Trade, neoclassical growth and heterogeneous firms*

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— still preliminary —

21st December 2007

Abstract

Existing trade models with heterogeneous firms successfully explain two empirical facts: exposure to trade leads, first, to a firm selection and, second, to an increase in industry–wide productivity. However, none of these models explains the following three additional empirical facts: first, exporting firms are more capital intensive than non–exporting firms. Second, the exit rate immediately with exposure to trade is higher for exporting firms than for non–exporting firms; nevertheless, exporting firms increase their *aggregate* production in the long–run. Third, the growth effect of exposure to trade is rather different between countries. This paper extends the model by Melitz (2003) by two factors of production and a Ramsey growth model and defines firm heterogeneity with respect to the factor input shares in production. This extension explains all five empirical facts.

JEL classification: F12, L11
Keywords: Intra–industry trade, two–factor model, firm heterogeneity in factor shares, neoclassical growth model

*I would like to thank Leon Bettendorf, Joseph Francois, Eckhard Janeba, Charles van Marrewijk and Bauke Visser for helpful comments. All errors are mine.

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1 Introduction

Recent econometric studies on the industry–wide consequences of trade reveal five empirical facts: first, exposure to trade leads to a firm selection in the sense that only part of a country's firms benefits from the export opportunities, while other firms have to exit the market (Clerides et al. (1998), Aw et al. (2000)). Second, exposure to trade increases industry–wide productivity due to resource reallocations (Bernard and Jensen (1999), Pavcnik (2002)). Third, exporters are more capital intensive than non–exporters (Bernard and Jensen (1999), Alvarez and López (2005)). Fourth, the exit rate with exposure to trade for the exporting firms exceeds the exit rate for non–exporting firms. Nevertheless, exposure to trade leads to a reallocation of resources such that the aggregate production of the 'more advanced' exporters increases and the industry–wide productivity increases.[1] Fifth, the growth effect of exposure to trade differs between countries, but is, at least, non–negative (Feenstra (2004), Alesina et al. (2005)).

Both the first and the second empirical fact can be explained very successfully by recent theoretical models which extend the Krugman (1980)–trade model by firm heterogeneity (cf., among others, Melitz (2003), Baldwin and Forslid (2004), Falvey et al. (2004), Bernard et al. (2007)). Two assumptions are central to these models: first, firms face fixed costs for serving the home and the foreign market. Second, firms do not know their total factor productivity until they pay a sunk market entry cost and enter the market. These two assumptions trigger a firm selection mechanism when a country enters international trade: as exporting is costly, only the more productive firms export and produce more in the open economy. Exposure to trade therefore increases competition for resources, which drives the least productive firms out of the market. Trade liberalization accordingly increases a country's average productivity. However, these theoretical contributions do not consider the third empirical fact, they get the opposite result with respect to the fourth empirical fact and they are silent on the growth effect of exposure to trade.

Baldwin and Robert–Nicoud (2005) address the growth effect of exposure to trade and extend the heterogeneous firms model by Melitz (2003) to an endogenous growth model in the spirit of Romer (1986, 1990) and Grossman and Helpman (1991). However, in

[1] Pavcnik (2002) uses data of the Chilean manufacturing sector and shows the following: in the period right after trade liberalization in Chile, 1979–1986, the exit rate for the exporting firms exceeds that for the non–exporting firms by almost one third. However, during the same period a factor reallocation towards the more productive exporters takes place so that the industry–wide productivity increases.

contrast to the empirical evidence, they get an exclusively negative growth effect of exposure to trade.

This paper presents an alternative extension of Melitz (2003). This extension explains all five empirical facts. The model setup is as follows:

first, this paper extends a heterogeneous firms trade model by a Ramsey growth model. In order to leave the analysis analytically tractable, this paper concentrates on the countries' steady states.

Second, all previous heterogeneous firms trade models assume heterogeneity with respect to total factor productivity.[2] This paper, in contrast, highlights the idea that heterogeneity with respect to total factor productivity can always be explained with the help of an *additional* factor of production, namely capital, which is employed in different input shares at given relative factor prices. If the time discount rate and the capital depreciation rate are chosen such that the relative price of labor exceeds unity, only the more capital intensive firms self–select into export markets.

Third, trade economists are not only interested in a comparison of steady states, but also in the firm dynamics with exposure to trade. In order to extract more information on the firm dynamics from the model, this paper makes the following assumption on firm behavior: the potential entrants *first* observe whether the established firms gain or lose with exposure to trade. Only *afterwards* the potential entrants decide whether to enter the market or not. With respect to the formal model setup, this assumption on firm behavior implies that the firm selection process with exposure to trade has to be split up into a first and a second selection process. The first selection process describes the reaction of the established firms to exposure to international trade. The subsequent entry decision of the potential entrants leads to the second selection process, which changes the countries' steady state capital/labor ratio. It is argued in this paper that the previous trade models with firm heterogeneity implicitly make a different assumption on firm behavior: in the previous models the potential entrants *immediately* enter the market with exposure to trade without first observing how exposure to trade influences the established firms. However, it is shown that this difference in firm behavior is crucial for the results in the present setup.

All other central components of the model are adopted from Melitz (2003), Baldwin and Forslid (2004), Falvey et al. (2004) and Bernard et al. (2007):

first, firms supply their goods in monopolistically competitive markets. Second, firms

[2]'Total factor productivity' reduces to 'labor productivity' in the single factor models by Melitz (2003), Falvey et al. (2004), Baldwin and Forslid (2004) and Baldwin and Robert–Nicoud (2005).

face uncertainty about their technologies, i. e., about their factor share parameters, before market entry. Third, there are three types of fixed costs. Market entry leads to one–time sunk costs, serving the home and the foreign market, respectively, leads to per period fixed costs.
The results show that, again, exposure to trade leads to a firm selection. However, the first selection process, which describes the reaction of the established firms to exposure to trade, is now supported by empirical evidence: exporting firms become larger, but their number *falls* when the country opens up to trade. If, instead, firm heterogeneity were defined with respect to total factor productivity, the first selection process would force the *non*–exporting firms to exit the market. Furthermore, it is shown that the subsequent second selection process with exposure to trade, which results from the entry decision of potential entrants, may lead to either a positive or a negative growth effect of exposure to trade, depending on the magnitudes of the export costs parameters. This paper therefore uses firm heterogeneity in order to explain why previous empirical studies on the growth effect of exposure to trade produced quantitatively rather different results.

2 Basic model

The steady states of two symmetric countries, the home country H and the foreign country F, are analyzed. Both countries are endowed with two factors of production, capital K and labor L, which are used to produce one differentiated good. The countries' labor endowments are constant over time. The countries' capital endowments are determined endogenously. As both countries are identical, the country index H or F is initially omitted. Furthermore, since only the steady state is analyzed, the time index is also dropped for the time being. The market for the differentiated good is characterized by large–group Dixit–Stiglitz monopolistic competition.

2.1 Production

A single firm i produces a unique variety of the differentiated good with the following modified *CES* production function

$$q(\phi_i) \;=\; \left(\phi_i^{1-\alpha} \cdot L_i^{\alpha} + (1-\phi_i)^{1-\alpha} \cdot K_i^{\alpha}\right)^{1/\alpha}, \tag{1}$$

where L_i and K_i denote the labor and capital inputs for firm i. This modified *CES* function yields the calibrated share form of the per unit cost function if all absolute prices are equal to unity. The calibrated share form of the cost function is taken from applied general equilibrium theory and simplifies further calculations considerably since only the firms' cost functions will be used.[3] The parameter ϕ_i denotes different technologies across firms. Firm i has the per unit cost function

$$c(\phi_i) \;=\; \Big(\phi_i \cdot w^{1-\sigma} + (1-\phi_i)\cdot r^{1-\sigma}\Big)^{1/(1-\sigma)}, \tag{2}$$

with w and r denoting the wage rate and the capital rental rate. The parameter σ represents the elasticity of substitution in production, which is given by $\sigma = 1/(1-\alpha)$. Furthermore, serving the domestic market leads to per period fixed costs $c(\phi_i)\cdot f$, which are produced with the same technology as the good itself.[4] The magnitude of f is identical across all firms. Given Dixit–Stiglitz preferences for the representative household, the profit maximizing price of firm i is given by $p(\phi_i)\cdot(1-1/\sigma) = c(\phi_i)$, where σ stands for the elasticity of substitution in the representative household's utility function. In order to avoid analytical complexities, the firms' production functions and the representative household's utility function share an identical value for σ. Furthermore, the number of firms is assumed to reach infinity in order to avoid integer restrictions.

2.2 Demand

Intratemporal preferences of the representative household are described by a *CES* love of variety utility function over the varieties of the differentiated good. This utility function leads to the following revenue function for a single firm i:

$$R(\phi_i) \;=\; P^{\sigma}\cdot Q \cdot p(\phi_i)^{1-\sigma}, \tag{3}$$

where $P = (\int_{\phi_H} p(\phi_H)^{1-\sigma} d\phi_H + \int_{\phi_F} (p(\phi_F)\cdot\tau)^{1-\sigma} d\phi_F)^{1/(1-\sigma)}$ denotes the aggregate price index and $Q = (\int_{\phi_H} q(\phi_H)^{\alpha} d\phi_H + \int_{\phi_F} (q(\phi_F)/\tau)^{\alpha} d\phi_F)^{1/\alpha}$ the aggregate consumption good. The index H (F) stands for the domestic (foreign) varieties supplied to the home market and τ, $\tau \geq 1$, denotes iceberg transport costs.

2.3 Aggregation

In each country, a continuum of heterogeneous firms in the differentiated goods sector exists. In order to keep the model still tractable, the mass of heterogeneous firms is

[3] Cf. Rutherford (2004) for the calibrated share form of a *CES* production function.

[4] Note that f stands for a per period fixed investment, e. g., for maintaining the machines for production or for maintaining the company buildings. This per period fixed investment is produced with firm i's common technology so that the actual per period fixed costs amount to $c(\phi_i)\cdot f$.

aggregated to a mass of average firms.[5] Aggregation proceeds in two steps.
First, the production side is analyzed. It can be shown that the following two versions of the model lead to identical absolute factor prices and aggregate factor income:

Version 1 — the disaggregated model

A mass N of heterogeneous firms produces according to the following per unit cost function:

$$c(\phi_i) = \left(\phi_i \cdot w^{1-\sigma} + (1-\phi_i) \cdot r^{1-\sigma}\right)^{1/(1-\sigma)}, \text{ with } \phi_i \in [0,1].$$

The demand for a single variety i is given by $q(\phi_i) = P^\sigma \cdot Q \cdot p(\phi_i)^{-\sigma}$;

Version 2 — the aggregated model

A mass $\widetilde{N}$ of average firms produces according to the following per unit cost function:

$$c(\widetilde{\phi}) = \left(\widetilde{\phi} \cdot w^{1-\sigma} + (1-\widetilde{\phi}) \cdot r^{1-\sigma}\right)^{1/(1-\sigma)}, \text{ with } \int_0^1 \phi \cdot g(\phi) d\phi = \widetilde{\phi},$$

where $g(\phi)$ is the distribution function for ϕ. ϕ is assumed to be uniformly distributed over the interval $[0,1]$. Each average firm's demand is given by $q(\widetilde{\phi}) = M_C/(\widetilde{N} \cdot c(\widetilde{\phi}) \cdot \sigma/(\sigma-1))$. M_C denotes aggregate factor income which is available for consumption. Furthermore, equations (40) and (52) in appendix A show that the aggregate price indices P and $\widetilde{P}$ in the disaggregated and the aggregated model, respectively, are identical.

The share parameters $\widetilde{\phi}$ and $1-\widetilde{\phi}$ will be labeled in the following as 'average labor share parameter' and 'average capital share parameter,' respectively.
Second, the aggregated model has to be extended by a Dixit–Stiglitz demand side, which leads to an identical welfare level as in the disaggregated version of the model. The equilibrium mass of average firms $\widetilde{N}$ is determined by a free entry/exit condition of the average firm, which is given by $f \cdot (\sigma - 1) = q(\widetilde{\phi})$.[6]

[5] See appendix A for details on aggregation. The aggregation procedure is such that both the original disaggregated model and the aggregated model not only lead to identical absolute factor prices and aggregate factor income, but also to an identical mass of varieties. A country's welfare is therefore identical in both the disaggregated and the aggregated version of the model. Furthermore, with a known distribution for the factor share parameter ϕ, the original disaggregated model can always be derived from the aggregated model.

[6] The free entry/exit condition will later be extended by sunk market entry costs. The demand for the average firm's good $q(\widetilde{\phi})$ depends on $\widetilde{N}$.

2.4 Dynamic structure

This paper endogenizes each country's long run capital endowment by means of the Ramsey growth model. In the short run, however, each country's capital endowment is constant. Both countries' labor endowment is constant.

Let the parameter t denote any single time period. Each household has an infinite time horizon. The representative household in each country maximizes its lifetime utility

$$U = \sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} \cdot u(Q_t), \tag{4}$$

subject to the production technologies and the factor endowments in each period. The parameter ρ denotes the time discount rate and u the intratemporal utility function. Each country's aggregate capital endowment in period t, $\overline{K}_t$, depends on the capital endowment in period $t-1$, $\overline{K}_{t-1}$, investment in $t-1$, I_{t-1}, and the per period depreciation rate δ: $\overline{K}_t = (1-\delta) \cdot \overline{K}_{t-1} + I_{t-1}$. Furthermore, the average firm's good is used for investment and the investment technology considers the varieties of the differentiated good to be perfect substitutes.

When the dual to this restricted maximization problem is formulated, the dynamic general equilibrium for the economy is defined by several zero profit and market clearing conditions. Most importantly, the zero profit condition for investment, the zero profit condition for the average good and the zero profit condition for the capital rental activity by households, which is the Euler equation, are sufficient to determine relative factor prices in the steady state. The extension to a growth model therefore also helps to leave the model analytically tractable since the factor market equilibrium conditions do not have to be considered explicitly in the steady state.

In order to simplify further calculations without changing the general results, assumption (A1) is made:

(A1): The elasticity of substitution in production and consumption is $\sigma = 2$.

The relative wage rate and the capital rental rate in the steady state then simplify to[7]

$$\frac{w_t}{r_t} = \frac{\widetilde{\phi}}{\rho + \delta - 1 + \widetilde{\phi}} \quad \text{and} \quad r_t = w_t \cdot \frac{\rho + \delta - 1 + \widetilde{\phi}}{\widetilde{\phi}}.$$

[7] See appendix B for the derivation of w_t/r_t in the steady state. The result that w_t/r_t in the steady state is independent of the factor market equilibrium conditions is originally due to Baxter (1992), pp. 737–739, but can also be derived from Lau et al. (2002), pp. 595–596. Furthermore, if the average firm's good were chosen as numéraire, r_t in the steady state would be equal to $\rho + \delta$.

In order to guarantee that the relative price of labor is positive and, at the same time, larger than unity, two further assumptions are made:

$$\text{(A2):}\quad \rho+\delta > 1-\widetilde{\phi} \qquad \text{and} \qquad \text{(A3):}\quad 1 > \rho+\delta.$$

Since $\widetilde{\phi}$ will be a variable of the model, it depends on the parameters ρ and δ. However, it can be shown that (A2) is fulfilled in the steady state equilibrium for 'sufficiently' large values for ρ and δ. (A3) implies that per unit production costs decline as the capital share parameter $1-\widetilde{\phi}$ increases.[8]

As only steady states will be analyzed and the labor endowment is assumed to be constant over time, the index t can henceforth be dropped. It can be shown that all aggregate variables of the model now depend only on ρ and δ, the wage rate w, the country's labor endowment $\overline{L}$ and the average labor share parameter $\widetilde{\phi}$:

$$\begin{aligned}
\text{price of average firm's good:}\quad p(\widetilde{\phi}) &= \frac{\sigma}{\sigma-1}\cdot c(\widetilde{\phi}) = 2\cdot w\cdot\frac{\rho+\delta-1+\widetilde{\phi}}{\widetilde{\phi}\cdot(\rho+\delta)}\\
\text{country's capital endowment:}\quad \overline{K} &= \frac{1-\widetilde{\phi}}{\widetilde{\phi}}\cdot\left(\frac{r}{w}\right)^{-\sigma}\cdot\overline{L} = \frac{(1-\widetilde{\phi})\cdot\widetilde{\phi}}{(\rho+\delta-1+\widetilde{\phi})^2}\cdot\overline{L}\\
\text{factor income:}\quad M &= w\cdot\overline{L}+r\cdot\overline{K} = w\cdot\overline{L}\cdot\frac{\rho+\delta}{\rho+\delta-1+\widetilde{\phi}}\\
\text{aggregate production:}\quad X &= \left(\widetilde{\phi}^{1-\alpha}\cdot\overline{L}^{\alpha}+(1-\widetilde{\phi})^{1-\alpha}\cdot\overline{K}^{\alpha}\right)^{1/\alpha} = \frac{\overline{L}\cdot\widetilde{\phi}\cdot(\rho+\delta)^2}{(\rho+\delta-1+\widetilde{\phi})^2}\\
\text{aggregate investment:}\quad I &= \delta\cdot\overline{K} = \delta\cdot\frac{(1-\widetilde{\phi})\cdot\widetilde{\phi}}{(\rho+\delta-1+\widetilde{\phi})^2}\cdot\overline{L}\\
\text{disposable factor income:}\quad M_C &= w\cdot\overline{L}+r\cdot\overline{K}-c(\widetilde{\phi})\cdot I\\
&= w\cdot\overline{L}\cdot\frac{(\rho+\delta)^2-\delta\cdot(1-\widetilde{\phi})}{(\rho+\delta)\cdot(\rho+\delta-1+\widetilde{\phi})},
\end{aligned}$$

where X also contains the production of fixed costs. Labor is chosen as the numéraire and w is set equal to unity for both countries as countries are symmetric.[9]

[8] (A3) is critical for the results in the sense that they just reverse if the relative price of labor were smaller than unity. The impact of (A3) will become clear in subsection 2.5, which describes the dynamic entry and exit process of firms. However, $w_t/r_t > 1$ does not necessarily imply that the analyzed countries are relatively labor scarce. Since the *relative* labor scarceness always depends on the trade partners' factor endowments, $w_t/r_t > 1$ is also compatible with relative labor abundance. The results in this paper therefore might refer to developed as well as to developing countries.

[9] Labor can be chosen as numéraire for one period only. Accordingly, the wage rate can be set equal to unity for one period only as well. The present value of both factor prices in the steady state then declines with the discount factor $1/(1+\rho)$. However, as only the model's steady state is analyzed, this simplification does not affect the model's quantity variables.

2.5 Firm entry and exit

The monopolistically competitive sector is populated by an unbounded mass of potential entrants into the market. Entering the market in t instantaneously requires an irreversible investment of $c(\phi_i) \cdot f_X$, which is included in the model as one–time sunk market entry costs. Again, in order to avoid analytical complexities, each firm produces f_X with an identical technology as the respective good itself.

After firm i entered the market, it has to draw in t its labor share parameter ϕ_i from a common exogenous cumulative distribution which is given by G and has positive support on $[0, 1]$.[10] After a firm gets to know its ϕ_i, it either starts serving the market in $t + 1$ or it exits the market. The firm starts serving the market if ϕ_i is 'sufficiently' small, i. e., if the capital share parameter $1 - \phi_i$ is 'sufficiently' large. Total per period profits, i. e., variable per period profits $R(\phi_i) - c(\phi_i) \cdot q(\phi_i)$ minus per period fixed costs $c(\phi_i) \cdot f$, are non–negative only for a 'sufficiently' large capital share parameter since $w/r > 1$. However, if the capital share parameter is 'small,' the firm immediately exits the market. In every period, each firm may be hit by a negative technology shock with probability θ, $0 < \theta < 1$. If a firm is hit by such a shock, it immediately exits the market.[11]

This type of market entry and exit decision establishes the threshold value ϕ^* for the labor share parameter and the probability of a successful market entry. All firms with $\phi_i > \phi^*$ exit the market immediately and never start serving the market as variable per period profits do not cover per period fixed costs $c(\phi_i) \cdot f$. All firms with $\phi_i \leq \phi^*$ are at least able to cover $c(\phi_i) \cdot f$ and start serving the market. Due to the uniform distribution of ϕ on $[0, 1]$, the probability of a successful market entry is likewise given by ϕ^*, and the labor share parameter of the average active firm is given by $\widetilde{\phi} = \phi^*/2$.

3 Equilibrium in the closed economy

The steady state equilibrium of the closed economy is identical to the one in a standard one–sector Ramsey growth model with given technologies. The present model is there-

[10] On the one hand, it is assumed that the one–time sunk market entry costs are produced with ϕ_i. On the other hand, it is assumed that firms draw their ϕ_i from a probability distribution *after* market entry. In order to bring both assumptions in line, it is supposed that the entering firms pay the irreversible resource input f_X with the help of a credit. After firms have drawn their ϕ_i, they pay the credit back in kind. The actual one–time sunk market entry costs then amount to $c(\phi_i) \cdot f_X$.

[11] The negative shock guarantees that in every period a constant amount of sunk market entry costs arises in the steady state. This sequence of the market entry and exit decision is adopted from Hopenhayn (1992) and Melitz (2003).

fore extended by further equations to determine the average labor share parameter $\widetilde{\phi}$. First, the zero profit condition ($Z.P.C.$) defines the threshold labor share parameter ϕ^*:

$$R(\phi^*) - q(\phi^*) \cdot c(\phi^*) \; = \; \frac{R(\phi^*)}{\sigma} \; = \; c(\phi^*) \cdot f, \tag{5}$$

where $q(\phi^*)$ denotes the production of the variety, which is produced with ϕ^*. The $Z.P.C.$ states that the variable per period profits of a firm with the threshold labor share parameter, $R(\phi^*)/\sigma$, have to equal the per period fixed costs of this firm, $c(\phi^*) \cdot f$. The firm producing with ϕ^* is called 'marginal firm.' All firms with a labor share parameter smaller than ϕ^* are characterized by $R(\phi_i)/\sigma > c(\phi_i) \cdot f$ since $w/r > 1$.

Second, if market entry and exit are unrestricted, the average firm's expected total per period profits have to equal the per period equivalent of the one–time sunk market entry costs. This free entry/exit ($F.E.C.$) condition is given by

$$G(\phi^*) \cdot \left(\frac{R(\widetilde{\phi})}{\sigma} - c(\widetilde{\phi}) \cdot f \right) \; = \; c(\widetilde{\phi}) \cdot f_M, \tag{6}$$

where f_M denotes the per period equivalent of the one–time sunk market entry costs and is implicitly defined by $f_X = f_M/(\rho + \mu + \rho \cdot \mu)$, $\mu = \theta/(1-\theta)$ as the firm has an infinite lifetime as well.[12] G denotes the distribution function for ϕ. $G(\phi^*)$ therefore equals the probability of $\phi_i \leq \phi^*$. If $\phi_i \leq \phi^*$, the firm starts serving the market and both variable profits and per period fixed costs occur. Given that the firm actually serves the market, it expects to have $R(\widetilde{\phi})/\sigma$ as variable per period profits and $c(\widetilde{\phi}) \cdot f$ as per period fixed costs.[13]

Finally, the aggregate production function specifies the relationship between the equilibrium mass of average firms, $\widetilde{N}$, and the amount produced by a single average firm, $q(\widetilde{\phi})$:

$$\widetilde{N} \cdot q(\widetilde{\phi}) \; = \; \overline{L} \cdot (1 - 1/\sigma) \cdot \frac{\widetilde{\phi} \cdot (\rho + \delta)^2}{(\rho + \delta - 1 + \widetilde{\phi})^2}, \tag{7}$$

[12] The parameter θ denotes the probability of a negative shock in each period. The negative shock forces the firm to exit the market immediately. Accordingly, the firm reaches period T with probability $(1-\theta)^{T-1}$. The discounted value of the per period equivalent of the sunk market entry costs in period t, $1/(1+\rho)^t \cdot c(\widetilde{\phi}) \cdot f_M$, therefore only occurs with probability $(1-\theta)^t$. Defining $1/(1+\mu) = 1-\theta$, which is equivalent to $\mu = \theta/(1-\theta)$, gives $f_X = f_M/(\rho + \mu + \rho \cdot \mu)$ from the formula of an infinite geometric series. f_M obviously increases with the probability of a negative shock.

[13] Equations (60)–(64) in appendix A demonstrate that $R(\widetilde{\phi}) = E\big(R(\phi)\big|\phi \leq \phi^*\big) = \int_0^{\phi^*} 1/\phi^* \cdot R(\phi) d\phi$. Furthermore, equations (65)–(72) in appendix A demonstrate that $c(\widetilde{\phi}) = E\big(c(\phi)\big|\phi \leq \phi^*\big) = \int_0^{\phi^*} 1/\phi^* \cdot c(\phi) d\phi$.

where the fraction $1/\sigma$ of aggregate production is used to produce fixed costs.

Equations (5)–(7) determine the values for $q(\widetilde{\phi})$, ϕ^* and $\widetilde{N}$ in the steady state in autarky. Both the zero profit condition (5) and the free entry/exit condition (6) can be simplified: first, profit maximizing behavior of firms implies that marginal revenue equals marginal costs, $p(\phi_i) \cdot (1 - 1/\sigma) = c(\phi_i)$. Second, the relationship between the amount produced by the marginal firm, $q(\phi^*)$, and the average firm, $q(\widetilde{\phi})$, is given by

$$\frac{q(\phi^*)}{q(\widetilde{\phi})} = \frac{p(\phi^*)^{-\sigma} \cdot P^{\sigma-1} \cdot M_C}{p(\widetilde{\phi})^{-\sigma} \cdot P^{\sigma-1} \cdot M_C} = \left(\frac{p(\widetilde{\phi})}{p(\phi^*)}\right)^2 \text{ since } \sigma = 2 \text{ due to (A1).} \tag{8}$$

Inserting the respective equilibrium prices gives:[14]

$$\frac{q(\phi^*)}{q(\widetilde{\phi})} = \left(\frac{w \cdot (\rho + \delta - 1 + \widetilde{\phi})/(\widetilde{\phi} \cdot (\rho + \delta))}{w \cdot (\rho + \delta - 1 + \widetilde{\phi})/(2 \cdot \widetilde{\phi} \cdot (\rho + \delta) - \widetilde{\phi})}\right)^2 = \left(\frac{2 \cdot (\rho + \delta) - 1}{\rho + \delta}\right)^2. \tag{9}$$

This equation shows that $q(\widetilde{\phi}) > q(\phi^*)$, as $\rho + \delta > 2 \cdot (\rho + \delta) - 1$ is fulfilled due to $1 > \rho + \delta$ by (A3).

Substituting $R(\phi^*) = p(\phi^*) \cdot q(\phi^*) = \sigma/(\sigma - 1) \cdot c(\phi^*) \cdot q(\phi^*)$ into equation (5) and considering that $q(\phi^*) = q(\widetilde{\phi}) \cdot ((2 \cdot (\rho + \delta) - 1)/(\rho + \delta))^2$ from equation (9) and $\sigma = 2$ from (A1) gives the following simplified *Z.P.C.*:

$$q(\widetilde{\phi}) = f \cdot \left(\frac{\rho + \delta}{2 \cdot (\rho + \delta) - 1}\right)^2. \tag{10}$$

Third, inserting $G(\phi^*) = \phi^*$, $R(\widetilde{\phi}) = p(\widetilde{\phi}) \cdot q(\widetilde{\phi}) = \sigma/(\sigma - 1) \cdot c(\widetilde{\phi}) \cdot q(\widetilde{\phi})$ and $\sigma = 2$ into equation (6) results in the following simplified *F.E.C.*:

$$q(\widetilde{\phi}) - f = \frac{f_M}{2 \cdot \widetilde{\phi}}. \tag{11}$$

Therefore, the steady state in autarky is described by the following system of equations:

$$\textit{Z.P.C.}: \quad q(\widetilde{\phi}) = f \cdot \left(\frac{\rho + \delta}{2 \cdot (\rho + \delta) - 1}\right)^2 \tag{12}$$

$$\textit{F.E.C.}: \quad q(\widetilde{\phi}) - f = \frac{f_M}{2 \cdot \widetilde{\phi}} \tag{13}$$

$$\text{aggregate production:} \quad \widetilde{N} \cdot q(\widetilde{\phi}) = \overline{L} \cdot (1 - 1/\sigma) \cdot \frac{\widetilde{\phi} \cdot (\rho + \delta)^2}{(\rho + \delta - 1 + \widetilde{\phi})^2}. \tag{14}$$

The zero profit condition shows that the equilibrium production by the average firm only depends on the model parameters. An increase in the labor endowment therefore only increases the mass of average firms.

[14] The price of the average good $p(\widetilde{\phi})$ is derived in subsection 2.4. The price of the marginal good $p(\phi^*)$ results from substituting r, which is derived in subsection 2.4, into the cost function (2).

4 Equilibrium in the open economy

As both countries are symmetric, only the home country H will be analyzed in detail. All home (foreign) variables are indexed with H (F).
Country H opens up to international markets and trades the differentiated good with the foreign country F. Exporting leads to extra costs. First, iceberg transport costs τ, $\tau \geq 1$, exist. Second, a domestic firm has to pay additional per period fixed costs $c(\phi_{iH}) \cdot f_{Ex}$ for serving the foreign market. Given $\tau > 1$ and/or $f_{Ex} > f$, not all firms find it profitable to export. Therefore, a second threshold value ϕ^*_{ExH} exists. ϕ^*_{ExH} is smaller than ϕ^*_H since $w/r > 1$. The additional threshold value ϕ^*_{ExH} determines whether an active domestic firm exports as well.
The exporting firms increase their production with exposure to trade due to Dixit–Stiglitz monopolistic competition. Since resources are fixed in the short run, opening the country up to trade therefore increases competition for scarce resources and immediately triggers a firm selection process. Due to heterogeneity, firms react differently to the increased competition for scarce resources and the composition of the average established firm changes when the country opens up to trade. Therefore, assumption (A4) on firm behavior is crucial both for the analysis of the firm dynamics with exposure to trade and for the resulting growth effects of exposure to trade:

(A4): The potential entrants *first* observe how the average established firm is influenced by exposure to trade, i. e., how the composition of the average established firm changes with exposure to trade. Only *afterwards*, the potential entrants decide whether to enter the market or not.

Assumption (A4) on firm behavior implies that the firm selection process with exposure to trade has to be split up into a first and a second selection process.
The first selection process analyzes the reaction of the *established* firms to exposure to trade, i. e., it analyzes which firms have to exit the market due to increased competition for scarce resources with exposure to trade. The *potential entrants* observe how the first selection process influences the average established firm. The 'average' refers to the average over both exporting and non–exporting firms.
The subsequent second selection process results from the following entry decision of the potential entrants: if the average established firm gains (loses) with the first selection process, the market entry of new firms is larger (smaller) than the mass of market exits due to the technological shock, so that the free entry/exit condition also holds in the open economy. The second selection process leads the country to the steady state in

the open economy.

If, instead, the potential entrants were to enter the market *immediately* after exposure to trade, the first selection process would be redundant. In fact, all previous papers on trade with firm heterogeneity in total factor productivity do not make assumption (A4) on firm behavior. Accordingly, these papers do not have to split up the selection process with exposure to trade into two steps.[15]

The separation into a first and a second selection process can be shown to be crucial in the present setup with heterogeneity in the factor intensities in production. The reason is as follows: without this separation, the free entry/exit condition in autarky, $G(\phi_H^*) \cdot \Big(R(\widetilde{\phi}_H)/\sigma - c(\widetilde{\phi}_H) \cdot f \Big) = c(\widetilde{\phi}_H) \cdot f_M$, would remain unchanged. It would simply have to be extended by the additional term $G(\phi_{ExH}^*) \cdot \Big(R_F(\widetilde{\phi}_{ExH})/\sigma - c(\widetilde{\phi}_{ExH}) \cdot f_{Ex} \Big)$ on the left hand side in order to get the free entry/exit condition with free trade. The variable $\widetilde{\phi}_H$ denotes the average labor share parameter over *all* established firms, $\widetilde{\phi}_{ExH}$ equals $\phi_{ExH}^*/2$ and $R_F(\widetilde{\phi}_{ExH})/\sigma$ denotes the variable export profits of the average domestic exporting firm. This additional term is positive since firms export only due to profit opportunities abroad. Therefore, the free entry/exit condition with free trade and without the first selection process would imply that the market entry of new firms is larger than the market exits due to the technological shock. It is shown in subsection 4.3 that the resulting second selection process would then lead to a positive growth effect. However, it is shown later in this section that in the present setup $G(\phi_H^*) \cdot \Big(R(\widetilde{\phi}_H)/\sigma - c(\widetilde{\phi}_H) \cdot f \Big)$ is smaller than $c(\widetilde{\phi}_H) \cdot f_M$ if the first selection process is considered. The resulting free entry/exit condition with free trade therefore does not necessarily imply that the market entry of new firms is larger than the market exits due to the technological shock. It is shown later in this section that the second selection process then may also lead to a negative growth effect.

[15] Cf., e. g., Melitz (2003), p. 1716: "[t]he increased labor demand by the more productive [exporting] firms and new entrants bids up the real wage and forces the least productive firms to exit." However, it is not considered that the "..increased labor demand by the more productive [exporting] firms..." with exposure to trade already leads to the first selection process, which influences the *established* firms. However, if it *were* assumed that the potential entrants decide on market entry not until they know how exposure to trade has influenced the established firms, it would have to be considered that the zero cutoff profit condition for the domestic market, equation (10) in Melitz (2003), does not hold any more due to the first selection process. Therefore, due to market exits, $r(\phi^*)/\sigma > f$ instead of $r(\phi^*)/\sigma = f$ after the first selection process. If it were separated between a first and a second selection process, which is required by assumption (A4) on firm behavior, the zero cutoff profit condition for the open economy in Melitz (2003) accordingly would be $\bar{\pi} = \Big(\widetilde{\phi}_H(\phi_H^*)/\phi_H^* \Big)^{\sigma-1} \cdot r(\phi_H^*)/\sigma - f + p_x \cdot n \cdot f_x \cdot k(\phi_x^*)$ and *not* $\bar{\pi} = f \cdot k(\phi_H^*) + p_x \cdot n \cdot f_x \cdot k(\phi_{xH}^*)$. However, subsection 4.1 of this paper demonstrates that the qualitative results in a Melitz (2003)–setup with heterogeneity in total factor productivity are robust with respect to assumption (A4) on firm behavior.

The remainder of section 4 is as follows. Subsection 4.1 describes the first selection process in the present model and contrasts it with the corresponding first selection process in a Melitz (2003)–type model with heterogeneity in total factor productivity. Subsection 4.1 therefore demonstrates that the different definition of firm heterogeneity in the present paper leads to a firm selection with exposure to trade, which is consistent with the fourth empirical fact stated in the introduction. Subsection 4.2 describes the equilibrium conditions for the open economy. These equilibrium conditions show that the present setup is also consistent with the third empirical fact stated in the introduction. Furthermore, the equilibrium conditions for the open economy in subsection 4.2 reveal how the second selection process with exposure to trade looks like. The second selection process is finally described in subsection 4.3. Subsection 4.3 therefore deals with the growth effect of exposure to trade and the fifth empirical fact stated in the introduction.

4.1 First selection process with exposure to trade

With respect to the production side, opening the country up to international trade is equivalent to an increase in the fixed costs of the exporting firms. The exporting firms also face an additional demand from abroad. However, a firm i starts exporting only if $R_F(\phi_{iH})/\sigma - c(\phi_{iH}) \cdot f_{Ex} \geq 0$, i. e., if total export profits are non–negative. The average exporting firm therefore ceteris paribus gains with exposure to trade. However, if each exporting firm produces a larger amount than in autarky, some firms have to exit the market since resources are fixed in the short run. But which firms will exit the market, the exporting firms or the non–exporting firms?

If the more capital intensive exporting firms produce more in the open economy, both w and r and the relative price of capital, r/w, increase with exposure to trade. While the resulting increase in aggregate factor income ceteris paribus benefits all active firms, the increase in r/w hurts the more capital intensive exporters. It can be shown that the *unique* equilibrium on goods and factor markets can be regained only if the mass of exporters decreases such that their aggregate production and goods and factor prices return to their autarky levels.[16] The *remaining* exporters definitely gain with the first

[16]Due to symmetry across countries, total demand for the average exporting firm's good rises from $q(\widetilde{\phi}_{ExH}) = M_{CH} \cdot P_H^{\sigma-1} \cdot \left(p(\widetilde{\phi}_{ExH})\right)^{-\sigma}$ to $q(\widetilde{\phi}_{ExH}) = M_{CH} \cdot P_H^{\sigma-1} \cdot \left(p(\widetilde{\phi}_{ExH})\right)^{-\sigma} \cdot \left(1+\tau^{1-\sigma}\right)$ with exposure to trade. The price index P_H for the home country therefore rises from

$$P_H = \left(\widetilde{N}_{NH} \cdot \left(p(\widetilde{\phi}_{NH})\right)^{1-\sigma} + \widetilde{N}_{ExH} \cdot \left(p(\widetilde{\phi}_{ExH})\right)^{1-\sigma}\right)^{1/(1-\sigma)}$$ to

$$P_H = \left(\widetilde{N}_{NH} \cdot \left(p(\widetilde{\phi}_{NH})\right)^{1-\sigma} + \widetilde{N}_{ExH} \cdot \left(1+\tau^{1-\sigma}\right) \cdot \left(p(\widetilde{\phi}_{ExH})\right)^{1-\sigma}\right)^{1/(1-\sigma)}$$ with exposure to trade, where

selection process since prices return to their autarky levels. The established less capital intensive non–exporters are unaffected by the first selection process since prices do not change with the first selection process. Opening the country up to international trade therefore immediately triggers a selection process against the more capital intensive exporters: exposure to trade ceteris paribus increases the production of each single exporting firm, but the increase in r/w reduces their mass proportionately.[17]

Charts a), b) and c) of figure 1 illustrate the first selection process. The horizontal axes display the range of ϕ, which leads to production and to exports, respectively. The vertical axes display total production $N(\phi_H) \cdot q(\phi_H)$ of firms with labor share parameter ϕ_H. As a lower ϕ_H leads to lower per unit costs, all firms with $\phi_H \in [0, \phi_H^*]$ serve the home market. All firms with $\phi_H \in [0, \phi_{ExH}^*]$ export as well in the open economy. Chart b) shows that exposure to trade ceteris paribus increases the production of each exporting firm. However, since resources are fixed in the short run, competition for capital reduces the mass of the more capital intensive exporting firms. Aggregate production of these firms therefore does not change during the first selection process, i. e., the country's average factor share parameters remain constant. However, aggregate home supply is now produced with a smaller average capital share parameter $1 - \widetilde{\phi}_H$ than in autarky — the domestic supply of each single more capital intensive exporting firm remains constant, but their mass decreases with the first selection process.

It is therefore a priori ambiguous whether the average over all remaining firms, i. e., the average over both the remaining exporting and non–exporting firms, gains or loses with the first selection process: the first selection process ceteris paribus benefits the average remaining firm since the average remaining *exporting* firm gains with the first selection process. However, the first selection process ceteris paribus harms the average remaining firm since aggregate home supply is now produced with a smaller average capital share parameter than in autarky. Subsection 4.3 analyzes which effect dom-

the subscripts N and Ex denote non–exporting and (potentially) exporting firms, respectively. P_H accordingly does not change if the mass of average exporting firms, $\widetilde{N}_{ExH}$, decreases by the factor $1/\left(1+\tau^{1-\sigma}\right)$ with the first selection process. The new *unique* equilibrium on goods and factor markets results. Only the free entry/exit condition is not necessarily fulfilled after the first selection process, which leads to the second selection process.

[17]Note that an equilibrium on goods and factor markets cannot be reached if part of the more capital intensive exporting firms just stops exporting with the increase in r/w and only concentrates on the domestic market. As long as part of the more capital intensive firms exports and as long as none of the exporters exits the market, the ratio of the country's average factor share parameters in production, $(1 - \widetilde{\phi}_H)/\widetilde{\phi}_H$, exceeds the country's relative capital endowment $\overline{K}_H/\overline{L}_H$ — which does not lead to an equilibrium on goods and factor markets. A situation in which no firm exports would equalize $(1 - \widetilde{\phi}_H)/\widetilde{\phi}_H$ and $\overline{K}_H/\overline{L}_H$, but is also no equilibrium because r/w would then be on its autarky level, which gives the more capital intensive firms an incentive to export.

inates and describes the subsequent second selection process, which results from the entry decision of the potential entrants. For the sake of completeness, chart d) in figure 1 already shows that the second selection process may either increase or decrease the threshold labor share parameters ϕ^*_{ExH} and ϕ^*_H.

If assumption (A4) on firm behavior were included in a Melitz (2003)–type model with only labor and heterogeneity in total factor productivity, the corresponding first selection process would look completely differently. Since fixed export costs exist, only the more productive firms export in a Melitz (2003)–type model. Exposure to trade again increases each exporting firm's total output, which increases aggregate factor demand. However, the resulting increase in the wage rate drives the least productive *non*–exporting firms out of the market. Opening the country up to trade therefore triggers a selection process against the less productive non–exporters in a Melitz (2003)–type model.[18] The average over all remaining firms, i. e., the average over both exporting and non–exporting firms, therefore definitely gains with the first selection process in a Melitz (2003)–type model: first, home supply is produced with a larger average total factor productivity after the first selection process and, second, the exporting firms gain with the first selection process anyway.[19]

4.2 Equilibrium

As exporting is costly, two types of firms exist in the equilibrium of the open economy. On the one hand, firms that serve only the domestic market and, on the other hand, firms that serve the domestic market and export as well. Again, the threshold value ϕ^*_H determines whether a firm starts production at all after it has entered the market. The additional threshold value ϕ^*_{ExH} determines whether a firm also exports. Partitioning with respect to export status implies $\phi^*_{ExH} < \phi^*_H$.

First of all, ϕ^*_H and ϕ^*_{ExH} are defined by the respective zero profit condition (*Z.P.C.*):

$$\text{\textit{Z.P.C.} for the home market:} \qquad \frac{R_H(\phi^*_H)}{\sigma} = c(\phi^*_H) \cdot f \tag{15}$$

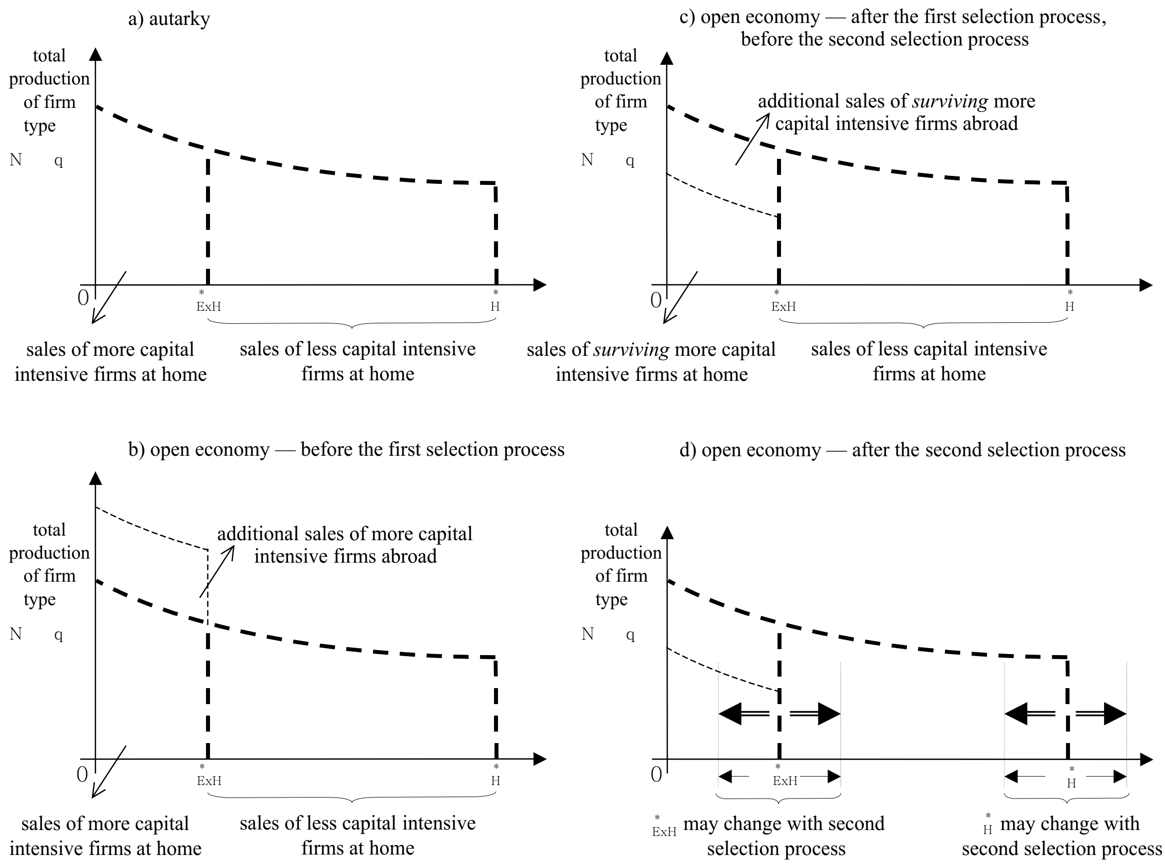
$$\text{\textit{Z.P.C.} for the foreign market:} \qquad \frac{R_F(\phi^*_{ExH})}{\sigma} = c(\phi^*_{ExH}) \cdot f_{Ex}. \tag{16}$$

The country H firm which produces with ϕ^*_H is called 'marginal firm for the home market.' The country H firm which produces with ϕ^*_{ExH} is called 'marginal firm for

[18] However, Pavcnik (2002), pp. 256–257, supports the first selection process in the present model.

[19] Therefore, *two* unambiguously positive effects on total profits of the average firm exist in the Melitz (2003)–type model. The results in Melitz (2003), Baldwin and Forslid (2004), Falvey et al. (2004) and Bernard et al. (2007) are therefore robust in a qualitative sense to excluding one of both positive effects, namely the first selection process.

Figure 1: First and second selection process with exposure to trade



the foreign market.' Dividing both *Z.P.C.* by each other gives

$$\frac{R_H(\phi_H^*)}{R_F(\phi_{ExH}^*)} = \frac{M_{CH} \cdot P_H^{\sigma-1} \cdot p(\phi_H^*)^{1-\sigma}}{M_{CF} \cdot P_F^{\sigma-1} \cdot \tau^{1-\sigma} \cdot p(\phi_{ExH}^*)^{1-\sigma}} = \frac{c(\phi_H^*) \cdot f}{c(\phi_{ExH}^*) \cdot f_{Ex}}, \tag{17}$$

$$\text{which is equal to } \left(\frac{p(\phi_{ExH}^*)}{p(\phi_H^*)}\right)^{-\sigma} = \tau^{\sigma-1} \cdot \frac{f_{Ex}}{f} \tag{18}$$

since countries are symmetric and $p(\phi_{iH}) \cdot (1 - 1/\sigma) = c(\phi_{iH})$. The ratio of the prices of both marginal firms, $p(\phi_{ExH}^*)/p(\phi_H^*)$, can be derived as

$$\frac{p(\phi_{ExH}^*)}{p(\phi_H^*)} = \frac{\left(\phi_{ExH}^* \cdot w^{1-\sigma} + (1-\phi_{ExH}^*) \cdot r^{1-\sigma}\right)^{\frac{1}{1-\sigma}}}{\left(\phi_H^* \cdot w^{1-\sigma} + (1-\phi_H^*) \cdot r^{1-\sigma}\right)^{\frac{1}{1-\sigma}}} = \frac{\phi_H^* + (1-\phi_H^*) \cdot \frac{\widetilde{\phi}_H}{\rho+\delta-1+\widetilde{\phi}_H}}{\phi_{ExH}^* + (1-\phi_{ExH}^*) \cdot \frac{\widetilde{\phi}_H}{\rho+\delta-1+\widetilde{\phi}_H}}, \tag{19}$$

where the second equality follows from $w = 1$, the steady state value of w/r as defined in subsection 2.4 and from $\sigma = 2$ by (A1). Further simplification results in

$$\frac{p(\phi^*_{ExH})}{p(\phi^*_H)} = \frac{\phi^*_H \cdot (\rho+\delta) - \phi^*_H + \widetilde{\phi}_H}{\phi^*_{ExH} \cdot (\rho+\delta) - \phi^*_{ExH} + \widetilde{\phi}_H} = \frac{\rho+\delta-0.5}{\phi^*_{ExH}/\phi^*_H \cdot (\rho+\delta-1)+0.5}, \tag{20}$$

where the second equality uses the fact that $\widetilde{\phi}_H = 0.5 \cdot \phi^*_H$. Equation (18) therefore can be written alternatively as:

$$\left(\frac{p(\phi^*_{ExH})}{p(\phi^*_H)}\right)^{-2} = \left(\frac{\phi^*_{ExH}/\phi^*_H \cdot (\rho+\delta-1)+0.5}{\rho+\delta-0.5}\right)^2 = \tau^{2-1} \cdot \frac{f_{Ex}}{f}. \tag{21}$$

The ratio for the threshold labor share parameters, which equals the probability that the average firm exports, is thus given by:

$$\frac{\phi^*_{ExH}}{\phi^*_H} = \frac{(\rho+\delta-0.5) \cdot \tau^{0.5} \cdot (f_{Ex}/f)^{0.5} - 0.5}{\rho+\delta-1}. \tag{22}$$

Therefore, the *Z.P.C.* for the foreign market will be dropped and the function $\phi^*_{ExH} = \phi^*_{ExH}(\phi^*_H)$ will be taken instead. ϕ^*_{ExH} is positive only if $(\rho+\delta-0.5)\cdot(\tau\cdot f_{Ex}/f)^{0.5}-0.5 < 0$, as $1 > \rho+\delta$ by (A3). If $\tau \geq 0.25 \cdot f/(f_{Ex} \cdot (\rho+\delta-0.5)^2)$, ϕ^*_{ExH} equals zero and international trade ceases. If $\tau > f/f_{Ex}$, the ratio ϕ^*_{ExH}/ϕ^*_H is smaller than unity, i. e., partitioning of firms with respect to the export status takes place. Furthermore, equation (22) shows that ϕ^*_{ExH}/ϕ^*_H is independent of the equilibrium average labor share parameter $\widetilde{\phi}_H$.

Since ϕ is uniformly distributed over the interval $[0, 1]$, the share of potentially exporting firms in autarky results as $\widetilde{N}_{ExH}/(\widetilde{N}_{ExH} + \widetilde{N}_{NH}) = \phi^*_{ExH}/\phi^*_H$ for any equilibrium $\widetilde{\phi}_H$. The share of potentially non–exporting firms in autarky results as $\widetilde{N}_{NH}/(\widetilde{N}_{ExH} + \widetilde{N}_{NH}) = 1 - \phi^*_{ExH}/\phi^*_H$. The share of exporting firms with free trade results as $\widetilde{N}_{ExH}/(1+\tau^{1-\sigma})\Big/\Big(\widetilde{N}_{ExH}/(1+\tau^{1-\sigma})+\widetilde{N}_{NH}\Big)$ since the mass of exporting firms decreases by the factor $1/(1+\tau^{1-\sigma})$ with the first selection process with exposure to trade, since the mass of non–exporting firms remains constant with the first selection process and since ϕ^*_{ExH}/ϕ^*_H does not depend on the equilibrium $\widetilde{\phi}_H$. The share of non–exporting firms with free trade accordingly results as $\widetilde{N}_{NH}\Big/\Big(\widetilde{N}_{ExH}/(1+\tau^{1-\sigma})+\widetilde{N}_{NH}\Big)$.

Second, the free entry/exit condition (*F.E.C.*) has to be extended by the expected variable export profits and the expected per period fixed costs for serving the foreign market. If the components of the *F.E.C.* are displayed separately for the average exporting firm and the average non–exporting firm, the *F.E.C.* in the open economy results as:

$$G(\phi_H^*)\cdot\left(\frac{R_H(\widetilde{\phi}_{ExH})+R_H(\widetilde{\phi}_{ExH})\cdot\tau^{1-\sigma}}{\sigma}\cdot\frac{\widetilde{N}_{ExH}/(1+\tau^{1-\sigma})}{\frac{\widetilde{N}_{ExH}}{1+\tau^{1-\sigma}}+\widetilde{N}_{NH}}+\frac{R_H(\widetilde{\phi}_{NH})}{\sigma}\cdot\frac{\widetilde{N}_{NH}}{\frac{\widetilde{N}_{ExH}}{1+\tau^{1-\sigma}}+\widetilde{N}_{NH}}\right)$$

$$-\ G(\phi_H^*)\cdot c(\widetilde{\phi}_{ExH})\cdot(f+f_{Ex})\cdot\frac{\widetilde{N}_{ExH}/(1+\tau^{1-\sigma})}{\frac{\widetilde{N}_{ExH}}{1+\tau^{1-\sigma}}+\widetilde{N}_{NH}}-G(\phi_H^*)\cdot c(\widetilde{\phi}_{NH})\cdot f\cdot\frac{\widetilde{N}_{NH}}{\frac{\widetilde{N}_{ExH}}{1+\tau^{1-\sigma}}+\widetilde{N}_{NH}}$$

$$=\ f_M\cdot\left(c(\widetilde{\phi}_{ExH})\cdot\frac{\widetilde{N}_{ExH}/(1+\tau^{1-\sigma})}{\frac{\widetilde{N}_{ExH}}{1+\tau^{1-\sigma}}+\widetilde{N}_{NH}}+c(\widetilde{\phi}_{NH})\cdot\frac{\widetilde{N}_{NH}}{\frac{\widetilde{N}_{ExH}}{1+\tau^{1-\sigma}}+\widetilde{N}_{NH}}\right). \tag{23}$$

where $\widetilde{\phi}_{ExH}$ denotes the average labor share parameter of the exporting domestic firms, while $\widetilde{\phi}_{NH}$ denotes the average labor share parameter of the non–exporting domestic firms. $R_H(\widetilde{\phi}_{ExH})\cdot\tau^{1-\sigma}/\sigma$ stands for the variable export profits of the average domestic exporting firm due to symmetry across countries. The weighting factors $\widetilde{N}_{ExH}/(1+\tau^{1-\sigma})\Big/\Big(\widetilde{N}_{ExH}/(1+\tau^{1-\sigma})+\widetilde{N}_{NH}\Big)$ and $\widetilde{N}_{NH}\Big/\Big(\widetilde{N}_{ExH}/(1+\tau^{1-\sigma})+\widetilde{N}_{NH}\Big)$ denote the relative importance of average domestic exporting and non–exporting firms, respectively.

Finally, the aggregate production function specifies the relationship between the equilibrium mass of average domestic firms, $\widetilde{N}_H$, and the amount produced by a single average domestic firm. $\widetilde{N}_H$ stands for the average over all firms, both exporting and non–exporting firms:

$$\widetilde{N}_H\cdot\left(q_{HH}(\widetilde{\phi}_{ExH})\cdot(1\ +\ \tau^{1-\sigma})\cdot\frac{\widetilde{N}_{ExH}/(1+\tau^{1-\sigma})}{\frac{\widetilde{N}_{ExH}}{1+\tau^{1-\sigma}}+\widetilde{N}_{NH}}+q_{HH}(\widetilde{\phi}_{NH})\cdot\frac{\widetilde{N}_{NH}}{\frac{\widetilde{N}_{ExH}}{1+\tau^{1-\sigma}}+\widetilde{N}_{NH}}\right)$$

$$=\ \overline{L}_H\cdot\frac{(1-1/\sigma)\cdot\widetilde{\phi}_H\cdot(\rho+\delta)^2}{(\rho+\delta-1+\widetilde{\phi}_H)^2}. \tag{24}$$

$q_{HH}(\phi_{iH})$ denotes domestic supply of a domestic firm i with labor share parameter ϕ_{iH} and $q_{HH}(\phi_{iH})\cdot\tau^{1-\sigma}$ denotes exports of this firm due to symmetry across countries.[20]

Equations (15), (23) and (24), can be further simplified: first, $p(\phi_{iH})\cdot(1-1/\sigma)=c(\phi_{iH})$ follows from profit maximizing behavior of firms. Second, the domestic supply of the marginal firm for the home market is again determined by the domestic supply of the average firm since $q_{HH}(\phi_H^*)/q_{HH}(\widetilde{\phi}_H)=(2\cdot(\rho+\delta)-1)^2/(\rho+\delta)^2$ holds as in autarky. Third, $q_{HH}(\widetilde{\phi}_{ExH})\cdot\widetilde{N}_{ExH}/(\widetilde{N}_{ExH}+\widetilde{N}_{NH})+q_{HH}(\widetilde{\phi}_{NH})\cdot\widetilde{N}_{NH}/(\widetilde{N}_{ExH}+\widetilde{N}_{NH})=q_{HH}(\widetilde{\phi}_H)$ due to the aggregation procedure described in appendix A. Finally, $G(\phi_{ExH}^*)/G(\phi_H^*)=$

[20] If the variable $q_{HF}(\widetilde{\phi}_{iH})$ denotes exports, $q_{HF}(\widetilde{\phi}_{iH})/\tau$ actually arrives abroad. Therefore, foreign demand for a single domestic variety is given by $q_{HF}(\widetilde{\phi}_{iH})/\tau=M_{CF}\cdot P_F^{\sigma-1}\cdot\big(p(\widetilde{\phi}_{iH})\cdot\tau\big)^{-\sigma}$.

ϕ^*_{ExH}/ϕ^*_H. The free trade equilibrium values for $q_{HH}(\widetilde{\phi}_H)$, $\widetilde{\phi}_H$, ϕ^*_{ExH} and $\widetilde{N}_H$ are therefore described by equation (22), together with the following three equations:[21]

$$\text{\textit{Z.P.C.} for the home market:} \quad q_{HH}(\widetilde{\phi}_H) = f \cdot \left(\frac{\rho+\delta}{2\cdot(\rho+\delta)-1}\right)^2 \tag{25}$$

$$\begin{aligned} \text{\textit{F.E.C.}:} \quad & q_{HH}(\widetilde{\phi}_H) - f \\ & - \frac{\phi^*_{ExH}}{\phi^*_H} \cdot \frac{c(\widetilde{\phi}_{ExH})}{c(\widetilde{\phi}_H)} \cdot \left(\frac{-\tau^{1-\sigma}}{1+\tau^{1-\sigma}} \cdot \left(\frac{f_M}{2\cdot\widetilde{\phi}_H} + f\right) + \frac{f_{Ex}}{1+\tau^{1-\sigma}}\right) = \frac{f_M}{2\cdot\widetilde{\phi}_H} \end{aligned} \tag{26}$$

$$\begin{aligned} \text{aggregate production:} \quad & \widetilde{N}_H \cdot q_{HH}(\widetilde{\phi}_H) \cdot \frac{\widetilde{N}_{ExH} + \widetilde{N}_{NH}}{\frac{\widetilde{N}_{ExH}}{1+\tau^{1-\sigma}} + \widetilde{N}_{NH}} \\ & = \overline{L}_H \cdot (1-1/\sigma) \cdot \frac{\widetilde{\phi}_H \cdot (\rho+\delta)^2}{(\rho+\delta-1+\widetilde{\phi}_H)^2}. \end{aligned} \tag{27}$$

Compared to autarky, the zero profit condition for the home market did not change. Obviously, the *F.E.C.* is the most critical equation, as it determines the equilibrium average labor share parameter $\widetilde{\phi}_H$. The impact of exposure to international trade on $\widetilde{\phi}_H$ is evident if the *F.E.C.* for both situations are compared:

$$\text{autarky:} \quad q(\widetilde{\phi}) - f = \frac{f_M}{2\cdot\widetilde{\phi}} \tag{28}$$

$$\begin{aligned} \text{free trade:} \quad & q_{HH}(\widetilde{\phi}_H) - f - \frac{\phi^*_{ExH}}{\phi^*_H} \cdot \frac{c(\widetilde{\phi}_{ExH})}{c(\widetilde{\phi}_H)} \cdot \left(\frac{-\tau^{1-\sigma}}{1+\tau^{1-\sigma}} \cdot \left(\frac{f_M}{2\cdot\widetilde{\phi}_H} + f\right) + \frac{f_{Ex}}{1+\tau^{1-\sigma}}\right) \\ & = \frac{f_M}{2\cdot\widetilde{\phi}_H}. \end{aligned} \tag{29}$$

Given that ϕ^*_{ExH} is larger than zero, i. e., $\phi^*_{ExH}/\phi^*_H > 0$, exposure to trade alters the average labor share parameter $\widetilde{\phi}_H$. The change in $\widetilde{\phi}_H$ influences the households' investment behavior and therefore changes the country's steady state capital endowment.

4.3 Second selection process with exposure to trade

It finally has to be analyzed whether the average labor share parameter which results from equation (29) is smaller or larger than the average labor share parameter which results from equation (28). Subsection 4.1 explained that, first, the more capital intensive exporting firms gain with the first selection process since $R_F(\phi_{iH})/\sigma - f_{Ex} \cdot c(\phi_{iH}) \geq 0$ holds for any remaining exporting firm i after the first selection process. Second, that aggregate home supply is produced with a smaller average capital share parameter

[21] Appendix C demonstrates the equivalence between equations (23) and (26).

after the first selection process since aggregate production of the more capital intensive exporting firms does not change with the first selection process, but part of their production is exported. The first (second) effect increases (decreases) total per period profits of the average remaining firm.

First, take the case in which the average remaining firm gains with the first selection process. The *F.E.C.*, equation (29), then implies that the mass of new entrants will be larger than the mass of shock–induced firm exits. The mass of active firms $\widetilde{N}_H$ accordingly increases. The *Z.P.C.*, equation (25), shows that $\widetilde{\phi}_H = \phi^*_H/2$ has to decrease with the increase in $\widetilde{N}_H$ since $q_{HH}(\widetilde{\phi}_H) = M_{CH}/\Big(\widetilde{N}_H \cdot p(\widetilde{\phi}_H)\Big)$ and $\partial p(\widetilde{\phi}_H)/\partial\widetilde{\phi}_H > 0$ since $w/r > 1$. Due to more competition with the increase in $\widetilde{N}_H$, only firms with ϕ_{iH} strictly below the initial threshold value ϕ^*_H accordingly successfully enter the market. Resources are therefore reallocated towards firms with a higher capital share parameter $1 - \phi_{iH}$ in this first case. The resulting decrease in $\widetilde{\phi}_H = \phi^*_H/2$ reduces the probability for a successful market entry, so that the *F.E.C.*, equation (29), is fulfilled again. The country's average labor (capital) share parameter therefore decreases (increases) with the second selection process if the average remaining firm gains with the first selection process.

Second, take the case in which the average remaining firm loses with the first selection process. The previous line of argument turns around and the average labor (capital) share parameter increases (decreases) with the second selection process.

Since empirical evidence indeed reports a factor reallocation towards firms with lower per unit costs with exposure to trade (Bernard and Jensen (1999), Pavcnik (2002)), the first case receives more empirical support. But when does the average remaining firm gain (lose) with the first selection process with exposure to trade? The average remaining firm gains (loses) whenever the additional term in the *F.E.C.* with free trade, $\phi^*_{ExH}/\phi^*_H \cdot c(\widetilde{\phi}_{ExH})/c(\widetilde{\phi}_H) \cdot \Big(-\tau^{1-\sigma} \cdot \Big(f_M/(2 \cdot \widetilde{\phi}_H) + f\Big) + f_{Ex}\Big)\Big/(1+\tau^{1-\sigma})$, is negative (positive). If the additional term is negative (positive), the average labor share parameter $\widetilde{\phi}_H = \phi^*_H/2$ has to decrease (increase) in order to equalize the total per period profits of the average active firm and the per period equivalent of the one–time sunk market entry costs again. A decrease (an increase) in $\widetilde{\phi}_H$ increases (decreases) the country's steady state capital endowment since a decrease (an increase) in $\widetilde{\phi}_H$ implies a subsequent rise (decline) in aggregate investment. Inserting $f_M/(2 \cdot \widetilde{\phi}_H) + f = f \cdot (\rho+\delta)^2/(2 \cdot (\rho+\delta) - 1)^2$ from equations (12) and (13) into the additional term of the *F.E.C.* with free trade leads to the following assessment of exposure to trade:

$$\text{positive growth effect/endowment gain:} \qquad \tau \;<\; \frac{(\rho+\delta)^2}{(2 \cdot (\rho+\delta) - 1)^2} \cdot \frac{f}{f_{Ex}}$$

$$\text{no growth effect/no endowment change:} \qquad \tau = \frac{(\rho+\delta)^2}{(2\cdot(\rho+\delta)-1)^2}\cdot\frac{f}{f_{Ex}}$$
$$\text{negative growth effect/endowment loss:} \qquad \tau > \frac{(\rho+\delta)^2}{(2\cdot(\rho+\delta)-1)^2}\cdot\frac{f}{f_{Ex}}.$$

Obviously, the countries' steady state capital endowments increase (decrease) with exposure to trade if τ and/or f_{Ex} are small (large). In this case, additional variable export profits are large (small) and additional fixed export costs are small (large).

4.4 Trade and steady state welfare

Intratemporal utility of the representative domestic household is given by $u(Q_H) = Q_H^{1-\sigma_I}/(1-\sigma_I)$, where σ_I denotes the intertemporal elasticity of substitution. Total consumption of the aggregate good results as $Q_H = M_{CH}/P_H$. For a given ϕ_H^*, the price index P_H for the home country in autarky and with free trade, respectively, is given by

$$P_H^{Aut.} = \left(\widetilde{N}_{ExH}^{Aut.}\cdot p(\widetilde{\phi}_{ExH})^{1-\sigma} + \widetilde{N}_{NH}^{Aut.}\cdot p(\widetilde{\phi}_{NH})^{1-\sigma}\right)^{1/(1-\sigma)} = (\widetilde{N}_H^{Aut.})^{1/(1-\sigma)}\cdot p(\widetilde{\phi}_H) \tag{30}$$

$$\begin{aligned} P_H^{FT} &= \Bigg(\underbrace{\frac{\widetilde{N}_{ExH}^{Aut.}}{1+\tau^{1-\sigma}}}_{=\widetilde{N}_{ExH}^{FT}}\cdot p(\widetilde{\phi}_{ExH})^{1-\sigma}\cdot(1+\tau^{1-\sigma}) + \underbrace{\widetilde{N}_{NH}^{Aut.}}_{=\widetilde{N}_{NH}^{FT}}\cdot p(\widetilde{\phi}_{NH})^{1-\sigma}\Bigg)^{1/(1-\sigma)} \\ &= (\widetilde{N}_H^{Aut.})^{1/(1-\sigma)}\cdot p(\widetilde{\phi}_H), \end{aligned} \tag{31}$$

where both equations (30) and (31) are simplified with the help of the aggregation procedure as described in appendix A. Furthermore, equation (31) uses the assumption of symmetry across countries and the superscripts *Aut.* and *FT* indicate variables in autarky and with free trade. For a given ϕ_H^*, i. e., $M_{CH}^{Aut.} = M_{CH}^{FT}$, aggregate consumption of the home country in both situations is therefore given by:

$$Q_H^{Aut.} = \frac{M_{CH}^{Aut.}\cdot\widetilde{N}_H^{Aut.}}{p(\widetilde{\phi}_H)} = Q_H^{FT} = \frac{M_{CH}^{FT}\cdot\widetilde{N}_H^{Aut.}}{p(\widetilde{\phi}_H)}, \quad \text{since } \sigma = 2 \text{ by (A1)}. \tag{32}$$

The aggregate production function in autarky, equation (14), implicitly specifies $\widetilde{N}_H^{Aut.}$ for a given ϕ_H^*:

$$\widetilde{N}_H^{Aut.} = \frac{\overline{L}_H\cdot(1-1/\sigma)}{q_{HH}(\widetilde{\phi}_H)}\cdot\frac{\widetilde{\phi}_H\cdot(\rho+\delta)^2}{(\rho+\delta-1+\widetilde{\phi}_H)^2}. \tag{33}$$

Inserting the expression for $\widetilde{N}_H^{Aut.}$ into equation (32) results in

$$Q_H^{Aut.} = \frac{M_{CH}^{Aut.}}{p(\widetilde{\phi}_H)}\cdot\frac{\overline{L}_H\cdot(1-1/\sigma)}{q_{HH}(\widetilde{\phi}_H)}\cdot\frac{\widetilde{\phi}_H\cdot(\rho+\delta)^2}{(\rho+\delta-1+\widetilde{\phi}_H)^2} = Q_H^{FT}. \tag{34}$$

Equation (34) shows that intratemporal utility does not change with exposure to trade if $\widetilde{\phi}_H^{Aut.} = \widetilde{\phi}_H^{FT}$. The comparative steady state analysis therefore can concentrate on the growth effect of exposure to trade, i. e., on the change in $\widetilde{\phi}_H$ with exposure to trade. First, disposable factor income M_{CH} increases with a rising average capital share parameter $1-\widetilde{\phi}_H$ since an increase in the average capital share parameter raises a country's capital endowment.[22] The positive effect on M_{CH} dominates the negative effect which results from an increase in investment with a rising capital endowment. Second, the price of the average good $p(\widetilde{\phi}_H)$ decreases with a rising average capital share parameter because $w/r > 1$. Finally, the mass of varieties as defined by equation (33) increases with a rising average capital share parameter since $q_{HH}(\widetilde{\phi}_H^{Aut.}) = q_{HH}(\widetilde{\phi}_H^{FT})$, as defined by equations (12) and (25), do not depend on the average capital share parameter in equilibrium. In summary, steady state welfare unambiguously increases (decreases) with an increase (decrease) in the average capital share parameter.

The consequences of exposure to trade on growth and steady state welfare are visualized in figure 2. According to this analysis, trade is only beneficial for a country if variable and/or fixed export costs are 'sufficiently' small.[23]

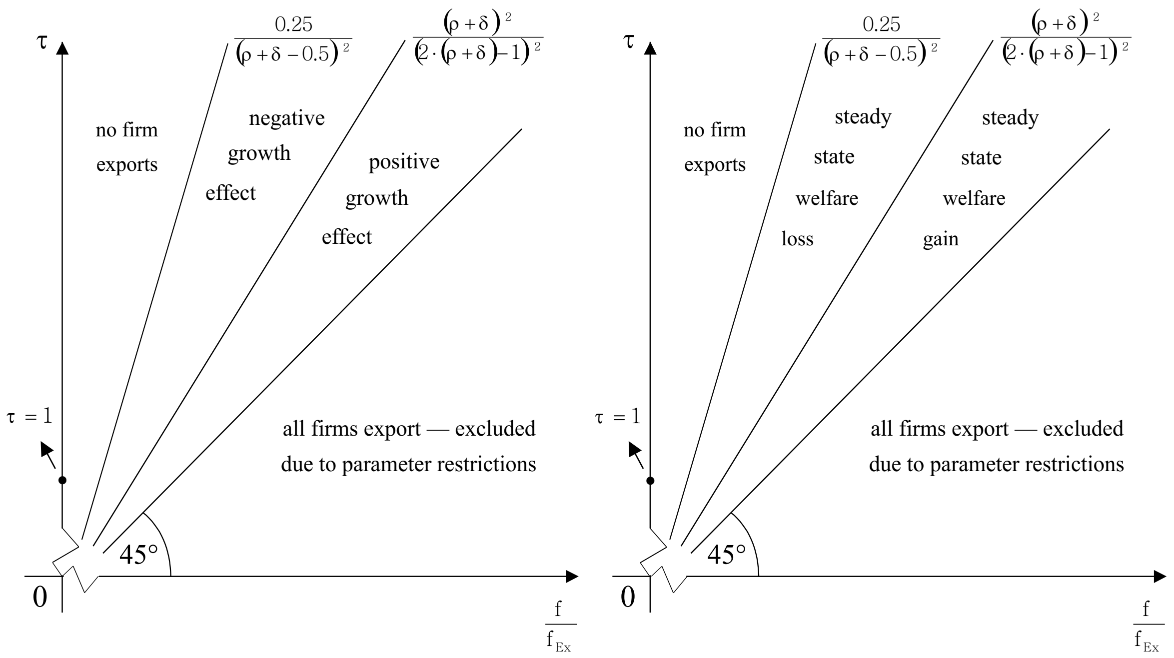
5 Conclusions

This paper extends previous trade models with firm heterogeneity (e. g., Melitz (2003), Baldwin and Forslid (2004), Bernard et al. (2007)) along three dimensions: first, it introduces heterogeneity across firms by means of different capital/labor shares in production at given relative factor prices. Previous trade models with firm heterogeneity concentrate on heterogeneity with respect to total factor productivity. The setup in this paper provides a neoclassical rationale for different factor productivities, which receives empirical support as firms in the same sector indeed differ with respect to their capital share parameter in production. Second, this paper endogenizes the countries' capital endowments by means of the standard Ramsey growth model. This dynamic framework takes the average capital share parameter to evaluate the growth and steady state welfare effects of globalization. Third, the firm selection process with exposure to trade follows a two–step procedure in which the potential entrants decide on mar-

[22]Cf. appendix D for the derivation of this and the following partial derivatives.

[23]Including the adjustment path to the new steady state does not change the welfare assessment of exposure to trade, at least for reasonable parameter values. If the representative household's steady state utility increases (decreases) with exposure to trade, the representative household's lifetime utility increases (decreases) as well. A corresponding numerical analysis is available from the author upon request.

Figure 2: Growth and steady state welfare consequences of exposure to trade



ket entry not until they have learned how exposure to trade influences the established firms. The first selection process describes the immediate reaction of the established firm to globalization, while the second selection process results from the subsequent entry decision of the potential entrants.

The results first demonstrates that, due to the specific definition of firm heterogeneity, the exporting firms become larger, but their mass decreases with the first selection process with exposure to trade. This outcome receives more empirical support than the outcome of previous heterogeneous firms trade models, where the mass of the *non*–exporting firms decreases immediately with exposure to trade.

Second, the results show that the subsequent second selection process with exposure to trade leads to ambiguous growth effects. The sign and magnitude of the growth effect solely depends on the export cost parameters. On the one hand this paper therefore may explain why previous empirical studies on the growth effect of exposure to trade yielded quantitatively rather different results. On the other hand, this paper provides an alternative to Baldwin and Robert–Nicoud (2005), who get a negative growth effect of exposure to trade, irrespective of the model parameters. However, an exclusively negative growth effect of exposure to trade is not supported by empirical evidence.

Appendix — not for publication

A Aggregation

This appendix shows that any general equilibrium model with $N_X + N_Y = N$ heterogeneous firms can be aggregated to a model with $\widetilde{N}$ average firms. The aggregation procedure is split into two steps. First, it is assumed that the equilibrium number of firms is given exogenously. The N heterogeneous firms are then aggregated to a single average firm. Second, it is explained how the single average firm is split into $\widetilde{N}$ average firms, where $\widetilde{N}$ is determined endogenously by a free entry/exit condition.

Type–X firms may represent the more capital intensive firms and type–Y firms may represent the more labor intensive firms. Each firm produces a single variety of a differentiated good. In principle, N could go to infinity. However, since ϕ is assumed to be uniformly distributed over $[0, 1]$ the aggregation procedure is explained with $N_X = N_Y = 1$ and $N = 2$ in order to keep the exposition easily tractable. Furthermore, it is assumed that goods X and Y are aggregated with a *CES* technology to give the aggregate consumption good Q. Good Q directly enters into the representative household's intratemporal utility function.

First of all, the general equilibrium factor prices w and r and aggregate factor income are determined with the help of the following production functions:

$$X = \Big(\phi_X^{1-\alpha} \cdot L_X^{\alpha} + (1-\phi_X)^{1-\alpha} \cdot K_X^{\alpha}\Big)^{1/\alpha} \tag{35}$$

$$Y = \Big(\phi_Y^{1-\alpha} \cdot L_Y^{\alpha} + (1-\phi_Y)^{1-\alpha} \cdot K_Y^{\alpha}\Big)^{1/\alpha} \tag{36}$$

$$Q = \Big(X^{\alpha} + Y^{\alpha}\Big)^{1/\alpha}, \tag{37}$$

where K_X, L_X, K_Y and L_Y represent the input of capital and labor in the production of goods X and Y. These factor inputs exclude the resources that are needed to produce fixed costs. Assuming large group monopolistic competition, the corresponding prices $p(\phi_X)$, $p(\phi_Y)$ and $p_Q \equiv P$ result as follows:

$$p(\phi_X) \cdot (1 - 1/\sigma) = \underbrace{\Big(\phi_X \cdot w^{1-\sigma} + (1-\phi_X) \cdot r^{1-\sigma}\Big)^{1/(1-\sigma)}}_{\equiv c(\phi_X)} \tag{38}$$

$$p(\phi_Y) \cdot (1 - 1/\sigma) = \underbrace{\Big(\phi_Y \cdot w^{1-\sigma} + (1-\phi_Y) \cdot r^{1-\sigma}\Big)^{1/(1-\sigma)}}_{\equiv c(\phi_Y)} \tag{39}$$

$$p_Q \equiv P = \Big((1/N) \cdot p(\phi_X)^{1-\sigma} + (1/N) \cdot p(\phi_Y)^{1-\sigma}\Big)^{1/(1-\sigma)} \cdot N^{1/(1-\sigma)}. \tag{40}$$

The prices $p(\phi_X)$ and $p(\phi_Y)$ include a monopolistic markup over marginal costs. Obviously, $N = 2$ in the present case. The demand functions are given by

$$Q_X = p(\phi_X)^{-\sigma} \cdot P^{\sigma-1} \cdot M, \qquad Q_Y = p(\phi_Y)^{-\sigma} \cdot P^{\sigma-1} \cdot M, \qquad Q = M/P, \tag{41}$$

where M denotes aggregate factor income. Finally, the factor market equilibrium conditions are given by

$$\begin{aligned}
&\Big(\phi_X \cdot w^{1-\sigma} + (1-\phi_X)\cdot r^{1-\sigma}\Big)^{\sigma/(1-\sigma)} \cdot \phi_X \cdot w^{-\sigma} \\
&\cdot\Big(\underbrace{p(\phi_X)^{-\sigma}\cdot \Big((1/N)\cdot p(\phi_X)^{1-\sigma} + (1/N)\cdot p(\phi_Y)^{1-\sigma}\Big)^{\sigma/(1-\sigma)} \cdot N^{\sigma/(1-\sigma)} \cdot Q}_{=p(\phi_X)^{-\sigma}\cdot P^{\sigma}\cdot M/P = N_X \cdot x_i = Q_X} + N_X \cdot f_X\Big) \\
&+\Big(\phi_Y \cdot w^{1-\sigma} + (1-\phi_Y)\cdot r^{1-\sigma}\Big)^{\sigma/(1-\sigma)} \cdot \phi_Y \cdot w^{-\sigma} \\
&\cdot\Big(\underbrace{p(\phi_Y)^{-\sigma}\cdot \Big((1/N)\cdot p(\phi_X)^{1-\sigma} + (1/N)\cdot p(\phi_Y)^{1-\sigma}\Big)^{\sigma/(1-\sigma)} \cdot N^{\sigma/(1-\sigma)} \cdot Q}_{=p(\phi_Y)^{-\sigma}\cdot P^{\sigma}\cdot M/P = N_Y \cdot y_i = Q_Y} + N_Y \cdot f_Y\Big) = \overline{L},
\end{aligned} \tag{42}$$

$$\begin{aligned}
&\Big(\phi_X \cdot w^{1-\sigma} + (1-\phi_X)\cdot r^{1-\sigma}\Big)^{\sigma/(1-\sigma)} \cdot (1-\phi_X) \cdot r^{-\sigma} \\
&\cdot\Big(\underbrace{p(\phi_X)^{-\sigma}\cdot \Big((1/N)\cdot p(\phi_X)^{1-\sigma} + (1/N)\cdot p(\phi_Y)^{1-\sigma}\Big)^{\sigma/(1-\sigma)} \cdot N^{\sigma/(1-\sigma)} \cdot Q}_{=p(\phi_X)^{-\sigma}\cdot P^{\sigma}\cdot M/P = N_X \cdot x_i = Q_X} + N_X \cdot f_X\Big) \\
&+\Big(\phi_Y \cdot w^{1-\sigma} + (1-\phi_Y)\cdot r^{1-\sigma}\Big)^{\sigma/(1-\sigma)} \cdot (1-\phi_Y) \cdot r^{-\sigma} \\
&\cdot\Big(\underbrace{p(\phi_Y)^{-\sigma}\cdot \Big((1/N)\cdot p(\phi_X)^{1-\sigma} + (1/N)\cdot p(\phi_Y)^{1-\sigma}\Big)^{\sigma/(1-\sigma)} \cdot N^{\sigma/(1-\sigma)} \cdot Q}_{=p(\phi_Y)^{-\sigma}\cdot P^{\sigma}\cdot M/P = N_Y \cdot y_i = Q_Y} + N_Y \cdot f_Y\Big) = \overline{K},
\end{aligned} \tag{43}$$

where the demand for labor and capital by sectors X and Y is derived by Shephard's Lemma. The terms $N_X \cdot f_X$ and $N_Y \cdot f_Y$ denote aggregate fixed investments by type–X firms and type–Y firms, respectively. Since the free entry/exit condition is fulfilled for both types of firms in general equilibrium, $Q_X = f_X \cdot (\sigma - 1)$ and $Q_Y = f_Y \cdot (\sigma - 1)$ holds. Inserting these expressions for Q_X and Q_Y into the factor market equilibrium conditions and considering that

$$\begin{aligned}
P &= \Big(\frac{1}{N}\cdot p(\phi_X)^{1-\sigma} + \frac{1}{N}\cdot p(\phi_Y)^{1-\sigma}\Big)^{1/(1-\sigma)} \cdot N^{1/(1-\sigma)} \\
&= \Big(\frac{1}{N}\cdot c(\phi_X)^{1-\sigma} + \frac{1}{N}\cdot c(\phi_Y)^{1-\sigma}\Big)^{1/(1-\sigma)} \cdot \frac{\sigma}{\sigma-1} \cdot N^{1/(1-\sigma)},
\end{aligned} \tag{44}$$

with $p(\phi_k) = \sigma/(\sigma-1) \cdot c(\phi_k)$, $k = X, Y$, results in:

$$\begin{aligned}
&\Big(\phi_X \cdot w^{1-\sigma} + (1-\phi_X) \cdot r^{1-\sigma}\Big)^{\sigma/(1-\sigma)} \cdot \phi_X \cdot w^{-\sigma} \\
&\cdot c(\phi_X)^{-\sigma} \cdot \Big((1/N) \cdot c(\phi_X)^{1-\sigma} + (1/N) \cdot c(\phi_Y)^{1-\sigma}\Big)^{\sigma/(1-\sigma)} \cdot N^{\sigma/(1-\sigma)} \cdot Q \cdot \frac{\sigma}{\sigma-1} \\
&+\Big(\phi_Y \cdot w^{1-\sigma} + (1-\phi_Y) \cdot r^{1-\sigma}\Big)^{\sigma/(1-\sigma)} \cdot \phi_Y \cdot w^{-\sigma} \\
&\cdot c(\phi_Y)^{-\sigma} \cdot \Big((1/N) \cdot c(\phi_X)^{1-\sigma} + (1/N) \cdot c(\phi_Y)^{1-\sigma}\Big)^{\sigma/(1-\sigma)} \cdot N^{\sigma/(1-\sigma)} \cdot Q \cdot \frac{\sigma}{\sigma-1} = \overline{L}, \quad (45)
\end{aligned}$$

$$\begin{aligned}
&\Big(\phi_X \cdot w^{1-\sigma} + (1-\phi_X) \cdot r^{1-\sigma}\Big)^{\sigma/(1-\sigma)} \cdot (1-\phi_X) \cdot r^{-\sigma} \\
&\cdot c(\phi_X)^{-\sigma} \cdot \Big((1/N) \cdot c(\phi_X)^{1-\sigma} + (1/N) \cdot c(\phi_Y)^{1-\sigma}\Big)^{\sigma/(1-\sigma)} \cdot N^{\sigma/(1-\sigma)} \cdot Q \cdot \frac{\sigma}{\sigma-1} \\
&+\Big(\phi_Y \cdot w^{1-\sigma} + (1-\phi_Y) \cdot r^{1-\sigma}\Big)^{\sigma/(1-\sigma)} \cdot (1-\phi_Y) \cdot r^{-\sigma} \\
&\cdot c(\phi_Y)^{-\sigma} \cdot \Big((1/N) \cdot c(\phi_X)^{1-\sigma} + (1/N) \cdot c(\phi_Y)^{1-\sigma}\Big)^{\sigma/(1-\sigma)} \cdot N^{\sigma/(1-\sigma)} \cdot Q \cdot \frac{\sigma}{\sigma-1} = \overline{K}, \quad (46)
\end{aligned}$$

However, if it is considered that

$$\Big(\phi_X \cdot w^{1-\sigma} + (1-\phi_X) \cdot r^{1-\sigma}\Big)^{\sigma/(1-\sigma)} = c(\phi_X)^{\sigma} \quad (47)$$

$$\Big(\phi_Y \cdot w^{1-\sigma} + (1-\phi_Y) \cdot r^{1-\sigma}\Big)^{\sigma/(1-\sigma)} = c(\phi_Y)^{\sigma}, \quad (48)$$

the factor market equilibrium conditions can be simplified further to

$$\begin{aligned}
&\phi_X \cdot w^{-\sigma} \cdot \left(\frac{1}{N} \cdot c(\phi_X)^{1-\sigma} + \frac{1}{N} \cdot c(\phi_Y)^{1-\sigma}\right)^{\sigma/(1-\sigma)} \cdot N^{\sigma/(1-\sigma)} \cdot Q \cdot \frac{\sigma}{\sigma-1} \\
&+\phi_Y \cdot w^{-\sigma} \cdot \left(\frac{1}{N} \cdot c(\phi_X)^{1-\sigma} + \frac{1}{N} \cdot c(\phi_Y)^{1-\sigma}\right)^{\sigma/(1-\sigma)} \cdot N^{\sigma/(1-\sigma)} \cdot Q \cdot \frac{\sigma}{\sigma-1} = \overline{L} \quad (49)
\end{aligned}$$

and

$$\begin{aligned}
&(1-\phi_X) \cdot r^{-\sigma} \cdot \left(\frac{1}{N} \cdot c(\phi_X)^{1-\sigma} + \frac{1}{N} \cdot c(\phi_Y)^{1-\sigma}\right)^{\sigma/(1-\sigma)} \cdot N^{\sigma/(1-\sigma)} \cdot Q \cdot \frac{\sigma}{\sigma-1} \\
&+(1-\phi_Y) \cdot r^{-\sigma} \cdot \left(\frac{1}{N} \cdot c(\phi_X)^{1-\sigma} + \frac{1}{N} \cdot c(\phi_Y)^{1-\sigma}\right)^{\sigma/(1-\sigma)} \cdot N^{\sigma/(1-\sigma)} \cdot Q \cdot \frac{\sigma}{\sigma-1} = \overline{K}. \quad (50)
\end{aligned}$$

Furthermore, the expression $((1/N) \cdot c(\phi_X)^{1-\sigma} + (1/N) \cdot c(\phi_Y)^{1-\sigma})^{\sigma/(1-\sigma)} \cdot N^{\sigma/(1-\sigma)} \cdot Q$ in the factor market equilibrium conditions can be simplified as well: inserting the expressions for $c(\phi_X)$ and $c(\phi_Y)$ into equation (44) gives

$$\begin{aligned}
P = &\left(\frac{1}{N} \cdot \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \cdot \Big(\phi_X \cdot w^{1-\sigma} + (1-\phi_X) \cdot r^{1-\sigma}\Big)\right. \\
&\left.+\frac{1}{N} \cdot \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \cdot \Big(\phi_Y \cdot w^{1-\sigma} + (1-\phi_Y) \cdot r^{1-\sigma}\Big)\right)^{1/(1-\sigma)} \cdot N^{1/(1-\sigma)}, \quad (51)
\end{aligned}$$

which leads to

$$P = \frac{\sigma}{\sigma - 1} \cdot \left(\widetilde{\phi} \cdot w^{1-\sigma} + (1 - \widetilde{\phi}) \cdot r^{1-\sigma}\right)^{1/(1-\sigma)} \cdot N^{1/(1-\sigma)} = \left(N \cdot p(\widetilde{\phi})^{1-\sigma}\right)^{1/(1-\sigma)} \quad (52)$$

if $\widetilde{\phi} \equiv \Big((1/N) \cdot \phi_X + (1/N) \cdot \phi_Y\Big)$ and $1 - \widetilde{\phi} \equiv \Big((1/N) \cdot (1 - \phi_X) + (1/N) \cdot (1 - \phi_Y)\Big)$. Therefore,

$$\begin{aligned}((1/N) \cdot c(\phi_X)^{1-\sigma} + (1/N) \cdot c(\phi_Y)^{1-\sigma})^{\sigma/(1-\sigma)} \cdot N^{\sigma/(1-\sigma)} \cdot Q \; &= \\ \left(\widetilde{\phi} \cdot w^{1-\sigma} + (1 - \widetilde{\phi}) \cdot r^{1-\sigma}\right)^{\sigma/(1-\sigma)} &\cdot \frac{M}{P} \cdot N^{\sigma/(1-\sigma)}. \quad (53)\end{aligned}$$

Inserting the expression for P, which is derived in equation (52), gives

$$\begin{aligned}&\left(\widetilde{\phi} \cdot w^{1-\sigma} + (1 - \widetilde{\phi}) \cdot r^{1-\sigma}\right)^{\sigma/(1-\sigma)} \cdot \frac{M}{P} \cdot N^{\sigma/(1-\sigma)} \; = \\ &\frac{\left(\widetilde{\phi} \cdot w^{1-\sigma} + (1 - \widetilde{\phi}) \cdot r^{1-\sigma}\right)^{\sigma/(1-\sigma)} \cdot M \cdot N^{\sigma/(1-\sigma)}}{\frac{\sigma}{\sigma-1} \cdot \left(\widetilde{\phi} \cdot w^{1-\sigma} + (1 - \widetilde{\phi}) \cdot r^{1-\sigma}\right)^{1/(1-\sigma)} \cdot N^{1/(1-\sigma)}} = \frac{\frac{\sigma-1}{\sigma} \cdot M \cdot 1/N}{\widetilde{\phi} \cdot w^{1-\sigma} + (1 - \widetilde{\phi}) \cdot r^{1-\sigma}}. \quad (54)\end{aligned}$$

If $c(\widetilde{\phi})$ denotes the marginal costs of the average firm, the following equation holds:

$$c(\widetilde{\phi})^{\sigma-1} \;=\; \left(\widetilde{\phi} \cdot w^{1-\sigma} + (1 - \widetilde{\phi}) \cdot r^{1-\sigma}\right)^{-1}. \quad (55)$$

The factor market equilibrium conditions can therefore be simplified to

$$c(\widetilde{\phi})^{\sigma-1} \cdot M \cdot \left(\phi_X \cdot w^{-\sigma} \cdot \frac{1}{N} + \phi_Y \cdot w^{-\sigma} \cdot \frac{1}{N}\right) \;=\; \overline{L} \quad (56)$$

$$\text{and} \quad c(\widetilde{\phi})^{\sigma-1} \cdot M \cdot \left((1 - \phi_X) \cdot r^{-\sigma} \cdot \frac{1}{N} + (1 - \phi_Y) \cdot r^{-\sigma} \cdot \frac{1}{N}\right) \;=\; \overline{K}. \quad (57)$$

However, as $\widetilde{\phi} = (\phi_X + \phi_Y)/N$, the factor market equilibrium conditions result in

$$\widetilde{\phi} \cdot w^{-\sigma} \cdot c(\widetilde{\phi})^{\sigma-1} \cdot M \;=\; \overline{L} \quad (58)$$

$$\text{and} \quad (1 - \widetilde{\phi}) \cdot r^{-\sigma} \cdot c(\widetilde{\phi})^{\sigma-1} \cdot M \;=\; \overline{K}. \quad (59)$$

Obviously, with respect to the production side, the monopolistic competition model with $N_X + N_Y = N$ heterogeneous firms is identical to a model with a single perfectly competitive average firm. The average firm faces a demand of $Q(\widetilde{\phi}) = M/c(\widetilde{\phi})$. Both models therefore lead to identical factor prices and an identical aggregate factor income.

However, in order to return to a model with monopolistic competition, the single average firm has to be split into $\widetilde{N}$ monopolistic competitive average firms. The equilibrium mass of average firms can be determined with the help of the free entry/exit condition of the average firm.

Total per period profits of a single firm i in the original disaggregated model with heterogeneous firms are given by $\pi_i = R(\phi_i)/\sigma - c(\phi_i) \cdot f$. The economy is in an equilibrium, i. e., no additional firms enter the market, whenever the expected total profits of a potential entrant are equal to zero, i. e., if $\int_0^{\phi^*} 1/\phi^* \cdot (R(\phi)/\sigma - c(\phi) \cdot f) d\phi = 0$. It can be shown that the expected total profits of a potential entrant reduce to the total profits of a firm with the average labor share parameter $\widetilde{\phi}$, $\pi(\widetilde{\phi}) = R(\widetilde{\phi})/\sigma - c(\widetilde{\phi}) \cdot f$, since the number of firms goes to infinity. First of all, it is straightforward to show that

$$\frac{R(\widetilde{\phi})}{\sigma} = \int_0^{\phi^*} \frac{1}{\phi^*} \cdot \frac{R(\phi)}{\sigma} d\phi, \tag{60}$$

with $\widetilde{\phi} = \phi^*/2$ as ϕ is assumed to be uniformly distributed over the interval $[0, 1]$. Since this equality also holds for a finite number of firms, it will be explained for the case of two firms for simplicity of exposition. Firm 1 is assumed to produce with $\phi_1 = 0$ and firm 2 is assumed to produce with $\phi_2 = \phi^*$. $\widetilde{\phi}$ is equal to $\phi^*/2 = (0 + \phi^*)/2$. Using $R(\phi_i)/\sigma = 1/\sigma \cdot P^{\sigma-1} \cdot M \cdot (p(\phi_i))^{1-\sigma}$, the following holds:[24]

$$\begin{aligned} \frac{R(\widetilde{\phi})}{\sigma} &= \frac{1}{\sigma} \cdot P^{\sigma-1} \cdot M \cdot (p(\widetilde{\phi}))^{1-\sigma} \\ &= \frac{1}{\sigma} \cdot P^{\sigma-1} \cdot M \cdot \Big(0.5 \cdot p(\phi_1 = 0)^{1-\sigma} + 0.5 \cdot p(\phi_2 = \phi^*)^{1-\sigma}\Big), \end{aligned} \tag{61}$$

where the second equality is the critical one. Since the price only includes a constant markup over the per unit costs, the second equality is fulfilled if

$$c(\widetilde{\phi})^{1-\sigma} = 0.5 \cdot \Big(c(\phi_1 = 0)^{1-\sigma} + c(\phi_2 = \phi^*)^{1-\sigma}\Big). \tag{62}$$

Equation (62) holds since

$$c(\widetilde{\phi})^{1-\sigma} = \widetilde{\phi} \cdot w^{1-\sigma} + (1 - \widetilde{\phi}) \cdot r^{1-\sigma} \quad \text{and} \tag{63}$$

$$\begin{aligned} 0.5 \cdot \Big(c(\phi_1)^{1-\sigma} + c(\phi_2)^{1-\sigma}\Big) &= 0.5 \cdot \Big(\phi_1 \cdot w^{1-\sigma} + (1 - \phi_1) \cdot r^{1-\sigma}\Big) \\ &\quad + 0.5 \cdot \Big(\phi_2 \cdot w^{1-\sigma} + (1 - \phi_2) \cdot r^{1-\sigma}\Big) \\ &= 0.5 \cdot (\phi_1 + \phi_2) \cdot w^{1-\sigma} + 0.5 \cdot (2 - \phi_1 - \phi_2) \cdot r^{1-\sigma} \\ &= \widetilde{\phi} \cdot w^{1-\sigma} + (1 - \widetilde{\phi}) \cdot r^{1-\sigma}, \end{aligned} \tag{64}$$

[24]Note that equations (44) and (52) demonstrate that the price indices in the disaggregated and the aggregated model, P and $\widetilde{P}$, respectively, are identical.

with $\widetilde{\phi} = \phi^*/2 = (0 + \phi^*)/2$.
Second, it has to be shown that

$$c(\widetilde{\phi}) \cdot f \;=\; f \cdot \int_0^{\phi^*} \frac{1}{\phi^*} \cdot c(\phi) d\phi \;\equiv\; \widetilde{c}(\phi) \cdot f. \tag{65}$$

Equation (65) only holds approximately for a finite number of firms, but it holds exactly if the number of firms approaches infinity. Therefore, it has to be shown that

$$c(\widetilde{\phi}) \cdot \Upsilon \;=\; \lim_{N \to \infty} \frac{1}{N} \cdot \sum_{i=1}^{N} c(\phi_i) \;=\; \int_0^{\phi^*} \frac{1}{\phi^*} \cdot c(\phi) d\phi \;=\; \widetilde{c}(\phi), \tag{66}$$

where the second equality is due to the definition of a Riemann integral and Υ is a constant. First of all, since $\sigma = 2$ and $w = 1$ the cost function of firm i results as

$$c(\phi_i) \;=\; \Big(\phi_i + (1 - \phi_i) \cdot r^{1-\sigma}\Big)^{1/(1-\sigma)} \;=\; \Big(\phi_i \cdot (1 - 1/r) + 1/r\Big)^{-1}. \tag{67}$$

Therefore,

$$\begin{aligned} \frac{1}{\phi^*} \cdot \int_0^{\phi^*} c(\phi) d\phi \;=\; \widetilde{c}(\phi) \;&=\; \frac{1}{\phi^*} \cdot \int_0^{\phi^*} \Big(\phi \cdot (1 - 1/r) + 1/r\Big)^{-1} d\phi \\ &=\; \frac{1}{\phi^*} \cdot \frac{1}{1 - 1/r} \cdot \Big[\ln\left(\phi \cdot (1 - 1/r) + 1/r\right)\Big]_0^{\phi^*} \\ &=\; \frac{1}{\phi^*} \cdot \frac{1}{1 - 1/r} \cdot \Big(\ln\left(\phi^* \cdot (1 - 1/r) + 1/r\right) - \ln(1/r)\Big). \end{aligned} \tag{68}$$

Furthermore, subsection 2.4 shows that the steady state value of the capital rental rate is given by

$$r \;=\; \frac{\rho + \delta - 1 + \phi^*/2}{\phi^*/2} \;=\; \frac{2 \cdot a + \phi^*}{\phi^*} \quad \text{if } a \equiv \rho + \delta - 1. \tag{69}$$

It follows that

$$1 - \frac{1}{r} \;=\; 1 - \frac{1}{(2 \cdot a + \phi^*)/\phi^*} \;=\; \frac{2 \cdot a}{2 \cdot a + \phi^*}. \tag{70}$$

Therefore, equation (68) can be transformed to give

$$\begin{aligned} \widetilde{c}(\phi) \;&=\; \frac{1}{\phi^*} \cdot \frac{2 \cdot a + \phi^*}{2 \cdot a} \cdot \left(\ln\left(\phi^* \cdot \frac{2 \cdot a}{2 \cdot a + \phi^*} + \frac{\phi^*}{2 \cdot a + \phi^*}\right) - \ln \frac{\phi^*}{2 \cdot a + \phi^*}\right) \\ &=\; \frac{2 \cdot a + \phi^*}{2 \cdot a \cdot \phi^*} \cdot \left(\ln \frac{\phi^* \cdot (2 \cdot a + 1)}{2 \cdot a + \phi^*} - \ln \frac{\phi^*}{2 \cdot a + \phi^*}\right) \\ &=\; \frac{2 \cdot a + \phi^*}{2 \cdot a \cdot \phi^*} \cdot \ln \frac{\phi^* \cdot (2 \cdot a + 1)}{\phi^*}. \end{aligned} \tag{71}$$

The ratio $\widetilde{c}(\phi)/c(\widetilde{\phi})$ therefore results as

$$\begin{aligned}\frac{\widetilde{c}(\phi)}{c(\widetilde{\phi})} &= \frac{(2\cdot a+2\cdot\widetilde{\phi})/(2\cdot a\cdot 2\cdot\widetilde{\phi})\cdot\ln(2\cdot a+1)}{(2\cdot a+2\cdot\widetilde{\phi})/(2\cdot\widetilde{\phi}\cdot(a+1))} \\ &= \frac{\rho+\delta}{2\cdot(\rho+\delta-1)}\cdot\ln(2\cdot\rho+2\cdot\delta-1) \;=\; \Upsilon. \qquad (72)\end{aligned}$$

Most importantly, the constant Υ does not depend on ϕ^*. Υ therefore can be omitted from the analysis without changing the comparative steady state results.

B Steady state factor prices

The dual to the household's restricted maximization problem as described in section 2.4 leads to several zero profit conditions. Most importantly, the zero profit conditions (*Z.P.C.*) for investment, the capital rental activity and the average good are sufficient to determine the relative factor prices in the steady state:[25]

$$Z.P.C.\text{ for investment:}\quad p(\widetilde{\phi}_t) = \frac{1}{1+\rho}\cdot p_t^K \text{ and } p(\widetilde{\phi}_t) = p_{t+1}^K \qquad (73)$$

$$Z.P.C.\text{ for capital rental activity:}\quad p_t^K = r_t+(1-\delta)\cdot p_{t+1}^K \qquad (74)$$

$$Z.P.C.\text{ for average/investment good:}\quad p(\widetilde{\phi}_t) = \left(\widetilde{\phi}\cdot w_t^{1-\sigma}+(1-\widetilde{\phi})\cdot r_t^{1-\sigma}\right)^{1/(1-\sigma)}. \qquad (75)$$

$p(\widetilde{\phi}_t)$ denotes the price of the average/investment good and p_t^K and p_{t+1}^K the price per unit of capital in period t and $t+1$. Although $\widetilde{\phi}$ is not fixed in this model with heterogeneous firms, it is written without an index t to simplify the exposition. Manipulation of equations (73) and (74) and transforming equation (75) leads to

$$r_t = (\rho+\delta)\cdot p(\widetilde{\phi}_t) \qquad (76)$$

$$w_t = \left(\frac{p(\widetilde{\phi}_t)^{1-\sigma}}{\widetilde{\phi}} - \frac{1-\widetilde{\phi}}{\widetilde{\phi}}\cdot r_t^{1-\sigma}\right)^{1/(1-\sigma)}. \qquad (77)$$

Solving equations (76) and (77) for the relative price of labor gives

$$\frac{w_t}{r_t} = \left(\frac{(\rho+\delta)^{\sigma-1}-1+\widetilde{\phi}}{\widetilde{\phi}}\right)^{1/(1-\sigma)}. \qquad (78)$$

Therefore, the relative price of labor only depends on the model parameters and on the average labor share parameter.

[25] Cf. for this result Baxter (1992), pp. 737–739, and Lau et al. (2002), pp. 595–596.

C The equivalence between equations (23) and (26)

The *expected* variable profits from market entry in the open economy equal the *average* variable profits over all active firms in the open economy, multiplied by $G(\phi_H^*)$. The value $G(\phi_H^*)$ equals the probability of being active after market entry. The average variable profits over all active firms in the open economy result from the weighted sum of the variable profits of the average exporting firm, $R_H(\widetilde{\phi}_{ExH}) \cdot (1+\tau^{1-\sigma})/\sigma$, and the variable profits of the average non–exporting firm in the open economy, $R_H(\widetilde{\phi}_{NH})/\sigma$. The expected variable profits from market entry in the open economy therefore result as

$$
\begin{aligned}
&G(\phi_H^*) \cdot \frac{R_H(\widetilde{\phi}_{ExH}) \cdot (1+\tau^{1-\sigma})}{\sigma} \cdot \frac{\widetilde{N}_{ExH}/(1+\tau^{1-\sigma})}{\frac{\widetilde{N}_{ExH}}{1+\tau^{1-\sigma}} + \widetilde{N}_{NH}} + G(\phi_H^*) \cdot \frac{R_H(\widetilde{\phi}_{NH})}{\sigma} \cdot \frac{\widetilde{N}_{NH}}{\frac{\widetilde{N}_{ExH}}{1-\tau^{1-\sigma}} + \widetilde{N}_{NH}} \\
&= G(\phi_H^*) \cdot \left(\frac{R_H(\widetilde{\phi}_{ExH})}{\sigma} \cdot \frac{\widetilde{N}_{ExH}}{\widetilde{N}_{ExH} + \widetilde{N}_{NH}} + \frac{R_H(\widetilde{\phi}_{NH})}{\sigma} \cdot \frac{\widetilde{N}_{NH}}{\widetilde{N}_{ExH} + \widetilde{N}_{NH}} \right) \cdot \frac{\widetilde{N}_{ExH} + \widetilde{N}_{NH}}{\frac{\widetilde{N}_{ExH}}{1+\tau^{1-\sigma}} + \widetilde{N}_{NH}},
\end{aligned}
\tag{79}
$$

where the weighting factors $\widetilde{N}_{ExH}/(1+\tau^{1-\sigma}) \Big/ \Big(\widetilde{N}_{ExH}/(1+\tau^{1-\sigma}) + \widetilde{N}_{NH}\Big)$ and $\widetilde{N}_{NH} \Big/ \Big(\widetilde{N}_{ExH}/(1+\tau^{1-\sigma}) + \widetilde{N}_{NH}\Big)$ denote the relative importance of average domestic exporting and non–exporting firms, respectively. The bracket on the right hand side of equation (79) is equal to $R_H(\widetilde{\phi}_H)/\sigma$, with $\widetilde{\phi}_H = \widetilde{\phi}_{ExH} \cdot \phi_{ExH}^*/\phi_H^* + \widetilde{\phi}_{NH} \cdot (1 - \phi_{ExH}^*/\phi_H^*)$, due to the aggregation procedure as described in appendix A. Compared to autarky, exposure to trade therefore increases the variable profits of the average active firm by the factor $\Big(\widetilde{N}_{ExH} + \widetilde{N}_{NH}\Big) \Big/ \Big(\widetilde{N}_{ExH}/(1+\tau^{1-\sigma}) + \widetilde{N}_{NH}\Big)$.

Second, total *expected* fixed costs from market entry in the open economy equal the *average* per period fixed costs over all active firms in the open economy, multiplied by $G(\phi_H^*)$, plus the average sunk market entry costs over all active firms in the open economy. Using the same weighting factors as for equation (79), total expected fixed costs from market entry in the open economy result as

$$
\begin{aligned}
&\Big(f_M + G(\phi_H^*) \cdot f\Big) \cdot \left(c(\widetilde{\phi}_{ExH}) \cdot \frac{\widetilde{N}_{ExH}/(1+\tau^{1-\sigma})}{\frac{\widetilde{N}_{ExH}}{1+\tau^{1-\sigma}} + \widetilde{N}_{NH}} + c(\widetilde{\phi}_{NH}) \cdot \frac{\widetilde{N}_{NH}}{\frac{\widetilde{N}_{ExH}}{1+\tau^{1-\sigma}} + \widetilde{N}_{NH}} \right) \\
&\quad + G(\phi_H^*) \cdot c(\widetilde{\phi}_{ExH}) \cdot f_{Ex} \cdot \frac{\widetilde{N}_{ExH}/(1+\tau^{1-\sigma})}{\frac{\widetilde{N}_{ExH}}{1+\tau^{1-\sigma}} + \widetilde{N}_{NH}} \\
&= \Big(f_M + G(\phi_H^*) \cdot f\Big) \cdot \left(c(\widetilde{\phi}_{ExH}) \cdot \frac{\widetilde{N}_{ExH}}{\widetilde{N}_{ExH} + \widetilde{N}_{NH}} + c(\widetilde{\phi}_{NH}) \cdot \frac{\widetilde{N}_{NH}}{\widetilde{N}_{ExH} + \widetilde{N}_{NH}} \right) \cdot \frac{\widetilde{N}_{ExH} + \widetilde{N}_{NH}}{\frac{\widetilde{N}_{ExH}}{1+\tau^{1-\sigma}} + \widetilde{N}_{NH}}
\end{aligned}
$$

$$+\Big(f_M + G(\phi_H^*)\cdot f\Big)\cdot\left(c(\widetilde{\phi}_{ExH})\cdot\frac{\widetilde{N}_{ExH}/(1+\tau^{1-\sigma})}{\frac{\widetilde{N}_{ExH}}{1+\tau^{1-\sigma}}+\widetilde{N}_{NH}} - c(\widetilde{\phi}_{ExH})\cdot\frac{\widetilde{N}_{ExH}}{\frac{\widetilde{N}_{ExH}}{1+\tau^{1-\sigma}}+\widetilde{N}_{NH}}\right)$$

$$+G(\phi_H^*)\cdot c(\widetilde{\phi}_{ExH})\cdot f_{Ex}\cdot\frac{\widetilde{N}_{ExH}/(1+\tau^{1-\sigma})}{\frac{\widetilde{N}_{ExH}}{1+\tau^{1-\sigma}}+\widetilde{N}_{NH}}. \quad (80)$$

Due to the aggregation procedure as described in appendix A, the right hand side of equation (80) can be simplified to

$$\Big(f_M + G(\phi_H^*)\cdot f\Big)\cdot c(\widetilde{\phi}_H)\cdot\frac{\widetilde{N}_{ExH}+\widetilde{N}_{NH}}{\frac{\widetilde{N}_{ExH}}{1+\tau^{1-\sigma}}+\widetilde{N}_{NH}}$$

$$+c(\widetilde{\phi}_{ExH})\cdot\frac{\widetilde{N}_{ExH}}{\frac{\widetilde{N}_{ExH}}{1+\tau^{1-\sigma}}+\widetilde{N}_{NH}}\cdot\left(\frac{-\tau^{1-\sigma}}{1+\tau^{1-\sigma}}\cdot\Big(f_M+G(\phi_H^*)\cdot f\Big)+G(\phi_H^*)\cdot\frac{f_{Ex}}{1+\tau^{1-\sigma}}\right).$$

Therefore, if $G(\phi_H^*)$ is set equal to ϕ_H^* due to the uniform distribution of ϕ on $[0,1]$, equation (23) can alternatively be written as

$$\frac{R_H(\widetilde{\phi}_H)}{\sigma}\cdot\frac{\widetilde{N}_{ExH}+\widetilde{N}_{NH}}{\frac{\widetilde{N}_{ExH}}{1+\tau^{1-\sigma}}+\widetilde{N}_{NH}} = \left(\frac{f_M}{2\cdot\widetilde{\phi}_H}+f\right)\cdot c(\widetilde{\phi}_H)\cdot\frac{\widetilde{N}_{ExH}+\widetilde{N}_{NH}}{\frac{\widetilde{N}_{ExH}}{1+\tau^{1-\sigma}}+\widetilde{N}_{NH}}$$

$$+c(\widetilde{\phi}_{ExH})\cdot\frac{\widetilde{N}_{ExH}}{\frac{\widetilde{N}_{ExH}}{1+\tau^{1-\sigma}}+\widetilde{N}_{NH}}\cdot\left(\frac{-\tau^{1-\sigma}}{1+\tau^{1-\sigma}}\cdot\left(\frac{f_M}{2\cdot\widetilde{\phi}_H}+f\right)+\frac{f_{Ex}}{1+\tau^{1-\sigma}}\right). \quad (81)$$

Using $\sigma = 2$ from assumption (A1), $R_H(\widetilde{\phi}_H) = \sigma/(\sigma-1)\cdot c(\widetilde{\phi}_H)\cdot q_{HH}(\widetilde{\phi}_H)$ and the fact that $\phi_{ExH}^*/\phi_H^* = \widetilde{N}_{ExH}/(\widetilde{N}_{ExH}+\widetilde{N}_{NH})$ for any given ϕ_H^*, equation (81) reduces to equation (26):

$$q_{HH}(\widetilde{\phi}_H) - f - \frac{\phi_{ExH}^*}{\phi_H^*}\cdot\frac{c(\widetilde{\phi}_{ExH})}{c(\widetilde{\phi}_H)}\cdot\left(\frac{-\tau^{1-\sigma}}{1+\tau^{1-\sigma}}\cdot\left(\frac{f_M}{2\cdot\widetilde{\phi}_H}+f\right)+\frac{f_{Ex}}{1+\tau^{1-\sigma}}\right) = \frac{f_M}{2\cdot\widetilde{\phi}_H}. \quad (82)$$

D Average capital share parameter and steady state welfare

This appendix demonstrates that a country's steady state welfare reacts positively to an increase in the average capital share parameter. Aggregate consumption is given by $Q_H^k = M_{CH}^k\cdot\widetilde{N}_H^k/p(\widetilde{\phi}_H^k)$, $k = Aut., FT$. The components of Q_H^k in the steady state are defined in subsection 2.4. The partial derivatives of M_{CH}^k, $\widetilde{N}_H^k$ and $p(\widetilde{\phi}_H^k)$ with respect to the average labor share parameter $\widetilde{\phi}_H^k$ result as follows:

$$\frac{\partial M_{CH}^k}{\partial\widetilde{\phi}_H^k} = \overline{L}_H\cdot\frac{\delta\cdot(\rho+\delta)\cdot(\rho+\delta-1+\widetilde{\phi}_H^k)-((\rho+\delta)^2-\delta+\delta\cdot\widetilde{\phi}_H^k)\cdot(\rho+\delta)}{(\rho+\delta)^2\cdot(\rho+\delta-1+\widetilde{\phi}_H^k)^2} < 0,$$

$$\text{as } 0 < \rho^2+\rho\cdot\delta \quad (83)$$

$$
\begin{aligned}
\frac{\partial p(\widetilde{\phi}_H^k)}{\partial \widetilde{\phi}_H^k} &= \frac{\sigma}{\sigma-1} \cdot \frac{\widetilde{\phi}_H^k \cdot (\rho+\delta) - (\rho+\delta-1+\widetilde{\phi}_H^k) \cdot (\rho+\delta)}{(\widetilde{\phi}_H^k \cdot (\rho+\delta))^2} \\
&= \frac{\sigma}{\sigma-1} \cdot (\rho+\delta) \cdot \frac{1-(\rho+\delta)}{(\widetilde{\phi}_H^k \cdot (\rho+\delta))^2} \; > \; 0, \text{ as } 1 \; > \; \rho+\delta \text{ by (A3)} \qquad (84) \\
\frac{\partial \widetilde{N}_H^k}{\partial \widetilde{\phi}_H^k} &= \frac{\overline{L}_H \cdot (1-1/\sigma)}{q(\widetilde{\phi}_H^k)} \cdot (\rho+\delta)^2 \cdot (\rho+\delta-1+\widetilde{\phi}_H^k) \cdot \frac{\rho+\delta-1-\widetilde{\phi}_H^k}{(\rho+\delta-1+\widetilde{\phi}_H^k)^4} \; < \; 0, \\
&\qquad \text{as } \rho+\delta > 1-\widetilde{\phi}_H^k \text{ by (A2) and } 1 > \rho+\delta \text{ by (A3).} \qquad (85)
\end{aligned}
$$

These results imply that $\partial Q_H^k / \partial(1-\widetilde{\phi}_H^k) > 0$, $k = Aut., FT$.

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