Adverse Selection in Annuity Markets: Evidence from the British Life Annuity Act of 1808

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Abstract

We look for evidence of adverse selection using data from an 1808 Act of British Parliament which effectively opened up a market for life annuities. We develop statistical techniques for analyzing the awkwardly condensed data, and our analysis indicates significant levels of self-selection. The evidence for adverse selection is particularly strong among a sub-sample of annuitants whose annuities were purchased by profit-seeking speculators. We find evidence of additional selection among self-nominated annuitants when the annuities were re-priced to reflect the empirical longevity of the early annuitants. This pattern of selection effects is reminiscent of the early stages of an Akerlovian "death spiral," suggesting that adverse selection may be an important factor underlying the paucity of annuity sales in contemporary annuity markets. But the magnitudes of the selection effects suggest that it is unlikely to be a complete explanation.

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1 Introduction

In 1808, the British Parliament passed the Life Annuity Act, effectively opening a market for government-provided life annuities. The unique features of the Act – including surviving data on the mortality histories of the annuitants – provide an unusual opportunity to explore the empirical importance and consequences of informational asymmetries.

Informational asymmetries have played an important role in economic theory since the seminal works of Akerlof (1970), Spence (1973), and others. Studying the empirical importance of adverse selection has proved challenging in several respects. First, as emphasized by Chiappori and Salanié (2000), it is difficult to empirically distinguish between moral hazard and adverse selection.

Second, informational asymmetries between market participants are likely to apply to the econometrician as well; information which is not known by an insurance provider, for example, is unlikely to be observed by the econometrician. This has motivated the development of tests relying only on *ex-ante* observable information and *ex-post* outcomes. As described in Dionne et al. (2001), these test for a correlation between insurance choices and *ex-post* risk, conditioning on information observable to the insurance provider. Informational asymmetries can pose empirical problems for these tests as well, since econometricians may not even have access to all information *symmetrically* known by the market participants.

Third, existing empirical evidence for even the most robust consequences of informational asymmetries is less than definitive. For example, Cawley and Phillipson (1999) find no evidence of selection in life insurance markets, Cardon and Hendel (2001) find no evidence in health insurance markets, and Chiappori and Salanie (2000) and Dionne et al. (2001) find no evidence of adverse selection in auto insurance markets (overturning Pueltz and Snow's, 1994, suggestive evidence). Some economists have recently argued that this absence of evidence may be due to countervailing informational advantages of the insurance provider (Villeneuve, 2003) or else to a fourth empirical challenge: the confounding effects of "advantageous" selection (DeMeza and Webb, 2001, Cohen and Einav, 2007, and Finkelstein and McGarry, 2006).

Annuity markets provide a particularly interesting setting to study informational asymmetries, not least because many of these empirical challenges are less acute than in other insurance settings: moral hazard is plausibly negligible for annuities, and annuity providers do significantly less risk-classification as compared with other markets. Perhaps in part for these reasons, annuity markets are one of the few settings in which direct evidence of adverse selection has been documented (Finkelstein and Poterba, 2002, 2004, 2006). Unfortunately, the applicability and generality of this evidence is limited for at least two reasons. First, the evidence comes from a "compulsory" market, wherein individuals are legally obligated to annuitize their assets. Second, a substantial literature documents the so-called "annuity puzzle," the observation that voluntary annuity markets are extremely underdeveloped in spite of theoretical findings indicating that annuities should be extremely desirable for rational, forward looking, risk averse retirees. Understanding this puzzle is crucial for interpreting any evidence of informational asymmetries in annuity markets.

This paper uses the 1808 Act to provide a novel empirical look at adverse selection in the purely voluntary annuity market effectively opened thereby. In the spirit of Finkelstein and Poterba (2006), we provide evidence of adverse selection by comparing the population-average longevity with the longevities of several classes of annuitants. Our evidence is indicative of adverse selection *within* the risk-classification scheme employed by the annuity provider (the U.K. government); insofar as we do not have data on the entire set of information available to the U.K. government, however, we cannot provide a pure test for informational asymmetries in the sense of Dionne et al. (2001).

The paper proceeds as follows. Section 2 introduces and describes the 1808 Life Annuity Act and its subsequent evolution. The Act's history then guides a series of empirical tests presented in Section 3. The awkwardly condensed nature of the surviving data from the annuities sold under the act necessitates developing novel empirical techniques. Additional details on these techniques appear in an appendix.

The Act's history also provides a unique opportunity to study the first stages of the type of "death spiral" dynamics found by Cutler and Reber (1998) (but not by Buchmuller and DiNardo, 2002) in health insurance markets. The evidence from Section 3 is suggestive of a death spiral. Since evidence of a death spiral would potentially resolve the annuity puzzle, Section 4 describes this puzzle in greater detail. It then presents an illustrative theoretical model of such a market, and it takes a closer look at how our empirical results bear on the puzzle. Section 5 offers some brief concluding remarks.

2 The 1808 Act as a Testing Ground

Prior to the Life Annuity Act of 1808, the British government's debt consisted almost exclusively of Consols – coupon bonds with an infinite maturity. The explicit goal of the Act was to convert these perpetual obligations to finite-lived ones by allowing the exchange of Consols for life annuities.[1] Since Consols were tradable assets, this effectively opened up a market for government provided life-annuities.

The Act was originally designed to provide annuities at actuarially fair rates. The (semi-annual) payment provided by the annuity received in exchange for a Consol was computed using the market price of Consols – i.e., using long-term interest rates – and a life table specific to the age of the individual upon whose life the annuity payments were contingent (henceforth the *nominee*). The life table used, known as the Northampton table, was thought to capture population-average mortality.[2] We can thus test for selection effects by looking at whether the longevity of the nominees exceeded the population-average longevity predicted by this table. As we will see below, there is indeed striking evidence of this sort of adverse selection: the mortality rates of the nominees were significantly lower than those predicted by the Northampton tables.

Members of the British Parliament suspected the inapplicability of the Northampton tables from an early stage, but it was not until 1823 that the government took active steps to address it. At this point, Parliament commissioned John Finlaison to study the mortality experience of the nominees of this and several earlier (and less significant) life-contingent debt issues.[3]

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[1] There may well have been more subtle motivations leading to its passage. Spencer Perseval, addressing Parliament in 1808 (*viz* Hendricks, 1856), argued that the Act would allow the government to retire debt at favorable interest rates without causing interest rates to rise – an argument Murphy ridicules as indicating Persival's desire to "have his cake and eat it too" (Murphy, 1939, page 6) but which is plausible if the government believed it could extract surplus by filling a missing market. Alternatively, there could have been a political desire to align the interests of the retired monied classes with the government by providing for them a valued service – as argued by Weir (1989) for the French government-issued Tontines of the 18th century.

[2] This table, published in Baily (1813) was based on the mortality experience of the residents of the town of Northampton. The original pricing of the annuities also included a 2% "load" relative to the actuarially fair prices.

[3] For a more thorough discussion of the history of these life-contingent debt issues, see Hendriks (1856), Murphy (1939) and The Insurance Institute of London (1969).

His report developed a new set of life tables, known as the Finlaison tables, based on the observed mortality experience of the nominees. After some debate and a brief suspension of the life annuity program, Parliament determined to resume it with pricing based on these new tables.

These dynamics allow us to test for the first stages of a "death spiral:" if, subsequent to the re-pricing, the new mortality tables *still* significantly under-predicted the longevity of the subsequent nominees, then the government would have continued to lose money at the new prices, and additional rounds of re-pricing would have been required. This could potentially have led to an unraveling of much or all of the market. We will show that the post-repricing nominees were indeed significantly longer lived than even the Finlaison tables predicted.

An additional feature useful for our study is the fact that the Northampton life tables used for pricing annuities under the original act were pooled-gender tables. In contrast, the Finlaison tables were gender-specific. Since women are and were longer lived than men (especially at annuity purchasing ages), we are able to test for selection effects by comparing the gender composition of purchases prior to and subsequent to re-pricing.

Finally, the Act had a particularly odd feature, absent from modern annuity markets: the buyer of an annuity did not have to be the nominee. Rather, an individual could buy an annuity whose payments were contingent on *anybody else's* life. This provided an opportunity for "speculation" on lives: investors could find and nominate particularly healthy individuals, buy annuities contingent on their lives, and profit handsomely from the higher-than-average longevity of these nominees – particularly if they bundled a collection of nominees and resold shares of the resulting income streams.[4] Anecdotally supporting the presence of this type of adverse selection is the following quote, taken from an informal history of the life annuities sold under the 1808 Act written (J. Francis, 1853, p. 200):

> From 1809 this system continued. The speculators soon found out that the Government charge for a life annuity afforded a very remunerative investment, and the insurance offices made considerable profits by purchasing and re-selling them. The Commis-

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[4] There was historical precedent for this sort of speculation. For example, Velde and Weir (1992) describe a scheme known as the "trente demoiselles de Genéve" devised by Genevan bankers to nominate classes of healthy young girls and take advantage of a French life-contingent debt issue in the 1770s.

> sioners of Greenwich Hospital also selected many of the most healthy of their pensioners and bought large annuities on them.

We can exploit this feature to study differences between "passive" selection effects among self-nominees and "speculative" selection effects. As we will see shortly, speculative selection appears to have been much stronger: speculators were quite successful at selecting particularly long-lived individuals, and they appear to have profited handsomely at the government's expense by doing so.

3 Data and Analysis

This section describes the sources and form of the data available for analysis. It then develops empirical techniques for using that data to test for adverse selection. Finally, it describes the results of that test. An analysis of the implications of these results for death-spiral dynamics and the annuity puzzle appears in the subsequent section.

3.1 The Data

Data is available from two reports commissioned by Parliament. Both reports were commissioned to examine the profitability of the annuities sold under the 1808 Act. The first is John Finlaison's 1829 report. His report contains data on annuities sold between 1808 and 1826. The second is an 1860 report by John Finlaison's son, Alexander Glen Finlaison. It examines annuities sold between 1808 and 1850. Both reports focus on developing mortality tables for annuity buyers; consequently, they report data in a highly aggregated form.

John Finlaison's 1829 report contains a data set for each gender. Each set consists of three columns of data. The first column is a list of the number of annuities sold between 1808 and 1826 at each nominee age.[5] The second column gives the number of nominee deaths (between 1808 and January, 1826) at each age. The final column reports the distribution of ages, in January 1826, of all nominees still living at that time.

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[5] As discussed above, annuity buyers were not constrained to nominate their own lives. The data we analyze is based on the ages of the *nominee* of a given contract, not the contract owner.

The 1860 report is in a similar format, but it contains five distinct three-column classes of data. The first three data sets describe three distinct classes of "the nominees of those parties who speculated in life annuities" (A. G. Finlaison, 1860, page 14), henceforth *speculative* nominees. The annuities described in these sets were sold to an investor or investment group seeking to profit by selecting particularly healthy older men and purchasing annuities contingent on their lives. Each of these three data sets refers to annuities purchased after 1828; hence, these nominees do not appear in John Finlaison's data set. Each of the three sets appears to represent one or more distinct investment portfolio put together by a group of speculators. The first contains 353 nominees aged 59 to 64. The second contains 288 nominees aged 73-84, and the third contains 34 nominees aged 85-92.

The final two data sets in A. G. Finlaison's 1860 report (one for each gender) contain all nominees nominated between 1808 and the end of 1850, *excluding* the "speculative" nominees described in the preceding paragraph. We henceforth refer to these nominee classes as "non-speculative." It is important to note, however, that that they include any speculative nominees nominated between 1808 and 1828 as well as any speculator nominated annuitants from after 1828 that the government failed to identify as such.

The first column for each of the classes of data in the 1860 report contains the number of nominees between 1808 and 1850 at each age. For the non-speculative classes and the youngest speculative class, the second column records the number of nominee deaths between 1808 and May 8, 1854 at each age of death. For the two older speculative classes, the second column reports death ages between 1808 and June 10th, 1856. The final column reports the distribution of ages, on December 31, 1850, of all nominees still alive on May 8, 1854 (respectively, on June 10, 1856 for the two older classes of speculative nominees). We illustrate the data in Tables 1 and 2 and Figure 1, described below.

Table 1 summarizes the data from the two reports. From the 1829 report, we see that, of the 6892 annuities purchased between 1808 and Jan 1, 1826, 2077, or 30%, were male, and 5344, or 77.5% were still alive on Jan 1, 1826. From the 1860 report, we see that a total of 16137 annuities were purchased between 1808 and Dec 31, 1850 (excluding the speculative nominee classes). Of these nominees, 34% were male, and 41% were still living on May 8, 1854. Among the speculative nominees from the 1860 data set, we see that all but

Table 1: Data Summary

	1829 Report	
	Purchased, 1808-1826	Living, January 1 1826
Male	2077	1484
Female	4815	3860
Total	6892	5344
	1860 Report, Non-Speculative Nominees	
	Purchased, 1808-1850	Living, May 8 1854
Male	5542	2256
Female	10595	4352
Total	16137	6608
	1860 Report, Speculative Nominees (Males)	
	Purchased, 1828-1850	Living, May 8 1854 (*June 10, 1856*)
Age 59-64	353	298
Age 73-84	288	*1*
Age 85-92	34	*0*

Table 1: Summary of the available data. The first data column reports the number of individuals on whom annuities were purchased. The second reports the number still living at the time of observation. The data are broken down into three classes: data on annuities purchased from 1808-1826; data on annuities purchased from 1808-1850 *excluding* annuities purchased between 1828 and 1850 by "speculators"; and annuities purchased by speculators between 1828 and 1850. (sources: J. Finlaison, 1829; A. G. Finlaison, 1860)

one of the nominees from the two older nominee classes had died by June 10, 1856 [6], while nearly 85% of the nominees in the younger class were still living on May 8, 1854.

Table 1 provides the first suggestive evidence of selection effects: the composition of male and female nominees changed in response to the movement from gender neutral pre-1828 pricing to gender-specific post-1828 pricing. To wit: males constituted approximately 30% of all early (pre-1826) nominees (i.e., $\frac{2077}{2077+4815}$). In contrast, they accounted for about 41% of all later nominees (i.e., $\frac{(5542-2077)+353+288+34}{(5542-2077)+353+288+34+(10595-4815)}$). In other words, relatively more favorable pricing for men unsurprisingly shifted the purchase patterns away from women and towards men.

Table 2 presents the entire data set for the youngest class of speculative nominees; the data for the other classes of nominees is in similar form, but we omit them to save space. To reiterate: the data is only available in highly aggregated form. *All* we observe is the total number of initial nominees, dead nominees, and living nominees at each age. Notably, we lack information both on *when* they were nominated and on whether and when any *particular* nominee died.

Figure 1 plots the three columns of data for the non-selected males nominated during the 1808-1850 period. The tall, dark curve graphs the distribution of ages of the nominees at the time of nomination. It shows a modal nomination age of about 60 and a range of nominee ages from the late teens to the early 90s. The rightmost curve shows the age, at death, of the males who had died by May 8, 1854. The lowest curve depicts the distribution of ages on Dec 31, 1850 of those nominees who had not died by May 8, 1854.

3.2 Shortcomings of the Data

Each of the 5542 non-speculative male nominees in the 1808-1850 data set implicitly appears twice in this data: his age at nomination is recorded, and *either* his age at death or his age in 1850 is recorded. But we lack information providing any connection between these two appearances of this same individual – for example, whether and when any particular age 65 nominee died.

This form of the data is well suited for estimating a mortality table –

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[6] The one remaining nominee died at the age of 102 in March, 1857 (A.G. Finlaison, 1860, page 88).

Table 2: Age 59-64 Speculative Nominee Data Set

Age	Number Nominated (1828-1850)	Number Died (by 1854)	Number Living in 1854 (by age in 1850)
59	7	0	0
60	50	0	4
61	61	1	13
62	91	3	25
63	81	3	17
64	63	7	17
65	0	8	2
66	0	6	16
67	0	8	34
68	0	9	36
69	0	5	43
70	0	3	42
71	0	1	22
72	0	1	4
73	0	0	7
74	0	0	9
75	0	0	4
76	0	0	3

Table 2: Complete data from the youngest speculative nominee class. Column 2 reports the number of nominees of each age (nominated between 1828 and 1850). Column 3 reports the number of these nominees (by age of death) who had died by May 8, 1854. Column 4 reports the number of the nominees (by age on Dec 31, 1850) who had not died by May 8, 1854. Source: A.G. Finlaison (1860).

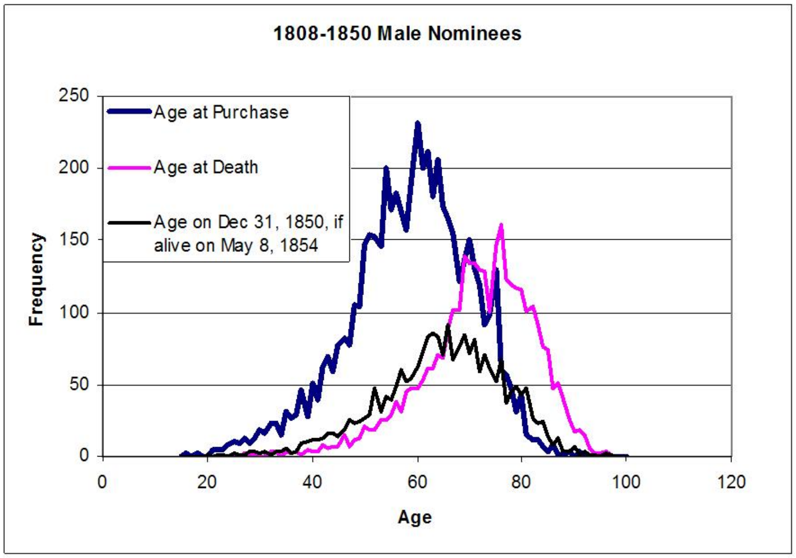

Figure 1: Non-Speculative Male Nominees, 1808-1850. Source: A.G. Finlaison (1860).

as was the purpose of the two reports – but it has significant shortcomings as a tool for testing for adverse selection. In principle, to test for selection effects, we simply want to compare the mortality experience of the nominees with their expected mortality experience based on a reference "unselected population." If we had disaggregated data reporting the purchase age, the purchase year, and the death age, if applicable, of each nominee, testing for selection would thus be completely straightforward. The aggregated nature of the data makes developing such a test more challenging.

3.3 A Consistent Test for Adverse Selection

We wish to test the question: Did nominees live longer than we would have expected? To that end, we first develop a test for the simpler case that applies to the two older nominee classes: when we know the death ages of *all* nominees (not just those who had died by 1826 or 1854). We then consider how to extend this case to the remaining classes, where a subset of death ages is known and the age distribution of survivors is known.

Take as given a population mortality table $\overrightarrow{q} = (q_0, q_1, q_2, \cdots, q_{109}, q_{110})$, where q_t denotes the age-t mortality hazard (i.e., the probability of dying before turning $t+1$, conditional on having reached age t) and where we assume that nobody survives beyond age 110. View the first column of data from each nominee class as a known vector $\overrightarrow{e} = (e_0, e_1, \cdots, e_{109}, e_{110})$, where e_t is the number of age t nominees. Let the random vector $\overrightarrow{d} = (d_0, d_1, \cdots, d_{109}, d_{109})$ denote the distribution of the actual death ages of all of these nominees, and let $\overline{d}$ denote the average death age of the nominees, i.e. $\overline{d} = \frac{\sum_{n=0}^{n=110} n d_n}{\sum_{n=0}^{n=110} d_n}$. When the realized value of $\overrightarrow{d}$, and hence of $\overline{d}$, is known, we can simply test whether $\overline{d}$ is significantly greater than we would have expected, given the ages at nomination and the population mortality table. Specifically, the test statistic

$$Z = \frac{\overline{d} - E\left[\overline{d}|\overrightarrow{e}, \overrightarrow{q}\right]}{\sqrt{V\left[\overline{d}|\overrightarrow{e}, \overrightarrow{q}\right]}}, \tag{1}$$

has a standard normal distribution (asymptotically) where $E\left[\overline{d}|\overrightarrow{e}, \overrightarrow{q}\right]$ and $V\left[\overline{d}|\overrightarrow{e}, \overrightarrow{q}\right]$ are, respectively, the expected value and variance of $\overline{d}$ given the mortality table $\overrightarrow{q}$ and the nomination ages $\overrightarrow{e}$. We can explicitly compute these by simulation or with analytic methods. High Z scores are taken as evidence of an adversely selected nominee pool.

With the exception of the two older classes of speculative nominees, we do not know $\overrightarrow{d}$. Instead, we know a vector $\overrightarrow{d^{early}}$ of death ages of individuals who died prior to 1826 (or 1854) and a vector $\overrightarrow{l}$ of ages of the individuals who were still alive.[7] This requires that we modify the simple test based on Equation 1. Our modified test compares the expected mean death age *conditional* on $\overrightarrow{d^{early}}$ and $\overrightarrow{l}$ with the the unconditional expected mean death age.

Specifically, the appendix establishes that the test statistic

$$Z' = \frac{E[\overline{d}|\overrightarrow{l}, \overrightarrow{d^{early}}, \overrightarrow{q}] - E[\overline{d}|\overrightarrow{e}, \overrightarrow{q}]}{\sqrt{V[\overline{d}|\overrightarrow{e}, \overrightarrow{q}] - V[\overline{d}|\overrightarrow{l}, \overrightarrow{d^{early}}, \overrightarrow{q}]}} \qquad (2)$$

has an asymptotic standard normal distribution.

Each term appearing on the right hand side of Equation 2 can be directly computed to arbitrary precision by simulation. We compute $E[\overline{d}|\overrightarrow{e}, \overrightarrow{q}]$ and $V[\overline{d}|\overrightarrow{e}, \overrightarrow{q}]$ for a given mortality table $\overrightarrow{q}$ by simulating a large number (in practice: 4000) of final death profiles for the nominee population $\overrightarrow{e}$ and computing the sample mean and variance of the average final death age for this sample. Similarly, we compute $E[\overline{d}|\overrightarrow{l}, \overrightarrow{d^{early}}, \overrightarrow{q}]$ and $V[\overline{d}|\overrightarrow{l}, \overrightarrow{d^{early}}, \overrightarrow{q}]$ by simulating a large number of final death profiles $\overrightarrow{d^{late}}$ for the still-living population $\overrightarrow{l}$. We then compute the sample mean and variance of the final average death age for the combination of this population and the early-dying population $\overrightarrow{d^{early}}$.

Our results are thus based on a series of Z-tests – based on Equation (1) or (2) – for whether the observed annuitant longevity significantly exceeds the longevity predicted by an exogenously given life table.

3.4 Results

We first test for selection effects by testing whether early nominees lived longer than predicted by the Northampton table. The results appear in Table 3. We see that the early nominees of both genders are conditionally

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[7] For the 1860 data sets, it is important to note the discrepancy between the third column of data and $\overrightarrow{l}$. This occurs because the data report the age as of December 31, 1850, but we know that all these individuals in fact lived until May 8, 1854. Hence, we obtain the 'correct' $\overrightarrow{l}$ by aging the population by slightly over 3 years.

Table 3: Testing for Adverse Selection – Northampton Tables

	$E(\bar{d})$		$\hat{\sigma}$	Z-Score	P-Value
	Unconditional	Conditional			
Males	72.33	72.96	0.124	5.03	$\mathcal{O}(10^{-7})$
Females	71.31	72.84	0.083	18.9	$\mathcal{O}(10^{-75})$

Table 3: Testing for adverse selection among early nominees. This table reports the results of a test for whether the individuals nominated for annuities lived longer than predicted by the Northampton life tables described in the text. $E(d)$ refers to the expected average nominee death age, once all nominees have died. The columns labeled "Unconditional" and "Conditional" report the unconditional expected average death age and the expected average death age *conditional* on the observed mortality history through 1826. The column labeled $\hat{\sigma}$ is the estimated standard deviation of the difference between these two columns. P-value refers to a one tailed test.

predicted to live significantly longer than predicted by the population average mortality tables, with males and females living an estimated average of .63 (i.e., 72.96 – 72.33) and 1.53 years longer, respectively. This is strongly suggestive of selection amongst the early nominees.[8]

Our second test amounts to a test for additional selection amongst annuitants nominated subsequent to repricing in 1828. This involves testing whether various populations of nominees lived longer than predicted by the Finlaison life tables used for pricing after 1828. Table 4 reports the results.

The final two rows of Table 4 help verify the validity of the Finlaison life tables for describing the mortality of the 1808-1826 nominee population. They indicate that males are conditionally predicted to live an estimated average of .002 years fewer than the Finlaison tables would suggest, while females are conditionally predicted to live an estimated average of .009 fewer years. Since both are statistically (and economically) insignificant, there is indeed no evidence that these early nominees lived longer (or shorter) than Finlaison's tables predict.

The first two rows of Table 4 report the results of testing whether the "non-speculative" nominees from 1808-1850 lived longer than the Finlaison tables predict. The first row of indicates that non-speculative 1808-1850 male nominees are conditionally predicted to live an average of about 1 year longer than would be unconditionally predicted, while Females nominated between 1808 and 1850 are conditionally predicted to live an average of about .33

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[8] Note that this conditional prediction likely understates the *degree* of adverse selection, since the conditional prediction is made under the assumption that the survivors will age according to the Northampton tables.

Table 4: Testing for Adverse Selection – Finlaison Tables

	$E(\bar{d})$		$\hat{\sigma}$	Z	P
	Unconditional	Conditional			
N-S Males, 1850	73.79	74.78	0.109	9.10	$\mathcal{O}(10^{-20}$
N-S Females, 1850	76.23	76.56	0.088	3.73	$\mathcal{O}(10^{-4}$
S Males, 59-64	74.84	77.72	0.320	9.01	$\mathcal{O}(10^{-19})$
S Males, 73-84	83.40	84.72	0.230	5.10	$\mathcal{O}(10^{-7})$
S Males, 85+	89.88	91.62	0.329	5.28	$\mathcal{O}(10^{-8})$
Males, 1826	73.87	73.87	.131	-0.02	0.509
Females, 1826	76.12	76.11	0.078	-0.11	0.545

Table 4: This table reports the results of a test for whether various classes of individuals nominated for annuities lived longer than predicted by the Finlaison life tables described in the text. "N-S" and "S" refer to non-speculative and speculative classes, respectively. $E(d)$ refers to the expected average nominee death age, once all nominees have died. The columns labeled "Unconditional" and "Conditional" report the unconditional expected average death age and the expected average death age *conditional* on the observed mortality history through 1854, 1856, or 1826 (for the first four rows, the fifth row, and the final two rows, respectively). The column labeled $\hat{\sigma}$ is the estimated standard deviation of the difference between these two columns. Z and P values refer to a 1-tailed test of whether the *conditional* number of years lived is significantly longer than the *unconditional* number of years lived?

years longer. Both are statistically significant at standard levels.[9]

The middle 3 rows of Table 4 indicate that all three classes of speculative nominees on whom we have data lived significantly longer than the Finlaison tables would suggest.[10] There is thus strong evidence of selection effects

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[9] The pre-1828 nominees are a subset of these nominees. Hence, this is a pure test for *additional* selection amongst post-1828 nominees only insofar as the Finlaison tables accurately describe the mortality of early nominees.

For similar reasons, these results *understate* the degree of adverse selection among the post re-pricing nominees insofar as they involve pooling the un-selected early nominees with the selected later nominees.

[10] Speculators appear to have selected *older* nominees for a simple reason, discussed by Francis (1853): The Finlaison tables were developed based on actual mortality experiences of early nominees. Payouts from early annuities were capped at age 75, which meant that there were extremely few *nominees* above this age prior to 1828. Finlaison's mortality estimates for aged individuals are thus derived from the mortality experience of individuals who were nominated at a young age and subsequently grew old – likely significantly underestimating the longevity of *selected* older lives.

Francis relays anecdotes suggesting that speculators understood this well. Apparently, they even combed the countryside looking for hale older men with valid birth certificates and even resorted to paying local surgeons and pastors to keep them healthy – a rare case

Table 5: Estimated Longevity Enhancements		
Panel A: Older Nominee Classes		
	LEF	
Selected Males, 73-84	25.37%	
Selected Males, 85+	94.02%	
Panel B: Other Nominee Classes		
	Estimated LEF Bounds	
	Lower Bound	Upper Bound
Selected Males, 59-64	22.56%	122.91%
Non-Selected 1850 Males	6.58%	10.74%
Non-Selected 1850 Females	1.74%	2.66%

Table 5: This table reports estimates of the "longevity enhancement factor" (LEF) described in the text. This measures the percentage by which the longevity of a given group of nominees exceeded the longevity predicted by Finlaison's mortality tables. Panel A reports the *actual* longevity enhancement factors for the oldest two groups of selected nominees from A.G. Finlaison's data set. Panel B reports estimated upper and lower bounds for the longevity enhancement factors of the non-selected groups and the youngest selected group of nominees.

among speculator-selected nominees. It is worth emphasizing that this evidence falls short of directly indicating adverse selection in the sense of Dionne et al. (2001), since there could have been other symmetrically known information (e.g., township of birth). So we do not and cannot know whether the positive correlation between longevity and the decision to purchase an annuity would be present if we conditioned on this information.

Table 4 indicates that both the "selected" and "non-selected" nominees from A.G. Finlaison's data set display higher longevity than the Finlaison mortality tables would indicate. Table 5 attempts to measure how much higher the longevity was by computing a "longevity enhancement factor" (LEF) for each of the five groups in the data set. This measures the percentage by which actual longevity exceeded the anticipated longevity. Formally:

$$LEF = \frac{Actual\ average\ death\ age - Expected\ average\ death\ age}{Expected\ average\ death\ age\ - Average\ nomination\ age}. \tag{3}$$

The top panel of Table 5 indicates that nominees from the middle and oldest selected nominee classes lived an average of 25.37% and 94.02% longer from their nomination date than the Finlaison tables indicated they would.

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of moral hazard in annuity markets.

For the other groups, the structure of the data prevents a direct computation of a LEF. Instead, the bottom panel of Table 5 reports an upper and lower bound on the LEF for the non-selected males, females, and the youngest group of selected nominees. The lower bound is computed under the assumption that the nominees still living at the time of observation will age according to the Finlaison tables – in effect assuming that all the longevity "enhancement" has already taken place. The upper bound is computed under the assumption that the nominees still living at the time of observation will have the same LEF applied to the remainder of their lifetimes.[11]

Table 5 indicates that the longevity of the speculative classes were substantially *more* enhanced – relative to expectations – than the longevity of the non-speculative classes. The non-selected males are estimated to have lived between 6.58% and 10.74% longer than the Finlaison tables predicted. In contrast, the selected groups are estimated to live more than 20% and as much as 122.91% longer than the Finlaison tables predicted. The longevity of the non-selected female nominee class appears to be even less selected than the males, with a LEF estimated to be between 1.73% and 2.66%.[12]

The relative enhancement of the non-selected males and the non-selected females likely stems from the combination of gender-blind early pricing and the fact that the pre-1828 data on which the Finlaison tables were based did not distinguish between "speculative" and "non-speculative" nominees. Since females are (and were) longer lived than males at typical annuity purchasing ages, most speculative nominees from the gender-neutral pricing era are likely to have been female. Table 4 clearly indicates that speculators were quite good at selecting long-lived nominees. Hence, Finlaison's female life tables probably reflect the longevity of a substantially more selected pool of nominees than the Finlaison's male life tables, and the additional selection

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[11] Formally, the upper bound LEF solves:

$$N_T \cdot (A_N + (E[A_F] - A_N)(1+LEF)) = N_D \cdot A_D + N_L \cdot (A_L + (E[A_{LD}|\vec{l}] - A_L)(1+LEF)), \quad (4)$$

where: N_T, N_D, and N_L are the number of nominees, the number of nominees who had died by the time of observation, and the number of living nominees, respectively; A_N, A_D, and A_L are the average age of the nominees, the dead nominees, and the living nominees, respectively; and $E[A_F]$ and $E[A_{LD}|\vec{l}]$ are the expected average death age the nominees at the time of nomination and the expected average death age of of the still living nominees at the time of observation.

[12] These qualitative observations are robust to accounting for the fact that all of the longevity "enhancement" is attributable to the post-1826 nominees (in light of Table 4).

effects after repricing are thus relative smaller for females.

4 Modern Annuity Markets and the Annuity Puzzle

There is a substantial and growing literature on the so-called "annuity puzzle," the observation very few individuals voluntarily choose to annuitize their retirement assets in spite of the benefits economic theory suggests they should provide. On the theoretical side, Yaari (1965) showed that full annuitization should be optimal under specialized circumstances, and Davidoff et al. (2005) have recently shown that the theoretical prediction of significant annuitization by optimizing individuals is quite general. On the empirical side, in contrast, Poterba et al. (2003), Johnson et al. (2004) and others have documented a marked paucity of life annuity purchases by households in the Health and Retirement Survey. This paucity is corroborated by industry sales data which indicate a paltry \$5.9 billion in immediate annuity sales in in the U.S. in 2006 (LIMRA, 2007) and by international evidence (James and Vittas, 2004).

A number of papers have attempted to explain the annuity puzzle. Potential explanations include pre-existing (public or company pension) annuities, unfavorable pricing resulting from adverse selection or administrative loading (Mitchell et al., 1999), within-household risk pooling for married couples (Brown and Poterba, 2000), bequest motives (Friedman and Warshawsky, 1990), higher returns from alternative assets (Milevsky, 1998), the need for liquidity to cover health shock expenditures (Sinclair and Smetters, 2004), and the option value of delaying annuitization (Dushi and Webb, 2004). Whether some combination of these explanations can fully resolve the puzzle remains an open question.

The typical approach in this literature is to posit that households are life-cycle utility maximizers – typically with constant relative risk aversion, additively separable utility functions – and to compute the value such individuals would place on having access to private annuity markets. There is some empirical support for this approach: Brown (2000) shows that individuals' stated intentions to annuitize are indeed correlated with the theoretical value they would place on annuitization according to such life-cycle models. Nevertheless, a distinct weakness of this approach for addressing the annuity puzzle is that quantitative conclusions regarding the value and levels of annuitization rely heavily on functional form utility assumptions. Our empirical methodology is free from functional form assumptions and therefore helps speak to the importance of adverse selection of a possible resolution to the puzzle.

We now present an abstract model of an annuity market. This illustrative model suggests three distinct possibilities: First, the market may exhibit no adverse selection whatsoever. This happens when providers can break even while selling annuities to the entire population of potential annuitants. Second, the market may suffer from adverse selection but still remain "thick," with firms earning normal profits selling annuities to all but a reasonably small number of the least healthy potential annuitants. Finally, the market may be nearly or completely destroyed by an Akerlovian death spiral, wherein annuity providers set progressively higher prices on their products, only to find that additional adverse selection renders each new price unprofitable, and the entire market eventually unravels.

4.1 A Stylized Model of Adverse Selection in Annuity Markets

Consider a sequence of periods $t = 0, 1, 2, ...$ In period 0, a continuum of individuals retires. Each individual has unit wealth and has the option of exchanging her wealth for a life annuity with a constant per-period payout of a. In periods $1, 2, ..., t, ...$, individuals consume, if they are still alive. An individual who purchases an annuity with her wealth will thus have a per-period consumption of a for as long as she lives. Individuals who do not purchase an annuity consume in later periods by saving at an interest rate r. We take $r = 0$, purely for simplicity. Also for simplicity, we assume that, conditional on reaching period t, an individual i has a privately known and period-invariant probability S_i of surviving to period $t+1$. Individuals differ only in their survival probabilities, and we take the distribution of S_i to be uniform on $[0, \overline{S}]$, where $\overline{S} < 1$.

Preferences over consumption streams $c_1, c_2, \cdots, c_t, \cdots$ are given by:

$$V(c_1, c_2, \cdots, c_t, \cdots) = \sum_{t=1}^{\infty} S_i^t \ln(c_t). \tag{5}$$

Individuals who choose to purchase an annuity thus receive utility

$$V_i^*(a) = \sum_{t=1}^{\infty} S_i^t \ln(a) = \frac{S_i \ln(a)}{1 - S_i}. \tag{6}$$

Individuals who do not choose to purchase an annuity solve

$$\begin{aligned} V_i^A \equiv Max \sum_{t=1}^{\infty} S_i^t \ln(c_t) \\ \text{s.t. } \textstyle\sum_{t=1}^{\infty} c_t \ \leq \ 1. \end{aligned} \tag{7}$$

Solving Equation 7 yields $c_t^* = S_i^{t-1}(1 - S_i)$, and, $V_i^A = \frac{S_i}{1-S_i}\left(\ln(1 - S_i) + \frac{S_i}{1-S_i}\ln(S_i)\right)$. Individuals offered annuities paying a per period will thus choose to annuitize whenever

$$\ln(a) > \ln(1 - S_i) + \frac{S_i}{1 - S_i}\ln(S_i). \tag{8}$$

The right-hand side of Expression 8 is decreasing in S_i. Thus, for any given annuity payment $a < 1$, individuals with survival probabilities above the cutoff value $S^*(a)$ solving

$$\ln(a) = \ln(1 - S^*(a)) + \frac{S^*(a)}{1 - S^*(a)}\ln(S^*(a)), \tag{9}$$

will purchase an annuity, while those with survival probabilities below $S^*(a)$ will choose not to purchase annuities. (If $a > 1$, then the everyone wishes to buy an annuity and we take $S^*(a) = 0$.)

We assume that annuities are sold in a competitive market, so that firms must break even on annuity sales.[13] Selling an annuity with payment a to an individual i yields expected profits $1 - \sum_{t=1}^{\infty} S_i^t a = 1 - \frac{S_i}{1-S_i}a$. Equilibrium is then characterized by a value a such that $E\left[\ \frac{S_i}{1-S_i}a \ \middle|\ S_i > S^*(a)\ \right] = 1$.

To solve for equilibrium, we first determine the annuity payment $a^*(\tilde{S})$ that would allow a firm to break even for a given cutoff $\tilde{S}$:

$$a^*(\tilde{S}) = \left[\int_{\tilde{S}}^{\overline{S}} \frac{S_i}{1 - S_i}\frac{dS_i}{\overline{S} - \tilde{S}}\right]^{-1} = \left[\frac{1}{\overline{S} - \tilde{S}}\ln\left(\frac{1 - \tilde{S}}{1 - \overline{S}}\right) - 1\right]^{-1}. \tag{10}$$

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[13] Adding a loading factor does not change the qualitative results.

We can find the unique equilibrium using the following iterative technique. First, "guess" that $\tilde{S} = \tilde{S}_0 \equiv 0$, so that everybody annuitizes. Second, use Equation 10 to compute the payout rate $a_0 \equiv a^*(\tilde{S}_0)$ at which firms would break even, given that everyone annuitizes. Third, use Equation 9 to compute $\tilde{S}_1 = S^*(a_0)$ – the shortest-lived individual who would voluntarily buy annuities with this payout rate. If $\tilde{S}_1 = \tilde{S}_0 \equiv 0$, then firms can sell annuities to all individuals and still break even, and the market does not suffer from adverse selection problems. Otherwise, some short-lived individuals – those with $S_i < \tilde{S}_1$ – are driven from the market. In this case, "guess" that $\tilde{S} = \tilde{S}_1$ and repeat the steps by finding the break-even payout rate $a_1 \equiv a^*(S_1)$ if only those individuals with $S_i > \tilde{S}_1$ annuitize, and then the "cutoff" survival probability $\tilde{S}_2 = S^*(a_1)$. Iterating this process to convergence yields an equilibrium pair $(a_\infty, \tilde{S}_\infty)$. The payout rate a_∞ allows firms to break even, given that annuities are sold to the set of individuals with $S_i > \tilde{S}_\infty$ – precisely the set who would voluntarily purchase annuities offering the payout rate a_∞.

The equilibrium depends on the distribution of risk types – i.e., on $\overline{S}$. For low values of $\overline{S}$, the market does not suffer from adverse selection. For example, when $\overline{S} = .8$, the break-even payout rate for the whole population is $a_0 = 1.36$, which makes annuities desirable for any rational individual.

For sufficiently high values of $\overline{S}$, the market is killed by a "death spiral." For example, when $\overline{S} = .999$, the fair payout rate for serving the whole market is $a_0 \approx .17$. A payout rate this low makes annuitization undesirable for all but the longest-lived 37% of individuals ($\tilde{S}_1 \approx .63$), so the first round of adverse selection "kills" about 63% of the market. To break even with the remaining 37%, firms must lower their payout rate to $a_1 \approx .067$, which drives another 20% of the population from the market ($\tilde{S}_2 \approx .83$). The next round drives another 8% of the population out of the market ($\tilde{S}_3 \approx .91$), and, in equilibrium, just over 1% of individuals end up buying annuities ($\tilde{S}_\infty \approx .99$): adverse selection essentially destroys the market.

For intermediate values of $\overline{S}$ the market suffers from adverse selection, but the iterative process converges quickly and the market is still "thick" in equilibrium: there is no "death spiral." When $\overline{S} = .85$, for example, the payout rate to serve the whole market is $a_0 \approx .96$. This payout rate drives individuals with survival probabilities below $\tilde{S}_1 \approx .0067$ from the market, forcing firms to raise prices by a very small amount, driving a few more individuals from the market, and so forth. But, in equilibrium, a full 99.2% of individuals buy annuities.

This model, while clearly overly simplistic, captures the three key possibilities: first, adverse selection could be completely absent from annuity markets; second, adverse selection could be present, but annuity markets could nevertheless function reasonably well; or, third, adverse selection could essentially destroy the annuity market.

4.2 Applying the Model to the 1808 Act

The historical evolution of the 1808 Life Annuity Act closely mimics the first stages of the equilibrium computation process modeled above. The government initially priced annuities at a price consistent with population-average mortality. Table 3 indicates that the annuitant pool they selected had significantly higher longevity than the population average. In 1829, they repriced the annuities based on the mortality of these selected annuitants. Table 4 indicates that the new annuitant pool had significantly higher longevity than even this new table indicated.

This pattern is inconsistent with the first possible outcome of the simplistic model: adverse selection appears to have played a significant role in this market. Unfortunately, the 19th century U.K. government was not so kind to 21st century researchers as to continue to re-price annuities to determine if the market would be destroyed by a death spiral or would settle into a reasonably "thick" market, so definitively distinguishing between the other two possibilities is impossible.

Nevertheless, we are inclined to tentatively interpret the results as at least moderately suggestive of the second case: the *absence* of a death spiral. While the non-selected females from A.G. Finlaison's data set were statistically significantly longer-lived than the Finlaison mortality tables indicated, the absolute magnitude of the longevity enhancement is modest. Table 4 indicates that, on average, nominees lived approximately 4 months longer than expected. This suggests that additional "rounds" of re-pricing would be unlikely to raise prices so dramatically as to significantly reduce the population of annuity buyers.[14] Furthermore, despite the significant increase in prices female nominees faced after the 1829 re-pricing, the market for these annuities remained strong: using the data from Table 1, we can infer that

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[14] Back of the envelope calculations using the toy model above yield no more than a 15% reduction in the size of the annuity market when the first round of re-pricing increases longevity by 3%. The empirical relevance of calculations based on such simply calibrated models are, of course, highly suspect.

there were an average of 267.5 female nominees per year prior to 1826, and an average of 240.8 female nominees per year subsequent to 1826.

5 Discussion and Conclusions

The United Kingdom's 1808 Life Annuity Act effectively opened up a market for government-provided life annuities. Analysis of data on purchases and subsequent mortality rates suggests that the market was characterized by two types of adverse selection: classic "self-selection" whereby individuals purchasing annuities for themselves were healthier than the average individual in the population, and "speculative" selection, whereby pecuniary-minded individuals or institutions took advantage of an odd feature of the act which allowed them to purchase annuities contingent on the lives of *others.*

Our results indicate that self-selection of annuitants was substantial: the annuitant pool between 1808 and 1826 was dramatically longer lived than the population average mortality table used to price annuities suggested. Furthermore, a 1829 attempt by the government to re-price annuities to reflect the higher longevity of the early annuitants resulted in an annuitant pool that was even longer lived. While this is reminiscent of the early stages of a death spiral, the degree to which the later annuitants were selected and the robustness of the market size suggests that, while adverse selection was clearly an important concern, it is unlikely to have completely undermined the viability of the market. This finding reinforces the view that an adverse selection driven death spiral is unlikely to explain the dearth of annuity sales in contemporary market commonly referred to as the "annuity puzzle." Adverse selection is thus likely to play an important but non-decisive role in resolving the puzzle.

Our analysis also indicates that speculators effectively selected particularly long-lived individuals to nominate for annuities and profited considerably from the Act, at the expense of the government. This recommends care to modern policy makers when contemplating expanding choice in government provided services: when choices can be made by pecuniary-minded individuals or institutions, selection effects may be significantly exacerbated and may impose particularly large costs on the government.

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6 Appendix

This appendix shows that the test statistic Z', defined in Equation (2) has an asymptotically standard normal distribution. To do so, it first establishes that $\overrightarrow{Y} =\equiv E\left[\frac{\overrightarrow{d}}{N}|\overrightarrow{d^{early}}, \overrightarrow{e}, \overrightarrow{l}, \overrightarrow{q}\right]$, viewed as a function of $\overrightarrow{d^{early}}$ and $\overrightarrow{l}$, is normally distributed with a variance which is consistently estimated by $\left(V\left[\overrightarrow{d}/N|\overrightarrow{e}, \overrightarrow{q}\right] - V\left[\overrightarrow{d^{late}}/N|\overrightarrow{l}, \overrightarrow{e}, \overrightarrow{q}\right]\right)$, where N is the number of annuity buyers.[15] Since we will take the vector of age-specific mortality hazards $\overrightarrow{q}$ and the vector $\overrightarrow{e}$ of nominees as a given throughout this appendix, we will drop them for notational convenience henceforth.

Recall that $\overrightarrow{d}$ is a random vector describing the number of individuals who died at at each age. It is equal to $\overrightarrow{d^{early}} + \overrightarrow{d^{late}}$, where $\overrightarrow{d^{early}}$ is a vector-valued random variable whose elements describe the number of individuals who had died, at each age, by a fixed death observation date (e.g, May 8th, 1854), and $\overrightarrow{d^{late}}$ is, similarly, a vector describing the death ages for those who died after that fixed date. Finally, recall that $\overrightarrow{l}$ is a vector-valued random variable describing the ages of the individuals who had not died by the death observation date.

Note that $E\left[N\overrightarrow{Y}\right] = E\left[\overrightarrow{d}\right]$, that $N\overrightarrow{Y} = \overrightarrow{d^{early}} + E\left[\overrightarrow{d^{late}}|\overrightarrow{l}\right]$, and that

$$V\left[N\overrightarrow{Y}\right] = V\left[\overrightarrow{d^{early}}\right] + V\left[E\left[\overrightarrow{d^{late}}|\overrightarrow{l}\right]\right] + 2Cov\left[\overrightarrow{d^{early}}, E\left[\overrightarrow{d^{late}}|\overrightarrow{l}\right]\right] \tag{11}$$

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[15] For asymptotics, we have in mind that the sample of nominees from (e.g.) 1808-1826 was a a representative sample drawn from some distribution of all possible nominees.

(N.b.: $E\left[\overrightarrow{d^{late}}|\overrightarrow{l}\right]$ is a random variable since it depends on the realization of $\overrightarrow{l}$.)

We now prove two simple lemmas using the law of iterated expectations:

Lemma 1 $Cov\left[\overrightarrow{d^{early}}, E\left[\overrightarrow{d^{late}}|\overrightarrow{l}\right]\right] = Cov\left[\overrightarrow{d^{early}}, \overrightarrow{d^{late}}\right]$.

Proof.

$$\begin{aligned}
Cov\left[\overrightarrow{d^{early}}, \overrightarrow{d^{late}}\right] &= E\left[\left(\overrightarrow{d^{early}} - E\left[\overrightarrow{d^{early}}\right]\right)\left(\overrightarrow{d^{late}} - E\left[\overrightarrow{d^{late}}\right]\right)'\right] \\
&= E\left[E\left[\left(\overrightarrow{d^{early}} - E\left[\overrightarrow{d^{early}}\right]\right)\left(\overrightarrow{d^{late}} - E\left[\overrightarrow{d^{late}}\right]\right)' |\overrightarrow{d^{early}}, \overrightarrow{l}\right]\right] \\
&= E\left[\left(\overrightarrow{d^{early}} - E\left[\overrightarrow{d^{early}}\right]\right) E\left[\left(\overrightarrow{d^{late}} - E\left[\overrightarrow{d^{late}}\right]\right)' |\overrightarrow{d^{early}}, \overrightarrow{l}\right]\right] \\
&= E\left[\left(\overrightarrow{d^{early}} - E\left[\overrightarrow{d^{early}}\right]\right)\left(E\left[\overrightarrow{d^{late}}|\overrightarrow{l}\right] - E\left[\overrightarrow{d^{late}}\right]\right)'\right] \\
&= Cov\left[\overrightarrow{d^{early}}, E\left[\overrightarrow{d^{late}}|\overrightarrow{l}\right]\right].
\end{aligned}$$

■

Lemma 2 $V\left[E\left[\overrightarrow{d^{late}}|\overrightarrow{l}\right]\right] = V\left[\overrightarrow{d^{late}}\right] - E\left[V\left[\overrightarrow{d^{late}}|\overrightarrow{l}\right]\right]$.

Proof.

$$\begin{aligned}
V\left[\overrightarrow{d^{late}}\right] &= E\left[\left(\overrightarrow{d^{late}} - E\left[\overrightarrow{d^{late}}\right]\right)\left(\overrightarrow{d^{late}} - E\left[\overrightarrow{d^{late}}\right]\right)'\right] \\
&= E\left[E\left[\left(\overrightarrow{d^{late}} - E\left[\overrightarrow{d^{late}}\right]\right)\left(\overrightarrow{d^{late}} - E\left[\overrightarrow{d^{late}}\right]\right)' |\overrightarrow{l}\right]\right] \\
&= E\left[E\left[\overrightarrow{d^{late}}\overrightarrow{d^{late\prime}} - \overrightarrow{d^{late}}E\left[\overrightarrow{d^{late}}\right]' - E\left[\overrightarrow{d^{late}}\right]\overrightarrow{d^{late\prime}} + E\left[\overrightarrow{d^{late}}\right]E\left[\overrightarrow{d^{late}}\right]' |\overrightarrow{l}\right]\right] \\
&= E\left[E\left[\overrightarrow{d^{late}}\overrightarrow{d^{late\prime}} - E\left[\overrightarrow{d^{late}}|\overrightarrow{l}\right]E\left[\overrightarrow{d^{late}}|\overrightarrow{l}\right]' |\overrightarrow{l}\right]\right] \\
&\quad + E\left[E\left[\overrightarrow{d^{late}}|\overrightarrow{l}\right]E\left[\overrightarrow{d^{late}}|\overrightarrow{l}\right]'\right. \\
&\quad \left. - E\left[\overrightarrow{d^{late}}|\overrightarrow{l}\right]E\left[\overrightarrow{d^{late}}\right]' - E\left[\overrightarrow{d^{late}}\right]E\left[\overrightarrow{d^{late}}|\overrightarrow{l}\right]' + E\left[\overrightarrow{d^{late}}\right]E\left[\overrightarrow{d^{late}}\right]'\right] \\
&= E\left[V\left[\overrightarrow{d^{late}}|\overrightarrow{l}\right]\right] + V\left[E\left[\overrightarrow{d^{late}}|\overrightarrow{l}\right]\right].
\end{aligned}$$

Re-arranging completes the proof. ■

From Lemma 2 and Equation 11, we have:

$$V\left[N\overrightarrow{Y}\right] = V\left[\overrightarrow{d^{early}}\right]+V\left[\overrightarrow{d^{late}}\right]-E\left[V\left[\overrightarrow{d^{late}}|\overrightarrow{d^{early}},\overrightarrow{l}\right]\right]+2Cov\left[\overrightarrow{d^{early}},E\left[\overrightarrow{d^{late}}|\overrightarrow{d^{early}},\overrightarrow{l}\right]\right].$$

Then, using Lemma 1 and the fact that $\overrightarrow{d} = \overrightarrow{d^{early}} + \overrightarrow{d^{late}}$ gives

$$\begin{aligned} V\left[N\overrightarrow{Y}\right] &= V\left[\overrightarrow{d^{early}}\right] + V\left[\overrightarrow{d^{late}}\right] + 2Cov\left[\overrightarrow{d^{early}},\overrightarrow{d^{late}}\right] - E\left[V\left[\overrightarrow{d^{late}}|\overrightarrow{d^{early}},\overrightarrow{l}\right]\right] \\ &= V\left[\overrightarrow{d}\right] - E\left[V\left[\overrightarrow{d^{late}}|\overrightarrow{d^{early}},\overrightarrow{l}\right]\right]. \end{aligned}$$

Note that $E\left[V\left[\frac{\overrightarrow{d^{late}}}{N}|\overrightarrow{d^{early}},\overrightarrow{l}\right]\right]$ is consistently estimated by $V\left[\frac{\overrightarrow{d^{late}}}{N}|\overrightarrow{l}\right]$, since $\frac{\overrightarrow{l}}{N}$ converges in probability. Hence, $V[\vec{Y}]$ is consistently estimated by $V[\overrightarrow{d}/N]-V\left[\frac{\overrightarrow{d^{late}}}{N}|\overrightarrow{l},\overrightarrow{d^{early}}\right]$.

The expected mean death age, conditional on the data and the mortality table –i.e., $E[\overline{d}|\overrightarrow{d}^{\,early},\overrightarrow{l}\,]$ – is equal to $\overrightarrow{n}\cdot\overrightarrow{Y}$, where $\overrightarrow{n}$ denotes a vector $(0,1,2,\cdots)'$ of ages. Hence, from above, its variance is consistently estimated by $\overrightarrow{n}'\left(V\left[\overrightarrow{d}/N\right] - V\left[\overrightarrow{d^{late}}/N|\overrightarrow{l}\right]\right)\overrightarrow{n}$. Its mean value is the *unconditional* expected average death age $E\left[\overline{d}\right]$. Hence, the test statistic

$$Z'' = \frac{E[\overline{d}|\overrightarrow{d}^{\,early},\overrightarrow{l}\,] - E\left[\overline{d}\right]}{\sqrt{\overrightarrow{n}'\left(V\left[\overrightarrow{d}/N\right] - V\left[\overrightarrow{d^{late}/N}|\overrightarrow{d^{early}},\overrightarrow{l}\right]\right)\overrightarrow{n}}} \tag{12}$$

is asymptotically distributed as a standard normal random variable.

Recognizing $\overrightarrow{n}'\left(V\left[\overrightarrow{d}/N\right]\right)\overrightarrow{n}$ and $\overrightarrow{n}'\left(V\left[\overrightarrow{d^{late}/N}|\overrightarrow{d^{early}},\overrightarrow{l}\right]\right)\overrightarrow{n}$ as being equal to $V[\overline{d}]$ and $V[\overline{d}|\overrightarrow{d^{early}}]$, respectively, we see that Z' (from Equation 2) and Z'' are equal. Hence, Z' has an asymptotic standard normal distribution as well.