Optimal Security Investments and Catastrophic Terrorism[1]

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Abstract

In the aftermath of 9/11, concern over security has increased dramatically in both the public and the private sector. Yet no clear algorithm exists to inform firms on the amount and the timing of security investments to mitigate catastrophic risks. The goal of this paper is to devise an optimum investment strategy for firms to mitigate exposure to catastrophic risks, focusing on how much to invest and when to invest. The latter question addresses the issue of whether postponing a risk mitigating decision is an optimal strategy or not. Accordingly we develop and estimate two models; a one-period model and a multi-period model. We caliberate these models using probability measures for catastrophic risks associated with food, from our prior research, together with the results of a "Benchmarking and Assessment Survey" of food companies. The limitations on the availability of catastrophic risk insurance and their high level of deductibility, together with the one-time nature of the alternative risk-mitigating investments suggest that such investments should be undertaken whether or not catastrophic risk insurance is available, particularly since these investments have a large impact on risk financing. Such investments may protect long term reputations and brand ratings in addition to mitigating potential catastrophic losses.

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1 Introduction

In the aftermath of September 11, the greatest terrorist attack of all time, and Katrina, the greatest natural disaster in recent history, insurance and reinsurance companies have become increasingly reluctant to cover catastrophic risks. The numerous difficulties that have plagued catastrophic risk insurance and particularly terrorism risk insurance (c.f., Kunreuther. et. al., 2005 and Auerswald, et. al. 2006) lead us to the need to consider what firms and businesses can do by way of ex-anti investments in risk mitigating strategies. However, little is known about how much investments for self-protection (self-insurance), or other hedging strategies, firms need when insurance markets fail to provide the necessary protection.

One of the principal reasons why catastrophic and especially terrorism insurance markets have not been adequate is the difficulty that exists in assessing low frequency high impact risks and the uncertainties that are associated with measuring such risks. In a previous line of research conducted by Mohtadi and Murshid (2004-2006) and broadly identified as "Risk Metrics Project," [3] we have attempted to take a first step at addressing this shortcoming by developing quantitative probability measures using a novel statistical methodology to calculate catastrophic risks from intentional contamination of food by chemical or biological agents. While it is hoped that these new measures find their way to the insurance industry and are ultimately adopted, that task is beyond us at the present time. In the meantime, however, the probability measures developed under this project which are based on primary data also collected under this project (Mohtadi and Murshid, 2006a), may be used to find a "best practice" strategy. For the purposes of this research, a best practice strategy consists of finding the optimum level of investments and the optimum timing of such investments that firms wishing to mitigate the impact of catastrophic risks should undertake or are undertaking. This is the task of the present study. This work is made possible by developing an analytical model that is calibrated by the data, not only from the probability measures of the Risk Metrics Project, but also from another project involving a survey of 217 food companies consisting of 59 food

[3] This research was sponsored by the University of Minnesota's National Center for Food Protection and Defense (NCFPD) and the Department of Homeland Security (DHS) and has led to a number of publications and working papers several of which are cited at the end of this paper.

retailers, 124 food manufacturers and 34 logistics providers that was jointly developed and administered by Universities of Minnesota, Michigan State and Georgia Institute of Technology. This is known as the "Benchmarking and Assessment Survey."[4] Some of the econometric results from that survey, which are found from a working paper by Agiwal, Mohtadi and Kinsey (2007), are then used in the present study.(For reasons discussed later, we will only use the survey results from the food suppliers/manufacturers and retailers/wholesalers/foodservice providers). Decisions of whether to invest at the present time or postpone the investment decision to a future time period are important considerations that have a quasi option value approach. However, such option value formulation provides a decision rule, given the amount of security investment, whereas we are interested in finding the optimum value of such investment under both a single-period and a multi-period scenario. Hence we extend our simple analytical model to incorporate the benefits of investments from future time periods to arrive at the optimal for a risk neutral decision maker. Nonetheless, we *are* able to address not only the question of optimum level of investments, but also the optimal timing of the investments as well.

Our results suggest that in comparison to a regulated rate for terrorism risk insurance, the optimum level of investments to mitigate risk is somewhat larger, but does depend on the *level* of risk. However, the limitations that exist on the availability of catastrophic insurance at these regulated rates, and their high level of deductibility, together with the one-time nature of the alternative risk-mitigating investment strategies suggest that such investments should be undertaken whether or not catastrophic risk insurance is available, particularly as their impact on risk financing may not be negligible at all.

Following the development of the analytical one-period model in section 2, the estimation method and the results are presented in section 3. In Section 4 we extend the basic single-period model to account for multiple periods in a finite time horizon. Here we address the question of whether postponing a risk mitigation investment is an optimal strategy or not. Section 5 makes some concluding observations based on the comparison of our results with those of catastrophic insurance.

2 A Simple Analytical Model

Consider investing K in infrastructure/ business practice/ etc. *ex-ante* to improve responsiveness to a catastrophic event, should such an event occur. Let L be the nominal

[4] This work is also sponsored by the University of Minnesota's National Center for Food Protection and Defense (NCFPD) and the Department of Homeland Security (DHS) and is still ongoing.

loss in the case of an event. Then an initial investment of K, could reduce the impact of the loss to a fraction $\lambda(K)$ such that $\lambda(K)L < L$ and $\lambda(K) < 1$. Higher values of initial investments reduce the impact of the loss L so that $\lambda'(K) < 0$. Thus, $\lambda(K)$ may be interpreted as the *"loss coefficient"*. However, this reduction in loss is bounded from below (by zero) as the law of diminishing marginal productivity would imply. Thus without loss of generality one can assume that $\lambda^{"}(K) > 0$ and that $\underset{\lim k\to\infty}{\lambda(K)} \longrightarrow 0$, as shown below:

Many functional forms satisfy the above requirement. We shall return to this issue shortly.

If π_L is the probability of an event of magnitude L, then the expected loss in the absence of investment is

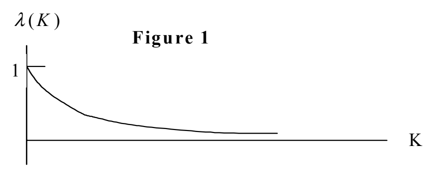


Figure 1

$$E(L|\pi_L)^{no\ invest} = \pi_L L \quad (1)$$

In the case of investing K the expected loss is:

$$E(L|\pi_L)^{invest} = \pi_L \lambda(K)L + K \quad (2)$$

Thus, while exposure to the event cannot be fully avoided, prior mitigating action can reduce the extent of loss from the event. Suppose the overall stream of profits from production is Π_o. Then expected profits from investing and non-investing in security measures are, respectively, $E(\Pi^{invest}) = \Pi_o - E(L|\pi_L)^{invest}$ and $E(\Pi^{no\ invest}) = \Pi_o - E(L|\pi_L)^{no\ invest}$. Then expected *net* gain from investing, given the probability π_L, is denoted by $G(\pi_L)$ and is found from

$$G(\pi_L) = E(L|\pi_L)^{no\ invest} - E(L|\pi_L)^{invest} = \pi_L[1 - \lambda(K)]L - K \quad (3)$$

Notice that $G(\pi_L)$ is $\gtreqless 0$ if $\pi_L \gtreqless \bar{\pi}$ where $\bar{\pi}$ is the threshold value of event probability and is given by $\bar{\pi} \equiv (K/L).(1/[1-\lambda(K)]$. This implies that for events with very low probability, a firm could lose by investing K!

Catastrophic events, which are the focus of this work, are by nature low frequency high impact events. Developing accurate metrics of catastrophic risk is not easy. The challenge arises because of the need to extrapolate from observed levels in data to unobserved levels. Classical statistical methods are not well-suited for this task (Coles 2001, Embrechts et al. 1997). Instead, the appropriate approach is to estimate catastrophic risk using extreme value (EV) analysis. By exploiting limiting arguments extreme value models can provide an approximate description of the stochastic behavior of extremes. This is both discussed and measured in Mohtadi and Murshid (2006b, 2006c, 2007b). Thus, $\pi(L) \sim f(L)$,where $f(L)$ is an EV probability distribution function. The expected gain from an event of that leads to a loss of size in the range L_o to L_1 is obtained from the EV probability distribution function such that,

$$G(\pi_{L_0<L<L_1}) = [1-\lambda(K)]\int_{L_o}^{L_1} f(L).L.dL - K \tag{4}$$

Owing to the complexity of the term inside the integral, we will approximate G by replacing L inside the integral with its linear mean, $\bar{L} = (L_o + L_1)/2$ to get:

$$G(\pi_{L_0<L<L_1}) \cong [1-\lambda(K)]\bar{L}[F(L|_{L>L_o}) - F(L|_{L>L_1})] - K \tag{5}$$

where $F(L)$ is the "anti-cumulative" distribution function of L , i.e., 1-cumulative density function of L.

2.1 Optimum Investments

Maximizing the net gains from security investments, $\underset{(K)}{Max} G(\pi_{L_0<L<L_1})(K)$, leads to a value K^* that satisfies the first order condition below[5]:

$$\lambda'(K^*)\bar{L}[F(L|_{L>L_o}) - F(L|_{L>L_1})] = -1 \tag{6}$$

[5] The second order condition is satisfied since $\lambda" > 0$

Deriving an explicit expression for optimum capital expenditures, K^*, depends on the functional forms used. But such functions must all satisfy the criteria discussed earlier. If we use the following form,

$$\lambda(K) = \frac{1}{1+\theta K^{\alpha}} \quad \theta > 0,\ 0 < \alpha < 1 \tag{7}$$

then the requirement that $\lambda' < 0$, $\lambda^{"} > 0$, $\lambda(0) = 1$, and $\lim_{k\to\infty} \lambda(K) \longrightarrow 0$, are all satisfied. Several functional specifications for such a loss coefficient with decreasing marginal rate of return as specified above exist in literature (Gordon and Loeb, 2002, Hausken 2006). In particular, Gordon and Loeb (2002), in relating reductions in security breach as a function of information security investments, have implied a version of $\lambda(K)$ that is similar to the one here. For our specification in 7, the first order condition yields:

$$(1+\theta K^{*\alpha})^2 = [F(L|_{L>L_o}) - F(L|_{L>L_1})]\bar{L}\theta\alpha K^{*\alpha-1} \tag{8}$$

We can use equation 8 along with 7 to both simplify the expression for K^* and also to substitute for the value of θ in terms of λ. The reason for the latter is that, as we shall see below, the empirical results from the "Benchmarking Survey" are closely associated with the values of λ rather than θ. Substituting from 7 into 8 we find:

$$\frac{1}{\lambda^2} = [F(L|_{L>L_o}) - F(L|_{L>L_1})]\bar{L}\theta\alpha K^{*\alpha-1} \tag{9}$$

In turn, we find from 7 that $\theta K^{\alpha} = (1-\lambda)/\lambda$. Substituting this value into 9 we find the optimum level of investments K^* to be:

$$K^* = \alpha\bar{L}[F(L|_{L>L_o}) - F(L|_{L>L_1})]\lambda(1-\lambda) \tag{10}$$

3 Estimation and Results

There are two values to use to estimate K^* in 10. The probability values of $F(L|_{L>L_o})$ and the parameter λ.First, we will discuss estimating $F(L|_{L>L_o})$. These values are derived from data complied by Mohtadi and Murshid (2006a). This work compiles 448 incidents of Chemical Biological and Radionuclear (CBRn) attacks from 1960s to 2005. Mohtadi and Murshid (2006b and 2006c) then calculate the probability of any such attack worldwide of a given magnitude or larger (in terms of the number of injuries) for different time horizons. These probability values are presented in the third column of Table 1 below.

The fourth and the fifth column then convert this anti-cumulative probability to the probability values $F(L|_{L_o<L<L_1})$ compatible with equation 10. Although any attack on the food sector will be of a CBRn nature, not every CBRn attack involves food. In fact, only about 60 of the CBRn incidents revolved around food (For a full chronology see Mohtadi and Murshid 2006a). Assuming a uniform distribution of CBRn attacks on food and non-food as our first prior, the above probabilities are adjusted in column five by the factor 60/448 or 0.134. This is presented in the final column of the table.

Table 1
Probabilities of a CBRN Attack of Various Magnitudes (based on Extreme Value Analysis) and Extrapolated Probabilities of Attacks on Food Sources

Number of casualties	Time Horizon	Anti-Cumulative Prob. of CBRN at various casualty levels $F(L \geq L_0)$	# of casualties L_0 to L_1	Prob. of a CBRN event between L_0 to L_1 $F(L \geq L_0) - F(L \geq L_1)$	Adjusted prob. for attacks on food
1000	Current risk	0.310	1000-5000	0.143	0.019
5000		0.167	5000-10000	0.034	0.005
10000		0.133	10000-15000	0.055	0.007
15000		0.078			
1000	5-year forecast	0.546	1000-5000	0.251	0.034
5000		0.295	5000-10000	0.071	0.009
10000		0.225	10000-15000	0.095	0.013
15000		0.130			
1000	10-year forecast	0.732	1000-5000	0.211	0.028
5000		0.520	5000-10000	0.110	0.015
10000		0.410	10000-15000	0.119	0.016
15000		0.291			
1000	20-year forecast	0.863	1000-5000	0.095	0.013
5000		0.768	5000-10000	0.057	0.008
10000		0.712	10000-15000	0.077	0.010
15000		0.634			

Source for column 3: Mohtadi and Murshid (2006b) and additional extrapolations

Before we proceed further, several features of Table 1 deserve attention. First, there are two key features of the anti-cumulative probability distribution of an event associated with x number of casualties or greater, in column 3 of Table 1, that deserve a mention. One; for a given time horizon, the anti-cumulative probability falls, as one would expect: The more severe an event, the less likely it is to occur.[6] Two; moving across time horizons, the anti-cumulative probability for a given x rises with longer time horizons! This interestingly but alarming result is a unique feature of the data and the fact that with the recent experience with terrorism and bioterrorism events, more sever events have become more likely in time (i.e., the tail parameter of the extreme value distribution shows a time trend. See Mohtadi-Murshid 2006b and 2006b for more details). Thus, an event leading

[6] However, we must add these tail events still turn out to be *fatter* than a normal density tail, a feature of these extreme value distributions discussed in Mohtadi-Murshid 2006b.

to casualty level of 15000 or more has a chance of under 1 percent in the current time horizon, but over 63 percent in a 20-year time frame!

Second, there are also two key features of the incremental probability distribution associated with area segments between two loss levels in column 5 that also deserve attention: One; for any given time horizon, the probability segments first fall with severity and then rise, exhibiting a U-like behavior. Closer scrutiny reveals that this pattern results because in column 3 the decline in the event probability, from a 5000+ to a 10,000+ casualty event, is much smaller than the decline from a 10,000+ to a 15,000+ casualty event. This pattern turn out to have significant implications for the amount of investment food companies have to make as a fraction of potential losses they would like to avoid, and produces a U shaped curve (Figure 3) that we will come to later. Two; moving across time horizons, with one exception, the probability segments for any given segment (say L_1 to L_2) *rises* for the first three time horizons (current, 5-year, 10-year), but *fall* for the last time horizon.[7] The reason for the fall seems to be the very large probability of the fat tail event in the last time horizon (15000 or more casualties). This large probability of 63% seems to take away areas from the other segments in this time horizon, again consistent with the time trend we have found for more extreme events.

Next, the "levels of injury" must be translated into dollar loss. To do this, we rely on a simple conversion factor from the forensic literature, where the value of $1 million is used for the loss of life. In the case of injuries, which by far dominate the data, we will adjust the conversion factor to $0.5 million. The assumption is that an injury or death will generate a liability for which the firm is ultimately responsible. We recognize that this is an overly simplified approach. For example, the economic impact on the firm might include such factors as reputation effects, loss of future markets, etc., and this impact might be vastly greater than the liability loss associated with injury or death. But in defense of this approach, it may be pointed out that those effects are indirect effects whereas the focus at this point is the direct effects. In any case, the result will be that the size of the expected gain (expected loss avoidance) from risk mitigating investments by firms may be significantly larger than the values obtained here and that our estimates will be only a conservative lower bound for what is actually needed.

Values of λ are estimated from the results of a survey of food companies. As was mentioned at the introduction, the survey questionnaire was jointly developed and administered by Universities of Minnesota, Michigan State and Georgia Institute of Tech-

[7] The minor exception is the first segment (1000-5000 casualties) only rise for the first two time horizons (current and 5-year) but not the third (or fourth).

nology. This questionnaire was sent out to participants in the U.S. food supply chain such as, food service retailers and wholesalers, grocery retailers and wholesalers, food service distributors, food manufacturers, processors and distributors and logistics providers. There were total of 217 firms consisting of 124 suppliers and manufacturers, 59 retailers/wholesalers/foodservice providers and 34 logistic providers Responses were on more than 100 questions. The responsibility of filling out the survey lay with individuals in Operations, Supply chain, Quality Assurance, Security, or Risk Management departments.

The primary focus of this survey was to collect firm level responses on their various security practices. Two other recent studies on food security practices, one on food service operations in Kansas (Yoon and Shanklin, 2007), and one on resilience in the U.K. food and drink industry (Peck, 2006), have also used survey data on security practices. However their scope of focus and the methodology is distinct from ours in that they have either a narrower focus (food service operations in schools and hospital in Kansas) or a qualitative case-study approach (detailed interviews with only 25-30 organizations in the U.K. food and drink industry).

The questionnaire on Security Benchmarking is divided into three distinct sections. The first section (I) asks various questions with respect to the security practices that the firms follow as prevention, detection, recovery and response measures to build resiliency within the firms as well as along the supply chain. The second section (II) focuses on the firm characteristics such as annual revenues, market size, supply chain scope, number of employees nationally and internationally, and also the employment profile of the employee primarily responsible for filling out this questionnaire. The third section (III) captures the impact of changes in security investments of the firms on *outcome* measures such as whether that was increased resilience, reduced risk profile, or reduced number of security incidents experienced, as a result of security investments by the firms. Given our interest in developing a measure of the parameter λ, our focus will be on *this* last section alone.

Out of the 217 observations, two had to be dropped due to incompleteness, resulting in 215 observations. In addition, owing to the variance in the nature of questions asked from the logistics providers under section III (which is the relevant section for our purpose), and the inherently different nature of their security concerns, this group was also dropped from our present observations, leading to a total of 181 food manufacturers and retailers. The respondents' profile is given in Table 2.

Table 2: Descriptive statistics of firm chracteristics for each firm type

Firm Type		Number of Employees in U.S.	Number of Employees Internationally	Annual Revenue	Market Area	Supplier Base
Suppliers/ Manufacturers	median	1001-5000	501-1000	$100M-$500M	Global	Global
	max	50,000+	50,000+	$1B+	Global	Global
	min	0-100	0-100	>$20M	Local	Local
Retailers/Wholesalers/ Foodservice	median	1001-5000	0-100	$500M-$1B	Regional	National
	max	50,000+	50,000+	$1B+	Global	Global
	min	0-100	0-100	>$20M	Local	Local

Section III of the survey which, as mentioned, is our focus here, entails a large number of questions for which responses are categorical and range from 1 to 5. One of the most significant series of questions are questions of the following type that would allow for the estimate of a "response function from risk mitigating investments to actual results. An example of these types of questions is the following question:

"Our firm's security investment has resulted in———security incidents."

Responses to questions of this type, to be inserted in the blank space, ranged from the value of 1 (significantly reduced) to 5 (significantly increase) and 6 (N/A). These responses indicated a significant reduction in reported incidents by firms that have made security investments. While there is a possibility that an action taken by a firm could lead to the *perception* of success by a respondent, whether or not the actual number of incidents has declined, this issue cannot be settled in this paper and is partly addressed in the ongoing work by Agiwal, Mohtadi and Kinsey (2007). For now, however, we assume that the survey results do capture objectively the relation between security investments and their outcome, in terms of reduced risk. With this caveat, the results from Agiwal, Mohtadi and Kinsey (2007), when broken down into food manufacturers and food retailers, yield the following means and variances as well as the associated value of λ. These value are entered based on the discussion that follows Table 3 below:

Table 3

	mean	variance	associated λ *
food manufacturing	1.72	.47	0.67
food retail	1.73	.46	0.67

* See below for the calculation of λ

It would be reasonable to assume that reductions in security incidents are linearly proportional to reductions in expected losses from such incidents. This is given by the expression $[1-\lambda]L$. However, the mapping from the values in the survey to the actual values of λ is somewhat arbitrary. For the lack of a more rigorous criterion, we will assume that an answer of "1" to the survey question, i.e., significant reduction in the number of security incidents, implies a reduction in incidents by 2/3 ($\lambda = 2/3$) and that an answer of 2 implies a reduction by 1/2 ($\lambda = 1/2$). Interpolating, the mean value of 1.7 from the above table would imply that $\lambda \simeq .67$. This is indicated in Table 3.

We now have sufficient information to estimate the optimum value of security investments a firm would need as self-insurance. For $\lambda = 0.67$, and for the various levels of injury and time horizons given and the associated probabilities in Table 1 the optimum level of risk mitigating investments can be calculated from equation 10 (using a values of 0.5). The results are reported in the table below:

Table 4

Estimated Values of secuity investment for different loss levels and Time Horizons

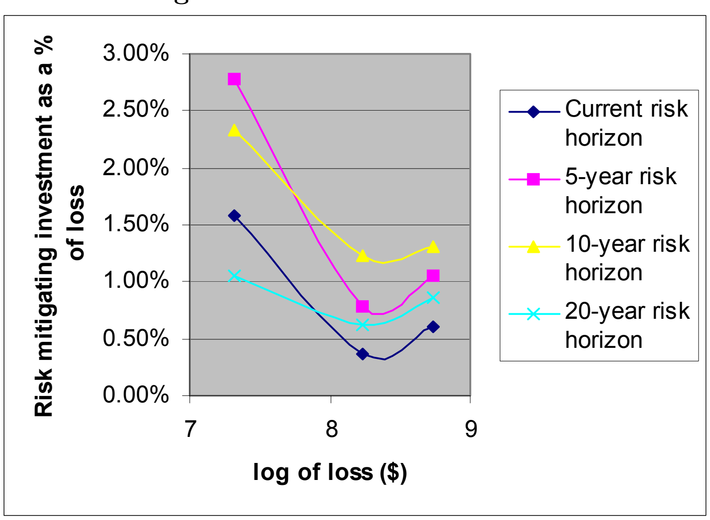
	Range of casualties Lo to L1	Mean # of casualties	monetary value of loss coming from injury (in $ million)	probability of loss between Lo and L1	Optimum investments K* in $ million	Optimum investments K* as % of loss
Current risk	1000-5000	3000	1,500.00	0.019	3.18	0.21%
	5000-10000	7500	3,750.00	0.005	1.87	0.05%
	10000-15000	12500	6,250.00	0.007	5.08	0.08%
5-year forecast	1000-5000	3000	1,500.00	0.034	5.57	0.37%
	5000-10000	7500	3,750.00	0.009	3.92	0.10%
	10000-15000	12500	6,250.00	0.013	8.79	0.14%
10-year forecast	1000-5000	3000	1,500.00	0.028	4.70	0.31%
	5000-10000	7500	3,750.00	0.015	6.13	0.16%
	10000-15000	12500	6,250.00	0.016	11.02	0.18%
20-year forecast	1000-5000	3000	1,500.00	0.013	2.10	0.14%
	5000-10000	7500	3,750.00	0.008	3.16	0.08%
	10000-15000	12500	6,250.00	0.010	7.15	0.11%

Source: Based on Table 1 and analysis in the text

Focusing on the last column of Table 4, a one-time capital investment aimed at mitigating extreme risk ranges from 37 cents per $100.00 in a 10-year time-horizon, for a potential loss of 1.5 billion dollars, to 5 cents per $100.00, in the current time-horizon, associated with a loss of 3.75 billion dollars. The larger percentage associated with the smaller loss seems puzzling at first. The key to the puzzle is the associated probabilities. The event with a higher risk mitigation cost has a probability of 3.4% while the event with the lower percentage cost has a probability of only 0.5 percent.

An interesting feature of Table 4, which is also reflected in Figure 2, is the fact that for any given time horizon, investments in risk mitigation as a percent of the potential loss, decline for a range of losses only to rise beyond that range, producing an inverted U pattern. As mentioned in conjunction with Table 1, this pattern has to do with the probability segments involved in both tables: for any given time horizon, the probability segments first fall with severity and then rise, producing the U shaped curve. Closer scrutiny revealed that this pattern results because the decline in the event probability, from a 5000+ to a 10,000+ casualty event, is much *smaller* than the decline from a 10,000+ to a 15,000+ casualty event (See Table 1, column 3). This in turn has to do with the larger than normal likelihoods of low-frequency, high impact "tail" events, captured in Extreme Value Statistical Analysis. A second feature of Figure 3 is the nature of the shifts in the four curves. Specifically, notice that at the high end of the risk scale, while extreme events associated with high monetary loss or high casualty (right end of X axis) become more costly with longer time horizons because they become more likely (recall the time trend associated with fat tails discussed earlier), the pattern seems to break down for 20-year time horizons. This anomaly is connected with the feature of Table 1 discussed earlier, namely that for this time horizon the much higher likelihood of an event with 15,000+ casualty takes away from previous segments including the probability of 10,000-15000 casualty event. A third feature of the figure, namely the crossing of the curves, is not as critical as it simply reflects the various trade-offs between time horizons and event likelihood that occurs at the *intermediate* segments of risk.

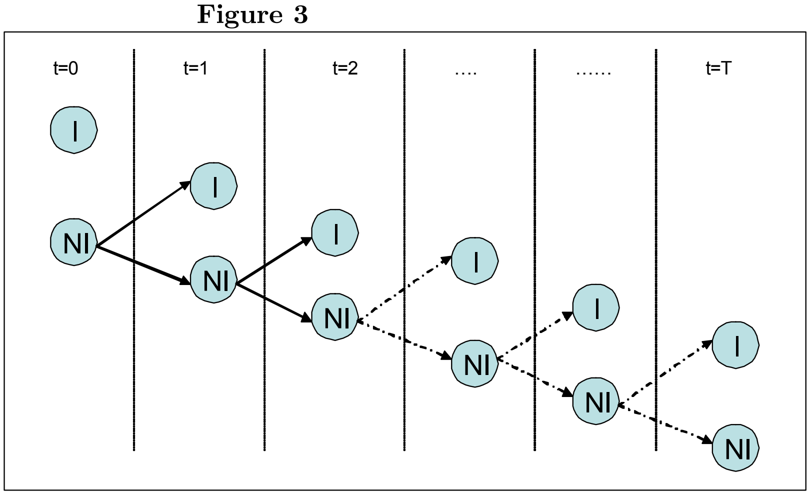
Figure 2



4 A Multi-period Extension

Firms may choose to postpone investing in risk mitigation. The question that arises is whether such a strategy is an optimal strategy or not. To answer this question we consider a dynamic extension of the previous model. Consider a firm with a finite time-horizon, say T, with the option to invest at the beginning of each time period, t, or to postpone the decision to future time periods. Since investing is an absorbing state, the decision tree of the firm can be represented by the following chart:

Figure 3



In turn, this can be expressed, for any time t, as the number of periods, t-1, the firm has not invested, followed by the number of period T-t starting from the time that the firm committed itself to security investments. This is presented in the following table:

Table 5

Period	Investment Decision Alternative Scenarios					
t=1	Not Invest	Not Invest	Not Invest	…………..	Not Invest	Invest
t=2	Not Invest	Not Invest	Not Invest		Not Invest	Invest
t=3	Not Invest	Not Invest	Not Invest		Invest	Invest
.	.	.	.		.	.
.	.	.	.		.	.
.	.	.	.		.	.
t=T-1	Not Invest	Not Invest	Invest		Invest	Invest
t=T	Not Invest	Invest	Invest	…………..	Invest	Invest

Letting the probability of a finite loss, L, occurring in period t be given by π_t, the expected losses/gains arising from making the security investments in t must include the cost of investments, K, assumed to be made at the beginning of the time period, t, and the reduced losses, $\lambda(K)L$. Based on this and figure 5, the present value of expected losses upon investing are given by,

$$PVE(L|\pi_t)^{invest} = \sum_{x=1}^{x=t-1} \delta^x \pi_x L + \sum_{x=t}^{x=T} \delta^x \pi_x \lambda(K) L + \delta^{t-1} K \tag{11}$$

The present value of expected losses when no investments are made at all, are given by,

$$PVE(L|\pi)^{no\ invest} = \sum_{x=1}^{x=T} \delta^x \pi_x L \tag{12}$$

Thus, the present value of expected net gain from investing, as arrived at in equation (3) earlier, is given by,

$$\begin{aligned} PVE(G) &= PVE(L|\pi_t)^{no\ invest} - PVE(L|\pi)^{invest} = \\ &\sum_{x=1}^{x=T} \delta^x \pi_x L - \sum_{x=1}^{x=t-1} \delta^x \pi_x L - \sum_{x=t}^{x=T} \delta^x \pi_x \lambda(K) L - \delta^{t-1} K \\ &= \bar{L} \sum_{x=1}^{x=t-1} \delta^x \pi_x - \lambda(K) \bar{L} \sum_{x=t}^{x=T} \delta^x \pi_x - \delta^{t-1} K \\ &= (1 - \lambda(K)) \sum_{x=t}^{x=T} \delta^x \pi_x L - \delta^{t-1} K \end{aligned} \tag{13}$$

The firm must maximize PVE(G) by optimally choosing the investment level K*, and the critical time period t* at which to invest:

$$\underset{\{K,t\}}{Max} PVE(G) \Rightarrow t^*, K^* \tag{14}$$

Consider a 10-year time horizon from the previous section. The probabilities of attacks on food supply for this horizon were given in Table 4. From that table, the probability values for current risk, 5-year risk forecast, and 10-year risk forecast are used to interpolate and obtain the values of π_t for $t = 1..10$. This assumes that on average there are no further anticipated changes in the probability of catastrophic events occurring. There could certainly be changes in the probabilities contingent upon new information, but interpolating values of the forecasts implies that all available information is incorporated into the forecasts at the time the decision is made.

Parallel to section 3, for catastrophic losses of sizes in the range L_o to L_1 the EV probability distribution function is obtained from Table 4 representing π_x. As before, we replace, L with its linear mean $\bar{L}$ yielding the present value of expected gains from investing in period t as,

$$PVE(G) = (1 - \lambda(K))\,\bar{L} \sum_{x=t}^{x=T} \delta^x \pi_x - \delta^{t-1} K \tag{15}$$

First, let us focus on the optimum investment level $K^*(t)$. For this, we have the analytical solution as follows:

$$-\lambda'(K^*)\bar{L} \sum_{x=t}^{x=T} \delta^x \pi_x = \delta^{t-1} \tag{16}$$

Using the form of $\lambda(K)$ as given by equation 7, λ' can be expressed in terms of λ yielding:

$$K^*(t) = \lambda(1 - \lambda)\bar{L} \sum_{x=t}^{x=T} \delta^{x-t+1} \pi_x \tag{17}$$

Next, let us consider the optimum timing of such investments. In principle, maximizing for t per 14 should yield an equation $t^*(K)$ which together with 17 could be solved to yield, t^* and K^*. However, in practice, $t^*(K)$ may only be obtained numerically. We conducted this exercise for a 10-year horizon and for the loss values and their corresponding probability values given by table 4. Surprisingly, we found that unless the values of K are exceedingly larger than in the allowable range for which $K^*(t)$ with $t = 1...10$, (see table below for this range) *the optimum timing* t^* *is always the very first period.* The estimation results are shown in Table 6.

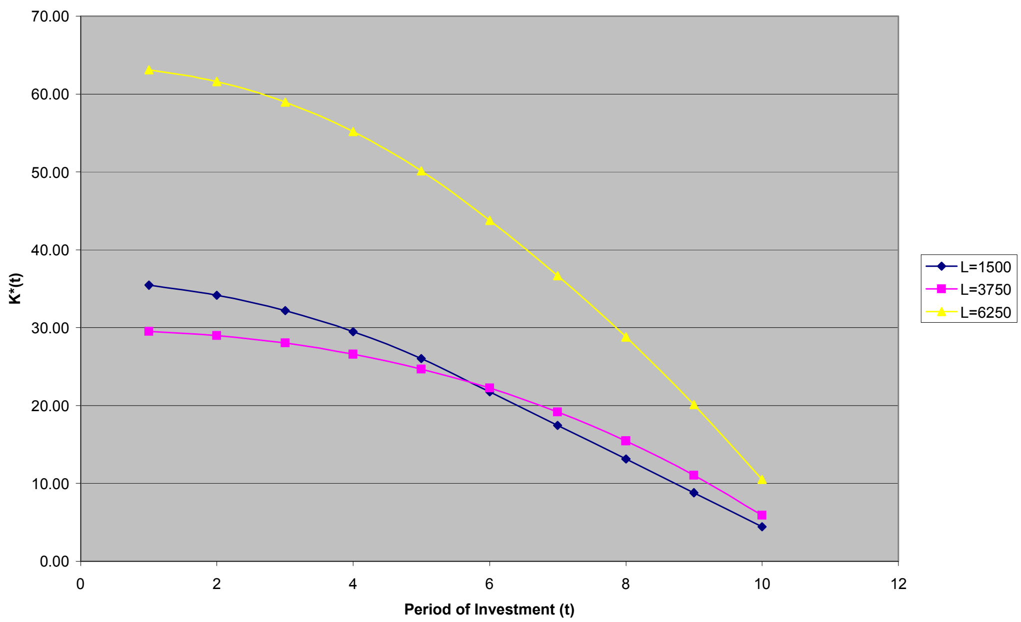
Table 6

Optimal Security Investments at different Loss Levels and Time-Points over a 10-year Horizons

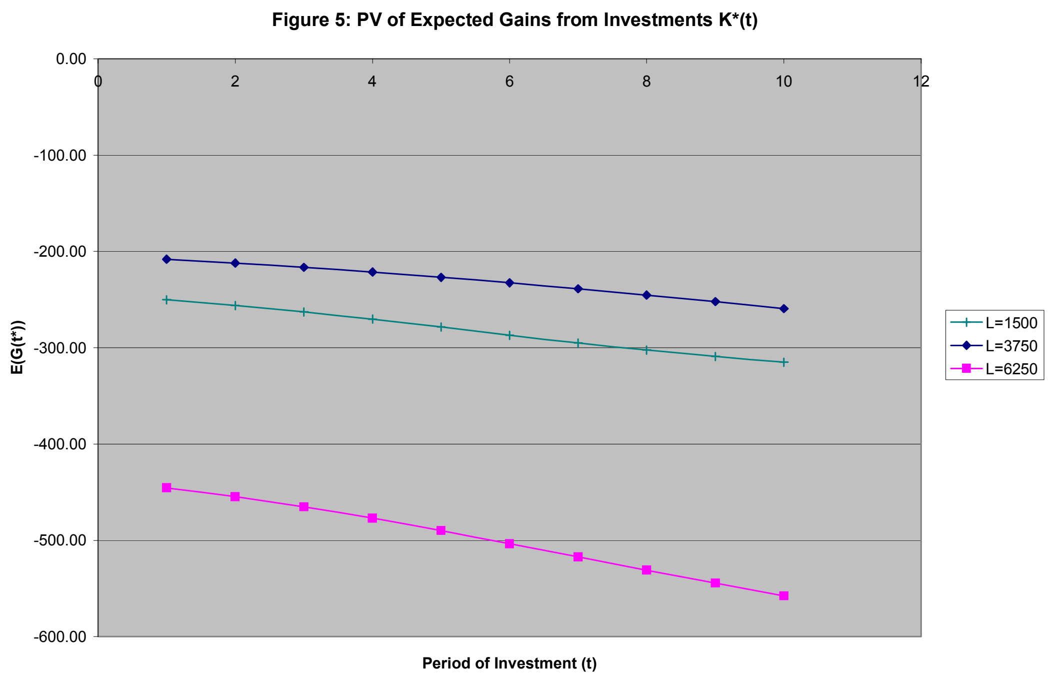
Range of casualties Lo to L1	Mean # of casualties	Monetary value of loss coming from injury (in $ million)	Probability of loss between Lo and L1	Investments made at the beginning of period t	PV of Optimum investments K*(t) in $ million	PV of Optimum investments K*(t) as % of PV of loss	PV of Expected Gains from Optimum Investments E(G(K*(t)))
1000-5000	3000	1,500.00	0.0190	1	35.45	0.31%	-250.28
			0.0228	2	34.16	0.30%	-256.22
			0.0265	3	32.19	0.28%	-262.98
			0.0303	4	29.49	0.26%	-270.46
			0.0340	5	26.02	0.23%	-278.57
			0.0328	6	21.76	0.19%	-287.23
			0.0316	7	17.46	0.15%	-295.17
			0.0304	8	13.14	0.11%	-302.43
			0.0292	9	8.79	0.08%	-309.07
			0.0280	10	4.41	0.04%	-315.13
5000-10000	7500	3,750.00	0.0050	1	29.49	0.10%	-208.24
			0.0065	2	28.97	0.10%	-212.15
			0.0080	3	28.01	0.10%	-216.61
			0.0095	4	26.58	0.09%	-221.54
			0.0110	5	24.67	0.09%	-226.91
			0.0116	6	22.23	0.08%	-232.64
			0.0122	7	19.17	0.07%	-238.81
			0.0128	8	15.46	0.05%	-245.36
			0.0134	9	11.05	0.04%	-252.24
			0.0140	10	5.91	0.02%	-259.40
10000-15000	12500	6,250.00	0.0070	1	63.11	0.13%	-445.59
			0.0085	2	61.59	0.13%	-454.71
			0.0100	3	58.96	0.12%	-465.23
			0.0115	4	55.16	0.12%	-476.99
			0.0130	5	50.11	0.11%	-489.84
			0.0136	6	43.77	0.09%	-503.64
			0.0142	7	36.68	0.08%	-517.35
			0.0148	8	28.80	0.06%	-530.95
			0.0154	9	20.09	0.04%	-544.42
			0.0160	10	10.50	0.02%	-557.73

To elaborate on this results, notice also that the corresponding optimum $K^*(t)$ values are the maximum for these first periods. This is also shown in Figure 4. Despite these large expenditures of capital, it is always better to invest in protection against catastrophic risk at the *beginning* of the period, rather than postpone such investments as shown by the fact that net profits are largest (least negative) when t=1 (last column).

Figure 4: PV of Optimum Investments



While larger $K^*(t)$ for smaller t values can be explained by lower present values of the cost of a fixed one-time investments, compared to the gains (in terms of reduced risk) over repeated periods, the case of $t^* = 1$ requires some further elaboration. Focusing on equation 15, evidently, as the period of investment is postponed (moves closer to the time horizon T—in this case T=10), the repeated nature of loss is sufficiently overwhelming as to favor an early investment decision rather than a later one. Interestingly, we tried many different parameters and the $t^* = 1$ results remained quite robust. This result is depicted in Figure 5.



Finally, it is instructive to compare the one-period and multi-period models with respect to the size of risk-mitigating investments as a *percentage* of the initial loss. Comparing the corresponding columns in both Tables 4 and 6 (next-to-last column in 6 and last column in 4), we see that, as a fraction of the original loss, risk mitigating investment levels in the multi-period model are very close to those in the one-period model. Notice, however, that the actual level of investments in this case is substantially larger (of the order of 10 times). This is of course not surprising since $K^*(t)$ in the multi-period model mitigates the risk of a repeated exposure to risk over the entire 10-year horizon.

5 Comparison with Catastrophic Insurance: Concluding Remarks

How do expenditures aimed at risk mitigation compare with the purchase of catastrophic risk insurance? To gain some insights into the answer to this question consider the fact that in 2002 the Insurance Service Office assigned insurance cost of approximately 10 cents per $100 loss for the highest risk cities, but after discussions with the regulators, these rates were later adjusted downwards to less than 3 cents per $100 of loss (See Auerswald, et. al. 2006, pp. 283). This suggests that loss mitigation costs are as high or higher than the cost of catastrophic insurance. While risk mitigation and insurance are not entirely comparable strategies due to continued risk exposure under risk mitigation strategies, still the high deductibles of catastrophic insurance make the two approaches more comparable than might appear at the first glance. In this respect, several points need to be stressed.

First, risk mitigation investments are one-time actions. This is true both in the single period and the multi-period model. For example, a one time capital expenditure of 5 cents per $100 from table 2 to mitigate the risk of an event that could cause $3.75 billion loss in the current time horizon, pays off compared to the purchase of catastrophic insurance after the fifth year. The key of course is that risk mitigation is not fool proof so that risk is not eliminated, but only reduced. As mentioned, however, this fact needs to be balanced against the fact that catastrophic risk insurance often entails large deductibles and, therefore, at least up to the deductible level, risk mitigation expenditures remain relevant whether or not catastrophic insurance is purchased.

Second, the availability of catastrophic insurance has become more limited after 9/11 attacks, due to a variety of complicating factors, including (a) the increased risk trend (for evidence see Mohtadi and Murshid 2007a; for discussion see Bogen and Jones, 2006), (b) the large size of the losses, requiring far greater pooling of resources than has been available to insurers and reinsures, (c) the uncertainties associated with calculating the probabilities of catastrophic insurance[8] and (d) the asymmetric information between insurance firms and the insured on the one hand, and the reinsurance firms and insurance firms on the other, resulting in a double moral hazard problem (Auerswald, ibid)

Third, as stated previously, issues such as reputation effects cannot be overlooked. These affects cannot be easily addressed by the purchase of insurance, especially if they are the outcome of inadequate risk mitigation by firms in the first place.

Fourth, the adequate coverage of catastrophic insurance together with the large potential losses and the associated "risk externalities" call for the involvement of the Federal, state and local governments (see Auerswald, et. al. 2006, pp. 284), as this becomes a clear case of market failure. In fact the Terrorism Risk Insurance Act (TRIA) and its eleventh hour extension in December of 2005 is aimed at addressing the gaps in this

[8] On this point, we hope that this contribution will be of value. We argue that terrorism risk is quantifiable in the same manner as weather risk may be.

market. However, many obstacles remain and catastrophic insurance remains a highly imperfect tool with limited or no availability in many cases. Thus, until and unless these issues are resolved risk mitigation becomes a important and essential strategy.

Fifth, while risk mitigation is not well correlated with insurance costs, due to moral hazard problems associated with asymmetric information between the insured and the insurer (again see Auerswald, et. al. 2006, pp. 283), it is likely that catastrophic risk mitigation will have a positive effect on the cost of risk financing, if not risk insurance. For example, Moody's risk ratings are highly affected by the ability of firms to prepare for and respond to risk. For all these reasons the importance of catastrophic risk mitigation strategies cannot be overstated and such strategies must be an essential part of firms' overall strategy.

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