Beyond Modern Portfolio Theory to Modern Investment Technology

Contingent Claims Analysis and Life-Cycle Finance

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ABSTRACT

This paper explores the application of contingent claims analysis (CCA) to two "hot" issues in life-cycle finance: (1) investing for retirement and (2) deciding when, if ever, to switch careers. Participants in individual retirement accounts do not have the time or the knowledge to make their own investment decisions. Today they are defaulted into life-cycle mutual funds that pass all risk directly through to the participant. We use CCA to demonstrate how financial firms can design and produce guaranteed contingent benefit contracts that improve participant welfare at no additional cost to the system. These contracts combine features of traditional annuities (i.e., lifelong income benefits) and structured investment products (e.g., guaranteed minimum plus "upside" equity participation). In exploring the career-choice issue in the second part of the paper, we use CCA in a somewhat different way. The decision to switch careers is analogous to deciding when to exercise an American-style option to swap one asset for another. By applying the methods used to analyze the option-exercise decision to the career-switching problem, we gain some new insights beyond those derived from the traditional dynamic programming approaches.

Introduction

This paper explores the application of contingent claims analysis (CCA) to two important issues in life-cycle finance: (1) investing for retirement and (2) deciding when, if ever, to switch careers. Contingent claims analysis is a methodology that grew out of the option pricing theory of Black and Scholes (1973) and Merton (1973). They derived the option pricing model by showing that there is a self-financing dynamic trading strategy that replicates the payoffs from a call option. In the absence of transaction costs, the law of one price and the force of arbitrage imply that the cost of the initial replicating portfolio is the price of the option. That same approach applies to any derivative security or contingent contract based on traded assets. For every dynamic trading strategy there exists an equivalent contingent contract.[1]

In reality most investors face substantial transactions costs and cannot trade even approximately continuously as done in the theoretical models. But in a modern, well-developed financial system, the lowest-cost transactors may have marginal trading costs *close to* zero, and can trade *almost* continuously. Thus, the lowest-cost producers of contingent contracts can approximate reasonably well the dynamic trading strategy, and in a competitive environment their cost of replicating the contingent payoffs is approximately the theoretical price.[2]

In the first part of this paper, we make use of this idea to address the problem faced by consumers saving and investing for retirement. Today's retirement saving plans consist of mutual funds that pass all investment risk through to plan participants. They are being replaced by semi-customized contingent contracts designed and guaranteed by financial intermediaries. From contingent-contract-equivalence it follows that a low-transaction-cost financial

[1]This contingent-contract equivalence to dynamic portfolio strategies can be derived by running the option-pricing derivation "in reverse." This type of procedure is developed in Haugh and Lo (2001) and Merton (1989, pp. 250–254, 1992, pp. 450–464).

[2] For a more complete development of this theme, see Merton (1989) and Merton and Bodie (2005).

intermediary can sell to high-transaction-cost customers fully hedged ("immunized") contracts that have the contingent payoffs associated with an optimized portfolio strategy. The intermediary pursues the dynamic trading strategy at its lower transaction costs and provides the specified contractual benefits to its customers.

In part 2 of this paper we apply CCA to the issue of career choices. The ability to change careers over one's working life is isomorphic with an option to exchange one asset for another. Having this option significantly affects the value of one's human capital, the largest asset that most individuals possess over much of their adult lives. Choosing the optimal strategy involves selecting an initial career and then switching later in life, if the conditions warrant it. Moreover, the switching option affects the consumer's optimal portfolio mix of safe and risky assets. CCA provides a methodology for studying and evaluating all of these effects.

For the sake of clarity of exposition, throughout this paper we illustrate the main ideas using simple binomial models.[3] To implement the kind of financial engineering required in the real world, however, much more mathematically complex models are required.[4]

Part 1. Retirement Saving Plans and Contingent Contracts

One need not have a crystal ball to envision a future in which financial firms produce personal retirement benefit contracts. The process is already underway. Several major financial firms have launched products of this sort, including one that is called Personal Pension Builder™.[5] Figure 1 illustrates the design and production of such contracts as a three stage process.[6] In stage 1 the firm determines its production cost schedule (i.e., the cost of state-contingent consumption) taking as inputs the prices of stocks, bonds, and other securities traded

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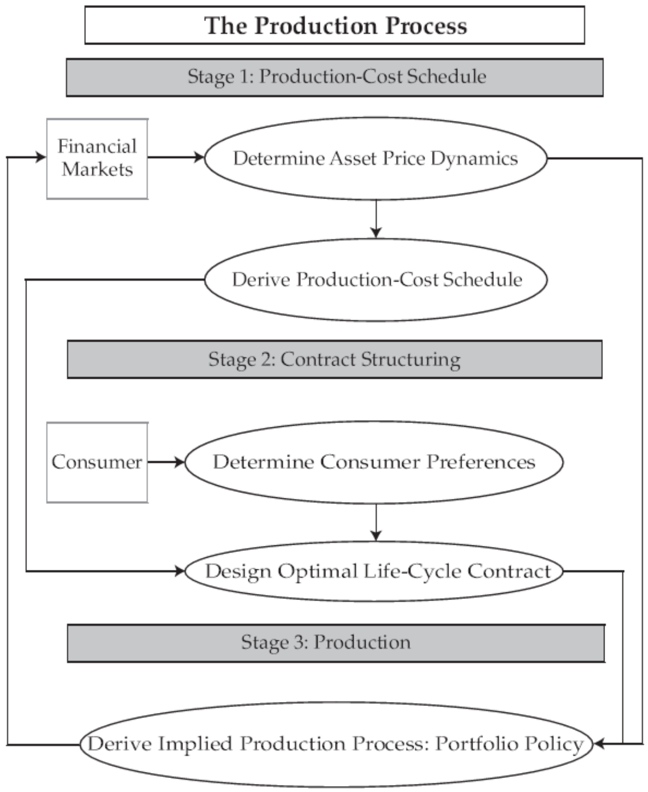
[3] For a full treatment of binomial models applied to options, see Cox, Ross, and Rubinstein (1979).
[4] For an introduction to advanced mathematical techniques applied to CCA, see Treussard (2008).
[5] MetLife "Creating Lifelong Income" at http://www.whymetlife.com/broker/download.asp?view=63 .
[6] This figure is taken from Bodie, Treussard, and Willen (2007).

in markets. Effectively, it computes implicit prices for elementary time-state securities, also known as Arrow–Debreu prices (Arrow 1953; Debreu 1959).

Figure 1

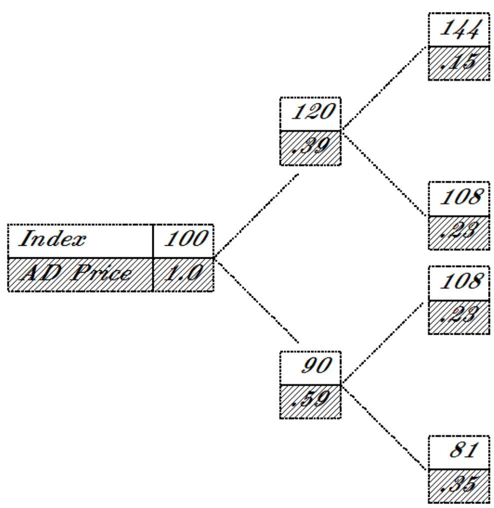


In stage 2, the firm designs "standard-model" contracts, by maximizing expected utility for "representative" consumers in various categories (e.g., single versus married) under a set of circumstances that are faced by most of them, such as the prevailing tax code and the mandatory social insurance system. As is routinely done today with insurance policies, these standard-model contracts can be modified by adding optional features to suit the special circumstances of different types of consumers. After the contract specifications are finalized, the firm commences stage 3, the process of dynamic replication.

Credible contract guarantees are an essential feature of these products.[7] To assure customers that their contingent benefits will not be subject to the risk of contract default, the firm must have sufficient risk capital. Regulation by the government or "self-regulatory" industry bodies may be desirable to ensure that this is so.

To clarify the design and production process for such personal pension contracts, consider the simple binomial model illustrated in Figure 2. There is a single consumption good, and prices and interest rates are measured in units of that good. There are three periods separated by three points in time: t = 0, 1, 2. There is a single risky asset – a stock, and in each period it can either go up by *u* or down by *d*. Each has equal probability of .5. In addition there is a safe asset that pays a fixed real rate of interest, *r*.

Figure 2



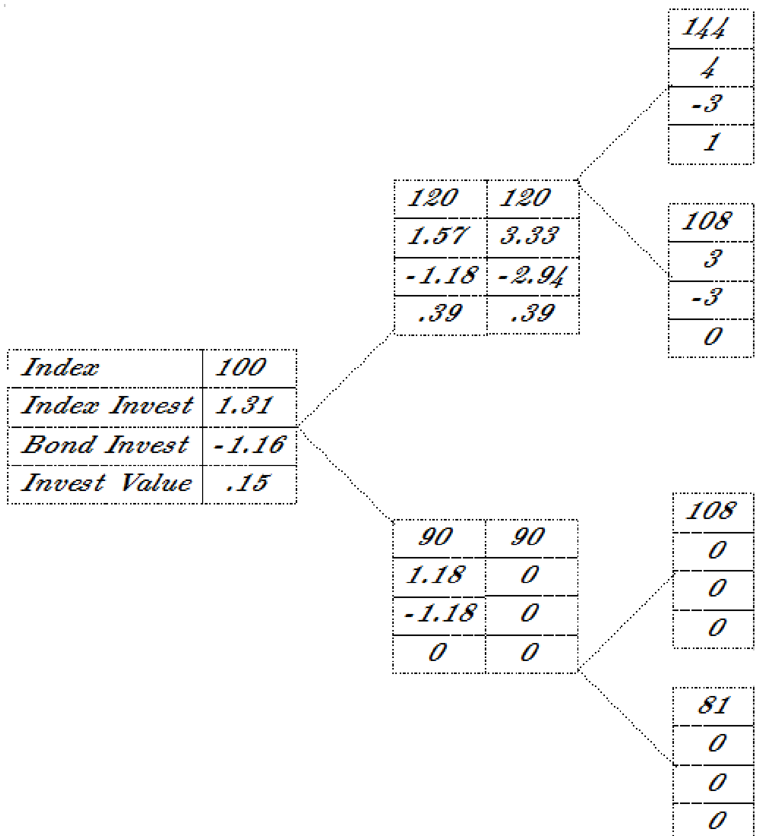
Each node in Figure 2 is a different time-state of the economy. Arrow-Debreu securities are not traded in the market, but the intermediary can synthesize them through dynamic trading in the stock and risk-free bonds and figure out the production costs for each one. Figure 3 shows

[7] For a discussion of the critical role of financial guarantees, see Merton and Bodie (1992).

the dynamic trading strategy required to produce an AD security that pays 1 consumption unit in time-state *uu* and 0 in all other time-states.

The initial portfolio at t=0, consists of 1.31 shares of stock, each costing 1 consumption unit, and a short position in risk free bonds equal in value to 1.16 consumption units. The net cost of this portfolio is .15 consumption units, and by the law of one price, this is the price of a *uu* AD security.

Figure 3



Rebalancing of the portfolio occurs at t=1. If the stock index goes up, the portfolio has a value of .39 consumption units, and the long position in stock is increased to 3.33 consumption units and the short bond position is increased to 2.94. No new infusion of cash is required. If the stock index goes down, the portfolio is liquidated – the proceeds from the sale of the stock is exactly enough to pay off the principal and interest on the bonds. At t=2, the payoff will be 1 consumption unit in the *uu* state, and zero in all other states.

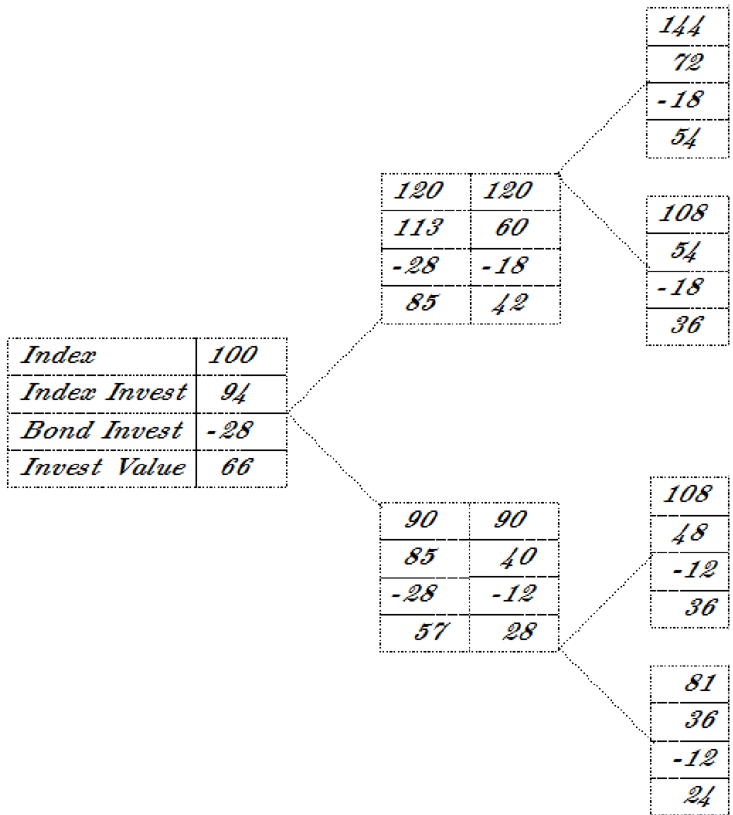
Standard-Model Contracts

Once the financial firm has determined the costs of its basic building-block securities, it is ready to structure standard-model life-cycle contracts for different categories of customers. For example, let's say it decides to offer a standard-model contract designed for people with logarithmic utility and a rate of time preference equal to zero. Its production cost is 100 units of consumption. It finds the contingent benefits in each time-state by maximizing expected utility for an individual whose total wealth – human plus financial — is 100 units of consumption. The solution is as shown in the first column of Table 1. Figure 4 describes the investment strategy followed by the financial firm to produce the optimal consumption contract for log consumers.

Table 1. Benefit Structure for Standard-Model Contingent Consumption Contracts

Time-state	Log Utility	Safe Contract	Log Utility with Habit Formation in Old Age
0	33.33	33.33	33.33
u	42.50	34	40.14
d	28.33	34	26.76
uu	54.18	34.68	54.18
ud	36.12	34.68	40.14
du	36.12	34.68	36.12
dd	24.08	34.68	26.76

Figure 4.



When multiple sources of risk are considered – such as when interest rate risk is introduced – the binomial model is no longer adequate (See, for instance, Merton, 1992, pp. 346-347). Yet exact solutions can still be derived via the more powerful mathematics of continuous-time modeling. In the continuous-time limit, the replicating portfolio strategy associated with the contingent consumption contract is directly related to the rate of change in the dollar value of the contract as stock indices and interest rates vary, or its slope with respect to the underlying source of uncertainty. This slope can be computed as a Malliavin derivative,

$$\mathcal{D}_s F(W) = \lim_{\varepsilon \to 0} \frac{F(W + \varepsilon 1_{[s,S]}) - F(W)}{\varepsilon},$$

which measures the change in the value of a Brownian functional *F(W)* as the path of uncertainty (the Brownian motion) is perturbed by an infinitesimally small amount ε beyond the current date

(See Detemple, Garcia, and Rindisbacher, 2003, and Treussard, 2008, for a complete development of these tools.).

Part 2. The option to switch careers

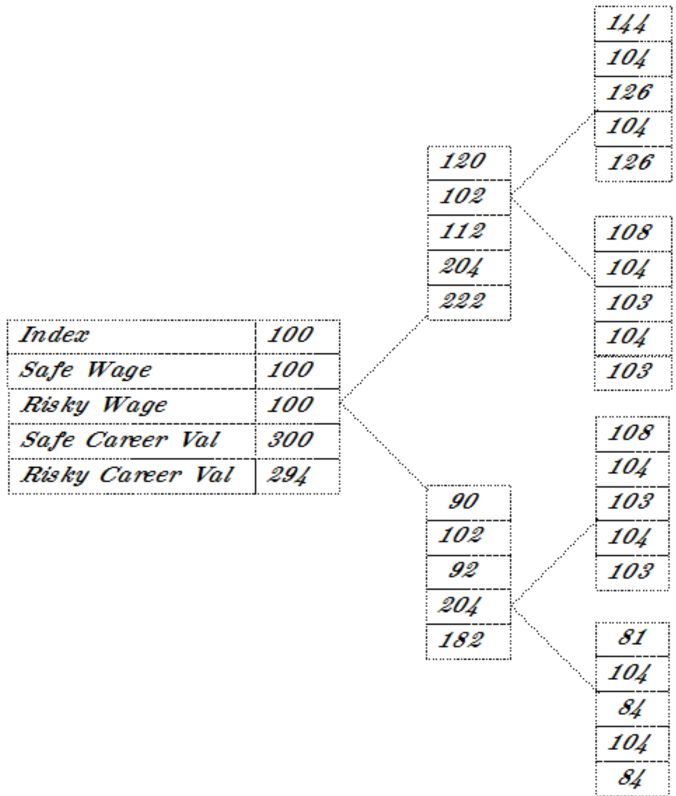
For most people, income from work is the major source of funding for lifetime consumption. The present value of an individual's lifetime potential earnings from work is called human capital. Bodie, Merton, and Samuelson (1992), modeled human capital as equivalent to a portfolio of risky stocks and risk-free bonds. In recent work, Ruffino and Treussard (2007) and Treussard (2007) have modeled the ability to switch careers as an American-style spread option.[8]

In their models, a career is characterized by a stochastic process for labor income with a certain mean and standard deviation. People will choose to switch when an alternative career exceeds the value of their existing career by more than some threshold amount. Ruffino and Treussard show that if people can switch careers it increases the value of their human capital and reduces its risk.

Let's look at the option to switch careers in a simple binomial model illustrated in Figure 5. There are two careers, one safe and the other risky. Labor income in the safe career grows at a constant rate over each of the two periods. Fluctuations in labor income for the risky career are perfectly correlated with a traded stock index. The individual must decide in which occupation to start her career, knowing that she may switch to the alternative career once over her lifetime at some switching cost. Let us first consider each career separately, ignoring the option to switch. In Figure 5 the last two entries in each vector represent the present value of either staying in the safe or risky career until retirement, where Arrow-Debreu prices are used to value the risky income stream. From Figure 5 we see that the safe career has the higher present value at time 0.

[8] Unlike European options that can be exercised only at their maturity dates, American options can be exercised any time up to (and including) maturity.

Figure 5



Now imagine that the individual considers the option to change careers, when faced with a switching cost of 10 units of consumption. For her to decide into which occupation to go, she must determine the value of her human capital in either case, including the value of her option to switch careers. At each node, the individual has the choice between switching careers and remaining in her current occupation for one more period, which implies a decision criterion

Option Val_t = Max[Alt $Career_t$ – Initial $Career_t$ – Cost, π^u x Option Val^u_{t+1} +π^d x Option Val^d_{t+1}],

where π^u and π^d are Arrow-Debreu prices. [9]

[9] The immediate exercise payoff being (Alt $Career_t$ – Initial $Career_t$ – Cost) renders this American option of the exotic spread type.

Table 2 below reports option values, optimal career option exercise policies, take-home income streams (net of incurred switching costs), and the resulting value of human capital depending on whether the individual starts out in the safe or risky career.

Table 2

	Start in Safe Career				**Start in Risky Career**			
State	Value of Switching Option	Switch?	Labor Income	Value of Human Capital	Value of Switching Option	Switch?	Labor Income	Value of Human Capital
0	3.11	No	100	**302.99**	7.24	No	100	**300.95**
u	8	**Yes**	102.14	211.92	0	No	112.14	221.92
d	0	No	101.98	203.92	12.23	**Yes**	91.98	193.92
uu	11.76	**Yes**	125.76	125.76	0	No	125.76	125.76
ud	0	No	102.96	102.96	0	No	102.96	102.96
du	0	No	104	104	0	No	104	104
dd	0	No	104	104	9.70	**Yes**	104	104

Thus, with switching costs equal to 10 consumption units, the individual would maximize her human capital by going into the safe career first and switching to the risky career at t =1, if the *u* state occurs. If switching costs are lower, however, the individual may find it optimal to start out in the risky career. This is reflected in Table 3, where switching costs are set equal to zero.

Table 3

	Start in Safe Career				Start in Risky Career			
State	Value of Switching Option	Switch?	Labor Income	Value of Human Capital	Value of Switching Option	Switch?	Labor Income	Value of Human Capital
0	6.99	No	100	**306.87**	13.39	No	100	**307.11**
u	18.00	**Yes**	112.14	221.92	0.61	No	112.14	222.53
d	0	No	101.98	203.92	22.23	**Yes**	101.98	203.92
uu	21.76	**Yes**	125.76	125.76	0	No	125.76	125.76
ud	0	No	102.96	102.96	1.03	**Yes**	104	104
du	0	No	104	104	1.03	**Yes**	104	104
dd	0	No	104	104	19.70	**Yes**	104	104

By starting out in the risky career, the individual captures an additional 1.04 consumption units of labor income after she exercises her career option in the *ud* state and subsequently earns the safe wage. This ability to capture early wage upside combined with the knowledge that one can fall back on a safe stream of income at a later date is also the source of Treussard and Ruffino's result that low-cost job switchers prefer to start out in riskier careers.

Conclusion

This paper explored the application of contingent claims analysis (CCA) to two "hot" issues in life-cycle finance: (1) investing for retirement and (2) deciding when, if ever, to switch careers. Participants in individual retirement accounts do not have the time or the knowledge to make their own investment decisions. Today they are defaulted into life-cycle mutual funds that pass all risk directly through to the participant. We use CCA to demonstrate how financial firms can design and produce guaranteed contingent benefit contracts that improve participant welfare at no additional cost to the system. These contracts combine features of traditional annuities (i.e., lifelong income benefits) and structured investment products (e.g., guaranteed minimum plus “upside” equity participation). In exploring the career-choice issue in the second part of the paper, we used CCA in a somewhat different way. The decision to switch careers is analogous to deciding when to exercise an American-style option to swap one asset for another. By applying the methods used to analyze the option-exercise decision to the career-switching problem, we gained some new insights beyond those derived from the traditional dynamic programming approaches. For the sake of clarity of exposition, throughout this paper we illustrated the main ideas using simple binomial models.

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