

# Dynamic Assignment of Objects to Queuing Agents

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## Online Appendix

### 1 Variants

#### 1.1 Stochastic queues

We consider a model where entry into the queue is stochastic. When the queue is of size 1, with probability  $s \in [0, 1]$  a new agent enters the queue. The baseline model corresponds to the case  $s = 1$ . When the queue is of size 2, it remains of size 2 with probability 1. Hence, after the object has either been allocated to one of the agents or discarded, the transition probabilities between different queue sizes are given by

	1	2
1	$1 - s$	$s$
2	0	1

In a FCFS mechanism, let  $V(1, 1)$  be the equilibrium value of agent 1 when the queue is of size 1 (before entry),  $V(1, 2)$  the equilibrium value of agent 1 when the queue is of size 2 and  $V(2, 2)$  the value of agent 2 when the queue is of size 2, with corresponding strategies  $q(1, 1)$ ,  $q(1, 2)$  and  $q(2, 2)$ . We compute

$$\begin{aligned} V(1, 1) &= (1 - s)[\pi + (1 - \pi)(1 - q(1, 1))V(1, 1)] \\ &\quad + s[\pi + (1 - \pi)(1 - q(1, 2))][(\pi + (1 - \pi)q(2, 2))V(1, 1) \\ &\quad + (1 - \pi)(1 - q(2, 2))V(1, 2)] - c, \\ V(1, 2) &= [\pi + (1 - \pi)(1 - q(1, 2))][(\pi + (1 - \pi)q(2, 2))V(1, 1) \\ &\quad + (1 - \pi)(1 - q(2, 2))V(1, 2)] - c, \\ V(2, 2) &= [\pi + (1 - \pi)q(1, 2)]V(1, 1) \\ &\quad + (1 - \pi)(1 - q(1, 2))[\pi + (1 - \pi)(1 - q(2, 2))V(2, 2)] - c. \end{aligned}$$

We consider equilibria where agent 1 adopts the same strategy when the queue is of size 1 and of size 2. When  $q(1, 2) = q(1, 1)$ ,  $V(1, 1) = V(1, 2)$  and we simplify notation by letting  $V(1) \equiv V(1, 1) = V(1, 2)$  and  $V(2) \equiv V(2, 2)$ . The situation then becomes similar to the deterministic queue model when the queue is of size 2. We compute equilibrium payoffs as in Table 1 and find that when  $q(1) = q(2) = 0$ ,  $\hat{V}(1) = 1 - \frac{c}{\pi}$ ,  $\hat{V}(2) = 1 - \frac{2c}{\pi(2-\pi)}$  and when  $q(1) = q(2) = 1$ ,  $\hat{V}(1) = \pi - c$  and  $\hat{V}(2) = \pi - 2c$ .

In the lottery, we compute the value when the queue is of size 1 and of size 2,  $V(1)$  and  $V(2)$  with corresponding strategies  $q(1)$  and  $q(2)$ . We obtain

$$\begin{aligned} V(1) &= (1-s)[\pi + (1-\pi)(1-q(1))V(1)] + s\left[\frac{\pi[1 + (1-\pi)(1-q(2))]}{2}\right. \\ &\quad \left. + \frac{V(1)[\pi + (1-\pi)q(2)][1 + (1-\pi)(1-q(2))]}{2} + (1-\pi)^2(1-q(2))^2V(2)\right] - c, \\ V(2) &= \frac{\pi[1 + (1-\pi)(1-q(2))]}{2} + \frac{V(1)[\pi + (1-\pi)q(2)][1 + (1-\pi)(1-q(2))]}{2} \\ &\quad + (1-\pi)^2(1-q(2))^2V(2) - c. \end{aligned}$$

We first compute equilibrium values when  $q(1) = q(2) = 0$  and obtain

$$\begin{aligned} \tilde{V}(1) &= \frac{\pi^2(2-\pi)(1-s) + \frac{\pi(2-\pi)s}{2} - c\pi(2-\pi) - cs(1-\pi)^2}{\pi(2-\pi)[1 - \frac{s}{2}(1-s)(1-\pi)]} \\ \tilde{V}(2) &= \frac{\frac{\pi(2-\pi)}{2} - (1-s)\frac{(1-2\pi)(2-\pi)}{2} - c(1 + (1-s)[\frac{\pi(2-\pi)}{2} - (1-\pi)]}{\pi(2-\pi)[1 - \frac{s}{2}(1-s)(1-\pi)]} \end{aligned}$$

We also compute equilibrium values when  $q(1) = q(2) = 1$  and obtain

$$\begin{aligned} \tilde{V}(1) &= \frac{s\pi + 2\pi(1-s) - 2c}{2-s}, \\ \tilde{V}(2) &= \pi - \frac{2c}{2-s}. \end{aligned}$$

In the baseline model when  $s = 1$ ,  $\hat{V}(1) = \tilde{V}(1)$  and  $\hat{V}(2) = \tilde{V}(2)$ . Straightforward computations show that  $\hat{V}(1) > \tilde{V}(1)$  and  $\hat{V}(2) < \tilde{V}(2)$  in both equilibria.

## 1.2 Heterogeneous waiting costs

We suppose that agents are divided into two categories: a fraction  $\lambda$  of agents with low waiting cost  $\underline{c}$ , and a fraction  $1 - \lambda$  of agents with high waiting cost  $\bar{c}$ . We suppose that the waiting cost is observable by the planner – for example, the

planner can verify whether an agent currently lives in an apartment or not, or the health status of an agent waiting for a transplant. Agents are now characterized by two variables: their rank in the waiting list, and their waiting cost. In order to select an optimal mechanism, the designer faces a trade-off between these two characteristics, and must choose which weight to assign to seniority and waiting cost in the offer sequence. We assume that the designer, after observing the waiting costs of the two agents,  $(c, c)$  chooses the probability that the most senior agent is proposed the object first,  $p(c, c)$ . Because it may be optimal to let the second agent choose first when he has a high waiting cost, we do not put any restriction on  $p(c, c)$ .

Suppose that each agent knows the waiting cost of the other agent in the queue. The strategy of each agent assigns to each of the four possible vectors of waiting costs  $(c, c)$  a point in  $\{0, 1\}$ . As each agent in the queue chooses four actions, the total number of strategies makes it intractable to characterize admissible equilibrium configurations as a function of the parameters.<sup>1</sup> In order to understand the trade-off between waiting costs and seniority rank, we focus attention on one specific equilibrium configuration: one where the low waiting cost  $\underline{c}$  is sufficiently low and the high waiting cost  $\bar{c}$  sufficiently high so that *all agents with low waiting cost are selective, and all agents with high waiting costs accept both objects*.

In this equilibrium, we first compute the value of the first agent when his waiting cost is low.

$$\begin{aligned} V_1(\underline{c}, \underline{c}) - \underline{c} &= \frac{p(\underline{c}, \underline{c}) + (1 - p(\underline{c}, \underline{c}))(1 - \pi) + p(\underline{c}, \underline{c})(1 - \pi)}{2 - \pi} \\ &+ \frac{(1 - p(\underline{c}, \underline{c}))A(p(\underline{c}, \underline{c}), p(\underline{c}, \bar{c}))}{2 - \pi} - \frac{c}{\pi(2 - \pi)}, \\ V_1(\underline{c}, \bar{c}) - \bar{c} &= p(\underline{c}, \bar{c})\pi + [p(\underline{c}, \bar{c})(1 - \pi) + 1 - p(\underline{c}, \bar{c})]A(p(\underline{c}, \underline{c}), p(\underline{c}, \bar{c})) - \underline{c}, \end{aligned}$$

where

$$\begin{aligned} A(p(\underline{c}, \underline{c}), p(\underline{c}, \bar{c})) &= \lambda V_1(\underline{c}, \underline{c}) + (1 - \lambda)V_1(\underline{c}, \bar{c}) \\ &= 1 - \frac{(\lambda + (1 - \lambda)\pi(2 - \pi))\underline{c}}{\pi(\lambda[p(\underline{c}, \underline{c}) + (1 - p(\underline{c}, \underline{c}))(1 - \pi)] + (1 - \lambda)p(\underline{c}, \bar{c})\pi^2(1 - \pi))}. \end{aligned}$$

Notice that the expected continuation value  $A(p(\underline{c}, \underline{c}), p(\underline{c}, \bar{c}))$  is increasing in both probabilities  $p(\underline{c}, \underline{c})$  and  $p(\underline{c}, \bar{c})$ . As  $A < 1$ , the values  $V_1(\underline{c}, \bar{c})$  and  $V_1(\underline{c}, \bar{c})$  are also

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<sup>1</sup>In principle, as each agent in the waiting list has  $2^4 = 16$  choices, the total number of strategy vectors is  $16 \times 16 = 256$ . Characterizing equilibrium configurations with such a large strategy set becomes intractable.

increasing in the probabilities  $p(\underline{c}, \underline{c})$  and  $p(\underline{c}, \bar{c})$ . We next compute the value of the first agent when his cost is high.

$$\begin{aligned} V_1(\bar{c}, \underline{c}) &= p(\bar{c}, \underline{c})\pi + (1 - p(\bar{c}, \underline{c}))\pi(1 - \pi) + \pi(1 - p(\bar{c}, \underline{c}))B(p(\bar{c}, \underline{c}), p(\bar{c}, \bar{c})) - \bar{c}, \\ V_1(\bar{c}, \bar{c}) &= p(\bar{c}, \bar{c})\pi + (1 - p(\bar{c}, \bar{c}))B(p(\bar{c}, \underline{c}), p(\bar{c}, \bar{c})) - \bar{c}, \end{aligned}$$

where

$$\begin{aligned} B(p(\bar{c}, \underline{c}), p(\bar{c}, \bar{c})) &= \lambda V_1(\bar{c}, \underline{c}) + (1 - \lambda)V_1(\bar{c}, \bar{c}) \\ &= \pi \frac{\lambda(p(\bar{c}, \underline{c}) + (1 - p(\bar{c}, \underline{c}))(1 - \pi) + (1 - \lambda)p(\bar{c}, \bar{c})) - c}{\lambda(1 - \pi(1 - p(\bar{c}, \underline{c}))) + (1 - \lambda)p(\bar{c}, \bar{c})}. \end{aligned}$$

Notice that the expected continuation value  $B(p(\bar{c}, \underline{c}), p(\bar{c}, \bar{c}))$  is increasing in  $p(\bar{c}, \underline{c})$  and  $p(\bar{c}, \bar{c})$ . As  $B < \pi$ , the values  $V_1(\bar{c}, \underline{c})$  and  $V_1(\bar{c}, \bar{c})$  are also increasing in the probabilities  $p(\bar{c}, \underline{c})$  and  $p(\bar{c}, \bar{c})$ . Turning to the second agent we compute his value when his cost is low as

$$\begin{aligned} V_2(\underline{c}, \underline{c}) &= \frac{p(\underline{c}, \underline{c})(1 - \pi) + 1 - p(\underline{c}, \underline{c}) + p(\underline{c}, \underline{c})}{2 - \pi} \\ &\quad + \frac{(1 - p(\underline{c}, \underline{c}))(1 - \pi)A(p(\underline{c}, \underline{c}), p(\underline{c}, \bar{c}))}{2 - \pi} - \frac{\underline{c}}{\pi(2 - \pi)}, \\ V_2(\bar{c}, \underline{c}) &= p(\bar{c}, \underline{c})A(p(\underline{c}, \underline{c}), p(\underline{c}, \bar{c})) + (1 - p(\bar{c}, \underline{c}))[\pi + (1 - \pi)A(p(\underline{c}, \underline{c}), p(\underline{c}, \bar{c}))] - \underline{c} \end{aligned}$$

It is interesting to note that  $V_2(\underline{c}, \underline{c})$  is increasing in  $p(\underline{c}, \bar{c})$  but non monotonic in  $p(\underline{c}, \underline{c})$ . The value  $V_2(\bar{c}, \underline{c})$  is increasing in  $p(\underline{c}, \underline{c})$  and  $p(\underline{c}, \bar{c})$  but decreasing in  $p(\bar{c}, \underline{c})$ . For the second agent with high costs

$$\begin{aligned} V_2(\underline{c}, \bar{c}) &= p(\underline{c}, \bar{c})[\pi B(p(\bar{c}, \underline{c}), p(\bar{c}, \bar{c})) + (1 - \pi)\pi] + (1 - p(\underline{c}, \bar{c}))\pi - \bar{c}, \\ V_2(\bar{c}, \bar{c}) &= p(\bar{c}, \bar{c})B(p(\bar{c}, \underline{c}), p(\bar{c}, \bar{c})) + (1 - p(\bar{c}, \bar{c}))\pi - \bar{c}. \end{aligned}$$

We observe that  $V_2(\bar{c}, \underline{c})$  is increasing in  $p(\bar{c}, \underline{c})$  and  $p(\bar{c}, \bar{c})$  but decreasing in  $p(\underline{c}, \bar{c})$  and  $V_2(\bar{c}, \bar{c})$  is increasing in  $p(\bar{c}, \underline{c})$  and non monotonic in  $p(\bar{c}, \bar{c})$ .

Contrary to the case of homogenous waiting costs, the value of the second agent is not necessarily increasing in the probability that the first agent is proposed the object. In order to illustrate this fact, we consider the special case where  $\lambda = \pi = \frac{1}{2}$  and compute the values of the probabilities which maximize the expected value of the second agent,  $EV_2 = \frac{1}{4}V_2(\underline{c}, \underline{c}) + V_2(\bar{c}, \underline{c}) + V_2(\underline{c}, \bar{c}) + V_2(\bar{c}, \bar{c})$ .

In order to maximize  $EV_2$  it is sufficient to maximize

$$E = -\frac{7\underline{c}(5 + 2p(\underline{c}, \underline{c}) + p(\bar{c}, \underline{c}))}{6(2 + 2p(\underline{c}, \underline{c}) + 3p(\underline{c}, \bar{c}))} - \frac{p(\underline{c}, \bar{c}) + 2p(\bar{c}, \bar{c})}{4} \\ + \frac{p(\underline{c}, \bar{c}) + 2p(\bar{c}, \bar{c})}{2} \frac{1 + p(\bar{c}, \underline{c}) + 2p(\bar{c}, \bar{c}) - 8\bar{c}}{2 + 2p(\bar{c}, \underline{c}) + 4p(\bar{c}, \bar{c})}.$$

We can check that  $\frac{\partial E}{\partial p(\underline{c}, \underline{c})} > 0$  and  $\frac{\partial E}{\partial p(\bar{c}, \bar{c})} < 0$ . In addition, for sufficiently large values of  $\bar{c}$  and sufficiently low values of  $\underline{c}$ ,  $\frac{\partial E}{\partial p(\underline{c}, \bar{c})} < 0$  and  $\frac{\partial E}{\partial p(\bar{c}, \underline{c})} > 0$ . Hence, in order to maximize the expected value of the second agent, the mechanism designer chooses  $p(\underline{c}, \underline{c}) = p(\bar{c}, \underline{c}) = 1$  and  $p(\underline{c}, \bar{c}) = p(\bar{c}, \bar{c}) = 0$ . *The mechanism should always give the object to the second agent when he has a high waiting cost and to the first agent when he has a low waiting cost.*

### 1.3 Last Come First Serve

We consider the Last Come First Serve rule for a waiting list of arbitrary size  $n$ . Consider a pure strategy equilibrium where agent  $i$  accepts the low quality object. We claim that all agents preceding  $i$  in the seniority order must accept the object as well. Because in the LCFS rule, these agents never have an opportunity to pick the object, they never have to make decisions along the equilibrium path. In the spirit of a sequential equilibrium, in order to guarantee that information sets where agents make decisions are reached, we suppose that the designer chooses the LCFS rule with probability  $1 - \epsilon$  and the uniform lottery with probability  $\epsilon$ . Let  $\omega^u(i) > 0$  and  $\gamma^u(i) > 0$  be the probability that  $i$  receives and picks the object under the uniform rule. Then, for any  $j < i$ ,

$$V(j) = \frac{\epsilon\pi \sum_{t=1}^j \omega^u(t) - jc}{\sum_{t=1}^j \gamma^u(t)}.$$

As  $\epsilon \rightarrow 0$ ,  $V(j) \rightarrow -\infty$  so that agent  $j$  always accepts the low value object. Next we observe that the equilibrium  $E^0$  where all agents accept the low value object exists for all values of  $c$  and  $\pi$ . Consider the last agent. If he rejects the low value object, he moves to rank  $n - 1$  next period. But because  $q(n) = 1$ , the continuation value  $V(n - 1) \rightarrow -\infty$  so that agent  $n$  has no incentive to reject the low value object. Other equilibria where the last agent in the queue are selective also exist for different values of the parameter. In fact, whenever  $\pi(1 - \pi)^{n-i} - ic \geq 0$  for  $i \geq 2$ , there exists a pure strategy equilibrium where all agents  $j = i + 1, \dots, n$  choose  $q(j) = 0$  and all agents  $j = 1, \dots, i$  choose  $q(j) = 0$ . Agents  $j = i + 1, \dots, n$  have a value

$$V(j) = \pi \sum_{t=0}^{j-i} (1 - \pi)^{n-i-t} - jc \geq (j - i)[\pi(1 - \pi)^{n-i} - ic] \geq 0,$$

so, upon rejecting the low value object, each agent  $j$  has a continuation value equal to  $V(j-1) \geq 0$ . This shows that all agents  $j = i+1, \dots, n$  optimally choose  $q(j) = 0$ . For all agents  $j = 1, \dots, i$ ,  $V(j-1) \rightarrow -\infty$  so all agents  $j = 1, \dots, i$  optimally choose  $q(j) = 1$ . Finally notice that if  $\pi(1 - \pi)^n - c \geq 0$ , there exists an equilibrium where all agents choose  $q(j) = 0$ , as  $V(j-1) \geq 0$  for all  $j \geq 2$  and  $V(1) \geq 0$ .

#### 1.4 Assignment with prior application

In the baseline model, we suppose that agents are given the opportunity to accept the object in sequence. This sequential assignment rule is time consuming as some agents choose to reject the object which is proposed to them. We consider an alternative assignment rule, where agents apply for the object after observing the value. The planner then chooses which applicant is assigned the object using a probabilistic priority mechanism  $p$ .

We consider a mechanism where, after learning his value, each agent announces  $a(i) \in \{0, 1\}$  where  $a(i) = 1$  means that the agent applies to the object. A random order  $\rho$  is drawn by the mechanism designer and the first agent in the order  $\rho$  who chooses  $a(i) = 1$  is assigned the object. Note that an agent with high value always applies to the mechanism and accepts the object in the sequential mechanism. An agent with low value applies in the mechanism with prior applications if and only if the expected value of participating is higher than the value of not participating.

The mechanism with prior application generates the same equilibrium values and continuation values for the two agents as the sequential mechanism. The top agent with low value chooses to participate if and only if

$$(1 - p)(\pi + (1 - \pi)q(2))V(1) \geq V(1)$$

or

$$V(1) \leq 0,$$

so that agent 1 participates when the value is low if and only if  $V(1) \leq 0$ , as in the sequential allocation model. The second agent with low value chooses to participate if and only if

$$p(\pi + (1 - \pi)q(1))V(1) \geq (\pi + (1 - \pi)q(1))V(1) + (1 - \pi)(1 - q(1))V(2)$$

or

$$(1-p)(\pi + (1-\pi)q(1))V(1) + (1-\pi)(1-q(1))V(2) \leq 0,$$

as in the sequential allocation model.

## 1.5 Information about the sequence

In the analysis, we suppose that agents know their position in the waiting list, but ignore the sequence in which offers are made. We now consider an alternative model where agents are told the sequence of offers. In the two-agent queue, this information only affects the decision of the second agent. For this agent, we distinguish between two states at which decisions must be made: state (2,1) when agent 2 knows that she is the first in the sequence of offers, and state (2,2) where agent 2 knows that she is the second in the sequence of offers. Notice that we do not need to distinguish between different continuation values at the two states, and will instead only compute the expected continuation value  $V(2)$  of the second agent before the offer sequence is drawn. The continuation value in state (2,1) is  $\pi V(1) + (1-\pi)V(2)$ , whereas the continuation value in state (2,2) is simply  $V(2)$ . As  $V(1) \geq V(2)$ , the continuation value of agent 2 is higher at state (2,1) than at state (2,2). The equilibrium in which agents 1 and 2 are selective in all circumstances results in continuation values

$$\begin{aligned} V^3(1) &= 1 - \frac{c}{\pi(1-\pi+p\pi)}, \\ V^3(2) &= 1 - \frac{2c}{\pi(2-\pi)}. \end{aligned}$$

As the behavior of the second agent in the two states are identical, the values are identical to the values in the baseline model when the sequence is not known. This equilibrium exists as long as  $V^3(2) - c \geq 0$  or  $c \leq \frac{\pi(2-\pi)}{2}$ . Next consider an equilibrium where agent 2 is selective in state (2,1) but not in state (2,2). The values are given by

$$\begin{aligned} V^2(1) &= 1 - \frac{c}{\pi(1-\pi+p\pi)} - c, \\ V^2(2) &= p[\pi V^2(1) + \pi(1-\pi)] + (1-p)[\pi + \pi(1-\pi)]V^2(1) \\ &\quad + (1-\pi)^2 V^2(2) - c, \\ &= \frac{\pi(2-\pi) - 2c}{1 - (1-p)(1-\pi)^2} \end{aligned}$$

This equilibrium exists if  $V^2(2) \leq 0$  and  $\pi V^2(1) + (1 - \pi)V^2(2) \geq 0$  or  $\pi(2 - \pi) < 2c$  and  $\pi - \frac{c}{1 - \pi + p\pi} + (1 - \pi)\frac{\pi(2 - \pi) - 2c}{1 - (1 - p)(1 - \pi)^2} \geq 0$ . Observe that both  $V^2(1)$  and  $V^2(2)$  are increasing in  $p$  in the range of parameters for which the equilibrium exists. Next, consider an equilibrium where agent 2 is never selective. This results in the values

$$\begin{aligned} V^1(1) &= 1 - \frac{c}{p\pi}, \\ V^1(2) &= \pi(1 + p - p\pi) - 2c, \end{aligned}$$

as in the baseline case. This equilibrium exists if  $\pi V^2(1) + (1 - \pi)V^2(2) \leq 0$  and  $V^1(1) - c \geq 0$  or  $c \geq \frac{p\pi + p(1 - \pi)\pi(1 + p(1 - \pi))}{1 + 2p(1 - \pi)}$  and  $c \leq p\pi$ . Finally, in an equilibrium where no agent is selective, values are given by

$$\begin{aligned} V^0(1) &= \pi - \frac{c}{p}, \\ V^0(2) &= \pi - 2c. \end{aligned}$$

As in the baseline case, and the equilibrium exists if and only if  $c \geq p\pi$ . We now compare the two regimes where agents are informed and not informed about the sequence. We check that the utility of both agents are increasing in the degree of selectivity of the equilibrium,  $V^3(1) = V^2(1) > V^1(1) > V^0(1)$  and  $V^3(2) > V^2(2) > V^1(2) > V^0(2)$ .

We also need to compare the parameter regions under which different equilibria exist. Notice that the parameter regions where the selective equilibrium (equilibrium 3) exists in the informed case is a subset of the parameter region under which the selective equilibrium (equilibrium 2) exists in the baseline case. However, the parameter region under which either equilibrium 2 or 3 exists is a superset of the region under which equilibrium 2 exists in the baseline case. Similarly, the parameter region under which equilibrium 1 exists in the informed case is a subset of the region under which the equilibrium exists in the baseline case, but the region under which either equilibrium 1 or 2 exists is a superset of the region under which equilibrium 1 exists in the baseline case. The parameter region where equilibrium 0 exists is identical in the two regimes. We illustrate these different regions when  $\pi = \frac{1}{2}$  in Figure 3.

Giving information about the sequence makes agent 2 more selective when she is first in the sequence but less selective when she is second. For agent 1, this always results in a positive effect on value, as it increases the parameter region for which agent 1 is selective when choosing first, increasing the opportunity that agent 1 gets to pick the object. The balance between the two effects on agent 2's expected

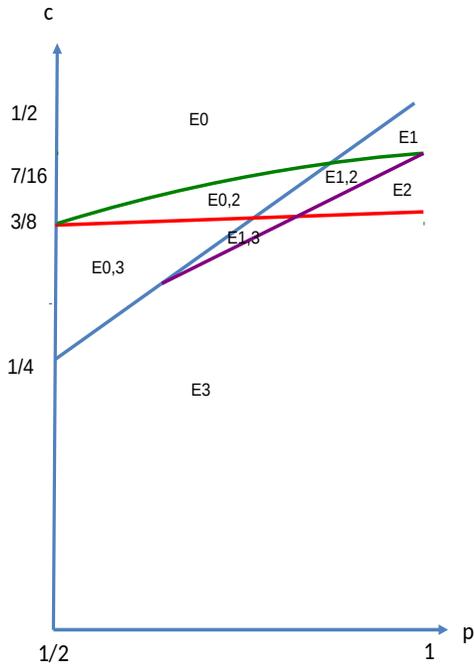


Figure 1: The two agent discrete model with information about the sequence

value is ambiguous. There are parameter regions for which the selective equilibrium exists in the baseline case but not in the informed regime, making agent 2 worse off, and parameter regions where equilibrium 2 exists in the informed regime but not in the baseline case, making agent 2 better off.

## 1.6 Eviction from the waiting list

We consider the effect of an eviction mechanism, where agents are taken away from the queue if they refuse an object with positive probability. Let  $\beta(1)$  and  $\beta(2)$  denote the probability that the first – respectively the second – agent remains in the queue if they refuse the object. In an equilibrium where both agents are selective, the equilibrium values are given by

$$\begin{aligned}
V^2(1) &= \frac{p\pi + (1-p)(1-\pi)\pi - c}{p\pi + (1-p)(1-\pi)\pi + (1-\beta(1))[p(1-\pi) + (1-p)(1-\pi)^2]}, \\
V^2(2) &= \frac{p\pi(1-\pi) + [p\pi + (1-p)\pi\beta(2)][V^2(1) - c] - c}{1 - \beta(2)(1-\pi)^2}.
\end{aligned}$$

Clearly,  $V^1(1)$  is increasing in  $\beta(1)$  and  $V^2(2)$  is increasing in  $\beta(1)$  and  $\beta(2)$ . Evicting the agents from the queue decreases their expected continuation values, making them less likely to be selective. In the equilibrium where only agent 1 is selective, the values become

$$\begin{aligned}
V^1(1) &= \frac{p\pi - c}{p\pi + (1-\beta(1))p(1-\pi)}, \\
V^1(2) &= p\pi(V^1(1) - c) + p\pi(1-\pi) + (1-p)\pi - c.
\end{aligned}$$

The values  $V^1(1)$  and  $V^1(2)$  are both increasing in  $\beta(1)$ . Finally, the values in the equilibrium where both agents accept both objects are clearly unaffected by the eviction probabilities. We thus observe that introducing eviction probabilities *reduces the values of the agents in the queue* and makes them less likely to be selective. As agents are less selective, the misallocation probability increases and the expected waste decreases. Hence, introducing eviction probabilities can only reduce the welfare of agents currently in the queue. It also accelerates the turnover in the queue, improving the well being of agents who are waiting to be included in the queue.