Microstructure Bluffing with Nested Information*

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Abstract

We analyze a model of trading under incomplete information similar to Chakraborty and Yilmaz (2004a, 2004b). We show that the possible presence of an informed insider in the market with long-lived private information generates an incentive for the insider to bluff or manipulate, i.e., undertake unprofitable trades early on in order to undertake profitable future trades at more favorable prices. In contrast to previous work, where the insider bluffs in order to add noise to the market's problem of inferring the fundamentals from the observed order flow, in the present paper the insider has an added incentive to manipulate. This arises from the presence of a large number of competitive rational traders who hold coarser information than the insider but finer information than the market maker, making it in their interest to "follow" the insider's trades in equilibrium.

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1 Introduction

Chakraborty and Yılmaz (2004a, 2004b) consider strategic models of repeated trading by an informed insider with long-lived private information in markets with risk-neutral competitive market makers. They show that, if market makers face uncertainty about the existence of informed trades in the order flow then, with sufficiently many periods of trading, the insider will bluff or manipulate in every equilibrium in the precise sense ruled out in Kyle (1985, pp. 1323), i.e., "by first destabilizing prices with unprofitable trades made at the nth auction, then recouping the losses and much more with profitable trades at future auctions". By manipulating, the insider adds noise to the market's inference problem and makes prices less sensitive to his trades.

In this note we extend this insight and demonstrate how the scope for profitable bluffing is enhanced by the presence of other rational informed traders in the market. These traders have superior information compared to the market makers, i.e., they know if there is any informed trading in the observed order flow although like the market makers they do not know the precise nature of the information. Due to this informational advantage they are better able to update on the information content of early period order flows, relative to market makers, giving rise to profitable trading opportunities via mimicking or *following* the earlier trades of the insider in later periods. Competition among these followers then leads prices to quickly incorporate all private information, eliminating almost all trading profits for the insider in later periods of trading. We show that such competitive pressure from followers may lead the insider to bluff the followers in equilibrium, i.e., undertake unprofitable early trades in order to trade in the future at favorable expected prices.

More precisely, we show that whenever the insider is not expected to bluff in equilibrium, followers will find it in their interest to trade in the direction of the first period order flow in the second period. This allows the insider to bluff by buying in the first period an asset that he knows is overvalued. He then profitably unwinds his position in the second period by trading in the direction of his information, but *against* the direction of anticipated follower order flow. When the follower order flow is large, the profits from such bluffing will be greater than the profits from not bluffing, i.e., from trading in the direction of his information in both periods. In particular, this is due to the fact that he will be trading in the *same* direction as follower order flow in the second period if he does not bluff. It follows that the insider must be expected to bluff in any equilibrium, i.e., make unprofitable early trades against the direction of his information profitably unwinding it later. By bluffing the insider lowers the competitive pressure from followers on his trading profits.

Although our results do not depend on a precise interpretation of followers, one may think of the traders in our model as possessing different amounts of information about a potential takeover target, along the lines of Bagnoli and Lipman (1996) and Vila (1989). In contrast to these papers however, our work concerns pure trade-based manipulation (see Allen and Gale, 1992). Our results are also conceptually related to the literature on the effect of disclosure regulations on market manipulation (Fishman and Hagerty (1995), John and Narayanan (1997), and Huddart, Hughes and Levine (2001)), except that information about the insider's past trades is not driven by exogenous regulation in our model but rather generated endogenously via the interaction of differentially informed traders in the marketplace.

2 Model

We consider a market for one asset with risk-neutral agents and a continuum of possible trade sizes. The current price of the asset is scaled to zero. The long-term or fundamental value of the asset is $v \in \{-1, 1\}$, with equally likely priors.

There are four kinds of traders in the market. The first is a strategic informed trader (the insider or leader) who knows the realized value of v. The insider is a rational informed trader $\alpha \in (0, 1)$. With probability $1 - \alpha$ the leader is a noise or liquidity trader whose trades are uncorrelated with fundamentals, represented by a probability density $g(x)$ and associated distribution $G(x)$, where x stands for a trading position. We suppose that $g(x)$ is continuous, strictly positive for all $x \in [-1, 1]$ and symmetric around zero. Indeed, in what follows we assume for simplicity that g is the uniform distribution and that noise trades are distributed independently across periods.

Apart from the leader there is also a second class of rational informed trader in the market called followers, indexed collectively by F. To simplify the arguments, we assume that each follower is infinitesimal, although in the aggregate there are a continuum of followers with total mass $\Delta > 1$.[1] Followers know whether the leader is informed (signal $s = I$) or noise ($s = N$), although they do not know the nature of his information. Finally, there are competitive market makers who know only the priors.

There are two periods of trading $t \in \{1, 2\}$ and each period all traders simultaneously

[1] The assumption of a continuum of followers implies that each follower's trades has no impact on prices, although in the aggregate followers may. This can be thought of as the limit of a large number of followers each with a small trade size.

submit market orders after observing the previous net order flow and prices. Subsequently, market makers set the price for that period after observing that period's net order flow. Denote by $x_t^I(v) \in \mathbb{R}$ the order of the leader in period t as a function of the realized value v, by $x_t^F(s) \in \mathbb{R}$ the aggregate order of followers as a function of their information s, and by $x_t^N \in \mathbb{R}$ that from noise.[2] If $x_t^i > 0$ (resp., < 0) we say that the order by $i = I, F, N$ is a *buy* (resp., *sell*), with $x_t^i = 0$ a *no-trade*. The net order flow x_t in any period is the sum of the orders from the different groups of traders with h_t the history of net order flows up to and including period t.

The price in period t is is denoted by $p(h_t)$ set by competitive market makers to equal the expected value of the asset conditional on the observed order flow. We let $\mu(h_t, s) = \Pr[v = 1|h_t, s]$ denote the follower's posterior beliefs given a history h_t and a signal s. Let $Q(x|v)$ and $R(x|v, h_1)$ be the cumulative distribution functions, for periods 1 and 2 respectively, that represents the informed trader's (behavior) strategy as a function of v and the prior history of net order flows. When Q and R admit densities we will denote them by q and r respectively.

Since $v \in \{-1, 1\}$ and we must have $-1 \leq p(h_t) \leq 1$ for all h_t, t in any equilibrium. Since an informed trader will never undertake an expected loss making trade in the final period, we must have $R(x|1, h_1) = 0$ for all $x < 0$ and $1 - R(x|-1, h_1) = 0$ for all $x > 0$, in any equilibrium. We say that the leader's strategy involves bluffing if he makes a loss in period 1, presumably to obtain better prices later.

Definition 1 *The insider is bluffing if* $Q(x|1) > 0$ *for some* $x < 0$ *or* $Q(x|-1) < 1$ *for some* $x > 0$*. Otherwise he is not bluffing.*

In what follows we use the term non-bluffing equilibrium to mean an equilibrium in which the leader is not bluffing.

3 Analysis

In this section we demonstrate our main result that the leader will bluff in every equilibrium. To this end, we begin by stating some necessary conditions of follower behavior in any (candidate) equilibrium where the leader is not bluffing.

[2] Excluding noise, the other orders will also depend in general on the observed history of previous orders from all market participants, but we suppress that dependence for notational ease.

Lemma 1 *In any candidate non-bluffing equilibrium,*

a *when $s = I$, following a first period buy (resp., sell) $p(h_2) = 1$ (resp., $p(h_2) = -1$) a.s. and $x_2^F(I) - x_2^F(N) \geq 1$ (resp. ≤ -1).*

b *$x_1^F(s)$ does not depend on s and, without loss of generality, we may take $x_1^F(s) = 0$ for all s.*

Proof. See the Appendix.■

Part (a) follows from the fact that followers infer the value of v from the first period order flow and, since there are many of them, compete away any profits from this information in period 2. Part (b) shows that the only way followers can earn non-negative expected profits is by not revealing their information through their first period trades. In particular, the followers' first period demand does not reflect their information in any non-bluffing equilibrium implying that whenever there exists a non-bluffing equilibrium where followers trade non-zero amounts in period 1, there exists another non-bluffing equilibrium where followers do not trade at all in the first period.

We turn now to characterizing the leader's behavior in the first period in a candidate non-bluffing equilibrium. From Lemma 1b, the first period net order flow is equal to the leader's order, without loss of generality. Furthermore, since the noise is distributed without a mass, the informed trader must play a mixed strategy without any mass in such a candidate equilibrium. To see this first consider the case in which an informed leader buys (sells) whenever his type is $v = 1$ (resp., $v = -1$). If there is a mass at any $x_1 = x_1^I \neq 0$, the trade reveals the existence and the information of the leader and thus market makers set the price equal to 1 (resp., -1) in *both* periods following such an order, implying that the leader earns zero profits in any equilibrium. But this a contradicts the fact that he can always trade an amount on which his strategy does not put positive probability, but is in the support of noise, for strictly positive profits. Similarly, a mass at $x_1^I = 0$ reveals the existence of an leader and thus market makers set the price equal to 1 (resp., -1) in the second period when $x_2^I > 0$ (resp., $x_2^I < 0$), eliminating all potential profits for the leader, a contradiction. Therefore, the informed trader must play a mixed strategy without any mass in a (non-bluffing) equilibrium with $q(x|v)$ the density representing this strategy.

By Bayes rule, following a first-period order $x_1^I = x \in (0, 1]$, the market-maker's updated probability that $v = 1$ in the candidate non-bluffing equilibrium is

$$\Pr[v = 1|x] = \frac{\frac{1}{2}[\alpha q(x|1) + (1-\alpha)g(x)]}{\frac{1}{2}[\alpha q(x|1) + (1-\alpha)g(x)] + \frac{1}{2}(1-\alpha)g(x)} \tag{1}$$

Similarly, when $x < 0$

$$\Pr[v = 1|x] = \frac{\frac{1}{2}(1-\alpha)g(x)}{\frac{1}{2}(1-\alpha)g(x) + \frac{1}{2}[\alpha q(x|-1) + (1-\alpha)g(x)]} \tag{2}$$

When $v = 1$, the first period profit for informed leader due to trading $x_1^I = x$ is

$$x[1 - p(x)] = x\frac{2(1-\alpha)g(x)}{\alpha q(x|1) + 2(1-\alpha)g(x)}$$

using $p(x) = E[v|x]$ and (1). Since he must be indifferent among all trade sizes x with $q(x|1) > 0$, and since the second period profit is zero following any non-zero order in the first period (by Lemma 1a), his total profit is

$$x\frac{2(1-\alpha)g(x)}{\alpha q(x|1) + 2(1-\alpha)g(x)} = k$$

where $k > 0$ is a constant equal to the leader's expected profits. Arranging terms we obtain an expression for

$$q(x|1) = 2\frac{1-\alpha}{\alpha}g(x)\left[\frac{x}{k} - 1\right]$$

Notice that the support of q is $(k, 1]$ and also that the value of k can be obtained from the identity $\int_k^1 q(x|1)dx = 1$. Analogously, when $v = -1$, the leader's first period mixed strategy is seen to be

$$q(x|-1) = 2\frac{1-\alpha}{\alpha}g(x)\left[-\frac{x}{k'} - 1\right]$$

with support on $[-1, -k')$, where k' is the expected profits for the leader when $v = -1$ obtained from the identity $\int_{-1}^{-k'} q(x|-1)dx = 1$.

Now consider the following deviation strategy for the leader when $v = 1$. He first sells, i.e., trades $x_1^I = -k - \varepsilon$, where $\varepsilon > 0$ and small. The price is $p(-k-\varepsilon) \to 0$ for arbitrarily small ε, using (2) since $q(-k-\varepsilon|-1) \to 0$ but $g(-k-\varepsilon) > 0$. Therefore, his loss in the first period is approximately k. However following this deviation, followers conclude that $v = -1$, upon observing the first period order flow $x_1^I < 0$. In the second period, using Lemma 1a, if the leader buys the amount $1 + x_2^F(N) - x_2^F(I)$, then the market makers conclude that all trading is driven by noise. Consequently, the price at which orders execute in the second period is 0. The overall deviation profits are $1 + x_2^F(N) - x_2^F(I) - k$, greater than the non-bluffing profit k, since $x_2^F(N) - x_2^F(I) \geq 1 > k$ following a first period sell when $s = I$, by Lemma 1a.

Since the leader has a strictly profitable deviation whenever the market makers and followers expect him not to bluff, every equilibrium must involve bluffing and we are ready to state our first result.[3]

[3] The existence of an equilibrium in our game follows from Kim and Yannelis (1997).

Proposition 1 *For any $\alpha \in (0,1)$, the insider must bluff in every equilibrium.*

The existence of followers creates incentives for the leader to bluff. Indeed, as Lemma 1 shows, if the leader does not bluff then followers compete away all profits in the second period. By bluffing, the leader creates noise in the inference problem for followers (and hence the market) thereby creating the possibility that he may trade in period 2 at favorable prices.

In order to see the impact of followers on bluffing, it is useful to consider the model in the absence of followers. In this case, the insider's expected profit in a non-bluffing (candidate) equilibrium is a continuously decreasing function of α. It converges to 2 as α goes to zero while it approaches to zero as α goes to one. On the other hand, the expected profits due to any bluffing strategy is bounded above by 1. For bluffing to occur in every equilibrium in the absence of followers, we then need α to be sufficiently large. In such cases, the market makers attach a very high probability to trades being driven by information resulting in low profits for the insider. By bluffing, the insider then adds noise to the price formation process, along the lines of Chakraborty and Yılmaz (2004a, 2004b). In this paper we show that the incentives to bluff are heightened when there are a large number of rational traders with information that is nested between that of the insider and the market, in the sense that bluffing necessarily occurs for any α.

4 Conclusion

We consider a simple dynamic model of trading by an informed leader in a model where a large number of rational traders compete and reduce the ability of the leader to hide the information content of his trades. For precisely this reason the leader will make unprofitable bluffing trades early on and later profitably unwind his position. These results add to our understanding of microstructure conditions under which bluffing is likely.

With regard to extensions, the illustrative model above may serve as a workhorse to analyze related scenarios. For instance, we conjecture that the same result will continue to hold when followers have (noisy) information about market fundamentals that the insider may use to profitably bluff. More interestingly, a nested information structure may give rise to incentives for "uninformed" traders to bluff. In particular, a strategic insider who does not have any information about fundamentals different from the market makers' or follower's information may find it optimal to mimic an informed insider and move prices away from the fundamentals and profit later by reversing his initial position at more favorable prices.

5 Appendix

Proof of Lemma 1: (a) Without loss of generality, we consider the case $x_1^I > 0$. In a non-bluffing equilibrium when $s = I$, following an order of $x_1^I = z = x_1 - x_1^F > 0$, the followers conclude that the leader's type is $v = 1$, i.e., $\mu(z, I) = 1$.

Suppose $p(h_2) < 1$ for a positive measure of h_2, given z and $s = I$. Then such histories are generated by strictly positive probability conditional also on $s = N$ and z. Furthermore, $E[p(h_2)|z, N] > 0$ implying that followers will sell when $s = N$, i.e., $x_2^F(N) < 0$. It follows that the maximum possible second period net order flow conditional on $s = N$ is equal 1, while the minimum possible order flow conditional on $s = I$ is $x_2^F(I)$. For the market makers to put positive weight on both $s = I$ and $s = N$ after some histories following z, we then need $x_2^F(I) \leq 1$. But since $E[p(h_2)|z, I] < 1$ by supposition, followers earn strictly positive profits when $s = I$ and we cannot have $x_2^F(I) < \Delta$, a contradiction. It then follows that market makers will be able to discern the state $s = I$ from $s = N$ upon observing the second period order flow with probability 1, implying that $x_2^F(I) - x_2^F(N) \geq 1$. ■

(b) For reasons discussed in Section 3, the informed trader's first period strategy represented by the distribution $Q(.|v)$ must be atomless and admit a density $q(.|v)$ with support denoted by Z_v for all $v \in \{-1, 1\}$. Since the candidate equilibrium is non-bluffing $z \geq 0$ (resp., $z \leq 0$) if $z \in Z_1$ (resp., Z_{-1}).

Let $\delta = x_1^F(I) - x_1^F(N)$ and let $z = x_1 - x_1^F(I) = x_1 - x_1^F(N) - \delta$ so that for any given first period order flow $z = x_1^I(v) = x_1^N - \delta$. Then, for all $z \in [-1-\delta, 1-\delta] \cup Z_1 \cup Z_{-1}$, the price as a function of z is

$$p(z) = \frac{\alpha[q(z|1) - q(z|-1)]}{\alpha q(z|1) + 2(1-\alpha)g(z+\delta) + \alpha q(z|-1)}$$

by Bayes' Rule.

We wish to show that $\delta = 0$ in any candidate non-bluffing equilibrium. Suppose to the contrary $\delta > 0$. We proceed in cases.

Case 1: $0 < \delta \leq 1$

In this case, the first period prices that can arise on the path of play in any non-bluffing equilibrium are given by

$$p(z) = \begin{cases} 1 & \text{if } z > 1-\delta, z \in Z_1 \\ \frac{\alpha[q(z|1)-q(z|-1)]}{\alpha q(z|1)+2(1-\alpha)g(\delta)+\alpha q(z|-1)} & \text{if } z \in [-1-\delta, 1-\delta] \\ -1 & \text{if } z < -1-\delta, z \in Z_{-1} \end{cases} \tag{3}$$

and arbitrary otherwise.

Now the indifference condition for type $v = 1$ of the leader says that for all $z \in Z_1$,

$$z[1 - p(z)] = k_1 > 0$$

where the strict inequality follows from inspecting (3) and observing that the leader can always buy an order of size $z \in [0, 1 - \delta)$ at a price $p(z) \in [0, 1)$ for strictly positive expected profits. It then follows that $\sup Z_1 \leq 1 - \delta$ and $\inf Z_1 \geq k_1$ so that solving the indifference condition

$$z[1 - p(z)] = z \frac{2(1 - \alpha) g(z + \delta)}{\alpha q(z|1) + 2(1 - \alpha) g(z + \delta)} = k_1$$

for $q(z|1)$ yields

$$q(z|1) = 2 \frac{1 - \alpha}{\alpha} g(z + \delta) \left[\frac{z}{k_1} - 1 \right]; z \in [k_1, 1 - \delta]$$

showing also that q has convex support.

Similarly exploiting type $v = -1$'s indifference condition

$$z[-1 - p(z)] = k_{-1} > 0$$

we obtain

$$q(z| - 1) = 2 \frac{1 - \alpha}{\alpha} g(z + \delta) \left[- \frac{z}{k_{-1}} - 1 \right]; \; z \in [-1 - \delta, -k_{-1}]$$

If we substitute the expressions for $q(.|1)$ and $q(.|-1)$ into expression (3) we see that the price

$$p(z) = \begin{cases} -1 - \frac{k_{-1}}{z} & \text{if } z \in [-1 - \delta, -k_{-1}] \\ 0 & \text{if } z \in (-k_{-1}, k_1) \\ 1 - \frac{k_1}{z} & \text{if } z \in [k_1, 1 - \delta] \end{cases}$$

for all z that can arise on the path of play in the candidate non-bluffing equilibrium. Furthermore, the constants k_1 and k_{-1} are obtained from the identities

$$\int_{k_1}^{1-\delta} q(z|1) dz = 1 \text{ and } \int_{-(1+\delta)}^{-k_{-1}} q(z| - 1) dz = 1$$

Using the fact that g is uniform, the last expression yields

$$\frac{k_{-1}}{k_1} = \left[\frac{1 + \delta - k_{-1}}{1 - \delta - k_1} \right]^2$$

allowing us to conclude that $0 < k_{-1} - k_1 < 2\delta$, a fact that we use later.

We now compute the expected first period price for each follower signal $s \in \{N, I\}$:

$$E[p(x_1)|N] = \frac{1-\alpha}{2\alpha}\left[\int_{-(1+\delta)}^{-k_{-1}}(-1-\frac{k_{-1}}{z})dz + \int_{k_1}^{1-\delta}(1-\frac{k_1}{z})dz\right]$$

And

$$E[p(x_1)|I] = \frac{1-\alpha}{2\alpha}\left[\int_{-(1+\delta)}^{-k_{-1}}(-1-\frac{k_{-1}}{z})(-1-\frac{z}{k_{-1}})dz + \int_{k_1}^{1-\delta}(1-\frac{k_1}{z})(\frac{z}{k_1}-1)dz\right]$$

Performing the integrations, it is easy to verify that $E[p(x_1)|N] + E[p(x_1)|I]$ equals

$$\frac{1-\alpha}{2\alpha}\left[-(k_{-1}-k_1-2\delta) - \frac{1}{2k_{-1}}(k_{-1}^2-(1+\delta)^2) + \frac{1}{2k_1}((1-\delta)^2-k_1^2)\right] > 0 \qquad (4)$$

where the inequality follows from recalling that $k_{-1} - k_1 < 2\delta$, $k_{-1} < 1+\delta$ and $k_1 < 1-\delta$.

Notice next that since $\delta = x_1^F(I) - x_1^F(N) < 1 < \Delta$, the total mass of followers, we must have $x_1^F(s) \in (-\Delta, \Delta)$ for some $s \in \{I, N\}$. For such s, followers must be indifferent between trading or not, i.e., must earn zero expected profits implying $E[p(x_1)|s] = 0$. Using (4) we then have $E[p(x_1)|s'] > 0$ for $s' \neq s$. But then $E[p(x_1)] = \alpha E[p(x_1)|I] + (1-\alpha)E[p(x_1)|N] > 0$, a contradiction with the law of iterated expectations implying that $E[p(x_1)] = E[E[v|x_1]] = E[v] = 0$. This completes the proof for this case.

Case 2: $\delta > 1$.

In this case, it is easy to see that conditional on $s = N$ followers know that the market makers cannot attach positive probability to the type $v = 1$ of the leader upon observing any first period order flow that can arise. It follows that, given $s = N$, $p \leq 0$ with probability 1 and $p < 0$ with positive probability on the path of play. But then $E[p(x_1)|N] < 0$ so that followers earn strictly positive profits when $s = N$ and that $x_1^F(N) = \Delta$. But then we cannot have $x_1^F(I) > x_1^F(N)$.

The proof for the case $\delta < 0$ is symmetric, with the two types of the informed trader switching roles relative to the case $\delta > 0$. For the last part of the claim, notice that if $\delta = 0$, then $k_1 = k_{-1}$ and $E[p(x_1)|N] = 0 = E[p(x_1)|I]$ so that any non-informative submission strategy satisfying $x_1^F(I) = x_1^F(N)$ can be part of such a equilibrium.■

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