Supplemental Appendix
Tying with Network Effects

Jay Pil Choi, Doh-Shin Jeon and Michael D. Whinston

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In this Supplemental Appendix, we provide four extensions: multihoming consumers, asymmetric network effects, product differentiation with partial market foreclosure, and network effects in market A (the tying market).

For completeness and self-containment of the Supplemental Appendix, we restate several Lemmas and Propositions from the main text. Specifically, Proposition 4, Lemma 3, and Proposition 5 from the main text are relabeled here as Proposition A.1, Lemma B.1, and Proposition B.1, respectively.

A Analysis of Tying with Multihoming Consumers

In this Appendix, we provide an extension which allows for a positive fraction of consumers who can multihome.

A.1 Model

Let $\mu \in (0,1)$ denote the fraction of consumers who can multihome at no cost, while the remaining consumers single-home, as in our main model. We assume that the ability to multihome is independent of a consumer's valuation for product A. As in Doganoglu and Wright (2006) and Jullien, Pavan and Rysman (2021), a multihoming consumer's gross market B surplus from consumption of both $B1$ and $B2$ equals the higher of the standalone benefits from the two products, $\max\{v_1, v_2\}$, plus β times the total number of consumers she can interact with through use of $B1$ and $B2$ (equal to the network size of $B1$ plus the network size of $B2$ minus the number of multihomers who consume both $B1$ and $B2$). In addition, we make the following assumptions:

• In market B, we maintain the current Assumption 1:

$$\Delta \equiv v_2 - v_1 > 0, \Delta < \beta < \frac{1}{2g(x)} \text{ for all } x \in [0, \overline{x}].$$

• In market A, we assume that α is sufficiently large so that firm 1 serves all consumers with the price of $p_A = \alpha$ under independent pricing; i.e., that condition (4) in the main text holds:

$$\alpha \geq \frac{1 - G(0)}{g(0)} = \frac{1}{g(0)}.$$

A.2 Independent pricing with multihoming consumers

A.2.1 Market A

Under the full market coverage condition (4), firm 1 serves all consumers in market A and hence the profit of firm 1 in market A is α.

A.2.2 Market B

Observe first that following price offers (p_{B1}, p_{B2}) such that $max\{v_1 + \beta - p_{B1}, v_2 + \beta - p_{B2}\} \geq 0$, there is always a NE consumer response of the following form:

- If $p_{B2} - p_{B1} < \Delta$: There is a NE consumer response in which all consumers buy $B2$ and none buy $B1$, and all enjoy a market B surplus equal to $v_2 + \beta - p_{B2}$.

- If $p_{B2} - p_{B1} > \Delta$: There is a NE consumer response in which all consumers buy $B1$ and none buy $B2$. (Note that since $p_{B1} \geq 0$ we have $p_{B2} > \Delta$, which implies that multihomers do not want to also buy $B2$.) All consumers enjoy a market B surplus equal to $v_1 + \beta - p_{B1}$.

- If $p_{B2} - p_{B1} = \Delta$: Both of the above NE consumer responses are equilibria. In addition, if $(p_{B1}, p_{B2}) = (0, \Delta)$, then there are also equilibria in which some (or all) multihomers buy both $B1$ and $B2$ while single-homers either all buy $B1$ or all buy $B2$. Every consumer receives the same payoff (equal to $v_1 + \beta - p_{B1} = v_2 + \beta - p_{B2}$) in all of these NE consumer responses.

In each of these cases, the NE consumer responses described above result in the largest possible market B surplus for every consumer given the price offers (p_{B1}, p_{B2}). In the first two cases, they are the unique NE consumer response satisfying our Pareto dominance refinement. In the last case, all of the NE consumer responses described are Pareto undominated NE consumer responses. However, the response in which all consumers buy only $B2$ gives firm 1 the lowest possible payoff among these and so is the NE response we select.[1]

Now consider equilibrium price offers given these consumer responses. In any equilibrium some consumer must be making a purchase: if not, then it must be that $v_i+\beta-p_{Bi} \leq 0$ for $i = 1, 2$ (otherwise there is a NE consumer response in which all consumers buy the same product and it gives strictly positive surplus). However, in that case, firm 2 would have a profitable deviation to charge $p_{B2} = \Delta$ which would attract all consumers since $v_2+\beta-\Delta > 0$ – a contradiction. Hence, in any equilibrium $max\{v_1+\beta-p_{B1}, v_2+\beta-p_{B2}\} \geq 0$.

If $p_{B2}-p_{B1} > \Delta$, then (by the discussion above) $B1$ is making all of the market B sales at price $p_{B1} \geq 0$. Firm 2 then has a profitable deviation to a price slightly below $p_{B1} + \Delta$, which leads all consumers to buy $B2$ – a contradiction. If, instead, $p_{B2} - p_{B1} < \Delta$, then firm 2 is selling $B2$ to all consumers but could do so more profitably if it deviated to a higher price – another contradiction.

Hence, it must be that $p_{B2}-p_{B1} = \Delta$ and (by the discussion above) that all consumers are buying only $B2$. However, if $p_{B1} > 0$, then firm 1 would have a profitable deviation that lowers p_{B1} slightly and leads all consumers to buy $B1$. Hence, in any equilibrium we must have $(p_{B1}, p_{B2}) = (0, \Delta)$ and all consumers buying only $B2$. As neither firm has a profitable deviation at those price offers, they are the unique equilibrium price offers.[2]

A.2.3 Summary

Under independent pricing, firm 1's profit is α and firm 2's profit is Δ.

[1]Our conclusion about the market B profits of the two firms in an independent pricing equilibrium would be unchanged if we did not select this NE consumer response, but rather any of these undominated NE consumer responses were possible.

[2]If we do not select the "all consumers buy only $B2$" NE consumer response when $p_{B2} - p_{B1} = \Delta$ we would derive the same equilibrium profits for the two firms because (i) if firm 2 was making strictly less than Δ despite the fact that $p_{B2} \geq \Delta$ then firm 2 would have a profitable deviation to a slightly lower price at which all consumers buy $B2$ and (ii) if firm 2 was making strictly more than Δ (and, hence, $p_{B2} > \Delta$, $p_{B1} > 0$, and firm 2 makes some sales of $B2$) then firm 1 would have a profitable deviation to a slightly lower price at which all consumers buy $B1$. Thus, we must have $p_{B2} = \Delta$, $p_{B1} = 0$, and all consumers purchasing $B2$. However, without the equilibrium selection, the same profits could arise in an equilibrium in which all consumers buy $B2$ at price $p_{B2} = \Delta$ but multihomers also buy $B1$ at a price of $p_{B1} = 0$.

A.3 Tying with multihoming consumers

We focus on the case of pure bundling to examine how our results generalize with the presence of consumer multihoming. We begin by examining consumer responses to firms' price offers, establishing a lemma that extends Lemma 1 in the paper to the case in which a positive fraction of consumers are capable of multihoming. As in the main text, we define $\widehat{P} = P - \alpha$. As we focus on an equilibrium in which all consumers buy the bundle, any p_{B2} strictly higher than Δ leads to zero profit for firm 2 as no multihomer will buy $B2$. For this reason, in what follows, we focus on price offers by firm 2 with $p_{B2} \in [0, \Delta]$ and consumer responses in which all multihoming consumers purchase $B2$.[3]

A.3.1 Consumer responses

For single-homing consumers, let $\psi_s(x, X_s, X_m|\widehat{P})$ be the payoff gain, given $\widehat{P}$, from purchasing the bundle over purchasing $B2$ for a single-homing type x consumer (i.e., whose willingness to pay for A is $\alpha + x$) if all single-homing consumers and all multihoming consumers whose types are respectively higher than X_s and X_m purchase the bundle. In a similar way, define $\psi_m(x, X_s, X_m)$ to be the payoff gain from purchasing the bundle *and* $B2$ over purchasing *only* $B2$ for a multihoming type x consumer. We have

$$\psi_s(x, X_s, X_m|\widehat{P}) = x + [(1-\mu)(1-2G(X_s)) - \mu G(X_m)]\,\beta - \Delta - (\widehat{P} - p_{B2}) \qquad \text{(A.1)}$$

and

$$\psi_m(x, X_s, X_m|\widehat{P}) = x + (1-\mu)(1-G(X_s))\beta - \widehat{P} \qquad \text{(A.2)}$$

Notice that $\psi_s(x, X_s, X_m|\widehat{P})$ is continuous in (x, X_s, X_m), increasing in x, and decreasing in X_s and X_m, and that $\psi_m(x, X_s, X_m|\widehat{P})$ shares the same characteristics except that it is independent of X_m. Also, $\psi_s(x, X_s, X_m) \leq \psi_m(x, X_s, X_m)$ for all (x, X_s, X_m). Intuitively, multihomers are more inclined to buy the bundle (in addition to $B2$) than are single-homers since, unlike for single-homers, for multihomers buying the bundle does not forgo network benefits from the presence of single-homers who buy $B2$ and does not forego the better stand-alone value v_2.

It is useful to observe the following:

Claim A.1 *Suppose that $p_{B2} \in [0, \Delta]$ and that no consumers buy the bundle. Then:*

[3] When $p_{B2} = \Delta$ multihomers are indifferent about buying $B2$ in addition to the bundle, but in equilibrium they must buy $B2$ for otherwise firm 2 would deviate to a slightly lower price.

(i) Single-homing consumers with $x = \overline{x}$ are indifferent between the bundle and B2 if and only if $\widehat{P} = \overline{x} - \beta - (\Delta - p_{B2})$.

(ii) Multihoming consumers with $x = \overline{x}$ are indifferent between buying both the bundle and B2 and buying only B2 if and only if $\widehat{P} = \overline{x}$.

Proof. *(i) The indifference condition for single-homing consumers is given by*

$$\psi_s(\overline{x}, \overline{x}, \overline{x}|\widehat{P}) = \overline{x} - \beta - \Delta - (\widehat{P} - p_{B2}) = 0.$$

(ii) The indifference condition for multihoming consumers is given by

$$\psi_m(\overline{x}, \overline{x}, \overline{x}|\widehat{P}) = \overline{x} - \widehat{P} = 0.$$

■

Claim A.2 *Suppose that $p_{B2} \in [0, \Delta]$ and that all consumers buy the bundle. Then:*

(i) Single-homing consumers with $x = 0$ are indifferent between the bundle and B2 if and only if $\widehat{P} = (1 - \mu)\beta - (\Delta - p_{B2})$.

(ii) Multihoming consumers with $x = 0$ are indifferent between buying both the bundle and B2 and buying only B2 if and only if $\widehat{P} = (1 - \mu)\beta$.

Proof. (i) The indifference condition for single-homing consumers is given by

$$\psi_s(0, 0, 0|\widehat{P}) = \beta(1 - \mu) - \Delta - \left(\widehat{P} - p_{B2}\right) = 0.$$

(ii) The indifference condition for multihoming consumers is given by

$$\psi_m(0, 0, 0|\widehat{P}) = (1 - \mu)\beta - \widehat{P} = 0.$$

■

We introduce an assumption to ensure that the values of $\widehat{P}$ identified in Claims A.1 and A.2 are well-ordered – specifically, to ensure that $\overline{x} - \beta - (\Delta - p_{B2}) > (1 - \mu)\beta$:

Assumption M: $\overline{x} > (2 - \mu)\beta + \Delta$

Next, using (A.1) and (A.2), define a *NE consumer response in cutoff strategies* with cutoffs $(\widetilde{X}_s, \widetilde{X}_m) \in (0, \overline{x})^2$, by

$$\psi_s(\widetilde{X}_s, \widetilde{X}_s, \widetilde{X}_m) = 0, \tag{A.3}$$

$$\psi_m(\widetilde{X}_m, \widetilde{X}_s, \widetilde{X}_m) = 0. \tag{A.4}$$

A NE consumer response in cutoff strategies is a NE consumer response in which single-homers with $x \geq \widetilde{X}_s$ and multihomers with $x \geq \widetilde{X}_m$ buy the bundle.

Claim A.3 *Suppose that $p_{B2} \in [0, \Delta]$. In an interior NE consumer response in cutoff strategies with cutoffs $(\widetilde{X}_s, \widetilde{X}_m) \in (0, \overline{x})^2$, we have $\widetilde{X}_s > \widetilde{X}_m$.*

Proof. From (A.3) and (A.4) (as well as (A.1) and (A.2)), whenever $(\widetilde{X}_s, \widetilde{X}_m) \in (0, \overline{x})^2$, we have

$$\widetilde{X}_s - \widetilde{X}_m = \beta[(1-\mu)G(\widetilde{X}_s) + \mu G(\widetilde{X}_m)] + (\Delta - p_{B2}) > 0. \tag{A.5}$$

■

Next, we observe that there is at most one interior pair of cutoffs – i.e., there is a unique solution to (A.3) and (A.4) – and it is increasing in $\widehat{P}$:

Claim A.4 *There is a unique solution to equations (A.3) and (A.4) and it is increasing in $\widehat{P}$.*

Proof. Totally differentiating (A.5), we have

$$\frac{d\widetilde{X}_m}{d\widetilde{X}_s} = \frac{1 - \beta(1-\mu)g(\widetilde{X}_s)}{1 + \beta\mu g(\widetilde{X}_m)} > 0$$

by Assumption 1. As $\frac{d\widetilde{X}_m}{d\widetilde{X}_s} > 0$, there is a one-to-one relationship between $\widetilde{X}_s$ and $\widetilde{X}_m$, and we can write

$$\widetilde{X}_m = \varphi(\widetilde{X}_s),$$

where $\varphi'(\widetilde{X}_s) > 0$.

Let us define $H(X_s) = X_m + [(1-\mu)(1 - G(X_s)]\,\beta$ subject to $X_m = \varphi(X_s)$. Then, any interior equilibrium $(\widetilde{X}_s, \widetilde{X}_m) = (\widetilde{X}_s, \varphi(\widetilde{X}_s))$ satisfies

$$H(\widetilde{X}_s) = \varphi(\widetilde{X}_s) + \Big[(1-\mu)(1 - G(\widetilde{X}_s)\Big]\,\beta = \widehat{P},$$

by conditions (A.2) and (A.4).

To demonstrate the uniqueness of the interior equilibrium, we establish that $H(\widetilde{X}_s)$ is a strictly increasing function.

$$\begin{aligned} H'(\widetilde{X}_s) &= \varphi'(\widetilde{X}_s) - (1-\mu)g(\widetilde{X}_s)\beta \\ &= \frac{1-\beta(1-\mu)g(\widetilde{X}_s)}{1+\beta\mu g(\widetilde{X}_m)} - (1-\mu)g(\widetilde{X}_s)\beta \\ &= \frac{1}{1+\beta\mu g(\widetilde{X}_m)}\left[1-\beta(1-\mu)g(\widetilde{X}_s)(2+\beta\mu g(\widetilde{X}_m))\right] \end{aligned}$$

From Assumption 1, we have $1 > 2\beta g(x)$ for any x. Hence,

$$\begin{aligned} 1-\beta(1-\mu)g(\widetilde{X}_s)(2+\beta\mu g(\widetilde{X}_m)) &> 1 - \frac{(1-\mu)}{2}(2+\frac{\mu}{2}) \\ &= 1-(1-\mu) - \frac{\mu(1-\mu)}{4} \\ &> \mu(1-\frac{(1-\mu)}{4}) > 0 \end{aligned}$$

This implies that there can be only one interior equilibrium that satisfies (A.3) and (A.4). In addition, as $\widehat{P}$ increases, $\widetilde{X}_s$ increases and hence $\widetilde{X}_m$ increases as well. ■

The above claims suggest the following pattern of consumer responses when $p_{B2} \in [0, \Delta]$: At very high values of $\widehat{P}$ no consumers buy the bundle. As $\widehat{P}$ falls, multihomers of type $\overline{x}$ are the first to find purchase of the bundle (in addition to $B2$) to be a dominant strategy. They do so if $\widehat{P} < \overline{x}$. As $\widehat{P}$ falls further and additional multihomers buy the bundle (by iterated dominance), single-homers of type $\overline{x}$ come to find purchase of the bundle to be optimal (regardless of what any other single-homers and the remaining multihomers do). This happens at a bundle price $\widehat{P}$ (which we denote by $\overline{P}_{int}$), which is above $\overline{x} - \beta - (\Delta - p_{B2})$, the bundle price $\widehat{P}$ at which single-homers of type $\overline{x}$ find it dominant to buy the bundle if *no* other consumers are doing so. As $\widehat{P}$ declines further, consumer responses are interior until $\widehat{P}$ falls to a level (which we denote by $\underline{P}^{int}$) at which multihomers of type $x = 0$ find it optimal to buy the bundle given the other single- and multihoming consumers who are definitely buying the bundle. Because some single-homing consumers still are buying $B2$, $\underline{P}^{int}$ is strictly below $(1-\mu)\beta$, the bundle price at which they would find it optimal to do so if *all* other consumers were buying the bundle.[4]

[4] When $p_{B2} = \Delta$, single-homers and multihomers all come to buy the bundle at the same price $\widehat{P}$ and we have $\underline{P}^{int} = (1-\mu)\beta$.

Finally, when $\widehat{P}$ falls to $(1-\mu)\beta-(\Delta-p_{B2})$ all single-homers also find buying the bundle to be optimal given that all other consumers definitely are doing so.

Interior consumer responses arise for the range of bundle prices $\widehat{P} \in (\underline{P}_{int}, \overline{P}_{int})$. As noted in the previous paragraph, the upper end of this range, $\overline{P}_{int}$, is the level of $\widehat{P}$ at which just enough multihomers are buying the bundle so that single-homing consumers of type $\overline{x}$ are indifferent between the bundle and $B2$ if no other consumers other than those multihomers buy the bundle. The required cutoff type of multihomers, which we denote by X_m^{int} is given by the solution to $\psi_s(\overline{x}, \overline{x}, X_m^{int}|\overline{P}^{int}) = 0$. When no single-homing consumers are buying the bundle, a multihomer of type X_m^{int} is indifferent about buying the bundle when

$$\psi_m(X_m^{int}, \overline{x}, X_m^{int}|\widehat{P}) = 0 \leftrightarrow X_m^{int} = \widehat{P}.$$

Thus, $\overline{P}_{int}$ is the unique solution to[5]

$$\psi_s(\overline{x}, \overline{x}, \overline{P}^{int}|\overline{P}^{int}) = \overline{x} - \Big[(1-\mu) + \mu G(\overline{P}^{int})\Big]\beta - \Delta - \Big(\overline{P}^{int} - p_{B2}\Big) = 0. \qquad \text{(A.6)}$$

The lower end of the range of bundle prices that lead to interior consumer responses, $\underline{P}_{int}$, is the level of $\widehat{P}$ at which just enough single-homing consumers are buying the bundle so that multihoming consumers of type $x = 0$ are indifferent between buying both the bundle and $B2$ and buying $B2$ only if all of the remaining single-homers buy $B2$. The required cutoff type of single-homing consumers, which we denote by X_s^{int}, is given by the solution to $\psi_m(0, X_s^{int}, 0|\underline{P}^{int}) = 0$. The cutoff X_s^{int} and $\underline{P}_{int}$ are therefore the unique solution to the following two equations:[6]

$$\psi_m(0, X_s^{int}, 0|\underline{P}^{int}) = (1-\mu)(1-G(X_s^{int}))\beta - \underline{P}^{int} = 0.$$

and

$$\psi_s(X_s^{int}, X_s^{int}, 0|\underline{P}^{int}) = X_s^{int} + (1-\mu)(1-2G(X_s^{int}))\beta - \Delta - \big(\underline{P}^{int} - p_{B2}\big) = 0.$$

For $\widehat{P}$ above $\overline{P}^{int}$, single-homing consumers all buy $B2$ so if any multihomers buy the bundle it is those types $x > \widetilde{X}_m$ such that $\psi_m(\widetilde{X}_m, \overline{x}, \widetilde{X}_m|\widehat{P}) = 0$ if such a solution exists and $\widetilde{X}_m = \overline{x}$ if not. The cutoff $\widetilde{X}_m$ that satisfies this condition (which is $\widetilde{X}_m = \widehat{P}$) is

[5] There is a unique solution to this equation since it is positive at $\overline{P}_{int} = 0$, negative at $\overline{P}_{int} = \overline{x}$, and a decreasing function of $\overline{P}_{int}$.

[6] Uniqueness follows because substituting for $\underline{P}_{int}$ in the second equation it becomes $X_s^{int} - (1-\mu)G(X_s^{int}))\beta - \Delta + p_{B2} = 0$ which is non-positive at $X_s^{int} = 0$, non-negative at $X_s^{int} = \overline{x}$ by Assumption M, and an increasing function by Assumption 1. Observe that when $p_{B2} = \Delta$, $X_s^{int} = 0$ and $\underline{P}^{int} = (1-\mu)\beta$.

unique and strictly increasing in $\widehat{P}$ for $\widehat{P} < \overline{x}$.

Likewise, for $\widehat{P}$ below $\underline{P}^{int}$, multihomers all buy the bundle so if any single-homers buy $B2$ it is those types $x < \widetilde{X}_s$ such that $\psi_s(\widetilde{X}_s, \widetilde{X}_s, 0|\widehat{P}) = 0$ if such a solution exists and $\widetilde{X}_s = 0$ otherwise. The cutoff $\widetilde{X}_s$ that satisfies this condition is unique and increasing in $\widehat{P}$ for $\widehat{P} > (1-\mu)\beta - (\Delta - p_{B2})$.

Observe that the arguments above imply:

Claim A.5 *There is a unique NE consumer response in cutoff strategies.*

The following lemma summarizes these consumer response outcomes and shows that they are the unique outcome of iterated elimination of dominated strategies.

Lemma A.1 *Under Assumptions 1 and M and the full coverage condition (4), when firm 1 offers only a bundle for sale, given prices of P for the bundle and $p_{B2} \in [0, \Delta]$ for product $B2$, and defining $\widehat{P} = P - \alpha$, the unique outcome in consumers' choices that survives iterated deletion of dominated strategies is as follows:*

(i) If $\widehat{P} \geq \overline{x}$: all consumers purchase B2 only (i.e., $\widetilde{X}_s = \widetilde{X}_m = \overline{x}$).

(ii) If $\widehat{P} \in [\overline{P}^{int}, \overline{x})$: we have $0 < \widetilde{X}_m < \widetilde{X}_s = \overline{x}$, where $\widetilde{X}_m = \widehat{P}$.

(iii) If $\widehat{P} \in (\underline{P}^{int}, \overline{P}^{int})$: we have $0 < \widetilde{X}_m < \widetilde{X}_s < \overline{x}$ where $\widetilde{X}_s$ and $\widetilde{X}_m$ satisfy

$$\widetilde{X}_s + \left[(1-\mu)(1-2G(\widetilde{X}_s)) - \mu G(\widetilde{X}_m)\right]\beta - \Delta - \left(\widehat{P} - p_{B2}\right) = 0; \tag{A.7}$$

$$\widetilde{X}_m + (1-\mu)(1-G(\widetilde{X}_s))\beta - \widehat{P} = 0. \tag{A.8}$$

(iv) If $\widehat{P} \in ((1-\mu)\beta - (\Delta - p_{B2}), \underline{P}^{int}]$: we have $\widetilde{X}_s > 0$ and $\widetilde{X}_m = 0$ where $\widetilde{X}_s$ satisfies

$$\widetilde{X}_s + (1-\mu)(1-2G(\widetilde{X}_s))\beta - \Delta - (\widehat{P} - p_{B2}) = 0.$$

(v) If $\widehat{P} \leq (1-\mu)\beta - (\Delta - p_{B2})$: we have $\widetilde{X}_s = \widetilde{X}_m = 0$.

If $p_{B2} = \Delta$, then the above results hold with $(1-\mu)\beta - (\Delta - p_{B2}) = \underline{P}^{int} = (1-\mu)\beta$ – i.e., Case (iv) disappears.

Proof. We first define the iterations we use to establish iterated dominance. Given cutoffs (X_s^n, X_m^n) and a price $\widehat{P}$, we define the next cutoffs $X_s^{n+1} = \Gamma_s(X_s^n, X_m^n|\widehat{P})$ and $X_m^{n+1} = \Gamma_m(X_s^n, X_m^n|\widehat{P})$ as follows:

$$\Gamma_s(X_s^n, X_m^n|\widehat{P}) = \begin{cases} \overline{x} \text{ if } \psi_s(\overline{x}, X_s^n, X_m^n|\widehat{P}) \leq 0; \\ 0 \text{ if } \psi_s(0, X_s^n, X_m^n|\widehat{P}) \geq 0; \\ \{x|\psi_s(x, X_s^n, X_m^n|\widehat{P}) = 0\} \text{ otherwise,} \end{cases}$$

and

$$\Gamma_m(X_s^n, X_m^n|\widehat{P}) = \begin{cases} \overline{x} \text{ if } \psi_m(\overline{x}, X_s^n, X_m^n|\widehat{P}) \leq 0; \\ 0 \text{ if } \psi_m(0, X_s^n, X_m^n|\widehat{P}) \geq 0; \\ \{x|\psi_m(x, X_s^n, X_m^n|\widehat{P}) = 0\} \text{ otherwise.} \end{cases}$$

Observe that these are weakly increasing functions: if more other consumers are buying the bundle (corresponding to lower values of $(\overline{X}_s^n, \overline{X}_m^n)$), then any given type x of consumer is more willing to buy the bundle, weakly lowering the values $\Gamma_s(X_s^n, X_m^n)$ and $\Gamma_m(X_s^n, X_m^n)$.

We first define a sequence of iterated dominance cutoffs starting at $(\overline{X}_s^0, \overline{X}_m^0) = (\overline{x}, \overline{x})$. Observe that when $\widehat{P} < \overline{x}$, we have $\Gamma_m(\overline{X}_s^0, \overline{X}_m^0) < \overline{x}$ (i.e., multihomer types near $\overline{x}$ find buying the bundle dominant). This starts a decreasing sequence of cutoffs $\Gamma_m(\overline{X}_s^n, \overline{X}_m^n)$ that converge to some $(\overline{X}_s^*, \overline{X}_m^*)$ with $\overline{X}_m^* < \overline{x}$.

We next define a sequence of iterated dominance cutoffs starting at $(\underline{X}_s^0, \underline{X}_m^0) = (0, 0)$. Observe that when $\widehat{P} > (1-\mu)\beta - (\Delta - p_{B2})$, we have $\Gamma_s(\underline{X}_s^0, \underline{X}_m^0) > 0$ (i.e., single-homing consumer types near $x = 0$ find buying $B2$ dominant). This starts an increasing sequence of cutoffs $\Gamma_s(\underline{X}_s^n, \underline{X}_m^n)$ that converge to some $(\underline{X}_s^*, \underline{X}_m^*)$ with $\underline{X}_s^* > 0$ and $(\underline{X}_s^0, \underline{X}_m^0) \leq (\overline{X}_s^0, \overline{X}_m^0)$.

Observe that when $\widehat{P} \geq \overline{x}$ the increasing sequence begins but not the decreasing sequence, while the reverse is true when $\widehat{P} \leq (1-\mu)\beta - (\Delta - p_{B2})$.

Given the definition of the functions $\Gamma_s(\cdot)$ and $\Gamma_m(\cdot)$, the cutoffs $(\overline{X}_s^*, \overline{X}_m^*)$ and $(\underline{X}_s^*, \underline{X}_m^*)$ (whenever the corresponding decreasing or increasing sequence exists) both satisfy the conditions to be NE consumer responses in cutoff strategies. As there is always a unique such NE consumer response, the cutoffs $(\overline{X}_s^*, \overline{X}_m^*)$ and $(\underline{X}_s^*, \underline{X}_m^*)$ must both equal the cutoffs in that consumer response.

So the NE consumer responses in Cases (i)-(v) arise as a consequence of iterated elimination of dominated strategies.

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A.3.2 Equilibrium price offers

We examine conditions under which there is an equilibrium in which firm 1 sells the bundle to all consumers, setting a bundle price of $P^* = \alpha + (1-\mu)\beta$, and firm 2 sells $B2$ to all multihomers by setting $p^*_{B2} = \Delta$. In this outcome, the network size of the bundle is 1 while that of $B2$ is μ.

We consider, in turn, firm 1's and firm 2's incentives to deviate from these price offers.

Firm 1's deviation incentives

Since firm 1 is selling the bundle to all consumers when $\widehat{P} = (1-\mu)\beta$ and $p_{B2} = \Delta$, it has no incentive to lower $\widehat{P}$ below $(1-\mu)\beta$. So we focus on whether it would want to raise $\widehat{P}$ above $(1-\mu)\beta$. Since $\underline{P}^{inf} = (1-\mu)\beta$ when $p_{B2} = \Delta$, such a deviation leads to one of Cases (i)-(iii) in Lemma A.1. Clearly firm 1 will not deviate in a manner that leads to Case (i), in which its profit is zero. We next consider Cases (iii) and (ii) in turn.

Firm 1's profit as a function of $\widehat{P}$ is

$$\Pi_1(\widehat{P}) = (\alpha + \widehat{P}) \left[(1-\mu)(1-G(\widetilde{X}_s)) + \mu(1-G(\widetilde{X}_m))\right]$$

whose derivative is

$$\Pi_1'(\widehat{P}) = \left[(1-\mu)(1-G(\widetilde{X}_s)) + \mu(1-G(\widetilde{X}_m))\right] - \left(\alpha+\widehat{P}\right)\left[(1-\mu)g(\widetilde{X}_s)\frac{\partial \widetilde{X}_s}{\partial \widehat{P}} + \mu g(\widetilde{X}_m)\frac{\partial \widetilde{X}_m}{\partial \widehat{P}}\right]. \tag{A.9}$$

We first consider Case (iii) and show that $\Pi_1'(\widehat{P}) < 0$ at all $\widehat{P} \in ((1-\mu)\beta, \overline{P}^{int})$. To do so, we first establish the following result:

Claim A.6 $\frac{d\widetilde{X}_s}{d\widehat{P}} > 1$ *and* $\frac{d\widetilde{X}_m}{d\widehat{P}} > 1$.

Proof. By totally differentiating the two equations (A.3) and (A.4) characterizing the interior equilibrium, we have

$$\begin{bmatrix} 1-2(1-\mu)g(\widetilde{X}_s)\beta & -\mu g(\widetilde{X}_m)\beta \\ -(1-\mu)g(\widetilde{X}_s)\beta & 1 \end{bmatrix} \begin{bmatrix} \frac{d\widetilde{X}_s}{d\widehat{P}} \\ \frac{d\widetilde{X}_m}{d\widehat{P}} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

By applying Cramer's rule, we have

$$\frac{d\widetilde{X}_s}{d\widehat{P}} = \frac{\begin{vmatrix} 1 & -\mu g(\widetilde{X}_m)\beta \\ 1 & 1 \end{vmatrix}}{\begin{vmatrix} 1-2(1-\mu)g(\widetilde{X}_s)\beta & -\mu g(\widetilde{X}_m)\beta \\ -(1-\mu)g(\widetilde{X}_s)\beta & 1 \end{vmatrix}} = \frac{1+\mu g(\widetilde{X}_m)\beta}{1-2(1-\mu)g(\widetilde{X}_s)\beta - \mu(1-\mu)g(\widetilde{X}_m)g(\widetilde{X}_s)\beta^2} > 1$$

$$\frac{d\widetilde{X}_m}{d\widehat{P}} = \frac{\begin{vmatrix} 1-2(1-\mu)g(\widetilde{X}_s)\beta & 1 \\ -(1-\mu)g(\widetilde{X}_s)\beta & 1 \end{vmatrix}}{\begin{vmatrix} 1-2(1-\mu)g(\widetilde{X}_s)\beta & -\mu g(\widetilde{X}_m)\beta \\ -(1-\mu)g(\widetilde{X}_s)\beta & 1 \end{vmatrix}} = \frac{1-(1-\mu)G(\widetilde{X}_s)\beta}{1-2(1-\mu)g(\widetilde{X}_s)\beta - \mu(1-\mu)g(\widetilde{X}_m)g(\widetilde{X}_s)\beta^2} > 1$$

■

Now, for $\widehat{P} \in ((1-\mu)\beta, \overline{P}^{int})$, we have

$$\begin{aligned}
\Pi_1'(\widehat{P}) &= \left[(1-\mu)(1-G(\widetilde{X}_s)) + \mu(1-G(\widetilde{X}_m))\right] - (\alpha+\widehat{P})\left[(1-\mu)g(\widetilde{X}_s)\frac{d\widetilde{X}_s}{d\widehat{P}} + \mu g(\widetilde{X}_m)\frac{d\widetilde{X}_m}{d\widehat{P}}\right] \\
&< \left[(1-\mu)(1-G(\widetilde{X}_s)) + \mu(1-G(\widetilde{X}_m))\right] - (\alpha+\widehat{P})\left[(1-\mu)g(\widetilde{X}_s) + \mu g(\widetilde{X}_m)\right] \\
&= (1-\mu)\left[(1-G(\widetilde{X}_s) - (\alpha+\widehat{P})g(\widetilde{X}_s)\right] + \mu\left[(1-G(\widetilde{X}_m)) - (\alpha+\widehat{P})g(\widetilde{X}_m)\right] \\
&< 0
\end{aligned}$$

The first inequality follows from Claim 6 while the last inequality follows because the full market coverage assumption and the monotone hazard rate condition for $G(\cdot)$ imply that

$$\alpha \geq \frac{1-G(0)}{g(0)} > \frac{(1-G(\widetilde{X}_s))}{g(\widetilde{X}_s)} \text{ and } \alpha \geq \frac{1-G(0)}{g(0)} > \frac{(1-G(\widetilde{X}_m))}{g(\widetilde{X}_m)}$$

Thus, firm 1 has no incentive to deviate to any price $\widehat{P}$ that induces case (iii).

We next consider Case (ii). If there is a deviation that induces Case (ii), in which firm 1 sells only to multihoming consumers and $\widetilde{X}_m = \widehat{P}$, firm 1's profit becomes

$$\Pi_1(\widehat{P}) = \mu(\alpha+\widehat{P})(1-G(\widehat{P})) < (\alpha+\widehat{P})(1-G(\widehat{P})),$$

which is firm 1's profit under independent pricing. We know that it is maximized with $\widehat{P} = 0$ and thus it cannot be higher than α, which is strictly less than the profit with

$\widehat{P} = (1-\mu)\beta$.

Thus, firm 1 does not have any incentive to deviate.

Firm 2's deviation incentives

Firm 2 has no incentive to deviate to $p_{B2} > \Delta$, which leads to zero profit. We thus consider incentives to charge a lower price $p_{B2} < \Delta$.

We first show that any such deviation will lead to an interior equilibrium per Case (iii) of Lemma A.1 by showing that for any $p_{B2} \in [0, \Delta)$ we have $\overline{P}^{int} > \widehat{P} = (1-\mu)\beta$. To see this, note first from (A.6) that $\overline{P}^{int}$ is increasing in p_{B2}. The result then follows since (A.6) implies that when $p_{B2} = 0$ we have

$$\overline{P}^{int} = [\overline{x} - (1-\mu)\beta - \Delta] - \mu\beta G(\overline{P}^{int}) > (1 - \mu G(\overline{P}^{int}))\beta \geq (1-\mu)\beta = \widehat{P},$$

where the first inequality follows from Assumption M.

Given $\widehat{P} = (1-\mu)\beta$, firm 2's demand for any $p_{B2} \in [0, \Delta]$ can be written as

$$D_2(p_{B2}) = \mu + (1-\mu)G(\widetilde{X}_s).$$

Thus, firm 2's profit is given by

$$\pi_2(p_{B2}) = p_{B2} D_2(p_{B2}).$$

For there to be no incentives to deviate from $p_{B2} = \Delta$, a *sufficient* condition is

$$\frac{d\pi_2(p_{B2})}{dp_{B2}} > 0 \text{ for all } p_{B2} \in [0, \Delta].$$

We have

$$\begin{aligned}\frac{d\pi_2(p_{B2})}{dp_{B2}} &= D_2(p_{B2}) + p_{B2}\frac{dD_2(p_{B2})}{dp_{B2}} \\ &= \left[\mu + (1-\mu)G(\widetilde{X}_s)\right] + p_{B2}\left[(1-\mu)g(\widetilde{X}_s)\frac{d\widetilde{X}_s}{dp_{B2}}\right].\end{aligned}$$

Totally differentiating (A.3) and (A.4) gives

$$d\widetilde{X}_s = -dp_{B2} + \beta\left[(1-\mu)2g(\widetilde{X}_s)d\widetilde{X}_s + \mu g(\widetilde{X}_m)d\widetilde{X}_m\right]$$

$$d\widetilde{X}_m = \beta(1-\mu)g(\widetilde{X}_s)d\widetilde{X}_s,$$

which imply that

$$\frac{d\widetilde{X}_s}{dp_{B2}} = -\frac{1}{1-\beta\left[(1-\mu)2g(\widetilde{X}_s) + \mu g(\widetilde{X}_m)\beta(1-\mu)g(\widetilde{X}_s)\right]} < 0$$

since

$$\begin{aligned}
&\beta\left[(1-\mu)2g(\widetilde{X}_s) + \mu g(\widetilde{X}_m)\beta(1-\mu)g(\widetilde{X}_s)\right] \\
&< \beta\left[(1-\mu)\frac{1}{\beta} + \mu\frac{1}{2\beta}\beta(1-\mu)\frac{1}{2\beta}\right] \\
&= (1-\mu)(1+\frac{\mu}{4}) \\
&< (1-\mu)(1+\mu) < 1,
\end{aligned}$$

where the first inequality is from $g(\cdot) < 1/(2\beta)$ of Assumption 1.

Therefore, the sufficient condition above can be rewritten as

$$\mu + (1-\mu)G(\widetilde{X}_s) > p_{B2}(1-\mu)g(\widetilde{X}_s)\left|\frac{d\widetilde{X}_s}{dp_{B2}}\right|.$$

As $G(\widetilde{X}_s) \geq 0$ and $p_{B2} \leq \Delta$, the above condition is satisfied if

$$\mu > \Delta\left[(1-\mu)g(\widetilde{X}_s)\left|\frac{d\widetilde{X}_s}{dp_{B2}}\right|\right].$$

Note that by Assumption 1,

$$\begin{aligned}
\left|\frac{d\widetilde{X}_s}{dp_{B2}}\right| &= \frac{1}{1-\beta\left[(1-\mu)2g(\widetilde{X}_s) + \mu g(\widetilde{X}_m)\beta(1-\mu)g(\widetilde{X}_s)\right]} \\
&< \frac{1}{1-(1-\mu)(1+\frac{\mu}{4})}.
\end{aligned}$$

As $\Delta g(\widetilde{X}_s) < \frac{1}{2}$ by Assumption 1, the sufficient condition is satisfied whenever $\mu \gtrsim 0.52$.

The case of the uniform distribution

The condition above ($\mu \gtrsim 0.52.$) is a sufficient one that ensures firm 2 has no incentive to deviate, regardless of the distribution of $G(\cdot)$. However, it is not a tight condition and is far from necessary in most cases. To illustrate this, we consider the special case in which x is uniformly distributed over $[0, \overline{x}]$ with density $1/\overline{x}$. Then, Assumption 1 becomes

$$0 < \Delta < \beta < \frac{\overline{x}}{2}.$$

We can verify that

$$\widetilde{X}_s = \frac{P^* - \alpha + (\Delta - p_{B2})\left(1 - \beta\frac{\mu}{\overline{x}+\mu}\right) - \beta(1-\mu)}{1 - \frac{\beta}{\overline{x}}\left[2(1-\mu) + \frac{\mu(\overline{x}-(1-\mu))}{\overline{x}+\mu}\right]}$$

and

$$\frac{d\widetilde{X}_s}{dp_{B2}} = -\frac{1 - \beta\frac{\mu}{\overline{x}+\mu}}{1 - \frac{\beta}{\overline{x}}\left[2(1-\mu) + \frac{\mu(\overline{x}-(1-\mu))}{\overline{x}+\mu}\right]} < 0,$$

where both the numerator and the denominator are strictly positive because of $2\beta < \overline{x}$ under Assumption 1.

Thus, we have

$$\frac{d\pi_2(p_{B2})}{dp_{B2}} = 1 - (1-\mu)(1 - \frac{\widetilde{X}_s}{\overline{x}}) - \frac{p_{B2}(1-\mu)}{\overline{x}} \frac{1 - \beta\frac{\mu}{\overline{x}+\mu}}{1 - \frac{\beta}{\overline{x}}\left[2(1-\mu) + \frac{\mu(\overline{x}-(1-\mu))}{\overline{x}+\mu}\right]}$$

and

$$\frac{d}{dp_{B2}}\left(\frac{d\pi_2(p_{B2})}{dp_{B2}}\right) = -2\frac{(1-\mu)}{\overline{x}} \frac{1 - \beta\frac{\mu}{\overline{x}+\mu}}{1 - \frac{\beta}{\overline{x}}\left[2(1-\mu) + \frac{\mu(\overline{x}-(1-\mu))}{\overline{x}+\mu}\right]} < 0$$

Since firm 2's profit function is strictly concave in p_{B2}, it has no incentive to deviate if $\left.\frac{d\pi_2(p_{B2})}{dp_{B2}}\right|_{p_{B2}=\Delta} \geq 0$.

When firm 2's first-order condition is evaluated at $p_{B2} = \Delta$,

$$\left.\frac{d\pi_2(p_{B2})}{dp_{B2}}\right|_{p_{B2}=\Delta} = \mu - \frac{\Delta(1-\mu)}{\overline{x}} \frac{1 - \beta\frac{\mu}{\overline{x}+\mu}}{1 - \frac{\beta}{\overline{x}}\left[2(1-\mu) + \frac{\mu(\overline{x}-(1-\mu))}{\overline{x}+\mu}\right]},$$

which is weakly positive if and only if

$$\frac{\mu}{1-\mu} \geq \frac{\Delta}{\overline{x}} \frac{1-\beta\frac{\mu}{\overline{x}+\mu}}{1-\frac{\beta}{\overline{x}}\left[2(1-\mu)+\frac{\mu(\overline{x}-(1-\mu))}{\overline{x}+\mu}\right]}. \tag{A.10}$$

Hence, under the uniform distribution, firm 2 has no incentive to deviate when $\overline{x}$ is sufficiently large.

A.4 Comparison of tying and independent pricing equilibria

For completeness and self-containment of the Supplemental Appendix, we restate Proposition 4 from the main text as Propostion A.1. Summarizing, we have:

Proposition A.1 *Suppose that fraction $\mu \in (0,1)$ of consumers can multihome without any cost and that Assumptions 1 and M as well as the full coverage condition are satisfied. If $\mu \gtrsim 0.52$, tying leads to the following equilibrium*

$$P^* = \alpha + (1-\mu)\beta \text{ and } p^*_{B2} = \Delta,$$

in which all consumers buy the bundle and all multihoming consumers buy both the bundle and B2. Tying raises firm 1's profit relative to independent pricing but reduces firm 2's profit as well as consumer and total welfare.

Proposition A.1 shows that the mechanism we identified in our baseline model, through which firm 1 can profitably employ tying to leverage its market power in market A into market B, continues to operate despite the presence of multihoming consumers. However, firm 1's gain from doing so is more limited in this case, as tying leads to a quasi-installed base advantage only due to capturing the single-homing consumers through sales of the bundle (which gives a value advantage of $(1-\mu)\beta$). When *all* consumers can multihome costlessly, this ability goes away.

In addition, two somewhat surprising observations are worth noting. First, multihoming can make tying more profitable than single-homing. This occurs if

$$(1-\mu)\beta > \beta - \Delta \Leftrightarrow \Delta > \mu\beta.$$

Second, under the same condition, the multihoming reduces consumer surplus: Note that the net surplus that multihomers obtain from buying $B2$ is zero. Then, what matters for the consumer surplus comparison is the price of the bundle, which is higher under multihoming. So we have:

Corollary 1 *Multihoming increases the tying firm's profit and reduces consumer surplus relative to no multihoming if and only if* $\Delta > \mu\beta$.

The reason for this result is that with a sufficient share of multihoming consumers, firm 2 does not lower its price in response to firm 1's tie.

B Analysis of Tying with Asymmetric Network Effects

In this Appendix, we extend the analysis by considering the case in which $B1$ and $B2$ can have different network effects.

B.1 Model

Consider the case in which $B1$ and $B2$ also differ in their network effects, denoted by β_1 and β_2. The assumption that $B2$ is the superior product in market B now can be restated as $v_1 + \beta_1 < v_2 + \beta_2$. Defining $\Delta_v \equiv (v_2 - v_1)$ and $\Delta_\beta \equiv (\beta_2 - \beta_1)$, this assumption is equivalent to $\Delta_v + \Delta_\beta > 0$.

We modify Assumption 1 as follows:

Assumption 1A: $\Delta_v + \Delta_\beta > 0$, $\beta_1 > \Delta_v$, and

$$\beta_1 + \beta_2 < \frac{1}{g(x)} \text{ for all } x \in [0, \overline{x}].$$

The second part of Assumption 1A (i.e., $\beta_1 > \Delta_v$) states that the network effects associated with $B1$ more than offset its disadvantage in stand-alone value relative to $B2$. The last part of Assumption 1A ensures that demand for the bundle decreases as the bundle price increases, as we show below.

We also assume in this appendix that the condition for full coverage in market A holds, specifically that α is sufficiently large so that firm 1 serves all consumers with the price of $p_A = \alpha$ in market A under independent pricing:

$$\alpha \geq \frac{1 - G(0)}{g(0)} = \frac{1}{g(0)}.$$

B.2 Independent pricing

Consider the competition in market B. By a similar argument as in the main text, the equilibrium in market B has $p_{B1} = 0$ and $p_{B2} = (v_2 + \beta_2) - (v_1 + \beta_1) = \Delta_v + \Delta_\beta$, with all consumers buying $B2$.

Thus, under independent pricing, firm 1's total profit is α, and firm 2's total profit is $(v_2 + \beta_2) - (v_1 + \beta_1) = \Delta_v + \Delta_\beta > 0$.

B.3 Tying

We extend Lemma 1 as follows.

Lemma B.1 *When firm 1 offers only a bundle for sale, given prices of P for the bundle and $p_{B2} < v_2$ for product $B2$, and defining $\widehat{P} = P - \alpha$, the unique outcome in consumers' choices that survives iterated deletion of dominated strategies is as follows:*[7]

(i) If $\widehat{P} - p_{B2} \in (\beta_1 - \Delta_v, \overline{x} - \beta_2 - \Delta_v)$, consumers whose valuation for A is higher than $\widetilde{X} \in (0, \overline{x})$ purchase the bundle while consumers whose valuation is lower than $\widetilde{X}$ purchase $B2$, where $\widetilde{X}$ satisfies

$$\widetilde{X} + \beta_1 \left[1 - G(\widetilde{X})\right] - \beta_2 G(\widetilde{X}) - \Delta_v = (\widehat{P} - p_{B2}); \tag{B.1}$$

(ii) If $\widehat{P} - p_{B2} \leq \beta_1 - \Delta_v$, all consumers purchase the bundle (i.e., $\widetilde{X} = 0$);

(iii) If $\widehat{P} - p_{B2} \geq \overline{x} - \beta_2 - \Delta_v$, all consumers purchase $B2$ only (i.e., $\widetilde{X} = \overline{x}$).

Proof. The proof follows the logic of the proof of Lemma 1. We have

$$\psi(x, X) = x + \beta_1 [1 - G(X)] - \beta_2 G(X) - \Delta_v - (\widehat{P} - p_{B2});$$

$$\Psi(X) \equiv \psi(X, X) = X + \beta_1 [1 - G(X)] - \beta_2 G(X) - \Delta_v - (\widehat{P} - p_{B2}).$$

Under Assumption 1A, $\Psi'(X) = 1 - (\beta_1 + \beta_2) g(X) > 0$. We have

$$\Psi(\overline{x}) = \overline{x} - \beta_2 - \Delta_v - (\widehat{P} - p_{B2})$$

and

$$\Psi(0) = \beta_1 - \Delta_v - (\widehat{P} - p_{B2}),$$

where $\overline{x} - \beta_2 - \Delta_v > \beta_1 - \Delta_v$ under Assumption 1A. ■

[7] A similar argument to that in Remark 3 in the main text establishes that $\beta_1 - \Delta_v < \overline{x} - \beta_2 - \Delta_v$

We have the following proposition

Proposition B.1 *Under Assumption 1A and the full coverage assumption, pure bundling leads to an equilibrium in which all consumers buy the bundle and*

$$P^* = \alpha + \beta_1 - \Delta_v \text{ and } p^*_{B2} = 0.$$

Firm 1's profit is larger than under independent pricing. Tying harms firm 2 and consumers, and reduces aggregate welfare.

Proof. From the previous lemma, $\widehat{P}^* = \beta_1 - \Delta_v$ and $p^*_{B2} = 0$ lead all consumers to buy the bundle (i.e., $\widetilde{X} = 0$). Hence, firm 1 has no incentive to lower its price. In what follows, we show that firm 1 has no incentive to raise its price. Note first that a total differentiation of condition (B.1), which defines $\widetilde{X}$, leads to

$$\frac{\partial \widetilde{X}}{\partial \widehat{P}} = \frac{1}{1 - (\beta_1 + \beta_2)\, g(\widetilde{X})} > 1, \tag{B.2}$$

where the inequality follows from Assumption 1A. Given $p^*_{B2} = 0$, firm 1 chooses $\widehat{P}$ to maximize its profit, given by

$$\Pi(\widehat{P}) \equiv (\alpha + \widehat{P})(1 - G(\widetilde{X})).$$

The first-order derivative with respect to $\widehat{P}$ is given by

$$\Pi'(\widehat{P}) = (1 - G(\widetilde{X})) - (\alpha + \widehat{P})g(\widetilde{X})\frac{\partial \widetilde{X}}{\partial \widehat{P}}.$$

We next show that the first-order derivative is negative for any $\widehat{P} \geq \beta_1 - \Delta_v$, which implies choosing $\widehat{P} = \beta_1 - \Delta_v$ maximizes firm 1's profit. This is the case if the following inequality holds for any $\widehat{P} \geq \beta_1 - \Delta_v$:

$$\frac{1 - G(\widetilde{X})}{g(\widetilde{X})} \leq (\alpha + \widehat{P})\frac{\partial \widetilde{X}}{\partial \widehat{P}}.$$

Because of the monotone hazard rate assumption on $G(\cdot)$, the full coverage assumption, condition (B.2), and Assumption 1A (which assumes $\beta_1 - \Delta_v > 0$), for any $\widehat{P} \geq \beta_1 - \Delta_v$ we have

$$\frac{1 - G(\widetilde{X})}{g(\widetilde{X})} \leq \frac{1 - G(0)}{g(0)} < \alpha < \alpha + \widehat{P} < (\alpha + \widehat{P})\frac{\partial \widetilde{X}}{\partial \widehat{P}}.$$

Firm 1's profit exceeds α, its profit under independent pricing, while firm 2 earns zero profit. Regarding consumer surplus, recall that under independent pricing, consumers are indifferent between coordinating on $B1$ at $p_{B1} = 0$ and coordinating on $B2$ at $p_{B2} = \Delta_v + \Delta_\beta$. To compute consumer surplus under tying, we can decompose the bundle price $P^* = \alpha + \beta_1 - \Delta_v$ into two components: the price for A, given by $p_A = \alpha$ and the implicit price for $B1$, given by $p_{B1} = \beta_1 - \Delta_v$. Thus, holding the price of A constant at its level under independent pricing, tying effectively increases the price of $B1$ from 0 to $\beta_1 - \Delta_v > 0$.

B.4 Application to the Complementary-products Case

We apply the framework of asymmetric network effects to the case of complementary products. Consider the situation in Subsection IVB where firm 1's product $A1$ faces competition from $A2$. Here, we assume that the added value product $A1$ brings to the system over $A2$ has two components: it increases the stand-alone value of A by $\alpha + x$ for a consumer of type x (as in Subsection IVB), and it also enhances the value added by product Bi, raising it from $v'_i + \beta' N_i$ to $v_i + \beta N_i$, where $v_i > v'_i > 0$, for $i = 1, 2$ and $\beta > \beta' > 0$. In other words, under this formulation, consumers' valuations for the system $A2/Bi$ are $(v'_i + \beta' N_i)$ for $i = 1, 2$, whereas in Subsection IVB, we assumed $v_i = v'_i > 0$, for $i = 1, 2$ and $\beta = \beta' > 0$. Since this modification does not affect the analysis under independent pricing, we focus below on the tying case.

In the presence of tying, there is competition between two systems $A1/B1$ and $A2/B2$. Let P denote the price of firm 1's bundled system $A1/B1$, and let p_{B2} be the price of firm 2's product $B2$, which also serves as the price of the system $A2/B2$, as product $A2$ is provided competitively at a price of zero. Define $\Delta' \equiv v'_2 - v_1$, which can be negative. Since the value added by $B1$ to the system $A1/B1$ is $v_1 + \beta N_1$, whereas the value added by $B2$ to the system $A2/B2$ is $(v'_2 + \beta' N_2)$, it follows from Assumption 1 that these values satisfy the last two conditions of Assumption 1A: $\beta(> \Delta) > v'_2 - v_1$ and

$$\beta + \beta' < (2\beta <) \frac{1}{g(x)} \text{ for all } x \in [0, \overline{x}].$$

Therefore, we can apply Proposition B.1 to the competition between $A1/B1$ and $A2/B2$, leading to the following result.

Proposition B.2 *Suppose Assumption 1 and the full coverage condition in market A under independent pricing hold. When tying is allowed with complementary products* *and there is an inferior competitively-supplied alternative in market A, there is a unique equilibrium in which all consumers purchase the A1/B1 bundle and the equilibrium prices are given by*

$$P^* = \alpha + \widehat{P}^* = \alpha + (\beta - \Delta'),\ p^*_{B2} = 0.$$

Moreover, firm 1's profit, equal to $\alpha + (\beta - \Delta')$*, exceeds that under independent pricing. Both consumer surplus and social welfare decrease:*

$$\begin{aligned} \widetilde{CS} &= CS^* - (\beta - \Delta') < CS^* \\ \widetilde{AS} &= AS^* - \Delta < AS^*. \end{aligned}$$

Compared to Proposition 3 in the main text, the above result introduces an important twist: tying forces the superior complementary product $B2$ to be used alongside the inferior primary product $A2$, thereby reducing the overall added value of $B2$. As a result, the compensation that firm 1 must offer to consumers decreases from Δ to Δ', which can even be negative. This further strengthens firm 1's incentive to tie.

C Tying with Product Differentiation and Partial Market Foreclosure in the Tied Market

In this Appendix, we introduce horizontal differentiation in market B. We identify conditions under which, consistent with the main text, no independent pricing equilibrium exists because firm 1 has a profitable deviation to bundling. Moreover, in some cases, this deviation does not result in complete foreclosure of $B2$, as firm 1 allows for partial market access by firm 2.

C.1 Model

C.1.1 Market A

There is a mass one of consumers whose valuation for product A is $\alpha + x$ with $x \in [0, \overline{x}]$ distributed according to $G(\cdot)$ with density $g(\cdot) > 0$. We maintain Assumption 1.

We assume there is full coverage in market A. Specifically, α is sufficiently large so that firm 1 serves all consumers with the price of $p_A = \alpha$ in market A under independent pricing:

$$\alpha \geq \frac{1-G(0)}{g(0)} = \frac{1}{g(0)}.$$

C.1.2 Market B

We consider a Hotelling model with a linear transportation cost with parameter $t > 0$. Consumers are uniformly distributed along the interval $[0, 1]$. Firm 1 is located at point 0 and firm 2 at point 1. A consumer's location—denoted by y, representing the distance from 0—is assumed to be independent of her valuation for product A. We assume that the location of a consumer (the distance from 0), denoted by y, is independent from her valuation for product A. We impose a full coverage assumption, specifically that v_1 and v_2 are sufficiently large to ensure that every consumer purchases either $B1$ or $B2$.

We introduce the following assumption:

Assumption D: $t < \min\{\beta/2, \Delta, \beta - \Delta\}$.

This assumption reflects a relatively strong network benefits and value advantage for $B2$ compared to the degree of product differentiation in market B.

C.2 Independent pricing

The outcome in market A has $p_A = \alpha$, exactly as in the main text under the full coverage assumption.

Consider market B. We first establish conditions under which there is a NE consumer response in which all consumers buy $B2$ or all buy $B1$.

All buying $B2$ is a NE consumer response if the consumer with $y = 0$ prefers to buy $B2$ given that all other consumers buy $B2$. This is true if

$$v_2 + \beta - t - p_{B2} \geq v_1 - p_{B1}$$

or equivalently

$$\Delta + \beta - t \geq p_{B2} - p_{B1} \tag{C.1}$$

The payoff of a consumer located at y in this equilibrium is $v_2 + \beta - t(1 - y) - p_{B2}$.

Similarly, all consumers buying $B1$ is a NE consumer response if the consumer at $y = 1$ prefers $B1$ given that all other consumers are buying $B1$, which holds if:

$$p_{B2} - p_{B1} \geq \Delta - \beta + t \tag{C.2}$$

The payoff of a consumer located at y in this equilibrium is $v_1 + \beta - ty - p_{B1}$.

We next establish that there is no interior NE consumer response that is Pareto-undominated; all consumers either purchase $B1$ or $B2$ in any Pareto-undominated Nash equilibrium.

Lemma C.1 *Under Assumption D, there is no interior NE consumer response which is Pareto-undominated.*

Proof. Suppose that there is a NE consumer response which is interior with a critical consumer type $y^* \in (0, 1)$. Then, the following condition holds:

$$v_1 + (\beta - t)y^* - p_{B1} = v_2 + (\beta - t)(1 - y^*) - p_{B2} \tag{C.3}$$

Note that condition (C.3) implies that both conditions (C.1) and (C.2) hold. Consider the all-buy-$B2$ NE consumer response. Since the network effect for $B2$ is larger than in the interior NE consumer response, all consumers located at $y \in [y^*, 1]$ are better off in the all-buy-$B2$ NE consumer response. Now consider consumers with $y < y^*$. In the interior NE consumer response, the payoff of a consumer located at $y < y^*$ is $v_1 + \beta y^* - ty - p_{B1}$. However, equation (C.3) implies that

$$\begin{aligned} v_2 + \beta - (1 - y)t - p_{B2} &= v_1 + 2(\beta - t)y^* + ty - p_{B1} \\ &> v_1 + \beta y^* - ty - p_{B1}, \end{aligned}$$

where the inequality follows because $2(\beta - t) > \beta$ under Assumption D. Thus, all consumers are better off in the all-buy-$B2$ NE consumer response. ■

Lemma 1 implies that there can be only corner solutions in Pareto-undominated NE consumer response; all consumers purchase $B1$ or $B2$. The next lemma establishes that there is no equilibrium under independent pricing in which all consumers purchase $B1$.

Lemma C.2 *Under Assumption D, there is no equilibrium in which all consumers purchase $B1$.*

Proof. Suppose that there is an equilibrium in which all consumers purchase $B1$ at $p_{B1} \geq 0$ with firm 2's profit being zero. In such an equilibrium, the payoff of a consumer located at y is $v_1 + \beta - ty - p_{B1}$. We show that, in contradiction, under Assumption D firm 2 would have a profitable deviation to charge $p_{B2} \in (p_{B1}, p_{B1} + (\Delta - t))$ (note that $\Delta - t > 0$ by Assumption D) which attracts all consumers and gives firm 2 a strictly positive profit.

To see this, observe first that since when firm 2 deviates to p_{B2} satisfying $p_{B2} - p_{B1} < \Delta - t \leq \Delta + \beta - t$, condition (C.1) holds, so an all-buy-$B2$ NE consumer response exists. To complete the argument, we show that this all-buy-$B2$ NE consumer response Pareto dominates the outcome in which all consumers buy $B1$ (the only other possible undominated NE consumer response, according to Lemma C.1). When all consumers buy $B2$, the payoff of a consumer located at y is $v_2 + \beta - (1-y)t - p_{B2}$. Observe that

$$\begin{aligned} v_2 + \beta - (1-y)t - p_{B2} &= v_1 + \Delta + \beta - (1-y)t - p_{B2} \\ &= v_1 + \beta + ty - p_{B1} + [p_{B1} + \Delta - t - p_{B2}] \\ &> v_1 + \beta - ty - p_{B1}. \end{aligned}$$

Thus, we have a contradiction. ■

In conclusion, under independent pricing firm 1 has zero profit in market B under assumption D.

C.3 Tying

Let P be the price of the bundle and $\widehat{P} = P - \alpha$. Let n_1 be the network size of the bundle and $n_2 (= 1 - n_1)$ be that of product $B2$. Let $\psi(x, y, n_1)$ represent the payoff gain from purchasing the bundle over purchasing $B2$ for type-x consumer located at $y \in [0, 1]$ given the network sizes $(n_1, 1 - n_1)$:

$$\psi(x, y, n_1) = x - \Delta + \beta(2n_1 - 1) - t(2y - 1) - (\widehat{P} - p_{B2}).$$

Observe that all consumers with $(x, y) \in [0, \overline{x}] \times [0, 1]$ satisfying $x - t(2y - 1) \equiv z(x, y) \in [-t, \overline{x} + t]$ have the same gain. So we can reason in terms of z instead of (x, y). Let $H(\cdot)$ be the c.d.f. of z with density $h(\cdot)$.

Recall that Assumption 1 implies $\overline{x} > 2\beta$. This, together with Assumption D, implies $z(\overline{x}, 1) > z(0, 0)$.

When $G(\cdot)$ is a uniform distribution, $H(\cdot)$ is given by

$$H(z) = \begin{cases} 1 - \frac{(\overline{x}+t-z)^2}{4t\overline{x}} & \text{if } z \in (\overline{x}-t, \overline{x}+t] \\ \frac{z}{\overline{x}} & \text{if } z \in [t, \overline{x}-t] \\ \frac{(z+t)^2}{4t\overline{x}} & \text{if } z \in [-t, t) \end{cases}$$

Observe that

$$h(-t) = \left.\frac{dH}{dz}\right|_{z=-t} = 0.$$

Let $\psi(z, Z)$ represent the payoff gain from purchasing the bundle over $B2$ for a type-z consumer when all consumers whose z is higher than Z buy the bundle and the rest buy $B2$. We have

$$\psi(z, Z) = z - \Delta + \beta(1 - 2H(z)) - (\widehat{P} - p_{B2}).$$

Define $\Phi(Z)$ as follows:

$$\Phi(Z) \equiv \psi(Z, Z) = Z - \Delta + \beta(1 - 2H(Z)) - (\widehat{P} - p_{B2}), \tag{C.4}$$

where $\Phi(Z)$ is a strictly increasing function of Z as $h(Z) \leq \frac{1}{\overline{x}} < \frac{1}{2\beta}$. We have

$$\Phi(\overline{x}+t) = \overline{x} + t - \Delta - \beta - (\widehat{P} - p_{B2}),$$

$$\Phi(-t) = -t - \Delta + \beta - (\widehat{P} - p_{B2}),$$

where $\overline{x} + t - \Delta - \beta > -t - \Delta + \beta$ under Assumption 1.

Lemma C.3 *Suppose that Assumptions 1 and D hold. When firm 1 offers only a bundle for sale, given prices of P for the bundle and p_{B2} for product $B2$, and defining $\widehat{P} = P - \alpha$, the unique outcome in consumers' choices that survives iterated deletion of dominated strategies is as follows:*

(i) If $\widehat{P} - p_{B2} \in (-t - \Delta + \beta, \overline{x} + t - \Delta - \beta)$, consumers whose z is higher than $\widetilde{Z} \in (-t, \overline{x}+t)$ purchase the bundle while consumers whose z is lower than $\widetilde{Z}$ purchase $B2$ where $\widetilde{Z}$ is the unique solution to

$$\Phi(\widetilde{Z}) = \widetilde{Z} - \Delta + \beta(1 - 2H(\widetilde{Z})) - \left(\widehat{P} - p_{B2}\right) = 0. \tag{C.5}$$

(ii) If $\widehat{P} - p_{B2} \leq -t - \Delta + \beta$, all consumers buy the bundle (i.e., $\widetilde{Z} = -t$).

(iii) If $\widehat{P} - p_{B2} \geq \overline{x} + t - \Delta - \beta$, all consumers buy $B2$ (i.e., $\widetilde{Z} = \overline{x} + t$).

Proof. The proof is omitted as it parallels the proof of Lemma 1 in Appendix. ■

By totally differentiating condition (C.5), which defines $\widetilde{Z}$, we can derive

$$\frac{\partial \widetilde{Z}}{\partial \widehat{P}} = \frac{1}{1 - 2\beta h(\widetilde{Z})} \geq 1.$$

Firm 1's profit is

$$\widetilde{\Pi}_1(\widehat{P}, p_{B2}) = (\alpha + \widehat{P})(1 - H(\widetilde{Z})).$$

The first-order derivative of firm 1's profit with respect to $\widehat{P}$ is

$$\frac{\partial \widetilde{\Pi}_1}{\partial \widehat{P}} = (1 - H(\widetilde{Z})) - \left(\alpha + \widehat{P}\right) h(\widetilde{Z}) \frac{\partial \widetilde{Z}}{\partial \widehat{P}}. \tag{C.6}$$

From Lemma C.3, by charging $\widehat{P} = -t - \Delta + \beta$, firm 1 can sell the bundle to all consumers even if firm 2 charges $p_{B2} = 0$ and by doing so it realizes a profit of $\alpha - t - \Delta + \beta$, which Assumption D implies generates a larger profit than under independent pricing. Thus, no independent pricing equilibrium exists.

Moreover, we can show below that the first-order derivative of firm 1's profit with respect to $\widehat{P}$ is strictly positive when it is evaluated at $\widehat{P} - p_{B2} = -t - \Delta + \beta$ (i.e., $\widetilde{Z} = -t$):

$$\begin{aligned} & (1 - H(\widetilde{Z})) - \left(\alpha + \widehat{P}\right) h(\widetilde{Z}) \frac{\partial \widetilde{Z}}{\partial \widehat{P}}\bigg|_{\widetilde{Z}=-t} \\ = \; & (1 - H(-t)) - (\alpha + \widehat{P}) \frac{h(-t)}{1 - 2\beta h(-t)} \\ = \; & 1, \end{aligned}$$

where the last equality follows from the fact that $H(-t) = h(-t) = 0$. Therefore, in deviating to tying, firm 1 does not find it optimal to sell the bundle to all consumers and in any equilibrium firm 1's profit must exceed that under independent pricing.[8]

D Network Effects in Market A

In this Appendix, we examine what happens if we introduce network effects in market A. We show that when firm 1 has an incentive to serve all consumers in market A under

[8] By continuity, if $\beta - \Delta - t$ is negative but close to zero, tying is strictly unprofitable conditional on full foreclosure but becomes profitable under partial foreclosure.

independent pricing, adding the network effects in market A does not affect our conclusion about the profitability of tying.

D.1 Model

Let β_A denote the network effect parameter in market A. Market B remains as described in the baseline model. To reflect network effects in market A, we modify Assumption 1 as follows.

Assumption 1N:

$$\beta_A + 2\beta < \frac{1}{g(x)} \text{ for all } x \in [0, \overline{x}].$$

This condition ensures that demand for product A with independent pricing decreases with p_A and that demand for the bundle when firm 1 offers only a bundle decreases with the bundle price.

We also assume full coverage in market A. Specifically, that $\alpha + \beta_A$ are sufficiently large so that firm 1 serves all consumers with the price of $p_A = \alpha + \beta_A$ in market A under independent pricing:

$$\alpha + \beta_A \geq \frac{1}{g(0)}.$$

We show below that this is a sufficient condition for full coverage.

D.2 Independent pricing

D.2.1 Market A

Lemma D.1 *Under independent pricing, given price p_A, and defining $\widehat{p}_A = p_A - \alpha$, the unique outcome in consumers' choices that survives iterated deletion of dominated strategies in market A is as follows:*

(i) If $\widehat{p}_A \in (\beta_A, \overline{x})$,[9] consumers whose stand-alone valuation for A is higher than $\widetilde{X} \in (0, \overline{x})$ purchase A while consumers whose valuation is lower than $\widetilde{X}$ do not purchase A where $\widetilde{X}$ satisfies

$$\widetilde{X} + \beta_A \left[1 - G(\widetilde{X})\right] = \widehat{p}_A \tag{D.1}$$

(ii) If $\widehat{p}_A \leq \beta_A$, all consumers buy A (i.e., $\widetilde{X} = 0$).

(iii) If $\widehat{p}_A \geq \overline{x}$, no consumer buys A (i.e., $\widetilde{X} = \overline{x}$).

[9] Assumption 1N implies $\beta_A < \overline{x}$.

Proof. The proof follows the logic of the proof of Lemma 1. We define $\psi(x, X)$ to be the net gain from purchase of A for a type-x consumer when all consumer types above X buy A:

$$\psi(x, X) = x + \beta_A [1 - G(X)] - \widehat{p}_A;$$

We then define:

$$\Psi(X) \equiv \psi(X, X) = X + \beta_A [1 - G(X)] - \widehat{p}_A.$$

Under Assumption 1N, $\Psi'(X) = 1 - \beta_A g(X) > 0$.

The argument proceeds similarly to the proof of Lemma 1 in the Appendix. For instance, when $\widehat{p}_A < \overline{x}$ a type $\overline{x}$ consumer finds it optimal to buy A even if no other consumers do, initiating a decreasing iterated dominance sequence. Conversely, when $\widehat{p}_A > \beta_A$, a type $x = 0$ consumer finds it optimal not to buy A even if all other consumers do, triggering an increasing iterated dominance sequence. These two sequences converge to an interior $\widetilde{X}$ at which $\Psi(\widetilde{X}) = 0$ when $\widehat{p}_A \in (\beta_A, \overline{x})$.

■

Hence, let $\widetilde{X}(\widehat{p}_A)$ be defined by $\widetilde{X}$ satisfying (D.1). We have

$$\frac{d\widetilde{X}}{d\widehat{p}_A} = \frac{1}{1 - \beta_A g(\widetilde{X})} > 1.$$

Firm 1's profit is then given by

$$(\alpha + \widehat{p}_A)(1 - G(\widetilde{X}(\widehat{p}_A))).$$

Its first-order derivative with respect to $\widehat{p}_A$ is

$$(1 - G(\widetilde{X}(\widehat{p}_A))) - (\alpha + \widehat{p}_A) g(\widetilde{X}(\widehat{p}_A)) \frac{d\widetilde{X}}{d\widehat{p}_A}.$$

Setting $\widehat{p}_A = \beta_A$ is optimal for firm 1 if the first-order derivative is negative for $\widehat{p}_A \geq \beta_A$, which holds because of the monotone hazard rate assumption on $G(\cdot)$, the full coverage assumption and Assumption 1N:

$$\frac{1 - G(\widetilde{X}(\widehat{p}_A))}{g(\widetilde{X}(\widehat{p}_A))} \leq \frac{1 - G(0)}{g(0)} \leq \alpha + \beta_A < (\alpha + \widehat{p}_A) \frac{d\widetilde{X}}{d\widehat{p}_A}.$$

Therefore, in market A, firm 1's profit is $\alpha + \beta_A$.

In market B, firm 1's profit is zero and firm 2's profit is $\Delta > 0$. In summary, under independent pricing, firm 1's total profit is $\alpha + \beta_A$ and firm 2's profit is $\Delta > 0$.

D.3 Tying

We extend Lemma 1 as follows.

Lemma D.2 *When firm 1 offers only a bundle for sale, given prices of P for the bundle and p_{B2} for product B2, and defining $\widehat{P} = P - \alpha$, the unique outcome in consumers' choices that survives iterated deletion of dominated strategies is as follows:*

(i) If $\widehat{P} - p_{B2} \in (\beta_A + \beta - \Delta, \overline{x} - \beta - \Delta)$,[10] *consumers whose valuation for A is higher than $\widetilde{X} \in (0, \overline{x})$ purchase the bundle while consumers whose valuation is lower than $\widetilde{X}$ purchase B2, where $\widetilde{X}$ satisfies*

$$\widetilde{X} + (\beta_A + \beta)\left[1 - G(\widetilde{X})\right] - \beta G(\widetilde{X}) - \Delta = (\widehat{P} - p_{B2}) \tag{D.2}$$

(ii) If $\widehat{P} - p_{B2} \leq \beta_A + \beta - \Delta$, all consumers buy the bundle (i.e., $\widetilde{X} = 0$).
(iii) If $\widehat{P} - p_{B2} \geq \overline{x} - \beta - \Delta$, all consumers buy B2 (i.e., $\widetilde{X} = \overline{x}$).

Proof. The proof follows the logic of the proof of Lemma 1. We have

$$\psi(x, X) = x + (\beta_A + \beta)\left[1 - G(X)\right] - \beta G(X) - \Delta - (\widehat{P} - p_{B2});$$

$$\Psi(X) \equiv \psi(X, X) = X + (\beta_A + \beta)\left[1 - G(X)\right] - \beta G(X) - \Delta - (\widehat{P} - p_{B2}).$$

Under Assumption 1N, $\Psi'(X) = 1 - (\beta_A + 2\beta)g(X) > 0$. We have

$$\Psi(\overline{x}) = \overline{x} - \beta - \Delta - (\widehat{P} - p_{B2}),$$

$$\Psi(0) = \beta_A + \beta - \Delta - (\widehat{P} - p_{B2}).$$

The remainder of the proof closely follows the structure of the proof of Lemma 1. ■

As in the main text, when firm 2 charges $p_{B2} = 0$, this lemma implies that firm 1 can sell the bundle to all consumers at a bundle price $P = \alpha + \beta_A + \beta - \Delta$, which exceeds its profit under the independent pricing equilibrium. Consequently, an independent pricing equilibrium cannot be sustained, as firm 1 would have a profitable deviation by offering the bundle. Moreover, firm 1's profit in any equilibrium involving tying must strictly exceed its profit under independent pricing.

[10] Assumption 1N implies $\beta_A + \beta - \Delta < \overline{x} - \beta - \Delta$.

References

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