Supplemental Appendix

Recurring Auctions with Costly Entry: Theory and Evidence

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In this appendix, we collect the analyses and discussions omitted from the main text.[53]

Appendix B Details in Empirical Analysis

B.1 Likelihood Function

Four scenarios emerge as the outcomes of recurring auctions. The likelihood function for each scenario can be calculated as follows.

Scenario 1: No one enters for three consecutive auctions. This implies that no one's valuation is higher than the entry threshold in the last auction (v_3^*), so the likelihood is $L_1 = [F(v_3^*)]^N$.

Scenario 2: Only one potential buyer enters and wins at the reserve price. This implies that there is only one potential buyer whose private value is above the entry threshold. The probability for that event is $L_2 = \binom{N}{1}\ [F(v_{t-1}^*) - F(v_t^*\ )]F(v_t^*)^{N-1}$.

Scenario 3: There are multiple entrants, and the deal price is higher than the reserve price. We calculate the likelihood as the unconditional probability of N_e entrants, multiplied by the conditional density of the second-highest value being the deal price:

$$L_3 = \binom{N}{N_e} N_e(N_e-1)[F(v_t^*)]^{N-N_e} f(\hat{p})[F(\hat{p}) - F(v_t^*)]^{N_e-2}[F(v_{t-1}^*) - F(\hat{p})],$$

where $\hat{p}$ is the observed winning bid.

Scenario 4: Zero probability events given the equilibrium, such as the winning bid being lower than the predicted entry threshold. The likelihood is 0 for these events. However, this does not imply that the simulated likelihood is 0, since this is only for one particular simulation. If, in all simulations for property i, there is at least one simulation draw in which the outcome can be rationalized, the simulated likelihood $\frac{1}{S}\sum_s L_{si}$ would be positive.

[53]This note is not self-contained; it is the online appendix of the paper "Recurring Auctions with Costly Entry: Theory and Evidence."

B.2 Importance Sampling

In this section, we lay out the detailed steps of the simulated maximum likelihood approach with importance sampling. Specifically, we rewrite the integral in equation (12) as follows:

$$\int L_i(y_i|\Lambda_i)\phi(\Lambda_i|B,X_i)d\Lambda_i = \int L_i(y_i|\Lambda_i)\frac{\phi(\Lambda_i|B,X_i)}{g(\Lambda_i|X_i)}g(\Lambda_i|X_i)d\Lambda_i, \tag{B1}$$

where $g(\Lambda_i|X_i)$ is the importance sampling density, which does *not* depend on the parameters B. In practice, we pick an initial guess B_0 and use $\phi(\Lambda_i|B_0,X_i)$ as the importance sampling density.

We then simulate the right-hand side of (B1) by drawing $S = 1,000$ realizations of Λ_i according to the importance sampling density, $g(\Lambda_i|X_i)$. Compared with $\phi(\Lambda_i|B,X_i)$, the importance sampling density renders Λ_i draws independent of B. The simulation is given by

$$\frac{1}{S}\sum_s L_{is}(y_{is}|\Lambda_{is})\frac{\phi(\Lambda_{is}|B,X_i)}{g(\Lambda_i|X_i)}, \tag{B2}$$

where Λ_{is} denotes a representative draw. The benefit of the importance sampling approach can be clearly seen from (B2): When B changes, it is not necessary to draw a new set of realizations of Λ_i and reevaluate $L_{is}(y_{is}|\Lambda_{is})$. Instead, the same set of $S = 1,000$ simulations can be used and only $\phi(\Lambda_{is}|B,X_i)/g(\Lambda_i|X_i)$ needs to be reevaluated, which is significantly less time-consuming.

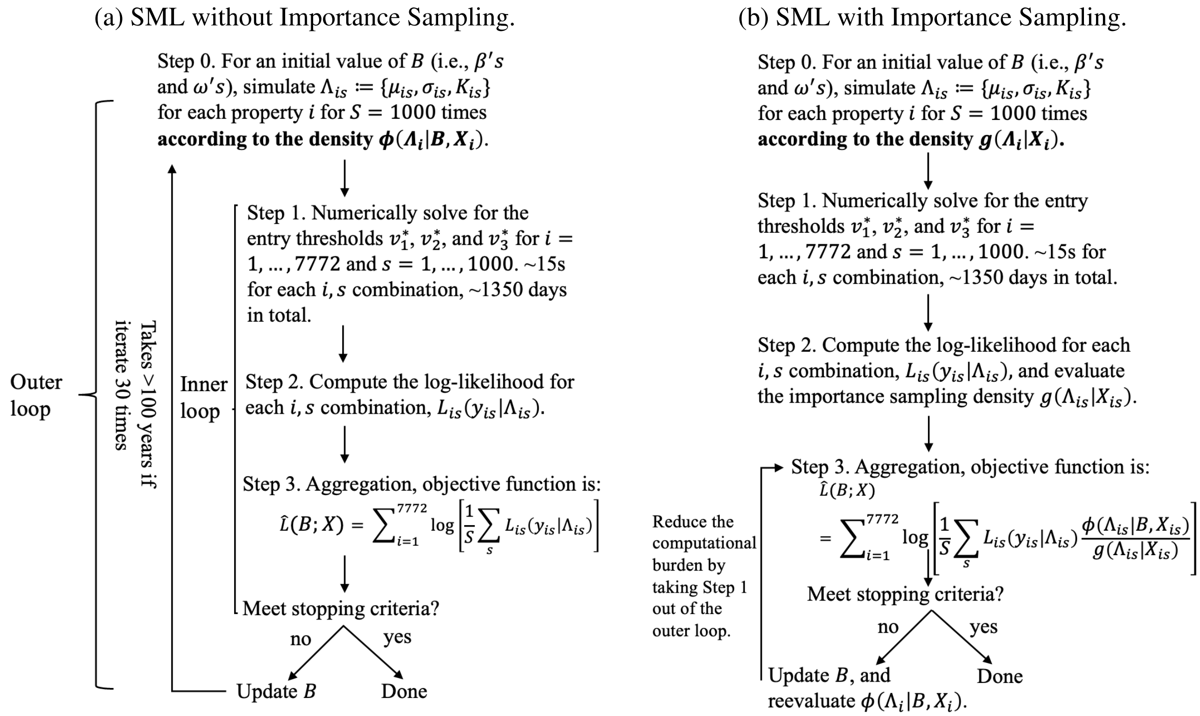
Figure B1 shows the details of the estimation steps. In Panel (a), we show a hypothetical scenario in which we estimate a model without using the importance sampling method. The flow chart in Panel (b) of Figure B1 shows the estimation steps of the simulated maximum likelihood method with importance sampling.

We use a computer cluster to evaluate $L_{is}(y_{is}|\Lambda_{is})$ for all properties and simulations in parallel, which further reduces the computation time for Step 1 in Figure B1(b). After obtaining the results, we search for B that maximizes the simulated likelihood. Standard errors are computed using a bootstrapping method in which *properties* are resampled 200 times.

B.3 Alternative Definitions of Potential Buyers

We examine the robustness of our results to alternative definitions of potential buyers. In the baseline setting, we compute the number of potential buyers as the number of individuals who have shown interest online, divided by 1000, plus the number of actual entrants. In this section, we change the factor from 1/1,000 to 1/500 and 1/1,500 and reestimate the recurring auction model.

Figure B1: Estimation Steps with and without Importance Sampling.



The estimation results reported in Table B1 and the counterfactual analysis results reported in Table B2 are similar to those obtained in the baseline setting, which attests to the robustness of our results.

B.4 Estimation Results by Year

In this section, we divide our sample into two groups: houses auctioned in 2017 and houses auctioned in 2018 and 2019. The estimation results reported in Table B3 suggest that coefficient estimates are similar across the two groups.

B.5 Estimation Results by Time Lag between First Two Auction Rounds

To further address concerns about potential buyers or their valuations changing over time, we compare parameter estimates for two subsamples: one with short time lags (bottom quartile, less than 24 days) between the first two auction rounds, and another with long time lags (top quartile, more than 44 days). The results reported in Table B4 indicate that the estimated parameters are very similar across subsamples with different time lags, which suggests that new entrants and changing

Table B1: Estimation Results for Alternative Definitions of Potential Buyers.

	constant	log (assess. price)	log(dist.)	area (100 m^2)	ω	mean
Panel A: Factor=1/500						
$\mu \sim TRN$	-0.288	1.002	-0.048	-0.038	0.167	4.268
$(X\beta_\mu, \omega_\mu, 1, 7)$	(0.022)	(0.005)	(0.003)	(0.010)	(0.003)	
$\sigma \sim TRN$	0.250	-0.023	0.021	0.018	0.096	0.196
$(X\beta_\sigma, \omega_\sigma, 0.01, 3)$	(0.020)	(0.004)	(0.002)	(0.008)	(0.003)	
$K \sim TRN$	-2.876	0.605	-0.251	0.337	0.480	0.513
$(X\beta_K, \omega_K, 0, 15)$	(0.154)	(0.031)	(0.020)	(0.045)	(0.014)	
Panel B: Factor=1/1500						
$\mu \sim TRN$	-0.158	0.989	-0.035	-0.029	0.157	4.364
$(X\beta_\mu, \omega_\mu, 1, 7)$	(0.015)	(0.003)	(0.002)	(0.007)	(0.003)	
$\sigma \sim TRN$	0.257	-0.032	0.022	0.035	0.104	0.188
$(X\beta_\sigma, \omega_\sigma, 0.01, 3)$	(0.018)	(0.004)	(0.002)	(0.008)	(0.004)	
$K \sim TRN$	-3.011	0.642	-0.249	0.304	0.478	0.514
$(X\beta_K, \omega_K, 0, 15)$	(0.132)	(0.027)	(0.018)	(0.039)	(0.013)	

Notes: (1) We use the same data and the same parameterization for Panel A and Panel B. The only difference lies in the definition of potential buyers. (2) Potential buyers' valuation is assumed to follow a truncated lognormal distribution: $v \sim TRLN(\mu, \sigma, 10^{-4}, 1200)$. (3) Standard errors in parentheses are obtained through bootstrapping 200 times . (4) The rightmost column shows the mean of $\Lambda = \{\mu, \sigma, K\}$.

valuations across rounds are not a major concern in the current setting.[54]

B.6 Balance Test for Attrited Houses

A balance test is performed to determine whether the observable characteristics of attrited houses differ significantly from those of other houses. As Table B5 shows, there are no significant differences in any of the observed variables we analyze between attrited houses and others.

B.7 Determinants of the Number of Bidders

Table B6 reports determinants of the number of entrants. The results suggest that the number of potential entrants has a significant and positive impact on the number of actual entrants. The number of potential entrants alone can explain 66% of the variation in the number of actual entrants

[54]Since the subsamples consist of observations conditional on the failure of the initial auction, estimation results are not expected to be similar to the baseline estimates obtained using the whole sample.

Table B2: Counterfactual Analyses for Alternative Definitions of Potential Buyers.

	Single-round ($T=1$)	Recurring ($T=2$)	Recurring ($T=3$)
Panel A: current reserve price			
Factor=1/500			
Mean efficiency (10K CNY)	112.79	131.79	132.71
Mean revenue (10K CNY)	103.37	120.30	121.09
Factor=1/1500			
Mean efficiency (10K CNY)	114.66	132.51	133.20
Mean revenue (10K CNY)	104.13	119.55	120.11
Panel B: optimal reserve price			
Factor=1/500			
Mean efficiency (10K CNY)	135.74	136.70	136.75
Mean revenue (10K CNY)	123.60	124.54	124.59
Factor=1/1500			
Mean efficiency (10K CNY)	137.25	137.95	137.99
Mean revenue (10K CNY)	123.16	123.85	123.89

Notes: (1) We report the mean revenue and efficiency at property level. (2) "Single-round (T=1)" refers to single-round auctions; "Recurring (T=2)" refers to 2-period recurring auctions; "Recurring (T=3)" refers to 3-period recurring auctions. (3) For Panel A, we use the current reserve prices, i.e., the reserve prices used in the estimation of the 3-period recurring auctions. A single-round auction is a 3-period recurring auction with the last 2 periods removed. A 2-period recurring auction is a 3-period recurring auction with the last period removed. For Panel B, we use the optimal reserve prices for efficiency and revenue, respectively, in each of the three cases in which $T=1$, $T=2$, and $T=3$.

Table B3: Estimation Results by Year.

	constant	$\log\left(\begin{array}{c}\text{assess.}\\ \text{price}\end{array}\right)$	log(dist.)	area (100 m^2)	ω	mean
Panel A: Year=2017						
$\mu \sim TRN$	-0.318	1.033	-0.025	-0.052	0.159	4.336
$(X\beta_\mu, \omega_\mu, 1, 7)$	(0.028)	(0.007)	(0.004)	(0.014)	(0.005)	
$\sigma \sim TRN$	0.246	-0.025	0.033	0.014	0.099	0.191
$(X\beta_\sigma, \omega_\sigma, 0.01, 3)$	(0.032)	(0.007)	(0.004)	(0.014)	(0.006)	
$K \sim TRN$	-3.476	0.719	-0.374	0.398	0.495	0.506
$(X\beta_K, \omega_K, 0, 15)$	(0.142)	(0.033)	(0.036)	(0.064)	(0.022)	
Panel B: Year=2018 or 2019						
$\mu \sim TRN$	-0.175	0.987	-0.044	-0.039	0.157	4.329
$(X\beta_\mu, \omega_\mu, 1, 7)$	(0.019)	(0.004)	(0.003)	(0.008)	(0.003)	
$\sigma \sim TRN$	0.257	-0.031	0.020	0.038	0.099	0.192
$(X\beta_\sigma, \omega_\sigma, 0.01, 3)$	(0.018)	(0.004)	(0.003)	(0.009)	(0.003)	
$K \sim TRN$	-2.890	0.618	-0.235	0.294	0.477	0.514
$(X\beta_K, \omega_K, 0, 15)$	(0.164)	(0.034)	(0.021)	(0.058)	(0.016)	

Notes: (1) #Obs.=1,807 in Panel A; #Obs.=5,965 in Panel B. (2) Potential buyers' valuation is assumed to follow a truncated lognormal distribution: $v \sim TRLN(\mu, \sigma, 10^{-4}, 1200)$. (3) Standard errors in parentheses are obtained through bootstrapping 200 times. (4) The rightmost column shows the mean of $\Lambda = \{\mu, \sigma, K\}$.

(as shown in the first column), which implies that the proxy is well constructed.

In the last column, we further explore the heterogeneous effects of the number of potential buyers across auction rounds. The results suggest that each additional potential buyer leads to an increase of 0.593 in the number of actual entrants in the first round. This effect rises to 0.775 (0.593 + 0.182) in the second round and falls to 0.45 (0.593 - 0.143) in the third round. The sorted entry pattern offers an explanation for the differential impact of the number of potential buyers on actual entrants across rounds. Alternatively, if sorting were absent and each auction independently drew a new set of potential buyers, the effect of the number of potential buyers on actual entrants would be homogeneous between the second and third rounds, since the reserve price remains the same.

The increase in the impact of the number of potential buyers on actual bidders during the second round and its decrease in the third round are also consistent with the institutional background and our model. Specifically, there is a 20% decrease in the reserve price in the second round, with no

Table B4: Estimation Results by Time Lag.

	constant	log(assess. price)	log(dist.)	area (100 m^2)	ω	mean
Panel A: Top quartile (time lag<24 days)						
$\mu \sim TRN$	-0.460	1.030	-0.033	-0.076	0.161	4.150
$(X_{\beta_\mu}, \omega_\mu, 1, 7)$	(0.092)	(0.020)	(0.016)	(0.042)	(0.008)	
$\sigma \sim TRN$	0.276	-0.023	0.028	-0.006	0.096	0.199
$(X_{\beta_\sigma}, \omega_\sigma, 0.01, 3)$	(0.133)	(0.025)	(0.017)	(0.051)	(0.015)	
$K \sim TRN$	-2.042	0.434	-0.201	0.415	0.478	0.591
$(X_{\beta_K}, \omega_K, 0, 15)$	(0.649)	(0.114)	(0.084)	(0.317)	(0.067)	
Panel B: Bottom quartile (time lag>44 days)						
$\mu \sim TRN$	-0.522	1.025	-0.027	-0.032	0.195	4.146
$(X_{\beta_\mu}, \omega_\mu, 1, 7)$	(0.069)	(0.016)	(0.011)	(0.037)	(0.014)	
$\sigma \sim TRN$	0.259	-0.024	0.012	0.038	0.106	0.219
$(X_{\beta_\sigma}, \omega_\sigma, 0.01, 3)$	(0.067)	(0.015)	(0.013)	(0.033)	(0.018)	
$K \sim TRN$	-1.553	0.351	-0.172	0.355	0.399	0.537
$(X_{\beta_K}, \omega_K, 0, 15)$	(0.392)	(0.086)	(0.074)	(0.159)	(0.052)	

Notes: (1) #Obs.=617 in Panel A and B; (2) Potential buyers' valuation is assumed to follow a truncated lognormal distribution: $v \sim TRLN(\mu, \sigma, 10^{-4}, 1200)$. (3) Standard errors in parentheses are obtained through bootstrapping 200 times. (4) The rightmost column shows the mean of $\Lambda = \{\mu, \sigma, K\}$.

Table B5: Balance Test for Attrited Houses.

	log(reserve)	log(assess. price)	area (100 m^2)	log(dist.)
attrition=1	0.011	-0.002	-0.013	0.001
	(0.062)	(0.060)	(0.035)	(0.096)
Year-by-month fixed effects	X	X	X	X
Observations	3891	3891	3891	3891
R-squared	0.056	0.050	0.031	0.041

Notes: (1) Standard errors in parentheses. (2) We remove houses sold in the first period from the balance test, since no attrition can happen in the first period.

further decrease in the third round. Therefore, we expect a significant drop in the entry threshold for the second round, while the decrease in the entry threshold between the second and third rounds is more moderate.

Table B6: Determinants of the Number of Bidders.

	# of bidders			
# of potential buyers	0.538	0.605	0.621	0.593
	(0.004)	(0.004)	(0.004)	(0.004)
round=2 $\cdot$ # of potential buyers				0.182
				(0.009)
round=3 $\cdot$ # of potential buyers				-0.143
				(0.037)
log(assessed price)		-0.936	-1.013	-1.053
		(0.029)	(0.029)	(0.029)
area (100 m^2)		0.379	0.510	0.534
		(0.049)	(0.049)	(0.048)
log (dist to city center)		-0.131	-0.141	-0.144
		(0.016)	(0.016)	(0.016)
round=2		1.608	1.676	0.196
		(0.048)	(0.048)	(0.088)
round=3		0.860	0.935	1.272
		(0.081)	(0.080)	(0.165)
Year-by-month fixed effects			X	X
Observations	11411	11411	11411	11411
R-squared	0.659	0.714	0.727	0.737

Notes: (1) Standard errors in parentheses. (2) We pool auctions in all three rounds in this table.

B.8 What Parameters Drive the Efficiency and Revenue Improvements?

To explore how auction-specific heterogeneity affects revenue and efficiency improvements from adding auction rounds, we regress the percentage improvements in efficiency and revenue—calculated for running $T = 3$ auctions instead of $T = 1$ auction using optimal reserve prices—against the mean value parameter μ, the entry cost K, the scale parameter σ, and the number of potential entrants. Results are reported in Table B7.

The findings are consistent with our recurring auction model. Properties with higher mean valuations and greater variance in bidders' value distributions are more likely to be sold, even in a single-round auction. As a result, the revenue and efficiency improvements from the reducing auction failure channel are limited. For properties where potential bidders face higher entry costs or the number of potential entrants is large, adding auction rounds helps potential buyers economize on entry costs and leads to improvements.

Table B7: Revenue and Efficiency Improvement by Adding Auction Rounds.

	Eff_improv	Rev_improv
μ	-0.561	-0.602
	(0.006)	(0.006)
K	0.839	0.898
	(0.015)	(0.017)
σ	-4.282	-4.421
	(0.094)	(0.102)
# of potential entrants	0.027	0.029
	(0.000)	(0.000)
Year-by-month fixed effects	X	X
Observations	7699	7699
R^2	0.699	0.690

Notes: (1) Standard errors in parentheses.

B.9 Varying Entry Costs

Table B8: Varying Entry Costs.

	Single-round ($T = 1$)	Recurring ($T = 2$)	Recurring ($T = 3$)
Panel A: current entry cost			
Mean efficiency (10K CNY)	114.28	132.48	133.26
Mean revenue (10K CNY)	104.06	119.98	120.62
Panel B: 90% of the current entry cost			
Mean efficiency (10K CNY)	114.67	132.70	133.42
Mean revenue (10K CNY)	104.41	120.18	120.77
Panel C: 110% of the current entry cost			
Mean efficiency (10K CNY)	113.89	132.26	133.10
Mean revenue (10K CNY)	103.71	119.77	120.47

Notes: (1) We report the mean revenue and efficiency at property level. (2) "Single-round (T=1)" refers to single-round auctions; "Recurring (T=2)" refers to 2-period recurring auctions; "Recurring (T=3)" refers to 3-period recurring auctions.