Supplementary online appendix to "Emission prices, biomass, and biodiversity in tropical forests"

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In this appendix we expand on some of the discussions in the paper.

1 Carbon prices and reforestation

See Pivello (2011) for a discussion of edge effects and Cochrane and Laurance (2002) for lower carbon-capture productivity in forest fragments of less than 100km^2. These papers document that carbon-capture is more effective if reforestation is implemented in areas much larger than the typical private land-holdings.

1.1 Model

The outcome of the minimization used in constructing the robustly optimal solution for land allocation is a special case of what is called smooth ambiguity aversion in the decision-theory literature. The smooth ambiguity decision model is typically justified without a formal link to robustness. See Klibanoff, Marinacci and Mukerji (2005) for an initial reference on smooth

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ambiguity, and see Hansen and Sargent (2024) for recent discussion of the conceptual linkages between preferences for prior robustness and smooth ambiguity aversion.

1.2 Results

Figure 1 illustrates the aggregate impact of emission prices on land-use.

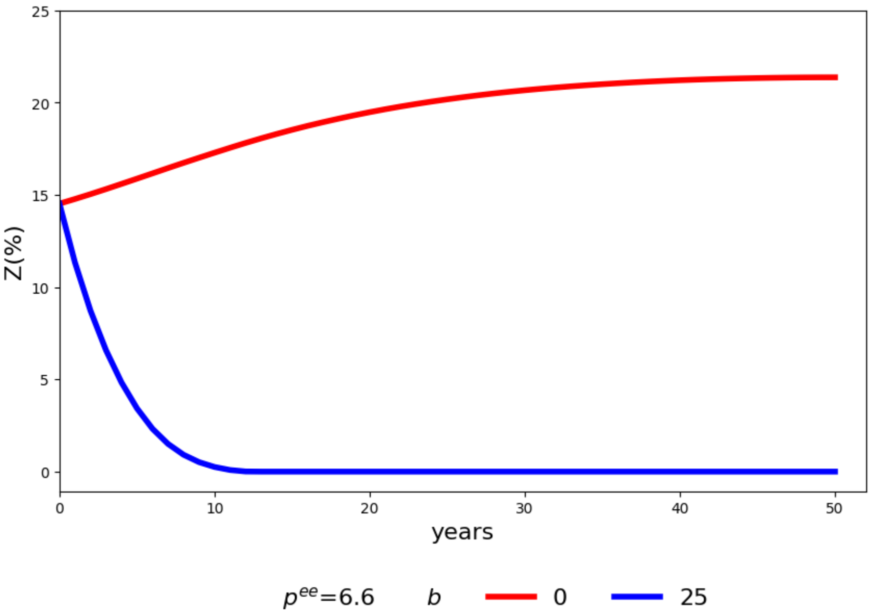

Figure 1: Evolution of the aggregate forest lost to agriculture for business-as-usual and for transfers that support an emissions price with $b = 25$. This figure uses computations in Assunção et al. (2023).

Here, we use the model with 1043 sites and with $P_t^a = \bar{P}^a$, the average under the stationary distribution of the fitted Markov process.

2 Biodiversity

There are many different ways to measure diversity, including species count, Hill indices, in which one parameter determines how to weight rare species relative to abundant species, and genetic diversity. Though species count is the simplest to measure and hence most used, in a widely cited review-article

Hooper et al. (2005) writes “Species’ functional characteristics strongly influence ecosystem properties. Functional characteristics operate in a variety of contexts, including effects of dominant species, keystone species, ecological engineers, and interactions among species (e.g., competition, facilitation, mutualism, disease, and predation). Relative abundance alone is not always a good predictor of the ecosystem-level importance of a species”.

In addition to the difficulty in measuring diversity, in contrast to CO_2 emissions, there is no agreed scientific model that connects biodiversity levels to variables that can be easily shown to matter for economic performance such as global temperature or the probability of extreme weather events, which would allow one to “price” biodiversity.

2.1 Emission price impact on biodiversity

Though Rozendaal et al. (2019) report the relatively fast convergence of species count under natural reforestation in tropical-forests, they also point out that convergence of the species-distribution is much slower.

To construct the η^i’s in Table 2 we use first Ter Steege et al. (2023) calculations of the number of different tree-species per hectare of original forest in pixels of 1° in the Amazon biome. We average these pixel values to estimate η^i’s potential tree-biodiversity per ha of each of our 1043 sites. The average of the η^i’s is 108 with a cross-sectional standard deviation of 53.

2.2 Biodiversity and Biomass

In the relationship estimated by O’Connor et al. (2017)

$$p_{bm} = (p_{bd})^{\mathsf{d}}, \tag{1}$$

p_{bd} is the proportion of original biodiversity that remains while p_{bm} is the proportion of the remaining biomass that actually survives, accounting for the change in species richness. The text in the paper is confusing because it refers to proportional loss instead of proportion remaining. The results reported in Table 4 are correct, however, as they were constructed using formula (1).

2.3 Preserving biodiversity

For analytical simplicity, we consider two territories, each of which has a common size and a common cost function for preserving biodiversity. Each territory has some unique species and common species. Territory one has $S_1 = U_1 + C$ species and territory two has $S_2 = U_2 + C$ species where U_i is the number of unique species to Territory i and C is the number of common species. Thus the total number of species across the two areas is $U_1 + U_2 + C$. We let Territory one have more species than Territory two: $U_1 > U_2 > 0$.

The preservation of biodiversity requires preservation of land. We denote the size of each territory by b, and we assume that the cost of saving a fraction f of a territory is $\mathsf{b}f$. Using the relationship between species saved and territory saved postulated by Arrhenius (1921), the cost of saving $\widetilde{S}_i$ species in location i with a total number S_i species is

$$\mathsf{b}\left(\frac{\widetilde{S}_i}{S_i}\right)^{\frac{1}{\mathsf{a}}} \tag{2}$$

for $0 < \mathsf{a} < 1$. For tropical forests, $\mathsf{a} = .25$ is often used. We assume that the proportions of unique and common species saved in a territory is the same as the proportions initially present. Since the costs are separable across locations, this specification abstracts from spatial linkages and is best viewed as being applicable when locations that are far apart.

We endow a "planner" with total budget $B > 0$ for preservation to allocate across territories in a socially efficient way. We restrict $B \leq 2b$, otherwise all species could be saved with a slack budget constraint. Let the budget allocated to territory one be λB and the budget allocated to territory two be $(1-\lambda)B$ for

$$0 \leq \lambda \leq 1. \tag{3}$$

Given a budget allocation, we deduce the total number of species saved in each location by inverting the cost function:

$$\widetilde{S}_1 = \left[\frac{\lambda B}{\mathsf{b}}\right]^{\mathsf{a}} S_1, \quad \widetilde{S}_2 = \left[\frac{(1-\lambda)B}{\mathsf{b}}\right]^{\mathsf{a}} S_2.$$

Since $\widetilde{S}_i \leq S_i$, we further restrict λ to satisfy:

$$1 - \frac{b}{B} \leq \lambda \leq \frac{b}{B} \tag{4}$$

These restrictions are weaker than those in (3) when $b > B$, but they are stronger when $b < B$. We view the ratio $\frac{B}{\mathsf{b}}$ as a measure of the abundance of the budget relative to the cost of preserving a single territory.

Our objective is to show that Arrhenius formula implies that if $\mathsf{a} < 1$, and the budget is scarce ($\frac{B}{\mathsf{b}} \leq 1$), the optimal choice will always imply $\lambda < 1$. Thus the planner would choose not to save as much of the more productive territory as she can afford.

Observe that a candidate choice of $\lambda < \frac{1}{2}$ is dominated by choice of $1 - \lambda$. This follows because with $1 - \lambda$ instead of λ, the number of idiosyncratic species is strictly greater and the distribution of common species across sites stays the same.[1]

Saved species originally common to both territories should overlap. Since we lack a model to determine which of the common species would survive in each territory, we assume that the species saved in Territory 2 are a subset of the species saved in Territory 1. This hypothesis biases against saving the less species rich Territory 2, since the gains from saving common species in Territory 2 is treated as null.

We will discuss an alternative approach in Remark 2 based on applying the Arrhenius formula separately to common and idiosyncratic species in each territory. In the remark, we will show that the resulting optimal solution corresponds to the optimal solution of our baseline case when there are no common species.

Accounting for the treatment of the overlapping species, the number of species that can be saved with the budget allocation is:

$$\left[\frac{\lambda B}{\mathsf{b}}\right]^{\mathsf{a}} U_1 + \left[\frac{(1-\lambda)B}{\mathsf{b}}\right]^{\mathsf{a}} U_2 + \left[\frac{\lambda B}{\mathsf{b}}\right]^{\mathsf{a}} C$$

In what follows we maximize this objective by choice of λ subject to (3) and (4).

We solve this problem in three steps.

2.3.1 $b \geq B$

In this case we may ignore the constraints (4). Since $(\mathsf{b}/B)^{\mathsf{a}}$ is a common scale factor in the three terms of the objective, for simplicity, we modify the

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[1] We owe this argument to Pengyu Chen.

optimization to be:

$$\max_{0\leq\lambda\leq 1} \lambda^{\mathsf{a}}(U_1 + C) + (1-\lambda)^{\mathsf{a}} U_2.$$

This objective is a concave function in λ.

Since $\mathsf{a} < 1$, it has a unique interior solution implied by the first-order conditions:

$$\mathsf{a}\lambda^{\mathsf{a}-1}(U_1 + C) - \mathsf{a}(1-\lambda)^{\mathsf{a}-1} U_2 = 0.$$

This in turn implies that

$$\left(\frac{\lambda}{1-\lambda}\right)^{\mathsf{a}-1} = \frac{U_2}{U_1 + C}$$

or

$$\frac{\lambda}{1-\lambda} = \left(\frac{U_1 + C}{U_2}\right)^{\frac{1}{1-\mathsf{a}}}.$$

The solution λ^* to this equation is

$$\lambda^* = \frac{(U_1 + C)^{\frac{1}{1-\mathsf{a}}}}{(U_2)^{\frac{1}{1-\mathsf{a}}} + (U_1 + C)^{\frac{1}{1-\mathsf{a}}}}. \tag{5}$$

Notice that the right side of (5) implies that $1/2 < \lambda^* < 1$. Moreover, λ^* is increasing in C.

Thus when the budget is scarce relative to the cost of preserving a territory as reflected by $B \leq b$, the planner opts to preserve some of the species-poor territory. This is true even if it is impossible to fully preserve the species rich-territory as would be the case when $b > B$. This "interiority" result follows from the slope of the relationship (2) at the origin.

Remark 1 *Suppose again that $b \geq B$. Consider the limiting case in which $\mathsf{a} = 1$. In this case, $\lambda = 1$, as given by the upper bound of the constraint set,* (3) *implying that the full budget is allocated to saving territory one. This outcome is analogous to a result in Weitzman (1998).*

2.3.2 $b < B$

In this case , λ^* given by (5) will continue to be valid provided that it satisfies:

$$\lambda^* \leq \frac{b}{B}.$$

In effect, this inequality puts an upper bound on the budget B given by:

$$B \leq \frac{b}{\lambda^*}$$

If this inequality fails to be true, then the constraint on λ binds, and the optimized λ is $\frac{b}{B}$. Again some of the budget is allocated to territory 2.

2.3.3 Model implications for species richness

In Figure 2, we show the dependence of the optimal budget allocation to territory one (λ) as a function of the relative richness of the unique species in the two territories (U_2/U_1). This figure gives four different curves depending on the ratio of the number of common species to the total number species in the two areas. Each of the curves is downward sloping, as to be expected. When the common species become prominent, more resources are allocated to territory one as reflected in a higher curve.

Remark 2 *An alternative to the approach just described treats common and idiosyncratic species separately. In this case, the number of common species saved depends on the fraction saved of Territory 1 plus the fraction saved of Territory 2. This sum is independent of* λ. *Provided that* $\mathsf{b} \geq B$, *a rescaled version of the maximization problem is*

$$\max_{\lambda}\{\lambda^{\mathsf{a}} U_1 + (1-\lambda)^{\mathsf{a}} U_2\},$$

with an optimal solution that is identical to the solution, λ^* *of equation* (5) *when* $C = 0$. *This is displayed as the blue line in Figure 2. This figure also confirms our original hypothesis. Imposing that the species saved in Territory 2 are a subset of the species saved in Territory 1 produces a less favorable outcome to preserving Territory 2.*

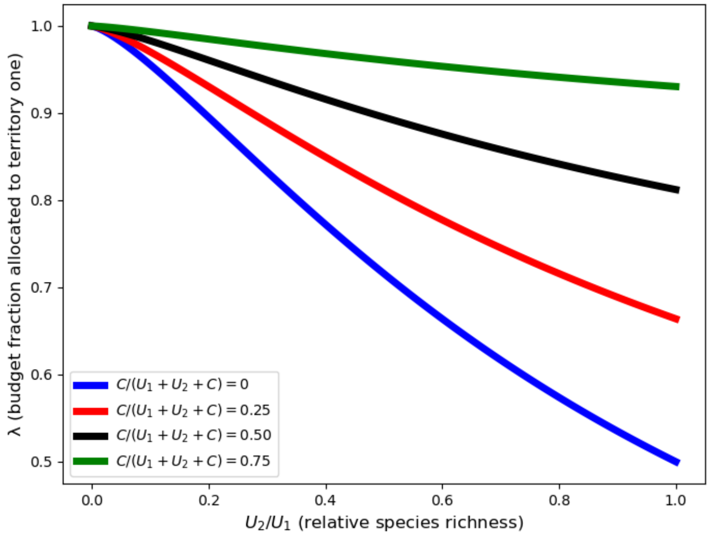

Figure 2: Optimal solution when $B < b$. The plots impose $\mathsf{a} = .25$ and they normalize U_1 to be 1.

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