# Online Appendix

## Monetary Cooperation during Global Inflation Surges

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## A Proofs and additional derivations

### A.1 Proof of Proposition 1

Substituting the constraints in the objective function reduces the global central bank's problem to

$$\max_{P_{i,0}} \frac{1}{\omega_{i,0}}\left(\frac{\alpha}{1-\alpha}+1-\omega_{i,0}\right)\log P_{i,0}-\chi\left(P_{i,0}\right)+t.i.p., \tag{A.1}$$

where t.i.p. collects all the terms not affected by monetary policy, subject to

$$P_{i,0} \leq \left(\frac{\omega_{i,0}}{\omega}\frac{1-\omega(1-\alpha)}{1-\omega_{i,0}(1-\alpha)}\right)^{\omega_{i,0}(1-\alpha)} \equiv P_{i,0}^{fe}, \tag{A.2}$$

where we have also used $\bar{Y}^T=\omega\bar{L}$, and intuitively $P_{i,0}^{fe}$ denotes the level of inflation consistent with full employment $L_{i,0}=\bar{L}$.

We start by proving that the solution features $P_{i,0}>1$. Imagine that the central bank sets $P_{i,0}\leq 1$. Then, since $\omega_{i,0}>\omega$, constraint (A.2) is slack. Moreover, recall that $\chi'\left(P_{i,0}\right)<0$ for $P_{i,0}<1$ and $\chi'\left(1\right)=0$, implying that the objective function is strictly increasing in $P_{i,0}$ when $P_{i,0}\leq 1$. This implies that it is optimal to set $P_{i,0}>1$.

Now notice that, since $\chi'\left(P_{i,0}\right)>0$ and $\chi''\left(P_{i,0}\right)>0$ for $P_{i,0}>1$, the objective function reaches its maximum at $P_{i,0}=\bar{P}_0$, defined by

$$\chi'\left(\bar{P}_{i,0}\right)\bar{P}_{i,0}=\frac{1}{\omega_{i,0}}\left(\frac{\alpha}{1-\alpha}+1-\omega_{i,0}\right). \tag{A.3}$$

Then if

$$\chi'\left(P_{i,0}^{fe}\right)P_{i,0}^{fe}\leq\frac{1}{\omega_{i,0}}\left(\frac{\alpha}{1-\alpha}+1-\omega_{i,0}\right), \tag{A.4}$$

it is optimal to set $P_{i,0}=P_{i,0}^{fe}$, i.e. and maintain the economy at full employment $L_{i,0}=\bar{L}$. Otherwise it is optimal to set $P_{i,0}=\bar{P}_{i,0}<P_{i,0}^{fe}$, which implies that $L_{i,0}<\bar{L}$.

### A.2 Proof of Lemma 1

To derive the consumption function, start by considering that from period $t=1$ on the economy enters a steady state in which $R=1/\beta$ and in every country consumption and output of both goods are constant. Let us denote by $C_i^T$ and $Y_i^T$ the consumption and output of tradable good in the final steady state in country $i$. Now iterate forward (11) and use the transversality condition to obtain

$$C_{i,0}^T + \frac{1}{R_0}\frac{C_i^T}{1-\beta} = Y_{i,0}^T + \frac{1}{R_0}\frac{Y_i^T}{1-\beta}, \tag{A.5}$$

where we have also used the fact that each country starts with zero assets. Now using the fact that

$$C_i^T = \beta R_0 C_{i,0}^T \frac{\omega}{\omega_{i,0}} \tag{A.6}$$

and rearranging the expression above gives

$$C_{i,0}^T = \frac{\omega_{i,0}(1-\beta)}{\omega_{i,0}(1-\beta+\omega\beta}\left(Y_{i,0}^T + \frac{1}{R_0}\frac{Y_i^T}{1-\beta}\right). \tag{A.7}$$

Since in the final steady state each country operates at full employment and both sectors have a linear production function, it is easy to check that

$$C_i^N = \frac{1-\omega}{\omega}C_i^T, \tag{A.8}$$

and

$$Y_i^T = \bar{L} - \frac{1-\omega}{\omega}C_i^T. \tag{A.9}$$

Combining this expression with (A.6), (A.7) and $\bar{Y}^T = \omega\bar{L}$ and rearranging gives expression (22).

### A.3 Central bank's objective function under free capital mobility

Households' expected utility in country $i$ is given by

$$\sum_{t=0}^{\infty}\beta^t\left(\omega_{i,t}\log\left(C_{i,t}^T\right) + (1-\omega_{i,t})\log\left(C_{i,t}^N\right) - \chi\left(\frac{P_{i,t}}{P_{i,t-1}}\right)\right). \tag{A.10}$$

Now consider that starting from period 1 on the economy enters a steady state with $C_i^N = C_t^T(1-\omega)/\omega$ and $P_{i,t} = P_{i,t-1}$. Hence, households' lifetime utility can be written as

$$\omega_{i,0}\log C_{i,0}^T + (1-\omega_{i,0})\log C_{i,0}^N - \chi\left(P_{i,0}\right) + \frac{\beta}{1-\beta}\log C_i^T + t.i.p, \tag{A.11}$$

where t.i.p. encapsulates all the terms not affected by monetary policy. Finally, using the expression above, $C_i^T = \beta R_0 C_{i,0}^T\omega/\omega_{i,0}$ and $C_{i,0}^N = Y_{i,0}^N$ gives the objective function (23).

### A.4 Proof of Proposition 2

Substituting some constraints in the objective function reduces the central bank's problem to

$$\max_{P_{i,0},C_{i,0}^T,Y_{i,0}^T}\frac{1}{1-\beta}\log C_{i,0}^T + \frac{1-\omega_{i,0}}{\omega_{i,0}}\log P_{i,0} - \chi\left(P_{i,0}\right) + t.i.p., \tag{A.12}$$

where t.i.p. collects all the terms not affected by monetary policy, subject to

$$C_{i,0}^T = \frac{\omega_{i,0}(1-\beta)}{\omega_{i,0}(1-\beta)+\beta}\left(Y_{i,0}^T + \frac{\bar{L}}{R_0(1-\beta)}\right). \tag{A.13}$$

$$P_{i,0}^{\frac{1}{\omega_{i,0}}} = \left(\frac{Y_{i,0}^T}{\bar{Y}^T}\right)^{\frac{1-\alpha}{\alpha}} \tag{A.14}$$

$$\alpha\left(Y_{i,0}^T\right)^{\frac{1}{\alpha}}\left(\bar{Y}^T\right)^{1-\frac{1}{\alpha}} + (1-\alpha)\bar{Y}^T + \frac{1-\omega_{i,0}}{\omega_{i,0}}C_{i,0}^T P_{i,0}^{\frac{1}{\omega_{i,0}}} \le \bar{L}. \tag{A.15}$$

We start by proving that the solution features $P_{i,0} \ge 1$. Imagine that the central bank sets $P_{i,0} < 1$. Then, since $\omega_0 \ge \omega$ and $R_0 \ge 1/\beta$, constraint (A.2) is slack. Moreover, recall that $\chi'(P_{i,0}) < 0$ for $P_{i,0} < 1$ and $\chi'(1) = 0$, implying that the objective function is strictly increasing in $P_{i,0}$ when $P_{i,0} < 1$. This implies that it is optimal to set $P_{i,0} \ge 1$.

Notice that the left-hand side of constraint (A.15) is increasing in $P_{i,0}$, since both $Y_{i,0}^T$ and $C_{i,0}^T$ are increasing in $P_{i,0}$. Define by $P_{i,0}^{fe} \ge 1$ the unique value of $P_{i,0}$ that makes (A.15) hold as an equality. In the main text, $P_{i,0}^{fe}$ is implicitly defined by equation (25).

Now imagine that the solution is such that constraint (A.15) is slack. Then the optimal $P_{i,0}$ satisfies

$$\chi'(P_{i,0}) = \frac{1}{\omega_{i,0}P_{i,0}}\left(\frac{\alpha}{1-\alpha}\frac{\omega_{i,0}}{\omega_{i,0}(1-\beta)+\beta}\frac{Y_{i,0}^T}{C_{i,0}^T} + 1 - \omega_{i,0}\right). \tag{A.16}$$

Intuitively, at an interior optimum the marginal disutility from increasing inflation is equated to the marginal benefit in terms of higher output and consumption. Notice that both sides of equation (A.16) are increasing in $P_{i,0}$. However, we are focusing on scenarios in which all the solutions to (A.16), except the smallest one, violate constraint (A.15). Then defining by $\bar{P}_{i,0}$ the smallest solution to (A.16), the optimal monetary policy is such that $P_{i,0} = \min\left(P_{i,0}^{fe}, \bar{P}_{i,0}\right) \ge 1$.

## A.5 Proof of Lemma 2

Since the reallocation shock is idiosyncratic $\omega_{i,0} > \omega$ and $R_0 = 1/\beta$. Now suppose that the trade balance is not in deficit, so that $C_{i,0}^T \le Y_{i,0}^T$. Equation (22) then implies that $Y_{i,0}^T \ge \bar{Y}\omega_{i,0}/\omega$. But then constraint (18) is satisfied only if

$$\omega_{i,0}^{1-\alpha}\left(1-\omega_{i,0}(1-\alpha)\right)^{\alpha} \le \omega^{1-\alpha}\left(1-\omega(1-\alpha)\right)^{\alpha}. \tag{A.17}$$

Notice that expression (A.17) holds as an equality if $\omega_{i,0} = \omega$, and the left-hand side of (A.17) is increasing in $\omega_{i,0}$. Then, since $\omega_{i,0} > \omega$, inequality (A.17) is violated. We have thus found a contradiction, implying that $C_{i,0}^T > Y_{i,0}^T$.

### A.6 Proof of Proposition 3

These results can be derived following the same steps outlined in the proof to Proposition 1. The only difference is that with free capital mobility it is optimal to set $L_{i,0} = \bar{L}$ if

$$\chi'\left(P_0^{fe}\right)P_0^{fe} \leq \frac{1}{\omega_0}\left(\frac{\alpha}{1-\alpha}\frac{\omega_0}{\omega_0(1-\beta)+\beta}+1-\omega_0\right), \tag{A.18}$$

otherwise $L_{i,0} < \bar{L}$.

Moreover, it is easy to see that $P_0^{fe}$ is not affected by cooperation. This implies that if condition (A.18) is satisfied then the uncooperative allocation coincides with the cooperative one. Instead, lack of cooperation lowers $\bar{P}_0$. This implies that whenever condition (A.18) is violated, the uncooperative allocation features less inflation and less employment than the cooperative one.

## B Production with capacity constraints

There is a continuum of mass one of competitive firms in each sector $j = T, NT$. Each firm needs to perform two tasks, say A and B, to produce. Let $L_A^j$ and $L_B^j$ the amount of labor allocated respectively to tasks A and B by the representative firm in sector $j$. Final output $Y^j$ is then[1]

$$Y^j = \left(\frac{L_A^j}{\alpha}\right)^{\alpha}\left(\frac{L_B^j}{1-\alpha}\right)^{1-\alpha}, \tag{B.1}$$

where $0 < \alpha < 1$ is a parameter determining the importance of the two tasks in the production process. Since labor is homogeneous, every worker is payed the same wage regardless of the task she fulfills.

Firms face a technological constraint, which limits the amount of labor that can be allocated to task B

$$L_B^j \leq (1-\alpha)\bar{Y}^j, \tag{B.2}$$

where the parameter $\bar{Y}^j > 0$ determines the severity of the capacity constraint. Notice that this constraint applies to period 0 only, and so should be understood as a short-run capacity constraint.

Denote by $L^j = L_A^j + L_B^j$ the total amount of labor employed in sector $j$. Now suppose that the capacity constraint does not bind in sector $j$. Since every worker is payed the same wage, the optimal allocation of labor by firms between the two tasks implies $L_A^j = \alpha L^j$, $L_B^j = (1-\alpha)L^j$ and $Y^j = L^j$. One can then see that the capacity constraint does not bind if $Y^j \leq \bar{Y}^j$. If this condition is violated, instead, $L_B^j = (1-\alpha)\bar{Y}^j$ and

$$Y^j = \left(\frac{L_A^j}{\alpha}\right)^{\alpha}\left(\bar{Y}^j\right)^{1-\alpha} = \left(\frac{L_{i,0}^j-(1-\alpha)\bar{Y}^j}{\alpha\bar{Y}^j}\right)^{\alpha}\bar{Y}^j, \tag{B.3}$$

where the second equality makes use of $L_A^j = L^j - (1-\alpha)\bar{Y}^j$.

[1]To simplify notation, in this appendix we omit the country and time subscripts.

The sectoral production functions thus take the form

$$Y^j = \begin{cases} L^j & \text{if } Y^j \leq \bar{Y}^j \\ \left(\frac{L^j-(1-\alpha)\bar{Y}^j}{\alpha\bar{Y}^j}\right)^{\alpha} \bar{Y}^j & \text{if } Y^j > \bar{Y}^j. \end{cases} \tag{B.4}$$

Output is thus linear in labor up to the threshold $\bar{Y}^j$. Once output exceeds $\bar{Y}^j$, labor productivity declines in the quantity produced. The implication is that sectoral Phillips curves are non-linear

$$P^j = \begin{cases} W & \text{if } Y^j \leq \bar{Y}^j \\ \left(\frac{Y^j}{\bar{Y}^j}\right)^{\frac{1-\alpha}{\alpha}} W & \text{if } Y^j > \bar{Y}^j. \end{cases} \tag{B.5}$$

Intuitively, due to perfect competition sectoral prices are equal to sectoral marginal costs. When capacity constraints do not bind, marginal costs are constant and prices fully inherit the nominal wage rigidity. When capacity constraints bind, marginal costs - and so prices - become increasing in the quantity produced. Hence, sectoral Phillips curves have a flat part corresponding to levels of output below $\bar{Y}^j$, and become upward-sloped thereafter. Sectoral supply curves are therefore convex, as documented empirically by Boehm and Pandalai-Nayar (2022).

In the main text we focus on scenarios in which $Y^T_{i,0} \geq \bar{Y}^T$ and $Y^N_{i,0} \leq \bar{Y}^N$, so that capacity constraints bind in the tradable sector, but not in the non-tradable one. Given that in our experiments production in the tradable sector expands - while production in the non-tradable sector contracts - compared to their steady state values, a sufficient condition for this to be the case is to assume $\bar{Y}^T = \omega\bar{L}$ and $\bar{Y}^N = (1-\omega)\bar{L}$.

# C Endogenous inflation cost

In this appendix, we introduce an endogenous inflation cost. We do so by assuming that firms need to pay a cost to adjust their prices, in the spirit of Rotemberg (1982). As is known at least since Erceg et al. (2000), the combination of prices adjustment costs and nominal wage stickiness breaks down the divine coincidence characterizing the baseline New Keynesian framework (Galí, 2009). In the context of our model, this implies that the optimal monetary policy may deviate from targeting full employment. This insight has been exploited by Guerrieri et al. (2021) in their analysis of reallocation shocks. In what follows, we adopt the modeling approach proposed by Bianchi and Coulibaly (2024).

To anticipate the bottomline of this analysis, we find that the presence of prices adjustment costs doesn't modify the results described in the main text. In fact, the optimal monetary policy problem ends up being very similar to the one derived under an exogenous utility cost from inflation.

Suppose that households' aggregate consumption is defined as

$$C_{i,t} = \left(\int_0^1 C_{i,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{1-\epsilon}},$$

where $C(j)$ denotes consumption of consumption good $j$, while $\epsilon > 1$ denotes the elasticity of substitution across these differentiated consumption goods. Optimal demand for each good $j$ implies

$$C_{i,t}(j) = \left(\frac{P_{i,t}}{P_{i,t}(j)}\right)^{\epsilon} C_{i,t},$$

where $P(j)$ is the price of good $j$, while the consumption price index is defined as $P_{i,t} = \left(\int_0^1 P_{i,t}(j)^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}}$.

Each good $j$ is produced by a monopolistic retailer, by aggregating tradable $(C(j)^T)$ and non-tradable $(C(j)^N)$ intermediate goods according to

$$C_{i,t}(j) = \left(\frac{C_{i,t}^T(j)}{\omega_{i,t}}\right)^{\omega_{i,t}} \left(\frac{C_{i,t}^N(j)}{1-\omega_{i,t}}\right)^{1-\omega_{i,t}}. \tag{C.1}$$

Cost minimization implies that the marginal production costs faced by retailers is given by

$$MC_{i,t} = \left(P_{i,t}^T\right)^{\omega_{i,t}} \left(P_{i,t}^N\right)^{1-\omega_{i,t}}. \tag{C.2}$$

Monopolistic retailers set the price of their good to maximize profits. Following Rotemberg (1982), we assume that in period $t = 0$ retailers face a quadratic adjustment cost from changing their price, specified as

$$\frac{\chi}{2}\left(\frac{P_{i,0}(j)}{P_{i,-1}(j)} - 1\right)^2 C_{i,t}, \tag{C.3}$$

in units of the final consumption good.[2] Though this does not affect the analysis, to parallel our assumption about wages stickiness we assume that firms do not incur any cost from changing prices from period $t = 1$ onward.

As it is standard in the literature, we assume that retailers set their price, and then commit to satisfy customers' demand for their products. Under these assumptions, each retailer sets its price by solving

$$\max_{P_{i,0}(j)} \left(P_{i,0} - MC_{i,0}\right)\left(\frac{P_{i,0}}{P_{i,0}(j)}\right)^{\epsilon} C_{i,0} - \frac{\chi}{2}\left(P_{i,0}(j) - 1\right)^2 C_{i,0}, \tag{C.4}$$

where, in accordance with the model of the main text, we have normalized $P_{i,-1}(j) = 1$. The optimality condition from price setting, evaluated in a symmetric equilibrium in which every retailer charges the same price, implies

$$P_{i,0} = \frac{\epsilon MC_{i,0}}{\epsilon - 1 + \chi P_{i,0}\left(P_{i,0} - 1\right)} = \frac{\epsilon \left(P_{i,0}^T\right)^{\omega_{i,0}} \left(P_{i,0}^N\right)^{1-\omega_{i,0}}}{\epsilon - 1 + \chi P_{i,0}\left(P_{i,0} - 1\right)}, \tag{C.5}$$

where the second equality makes use of the definition of retailers' marginal costs.

With respect to the expression for the CPI of the model in the main text, equation (2), there

[2]Notice that, as in Guerrieri et al. (2021) and Bianchi and Coulibaly (2024), we assume that prices adjustment costs are proportional to aggregate consumption. This assumption simplifies the algebra, but it is by no means crucial for our results. It could be justified on the ground that the costs of adjusting prices are increasing in the economic size, and so in the complexity, of the economy.

are two differences. Due to the presence of monopolistic power, retailers now charge a mark-up over their marginal production costs. Given the structure of our economy, and in particular the assumption of an inelastic labor supply, this difference does not affect our analysis. Second, due to the presence of pricing frictions, the consumer price index is less responsive to changes in the marginal production costs, i.e. to changes in $P^T_{i,0}$ and $P^N_{i,0}$.

An interesting special case is the limit $\epsilon \to +\infty$, in which consumption goods are close to perfect substitutes and the retailers' monopoly power vanishes. In this case, the consumer price index is identical to its definition in the main text (i.e. $P_{i,0} = \left(P^T_{i,0}\right)^{\omega_{i,0}} \left(P^N_{i,0}\right)^{1-\omega_{i,0}}$). Intuitively, when monopoly power is infinitesimally small, retailers have to charge a price equal to marginal costs to remain in the market.[3]

The rest of the framework is identical to the one in the main text (except that now there is no exogenous utility loss from inflation).

How does the presence of price adjustment costs affect households' utility? Intuitively, inflation now entails a productivity loss for the economy, which opens up a wedge between the intermediate goods used in production and the consumption enjoyed by households. More precisely, households' utility in the short run is now given by

$$\omega_{i,0} \log C^T_{i,0} + (1-\omega_{i,0}) \log C^N_{i,0} + \log\left(1 - \frac{\chi}{2}\left(P_{i,0}-1\right)^2\right), \tag{C.6}$$

where the last term captures the fact that a fraction of the intermediate inputs ends up being used to pay for the prices adjustment costs. The optimal monetary policy problem now consists in maximizing (C.6), subject to the same constraints specified in the main text, with the exception that constraint (19) is replaced by

$$P_{i,0} = \frac{\epsilon\left(P^T_{i,0}\right)^{\omega_{i,0}}}{\epsilon - 1 + \chi P_{i,0}\left(P_{i,0}-1\right)}. \tag{C.7}$$

When setting the optimal policy, the central bank will now face the following trade-off. On the one hand, increasing inflation may be good for welfare, insofar as it leads to higher employment and so higher production of intermediate goods. On the other hand, higher inflation reduces labor productivity, and so creates a wedge between the amount of intermediate goods produced and final consumption. If the productivity losses from inflation are small enough, it will be optimal for the central bank to let inflation rise as much as needed to maintain full employment. Otherwise, the optimal inflation rate corresponds to the one that maximizes consumption, by optimally trading off employment and labor productivity.

Notice that the productivity losses due to prices adjustment costs play the same role as the convex utility losses from price inflation that we assumed in the main text. In fact, under the

[3]One undesirable feature of this approximation is that retailers may make negative profits in the short run, because the cost of adjusting prices may be larger than the (infinitesimally small) profits earned. However, one could think that firms may accept some losses in the short run, in order to retain their customers base in the long run. Or one could assume that the government compensates firms for these short-run losses through lump-sum subsidies.

approximation $\epsilon \to +\infty$ the two models are essentially identical. Moving away from that approximation, the analysis becomes a bit more algebraically involved, because it is no longer possible to express $P_{i,0}$ as a closed-form function of $P_{i,0}^T$. That said, the two models retain exactly the same economic intuition.

# D A symmetric cost-push shock

Throughout the paper, we have focused on scenarios in which inflationary pressures are concentrated in the sector producing tradable goods. Self-oriented national central banks then have a strong incentive to tighten monetary policy to attract capital inflows, so as to reallocate production towards the low-inflation non-tradable sector. This is precisely the reason behind the coordination failure described in Section 4.3.[4]

In this appendix, we consider a case in which the global economy is hit by a cost-push shock inducing symmetric inflationary pressures in two sectors. We will show that in this case there are no gains from international monetary cooperation, intuitively because capital inflows no longer have a first-order impact on the inflation/employment trade-off faced by national central banks.

We consider an economy in which the slope of the Phillips curve is the same in both sectors. To do so, we assume that the production function in the non-tradable sector is

$$Y_{i,0}^N = \left( \frac{L_{i,0}^N - (1-\alpha)\bar{Y}^N}{\alpha \bar{Y}^N} \right)^\alpha \bar{Y}^N, \tag{D.1}$$

so that the price of the non-traded good is now given by

$$P_{i,0}^N = W_{i,0} \left( \frac{Y_{i,0}^N}{\bar{Y}^N} \right)^{\frac{1-\alpha}{\alpha}}. \tag{D.2}$$

To ensure symmetry across the two sectors, we set $\omega \bar{Y}^N = (1-\omega)\bar{Y}^T$. Moreover, to make things interesting, we consider a case in which capacity constraints bind when inflation is on target. This happens if $\bar{Y}^T < \omega \bar{L}$, which we assume to hold from now on.

We abstract from reallocation shocks by maintaining households' expenditure shares constant, that is $\omega_{i,t} = \omega$ for all $i$ and $t$. Instead, we generate global inflationary pressures by considering an increase in the short-run nominal wage. More precisely, we assume that all the countries have the same short-run nominal wage $W_{i,0} = W > 1$, and consider what happens as $W$ increases.

**International cooperation.** Let us start from deriving the equilibrium under international monetary cooperation. Just as in the main text, in this case central banks internalize that in every country trade has to balance, so that $C_{i,0}^T = Y_{i,0}^T$. Since the two sectors have symmetric production functions, with a bit of algebra one can show that output and consumption of both goods move

[4] For simplicity, in our baseline model we have taken the extreme assumption of a flat Phillips curve in the non-tradable sector. But the coordination failure result holds as long as the Phillips curve is flatter in the non-traded sector than in the traded one.

proportionally (i.e. $\omega C_{i,0}^N = (1-\omega) C_{i,0}^T$), and that the relative price of the two goods is constant

$$P_{i,0}^T = P_{i,0}^N = P_{i,0}. \tag{D.3}$$

Hence, the consumer price index is pinned down by

$$P_{i,0} = W \left( \frac{Y_{i,0}^T}{\bar{Y}^T} \right)^{\frac{1-\alpha}{\alpha}}. \tag{D.4}$$

The optimal monetary policy under cooperation then consists in setting $P_{i,0}$ to maximize households' utility

$$\log Y_{i,0}^T - \chi\left(P_{i,0}\right), \tag{D.5}$$

subject to (D.4) and

$$\alpha \left(Y_{i,0}^T\right)^{\frac{1}{\alpha}} \left(\bar{Y}^T\right)^{1-\frac{1}{\alpha}} + (1-\alpha)\bar{Y}^T \leq \omega \bar{L}. \tag{D.6}$$

Combining the two constraints implies that the price level consistent with full employment is

$$P_{i,0}^{fe} = \frac{W}{\alpha} \left( \frac{\omega \bar{L} - (1-\alpha)\bar{Y}^T}{\alpha \bar{Y}^T} \right)^{1-\alpha}. \tag{D.7}$$

Naturally, a higher nominal wage calls for a higher price level to maintain full employment. If the optimum is interior, instead, the optimal inflation rate is implicitly defined by

$$\chi'\left(\bar{P}_{i,0}\right) \bar{P}_{i,0} = \frac{\alpha}{1-\alpha}. \tag{D.8}$$

The optimal monetary policy under cooperation then consists in setting $P_{i,0} = \min\left(P_{i,0}^{fe}, \bar{P}_{i,0}\right)$.

**Uncooperative equilibrium.** We next turn to the equilibrium without international cooperation. For the same reasons described in the main text, there are no gains from cooperation when self-oriented national central banks choose to maintain their economies at full employment. We then focus on scenarios in which in the uncooperative equilibrium constraint (D.6) does not bind.

Following the same approach as in the main text, one can then show that self-oriented national central banks choose $P_{i,0}$ to maximize

$$\left(\omega + \frac{\beta}{1-\beta}\right) \log C_{i,0}^T + (1-\omega_{i,0}) \log Y_{i,0}^N - \chi\left(P_{i,0}\right), \tag{D.9}$$

subject to constraints

$$P_{i,0}^T = W \left( \frac{Y_{i,0}^T}{\bar{Y}^T} \right)^{\frac{1-\alpha}{\alpha}} \tag{D.10}$$

$$P_{i,0}^N = W \left( \frac{Y_{i,0}^N}{\bar{Y}^N} \right)^{\frac{1-\alpha}{\alpha}} \tag{D.11}$$

$$Y_{i,0}^{N}=\frac{1-\omega}{\omega}\frac{P_{i,0}^{T}}{P_{i,0}^{N}}C_{i,0}^{T} \tag{D.12}$$

$$P_{i,0}=\left(P_{i,0}^{T}\right)^{\omega}\left(P_{i,0}^{N}\right)^{1-\omega} \tag{D.13}$$

$$C_{i,0}^{T}=\frac{\omega}{\omega(1-\beta)+\beta}\left(Y_{i,0}^{T}(1-\beta)+\frac{\bar{L}}{R_0}\right). \tag{D.14}$$

After some algebra, one can show that the optimal monetary policy satisfies

$$\chi'\left(P_{i,0}\right)P_{i,0}\left(\frac{1-\alpha}{\alpha}-(1-\omega)(1-\alpha)\left(1-\frac{Y_{i,0}^{T}}{C_{i,0}^{T}}\frac{\omega(1-\beta)}{\omega(1-\beta)+\beta}\right)\right)= \tag{D.15}$$

$$=(1-\omega)(1-\alpha)+\frac{Y_{i,0}^{T}}{C_{i,0}^{T}}\frac{\omega\left(1-(1-\beta)(1-\alpha)(1-\omega)\right)}{\omega(1-\beta)+\beta}.$$

Evaluated around a symmetric equilibrium, this expression collapses to (D.8). Hence, the uncooperative equilibrium coincides with the cooperative one, and there are no welfare gains from international cooperation.

To understand the logic behind this result, consider that the aggregate Phillips curve can be written as

$$L_{i,0}=\bar{Y}^{T}\left(\frac{1-\alpha}{\omega}+\alpha\left(\frac{P_{i,0}}{W}\right)^{\frac{1}{1-\alpha}}\left(\left(\frac{C_{i,0}^{T}}{Y_{i,0}^{T}}\right)^{\omega-1}+\frac{1-\omega}{\omega}\left(\frac{C_{i,0}^{T}}{Y_{i,0}^{T}}\right)^{\omega}\right)\right). \tag{D.16}$$

Differentiating this expression with respect to $C_{i,0}^{T}/Y_{i,0}^{T}$ gives that around a symmetric equilibrium marginal changes in the trade deficit have no impact on the employment/inflation trade off faced by national central banks. This explains why self-oriented national central banks do not have an incentive to deviate from the cooperative optimal monetary policy. The key message is that the coordination failure that we describe arises when capital inflows ameliorate the domestic trade-off between inflation and employment, by reallocating production towards the sector with the flatter Phillips curve.

# E Supply disruptions

Since the start of the pandemic, the global economy has been harmed by several negative supply shocks - hitting particularly hard the tradable sector. First, the pandemic itself disrupted global supply chains and hampered international trade. Second, Russia's invasion of Ukraine caused a sharp spike in energy prices, disrupting production in the energy-intensive manufacturing sector.[5] Motivated by these facts, in this section we study the economy's response to a negative productivity shock affecting the tradable sector. As we will see, many of the insights that we derived for demand

[5] Of course, high energy prices have broader implications for to economy, besides disrupting production in the manufacturing sector. See Auclert et al. (2023) for an interesting recent paper studying several channels through which a rise in the price of energy affects open economies.

reallocation shocks also apply to this alternative disturbance. This result is not surprising, once one realizes that tradable-biased supply disruptions - just like surges in demand for tradables - generate global scarcity of traded goods.

To introduce supply shocks, we replace the production function in the tradable sector with

$$Y_{i,0}^T = \left( \frac{L_{i,0}^T - (1-\alpha)\xi \bar{Y}^T}{\alpha \xi \bar{Y}^T} \right)^{\alpha} \xi \bar{Y}^T.$$

The parameter $\xi$ determines productivity in the tradable sector in the short run. When $\xi < 1$ the tradable sector is hit by a supply disruption driving productivity below its steady state value. Profit maximization gives the pricing function

$$P_{i,0}^T = W_{i,0} \left( \frac{Y_{i,0}^T}{\xi \bar{Y}^T} \right)^{\frac{1-\alpha}{\alpha}}.$$

So a fall in $\xi$ - holding everything else constant - causes a rise in the price of tradables. Intuitively, lower productivity increases marginal costs and induces firms to charge higher prices.

We study a global supply disruption, i.e. a scenario in which every country is identical and $\xi$ unexpectedly falls below 1 in period $t = 0$. To isolate the impact of this negative supply shock, we abstract from demand reallocation by setting $\omega_{i,0} = \omega$. Since the analysis is very similar to what we have already seen, in this section we limit ourselves to sketch out a few results.

**International cooperation.** The optimal monetary policy problem under international cooperation is identical to the one derived in Section 3, with the exception that the term $\bar{Y}^T$ in constraints (16) and (18) is replaced by $\xi \bar{Y}^T$. The Phillips curve, which recall we define as the relationship between firms' labor demand and inflation, now takes the form

$$P_{i,0} = \left( \frac{L_{i,0}/(\xi \bar{L}) - \omega(1-\alpha)}{1 - \omega(1-\alpha)} \right)^{\omega(1-\alpha)}. \tag{E.1}$$

A drop in $\xi$ thus acts as a negative cost-push shock, worsening the trade off between inflation and employment faced by monetary authorities. The reason is simple. Lower productivity drags profits in the tradable sector down. To prevent a reduction in employment real wages need to fall to restore profitability.[6] Since nominal wages are rigid, a drop in real wages can only be attained through a rise in inflation.

If the inflation cost is small enough, the global central bank chooses to maintain full employment by setting $P_{i,0}$ equal to

$$P_{i,0}^{fe} \equiv \left( \frac{1/\xi - \omega(1-\alpha)}{1 - \omega(1-\alpha)} \right)^{\omega(1-\alpha)}. \tag{E.2}$$

Otherwise, it is optimal to equate the marginal welfare benefit from a rise in inflation to its marginal

[6] Even though the supply disruption hits exclusively the tradable sector, it may lead to a decline in employment in the non-traded sector too. In fact, lower economic activity in the tradable sector decreases households' income, and so their demand for non-traded goods. In turn, lower demand leads to a reduction in production and employment in the non-traded sector. A tradable-biased supply disruption thus depresses firms' demand for labor in both sectors.

cost

$$\chi'(P_{i,0})\,P_{i,0} = \frac{1}{\omega}\left(\frac{\alpha}{1-\alpha} + 1 - \omega\right). \tag{E.3}$$

The optimal monetary policy under international cooperation then consists in setting $P_{i,0} = \min\left(P_{i,0}^{fe}, \bar{P}_{i,0}\right)$, where $\bar{P}_{i,0}$ is the value of $P_{i,0}$ that solves equation (E.3).

Similar to a demand reallocation shock, a tradable-biased supply disruption thus causes a rise in inflation, and possibly a fall in employment.[7] The only substantial difference is that while a rise in $\omega_0$ leads to an increase in employment and production in the tradable sector, after a drop in $\xi$ production and employment in the traded sector contract.

**Uncooperative equilibrium.** Absent international cooperation, every country tries to smooth out the impact of the transitory negative productivity shock on consumption by borrowing from abroad, so as to run a trade deficit. However, in a symmetric equilibrium trade imbalances cannot arise. Instead, the world interest rate rises until the balanced-trade equilibrium - in which every country consumes exactly its own production of traded goods - is restored.

The implications for monetary policy are in line with those derived in Section 4.2. That is, lack of cooperation does not affect the amount of inflation needed to maintain full employment, which is still given by expression (E.2). But lack of cooperation reduces the marginal benefit attached by self-oriented central banks to a rise in inflation, because they perceive that access to international credit markets weakens the impact of higher inflation on domestic demand and employment. In fact, in the uncooperative equilibrium condition (E.3) is replaced by

$$\chi'(P_{i,0})\,P_{i,0} = \frac{1}{\omega}\left(\frac{\alpha}{1-\alpha}\frac{\omega}{\omega+\beta(1-\omega)} + 1 - \omega\right). \tag{E.4}$$

Comparing (E.3) and (E.4) shows that - when the optimal monetary policy is interior - self-oriented national central banks adopt a more contractionary monetary stance and tolerate less inflation compared to a benevolent global central bank. The insights derived in Section **??** thus extend to tradable-biased negative supply shocks.

**Gains from cooperation and international spillovers.** Exactly for the same reasons described in Section 4.3, lack of cooperation may thus lead to an overly tight monetary response to a global supply disruption. Efforts to contain domestic inflation through monetary tightenings, indeed, exacerbate the global scarcity of traded goods and impose negative externalities toward the rest of the world.

Supply disruptions biased toward the tradable sector thus have effects very similar to shocks boosting the global demand for tradable goods. What makes the two shocks similar is that they both lead to a global scarcity of tradable goods. It is then not surprising that some of the issues highlighted by this paper - such as monetary coordination problems arising from competitive appreciations - were present in the economic debate of the 1970s/1980s (Bruno and Sachs, 1985),

[7] To see this point, consider that if productivity is equal to its steady state value ($\xi = 1$), then the central bank can attain both full employment and zero inflation. It follows that if $\xi < 1$ it is optimal for the central bank to let inflation rise above its steady state value. Moreover, if the drop in $\xi$ is sufficiently large, then $P_{i,0}^{fe} > \bar{P}_{i,0}$ and the optimal monetary policy entails a drop in employment below $\bar{L}$.

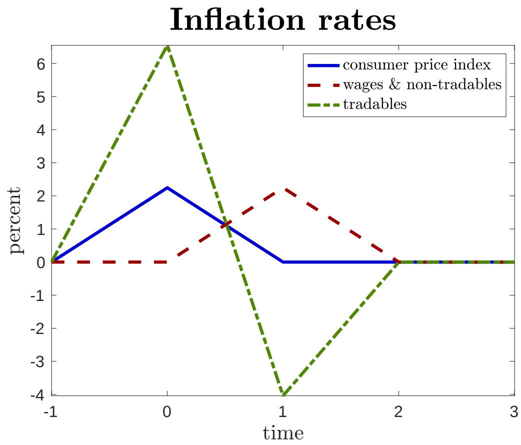


**Figure 6: Inflation dynamics around a global demand reallocation shock.** Notes: solid line refers to CPI inflation ($P_{i,t}/P_{i,t-1}$). Dashed line refers to wage inflation ($W_{i,t}/W_{i,t-1}$), which is equal to inflation in the non-traded sector ($P^N_{i,t}/P^N_{i,t-1}$). Dash-dotted line refers to inflation in the traded sector ($P^T_{i,t}/P^T_{i,t-1}$).

a period characterized by negative supply shocks affecting the tradable sector.

# F Wages and prices during the disinflation phase

We have already shown that real wages decline when demand reallocates. But what happens next? In our framework, once the shock abates real wages fully recover their value in the initial steady state.[8] The demand reallocation shock is thus followed by a period in which nominal wages grow faster than prices, so that real wages regain the ground lost during the shock period.

Since prices in the non-tradable sector are equal to wages, this reasoning implies that sectoral inflation evolves according to

$$\frac{W_{i,1}}{W_{i,0}} = \frac{P^N_{i,1}}{P^N_{i,0}} > 1 > \frac{P^T_{i,1}}{P^T_{i,0}}.$$

In words, the disinflation that follows the period of the shock is driven by a drop in tradable inflation below its steady state value. Inflation in the non-traded sector, instead, overshoots its long-run value in period $t = 1$. This adjustment is needed so that relative prices go back to their steady state value.

Figure 1 displays the inflation dynamics around a global reallocation shock. To draw this figure, we have assumed that $\omega_0 = 0.35$, that monetary policy maintains full employment $L_0 = \bar{L}$, and kept all the other parameters as in our running numerical example. The figure shows that the shock period ($t = 0$) is characterized by a sharp rise in inflation in the traded sector. During the disinflation, i.e. in $t = 1$, wage and non-tradable price inflation overshoot their long-run value, while the tradable sector experiences a period of deflation. Interestingly, consistent with these predictions, both in the United States and in the euro area the recent burst of inflation originated

[8] In reality, part of the decline in real wages caused by the reallocation shock may become permanent. For instance, this would happen if the demand reallocation shock was associated with a drop in investment, hurting future productivity growth. Benigno and Fornaro (2018) and Fornaro and Wolf (2023) provide frameworks in which temporary shocks may persistently affect real wages through this channel.

in the goods sector, and only later migrated to wages and to the service sector (Lane, 2023; Fornaro and Romei, 2024).

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