$

Therefore, σ^* may not be in $\mathcal{E}(\mathcal{P})$. Finally, consider any strategy profile σ such that $\mathscr{V}(\sigma) \geqslant \mathscr{V}(\sigma^*)$. By Assumption 2, (IC:NP-S) holds for all $s \in P$ in both, $\widehat{\mathcal{P}}$ and $\mathcal{P}$ (Since $\mathcal{P}^1 = \widehat{\mathcal{P}}^1$). Since σ^* is a maximal equilibrium under $\widehat{\mathcal{P}}$, $\sigma \notin \mathcal{E}(\widehat{\mathcal{P}})$. Therefore, $\exists s \in P$ for whom (IC:P-S) fails. Finally, for all $s \in P$,

$$\mathcal{P}_s^0(P)\Lambda_{\mathrm{b}} + (1 - \mathcal{P}_s^0(P))\Lambda_{\mathrm{o}} \geqslant \widehat{\mathcal{P}}_s^0(P)\Lambda_{\mathrm{b}} + (1 - \widehat{\mathcal{P}}_s^0(P))\Lambda_{\mathrm{o}}.$$

Therefore, $\sigma \notin \mathcal{E}(\mathcal{P})$. The proposition follows.

□

B2. More general payoffs

Let us rewrite the players' payoffs from Table 4. First, notice that this specification

	$A \geqslant \bar{n}+1$	$A \leqslant \bar{n}$
$a=1$	$u(\theta)+h(s)$	$h(s)$
$a=0$	$v(\theta)$	0

Table 6: More general payoffs

captures various canonical models in the literature. For instance, setting $u(\theta) = v(\theta) = \theta$ and $h(s) = -c$ is our baseline model, while $u(\theta) = v(\theta) = 1$, and $h(s) = -s$ (where s is interpreted as the group's cost, is Dziuda et al. (2021). A regime-change game without free-riding, as in Morris and Shin (1998) for instance, would have $u(\theta) = \theta$, $v(\theta) = 0, h(s) = -c$. Thus, the above specification incorporates common-value as well as private-value payoffs with and without free-riding. We show how our techniques can be used to analyze such environments, and how information similarity has a delicate effect even in such environments. To this end, let us restrict attention to symmetric strategies. The main result of the section is below.

Proposition 8: *Suppose $\mathcal{P}^\theta \succcurlyeq_{CAD} \widehat{\mathcal{P}}^\theta$ in some state θ where a regime change is desirable. Then,*

1. *In the encouragement environment, $\mathscr{V}^*(\mathcal{P}) \geqslant \mathscr{V}^*(\widehat{\mathcal{P}})$.*
2. *If $u(\theta) - v(\theta)$ is sufficiently small (but need not be zero), then it is possible to have $\mathscr{V}^*(\widehat{\mathcal{P}}) > \mathscr{V}^*(\mathcal{P})$.*
3. *If $v(\theta) = 0$, i.e., the game does not feature any free-riding motive, then $\mathscr{V}^*(\mathcal{P}) \geqslant \mathscr{V}^*(\widehat{\mathcal{P}})$.*

Proof. Let us revisit (IC:P) and (IC:NP) for this environment. Agents' participation constraint depends on Δ defined below:

$$\begin{aligned}\Delta(\theta, s) :=& Prob\left(\mathbf{A} > \bar{\boldsymbol{n}} \mid \boldsymbol{\theta} = \theta, \mathbf{S} = s, a = 1\right) u(\theta) + h(s)\\ &- Prob\left(\mathbf{A} > \bar{\boldsymbol{n}} \mid \boldsymbol{\theta} = \theta, \mathbf{S} = s, a = 0\right) v(\theta)\end{aligned}$$

An agent's best response is given by,

$$a(s) = \begin{cases} 1 & \text{if } \mathbb{E}_s[\Delta(\boldsymbol{\theta}, s)] > 0 \\ [0,1] & \text{if } \mathbb{E}_s[\Delta(\boldsymbol{\theta}, s)] = 0 \\ 0 & \text{if } \mathbb{E}_s[\Delta(\boldsymbol{\theta}, s)] < 0 \end{cases}$$

As done throughout the paper, we investigate the effect of changing similarity of information in some states while keeping the marginal distribution unchanged. To this end, let $\mathcal{P}^\theta, \widehat{\mathcal{P}}^\theta$ be two distributions in state θ such that $\mathcal{P}^\theta \succcurlyeq_{CAD} \widehat{\mathcal{P}}^\theta$. Consider a symmetric strategy profile $\sigma : \mathcal{S} \to \{0, 1\}$. For a fixed σ, define $\pi(\theta, s, \alpha) := Prob\left(\mathbf{A} > \bar{\boldsymbol{n}} \mid \boldsymbol{\theta} = \theta, \mathbf{S} = s, a = \alpha\right)$ to be the probability of a successful protest at a given (θ, s, a) triple.[29] Define the pivotal probability of an agent under$(\theta, s; \mathcal{P})$ as,

$$Piv(\theta, s; \mathcal{P}) := \pi(\theta, s, 1; \mathcal{P}) - \pi(\theta, s, 0; \mathcal{P}).$$

Then,

$$\begin{aligned}\mathbb{E}_s\left[\Delta(\boldsymbol{\theta}, s; \mathcal{P}) - \Delta(\boldsymbol{\theta}, s; \widehat{\mathcal{P}})\right] = Prob\left(\boldsymbol{\theta} = \theta \mid \mathbf{S} = s\right) \Big[&\big(u(\theta) - v(\theta)\big)\big(\pi(\theta, s, 1; \mathcal{P}) - \pi(\theta, s, 1; \widehat{\mathcal{P}})\big) \\ &+ \Big(Piv(\theta, s; \mathcal{P}) - Piv(\theta, s; \widehat{\mathcal{P}})\Big) v(\theta)\Big] \end{aligned} \tag{6}$$

We seek here a result similar to our Theorems 1 and 2 that rank equilibrium sets according to the set inclusion order. To have $\sigma \in \mathcal{E}(\widehat{\mathcal{P}}) \implies \sigma \in \mathcal{E}(\mathcal{P})$, we need that the above difference be $\geqslant 0$ for all $s \in P(\sigma)$ and it be $\leqslant 0$ for all $s \notin P(\sigma)$.

[29] We suppress the dependence on σ for notational simplicity.

First, notice that $\pi(\theta, s, 1; \mathcal{P}) \geqslant \pi(\theta, s, 1; \widehat{\mathcal{P}})$ for all $s \in P(\sigma)$. Under $\mathcal{P}$, it is more likely that the other group also participates when $s \in P(\sigma)$, and therefore, it is more likely that a regime change occurs. Similarly, $\pi(\theta, s, 1; \mathcal{P}) \leqslant \pi(\theta, s, 1; \widehat{\mathcal{P}})$ for all $s \in NP(\sigma)$ because, given that $s \in NP(\sigma)$, it is now *less* likely that the other group participates under $\mathcal{P}$ than under $\widehat{\mathcal{P}}$. Therefore,

$$\text{sign}\left(\big(u(\theta) - v(\theta)\big)\big(\pi(\theta, s, 1; \mathcal{P}) - \pi(\theta, s, 1; \widehat{\mathcal{P}})\big)\right) = \begin{cases} \text{sign}(u(\theta) - v(\theta)) & \text{if } s \in P(\sigma) \\ -\text{sign}(u(\theta) - v(\theta)). & \text{if } s \in NP(\sigma). \end{cases}$$

Therefore, if the regime change is desirable in state θ, i.e., when $u(\theta) \geqslant v(\theta)$, then the first term in (6) is positive if $s \in P(\sigma)$ and negative if $s \in NP(\sigma)$.

In the encouragement (discouragement) environment, an agent is more (less) likely to be pivotal when both groups participates than when only one participates. Therefore, in the encouragement environment

$$Piv(\theta, s; \mathcal{P}) - Piv(\theta, s; \widehat{\mathcal{P}}) \begin{cases} \geqslant 0 & \text{if } s \in P(\sigma) \\ \leqslant 0 & \text{if } s \in NP(\sigma). \end{cases}$$

And in the discouragement environment,

$$Piv(\theta, s; \mathcal{P}) - Piv(\theta, s; \widehat{\mathcal{P}}) \begin{cases} \leqslant 0 & \text{if } s \in P(\sigma) \\ \leqslant 0 & \text{if } s \in NP(\sigma). \end{cases}$$

Therefore, the combined effect of changing similarity in state θ depends on the following two factors:

1. Whether a regime change is desirable, $u(\theta) \geqslant v(\theta) \geqslant 0$, or undesirable, $u(\theta) \leqslant v(\theta) \leqslant 0$.
2. Whether we are in the encouragement environment or discouragement environment.

Notice that the sign of the first term in (6)—which encapsulates the extent of free-riding $(u(\theta) - v(\theta))$, and the marginal change in probability from increased similarity—does not depend on whether we are in encouragement or discouragement environment. When the extent of free-riding is strong, $u(\theta) - v(\theta)$ is small as in our baseline model (where it is zero), then the sign of (6) is determined essentially by the second term alone. In this case, where $u(\theta) - v(\theta)$ is zero, Theorem 2 established that we can have the maximal participation as well as welfare reduce when similarity increases. But then, a quick look at (6) highlights that this is not a knife-edge result in the sense that a similar reasoning would go through even if $u(\theta) - v(\theta)$ is strictly positive but sufficiently small. In such a case too the maximal participation can reduce when similarity increases in states where a regime change is desirable.

Let us look at the encouragement environment in contrast. Suppose that regime change is desirable in θ. Then, both the terms in (6) are positive if $s \in P(\sigma)$, and are negative if $s \in NP(\sigma)$. Therefore, $\sigma \in \mathcal{E}(\widehat{\mathcal{P}}) \implies \sigma \in \mathcal{E}(\mathcal{P})$. Thus, $\mathscr{V}^*(\mathcal{P}) \geqslant \mathscr{V}^*(\widehat{\mathcal{P}})$. □

B3. Informativeness of turnout

Consider our baseline model with two groups, two states, and a finite set of signals. For expositional simplicity, we consider here a slightly different variant of that model. In particular, we assume that $\mathbf{N}_1, \mathbf{N}_2$ are Poisson distributed with mean N, while $\bar{\boldsymbol{n}}$ is deterministic. That is, let $\psi(k, N) = \frac{\exp(-N)N^k}{k!}$ be the Poisson pdf that specifies the probability that nature chooses a group to have k agents. However, now assume that a strategic policymaker observes the realized turnout and then decides whether to change the regime. We restrict attention to symmetric strategies, and as before, we denote the associated participation and nonparticipation sets of any strategy by P and NP, respectively.

Given an information structure $\mathcal{P}$, agents' strategy σ, and aggregate turnout $\mathbf{A}$, the policymaker's belief about the state of the world is given by the likelihood function

$$\beta(\cdot; \mathcal{P}, P) := \frac{Prob(\boldsymbol{\theta} = 1 | \mathbf{A} = \cdot, P)}{1 - Prob(\boldsymbol{\theta} = 1 | \mathbf{A} = \cdot, P)}.$$

The policymaker changes the status quo only if she is sufficiently confident that the state is 1; that is, there is a cutoff belief, $\underline{\beta}$, such that the policymaker changes the status quo if $\beta(k) \geqslant \underline{\beta} > 0$. Therefore, the policymaker's preferences are (ordinally) aligned with the citizens'.

We define informativeness of turnout, given a strategy σ, and its associated participation set P, as in Ekmekci and Lauermann (2022):

$$I(P) := \mathbb{P}^1(P) - \mathbb{P}^0(P)$$

Define $\bar{P} := \{s \in \mathcal{S} : \mathbb{P}^1(\{s\}) > \mathbb{P}^0(\{s\})$. It is easy to see that the informativeness of any strategy is bounded $I(\bar{P})$. Therefore, for a given information structure $\mathcal{P}$ with fixed marginals, we say that *information aggregates* if $\mathbb{1}_{\bar{P}}$ is an equilibrium under $\mathcal{P}$.

We fix $\mathcal{P}^0$ to investigate when, if at all, increasing similarity of information facilitates information aggregation. To this end, define

$$\underline{l} := \frac{\mu}{1-\mu} \frac{\mathbb{P}^1(\bar{P})^2}{\mathcal{P}^0(\bar{P}, \bar{P})}$$
$$\bar{l} := \frac{\mu}{1-\mu} \frac{\mathbb{P}^1(\bar{P})}{\mathcal{P}^0(\bar{P}, \bar{P})}.$$

Proposition 9: *Suppose that $\mathbb{P}^1(\bar{P})\mathbb{P}^0(\bar{P}) > \mathcal{P}^0(\bar{P}, \bar{P})$.*[30]

1. *If $\underline{l} \leqslant \underline{\beta} < \bar{l}$, then information does not aggregate if $\mathcal{P}^1$ has conditionally independent signals (denoted by $\mathcal{P}^{1,CI}$) as long as $c > 0$; and $\exists c > 0$ and a signal $\mathcal{P}^1 \succcurlyeq_{CAD} \mathcal{P}^{1,CI}$, such that information aggregates under $\mathcal{P}^1$.*
2. *If $\underline{\beta} > \bar{l}$, then information does not aggregate for any $\mathcal{P}^1 \succcurlyeq_{CAD} \mathcal{P}^{1,CI}$ and any $c > 0$.*

[30] It is easy to generate examples in which this inequality is satisfied. For example, this inequality holds if the signals are independent conditional on the state.

Proof of Proposition 9. Substituting the expression for Poisson pdf, we get

$$\begin{aligned}\beta(k,\mathcal{P},\bar{P}) =& \frac{\mu}{1-\mu}\frac{\mathcal{P}^1(\bar{P},\bar{P})\psi(k,2N)+2\mathcal{P}^1(\bar{P},\bar{NP})\psi(k,N)}{\mathcal{P}^0(\bar{P},\bar{P})\psi(k,2N)+2\mathcal{P}^0(\bar{P},\bar{NP})\psi(k,N)}\\ =& \frac{\mu}{1-\mu}\frac{\mathbb{P}^1(\bar{P})+\mathcal{P}^1(\bar{P},\bar{P})(e^{-N}2^{k-1}-1)}{\mathbb{P}^0(\bar{P})+\mathcal{P}^0(\bar{P},\bar{P})(e^{-N}2^{k-1}-1)}\end{aligned}$$

Since 2^{k-1} is increasing in k,

$$\text{sign}\left(\frac{\partial\beta(k,\mathcal{P},\bar{P})}{\partial k}\right) = \text{sign}(\mathcal{P}^1(\bar{P},\bar{P})\mathbb{P}^0(\bar{P}) - \mathbb{P}^1(\bar{P})\mathcal{P}^0(\bar{P},\bar{P})).$$

When signals are conditionally independent in state 1, $\mathcal{P}^1(\bar{P},\bar{P}) = \mathbb{P}^1(\bar{P})^2$. Moreover, for any $\mathcal{P}^1 \succcurlyeq_{CAD} \mathcal{P}^{1,CI}$, $\mathcal{P}^1(\bar{P},\bar{P}) \geqslant \mathbb{P}^1(\bar{P})^2$. This implies

$$\text{sign}\left(\frac{\partial\beta(k,\mathcal{P},\bar{P})}{\partial k}\right) = \text{sign}(\mathbb{P}^1(\bar{P})\mathbb{P}^0(\bar{P}) - \mathcal{P}^0(\bar{P},\bar{P})) > 0.$$

The last inequality is true by hypothesis. Therefore, for any $\mathcal{P}$ such that $\mathcal{P}^1 \succcurlyeq_{CAD} \mathcal{P}^{1,CI}$, $\beta(k,\mathcal{P},\bar{P})$ is increasing in k. Moreover,

$$\lim_{k\to\infty}\beta(k,\mathcal{P},\bar{P}) = \frac{\mu}{1-\mu}\frac{\mathcal{P}^1(\bar{P},\bar{P})}{\mathcal{P}^0(\bar{P},\bar{P})}$$

When $\mathcal{P}^1 = \mathcal{P}^{1,CI}$,

$$\lim_{k\to\infty}\beta(k,\mathcal{P},\bar{P}) = \underline{l} < \underline{\beta}$$

Therefore, $\beta(k,\mathcal{P},\bar{P}) < \underline{\beta}$ for all k whenever $\mathcal{P}^1 = \mathcal{P}^{1,CI}$. Therefore, $\mathbb{1}_{\bar{P}} \notin \mathcal{E}(\mathcal{P})$ if $\mathcal{P}^1 = \mathcal{P}^{1,CI}$ as long as $c > 0$. That is, information aggregation fails when signals are conditionally independent.

In contrast, when $\mathcal{P}^1 = \mathcal{P}^{1,corr}$, where $\mathcal{P}^{1,corr}$ means signals being fully correlated in state 1,

$$\lim_{k\to\infty}\beta(k,\mathcal{P},\bar{P}) = \bar{l}.$$

If $\bar{l} > \underline{\beta}$, $\exists k^*$ such that $\beta(k,\mathcal{P},\bar{P}) > \underline{\beta}$ for all $k \geqslant k^*$. For any $\mathcal{P}^1 \succcurlyeq_{CAD} \mathcal{P}^{1,CI}$, by Lemma 6, $\mathcal{P}^1(\bar{P},\bar{P}) = \mathcal{P}^{1,CI}(\bar{P},\bar{P}) + \alpha$, for some $\alpha \geqslant 0$. We know that, when $\alpha = 0$, $\beta(k,\mathcal{P},\bar{P}) < \underline{\beta}$ for all k, and, when $\alpha = \mathbb{P}^1(\bar{P}) - \mathbb{P}^1(\bar{P})^2$, $\exists k^* \in \mathbb{N}$ such that $\beta(k,\mathcal{P},\bar{P}) > \underline{\beta}$ whenever $k \geqslant k^*$. Therefore, we can choose an $\alpha > 0$ small enough to construct $\mathcal{P}^1$ so that $\mathcal{P}^1(\bar{P},\bar{P}) = \mathcal{P}^{1,CI}(\bar{P},\bar{P}) + \alpha$ and $\beta(k,\mathcal{P},\bar{P}) \geqslant \underline{\beta}$ if and only if $k > k^* > 2N$. Therefore, the policymaker would use a threshold of $\bar{n} = k^* > 2N$ when $\mathcal{P}^1$ constructed using α described above. It is easy to check that $\psi(\bar{n},2N) > \psi(\bar{n},N)$ in this case. Claim 2 then establishes that,

$$\min_{s\in\bar{P}}\mu(s)\left[\mathcal{P}_s^1(\bar{P})\psi(\bar{n},2N) + \mathcal{P}_s^1(\mathcal{S}\backslash\bar{P})\psi(\bar{n},N)\right] > \max_{s\in\mathcal{S}\backslash\bar{P}}\mu(s)\mathcal{P}_s^1(\bar{P})\psi(\bar{n},N)$$

Therefore, by letting c to be strictly between the LHS and the RHS of the above, we get that $\mathbb{1}_{\bar{P}}$ is an equilibrium, for it satisfies (IC:P) for all $s \in \bar{P}$ and (IC:NP) for all $s \in \mathcal{S} \backslash \bar{P}$. Therefore, information aggregates under $\mathcal{P}$ wherein $\mathcal{P}^1 \succcurlyeq_{CAD} \mathcal{P}^{1,CI}$.

Finally, if $\underline{\beta} > \bar{l}$, then the policymaker would not change the status quo regardless of the turnout for any $\mathcal{P}^1 \succcurlyeq_{CAD} \mathcal{P}^{1,CI}$ establishing the last part of the Proposition.

Claim 2: *If $\psi(\bar{n}, 2N) > \psi(\bar{n}, N)$, then*

$$\min_{s \in \bar{P}} \mu(s) \left[\mathcal{P}^1_s(\bar{P})\psi(\bar{n}, 2N) + \mathcal{P}^1_s(\mathcal{S}\backslash\bar{P})\psi(\bar{n}, N)\right] > \max_{s \in \mathcal{S}\backslash\bar{P}} \mu(s)\mathcal{P}^1_s(\bar{P})\psi(\bar{n}, N)$$

for any $\mathcal{P}^1 \succcurlyeq_{CAD} \mathcal{P}^{1,CI}$.

Proof. First, by definition of $\bar{P}$, $\min_{s\in\bar{P}} \mu(s) > \max_{s\in\mathcal{S}\backslash\bar{P}} \mu(s)$. Also, $\mathcal{P}^{1,CI}_s(\bar{P}) = \mathbb{P}^1(\bar{P})$ is independent of s, and, $\mathcal{P}^1_s(\bar{P}) \geqslant \mathbb{P}^1(\bar{P})$ for all $s \in \bar{P}$ and $\mathcal{P}^1_s(\bar{P}) \leqslant \mathbb{P}^1(\bar{P})$ for all $s \notin \bar{P}$. Therefore, $\min_{s\in\bar{P}} \mathcal{P}^1_s(\bar{P}) \geqslant \max_{s\in\mathcal{S}\backslash\bar{P}} \mathcal{P}^1_s(\bar{P})$. The claim, then, follows due to $\psi(\bar{n}, 2N) > \psi(\bar{n}, N)$.

□

□

Given an information structure $\mathcal{P}$, we say an equilibrium σ^* has "*maximally informative turnout*" if $I(\sigma^*) \geqslant I(\sigma)$ for all $\sigma \in \mathcal{E}(\mathcal{P})$. We denote $I(\sigma^*)$ by $\mathcal{I}(\mathcal{P})$. Proposition 10 below, shows that increasing similarity of information can reduce the informativeness of turnout.

Proposition 10: *Suppose that $\mathcal{P}^1 \succcurlyeq_{CAD} \widehat{\mathcal{P}}^1$ and $\mathcal{P}^0 = \widehat{\mathcal{P}}^0$. Let $\bar{n}^*$ be the optimal participation threshold for the equilibrium with maximally informative turnout under $\widehat{\mathcal{P}}$.*

1. *If $\psi(\bar{n}^*, 2N) > 2\psi(\bar{n}^*, N)$, then $\mathcal{I}(\mathcal{P}) \geqslant \mathcal{I}(\widehat{\mathcal{P}})$ if $\max_{T\subset\mathcal{S}} \mathcal{P}^1(T,T) - \widehat{\mathcal{P}}^1(T,T)$ is sufficiently small.*
2. *If $\psi(\bar{n}^*, 2N) < \psi(\bar{n}^*, N)$, then it is possible that $\mathcal{I}(\mathcal{P}) < \mathcal{I}(\widehat{\mathcal{P}})$.*

Proof of Proposition 10. Let σ^* be the maximally informative equilibrium under $\widehat{\mathcal{P}}$. If the policymaker continues to use $\bar{n}^*$ as the cutoff, then σ^* continues to remain an equilibrium under $\mathcal{P}$ due to Theorem 1. While this takes care of the incentives of the participants, unlike the earlier arguments, we also need to ensure that a cutoff of $\bar{n}^*$ is indeed a best response for the policymaker. Since $\bar{n}^*$ is the cutoff for the maximally informative equilibrium $\beta(\bar{n}^*; \widehat{\mathcal{P}}) \geqslant \underline{\beta}$ and $\beta(k; \widehat{\mathcal{P}}) < \underline{\beta}$ for all $k < \bar{n}^*$. By Bayes' rule, we have,

$$\frac{\beta(k; \widehat{\mathcal{P}})}{1 - \beta(k; \widehat{\mathcal{P}})} = \frac{\mu}{1-\mu} \frac{\widehat{\mathcal{P}}^1(P,P)\psi(k,2N) + 2\widehat{\mathcal{P}}^1(P,NP)\psi(k,N)}{\widehat{\mathcal{P}}^0(P,P)\psi(k,2N) + 2\widehat{\mathcal{P}}^0(P,NP)\psi(k,N)}$$

Since $\psi(\bar{n}^*, 2N) > 2\psi(\bar{n}^*, N)$, $\mathcal{P}^1(P,P) = \widehat{\mathcal{P}}^1(P,P) + \alpha$ and $\mathcal{P}^1(P,NP) = \widehat{\mathcal{P}}^1(P,NP) - \alpha$ for some $\alpha > 0$ by Lemma 6, and $\mathcal{P}^0 = \widehat{\mathcal{P}}^0$, $\beta(\bar{n}^*; \mathcal{P}) > \beta(\bar{n}^*; \widehat{\mathcal{P}}) \geqslant \underline{\beta}$. However, it is now also possible that $\beta(k;\mathcal{P}) \geqslant \underline{\beta}$ for some $k < \bar{n}^8$. Simply lowering the threshold in this case is not an option either as it affects the incentives of the agents, possibly destroying σ^* as an equilibrium. However, when $\max_{T\subseteq\mathcal{S}} \mathcal{P}^(T,T) - \widehat{\mathcal{P}}^1(T,T)$ is sufficiently small, $\beta(k;\mathcal{P}) < \underline{\beta}$. Finally, since $\mathcal{I}(\cdot)$ only depends on the marginal distributions, we obtain the desired inequality.

For the second part, suppose that $N = 20$, $\mathcal{S} = \{0,1\}$, $c = 0.0368$, and $\underline{\beta} = 0.7281$. Signals are conditionally independent in state 0 with the marginal distribution $\mathbb{P}^0(1) = 0.3$. In state 1, $\widehat{\mathcal{P}}^1(1,1) = 0.66, \widehat{\mathcal{P}}^1(1,0) = 0.15$. $\mathcal{P}^1$ is constructed from $\widehat{\mathcal{P}}^1$ by using $\alpha = 0.05$. It is easy to see that $\bar{n}^*(\widehat{\mathcal{P}}) = 28$, while the same no longer constitutes an equilibrium under $\mathcal{P}$. In this case, if an informative equilibrium exists, it must involve mixing. It is easy to check that mixing can only happen on signal 1, and agents continue to not participate when they receive a signal 0. Therefore, informativeness under $\mathcal{P}$ is strictly lower than under $\widehat{\mathcal{P}}$.

□

B4. On optimal information similarity

Consider two extreme cases: conditionally independent signals and perfectly correlated signals. Suppose $Y = (Y_1, Y_2)$ where Y_i is distributed according to $\mathbb{P}^1$, and Y_1, Y_2 are independent. Denote this joint distribution by $\mathcal{P}^{1,CI}$. Analogously, let $Y = (Y_1, Y_2)$ be a random variable such that $Y_1 = Y_2$ a.s., and Y_i is distributed according to $\mathbb{P}^1$. We denote this joint distribution by $\mathcal{P}^{1,corr}$. Given a conditionally independent signal distribution $\mathcal{P}^{1,CI}$, define

$$CI^{\uparrow} := \{\mathscr{D} \in \Delta(\mathcal{S} \times \mathcal{S}) : \mathscr{D} \succcurlyeq_{CAD} \mathcal{P}^{1,CI}\}$$

as all the signal distributions that are more similar (in the CAD sense) than $\mathcal{P}^{1,CI}$. Recall that by definition of CAD, all such distributions have the same marginal, and in this case, $\mathcal{P}^0$ does not affect $\mathscr{V}(\cdot)$. Therefore, the designer solves the following problem:

$$\sup_{\mathcal{P}^1 \in CI^{\uparrow}} \mathscr{V}^*(\mathcal{P}^1, \mathcal{P}^0)$$

Proposition 11 (Optimal information similarity)**:** *An optimal information structure exists. In encouragement environments, fully correlated signals are optimal. In discouragement environments, if conditionally independent signals satisfy Condition M, then they are optimal. In other cases, intermediate levels of similarity can be optimal.*

Proof. We prove this using three steps. Steps 1 and 2 establish the existence of an optimal information structure, while Step 3 describes it.

Step 1: We show that $CI^{\uparrow}$ is weak-* compact. Consider a sequence $\{\mathscr{D}_m\}$ from $CI^{\uparrow}$ that converges to $\mathscr{D}$ in the sense that for all $f \in C(\mathcal{S} \times \mathcal{S})$, $\int f \mathrm{d}\mathscr{D}_m \to \int f \mathrm{d}\mathscr{D}$. Consider a symmetric $\alpha \in \mathbb{R}_+^{\mathcal{S}\times\mathcal{S}}$, i.e., $\alpha(i,j) = \alpha(j,i)$ for all i,j, and $\widehat{\mathscr{D}} \in \Delta(\mathcal{S}\times\mathcal{S})$.

Define,

$$\mathscr{D}(i,j) = \widehat{\mathscr{D}}(i,j) - \alpha(i,j)\mathbb{1}_{i\neq j} + \sum_{k\neq i} \alpha(i,k)\mathbb{1}_{i=j}$$

If $\mathscr{D} \in \Delta(\mathcal{S}\times\mathcal{S})$, then we say that "$\mathscr{D}$ is obtained from $\widehat{\mathscr{D}}$ by an ETI given by α", denoted by $\mathscr{D} = \widehat{\mathscr{D}} \biguplus \alpha$. Recall from the proof of Lemma 6 that an alternative characterization of the CAD order (from Proposition 1 in Meyer (1990)) is

$$\mathscr{D} \succcurlyeq_{CAD} \widehat{\mathscr{D}} \iff \exists \alpha \in \mathbb{R}_+^{\mathcal{S}\times\mathcal{S}} \text{ such that } \mathscr{D} = \widehat{\mathscr{D}} \biguplus \alpha.$$

Since $\mathscr{D}_m \succcurlyeq_{CAD} \mathcal{P}^{1,CI}$, we have a sequence $(\alpha_m) \in \mathbb{R}_+^{\mathcal{S}\times\mathcal{S}}$ such that $\mathscr{D}_m = \mathcal{P}^{1,CI} \biguplus \alpha_m$. Due to finiteness of $|\mathcal{S}\times\mathcal{S}|$, and boundedness of α_m, $\{\alpha_m\}$ has a convergent subsequence, $\{\alpha_{m_k}\}$. Let α be a limit of one such convergent subsequence. Let $\tilde{\mathscr{D}} := \mathcal{P}^{1,CI} \biguplus \alpha$. Suppose, for contradiction, that $\tilde{\mathscr{D}} \neq \mathscr{D}$. Then, there exists some $(x,y) \in \mathcal{S}\times\mathcal{S}$ such that $\tilde{\mathscr{D}}(x,y) \neq \mathscr{D}(x,y)$. Consider a continuous function $f \in C(\mathcal{S}\times\mathcal{S})$ such that $f(x',y') = 1$ if $x' = x$ and $y' = y$, and $f(x',y') = 0$, otherwise.[31] By construction, $\int_{\mathcal{S}\times\mathcal{S}} f \mathrm{d}\mathscr{D}_{m_k} \to \int_{\mathcal{S}\times\mathcal{S}} f \mathrm{d}\tilde{\mathscr{D}} \neq \int_{\mathcal{S}\times\mathcal{S}} f \mathrm{d}\mathscr{D}$. Hence, a contradiction. Therefore, $\tilde{\mathscr{D}} = \mathscr{D} = \mathcal{P}^{1,CI} \biguplus \alpha$, i.e., $\mathscr{D} \succcurlyeq_{CAD} \mathcal{P}^{1,CI}$, and therefore, $\mathscr{D} \in CI^{\uparrow}$. Therefore, $CI^{\uparrow}$ is closed. Finally, since $\mathcal{S}\times\mathcal{S}$ is compact, $\Delta(\mathcal{S}\times\mathcal{S})$ is weak$-^*$ compact. This makes $CI^{\uparrow}$ weak$-^*$ compact.

Step 2: Given compactness, consider a sequence $\{\mathscr{D}_m\} \in CI^{\uparrow}$ such that $\mathscr{V}(\mathscr{D}_m) \to \bar{\mathscr{V}} = \sup_{\mathscr{D}\in CI^{\uparrow}} \mathscr{V}^*(\mathscr{D})$. By compactness, we can (wlog) assume $\mathscr{D}_m$ converges to some $\mathscr{D} \in CI^{\uparrow}$. For each $\mathscr{D}_m$, let σ_m^* be a maximal turnout equilibrium, with participation and not-participation sets (P_m^*, NP_m^*). Due to the finiteness of $\mathcal{S}$, $|2^{\mathcal{S}}|$ is finite, and therefore, wlog, $P_m^* = P^*$ for a sufficiently large m.[32] Since the marginals are unchanged, we have $\mathscr{V}(\mathscr{D}_m) = 2N\mathbb{P}^1(P^*)$ for a sufficiently large m. Therefore, $\mathscr{V}(\mathscr{D}_m) = \bar{\mathscr{V}}$ for a sufficiently large m.

Step 3: Since more similarity increases maximal equilibrium turnout in encouragement environments (when $\Lambda_b > \Lambda_o$), we have $\mathscr{V}^*(\mathcal{P}^{1,corr}, \mathcal{P}^0) \geqslant \mathscr{V}^*(\mathcal{P}^1, \mathcal{P}^0)$ for any $\mathcal{P}^1 \in CI^{\uparrow}$. In discouragement environments (when $\Lambda_o > \Lambda_b$), since $\mathcal{P}^{1,CI}$ satisfies Condition M, any $\mathcal{P}^1 \succcurlyeq_{CAD} \mathcal{P}^{1,CI}$ has $\mathscr{V}^*(\mathcal{P}^1, \mathcal{P}^0) \leqslant \mathscr{V}^*(\mathcal{P}^{1,CI}, \mathcal{P}^0)$. To see that intermediate levels of similarity can be optimal otherwise, consider the example below.

Example 1: *Suppose that* $\Lambda_o = 0.042, \Lambda_b = 0.02$, *the cost of participation is* $c = 0.009$. *The signal structure is as follows:* $\mathcal{S} = \{1,2,3\}$, *and the prior* μ_0 *is such that the posterior signals* $\mu = [0.3, 0.45, 0.9]$. *Moreover, the marginal distribution in state* 1 *is* $\mathbb{P}^1 = [0.25, 0.3, 0.45]$.[33]

It is easy to check that the unique equilibrium is $\sigma = \mathbb{1}_{s=3}$ under $\mathcal{P}^{1,CI}$. In particular, the strategy profile $\mathbb{1}_{\{2,3\}}$ is not an equilibrium because (IC:NP) at signal 1 is violated:

$$\mu(1)\mathbb{P}^1(\{2,3\})\Lambda_o - c = 0.0045 > 0.$$

[31] Such a function obviously exists because of the finiteness of $\mathcal{S}$.

[32] To be precise, it may be necessary to pass onto a subsequence for this to be true.

[33] It is easy to check that $\exists$ μ_0 and $\mathcal{P}^0$ that yield the required posterior given the $\mathcal{P}^1$ above.

Therefore, an agent with a signal 1 would like to participate given that his group is not. However, if we change $\mathcal{P}^1$ by performing an ETI on the square $\{(1,1),(1,2),(2,1),(2,2)\}$ with $\alpha = 0.05$, we can support an equilibrium $\sigma = \mathbb{1}_{s\in\{2,3\}}$. This is because, now,

$$\mu(1)(\mathbb{P}^1(\{2,3\}) - \alpha)\Lambda_o - c = -0.0018 < 0.$$

Moreover, it is easy to check that (IC:P) is satisfied for signals 2 and 3 under both the information structures for the strategy profile $\mathbb{1}_{\{2,3\}}$. Finally, $\mathcal{P}^{1,corr}$ also cannot support $\mathbb{1}_{s\in\{2,3\}}$ as an equilibrium because of the violation of (IC:P) at signal 2: $\mu(2)\Lambda_b - c = -0.0009 < 0$.

□

Notice that the above argument also demonstrates that we can use similar arguments to analyze the case wherein the designer wishes to minimize expected participation instead of maximizing it. In this case, conditionally independent signals would turn out to be optimal in the encouragement environment.

B5. Changing information similarity along with changing marginals

In studying the effect of changing information similarity, we have assumed throughout that while the joint distribution of signals changes, the marginal distribution in any given state remains unaltered. In this section, we allow the marginal distribution to change as well. To study this case, it is useful to think of signals as posteriors, following Kamenica and Gentzkow (2011). That is, we assume that $\mathcal{S}$ is the set of posteriors with the natural order, and $\mathcal{P}^1, \mathcal{P}^0$ are two feasible joint distributions over posteriors. As before, we assume that $\mathcal{S}$ is finite. Similar to Definition 2, we say that $Y \succcurlyeq_{CAD} \widehat{Y}$ if only part (2) of that definition holds. That is, we require that, given own signal, an agent assigns a higher probability to the other agent receiving the signal, but we dispense away the requirement of the two signals having the same marginal distributions. We make the following additional assumption.

Assumption 3: *Define $g(y,x) := \mathcal{P}^1(\mathbf{S}_2 \geqslant y | \mathbf{S}_1 = x)$. For every y, $g(y,x_1) \geqslant g(y,x_2)$ if $y \leqslant x_2 \leqslant x_1$.*

While we omit the proof here, we can show that Assumption 3 is weaker than affiliation, i.e., if $\mathcal{P}^1$ is affiliated, Assumption 3 holds, but the converse is not true. (See Krishna (2009) for the definition of affiliation). Assumption 3 is also weaker than first-order stochastic dominance.

Lemma 8: *Suppose that $\Lambda_b > \Lambda_o$ and $\mathcal{P}^1$ satisfies Assumption 3. Then, the maximal equilibrium σ^* under $\mathcal{P}$ has a cutoff structure, i.e., $P(\sigma^*) = s^{\uparrow}$ for some some $s \in \mathcal{S}$.*[34]

[34] $s^{\uparrow}$ is the upper contour set, i.e., the set of signal realizations greater than or equal to s.

Proof. The proof follows similar steps as the proof of Lemma 2. Let σ be an equilibrium. Define $\underline{x}$ to be the minimum element of $P(\sigma)$, where $\mathcal{S}$ is endowed with the natural order of posteriors. Then, for every $s \geqslant \underline{x}$,

$$\begin{aligned}\mu(s)\left[\mathcal{P}^1_s(\underline{x}^{\uparrow})\Lambda_{\mathrm{b}} + (1-\mathcal{P}^1_s(\underline{x}^{\uparrow}))\Lambda_{\mathrm{o}}\right] &\geqslant \mu(s)\left[\mathcal{P}^1_{\underline{s}}(\underline{x}^{\uparrow})\Lambda_{\mathrm{b}} + (1-\mathcal{P}^1_{\underline{s}}(\underline{x}^{\uparrow}))\Lambda_{\mathrm{o}}\right] \\ &\geqslant \mu(\underline{x})\left[\mathcal{P}^1_{\underline{s}}(\underline{x}^{\uparrow})\Lambda_{\mathrm{b}} + (1-\mathcal{P}^1_{\underline{s}}(\underline{x}^{\uparrow}))\Lambda_{\mathrm{o}}\right] \geqslant c\end{aligned}$$

where the first inequality follows from Assumption 3 and the second inequality is due to $\mu(s) \geqslant \mu(\underline{x})$ whenever $s \geqslant \underline{x}$.

Therefore, (IC:P) is satisfied for all $s \geqslant \underline{x}$. Hence, following identical steps as in Lemma 2 with $P_1 = P_2 = \underline{s}^{\uparrow}$, we obtain that the maximal equilibrium is in symmetric, cut-off strategies. □

For the proposition below, we let $\mathcal{P}, \widehat{\mathcal{P}}$ be two joint distributions and let $\mathbb{P}^\theta, \widehat{\mathbb{P}}^\theta$ be the joint distribution of signals (and hence posteriors) they induce in state θ. Following Kuvalekar et al. (2023), we now define what it means for a random variable to be "more spread out" than another.[35]

Definition 8: *We say that a random variable Y is "more spread out around y" than $\widehat{Y}$ if, $G_Y(z) \geqslant G_{\widehat{Y}}(z)$ for all $z \leqslant y$ and $G_Y(z) \leqslant G_{\widehat{Y}}(z)$ for all $z \geqslant y$.*

Proposition 12: *Suppose that $\Lambda_b > \Lambda_o$, $\mathcal{P}^1 \geqslant_{CAD} \widehat{\mathcal{P}}^1$, and $\mathbb{P}^1$ is more spread out around $\tilde{\mu}$ than $\widehat{\mathbb{P}}^1$ for some $\tilde{\mu} \leqslant c$. Then, $\mathscr{V}^*(\mathcal{P}) \geqslant \mathscr{V}^*(\widehat{\mathcal{P}})$.*

Proof. By Theorem 1, if $\sigma \in \mathcal{E}(\widehat{\mathcal{P}})$, then $\sigma \in \mathcal{E}(\mathcal{P})$ whenever $\Lambda_{\mathrm{b}} > \Lambda_{\mathrm{o}}$. Notice that the proof of this part in Theorem 1 did not use the fact that the marginal distributions remained unchanged therein. By Lemma 8, the maximal equilibrium is in cutoff strategies for $\mathcal{P}$ and $\widehat{\mathcal{P}}$. Let $\sigma(\widehat{\mathcal{P}})$ denote the maximal equilibrium under $\widehat{\mathcal{P}}$. Since it has a cutoff structure, let $\hat{s}$ be the associated cutoff. An agent would never participate on a signal such that $\mu(s) < c$. Therefore, $\mu(\hat{s}) \geqslant c$. Hence,

$$\mathscr{V}^*(\widehat{\mathcal{P}}) = \widehat{\mathbb{P}}^1(\hat{s}^{\uparrow}) \leqslant \mathbb{P}^1(\hat{s}^{\uparrow})$$

where the inequality follows due to the spread-out ranking of $\mathbb{P}^1$ and $\widehat{\mathbb{P}}^1$. Thus, $\mathscr{V}^*(\mathcal{P}) \geqslant \mathscr{V}^*(\widehat{\mathcal{P}})$. □

Finally, unlike in Theorem 1, more information can strictly lower participation even in the encouragement environment without an additional condition such as the spread-out order.[36]

B6. Example from the introduction

In this section, we analyze the example from the introduction in more detail. Recall the example. The payoff matrix is as follows.

		Bob	
		work	shirk
Abe	work	$\theta - c, \theta - c$	$q\theta - c, q\theta$
	shirk	$q\theta, q\theta - c$	$0, 0$

Abe and Bob each receives a binary signal $\mathbf{S}_i \in \{0,1\}$, such that $\mathbf{S}_i = 0$ if $\boldsymbol{\theta} = 0$, and if $\boldsymbol{\theta} = 1$, the signals are drawn from some exchangeable joint distribution $\mathcal{P}^1$. Table 7 below describes $\mathcal{P}^1$.

$\boldsymbol{\theta} = 1$	$\mathbf{S}_{-i} = 0$	$\mathbf{S}_{-i} = 1$	marg
$\mathbf{S}_i = 0$	$(1-p)^2 + \alpha$	$p(1-p) - \alpha$	$1-p$
$\mathbf{S}_i = 1$	$p(1-p) - \alpha$	$p^2 + \alpha$	p
marg	$1-p$	p	1

Table 7: Probability distribution of signals when $\boldsymbol{\theta} = 1$

Notice that the marginal in state $\boldsymbol{\theta} = 1$ is given by $P(\mathbf{S}_i = 1|\boldsymbol{\theta} = 1) = p$, and $\mu(0) \in (\frac{1}{2}, 1)$. Higher α makes Abe and Bob's signals more similar conditional on the state being $\theta = 1$: Signals are independent when $\alpha = 0$ and perfectly correlated when $\alpha = p(1-p)$. Define

$$\tilde{p} := \mathcal{P}^1_1(\mathbf{S}_{-i} = 1) = p + \frac{\alpha}{p}.$$

Obviously, $\tilde{p} \geqslant p$ and the equality holds under conditional independence. Given the primitive p, we can say that the signals are more similar when $\tilde{p}$ is higher.

For simplicity, we assumed in the example that a player never works after seeing $\mathbf{S}_i = 0$, or formally,

$$\frac{c}{\mu(0)} > 1.$$

This assumption holds when $c > \frac{1-p}{1+1-p}$, or $p > \frac{1-2c}{1-c}$. If $c > \frac{1}{2}$, then this assumption always holds true, and when $c < \frac{1}{2}$, this assumption requires p to be sufficiently high. For simplicity, we assume

$$c > \frac{1}{2}.$$

Let us start with a numerical example. Suppose that $c = 0.6$ and $Prob(\mathbf{S}_i = 1|\theta = 1) = \frac{1}{3}$.

Consider signals that are independent conditional on $\boldsymbol{\theta} = 1$. Then, $\mathcal{P}^1_1(\mathbf{S}_{-i} = 1) = \frac{1}{3}$. If $\mathcal{P}^1_1(\mathbf{S}_{-i} = 1) < \frac{1}{2}$, a player's incentive to work is more influenced by whether she alone can make a difference, making the LHS of the incentive

[35] The notion of a "rotation order" by Johnson and Myatt (2006) is essentially identical. The only (minor) distinction is that the rotation order is defined in the context of demand shifts.

[36] An example is available on request.

constraint increasing in q. Therefore, if σ^1 is an equilibrium for some q, it is an equilibrium for any $q' > q$. This gives us the first result with $q^* = 0.8$.

Next, we increase the similarity of the signal as in the example from the introduction with $\alpha = \frac{2}{9}$. This makes $\mathcal{P}_1^1(\mathbf{S}_{-i} = 1) = 1$. If $\mathcal{P}_1^1(\mathbf{S}_{-i} = 1) > \frac{1}{2}$, a player's incentive to work is more influenced by whether the other player works, making the LHS on the incentive constraint decreasing in q. Therefore, if σ^1 is an equilibrium for some q, it is an equilibrium for all $q' < q$. This gives us the second result with $q^{**} = 0.4$.

Below, we show that the qualitative insight demonstrated in the introduction is valid even if we allowed asymmetric strategies or mixing. For any $\beta \in [0,1]$, define $I^\beta(\tilde{p}, q)$ to be the probability of being pivotal when working, given that the other player works with probability β when he receives a signal $\mathbf{S}_{-i} = 1$.

$$I^1(\tilde{p}, q) := \tilde{p}(1-q) + (1-\tilde{p})q.$$

$$I^0(\tilde{p}, q) := q.$$

For any $\beta \in (0,1)$,

$$I^\beta(\tilde{p}, q) := \tilde{p}(\beta(1-q) + (1-\beta)q) + (1-\tilde{p})q.$$

Note that $I^\beta(\tilde{p}, q)$ is decreasing in β if $q > \frac{1}{2}$ and increasing in β if $q < \frac{1}{2}$. If $I^0(\tilde{p}, q), I^1(\tilde{p}, q) < c$, then $I^\beta(\tilde{p}, q) < c$ for all β.

In the introduction, we only considered the symmetric pure strategy profile σ^1 in which both Abe and Bob work with probability 1 after seeing $\mathbf{S}_i = 1$. Now let σ^β denote symmetric mixed strategy profiles in which both players work with probability β after seeing $\mathbf{S}_i = 1$. Let σ^a denote an asymmetric strategy profile in which exactly one of Abe and Bob works after seeing $\mathbf{S}_i = 1$, and let σ^0 denote the strategy profile with no effort even after $\mathbf{S}_i = 1$. We say an equilibrium is maximal if it maximizes the total expected effort by the two players.

Proposition 13: *Assume $c > \frac{1}{2}$.*

1. *If $q < \frac{1}{2}$ then*
   (a) *for $\tilde{p} < c$, σ^0 is the maximal equilibrium*
   (b) *for $\tilde{p} > c$, there exists a threshold q^{**} such that, σ^1 is the maximal equilibrium whenever $q \in (0, q^{**})$ and σ^0 is the maximal equilibrium in $q \in (q^{**}, \frac{1}{2})$*

   The threshold q^{**} is increasing in $\tilde{p}$. Therefore, the region where σ^1 is the maximal equilibrium is increasing in the set order in $\tilde{p}$. Therefore, similarity helps participation.

2. *If $q > \frac{1}{2}$, then there exist two thresholds, $\widehat{q}, q^*$ such that the following holds:*
   (a) *for $\tilde{p} < 1-c$, σ^0 is the maximal equilibrium in $q \in (\frac{1}{2}, c)$, σ^a is the maximal equilibrium in $q \in (c, \widehat{q})$, σ^β is the maximal equilibrium in $q \in (\widehat{q}, q^*)$, σ^1 is the maximal equilibrium in $q \in (q^*, 1)$*

(b) for $\tilde{p} > 1-c$, σ^0 is the maximal equilibrium in $q \in (\frac{1}{2}, c)$, σ^a is the maximal equilibrium in $q \in (c, \widehat{q})$, σ^β is the maximal equilibrium in $q \in (\widehat{q}, 1)$.

The thresholds $\widehat{q}, q^*$ are increasing in $\tilde{p}$, and the mixing probability β is decreasing in $\tilde{p}$. Therefore, similarity hurts participation.

Proof. Define

$$q^*(\tilde{p}) := \max\left\{\min\left\{\frac{c-\tilde{p}}{1-2\tilde{p}}, 1\right\}, 0\right\}$$
$$q^{**}(\tilde{p}) := \max\{\min\left\{\frac{\tilde{p}-c}{2\tilde{p}-1}, 1\right\}, 0\}$$
$$\widehat{q}(\tilde{p}) := \max\left\{\min\left\{\frac{1}{2}\left(\frac{2c-1}{1-\tilde{p}}+1\right), 1\right\}, 0\right\}.$$

All three functions are non-decreasing in $\tilde{p}$. Since $q^*(0) = \widehat{q}(0) = c$, for any $\tilde{p}$, $q^*(\tilde{p}), \widehat{q}(\tilde{p}) \geqslant c$. Recall that $c > \frac{1}{2}$. On the other hand, for any $\tilde{p}$, $q^{**}(\tilde{p}) \leqslant q^{**}(1) = 1-c < \frac{1}{2}$. Also, note that $q^*(1-c) = 1$ and $q^{**}(c) = 0$.

Claim 3: *For any $\tilde{p} \in [p, 1]$, σ^1 is the maximal equilibrium if $q \geqslant q^*(\tilde{p})$ or if $q \leqslant q^{**}(\tilde{p})$. Moreover, if $\tilde{p} \in (1-c, c)$, then $q^*(\tilde{p}) = 1$ and $q^{**}(\tilde{p}) = 0$, and σ^1 is not an equilibrium for any $q \in [0, 1]$.*

Proof. σ^1 is an equilibrium if $I^1(\tilde{p}, q) \geqslant c$. If σ^1 is an equilibrium, then it is the maximal equilibrium. Suppose that $\tilde{p} \leqslant \frac{1}{2}$. Then, $I^1(\tilde{p}, q)$ is increasing in q. Therefore, there exists $q^*(\tilde{p})$ (defined above) such that for $q \geqslant q^*(\tilde{p})$, $I^1(\tilde{p}, q) \geqslant c$, i.e, σ^1 is the maximal equilibrium. Next, suppose that $\tilde{p} \geqslant \frac{1}{2}$. Then, $I^1(\tilde{p}, q)$ is decreasing in q. Therefore, there exists $q^{**}(\tilde{p})$ (defined above) such that for $q \leqslant q^{**}(\tilde{p})$, $I^1(\tilde{p}, q) \geqslant c$, i.e, σ^1 is the maximal equilibrium.

If $\tilde{p} < 1-c$, then $q^{**}(\tilde{p}) = 0$ and $q^*(\tilde{p}) \in (c, 1)$. Therefore, σ^1 is not an equilibrium for $q < \frac{1}{2}$, whereas it is an equilibrium only if q is sufficiently high. If $\tilde{p} = 1-c$, then $q^*(\tilde{p}) = \frac{c-\tilde{p}}{1-2\tilde{p}} = 1$, and σ^1 is an equilibrium only if $q = 1$. For $\tilde{p} \in (1-c, c)$, σ^1 is not an equilibrium for any q. If $\tilde{p} = c$, then $q^{**}(\tilde{p}) = \frac{\tilde{p}-c}{2\tilde{p}-1} = 0$, and σ^1 is an equilibrium only if $q = 0$. If $\tilde{p} > c$, then $q^{**}(\tilde{p}) \in (0, 1-c)$ and $q^*(\tilde{p}) = 1$. Therefore, σ^1 is not an equilibrium for $q > \frac{1}{2}$, whereas it is an equilibrium only if q is sufficiently low.

□

Claim 4: *For any $\tilde{p} \in [p, 1]$, σ^0 is the maximal equilibrium if $q \in (q^{**}(\tilde{p}), c)$.*

Proof. If $q < c \leqslant q^*(\tilde{p})$ and $q > q^{**}(\tilde{p})$, then σ^1 is not an equilibrium, i.e., $I^1(\tilde{p}, q) < c$. Moreover $I^0(\tilde{p}, q) = q < c$, which means σ^a cannot be an equilibrium either. Recall that if $I^1(\tilde{p}, q) < c$ and $I^0(\tilde{p}, q) < c$, then $I^\beta(\tilde{p}, q) < c$ for any β, which means σ^β cannot be an equilibrium either. Note that σ^0 is always an equilibrium for $q < c$. Therefore, it is the only equilibrium, and hence, the maximal equilibrium. □

Since $c > \frac{1}{2}$, it follows from the above two claims that for $q < \frac{1}{2}$, the maximal equilibrium is either σ^1 or σ^0. Recall that $q^{**}(\tilde{p}) = 0$ for all $\tilde{p} < c$. Therefore, for $\tilde{p} < c$ the maximal equilibrium is σ^0 for all $q \in [0, \frac{1}{2}]$ and for $\tilde{p} > c$ the maximal equilibrium is σ^1 for all $q \in [0, q^{**}(\tilde{p})]$ and σ^0 for all $q \in [q^{**}(\tilde{p}), \frac{1}{2}]$. This proves the first part of the proposition.

Claim 5: *For any $\tilde{p} \in [p, 1]$, σ^a is the maximal equilibrium if $q \in [c, \widehat{q}(\tilde{p})]$ and σ^β is the maximal equilibrium if $q \in [\widehat{q}(\tilde{p}), q^*(\tilde{p})]$.*

Proof. For any $\tilde{p}$, $\widehat{q}(\tilde{p}) \in [c, q^*(\tilde{p})]$. It follows from Claim 3 that σ^1 is not an equilibrium in the interval $[c, q^*(\tilde{p})]$. In this interval, σ^a is an equilibrium since $I^0(\tilde{p}, q) = q \geqslant c$. In this interval, there is also a mixed strategy equilibrium σ^β where the players are indifferent between exerting effort and not exerting effort after seeing signal 1, i.e., $I^\beta(\tilde{p}, q) = c$. This gives us the equilibrium probability of effort

$$\beta = \frac{q - c}{(2q - 1)\tilde{p}} = \frac{1}{\tilde{p}} \left(\frac{1}{2} - \frac{c - \frac{1}{2}}{2q - 1} \right).$$

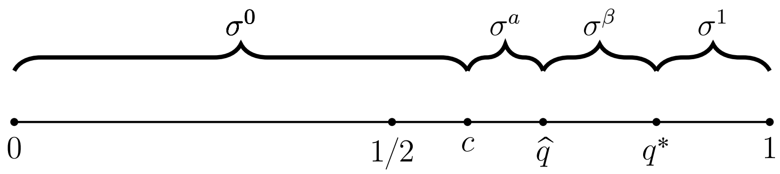
The expected effort under σ^a is p whereas that under σ^β is $2\beta p$. Therefore, in this interval, σ^β is the maximal equilibrium if $\beta \geqslant \frac{1}{2}$ and σ^a is the maximal equilibrium, otherwise. $\beta \geqslant \frac{1}{2}$ is equivalent to $q \geqslant \widehat{q}(\tilde{p})$. Since β is decreasing in $\tilde{p}$ and increasing in q, $\widehat{q}(\tilde{p})$ increasing in $\tilde{p}$.

□

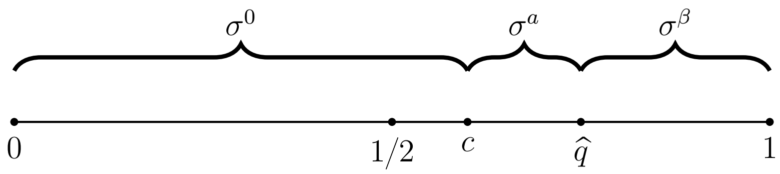
Recall that for any $\tilde{p}$, $\widehat{q}(\tilde{p}), q^*(\tilde{p}) \geqslant c > \frac{1}{2}$ and $q^*(\tilde{p}) = 1$ for $\tilde{p} > 1 - c$. Therefore, under $\tilde{p} < 1 - c$, for $q > \frac{1}{2}$, σ^0 is the maximal equilibrium in $q \in (\frac{1}{2}, c)$, σ^a is the maximal equilibrium in $q \in (c, \widehat{q}(\tilde{p}))$, σ^β is the maximal equilibrium in $q \in (\widehat{q}, (\tilde{p}) q^*(\tilde{p}))$, σ^1 is the maximal equilibrium in $q \in (q^*(\tilde{p}), 1)$. Finally, under $\tilde{p} > 1 - c$, $q^*(\tilde{p}) = 1$ and σ^1 is not an equilibrium at all. This proves the second part of the proposition.

□

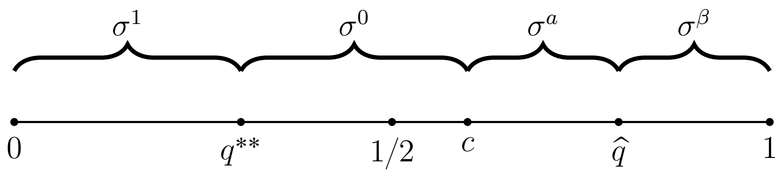
Figure 1 provides a visual description of how the maximal equilibrium changes with information similarity. In this simple example, under conditionally independent signals $\tilde{p} = p < 1 - c < \frac{1}{2}$, $I^1(\tilde{p}, q)$ increases in q. That is, a player has a higher incentive to work if he is more likely to complete the project on his own. Therefore, there is a $q^* > c$ such that σ^1 is an equilibrium for $q > q^*$. For $q < q^*$, σ^1 cannot be sustained as an equilibrium. However, if $q > c$, there is an asymmetric and mixed equilibrium. A higher $\tilde{p}$ means $I^1(\tilde{p}, q)$ is lower (since $q > \frac{1}{2}$). Thus, similar information reduces a player's incentive to work for high q, and it becomes more difficult to sustain σ^1 as an equilibrium (q^* increases). In fact, if $\tilde{p} > 1 - c$, then $q^* = 1$, i.e., σ^1 cannot be an equilibrium for any $q > \frac{1}{2}$. Even for the mixed strategy equilibrium, the probability of working must decrease to sustain an equilibrium. As $\tilde{p}$ increases further ($\tilde{p} > c$), then $I^1(\tilde{p}, q) > c$ for sufficiently small values of q ($q < q^{**}$). That is, players who are unlikely to complete the project on their own, are now willing to work because it is very likely ($\tilde{p} > c$), that when they see 1 the other also sees 1. As $\tilde{p}$ increases further, $I^1(\tilde{p}, q)$ increases, that is, it becomes easier to sustain σ^1 as an equilibrium for low values of q, and accordingly q^{**} increases.



$\tilde{p} < 1 - c$



$\tilde{p} \in (1 - c, c)$



$\tilde{p} > c$

Figure 1: Information Similarity and Maximal Equilibrium