Fixed Costs of Capital Adjustment in a Two Country Real Business Cycle Model

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- Preliminary and incomplete -

Abstract

The present paper analyzes the role of non-convex adjustment costs to capital in a stylized two country model of real business cycles. In contrast to the closed economy case, we find fixed adjustment costs to have a significant influence on investment decisions for each of the countries just as for the aggregated world economy. Like convex costs, fixed adjustment costs substantially limit capital re-allocation across countries. Because of this, they drive up the correlation of domestic investment and domestic saving, increase the correlation of output between countries, and decrease the variability of the trade balance. All this brings the model more in line with observed data. For the aggregated world economy, non-convex adjustment costs limit the variability of investment compared to the no-cost case, but the effect on the world economy is far less pronounced than on the individual economies.

Keywords: Non-convex Adjustment Costs, International Business Cycles, Investment

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1 Introduction

There is substantial empirical evidence that non-convexities in capital adjustment are not trivial at the micro level.[1] Specifically, the adjustment of capital at the plant level is

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[1]Evidence for US data comes for example from Caballero et al. (1995), Doms and Dunne (1998), Caballero and Engel (1999), Cooper et al. (1999), Abel and Eberly (2002). For evidence from Norwegian data see Nielsen and Schiantarelli (2003), See Bayer (2004a) or Bayraktar et al. (2005) for evidence from German data, and Bayer (2004b) for evidence from UK data.

characterized by long periods of relative inactivity interrupted by occasional bursts of investment. Intuitively, one may guess that this lumpy investment behavior has important macroeconomic consequences it overall dampens economic fluctuations but may lead to brisk expansions and sharp economic downturns at the same time, i.e. investment activity is extremely high or low too often. However, recent contributions suggest that the effect of non-convex or more specifically fixed adjustment costs completely washes out at the macro level when moving from partial equilibrium to general equilibrium analysis. There is neither a particular dampening effect of fixed adjustment costs, nor excess kurtosis in investment behavior. Hence, this research suggest that lumpy investment may be negligible for macroeconomic models of business cycles (Veracierto, 2002, Thomas, 2002, Khan and Thomas, 2003 and 2004).

Common to these papers is that they look at a single sector closed economy model. We re-evaluate the striking closed economy result on the basis of a model of international business cycles, which may be an adequate new angle to look at the issue for several reasons: Firstly, we may think of a general equilibrium, open economy model intuitively as a hybrid of partial and general equilibrium, because price movements are dampened by the possibility of trade. Secondly, adjustment costs to capital are essential in open economy RBC models to keep international re-allocation of capital within realistic bounds after country specific productivity shocks. So far the literature models them typically as convex costs, but empirical micro evidence suggests non-convex costs.[2] Thirdly, the open economy results can be read as a model of two non-atomistic sectors of a closed economy alternatively. For all of these three reasons, we extend the analysis of the influence of non-convex adjustment cost to capital on the cyclical behavior of macroeconomic variables to a two-country model of the world economy.

However, to allow for the alternative two sector interpretation, it becomes necessary to abstract from a number of distinct features of international economics like trading costs, or international specialization. This means that the modelled world economy is made up of two countries which produce a homogeneous capital-consumption good. This is the modelling strategy proposed by the seminal contributions of Backus, Kehoe, and Kydland (1992) or Baxter and Crucini (1993), but in contrast to these papers, we model both countries subject to a non-convex investment friction. Besides this friction the countries are interlinked by frictionless trade and asset markets.

In this model economy, firms exploit temporary productivity differences and heavily

[2] The convex adjustment costs assumption has been introduced to international RBC models by Baxter and Crucini (1993). For further examples, see van Wincoop (1996), Chari et al. (2002), Mazzenga and Ravn (2004), or Cuñat and Maffezzoli (2004).

shift production to the more productive country if adjustment costs are absent. This has lead Baxter and Crucini (1993) to suggest *convex* adjustment cost to capital as a model element to reduce investment fluctuations and indeed convex adjustment costs have become a crucial building block of international RBC models ever since.[3] However, convex adjustment costs reduce the ability to reproduce business cycles in closed economy models (Kydland and Prescott, 1982). By contrast, fixed adjustment costs show no influence on the business cycle behavior of a closed economy model.[4]

For the open economy model, fixed costs of capital adjustment turn out to have an impact on each single economy as well as on the world aggregate. Non-convex adjustment costs dampen aggregate fluctuations both, for each single country and for the world as an aggregate.[5] While this contrasts the results for fixed adjustment costs in closed economies, the macroeconomic consequences of fixed costs in open economies are very similar to those of quadratic adjustment costs.

The adjustment costs primarily affect capital re-allocations across countries. Only secondarily they influence world aggregate investments. Without adjustment costs, the fluctuation in productivity differences augments world wide productivity fluctuations. With adjustment costs, short run productivity differentials cannot be exploited by moving capital from one country to the other so easily. Since capital cannot be re-allocated to the more productive country cheaply, capital is employed less productively on average. The size of this distortion fluctuates along the business cycle and depends on changes in productivity disparities. This in turn results in a different cyclical behavior of world aggregate variables and marks therefore a stark contrast to previously established results for closed economies (Veracierto, 2002, Thomas, 2002, Khan and Thomas, 2003 and 2004).

In a closed economy, it is (domestic) consumer demand that drives production decisions, so that the consumer's desire for smooth consumption precludes the brisk expansions that a partial equilibrium intuition suggests with fixed adjustment costs. Moreover, this desired smooth consumption can be achieved relatively cheaply in equilibrium despite the fixed costs of capital adjustment. To meet the (aggregate) capital required to produce tomorrow's consumption plan, price movements let those firms invest a little more which are about to adjust anyway. If returns to scale are not too steeply decreas-

[3]See footnote 2.

[4]Bachman, Caballero, and Engel's (2006) results suggests that fixed adjustment costs are much larger than proposed by Khan and Thomas (2003, 2004) and do significantly improve the ability of a closed economy RBC model to replicate business cycle dynamics.

[5]The effect on the single countries is similar to the effect of fixed adjustment cost on capital reallocation across firms that Khan and Thomas (2004) find for a closed economy model with atomistic firms which have heterogeneous productivity.

ing, the welfare cost is small of such a policy and it can be used to closely replicate the frictionless consumption path.[6] In fact, Khan and Thomas (2003) find that the macroeconomic behavior is virtually indistinguishable between an economy with fixed adjustment costs and one without any adjustment cost.

Our results for the two country case suggest that this equilibrium mechanics does not hold for an open economy. There, households can achieve consumption smoothing also by trade. In contrast to the smoother consumption, investment and hence production fluctuates more in a two country model because firms exploit temporary productivity differences and heavily shift production to the more productive country, at least if adjustment costs are absent. Adjustment costs now reduce the propensity to shift capital to the more productive country and thereby dampen the aggregate fluctuations.

Since this causation chain for the dampening effect of adjustment costs is the same, whether adjustment costs are fixed or convex, we explicitely compare both model alternatives. We search quadratic adjustment costs that replicate the volatility of investment for each single country and compare the behavior of all other macroeconomic aggregates. In fact, we find that the model can be posed approximately equivalent in terms of quadratic costs. The national macroeconomic variables in both model specifications, quadratic and fixed adjustment costs, behave similarly. However, fixed adjustment costs more strongly dampen world aggregate investment-rates, while they do not effect all other world aggregate variables much different to the quadratic costs formulation.

This gives *non-convex* costs as a feature of an RBC model some appeal. Fixed costs affect an open economy model similar to convex costs, but they do not have the adverse effects of convex costs that Kydland and Prescott (1982) find. Both, convex and non-convex costs, limit capital re-allocation across countries. Due to this limiting effect on capital re-allocation across countries, non-convex adjustment costs drive up the correlation of domestic investment and domestic saving especially. Similarly, they also increase the output correlation between countries and limit the variability of the trade balance. Therefore, introducing non-convex adjustment costs brings the model more in line with observed data on international business cycles. Hence, non-convex adjustment costs appear to be a promising building block of an RBC model and not at all a superfluous element as one may have concluded from previous contributions.

Our modelling strategy and parameter choices is chosen as to keep our results as comparable as possible to those papers which find no role of non-convex costs in closed economies.[7] This allows us to identify international capital mobility as the factor which

[6] I would like to thank Eduardo Engel and Rüdiger Bachmann for pointing out this intuition to me.

[7] Bachman, Caballero and Engel (2006) criticize this parameter choice of the closed economy literature.

gives fixed adjustment costs a significant role in our model. Though, this identification strategy comes at a cost. It does not allow to replicate international business cycle moments as well as we might have been otherwise.[8] To be closely comparable we borrow the technological parameters from Khan and Thomas (2003), but these borrowed parameters, utility functions, and production functions perform badly in replicating some of the international business cycle moments, since they were designed to match a closed economy model with US data. Nonetheless, the introduction of non-convex costs improves the model performance and shapes theoretical business cycle moments towards observed moments in general.

For comparability, we analyze our two country economy within two settings for productivity: one in which productivity is uncorrelated and one setting where we use the same productivity process as in Backus, Kehoe, and Kydland (1992), which features correlated shocks and productivity spillover. This strategy allows us to elaborate the theoretical differences to the closed economy model most clearly. It marks a central difference to the otherwise closely related paper of Khan and Thomas (2004) that we find that adjustment costs are limiting the variability of investment for the aggregated world economy compared to the no-cost case. Khan and Thomas' (2004) paper introduces firm-specific productivity shocks to the earlier (Khan and Thomas, 2003) model, but comes to no different conclusions for the aggregate economy than for the model without firm specific shocks. This difference to Khan and Thomas' (2004) is insofar important as our international economy model may alternatively be read as a closed economy with significant sectors, while their model with atomistic firms (sectors) may be read as a model of a continuum of small open economies.

The remainder of this paper is organized as follows: Section 2 presents the model, Section 3 discusses the numerical solution method for our model, Section 4 explains parameter-choices, Section 5 presents and interprets our central findings and finally Section 6 concludes. An appendix displays the equilibrium forecasting rules involved in the numerical solution of the model.

2 Model

We model an international economy composed of two big countries. These two countries produce and consume a single homogeneous good, which also serves as capital like in the

They argue that adjustment costs are assumed to be substantially too low and households are assumed to be too risk averse.

[8] In particular, using the utility function specification of Khan and Thomas (2003), which is additive seperable in consumption and labor, induces perfect correlation of consumption in the two modelled countries.

seminal paper of Backus, Kehoe, and Kydland (1992). For this economy, we assume that there are fixed costs associated with undertaking capital investment as in Thomas (2002). Conversely, our model can be understood as Thomas' (2002) model, but extended by the introduction of a second economy. This second economy is a replica of the first one except that the productivities of both economies are not perfectly correlated. To model international economic relations, we assume that between both economies the consumption-capital good can be traded freely. Moreover, there is a complete asset market, i.e. there exists a contingent claim for any state of the world, this claim pays one unit of the consumption good in the specified state, and it can be freely traded between as well as within both economies.

Each economy $i = 1, 2$ has a measure-one continuum of firms that produce the consumption/capital good according to a production function

$$y = z_i F(k, n) \tag{1}$$

using capital k and labor n. Here, z_i is the total factor productivity in country i, which is stochastic and follows a bivariate Markov chain that approximates the geometrical VAR

$$\ln z_t = \begin{pmatrix} \ln z_1 \\ \ln z_2 \end{pmatrix}_t = B \ln z_{t-1} + \varepsilon_t,\ var(\varepsilon) = \Sigma. \tag{2}$$

The Markov chain has $j = 1...J$ states ζ_j for each country and the transition probabilities for productivity are given by:

$$\pi_{ijkl} = P\left(\ln z_{1t} = \zeta_k \wedge \ln z_{2t} = \ln \zeta_l | \ln z_{1t-1} = \zeta_i \wedge \ln z_{2t-1} = \zeta_j\right). \tag{3}$$

2.1 Firms

A firm decides whether or not to invest after production. In case a firm invests, it has to pay some fixed cost of ξ working hours (at a real wage w). We assume ξ to be a random number, uniformly distributed on the interval $\left[0, \bar{\xi}\right]$, but known to the firm before investment. The cumulative distribution function of ξ is denoted by $G(\xi)$. In case the firm does invest, its next period stock of capital k' results from

$$\gamma k' = k(1 - \delta) + i \tag{4}$$

with depreciation rate δ. Throughout this paper, primes indicate next period variables. The parameter γ reflects labor augmenting technological change, by which the model is 'deflated' as familiar in the RBC literature (see Khan and Thomas, 2003, p. 3, or King and Rebelo, 1999) for details. The cost of the investment are (in units of the consumption good) $i + w\xi$, which is the purchased capital i plus a fixed number of working hours ξ multiplied by the real wage w. In case the firm does not invest, capital only depreciates,

$$\gamma k' = k(1-\delta), \tag{5}$$

but the firm does not need to pay *any* adjustment cost. In other words, the fixed costs of capital adjustment are ξw.

Besides productivity and adjustment cost, also the distribution of capital affects the decision of the firm. This distribution is represented by the distribution function $\mu = (\mu_1, \mu_2)$, where μ_i is the distribution of capital in country i. Together with the productivity z these distributions determine labor demand and hence real wages $w(z,\mu)$. At the same time, they determine the price of the consumption good $p(z,\mu)$, since total production also depends on (z,μ).

Each firm takes prices, wages and productivity and their expected future values as given, and chooses how much labor it employs and how much capital it wants to be holding in the following period. Putting these elements together, we obtain the following Bellman equation, which determines firm value and the firm's investment policy:

$$V(k,\xi,\mu,(z_i,z_j)) = \max_n \left(z_i F(k,n) - wn + (1-\delta)k\right)p + \max\left(V^{\text{adj}}, V^{\text{no adj}}\right) \tag{6}$$

with

$$V^{\text{adj}} := -\xi wp + \max_{k'} -\gamma k' p + \beta \sum_{k=1}^{J}\sum_{l=1}^{J} \pi_{ijkl} \bar{V}\left(k', \mu', (z_k, z_l)\right),$$
$$V^{\text{no adj}} := -(1-\delta)kp + \beta \sum_{k=1}^{J}\sum_{l=1}^{J} \pi_{ijkl} \bar{V}\left(\frac{(1-\delta)}{\gamma}k, \mu', (z_k, z_l)\right),$$

and

$$\bar{V}(k,\mu,z) := \int_0^{\bar{\xi}} \bar{\xi}^{-1} V(k,\xi,\mu,z)\, d\xi.$$

Value is based on production $z_i F(k,n)$ plus the inherited capital from the previous period $(1-\delta)k$, minus labor costs wn all multiplied by the current price p. From this sum, the costs to install the new stock of capital are subtracted, which is $(1-\delta)kp$ or $\gamma k' p + \xi wp$ in case of no adjustment or adjustment of the capital stock respectively. Finally, the expected discounted future value $\beta \sum_{k=1}^{J}\sum_{l=1}^{J} \pi_{ijkl}\bar{V}(k',\mu',(z_k,z_l))$ is added.

The function μ' captures the *expected distribution* of capital in the following period. In equilibrium μ' must coincide with the distribution that later realizes, since this distribution depends only on decisions undertaken on the basis of information available in the current period. Under the assumption that asset markets are complete, the firm's discount factor β and the time preferences of households are identical.

This leaves us with a model for the firm's decisions which is basically Khan and Thomas' (2003) model. Therefore, we skip a detailed description how optimal policies can be derived, but refer the reader to the original paper. Following Khan and Thomas, we denote the associated policy functions for each country i by, $N_i^f = N_i^f (k_t, z_t, \mu_t)$ and $K_i^f (k, \xi, \mu, z)$.

2.2 Households

We assume that each economy is inhabited by a unit-measure of identical households. Also between economies the households are identical. In each economy households consume the homogeneous consumption good, with consumption in country i given by c_i and they offer labor n_i on the local labor market. The households hold and trade one period shares $\lambda(k)$ in domestic plants which they buy at price $\chi_{\lambda,i}$, and they trade contingent claims $\theta_{i,j}$ to maximize their lifetime utility W. To do so, they solve the following optimization problem

$$W\left(\lambda, \theta, \left(\zeta_i, \zeta_j\right), \mu\right) = \max_{c,n,\lambda',\theta'} U(c, 1-n) + \beta \sum_{m=1}^{J} \sum_{n=1}^{J} \pi_{ijmn} W\left(\lambda', \theta', \left(\zeta_m, \zeta_n\right), \mu'\right) \quad (7)$$

$$\text{s.t. } p(z,\mu)\left(c - w(z,\mu) n - \theta_{i,j}\right) \leq$$
$$\int \bar{V}(k,z,\mu) \lambda(dk) - \int \chi_\lambda (k,z,\mu) \lambda'(dk) - \sum_{m,n} \chi_{\theta_{m,n}}(z,\mu) \theta'_{m,n}, \quad z := \left(\zeta_i, \zeta_j\right)$$

We denote by $C_i(\lambda, \theta, z, \mu)$ the consumption of country i's households and by $N_i^h(\lambda, \theta, z, \mu)$ the labor supply of country i's households, by $\Lambda_i(k; \lambda, \theta, z, \mu)$ we denote the quantity of purchased shares and by $\Theta_i(z'; \lambda, \theta, z, \mu)$ the number of purchased contingent claims.

2.3 Equilibrium

This leads us to the definition of a recursive competitive equilibrium.

Definition 1 *A recursive competitive equilibrium is a set of functions*

$$\left(p, \chi_\theta, \chi_{\lambda,i=1,2}, w_{i=1,2}, V_{i=1,2}, N_{i=1,2}^f, K_{i=1,2}^f, W_{i=1,2}, C_{i=1,2}, N_{i=1,2}^h, \Lambda_{i=1,2}, \Theta_{i=1,2}\right)$$

such that the following 6 conditions hold:

1. *The firm-value functions $V_{i=1,2}$ fulfill the Bellman equation (6) with associated policy functions $N^f_{i=1,2}, K^f_{i=1,2}$.*

2. *The lifetime-utility functions of households $W_{i=1,2}$ fulfill the Bellman equation (7) with associated policy functions*
   $C_{i=1,2}, N^h_{i=1,2}, \Lambda_{i=1,2}, \Theta_{i=1,2}$.

3. *The market for shares clears:*

$$\Lambda_i\left(k', \mu_i; \theta, z, \mu\right) = \int_{\left\{(k,\xi)|k'=K^f_i(k,\xi,\mu,z)\right\}} G\left(d\xi\right)\mu_i\left(dk\right) = \mu'_i\left(k'\right), \text{ for } i = 1, 2.$$

4. *There is no net-holding of contingent claims $\Theta_1 + \Theta_2 = 0$.*

5. *The labor markets clear:*

$$N^h_i\left(\mu, \theta, z, \mu\right) = \bar{N}^f_i := \int \left(N^f_i\left(k; z, \mu\right) + \int_0^{\bar{\xi}} \frac{\xi}{\bar{\xi}} \mathbb{I}_{\left(\frac{1-\delta}{\gamma}k - K^f_i(k,\xi,\mu,z)\neq 0\right)} d\xi\right)\mu_i\left(dk\right),$$

   where $\mathbb{I}$ is an indicator function.

6. *The goods market clears:*

$$C_1 + C_2 = Y_1\left(z, \mu\right) + Y_2\left(z, \mu\right) - I_1\left(z, \mu\right) - I_2\left(z, \mu\right),$$

   where Y_i is the production of country i and I_i is its investment. This means

$$Y_i := \int z_i F\left(k, N^f_i\left(k; z, \mu\right)\right)\mu_i\left(dk\right)$$

   and

$$I_i := \int \int_0^{\bar{\xi}} \left(\left(1-\delta\right)k - K^f_i\left(k, \xi; \mu, z\right)\right)\bar{\xi}^{-1} d\xi \mu_i\left(dk\right).$$

These six primitive equilibrium conditions imply further conditions that characterize the equilibrium more directly. For the asset market to clear, the stochastic discount factor must be equal between both economies. Hence, we have

$$\frac{\frac{\partial}{\partial c}u_1\left(C'_1, 1 - N'_1\right)}{\frac{\partial}{\partial c}u_1\left(C_1, 1 - N_1\right)} = \frac{\frac{\partial}{\partial c}u_2\left(C'_2, 1 - N'_2\right)}{\frac{\partial}{\partial c}u_2\left(C_2, 1 - N_2\right)} \tag{8}$$

for any current and future state pair.

If we assume that the initial situation is symmetric, i.e. there are no net claims, both economies are equally large and both are equally productive, then equilibrium condition (8) implies further

$$\frac{\partial}{\partial c}u_1\left(C_1,1-N_1\right)=\frac{\partial}{\partial c}u_2\left(C_2,1-N_2\right). \tag{9}$$

Moreover, the household's choice between leisure and consumption implies

$$w_i=\frac{-\frac{\partial}{\partial n}u_i\left(C_i,1-N_i\right)}{\frac{\partial}{\partial c}u_i\left(C_i,1-N_i\right)} \tag{10}$$

as an equilibrium condition.

With these three equilibrium conditions, the equilibrium for the goods market can be fully characterized without an interlink to the asset markets, if only the numeraire is chosen appropriately. Normalizing prices so that the marginal lifetime utility of wealth ($p\theta$) is one in equilibrium, makes it possible–as in Khan and Thomas, (2003, 2004)–to dichotomize the economy and solve for the goods market equilibrium without determining all asset prices. This normalization yields

$$p\left(z,\mu\right)=\frac{\partial}{\partial c}u_1\left(C_1,1-N_1\right) \tag{11}$$

as our final equilibrium condition. A good's market equilibrium is then achieved if

$$p=\frac{\partial}{\partial c}u_1\left(C_1^f\left(p,w,z,\mu\right),1-\bar{N}_1^f\left(p,w,z,\mu\right)\right) \tag{12}$$

and

$$w_i=\frac{\frac{\partial}{\partial n}u_i\left(C_i^f\left(p,w,z,\mu\right),1-\bar{N}_i^f\left(p,w,z,\mu\right)\right)}{\frac{\partial}{\partial c}u_i\left(C_i^f\left(p,w,z,\mu\right),1-\bar{N}_i^f\left(p,w,z,\mu\right)\right)} \tag{13}$$

where $C_i^f\left(p,w\right)$ splits the total consumption goods C^* that are produced

$$C^*\left(p,w,z,\mu\right):=Y_1\left(p,w,z,\mu\right)+Y_2\left(p,w,z,\mu\right)-I_1\left(p,w,z,\mu\right)-I_2\left(p,w,z,\mu\right) \tag{14}$$

so that (9) holds.[9]

[9] In this notation we assume that prices and wages in (6) are treated as scalar variables instead of given functions of $\left(z,\mu\right)$.

3 Model Solution

Since we are able to dichotomize the economy in such a way, finding the equilibrium practically coincides with solving the Bellman equation (6) for a *rational forecasting rule* for future prices and the capital distribution μ'. However, μ is an infinite dimensional object, so that we have to rely on the method developed by Krussell and Smith (1997, 1998). This means we approximate μ by a finite set of moments $m \in R^n$. Given this set of moments, the main difficulty becomes to obtain a tractable forecasting rule $\Gamma : (z, m) \rightarrow (\hat{p}(z, m), \hat{w}(z, m), \hat{m}')$ and calculating the rational expectations $\bar{\Gamma}$.

To calculate a parameterized rational expectations function Γ, we simulate $T = 3050$ periods of the productivity process, which we keep fixed during the following algorithm:

First, we solve for the value function in equation (6) for an initial guess of Γ_0.[10] Then, we use this value function to generate policy functions from (6) for arbitrary current prices. These policy functions in prices p instead of (μ, z) determine the supply of consumption goods, so that we can use them together with (12) and (13) to compute equilibrium prices.[11] We do so for each of the 3050 periods constantly updating μ using the equilibrium investment policies under the realized equilibrium prices (and expectations Γ_0). This leaves us with a series of equilibrium realizations of μ_t, m_t, p_t, and w_t, $t = 1...T$ under the forecasting rule Γ_0.

From these realizations, we estimate a new forecasting rule Γ_1 using ordinary least squares (but dropping the first 50 observations to avoid an influence of the initialization). Then the algorithm starts over again, using Γ_1 as expectations function.[12] The algorithm stops when Γ_{s-1} and Γ_s do no longer differ significantly (based on an F-test), such that Γ_s is close to the rational expectation $\bar{\Gamma}$ for the chosen discretization of μ. Whether or not this discretization of μ carries enough information can be seen by looking at the R^2-statistics of the final forecasting regressions.

[10] We use Chow and Tsitsiklis (1991) multigrid algorithm to solve for V. Although this algorithm is linear in the number of grid points and computation time for a given number of dimensions of the arguments in V, still the curse of dimensionality restricts our possible choice of dimensions somewhat, as the computation time on the sparse initial grid growths exponentially. So does the memory requirement. The computer codes (MATLAB) are available upon request.

[11] It is important to note that we *do not* use the price expectations $p(z, m)$ in this step.

[12] See Khan and Thomas (2003) for further details. Also note that it is necessary to use the same productivity realization in every iteration of this algorithm.

4 Parameter choices

4.1 Expectations function, moments of μ

An important ingredient of this method to calculate an approximate equilibrium is the choice of the set of moments m and the parametric family of Γ. An important restriction is computational tractability. When the number of moments in m is n, i.e. $m \in \mathbb{R}^n$, the value function V has $3 + 2n$ dimensions within our two country setting (capital plus 2 productivities, and n elements of m for each country). This restricts our choice of n substantially. Fortunately however, it turns out that Γ forecasts already well with just the mean capital per country as elements of m. Since Khan and Thomas (2003) find in a similar setting that adding further moments does not alter the results when R^2 is already large, we do not include further moments.

The larger number of productivity states–J^2 in comparison to only J states in a closed economy setting–makes it necessary to deviate from Khan and Thomas' (2003) parameterization of Γ somewhat. We restrict the slope of the forecasting function to be one common parameter across productivities. This means that we chose the forecasting equation for $z_1 = \zeta_i$ and $z_2 = \zeta_j$

$$\ln y = \alpha_{y,i} + \alpha^*_{y,j} + \beta_y \ln m_1 + \beta^*_y \ln m_2, \tag{15}$$

where y is the variable that has to be forecasted which can be m'_1, m'_2 or p in our case. Since we model the world to be symmetric, in theory both countries should have the same influence on prices. Therefore, we further impose the constraint $\beta_p = \beta^*_p$.[13]

By contrast to our constrained specification, the original Khan and Thomas (2003) forecasting equation is

$$\ln y = \alpha_{y,i} + \beta_{y,i} \ln m \tag{16}$$

where both the slope and the intercept of the forecasting function depend on the state of productivity (z_i). Experimenting with the common slope constraint in the original Khan and Thomas (2003) closed economy model showed that results were robust against this simplification.

4.2 Functional forms

The second important group of parameter choices to be made concerns the functional form of production and utility functions. The two streams of literature, role of non-

[13] A similar argument applies to the mean stock of capital m_i if there are no adjustment costs. Thus, we impose this restriction also for m_i for the no adjustment-costs case.

Table 1: Parameters

Production function ϕ_k	ϕ_n	disutility from labor A	max. adjustment cost $\bar{\xi}$	depreciation δ	discount factor β	labor augm. techn. change γ
.325	.58	3.614	.002	.06	.9225	1.016

convexities in RBC models and international RBC models, differ somewhat in the typical choices. Hence, choosing functional forms is related to the question of which is the primary point of comparison. We opt for the non-convexities and against the international RBC models. We do so in order to not obscure the influence of international trade on investment with changes in functional forms. Therefore, parameter choices follow Khan and Thomas (2003) closely. We specify the utility function of households as additively separable with logarithmic utility in consumption and linear utility in leisure

$$u\left(c, 1-n\right) = \ln c - An. \tag{17}$$

This choice is also convenient since it fixes the real wage to $w_i = \frac{A}{p}$ and makes the calculation of equilibrium easier. Its downside is that this utility function directly implies perfect correlation of consumption between both countries. This is an artifact we would not see in any real data, where there is typically some correlation between consumption levels but far less than correlation between production (the 'quantity anomaly'). For the production function we choose a Cobb-Douglas function

$$z_i F\left(k, n\right) = z_i k^{\phi_k} n^{\phi_n}. \tag{18}$$

The adjustment cost are assumed to be uniformly distributed on $\left[0, \bar{\xi}\right]$.

4.3 Parameter values

These functional form choices leave us with a set of parameters, for which we choose values as displayed in Table 1. These values are the same as Khan and Thomas (2003) choose. Again, this is done for the sake of comparability.

We experiment with two alternative specifications for the stochastic productivity process: one specification in which the productivity processes for both countries are cross sectionally uncorrelated, but autocorrelated with an autocorrelation coefficient $\rho = 0.9225$, and one specification that features correlated shocks and productivity spillover.

In the case of uncorrelated productivity, total factor productivity follows in each country the geometrical AR-1 process

$$\ln z_{it} = \rho \ln z_{it-1} + \varepsilon_{it}, \tag{19}$$

where ε is the productivity innovation. The innovations ε_{it} are i.i.d. normally distributed and have a standard deviation of $\sigma_\varepsilon = 0.0134$. This is exactly the setting Khan and Thomas (2003) choose. To further match their setup as closely as possible in this experiment, we also set the number of states $J = 5$ and use their specified transition matrix. Khan and Thomas originally choose their parameter values as to match business cycle moments of the US economy.

Alternatively, we specify the model to feature correlated productivity shocks and spillover, such that Σ and B in (2) are non-unitary matrices. The matrices Σ and B are chosen as to represent an annualized version of the process considered by Backus, Kehoe, and Kydland (1992). Therefore, in (2) we set

$$\Sigma = 10^{-4} \times \begin{pmatrix} 2.445 & 1.143 \\ 1.143 & 2.445 \end{pmatrix} \text{ and } B = \begin{pmatrix} 0.7120 & 0.2642 \\ 0.2642 & 0.7120 \end{pmatrix}, \tag{20}$$

so that the productivity process is

$$\ln z_t = B \ln z_{t-1} + \varepsilon_t, \ var(\varepsilon_t) = \Sigma.$$

Pre-multiplication of $W = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}$ decomposes this process is into two orthogonal components, to which Tauchen's (1986) univariate algorithm can be applied to generate transition probability matrices and a grid for z^*. Since Σ and B are symmetric 2×2 matrices with constant entries on the main diagonal, we can write them as $B = W\Gamma_z W$ and $\Sigma = W\Gamma_\varepsilon W$, with Γ_z and Γ_ε diagonal matrices. Define $\ln z_t^* := W \ln z_t$. We obtain

$$\ln z_t^* = WB \ln z_{t-1} + W\varepsilon_t = \Gamma_z \ln z_{t-1}^* + \varepsilon_t^*, \ var(\varepsilon_t) = \Gamma_\varepsilon.$$

The first component represents the productivity difference between countries, whereas the second component of z^* reflects the productivity component common to both regions.

In this correlated productivity specification, productivity differences are very short lived. We obtain from Γ_z an autoregressive coefficient 0.4477 of productivity differences

5 Results

For our simulations, we consider a model *without any adjustment cost* as a *reference model.* For a closed economy Khan and Thomas (2003) report no difference of the fixed adjustment cost economy to this reference. By contrast, they find that convex adjustment cost significantly decrease the volatility of investment and output, but they decrease the volatility of investment more so than the volatility of output.

Similarly, Baxter and Crucini (1993) find in a two-country model that convex adjustment cost have an important influence. Convex cost strongly increase the cross country correlation of investment and they also increase the within country correlation of investment and savings. Therefore, Baxter and Crucini conclude that adjustment cost might help to explain in particular the Feldstein-Horioka (1980) puzzle.

To provide a rough comparison for our model simulations, Table 2 presents some summary statistics for the G7 economies to compare our theoretical results in the following.

5.1 Uncorrelated productivity shocks

We begin with the results for our model that compares most closely to the existing literature on non-convexities in general equilibrium. That is, we start with Khan and Thomas' (2003) model augmented by another country but with uncorrelated productivity shocks.

Table 3 gives the central results of this exercise and compares standard deviations and cross correlations between the frictionless and the fixed cost model with $\bar{\xi} = 0.002$ and with $\bar{\xi} = 0.02$.[14] The comparison is done for each country individually just as for the aggregated world economy (i.e. capital re-allocations cancel out and are not counted as investment).[15]

From this table, we see that investment is substantially *less* volatile under the fixed adjustment cost regime than without costs. In a sense, this resembles the finding of Khan and Thomas (2004) for firm-specific productivity shocks, where adjustment costs

[14] Khan and Thomas (2003) suggested $\bar{\xi} = 0.002$ to match the average adjustment frequency in their model. Structural empirical estimation like in Bayraktar et al. (2005) or Cooper and Haltiwanger (2005) support rather fixed costs of $\bar{\xi} = 0.02$. They estimate a fixed costs of around 7% of the stock of capital a firm holds. Average capital holdings in our model are $\exp(1.05)$, nominal wages are 3.614. Hence taking literally the estimates, we should set $\bar{\xi} = 2 \cdot \frac{0.07\exp(1.05)}{A} \approx 0.11$. But arguably, these studies do not take general equilibrium effects into account and hence overestimate adjustment costs in order to match investment fluctuations.

[15] Equilibrium forecasting rules are reported in the appendix.

Table 2: Standard deviations, and cross correlations for G7 economies

a) Standard deviations of macroeconomic variables [in %]

Country	output (Y)	investment rate ($\frac{I}{Y}$)	consumption (C)	net export rate ($\frac{NX}{Y}$)
United States	1.64	0.69	1.23	0.54
United Kingdom	3.96	1.05	4.05	1.30
France	2.23	0.75	2.72	0.72
Germany	3.92	1.19	4.47	0.96
Italy	4.49	1.05	4.40	1.33
Canada	2.83	1.10	2.48	1.03
Japan	2.83	1.30	3.05	0.92
average	**3.13**	**1.02**	**3.20**	**0.97**

b) Correlations with US / Within country

Country	Y	$\frac{I}{Y}$	$\frac{S}{Y}, \frac{I}{Y}$	$\frac{NX}{Y}, Y$	C, Y
United States			0.59	-0.14	0.82
United Kingdom	0.59	0.17	0.03	0.19	0.95
France	0.45	-0.18	0.07	-0.32	0.89
Germany	-0.18	-0.11	0.64	-0.83	0.94
Italy	0.35	-0.33	0.21	-0.26	0.82
Canada	0.63	0.15	0.61	-0.30	0.78
Japan	-0.02	-0.23	0.53	-0.53	0.76
average	**0.30**	**- 0.09**	**0.38**	**-0.31**	**0.85**

Variable in levels are log and HP-filtered. Rates are only HP- filtered. Data comes from the IFS Database of the IMF, is in local currency and covers the period 1961-1998 except for German trade data, which is only reported from 1980. Savings S are computed as output less government expenditure and private consumption.

Table 3: Simulation Results: Standard Deviations and Correlations, Model with Uncorrelated Shocks

a) Standard deviations of macroeconomic variables [in %]

	Y	$\frac{I}{Y}$	C	$\frac{NX}{Y}$
individual economies, no cost	8.07	24.99	1.32	27.81
individual economies, fixed cost $\bar{\xi} = 0.002$	6.53	15.45	1.23	17.01
individual economies, high fixed cost $\bar{\xi} = 0.02$	5.05	8.88	1.05	9.62
world economy, no cost	2.10	2.19	1.32	–
world economy, fixed cost	2.03	1.96	1.23	–
world economy, high fixed cost	1.83	1.58	1.05	–

b) Correlations:

	Cross country		Within country		
	Y_1, Y_2	$\frac{I_1}{Y_1}, \frac{I_2}{Y_2}$	$\frac{S}{Y}, \frac{I}{Y}$	$\frac{NX}{Y}, Y$	C, Y
individual economies, no cost	-0.94	-0.86	-0.30	0.50	0.11
individual economies, fixed cost	-0.90	-0.84	-0.13	0.40	0.09
individual economies, high fixed cost	-0.83	-0.86	0.06	0.33	0.12

If the variable is in levels, it is log and HP-filtered, rates are only HP filtered, both with weight 100.

substantially limit capital re-allocations across firms. However, the result deviates in an important point: in our setting the lower volatility at the single country level carries over to the world economy as an aggregate. It does not do so in Khan and Thomas' (2004) model with an infinite number of individual productivity shocks.

Looking at the results in more detail reveals some similarity between convex adjustment costs–as studied by Baxter and Crucini (1993)–and non-convex ones in shaping international business cycle behavior. In either case, adjustment costs have a strong influence in particular on the volatility of trade, as they decrease the amount of capital re-allocations across countries. In turn, they increase the correlation between investment and savings. Notwithstanding the dampening effect that adjustment costs already have, they are specified too small with adjustment costs being $\bar{\xi} = 0.002$ at maximum. Swings in the employed capital are still too large between countries.

The size of $\bar{\xi} = 0.002$ as maximal adjustment costs had been chosen by Khan and Thomas (2003) as to mimic adjustment hazards from micro studies with their closed economy model. In our open economy setup, the same costs yield a high adjustment frequency at the plant level because of an unrealistically large amount of re-allocation of capital between countries. With $\bar{\xi} = 0.02$ the influence of adjustment costs is more pronounced, but still the investment rate fluctuates far too much relative to the variance of output. Nonetheless, the largest swings in capital allocation are filtered out by the fixed adjustment cost even at the smaller costs. This can be seen if we look at Figure 1, which compares the frequencies of HP-filtered investment rates of the no-cost and the medium cost model. Adjustment costs prohibit large-scale re-allocation of capital from one country to the other, since this would make adjustment by a large fraction of plants necessary.

This inability to re-adjust a given stock of capital feeds back to the investment of the aggregate world economy: Without re-allocating capital, the world cannot fully exploit productivity differences, so that the average productivity of the world's stock of capital fluctuates less. As a result, aggregate world investments are less volatile, too.

Alternatively to the graphs in Figure 1, we may summarize our findings in the following regression:

$$\left(\frac{I_1}{Y_1}\right)_t = \rho \left(\frac{I_1}{Y_1}\right)_{t-1} + \alpha_1 \Delta \left(z_1 - \bar{z}\right)_t + \alpha_2 \Delta \bar{z}_t. \tag{21}$$

Here, $\bar{z}_t$ is the geometric mean of both productivities and Δ denotes first differences. In this regression, α_1 summarizes the expansive effect of country 1's specific productivity relative to the world average, while α_2 measures the effect of a general increase in the

Figure 1: Differences in the Distribution of Investment Rates, Model with Uncorrelated Productivity Shocks

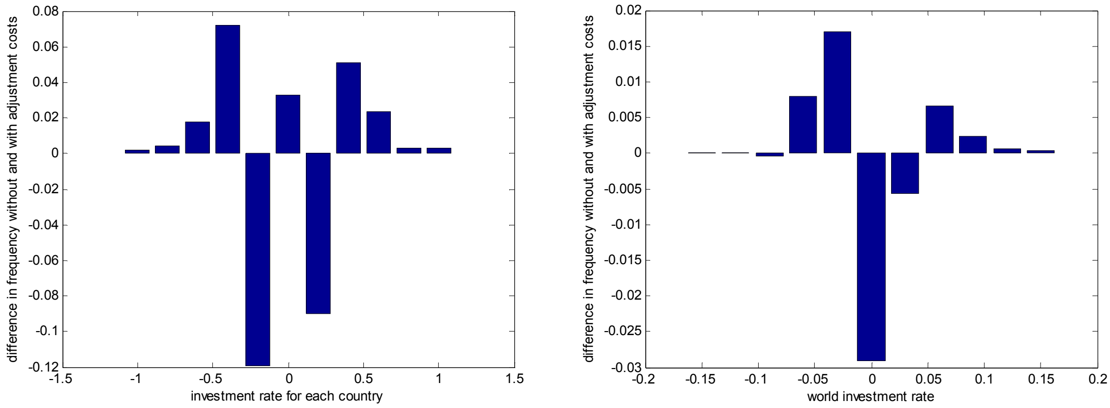

Table 4: Simulation Results: Regression of Investment on Productivity Shocks, Model with Uncorrelated Shocks

a) Investment of Individual Economies

	$\bar{\xi} = 0.02$	$\bar{\xi} = 0.002$	No Cost
productivity difference $\Delta\left(z_{1t} - \bar{z}_t\right)$	19.26	34.87	61.02
average productivity $\Delta \bar{z}_t$	2.02	1.56	0.70
autocorrelation $\left(\frac{I_1}{Y_1}\right)_{t-1}$	0.60	0.31	-0.05

b) Investment of the World Economy

	$\bar{\xi} = 0.02$	$\bar{\xi} = 0.002$	No Cost
productivity variance $\Delta\left(z_{1t} - \bar{z}_t\right)^2$	30.51	7.24	-19.78
average productivity $\Delta \bar{z}_t$	2.07	1.95	1.50
autocorrelation $\left(\frac{I_W}{Y_W}\right)_{t-1}$	0.75	0.65	0.46

Investment rates are HP-Filtered

world's productivity. Table 4 presents the results of the regression exercise.

The regression reveals that introducing adjustment costs makes investment much less sensitive to idiosyncratic elements. At the same time, it becomes more affected by aggregate productivity. Furthermore, movements in investment rates exhibit a substantially larger persistence.

To also summarize the world economy in a similar regression, we replace individual productivity by the variance of productivity. This shows a further difference between the reference model and the model with fixed costs: While investment decreases in the reference model when there is a (temporary) increase in the productivity difference between both countries, an increase in productivity differentials augments investment when there are fixed adjustment costs. In summary, non-convex adjustment costs display a significant influence on the behavior of our model.

When we compare the model to the data, however, we see that the model does not match the observed data in a number aspects. The savings and the investment rate are correlated far too little and both investment rates and output are negatively correlated in the simulation, while they are positively correlated in the data. From a more optimistic point of view, at least correlations move in the direction of the observed patterns of the data when adjustment costs are increased. Nonetheless, the overall mediocre performance of the model is not all too much surprising insofar, as we assumed the productivity processes to be uncorrelated across countries thus far.

5.2 Correlated Productivity

While the assumption of uncorrelated productivity is instructive for comparison with previous studies of closed economies, it is unrealistic in an open economy model and is not well suited for comparison of the model with observed data. Therefore, we also analyze a variant of the model which uses the productivity process specified by Backus, Kehoe, and Kydland (1992), so that it exhibits productivity spillover and correlated productivity shocks as explained before.

For the simulation with correlated productivities, Table 5 presents the main results. Again we consider three variants: no, medium and high fixed cost of capital adjustment.

By contrast to the uncorrelated productivity case, investment costs influence output fluctuations less and the influence is more concentrated towards investment- and trade-fluctuations on the individual country basis. This may reflect that productivity differences are only very short lived in our simulation.

The central effect of higher adjustment costs is that investment fluctuates less, which leads to less fluctuations in the trade rate. Savings and investment rates become more

Table 5: Simulation Results: Standard Deviations and Correlations, Model with Correlated Shocks

a) STANDARD DEVIATIONS OF MACROECONOMIC VARIABLES [IN %]

	Y	$\frac{I}{Y}$	C	$\frac{NX}{Y}$
individual economies, no cost	4.84	13.88	1.36	14.85
individual economies, fixed cost $\bar{\xi} = 0.002$	4.00	7.12	1.37	7.18
individual economies, high fixed cost $\bar{\xi} = 0.02$	3.48	2.92	1.37	2.29
world economy, no cost	2.85	2.22	1.36	–
world economy, fixed cost	2.76	1.30	1.37	–
world economy, high fixed cost	2.73	1.25	1.37	–

b) CORRELATIONS:

	CROSS COUNTRY		WITHIN COUNTRY		
	Y_1, Y_2	$\frac{I_1}{Y_1}, \frac{I_2}{Y_2}$	$\frac{S}{Y}, \frac{I}{Y}$	$\frac{NX}{Y}, Y$	C, Y
individual economies, no cost	-0.31	-0.95	-0.13	0.35	0.52
individual economies, fixed cost	-0.05	-0.93	0.16	0.17	0.64
individual economies, high fixed cost	0.25	-0.64	0.63	0.10	0.74

If the variable is in levels, it is log and HP-filtered, rates are only HP filtered, both with weight 100.

Figure 2: Differences in the Distribution of Investment Rates, Model with Correlated Productivity Shocks

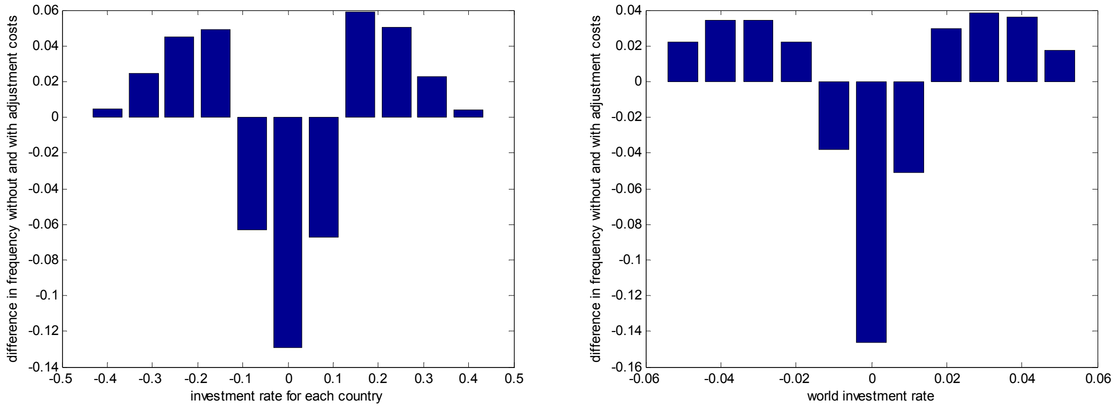

Table 6: Simulation Results: Regression of Investment on Productivity Shocks, Model with Correlated Shocks

a) Investment of Individual Economies

	$\bar{\xi} = 0.02$	$\bar{\xi} = 0.002$	No Cost
country specific $-\Delta z^*_{1t}$	1.90	4.94	8.92
common productivity Δz^*_{2t}	0.37	0.34	0.29
autocorrelation $\left(\frac{I_1}{Y_1}\right)_{t-1}$	0.62	0.35	-0.05

b) Investment of the World Economy

	$\bar{\xi} = 0.02$	$\bar{\xi} = 0.002$	No Cost
productivity variance Δz^{*2}_{1t}	0.75	2.08	-2.24
average productivity Δz^*_{2t}	0.39	0.39	0.33
autocorrelation $\left(\frac{I_W}{Y_W}\right)_{t-1}$	0.77	0.72	0.15

Investment rates are HP-filtered

correlated. Moreover, we see a strong increase in the correlation of output and consumption. Business cycles across countries positively co-move and the correlation of investment rates substantially increases (at least if we move to the high cost specification). Higher adjustment costs also make the trade rate less procyclical. Overall non-convex adjustment costs improve the model's fit with the data. Interestingly, the effect of adjustment costs is actually more pronounced in the correlated productivity setup if we compare the effect on the fluctuations of world investment in Tables 3 and 5.

However, overall the pattern of the influence of fixed adjustment cost does not change with introducing correlated productivity shocks. Inspecting the different distributions of investment rates in Figure 2 does only corroborate the previous findings. The same holds true for our regression exercise (21). For this regression, we can now directly draw on the orthogonal components of z, i.e. the z^* process: We estimate

$$\left(\frac{I_1}{Y_1}\right)_t = \rho \left(\frac{I_1}{Y_1}\right)_{t-1} - \alpha_1 \Delta z^*_{1t} + \alpha_2 \Delta z^*_{2t}. \tag{22}$$

Table 6 displays the results from this regression.

Larger cost of capital adjustment make the common component more important and temporary differences in productivity become less important for the investment of an individual country. This is anologous to what we found when productivity was set to be uncorrelated. On the level of the aggregated world economy, larger productivity differences translate into larger investments under significant fixed adjustment costs, while the opposite holds true for the frictionless model. Again this reflects the uncorrelated productivity case. Moreover, investment is the more autocorrelated, the higher adjustment costs are.

To sum up: the introduction of fixed costs of capital adjustment not only has significant impact on the business cycle properties of the macroeconomic variables in our model, but it also increases the ability to match observed patterns of data even with the simple model considered.[16]

5.3 Quadratic adjustment costs

As suggested in the before, the causation chain for the influence of fixed costs of capital adjustment is very similar to the one in a convex adjustment costs model. To understand this relationship more closely, we search quadratic adjustment costs specifications that

[16]This improvement of the model fit is of importance even more so as model parameters and structures were by no means chosen to maximize the ability of our model to reflect the data, but they were taken mostly from established *closed* economy models for the sake of comparability.

Table 7: Simulation Results: Standard Deviations and Correlations, Model with Correlated Shocks, quadratic adjustment costs

a) Standard deviations of macroeconomic variables [in %]

	Y	$\frac{I}{Y}$	C	$\frac{NX}{Y}$
individual economies, quadratic cost $\bar{\xi} = 0.00127$	3.92	7.10	1.37	7.14
individual economies, high fixed cost $\bar{\xi} = 0.012$	3.44	2.78	1.38	2.29
world economy, fixed cost	2.80	1.51	1.37	–
world economy, high fixed cost	2.78	1.50	1.38	–

b) Correlations:

	Cross country		Within country		
	Y_1, Y_2	$\frac{I_1}{Y_1}, \frac{I_2}{Y_2}$	$\frac{S}{Y}, \frac{I}{Y}$	$\frac{NX}{Y}, Y$	C, Y
individual economies, qudratic cost	0.02	-0.91	0.17	0.16	0.65
individual economies, high quadratic cost	0.31	-0.42	0.53	0.20	0.74

If the variable is in levels, it is log and HP-filtered, rates are only HP filtered, both with weight 100.

each yield the same ammount of investment variability at the country level as one of the two fixed adjustment cost specification yields [As of now the specification is not perfectly matched]. We consider only the model specification with correlated productivity shocks. Table 7 provides the results of this exercise. Quadratic adjustment costs are specified as

$$\bar{\xi} w_t \left(i_t / k_t \right)^2 .$$

In other words, compared to the fixed cost formulation, we replace the stochastic term ξ by the fixed term $\bar{\xi}$ and use the quadratic gross-investment rate to obtain the costs in terms of units of labor.

If we compare Table 7 to Table 5, we observe that allmost all macroeconomic variables show a variance covariance pattern that are very similar to the fixed adjustment cost model. Only world aggregate investments fluctuate more in the convex cost specification than in the fixed adjustment cost counterpart. Consequently, interpreting our model as a two-sector model, we see that fixed adjustment costs actually smooth aggregate investment fluctuations more strongly than quadratic adjustment costs if we keep the smoothing effect for the sectors constant.[17]

6 Conclusion

The present paper analyzes the role of non-convex adjustment cost in an open-economy computable-general-equilibrium model. The result of this computational experiment contrasts the findings for a closed economy, where non-convex adjustment cost have proven not to matter. In the open economy model, fixed adjustment cost substantially limit capital re-allocations from one country to the other, so that country specific investment varies much less. This lesser re-allocation is similar to the situation with firm-specific productivity shocks that Khan and Thomas (2004) study. In contrast to a situation with firm-specific shocks, however, it feeds back to the aggregate that there is a smaller ability to re-allocate capital according to short-term productivity differentials when shocks are country specific. Consequently, fixed adjustment costs also lead to a lower volatility of investment on the world aggregate level.

It is interesting and of importance to compare the findings from a model with firm-specific shocks to the model with country-specific shocks, insofar as the former, i. e. Khan and Thomas' (2004) model, might be interpreted in an open economy sense as a model with many small economies, while the latter, i.e. our model, can be interpreted conversely in a closed economy sense as a model with two significant sectors (as opposed to atomistic firms in their paper). The difference across both models is that the distribution of individual productivities is fixed in Khan and Thomas' (2004) model, while it changes over time and at business cycle frequency in the case of the two country model presented in this paper. Hence, the inability to re-allocate capital across differently productive units can only have an influence on the business cycle in the latter case.

For the international business cycle interpretation of our model, we find that fixed adjustment costs may help to explain some of the correlations in observed data, like high savings and investment correlations, low trade variance, or high cross-country correlations of output. The effect of non-convex adjustment cost is similar to convex costs in this respect. Both do differ mainly in their impact on a on the world aggregate investment just as they differ for closed economy model where convex adjustment costs are influetial while non-convex costs seem not to matter. In fact, our regression analysis re-

[17] This reinforces the finding of Bachman, Caballero and Engel (2006) who argue that if one tries to match sectoral investment fluctuations in a general equilibrium model with fixed adjustment costs, the "estimated" costs are much larger than proposed by Khan and Thomas (2003, 2004) and then the smoothing effect of adjustment costs is much larger.

veals that the effect of non-convex costs is very specific on re-allocation of capital across countries.

As a next step of research, it might be a interesting to find out whether non-convex costs and convex cost formulations differ more significantly in more complex models of international business cycle and are hence more helpful than convex costs to solve some puzzles in the open economy setting. We did only take a first step in this direction, since our approach did not try to actually match observed international business cycle statistics by modifying assumptions on utility functions, or production functions, by assuming the goods in both countries were imperfect substitutes and so forth. We took this restricted approach to isolate the effect international trade possibilities have on the role that fixed adjustment costs play in shaping the cyclical behavior of macroeconomic variables. This effect is significant and reopens the door for fixed adjustment costs to influence business cycle fluctuations.

7 Appendix - forecasting rules

The following tables report the equilibrium forecasting regressions for the various cases: no, high and medium adjustment costs and uncorrelated and correlated productivity.

Table 8: Forecasting rules no cost, uncorrelated shocks

Forecast of m_1

Value of the intercept $\alpha_{m_1,i} + \alpha^*_{m_1,j}$

Productivity state i,j	1	2	3	4	5
1	-0.019	-0.184	-0.343	-0.507	-0.662
2	0.156	-0.009	-0.167	-0.332	-0.486
3	0.326	0.161	0.003	-0.162	-0.316
4	0.502	0.337	0.178	0.014	-0.141
5	0.667	0.502	0.344	0.179	0.025

Slope parameters

β	0.412
β^*	0.412

Forecast of m_2

Value of the intercept $\alpha_{m_2,i} + \alpha^*_{m_2,j}$

Productivity state i,j	1	2	3	4	5
1	-0.019	0.156	0.326	0.502	0.667
2	-0.184	-0.009	0.161	0.337	0.502
3	-0.343	-0.167	0.003	0.178	0.344
4	-0.507	-0.332	-0.162	0.014	0.179
5	-0.662	-0.486	-0.316	-0.141	0.025

Slope parameters

β	0.412
β^*	0.412

Forecast of p

Value of the intercept $\alpha_{p,i} + \alpha^*_{p,j}$

Productivity state i,j	1	2	3	4	5
1	1.187	1.177	1.167	1.156	1.141
2	1.177	1.168	1.158	1.146	1.132
3	1.167	1.158	1.148	1.136	1.122
4	1.156	1.146	1.136	1.125	1.111
5	1.141	1.132	1.122	1.110	1.096

Slope parameters

β	-0.266
β^*	-0.266

Table 9: Forecasting rules medium cost, uncorrelated shocks

Forecast of m_1

Value of the intercept $\alpha_{m_1,i} + \alpha^*_{m_1,j}$

Productivity state i,j	1	2	3	4	5
1	-0.017	-0.111	-0.204	-0.301	-0.393
2	0.086	-0.007	-0.101	-0.198	-0.290
3	0.189	0.096	0.002	-0.095	-0.187
4	0.296	0.203	0.109	0.012	-0.080
5	0.398	0.305	0.211	0.114	0.022

Slope parameters

β	0.607
β^*	0.229

Forecast of m_2

Value of the intercept $\alpha_{m_2,i} + \alpha^*_{m_2,j}$

Productivity state i,j	1	2	3	4	5
1	-0.017	0.086	0.189	0.296	0.398
2	-0.111	-0.007	0.096	0.203	0.305
3	-0.204	-0.101	0.002	0.109	0.211
4	-0.301	-0.198	-0.095	0.012	0.114
5	-0.393	-0.290	-0.187	-0.080	0.022

Slope parameters

β	0.607
β^*	0.229

Forecast of p

Value of the intercept $\alpha_{p,i} + \alpha^*_{p,j}$

Productivity state i,j	1	2	3	4	5
1	1.186	1.177	1.168	1.158	1.146
2	1.177	1.168	1.159	1.149	1.137
3	1.168	1.159	1.150	1.140	1.128
4	1.158	1.150	1.141	1.131	1.119
5	1.146	1.137	1.129	1.119	1.106

Slope parameters

β	-0.318
β^*	-0.318

Table 10: Forecasting rules high cost, uncorrelated shocks

Forecast of m_1

Value of the intercept $\alpha_{m_1,i} + \alpha^*_{m_1,j}$

Productivity state i,j	1	2	3	4	5
1	-0.015	-0.070	-0.125	-0.182	-0.234
2	0.048	-0.007	-0.062	-0.119	-0.171
3	0.112	0.057	0.002	-0.056	-0.107
4	0.178	0.123	0.067	0.010	-0.041
5	0.238	0.183	0.127	0.070	0.019

Slope parameters

β	0.730
β^*	0.113

Forecast of m_2

Value of the intercept $\alpha_{m_2,i} + \alpha^*_{m_2,j}$

Productivity state i,j	1	2	3	4	5
1	-0.015	0.048	0.112	0.178	0.238
2	-0.070	-0.007	0.057	0.123	0.183
3	-0.125	-0.062	0.002	0.067	0.127
4	-0.182	-0.119	-0.056	0.010	0.070
5	-0.234	-0.171	-0.107	-0.041	0.019

Slope parameters

β	0.730
β^*	0.113

Forecast of p

Value of the intercept $\alpha_{p,i} + \alpha^*_{p,j}$

Productivity state i,j	1	2	3	4	5
1	1.184	1.176	1.168	1.158	1.146
2	1.176	1.169	1.160	1.150	1.138
3	1.168	1.160	1.152	1.142	1.130
4	1.158	1.151	1.142	1.132	1.120
5	1.145	1.138	1.129	1.119	1.107

Slope parameters

β	-0.364
β^*	-0.364

Table 11: Forecasting rules no cost, correlated shocks

Forecast of m_1

Value of the intercept $\alpha_{m_1,i} + \alpha^*_{m_1,j}$

Productivity state i,j	1	2	3	4	5
1	0.044	0.065	0.075	0.085	0.105
2	0.006	0.027	0.037	0.047	0.068
3	-0.023	-0.002	0.007	0.017	0.038
4	-0.053	-0.032	-0.023	-0.013	0.008
5	-0.094	-0.073	-0.063	-0.053	-0.033

Slope parameters

β	0.411
β^*	0.411

Forecast of m_2

Value of the intercept $\alpha_{m_2,i} + \alpha^*_{m_2,j}$

Productivity state i,j	1	2	3	4	5
1	-0.093	-0.073	-0.064	-0.054	-0.034
2	-0.052	-0.032	-0.022	-0.013	0.007
3	-0.022	-0.002	0.007	0.016	0.036
4	0.007	0.027	0.037	0.046	0.066
5	0.046	0.066	0.076	0.085	0.105

Slope parameters

β	0.411
β^*	0.411

Forecast of p

Value of the intercept $\alpha_{p,i} + \alpha^*_{p,j}$

Productivity state i,j	1	2	3	4	5
1	1.214	1.182	1.160	1.136	1.105
2	1.213	1.182	1.159	1.136	1.104
3	1.213	1.182	1.159	1.136	1.104
4	1.213	1.182	1.159	1.136	1.104
5	1.214	1.182	1.160	1.136	1.105

Slope parameters

β	-0.242
β^*	-0.242

Table 12: Forecasting rules medium cost, correlated shocks

Forecast of m_1

Value of the intercept $\alpha_{m_1,i} + \alpha^*_{m_1,j}$

Productivity state i,j	1	2	3	4	5
1	0.015	0.036	0.045	0.055	0.075
2	-0.009	0.012	0.021	0.031	0.051
3	-0.024	-0.003	0.006	0.016	0.036
4	-0.040	-0.019	-0.010	-0.000	0.020
5	-0.064	-0.043	-0.034	-0.024	-0.004

Slope parameters

β	0.550
β^*	0.274

Forecast of m_2

Value of the intercept $\alpha_{m_2,i} + \alpha^*_{m_2,j}$

Productivity state i,j	1	2	3	4	5
1	-0.065	-0.044	-0.034	-0.025	-0.004
2	-0.040	-0.019	-0.010	-0.000	0.020
3	-0.025	-0.004	0.006	0.015	0.036
4	-0.010	0.012	0.021	0.031	0.051
5	0.014	0.035	0.045	0.055	0.075

Slope parameters

β	0.550
β^*	0.274

Forecast of p

Value of the intercept $\alpha_{p,i} + \alpha^*_{p,j}$

Productivity state i,j	1	2	3	4	5
1	1.216	1.183	1.159	1.134	1.101
2	1.215	1.182	1.158	1.134	1.101
3	1.215	1.182	1.158	1.134	1.101
4	1.215	1.182	1.158	1.134	1.101
5	1.216	1.183	1.159	1.135	1.101

Slope parameters

β	-0.240
β^*	-0.240

Table 13: Forecasting rules high cost, correlated shocks

Forecast of m_1

Value of the intercept $\alpha_{m_1,i} + \alpha^*_{m_1,j}$

Productivity state i,j	1	2	3	4	5
1	-0.008	0.012	0.021	0.031	0.051
2	-0.018	0.002	0.011	0.020	0.041
3	-0.025	-0.005	0.004	0.014	0.034
4	-0.032	-0.012	-0.003	0.007	0.027
5	-0.042	-0.021	-0.013	-0.003	0.017

Slope parameters

β	0.700
β^*	0.127

Forecast of m_2

Value of the intercept $\alpha_{m_2,i} + \alpha^*_{m_2,j}$

Productivity state i,j	1	2	3	4	5
1	-0.043	-0.022	-0.012	-0.003	0.017
2	-0.033	-0.012	-0.002	0.007	0.028
3	-0.026	-0.005	0.005	0.014	0.034
4	-0.019	0.002	0.012	0.021	0.041
5	-0.009	0.012	0.022	0.031	0.051

Slope parameters

β	0.700
β^*	0.127

Forecast of p

Value of the intercept $\alpha_{p,i} + \alpha^*_{p,j}$

Productivity state i,j	1	2	3	4	5
1	1.216	1.182	1.158	1.134	1.100
2	1.216	1.182	1.158	1.134	1.100
3	1.216	1.182	1.158	1.134	1.100
4	1.216	1.182	1.158	1.134	1.100
5	1.216	1.182	1.158	1.134	1.100

Slope parameters

β	-0.239
β^*	-0.239

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