# A Structural Model of the Retail Market for Illicit Drugs by Manolis Galenianos and Alessandro Gavazza ONLINE APPENDICES 

## C Observable Drug Purity

In this Appendix, we prove the existence of equilibrium and derive its characterization when buyers observe drug quality before making a purchase. The analysis mirrors that of the baseline model.

## C. 1 The Buyers

We derive buyers' actions $(H(\cdot)$ and $B$ ) taking as given sellers' behavior $(F(\cdot)$ and $S)$.
A type- $z$ buyer takes as given other agents' actions and makes two decisions when meeting a new seller: whether to consume (reservation $\hat{R}_{z}$ ) and, if so, whether to match (reservation $R_{z}$ if unmatched and $q$ if matched with a $q$-seller). Notice that a buyer will never match with a seller whose product he does not want to consume. The value functions of a type- $z$ buyer satisfy:

$$
\begin{aligned}
r \bar{V}_{z} & =\alpha_{B}(\theta) \int_{0}^{\bar{q}} \max \left[z x-p+\max \left[V_{z}(x)-\bar{V}_{z}, 0\right], 0\right] d F(x), \\
r V_{z}(q) & =\gamma(z q-p)+\alpha_{B}(\theta) \int_{0}^{\bar{q}} \max \left[z x-p+\max \left[V_{z}(x)-V_{z}(q), 0\right], 0\right] d F(x)+\delta\left(\bar{V}_{z}-V_{z}(q)\right) .
\end{aligned}
$$

Comparing the static costs and benefits of consumption and equating the value functions deliver the reservation quality for becoming matched:

$$
R_{z}=\hat{R}_{z}=\frac{p}{z} .
$$

Thus, we can rewrite the value functions as follows:

$$
\begin{aligned}
r \bar{V}_{z} & =\alpha_{B}(\theta) \int_{\frac{p}{z}}^{\bar{q}}\left(z x-p+V_{z}(x)-\bar{V}_{z}\right) d F(x), \\
r V_{z}(q) & =\gamma(z q-p)+\alpha_{B}(\theta)\left(\int_{\frac{p}{z}}^{\bar{q}}(z x-p) d F(x)+\int_{q}^{\bar{q}}\left(V_{z}(x)-V_{z}(q)\right) d F(x)\right)+\delta\left(\bar{V}_{z}-V_{z}(q)\right) .
\end{aligned}
$$

Using integration by parts, the value of being unmatched satisfies:

$$
\begin{aligned}
r \bar{V}_{z} & =\alpha_{B}(\theta)\left(\left.\left(z q-p+V_{z}(q)-\bar{V}_{z}\right) F(q)\right|_{R_{z}} ^{\bar{q}}-\int_{R_{z}}^{\bar{q}}\left(z+V_{z}^{\prime}(x)\right) F(x) d x\right) \\
& =\alpha_{B}(\theta)\left(z\left(\bar{q}-R_{z}\right)+V_{z}(\bar{q})-V_{z}\left(R_{z}\right)-\int_{\frac{p}{z}}^{\bar{q}}\left(z+V_{z}^{\prime}(x)\right) F(x) d x\right),
\end{aligned}
$$

where we used $R_{z}=\frac{p}{z}$ and $\bar{V}_{z}=V_{z}\left(R_{z}\right)$ in the last equality. Using the fundamental theorem of calculus,
we obtain:

$$
\begin{aligned}
r \bar{V}_{z} & =\alpha_{B}(\theta)\left(\int_{\frac{p}{z}}^{\bar{q}}\left(z+V_{z}^{\prime}(x)\right) d x-\int_{\frac{p}{z}}^{\bar{q}}\left(z+V_{z}^{\prime}(x)\right) F(x) d x\right) \\
& =\alpha_{B}(\theta) \int_{\frac{p}{z}}^{\bar{q}}\left[z+V_{z}^{\prime}(q)\right](1-F(x)) d x .
\end{aligned}
$$

Differentiating the value of being matched with respect to $q$ and rearranging, we obtain:

$$
V_{z}^{\prime}(q)=\frac{\gamma z}{r+\delta+\alpha_{B}(\theta)(1-F(q))}
$$

Hence, combining the previous two equations, we obtain:

$$
r \bar{V}_{z}=z \alpha_{B}(\theta) \int_{\frac{p}{z}}^{\bar{q}}\left(1+\frac{\gamma}{r+\delta+\alpha_{B}(\theta)(1-F(x))}\right)(1-F(x)) d x .
$$

We now determine whether a buyer of type $z$ participates in the market, taking as given other agents' actions summarized by $\{F(\cdot), \theta\}$.

Notice that:

$$
\begin{aligned}
r \bar{V}_{0}= & -\alpha_{B}(\theta) p<0, \\
\frac{\partial r \bar{V}_{z}}{\partial z}= & \alpha_{B}(\theta) \int_{0}^{\bar{q}} x d F(x)+\alpha_{B}(\theta) \int_{R_{z}}^{\bar{q}} \frac{\gamma(1-F(x))}{r+\delta+\alpha_{B}(\theta)(1-F(x))} d x+ \\
& \frac{p}{z^{2}} \frac{\alpha_{B}(\theta) \gamma\left(1-F\left(R_{z}\right)\right)}{r+\delta+\alpha_{B}(\theta)\left(1-F\left(R_{z}\right)\right)}>0,
\end{aligned}
$$

which prove that there exists a $\tilde{z}(F, \theta)$ such that a buyer participates if and only if $z \geq \tilde{z}(F, \theta)$.
Aggregating the entry decision across buyers, we obtain:
Proposition 6 Given $F(\cdot)$ and $S$ :

1. If $\frac{p}{q} \geq \bar{z}$, then there is no buyer entry: $B=0$.
2. If $\frac{p}{\bar{q}}<\bar{z}$, then there is a unique buyer type $z^{* *} \leq \bar{z}$ such that all buyers with $z>z^{* *}$ participate in the market and all buyers with $z \leq z^{* *}$ do not.
3. The marginal buyer type is given by the solution to:

$$
\begin{equation*}
z^{* *} \alpha_{B}(\theta) \int_{\frac{p}{z^{* *}}}^{\bar{q}}\left(1+\frac{\gamma}{r+\delta+\alpha_{B}(\theta)(1-F(q))}\right)(1-F(q)) d q=K_{B} . \tag{C1}
\end{equation*}
$$

Proof. Buyers with $z \leq \frac{p}{\bar{q}}$ get no benefit from participating in the market and never enter. Consider a buyer with $z>\frac{p}{\bar{q}}$. Whether this buyer enters the market depends on how quickly he trades, which, in turn, depends on the buyer-seller ratio $\theta$. We have:

$$
\begin{aligned}
\lim _{\theta \rightarrow \infty} r \bar{V}_{z} & =0<K_{B}, \\
\lim _{\theta \rightarrow 0} r \bar{V}_{z} & =\lim _{\theta \rightarrow 0} \alpha_{B}(\theta) \int_{\frac{p}{z}}^{\bar{q}}(1-F(x)) d x>K_{B}, \\
\frac{\partial r \bar{V}_{z}}{\partial \theta} & =z \int_{\frac{p}{z}}^{\bar{q}}\left(\alpha_{B}^{\prime}(\theta)+\frac{\alpha_{B}^{\prime}(\theta)(r+\delta)}{\alpha_{B}(\theta)^{2}} \frac{\gamma}{\left(\frac{r+\delta}{\alpha_{B}(\theta)}+1-F(x)\right)^{2}}\right)(1-F(x)) d x<0 .
\end{aligned}
$$

Therefore, for each buyer of type $z$ with $z>\frac{p}{\bar{q}}$, there is a unique $\theta(z)$ such that he participates if $\theta \leq \theta(z)$ and stays out otherwise. Hence, the measure of buyers in the market is:

$$
B=\bar{B}(1-\bar{M}(z(\theta))) .
$$

Given $S$, this leads to

$$
\theta(z(\theta))=\frac{\bar{B}(1-\bar{M}(z(\theta)))}{S}
$$

We now show that, given $S$ and $F(\cdot)$, there is a unique $z^{* *}$ such that $z^{* *}=z\left(\theta\left(z^{* *}\right)\right)$. We show that as $z^{* *}$ increases, the participation value of the marginal type increases after taking into account the effect on $\theta$ :

$$
\frac{d r \bar{V}_{z^{* *}}}{d z^{* *}}=\frac{\partial r \bar{V}_{z^{* *}}}{\partial z^{* *}}+\frac{\partial r \bar{V}_{z^{* *}}}{\partial \theta}\left(-\bar{B} \bar{M}^{\prime}\left(z^{* *}\right)\right)>0
$$

Therefore, there is a unique $z^{* *}$ such that the unmatched value of the marginal buyer equals $K_{B}$, as in equation (C1).

Corollary 3 The number of buyers who participate in the market is $B=\bar{B}\left(1-\bar{M}\left(z^{* *}\right)\right)$.
As in the baseline model, the distribution of reservation qualities obtains:

$$
H(R)= \begin{cases}0 & \text { if } R \leq \underline{R} \\ \frac{1-\bar{M}\left(\frac{p}{R}\right)}{1-\bar{M}\left(z^{* *}\right)} & \text { if } R \in[\underline{R}, \bar{R}] \\ 1 & \text { if } R \geq \bar{R}\end{cases}
$$

where $\underline{R}=R_{z}=\frac{p}{z}$ and $\bar{R}=R_{z^{* *}}=\frac{p}{z^{* *}}$.

## C. 2 The Sellers

We derive sellers' actions, taking as given the measure of buyers who participate $B$ and the distribution of reservation qualities $H(\cdot)$.

Free entry determines the measure $S$ and type distribution $D(\cdot)$ of sellers in the market, subject to flow participation cost $K_{S}$. The problem of a seller of type $c$ is to choose a level of quality $q^{* *}(c)$ that maximizes her steady-state profits, which depend on the margin per transaction $(p-c q)$ and the steady-state flow of transactions $(t(q))$ :

$$
\pi_{c}(q)=(p-c q) t(q) .
$$

We first derive some necessary conditions on the distribution of offered qualities:
Lemma 2 In equilibrium, the quality distribution $F$ has support on a subset of $[\underline{q}, \bar{q}]$, where $\underline{q} \in[\underline{R}, \bar{R}]$, and is continuous on $[0, \bar{q}]$.
Proof. For $q \in[0, \underline{R})$, we have $t(q)=0$ and, therefore, $q \geq \underline{q}$ for some $\underline{q} \geq \underline{R}$. If $\underline{q}>\bar{R}$, then $t(q)=t(\bar{R})$ for $q \in[\bar{R}, \underline{q}]$, which implies that $\pi_{c}(\bar{R})>\pi_{c}(q)$ for $q \in(\bar{R}, \underline{q}]$. Therefore, $\underline{q} \leq \bar{R}$. The previous point proves that $F$ is constant (and, hence, continuous) on $[0, q]$. Standard arguments (as in Burdett and Mortensen, 1998) prove continuity on $[\underline{q}, \bar{q}]$.

We take $H(\cdot), F(\cdot)$ and $\theta$ as given and calculate the steady-state profits that a type-c seller would enjoy for any quality $q$. The main result is summarized in the next proposition.

Proposition 7 The steady-state profits of a seller of type $c$ who offers quality $q$ are:

$$
\pi_{c}(q)=\alpha_{B}(\theta) \theta H(q)(p-c q)\left(1+\frac{\gamma \delta}{\left(\delta+\alpha_{B}(\theta)(1-F(q))\right)^{2}}\right), \quad q \geq \underline{q} .
$$

Proof. To determine profits, we need to first determine the flow of a seller's transactions as a function of the quality that she offers. The rate at which an individual seller transacts with a new buyer equals the meeting rate times the probability that the seller's quality is above the buyer's reservation:

$$
t_{N}(q)=\alpha_{S}(\theta) H(q)=\theta \alpha_{B}(\theta) H(q) .
$$

The flow of transactions from regular buyers is:

$$
t_{R}(q)=\gamma l(q),
$$

where $l(q)$ is the steady-steady number of regular buyers of a seller offering $q$. The number of regular buyers per seller offering $q$ is:

$$
l(q)=\frac{(B-\bar{n}) G^{\prime}(q)}{S F^{\prime}(q)}
$$

where $\bar{n}$ is the number of unmatched buyers; $(B-\bar{n}) G^{\prime}(q)$ is the number of buyers who are matched with a seller offering $q$; and $S F^{\prime}(q)$ is the number of sellers offering quality $q$.

We now determine the number of unmatched buyers and their type distribution. In steady state, the flow of buyers from the unmatched to the matched state must equal the flow out of the matched state and into the unmatched state. Let $n(R)$ denote the number of buyers who are unmatched and whose type is less than $R$. The total number of unmatched buyers is, thus, given by $n(\bar{R}) \equiv \bar{n}$.

An unmatched buyer of type $R$ becomes matched after transacting with a seller who offers abovereservation quality, which occurs at rate $\alpha_{B}(\theta)(1-F(R))$. A matched buyer exits the matched state when his match is exogenously destroyed, which occurs at rate $\delta$. As a result, in steady state, the following holds:

$$
n^{\prime}(R) \alpha_{B}(\theta)(1-(F(R)))=\delta\left(B H^{\prime}(R)-n^{\prime}(R)\right) \Rightarrow n^{\prime}(R)=\frac{\delta B H^{\prime}(R)}{\delta+\alpha_{B}(\theta)(1-F(R))},
$$

which we can rewrite as:

$$
n(R)=\int_{\underline{R}}^{R} \frac{B \delta}{\delta+\alpha_{B}(\theta)(1-F(x))} d H(x) .
$$

Therefore, we have:

$$
\begin{aligned}
\bar{n} & =\int_{\underline{R}}^{\bar{R}} \frac{B \delta}{\delta+\alpha_{B}(\theta)(1-F(x))} d H(x), \\
B-\bar{n} & =B\left(1-\int_{\underline{R}}^{\bar{R}} \frac{\delta}{\delta+\alpha_{B}(\theta)(1-F(x))} d H(x)\right)=\int_{\underline{R}}^{\bar{R}} \frac{B \alpha_{B}(\theta)(1-F(x))}{\delta+\alpha_{B}(\theta)(1-F(x))} d H(x) .
\end{aligned}
$$

We now characterize $G(\cdot)$. The mass of matched buyers receiving quality up to $q$ is $(B-\bar{n}) G(q)$. An unmatched type- $R$ buyer flows into this group if $R \leq q$ and he samples a seller who offers quality less than $q$, which occurs at rate $\alpha_{B}(\theta)(F(q)-F(R))$. A buyer flows out of this group if the match is exogenously destroyed or if he samples a new seller whose quality if greater than $q$, which occurs at rate
$\delta+\alpha_{B}(\theta)(1-F(q))$. Equating these flows yields:

$$
\begin{aligned}
& \quad \alpha_{B}(\theta) \int_{\underline{R}}^{q}(F(q)-F(R)) d n(R)=(B-\bar{n}) G(q)\left(\delta+\alpha_{B}(\theta)(1-F(q))\right), \\
& \Rightarrow(B-\bar{n}) G(q) \frac{\alpha_{B}(\theta) B \delta \int_{\underline{R}}^{q} \frac{F(q)-F(x)}{\delta+\alpha_{B}(\theta)(1-F(x))} d H(x)}{\delta+\alpha_{B}(\theta)(1-F(q))} .
\end{aligned}
$$

Rearranging, we obtain:

$$
(B-\bar{n}) G^{\prime}(q)=\frac{\alpha_{B}(\theta) B \delta F^{\prime}(q) H(q)}{\left(\delta+\alpha_{B}(\theta)(1-F(q))\right)^{2}},
$$

which implies that the flow of transactions from regular buyers is:

$$
t_{R}(q)=\frac{\gamma \alpha_{B}(\theta) \theta \delta H(q)}{\left(\delta+\alpha_{B}(\theta)(1-F(q))\right)^{2}} .
$$

Combining results completes the proof of the Proposition.
We characterize the distribution of offered qualities, $F(\cdot)$ and the number of sellers who enter the market, taking as given the number of buyers $B$ and the distribution of their reservation values $H(\cdot)$.

Lemma 3 Consider sellers 1 and 2 with costs $c_{1}$ and $c_{2}$, and denote their actions by $q_{1}$ and $q_{2}$ and their profits by $\pi_{1}$ and $\pi_{2}$. Then: $c_{1}>c_{2} \Rightarrow q_{2}>q_{1}$ and $c_{1}>c_{2} \Rightarrow \pi_{2}>\pi_{1}$.

Proof. The proof for qualities is by contradiction. Suppose that $c_{1}>c_{2}$ and $q_{2} \leq q_{1}$. Recall that profits are given by $\pi_{c}(q)=(p-c q) t(q)$. Seller 1 chooses $q_{1}$ over $q_{2}$ and seller 2 chooses $q_{2}$ over $q_{1}$. Therefore:

$$
\begin{aligned}
\left(p-c_{1} q_{1}\right) t\left(q_{1}\right) \geq\left(p-c_{1} q_{2}\right) t\left(q_{2}\right) & \Rightarrow p\left(t\left(q_{1}\right)-t\left(q_{2}\right)\right) \geq c_{1}\left(t\left(q_{1}\right) q_{1}-t\left(q_{2}\right) q_{2}\right) \\
\left(p-c_{2} q_{2}\right) t\left(q_{2}\right) \geq\left(p-c_{2} q_{1}\right) t\left(q_{1}\right) & \Rightarrow p\left(t\left(q_{1}\right)-t\left(q_{2}\right)\right) \leq c_{2}\left(t\left(q_{1}\right) q_{1}-t\left(q_{2}\right) q_{2}\right) .
\end{aligned}
$$

which yields the desired contradiction, after noting that $c_{1}>c_{2}$.
Regarding profits, note that:

$$
\pi_{1}=\left(p-c_{1} q_{1} t\left(q_{1}\right)\right)<\left(p-c_{2} q_{1}\right) t\left(q_{1}\right) \leq\left(p-c_{2} q_{2}\right) t\left(q_{2}\right)=\pi_{2}
$$

which completes the proof.
We characterize the marginal seller type $c^{* *}$ and the lowest quality that is offered, $\underline{q}$ (we know from the previous Lemma that $\underline{q}$ is offered by the $c^{* *}$-seller). Two conditions need to be satisfied: first, $\underline{q}$ must give higher profits to $c^{* *}$ than any other quality level; second, $\underline{q}$ must cover the seller's flow cost. The proposition summarizes the result.

Proposition 8 Given $B$ and $H(\cdot)$, there is a unique marginal seller type $c^{* *}$ such that sellers with $c \leq c^{* *}$ participate in the market and sellers with $c>c^{* *}$ do not. The marginal seller type is determined by the solution to:

$$
\begin{equation*}
\alpha_{B}(\theta) \theta H\left(\underline{q}\left(c^{* *}\right)\right)\left(p-c \underline{q}\left(c^{* *}\right)\right)\left(1+\frac{\gamma \delta}{\left.\left(\delta+\alpha_{B}(\theta)\right)\right)^{2}}\right)=K_{S}, \tag{C2}
\end{equation*}
$$

where $\underline{q}(c)$ solves

$$
\begin{equation*}
c H(\underline{q})=H^{\prime}(\underline{q})(p-c \underline{q}) . \tag{C3}
\end{equation*}
$$

Proof. It is immediate from Lemma 3 that a marginal type exists; she offers the lowest quality level and her profits are equal to the entry cost $K_{S}$. The profits of a type- $c$ seller who offers the lowest quality are:

$$
\underline{\pi}_{c}(q)=\alpha_{S}(\theta) H(q)(p-c q)\left(1+\frac{\gamma \delta}{\left.\left(\delta+\alpha_{B}(\theta)\right)\right)^{2}}\right)
$$

The optimal choice of the lowest quality is given by the root of:

$$
\underline{\pi}_{c}^{\prime}(q)=\alpha_{S}(\theta)\left(1+\frac{\gamma \delta}{\left.\left(\delta+\alpha_{B}(\theta)\right)\right)^{2}}\right)\left(H^{\prime}(q)(p-c q)-c H(q)\right) .
$$

The second derivative of profits is always negative due to the log-concavity of $H(\cdot)$ :

$$
\underline{\pi}_{c}^{\prime \prime}(q)=\alpha_{S}(\theta)\left(1+\frac{\gamma \delta}{\left.\left(\delta+\alpha_{B}(\theta)\right)\right)^{2}}\right)\left(H^{\prime \prime}(q)(p-c q)-2 c H(q)-c H^{\prime}(q)\right)<0 .
$$

Therefore, the optimal quality choice for a type-c seller who offers the lowest quality $\underline{q}(c)$ is given by equation (C3).

The marginal seller type determines the buyer-seller ratio according to $\theta=\frac{B}{S \bar{D}\left(c^{* *}\right)}$. The profits of the seller who offers $\underline{q}(c)$ are decreasing in her cost $c$ :

$$
\frac{d \underline{\pi}_{c}(\underline{q}(c))}{d c}=\frac{\underline{\pi}_{c}(\underline{q})}{d \underline{q}} \underline{q}^{\prime}(c)+\frac{\partial \underline{\pi}_{c}(\underline{q}(c))}{\partial c}+\frac{\partial \underline{\pi}_{c}(\underline{q}(c))}{\partial \theta} \frac{d \theta}{d c}<0 .
$$

It is also immediate that:

$$
\begin{aligned}
& \lim _{c \rightarrow 0} \underline{\pi}(\underline{q}(c))>K_{S}, \\
& \lim _{c \rightarrow \infty} \frac{\pi(q(c))}{}<K_{S} .
\end{aligned}
$$

Therefore, there is a unique marginal seller type $c^{* *}$ who is determined by equation (C2).

Corollary 4 The number of sellers who participate in the market is $S=\bar{S} \bar{D}\left(c^{* *}\right)$. The distribution of their types is $D(c)=\frac{\bar{D}(c)}{\bar{D}\left(c^{* *}\right)}$.

We now determine sellers' optimal $q^{* *}(c)$ for $c<c^{* *}$.
Proposition 9 Given $H(\cdot)$ and $B$, the optimal quality choice for sellers of type $c$ is given by the solution to the differential equation

$$
\begin{equation*}
q^{* *} \prime(c)=\frac{2 H\left(q^{* *}(c)\right)\left(p-c q^{* *}(c)\right) \gamma \delta \alpha_{B}(\theta) D^{\prime}(c)}{\left[H^{\prime}\left(q^{* *}(c)\right)\left(p-c q^{* *}(c)\right)-c H\left(q^{* *}(c)\right)\right]\left(\delta+\alpha_{B}(\theta) D(c)\right)\left[\left(\delta+\alpha_{B}(\theta) D(c)\right)^{2}+\gamma \delta\right]}, \tag{C4}
\end{equation*}
$$

where the initial condition $q^{* *}\left(c^{* *}\right)=\underline{q}\left(c^{* *}\right)$.
The distribution of qualities is:

$$
F(q)=1-D\left(q^{* *-1}(q)\right) .
$$

Proof. To characterize the optimal quality offer $q^{* *}(c)$ for a type- $c$ seller with $c<c^{* *}$, we rewrite her profits as if she decides which other type $c^{\prime}$ to imitate rather than which quality to offer. In other words,
her profits from offering some quality $q^{\prime}$ are written in terms of imitating type $c^{\prime}$, who offers quality $q^{\prime}=q^{* *}\left(c^{\prime}\right)$. We have:

$$
\pi_{c}\left(c^{\prime}\right)=\alpha_{S}(\theta) H\left(q^{* *}\left(c^{\prime}\right)\right)\left(p-c q^{* *}\left(c^{\prime}\right)\right)\left(1+\frac{\gamma \delta}{\left(\delta+\alpha_{B}(\theta) D\left(c^{\prime}\right)\right)^{2}}\right)
$$

The advantage of formulating the choice in terms of $c^{\prime}$ rather on than $q^{\prime}$ is that the term in the denominator depends on the exogenous type distribution $D(\cdot)$ rather than on the endogenous quality distribution $F(\cdot)$. We can recover the quality distribution once we construct $q^{* *}(c)$.

We differentiate profits with respect to $c^{\prime}$ :

$$
\begin{aligned}
\pi_{c}^{\prime}\left(c^{\prime}\right)= & \alpha_{S}(\theta)\left(H^{\prime}\left(q^{* *}\left(c^{\prime}\right)\right) q^{* *} \prime\left(c^{\prime}\right)\left(p-c q^{* *}\left(c^{\prime}\right)\right)\left(1+\frac{\gamma \delta}{\left(\delta+\alpha_{B}(\theta) D\left(c^{\prime}\right)\right)^{2}}\right)\right. \\
& -\frac{c q^{* *}\left(c^{\prime}\right) H\left(q^{* *}\left(c^{\prime}\right)\right) \gamma \delta}{\left(\delta+\alpha_{B}(\theta) D\left(c^{\prime}\right)\right)^{2}}-H\left(q^{* *}\left(c^{\prime}\right)\right)\left(p-c q^{* *}\left(c^{\prime}\right) \frac{\gamma \delta 2\left(\delta+\alpha_{B}(\theta) D\left(c^{\prime}\right)\right) D^{\prime}\left(c^{\prime}\right)}{\left(\delta+\alpha_{B}(\theta) D\left(c^{\prime}\right)\right)^{3}}\right) .
\end{aligned}
$$

Equating the derivative to zero and setting $c^{\prime}=c$ leads to equation (C4), which, together with the initial condition $q^{* *}\left(c^{* *}\right)=\underline{q}\left(c^{* *}\right)$, defines $q^{* *}(c)$ for $c \in\left(0, c^{* *}\right)$.

## C. 3 Equilibrium

The proof of equilibrium existence is identical to that of the baseline model and is, therefore, omitted.

Table D1: The Effect of Penalties, Higher Meeting Rate and Lower Destruction Rate

|  | Baseline | Higher $\gamma$ | Lower $\delta$ |
| :--- | :---: | :---: | :---: |
| Fraction of Rip-offs (\%) | 15.862 | 1.357 | 0.674 |
|  | $[13.646 ; 16.812]$ | $[1.273 ; 1.441]$ | $[0.478 ; 0.770]$ |
| Average Pure Grams Per $\$ 100$ | 0.616 | 0.943 | 1.070 |
|  | $[0.597 ; 0.636]$ | $[0.933 ; 0.955]$ | $[1.051 ; 1.095]$ |
| St. Dev. Pure Grams Per $\$ 100$ | 0.271 | 1.134 | 0.858 |
|  | $[0.256 ; 0.279]$ | $[1.103 ; 1.164]$ | $[0.737 ; 0.902]$ |
| Active Buyers, in Millions | 3.431 | 1.043 | 1.083 |
|  | $[3.312 ; 3.530]$ | $[1.024 ; 1.053]$ | $[1.046 ; 1.111]$ |
| Active Sellers, in Millions | 0.290 | 1.043 | 1.083 |
|  | $[0.271 ; 0.295]$ | $[1.024 ; 1.053]$ | $[1.046 ; 1.111]$ |
| Fraction of Matched Buyers (\%) | 54.040 | 1.056 | 1.054 |
|  | $[52.420 ; 55.100]$ | $[1.029 ; 1.103]$ | $[0.997 ; 1.080]$ |
| Average Number of Purchases Per Month | 12.726 | 1.046 | 1.227 |
|  | $[12.228 ; 13.389]$ | $[1.026 ; 1.076]$ | $[1.176 ; 1.241]$ |
| Average Pure Grams Consumed per Month | 9.464 | 1.056 | 1.242 |
|  | $[9.057 ; 9.990]$ | $[1.034 ; 1.086]$ | $[1.192 ; 1.260]$ |

Notes-This table reports market outcomes in two counterfactual cases: 1) buyers' meeting rate with regular dealers $\gamma$ is 20 -percent higher than in the baseline case; and 2) the destruction rate $\delta$ is 20 percent lower than in the baseline case, expressed as ratios over the corresponding values in the baseline case. 95-percent confidence intervals in brackets.

## D Additional Counterfactuals and Sensitivity Analyses

We perform the following additional counterfactuals, with results displayed in Table D1: 1) a $20-$ percent increase in $\gamma$; and 2) a 20-percent decrease in $\delta$. As in Section 5.4, we report quantitative results for the population (i.e., without sample selection) as ratios of the corresponding values in the baseline case (i.e., a ratio larger than one implies an increase relative to the baseline case).

To understand the results of Table D1, notice that an increase in $\gamma$ increases buyers' frequency of consumption which, in turn, increases their value of participating in the market. This leads to an increase in the entry of buyers, similarly to the counterfactual with lower $K_{B}$ (of course, sellers' incentives are also affected, but this turns out to be of second-order importance). As in that counterfactual, more buyers enter the market, rip-offs increase and quality dispersion increases. Since sellers' free entry condition, equation (A2), does not change, the buyer-seller ratio exactly equals that of the baseline case. Therefore, a larger number of sellers participate in the market than in the baseline case, as well.

A reduction in $\delta$ increases the value of a long-term relationship, as it leads to less frequent exogenous dissolution. This change increases the attractiveness for sellers of acquiring regular customers and, as a result, more sellers offer positive quality, rip-offs decline, average quality increases and quality dispersion drops. The reduction in rip-offs and exogenous match dissolution leads to an increase in the proportion of buyers who are matched, thereby increasing buyers' value of participating in the market and, thus, their numbers. Since again sellers' free entry condition, equation (A2), does not change, the buyer-seller ratio exactly equals that of the baseline case. Therefore, a larger number of sellers participate in the market than in the baseline case, as well.

Table D2: Observable Quality: Urn-Ball Matching Function

|  | Baseline | ObSERVABLE $q$, | ObSERVABLE $q$, <br> Partial Eq. |
| :--- | :---: | :---: | :---: |
| General Eq. |  |  |  |

Notes-This table reports market outcomes in the counterfactual cases in which buyers can observe drugs' purity before purchasing, expressed as ratios over the corresponding values in the baseline case. 95 -percent confidence intervals in brackets.

Moreover, we perform several sensitivity analyses to the counterfactuals of Section 5.4. Overall, these additional analyses deliver results that are very similar results to those reported in Section 5.4, thus showing their robustness. More specifically:

- We perform our counterfactuals on observable quality and on penalties using alternative matching functions. The counterfactuals of Section 5.4 impose a Cobb-Douglas matching function. Tables D2 and D3 display the results obtained with the urn-ball matching function between $B$ buyers and $S$ sellers: $M(B, S)=\omega_{1} S\left(1-e^{-\omega_{2} \frac{B}{S}}\right)$, where we set the parameters $\omega_{1}$ and $\omega_{2}$ to match the estimate of $\alpha_{B}$ reported in Table 2 and the average number of transactions per seller reported in Section 5.3. Tables D4 and D5 display the results obtained with the telegraph line: $M(B, S)=$ $\frac{\omega_{3} B S}{\omega_{4} B+S}$, where we set the parameters $\omega_{3}$ and $\omega_{4}$ to match the estimate of $\alpha_{B}$ reported in Table 2 and the average number of transactions per seller reported in Section 5.3. Note that these alternative matching functions have obviously no effect in the observable- $q$, partial-equilibrium case, as the the numbers of buyers and of sellers are fixed at their baseline values.
- We perform our counterfactuals on penalties using alternative values of sellers' and buyers' fixed costs. Tables D6-D7 display the results in the cases in which we decrease sellers' and buyers' costs by 10 percent and 20 percent, respectively.

Table D3: The Effect of Penalties: Urn-Ball Matching Function

|  | BaSELINE | LOWER $K_{S}$ | LOWER $K_{B}$ |
| :--- | :---: | :---: | :---: |
| Fraction of Rip-offs $(\%)$ | 15.862 | 2.481 | 1.162 |
|  | $[13.646 ; 16.812]$ | $[2.374 ; 2.841]$ | $[1.080 ; 1.209]$ |
| Average Pure Grams Per $\$ 100$ | 0.616 | 0.712 | 0.972 |
|  | $[0.597 ; 0.636]$ | $[0.684 ; 0.726]$ | $[0.950 ; 0.983]$ |
| St. Dev. Pure Grams Per $\$ 100$ | 0.271 | 1.311 | 1.062 |
|  | $[0.256 ; 0.279]$ | $[1.282 ; 1.377]$ | $[1.029 ; 1.074]$ |
| Active Buyers, in Millions | 3.431 | 0.867 | 1.036 |
|  | $[3.312 ; 3.530]$ | $[0.850 ; 0.894]$ | $[1.008 ; 1.042]$ |
| ACtive Sellers, in Millions | 0.290 | 1.565 | 1.036 |
|  | $[0.271 ; 0.295]$ | $[1.552 ; 1.636]$ | $[1.008 ; 1.042]$ |
| Fraction of Matched Buyers (\%) | 54.040 | 1.001 | 0.996 |
|  | $[52.420 ; 55.100]$ | $[0.971 ; 1.051]$ | $[0.955 ; 1.008]$ |
| Average Number of Purchases Per Month | 12.726 | 1.077 | 0.991 |
|  | $[12.228 ; 13.389]$ | $[1.058 ; 1.125]$ | $[0.971 ; 1.015]$ |
| Average Pure Grams Consumed Per Month | 9.464 | 1.022 | 0.988 |
|  | $[9.057 ; 9.990]$ | $[1.005 ; 1.072]$ | $[0.969 ; 1.016]$ |

Notes-This table reports market outcomes in the counterfactual cases in which buyers' cost $K_{B}$ and sellers' cost $K_{S}$ are 15-percent lower than in the baseline case, respectively, expressed as ratios over the corresponding values in the baseline case. 95-percent confidence intervals in brackets.

Table D4: Observable Quality: Telegraph-Line Matching Function

|  | Baseline | ObSERVABLE $q$, <br> Partial Eq. | ObSERVABLE $q$, <br> GENERAL EQ. |
| :--- | :---: | :---: | :---: |
| Fraction of RIP-OFFS (\%) | 15.862 | 0.000 | 0.000 |
|  | $[13.646 ; 16.812]$ | $[0.000 ; 0.000]$ | $[0.000 ; 0.000]$ |
| Average Pure Grams Per $\$ 100$ | 0.616 | 1.204 | 1.197 |
|  | $[0.597 ; 0.636]$ | $[1.160 ; 1.261]$ | $[1.165 ; 1.216]$ |
| St. Dev. Pure Grams Per $\$ 100$ | 0.271 | 0.184 | 0.170 |
|  | $[0.256 ; 0.279]$ | $[0.156 ; 0.287]$ | $[0.142 ; 0.180]$ |
| Active Buyers, In Millions | 3.431 | 1.000 | 1.057 |
|  | $[3.312 ; 3.530]$ | $[1.000 ; 1.000]$ | $[1.037 ; 1.074]$ |
| Active Sellers, in Millions | 0.290 | 1.000 | 0.822 |
|  | $[0.271 ; 0.295]$ | $[1.000 ; 1.000]$ | $[0.807 ; 0.843]$ |
| Fraction of Matched Buyers (\%) | 54.040 | 1.054 | 0.997 |
|  | $[52.420 ; 55.100]$ | $[1.024 ; 1.109]$ | $[0.955 ; 1.037]$ |
| Average Number of Purchases per Month | 12.726 | 1.053 | 0.981 |
|  | $[12.228 ; 13.389]$ | $[1.040 ; 1.086]$ | $[0.960 ; 1.005]$ |
| Average Pure Grams Consumed per Month | 9.464 | 1.082 | 1.004 |

Notes-This table reports market outcomes in the counterfactual cases in which buyers can observe drugs' purity before purchasing, expressed as ratios over the corresponding values in the baseline case. 95 -percent confidence intervals in brackets.

Table D5: The Effect of Penalties: Telegraph-Line Matching Function

|  | BaSELINE | LOWER $K_{S}$ | LOWER $K_{B}$ |
| :--- | :---: | :---: | :---: |
| Fraction of Rip-offs $(\%)$ | 15.862 | 2.053 | 1.162 |
|  | $[13.646 ; 16.812]$ | $[1.956 ; 2.281]$ | $[1.082 ; 1.211]$ |
| Average Pure Grams PEr $\$ 100$ | 0.616 | 0.798 | 0.972 |
|  | $[0.597 ; 0.636]$ | $[0.777 ; 0.806]$ | $[0.948 ; 0.982]$ |
| St. Dev. Pure Grams Per $\$ 100$ | 0.271 | 1.270 | 1.062 |
|  | $[0.256 ; 0.279]$ | $[1.235 ; 1.318]$ | $[1.028 ; 1.076]$ |
| ACtive Buyers, in Millions | 3.431 | 0.915 | 1.036 |
|  | $[3.312 ; 3.530]$ | $[0.892 ; 0.934]$ | $[1.002 ; 1.040]$ |
| ACtive Sellers, in Millions | 0.290 | 1.415 | 1.036 |
|  | $[0.271 ; 0.295]$ | $[1.392 ; 1.453]$ | $[1.002 ; 1.040]$ |
| Fraction of Matched Buyers (\%) | 54.040 | 0.986 | 0.996 |
|  | $[52.420 ; 55.100]$ | $[0.931 ; 1.037]$ | $[0.939 ; 1.016]$ |
| Average Number of Purchases Per Month | 12.726 | 1.057 | 0.991 |
|  | $[12.228 ; 13.389]$ | $[1.013 ; 1.081]$ | $[0.965 ; 1.012]$ |
| Average Pure Grams Consumed Per Month | 9.464 | 1.025 | 0.988 |
|  | $[9.057 ; 9.990]$ | $[0.981 ; 1.047]$ | $[0.961 ; 1.014]$ |

Notes-This table reports market outcomes in the counterfactual cases in which buyers' cost $K_{B}$ and sellers' cost $K_{S}$ are 15-percent lower than in the baseline case, respectively, expressed as ratios over the corresponding values in the baseline case. 95-percent confidence intervals in brackets.

Table D6: The Effect of Penalties, Alternative Costs I

|  | Baseline | LOWER $K_{S}$ | LOWER $K_{B}$ |
| :--- | :---: | :---: | :---: |
| Fraction of Rip-OFFs (\%) | 15.862 | 1.445 | 1.109 |
|  | $[13.646 ; 16.812]$ | $[1.399 ; 1.545]$ | $[1.058 ; 1.141]$ |
| Average Pure Grams Per $\$ 100$ | 0.616 | 0.906 | 0.980 |
|  | $[0.597 ; 0.636]$ | $[0.898 ; 0.916]$ | $[0.963 ; 0.988]$ |
| St. Dev. Pure Grams Per $\$ 100$ | 0.271 | 1.134 | 1.041 |
|  | $[0.256 ; 0.279]$ | $[1.125 ; 1.173]$ | $[1.017 ; 1.056]$ |
| Active Buyers, In Millions | 3.431 | 0.956 | 1.024 |
|  | $[3.312 ; 3.530]$ | $[0.954 ; 0.972]$ | $[1.000 ; 1.029]$ |
| Active Sellers, in Millions | 0.290 | 1.180 | 1.024 |
|  | $[0.271 ; 0.295]$ | $[1.178 ; 1.200]$ | $[1.000 ; 1.029]$ |
| Fraction of Matched Buyers (\%) | 54.040 | 0.991 | 0.979 |
|  | $[52.420 ; 55.100]$ | $[0.967 ; 1.026]$ | $[0.959 ; 1.027]$ |
| Average Number of Purchases per Month | 12.726 | 1.011 | 0.988 |
|  | $[12.228 ; 13.389]$ | $[0.988 ; 1.036]$ | $[0.966 ; 1.019]$ |
| Average Pure Grams Consumed per Month | 9.464 | 0.995 | 0.987 |

Notes-This table reports market outcomes in the counterfactual cases in which buyers' cost $K_{B}$ and sellers' cost $K_{S}$ are 10-percent lower than in the baseline case, respectively, expressed as ratios over the corresponding values in the baseline case. 95-percent confidence intervals in brackets.

Table D7: The Effect of Penalties, Alternative Costs II

|  | BASELINE | LOWER $K_{S}$ | Lower $K_{B}$ |
| :---: | :---: | :---: | :---: |
| Fraction of Rip-offs (\%) | 15.862 | 1.890 | 1.211 |
|  | [13.646; 16.812] | [1.791; 2.073] | [1.107; 1.276] |
| Average Pure Grams per $\$ 100$ | 0.616 | 0.823 | 0.962 |
|  | [0.597; 0.636] | [0.808; 0.838 ] | [0.953; 0.977] |
| St. Dev. Pure Grams per $\$ 100$ | 0.271 | 1.233 | 1.078 |
|  | [0.256; 0.279] | [1.202; 1.299] | [1.038; 1.101] |
| Active Buyers, in Millions | 3.431 | 0.916 | 1.047 |
|  | [3.312; 3.530] | [0.902; 0.943] | [1.024; 1.059] |
| Active Sellers, in Millions | 0.290 | 1.432 | 1.047 |
|  | [0.271;0.295] | [1.409; 1.474] | [1.024; 1.059] |
| Fraction of Matched Buyers (\%) | 54.040 | 0.994 | 0.980 |
|  | [52.420; 55.100] | [0.955; 1.043] | [0.952; 1.005] |
| Average Number of Purchases per Month | 12.726 | 1.043 | 0.986 |
|  | [12.228; 13.389] | [1.014; 1.060] | [0.960; 1.012] |
| Average Pure Grams Consumed per Month | 9.464 | 1.012 | 0.984 |
|  | [9.057; 9.990] | [0.984; 1.031] | [0.956; 1.009] |

Notes-This table reports market outcomes in the counterfactual cases in which buyers' cost $K_{B}$ and sellers' cost $K_{S}$ are 20-percent lower than in the baseline case, respectively, expressed as ratios over the corresponding values in the baseline case. 95-percent confidence intervals in brackets.

