The Timing of Signaling: To Study in High School or in College?*

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Abstract

American students study harder in college than in high school while East Asian students study harder in high school than in college. This paper proposes a signaling explanation. Signaling may occur over time both in high school and in college, and societies may differ in the timing of signaling. Students work harder in the signaling stage determined by the society as a whole. A testable implication is that high ability workers in East Asia are more concentrated among a few colleges than their US counterparts. This implication is confirmed by top CEO education profile data in the US and Korea.

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1 Introduction

American students study harder in college than in high school while East Asian students study harder in high school than in college. Time use surveys show that average American students study 4.6 hours per week in high school and 9.4 hours in college while average Korean and Japanese students study 14 and 19 hours respectively in high school and 5.1 and 8.8 hours in college (American Time Use Survey (2003), Korean Time Use Survey (2004), Juster and Stafford (1991)). American students more than double their study hours in college while Korean and Japanese students decrease their study hours to less than a half.

Why do American students study more in college than in high school while East Asian students study more in high school than in college? This paper proposes a signaling explanation for this puzzle and provides its empirical evidence. It builds on the ideas of Spence (1973). The main point of departure is that signaling can take place over time both in high school and in college, and that societies may differ in when the signaling takes place. Students work harder in the signaling stage determined by the society as a whole.

I build a two-stage signaling model and show that there exist two equilibria. In one equilibrium named Asian equilibrium, signaling takes place in high school and students work harder in high school than in college. In this equilibrium, firms believe that college names signal workers' abilities better than college GPAs and this makes students study harder in high school; as students compete harder to enter better colleges, college names actually become better signals of workers' abilities. In the other equilibrium named US equilibrium, signaling takes place in college and students work harder in college than in high school. Firms believe that college GPAs signal workers' abilities better than college names and this makes students study harder in college; as workers compete harder to achieve better GPAs, college GPAs actually become better signals of workers' abilities.

My model generates two different kinds of explanations for why societies might have different signaling stages. The first explanation is a multiple-equilibria argument. My model shows that the two equilibria with different signaling stages coexist under certain conditions. Thus, two societies with identical fundamentals can have different signaling stages, and the signaling stage is selected only by the society's self-fulfilling belief. The second explanation is based on the differences in fundamentals between societies. The model shows that signaling is more likely to take place in high school if college-alumni networks are more important for job performance. A case can be made that this condition is more true for East Asian countries.

The theory also delivers a testable implication I can examine with data. If college names are better signals of workers' abilities in East Asia than in the US, high ability workers in East Asia have to be more concentrated among a few top colleges than their counterparts in the US. I examine this implication by looking at college distribution of the largest firms' CEOs in the US and Korea. These top CEOs are clearly high ability workers, and I find that the CEOs in Korea are substantially more concentrated among a few top colleges than the CEOs in the US. For example, 48 percent of the Korean CEOs are from Seoul National University, which accounts for just 0.4 percent of all college students. In contrast, a group of top US colleges, which accounts for the same percentage of college graduates, produces only 19 percent of the US CEOs.

My theory has implications for two important issues. The first issue concerns the debate over the causes of the mediocre performance of American high school students. It is a well documented fact that the high school performance of American students is not as good as that of their East Asian counterparts. For example, in a recent study by OECD (2000), American 15-year-olds were ranked 14th in science while Koreans ranked 1st and Japanese 2nd. While many factors may contribute to the mediocre performance of American students, undoubtedly one part of the explanation is simply that American high school students are not studying as hard as their East Asian counterparts.

My theory implies a trade-off between high school and college education performance, the levels of performance depending on when signaling takes place. If signaling were to occur in high school as in East Asia, US students would work more in high school and their high school performance would improve, but they would work less in college and their college performance would decline. The mediocre performance of US high school students may then not be as bad as it looks, for it is one of the reasons US higher education performance is so excellent.

The second issue concerns education productivity estimation literature that uses international test data for high school students (e.g., Heyneman and Loxley (1983), Hanushek and Luke (2001)). These studies examine the effect of different education systems on student performance. Any productivity study needs to control for all inputs, and in education one input is clearly how hard students are studying. Any differences in signaling stage can lead to differences in study time and this may bias estimates if not properly controlled. For example, these studies conclude that public education expenditure does not matter much for high school students' performance. Part of what drives this result is that most East Asian countries belong to the low spending group and yet their high school students do so well (See Woesmann (2003)). If their excellent performance is at least partly due to signaling occurring in high school, the coefficient for education expenditure will be underestimated.

There is a huge signaling literature following the seminal work of Spence (1973), generalized in many ways including signaling with many signals (e.g., Quinzii and Rochet (1985), Engers (1987), Cho and Sobel (1990)) and repeated signaling (e.g., Cho (1993), Milgrom and Roberts (1982), Kaya (2005)). This paper differs from the previous literature in that there is a group externality in signaling: one has to signal one's ability in the signaling stage determined by the society as a whole. For example, if signaling occurs in high school so that high and low ability workers are completely separated into different colleges, a high ability worker who deviates to a low ability college would not be able to signal his true ability in the college. Firms would believe that he is a low ability worker regardless of his college GPA.

Another novel feature of my model is that workers signal their abilities through their ranks: they choose performance levels in high school and in college but firms observe only their ranks in schools. This is a reasonable assumption because grades are often determined by students' relative ranks, not by their absolute performance. The role of this rank signaling assumption in the model is to limit the ranges of signaling variables (i.e., performances in high school and college) that are observable to firms and thus to prevent each worker from freely choosing his signaling stage. Different equilibria have different observable ranges of the signaling variables across the signaling stages.

The rest of the paper proceeds as follows. Section 2 presents the model. Section 3 characterizes equilibria in colleges and job market. Section 4 characterizes equilibria in high school and finds sufficient conditions under which each equilibrium exists. Section 5 presents a simulation result. Section 6 provides empirical evidence. Section 7 discusses other explanations for the puzzle. Section 8 concludes.

2 The Model

In this section I present the model of workers signaling their abilities both in high school and in college. Each worker decides how much to study in each school and which college to attend. There are three ability types of workers and two colleges, and this provides a minimal setting where signaling can take place both in high school and in college. There is a networking effect in colleges that improves workers' productivity by a fraction α of average student ability in the college. The networking effect coefficient α is the key parameter of the model, deciding which type of equilibrium exists. Colleges admit workers based on their ranks in high school performance, and firms observe each worker's college name and rank in the college and make wage offers based on these two signals.

2.1 Workers

Workers differ in two characteristics: innate ability and disutility of studying in high school. There are three ability types represented by $\theta^H, \theta^M, \theta^L$ where $\theta^H > \theta^M > \theta^L > 0$, and each ability type consists of a unit measure of workers. Workers are also heterogeneous in disutility of studying in high school. The disutility coefficient γ is distributed identically across all ability types, subject to a strictly increasing continuous cumulative distribution function $F : [\underline{\gamma}, \overline{\gamma}] \to [0, 1]$.

Each worker goes to high school and college, and the following utility function describes their preferences:

$$U_\gamma(n_h, n_c, w) \equiv \gamma \cdot v(n_h) + v(n_c) + w \text{ for all } \gamma \in [\underline{\gamma}, \overline{\gamma}] \text{ and } n_h, n_c, w \in \mathbb{R}_+,$$

where w is wage, and n_h and n_c are the time spent studying in high school and college respectively. The study disutility function $v : \mathbb{R}_+ \to \mathbb{R}_-$ is negative valued, strictly *decreasing,* twice differentiable, strictly concave, and satisfies the property that $\lim_{n\to 1} v(n) = -\infty$ and $v(n) = -\infty$ for $n \geq 1$. This property implies that no worker studies more than a unit measure of time either in high school or in college.

Note that study disutility increases in γ when study time is fixed. The heterogeneity in γ captures variation among students other than their innate abilities. For example, disutility coefficient γ would be high for those who just hate studying or do not have good educational environments. In this model, the heterogeneity in γ interferes with effective sorting in high school and allows an equilibrium where some high ability workers end up in a bad college. I assume for simplicity that there is no heterogeneity in disutility of studying in college.

2.2 Stage 1: High School

Each worker decides how much time to spend studying in high school. High school performance p_h depends on workers' ability and study time. For simplicity I assume a linear performance function:

$$p_h(n_h, \theta) = \theta n_h \text{ for all } \theta,\, n_h \in \mathbb{R}_+.$$

2.3 Stage 2: Colleges

There are two colleges – A and B. Each worker applies for one of the colleges, and each college admits *one-and-a-half* unit measure of its applicants who have the best high school performance.[2] Colleges A and B are *ex ante* identical, but *ex post* different in terms of student ability distribution. Without the loss of generality I assume that college A is the superior college with the better average student ability in equilibrium.

Every worker ends up in one of the colleges because the total measure of admission from both colleges is equal to that of all workers. Once in college, workers decide how much time to spend

[2]This cut-off rule in high school performance turns out to be the optimal strategy for each college maximizing its average students' ability, subject to the requirement that each college has to fill up their seats.

studying. The college performance p_c is determined in the same manner as in high school:

$$p_c(n_c, \theta) = \theta n_c \text{ for all } \theta,\, n_h \in \mathbb{R}_+.$$

There is a networking effect in college: job productivity of workers grows by α fraction of the average student ability in their colleges. In other words, job productivity of a college s student increases by $\alpha E(\theta|s)$ $(s = A, B)$. College friends at work can help each other to improve their job productivity. Moreover, one's job productivity increases even more if one's friends have better abilities. In this model the networking effect coefficient α is the key parameter that decides which type of equilibrium exists.

2.4 Stage 3: Job Market

There are two firms (or more) who have the same constant returns to scale technology: a college s graduate with ability θ produces $\theta + \alpha E(\theta|s)$, that is, his innate ability plus the networking gain in college. Firms can observe each worker's college name and his rank in the college and compete for the worker by simultaneously announcing their wage offers based on these two signals. The rank of a worker in a college is defined as the measure of the workers in the college who have strictly higher performance.

2.5 Equilibrium

A Bayesian Nash Equilibrium of the model consists of the list including each worker's study time and college choice and each firm's wage schedule, such that every player's strategy is the best response to the other players' strategies and the firms' beliefs are updated by the Bayesian rule whenever applicable. I focus on the following two types of equilibria in order to show the differences in signaling stage and their impact on study hours and educational performance.

Definition 1 *Asian equilibrium is the Pareto-dominant separating Bayesian Nash equilibrium, where college A has only high and medium ability workers and college B has only medium and low ability workers.*

Definition 2 *US equilibrium is the Pareto-dominant separating Bayesian Nash equilibrium, where each college has all three ability types of workers.*

Note the following four points in the equilibrium definitions. First, a college name is a better signal of a worker's ability in Asian equilibrium. In Asian equilibrium, firms can safely infer that college A graduates are at least of medium ability and that college B graduates are at best of medium ability. Second, specific shares of ability types within colleges do not matter for equilibrium classification. What matters is whether certain ability types exist with a positive measure or not. Third, the colleges in US equilibrium do not have to be identical. They may differ in their share of each ability type as long as each college has a positive measure of each ability type. Fourth, both equilibria are *Pareto-dominant* separating equilibria. It is a well known result that Spence signaling models produce a continuum of multiple equilibria. In order to obtain a unique outcome, most applied literature focus on the Pareto-dominant separating equilibrium outcome (or the Riley outcome). This equilibrium outcome is the only equilibrium outcome that survives the refinement of D1 criterion (see Banks and Sobel (1987), Cho and Kreps (1987), Cho and Sobel (1990)).

3 Colleges and Job Market

In this section, I start characterizing each equilibrium backward from the last stages: colleges and job market. This section has two key results. First, college students in US equilibrium work harder than their counterparts in Asian equilibrium. Second, the benefit of attending the better college (college A) is greater in Asian equilibrium than in US equilibrium.

There are two standard results to check before characterizing college equilibrium outcome. First, the single crossing property holds between college performance and wage. Higher ability workers need smaller wage compensation for the same marginal performance increase because it costs them less study time. Thus, higher ability workers can outrank lower ability workers by achieving performance that is too costly for the lower ability workers to imitate. Second, the firms offer each individual worker the wage equal to his expected productivity based on the signals (college name and rank in the college) due to their constant returns to scale technology and Bertrand competition for workers.

3.1 Colleges in Asian Equilibrium

In Asian equilibrium, college A has one unit measure of high ability workers and a half unit measure of medium ability workers. In any separating equilibrium, high ability workers outrank medium ability workers and thus high ability workers get rank 0 and medium ability workers get rank 1. Since firms pay expected productivity based on the signals, firms offer high ability workers' wage $\theta^H + \alpha \left\{ (2/3)\, \theta^H + (1/3)\, \theta^M \right\}$ to college A graduates with rank 0 and medium ability workers' wage $\theta^M + \alpha \left\{ (2/3)\, \theta^H + (1/3)\, \theta^M \right\}$ to college A graduates with rank 1.

In this Pareto-dominant (i.e., signaling-cost-minimizing) separating equilibrium, medium ability workers, the lowest ability type in this college, do not study at all $\left(P_A^L = 0\right)$ and high ability workers study just enough to weakly separate themselves away from medium ability workers. Therefore, high ability workers' performance P_A^H is determined by the medium ability workers' indifference condition between their equilibrium pay off and high ability workers' pay off: $v(P_A^H/\theta^M) + \theta^H = v(0) + \theta^M$.

Even though college A does not have any low ability workers in equilibrium, I still need to know how a low ability worker would behave if he deviated to college A, because he makes a college choice in high school. The low ability worker in college A would not perform better than medium ability workers because it costs him more study hours than it does medium ability workers to achieve the same level of performance. Since medium ability workers in college A do not study at all, the low ability worker would not study at all either $\left(P_A^L = 0\right)$ and get paid medium ability workers' wage with rank 1. Table 1 summarizes the equilibrium outcome in college A.

Ability	Performance	Rank	Wage
Low	0	1	$\theta^M + \alpha \left\{(2/3)\,\theta^H + (1/3)\,\theta^M\right\}$
Medium	0	1	$\theta^M + \alpha \left\{(2/3)\,\theta^H + (1/3)\,\theta^M\right\}$
High	P_A^H such that $v(P_A^H/\theta^M) + \theta^H = v(0) + \theta^M$	0	$\theta^H + \alpha \left\{(2/3)\,\theta^H + (1/3)\,\theta^M\right\}$

Table 1: College A in Asian Equilibrium

College B has a half unit measure of medium ability workers and one unit measure of low ability workers in Asian equilibrium. Thus, medium ability workers get rank 0 and low ability workers get rank 1/2, and firms offer wages $\theta^M + \alpha \left\{(1/3)\,\theta^M + (2/3)\,\theta^L\right\}$ to college B graduates with rank 0 and $\theta^L + \alpha \left\{(1/3)\,\theta^M + (2/3)\,\theta^L\right\}$ to college B graduates with rank 1/2. Low ability workers do not study at all $\left(P_B^L = 0\right)$ and medium ability workers study just enough to separate themselves away from low ability workers: $v(P_B^M/\theta^L) + \theta^M = v(0) + \theta^L$.

A high ability worker, if he deviated to college B, would not perform any better than medium ability workers because doing so would not give him any better rank. He would not perform any worse than medium ability workers either because it costs him less effort to achieve the same level of performance. Thus, the high ability worker would achieve the same performance as medium ability workers in the college and get paid the medium ability workers' wage. Note that the high ability worker can not affect the ranks of the other ability types in the college because he is of measure 0. Table 2 summarizes the college B outcomes.

Ability	Performance	Rank	Wage
Low	0	1/2	$\theta^L + \alpha \left\{(1/3)\,\theta^M + (2/3)\,\theta^L\right\}$
Medium	P_B^M such that $v(P_B^M/\theta^L) + \theta^M = v(0) + \theta^L$	0	$\theta^M + \alpha \left\{(1/3)\,\theta^M + (2/3)\,\theta^L\right\}$
High	$P_B^H = P_B^M$	0	$\theta^M + \alpha \left\{(1/3)\,\theta^M + (2/3)\,\theta^L\right\}$

Table 2: College B in Asian Equilibrium

College B in Asian equilibrium is the place where the rank-signaling assumption in college performance makes a difference. Suppose that firms observe raw college performance directly. The high ability wokrer who deviated to college B can signal his true ability by achieving performance that is too costly or even impossible for medium ability workers to imitate. Firms should believe that workers with these performance levels are of high ability, following the idea of the intuitive criterion in Cho and Kreps (1987). However, this is somewhat in conflict with the belief created by Bayesian update rule that college B has only medium and low ability workers in Asian equilibrium. The rank-signaling assumption resolves this possible conflict in belief by limiting the range of college performance that are observable to firms: there are only two levels of performance 0 and P_B^M to either of which all other performance is observationally identical to firms. Note that in equilibrium firms do not have an incentive to directly look at workers' college performances because their college performances are perfectly inferable from their college names and ranks in the colleges.

3.2 Colleges in US Equilibrium

In US equilibrium, both colleges have all three ability types but with different shares. Suppose that college s has σ_s^i measure of ability type i workers ($s = A, B$ and $i = H, M, L$). In college s, high, medium, and low ability workers get ranks 0, σ_s^H, and $\sigma_s^H + \sigma_s^M$ respectively, and firms offer wages $\theta^H + \alpha E(\theta|s)$, $\theta^M + \alpha E(\theta|s)$, and $\theta^L + \alpha E(\theta|s)$ to college s graduates with ranks 0, σ_s^H, and $\sigma_s^H + \sigma_s^M$. Low ability workers do not study at all, and the higher ability workers study just enough to separate themselves away from the lower ability workers. Table 3 summarizes the college outcomes in US equilibrium.

Ability	Performance	Rank	Wage
Low	0	0	$\theta^L + \alpha E(\theta\|s)$
Medium	P_s^M such that $v(P_s^M/\theta^L) + \theta^M = v(0) + \theta^L$	σ_s^H	$\theta^M + \alpha E(\theta\|s)$
High	P_s^H such that $v(P_s^H/\theta^M) + \theta^H = v(P_s^M/\theta^M) + \theta^M$	$\sigma_s^H + \sigma_s^M$	$\theta^H + \alpha E(\theta\|s)$

Table 3: College s in US Equilibrium (s = A,B)

Note that college performances P_s^i do not depend on college name s ($i = H, M, L$). The same ability workers perform equally in both colleges regardless of ability distribution as long as each college has all the three ability types with positive measures.

3.3 Asian Colleges vs. US Colleges

Now I compare Asian and US colleges outcomes. I first show that US students study harder than Asian students in college. In Asian equilibrium college A graduates are considered at least of medium ability while in US equilibrium college A graduates can be of any ability type. This belief allows college A students in Asian equilibrium to study less in college to signal their ability, thus lowering their college performances. Since performance is an increasing function in study time, it follows trivially that college A students spend more time studying in US equilibrium. The following result can be easily obtained by comparing Tables 1 and 2 with Table 3.

Proposition 1 *Every college student studies weakly more hours and performs weakly better in US equilibrium than in Asian equilibrium. In particular, high and medium ability students in college A study strictly more hours and perform strictly better in US equilibrium.*

The result in Proposition 1 is more generally true: students work harder in US equilibrium whenever the equilibria are fully separating and when the support of ability distribution within each college in an Asian equilibrium is strictly contained in the support of ability distribution within each college in a US equilibrium.[3] This result does not depend on the specific assumption about discrete types.

Now I show that the benefit of attending college A is greater in Asian equilibrium than in US equilibrium. In Asian equilibrium, there are two endogenous effects that make college A preferable to college B. First, there is the "networking" effect. The network productivity gain in college A is greater than the productivity gain in college B because the average ability of college A students is better than that of college B students. Second, there is a "sorting" effect which makes college A even more attractive. The sorting effect occurs because firms believe that college A graduates are at least of medium ability and college B graduates are at best of medium ability.

[3] I thank a referee for pointing this out.

In order to better understand this sorting effect, suppose that there is no networking effect ($\alpha = 0$). Low ability workers prefer college A because they can make medium ability workers' wage by attending college A. Medium ability workers get the same wage whether they attend college A or college B, but still prefer college A because they do not have to study at all in college A. High ability workers prefer college A because they would make medium ability workers' wage if they attended college B. This sorting effect can be verified algebraically by comparing the outcomes in Table 1 and Table 2 with holding $\alpha = 0$.

In US equilibrium, the sorting effect does not exist. In Table 3 the outcomes for both colleges are identical if the networking effect does not exist ($\alpha = 0$). The networking effect still makes college A weakly preferable to college B, but the size of the networking effect is smaller than that in Asian equilibrium because the average students' ability difference between college A and B is bigger in Asian equilibrium. Thus, the benefit of attending college A is greater in Asian equilibrium.[4]

Proposition 2 *The benefit of attending college A is strictly greater in Asian equilibrium than in US equilibrium.*

4 High School and the Existence of Equilibrium

In this section, I characterize equilibrium outcomes in high school. This section has three key results. First, high school students in Asian equilibrium study harder than their counterparts in US equilibrium. Second, workers in Asian equilibrium study harder in high school than in college while workers in US equilibrium study harder in college than in high school. Third, Asian equilibrium exists for a sufficiently large networking coefficient α and US equilibrium exists for a sufficiently small networking effect coefficient α.

Firms do not directly observe workers' high school performance. Instead, the firms observe workers' college names that are determined by their ranks in high school performance. The role of

[4]The benefit of attending college A is algebraically defined as $\left\{v\left(n_c^A\right) + w^A\right\} - \left\{v\left(n_c^B\right) + w^B\right\}$ where n_c^s is the amount of work in college and w^s is the wage, if he or she attends college s $(s = A, B)$.

this rank signaling assumption is to prevent complete signaling in high school stage by limiting the range of high school performances workers can choose: all workers choose either college A admission cut-off performance C_A or college B cut-off C_B because firms care only about workers' college names. If firms directly observed workers' high school performances, high and medium ability workers would be able to signal their abilities in high school by achieving high school performances that are too costly for lower ability types to imitate. Note that in equilibrium firms do not have an incentive to directly observe workers' high school performances because their high school performance are perfectly inferable from their college names.

4.1 High School in Asian Equilibrium

In Asian equilibrium, a half unit measure of medium ability workers attend college A and the other half unit measure of medium ability workers attend college B. More precisely, medium ability workers with $\gamma < \gamma_m$, where $F(\gamma_m) = 0.5$, attend college A because they have lower utility costs of achieving the cut-off C_A for college A admission, and medium ability workers with $\gamma > \gamma_m$ attend college B. A medium ability worker with γ_m is indifferent between the two colleges, and this uniquely determines college A cut off performance C_A:

$$\gamma_m v\left(C_A/\theta^M\right) + v(0) + \frac{\alpha}{3}\left(2\theta^H + \theta^M\right) = \gamma_m v\left(C_B\right) + v\left(P_B^M/\theta^M\right) + \alpha\left\{(2/3)\,\theta^L + (1/3)\,\theta^M\right\}.$$

The college B cut-off C_B is 0 because college B is less preferred and there are enough seats to accomodate all workers. I can rewrite the above equation as

$$\gamma_m\left\{v(0) - v\left(C_A/\theta^M\right)\right\} = v(0) - v\left(P_B^M/\theta^M\right) + \frac{2\alpha}{3}\left(\theta^H - \theta^L\right). \tag{1}$$

The college A cut-off C_A also has to satisfy the incentive compatibility conditions for the other types. In other words, the cut-off C_A has to be such that no high ability workers deviate to college B and no low ability workers deviate to college A. In order to show that no high ability worker deviates to college B, it suffices to show that the high ability worker with the highest study disutility coefficient $\bar{\gamma}$ doesn't deviate to college B. Analogously, I also need to show the low ability worker with $\underline{\gamma}$ doesn't deviate to college A.

Let $\underline{R}_A^L$ and $\bar{R}_A^H$ be the maximum high school performance levels which low ability workers with $\underline{\gamma}$ and high ability workers with $\bar{\gamma}$ are willing to achieve in order to attend college A. They are indifferent between attending college A with these reservation performances in high school and attending college B with 0 performance. Therefore, $\underline{R}_A^L$ and $\bar{R}_A^H$ are determined by

$$\underline{\gamma}\left\{v(0) - v\left(\underline{R}_A^L/\theta^L\right)\right\} = \theta^M - \theta^L + \frac{2\alpha}{3}\left(\theta^H - \theta^L\right), \tag{2}$$

$$\bar{\gamma}\left\{v(0) - v\left(\bar{R}_A^H/\theta^H\right)\right\} = v\left(P_A^H/\theta^H\right) - v\left(P_B^H/\theta^H\right) + \frac{2\alpha}{3}\left(\theta^H - \theta^L\right). \tag{3}$$

It follows from conditions $(1), (2)$, and (3) that $\underline{R}_A^L$, C_A, and $\bar{R}_A^H$ converge to θ^L, θ^M, θ^H respectively as α increases to infinity (note that college outcomes P_A^H and P_B^M do not depend on α), which implies that $\underline{R}_A^L < C_A < \bar{R}_A^H$ for sufficiently large α. For these α no low ability workers attend college A and no high ability workers attend college B, and thus there exists an Asian equilibrium.

Proposition 3 *There exists an Asian equilibrium for sufficiently large α.*

The networking effect plays a crucial role in Proposition 3. Without the networking effect, the single crossing property may not hold in high school stage and thus there may not exist an Asian equilibrium.[5] However, as the networking benefit, thus the benefit of attending college A, becomes larger and larger, all types are prepared to spend a lot of hours studying, and higher ability workers eventually outperform lower ability workers because they have higher upper bounds on their performances.

4.2 High School in US Equilibrium

In order for a US equilibrium to exist, both colleges A and B should have all three ability types. The heterogeneity in high school study disutility γ makes the existence of US equilibrium possible,

[5] For example, when there is no networking effect, the gains from entering the good college is higher for low ability types than for medium ability types. If low types get into the good college, they can pass off as the medium types at no extra effort. So the gain is $\theta^M - \theta^L$. If a medium type in the bad college joins the good college, the savings in signaling cost is $v(0) - v\left(P_B^M/\theta^M\right)$. But it follows from Table 2 that $\theta^M - \theta^L = v(0) - v\left(P_B^M/\theta^L\right) > v(0) - v\left(P_B^M/\theta^M\right)$.

where some high ability workers with high γ attend the inferior college B and some low ability workers with low γ attend the superior college A. Therefore, in order for a US equilibrium to exist, the heterogeneity in γ has to be sufficiently large relative to the heterogeneity in ability.

Assumption 1 *$\bar{\gamma}/\gamma_m > \theta^H/\theta^M$ and $\gamma_m/\underline{\gamma} > \theta^M/\theta^L$ where $F(\gamma_m) = 0.5$.*

Unlike Asian equilibrium, the ability distribution of workers across colleges in US equilibrium is not directly pinned down by the equilibrium definition but has to be endogenously determined. Since the sorting effect is not present in US equilibrium, the networking effect is the entire benefit of attending college A. Workers in high school observe the size of this networking effect and decide which college to attend, aggregately determining the ability distribution of the workers across colleges. This new ability distribution, in turn, determines the size of the new networking effect. In equilibrium, the initial networking effect has to coincide with the resulting networking effect.

When the networking effect coefficient $\alpha > 0$ is fixed, the networking effect is determined by the average student ability difference between the colleges. This cross-college ability difference $x \equiv E(\theta|A) - E(\theta|B)$ cannot be negative because college A students have better average ability. Further, the cross-college ability difference x is smaller than $(2/3)\left(\theta^H - \theta^L\right)$ which can be achieved only in Asian equilibrium. Let $\Pi \equiv [0, (2/3)\left(\theta^H - \theta^L\right)]$ denote the set of possible cross-ability difference and let $\psi(x) : \Pi \twoheadrightarrow \Pi$ be the new cross-college ability difference correspondence resulting from the workers' college choices, given the initial ability difference x.

Suppose $x = 0$. There is no networking effect and workers are indifferent between college A and college B. The cut-off performances for admission have to be the same across the colleges, and workers randomly choose their colleges. Thus, the new ability distribution of workers is not unique and the resulting ability difference can be any number in Π, which implies

$$\psi(0) = \Pi.$$

Suppose $x > 0$. Given x and $C_A > 0$, there exists a unique critical disutility coefficient $\tilde{\gamma}^i(x, C_A) \in \mathbb{R}_{++}$ for each ability type, such that workers with $\tilde{\gamma}^i(x, C_A)$ are indifferent between college A and college B ($i = H, M, L$):

$$\tilde{\gamma}^i(x, C_A) \cdot v(C_A/\theta^i) + \alpha E(\theta|A) = \tilde{\gamma}^i(x, C_A) \cdot v(0) + \alpha E(\theta|B) \quad \text{for } x > 0,\ i = H, M, L.$$

Solving the above equation for $\tilde{\gamma}^i$, I obtain

$$\tilde{\gamma}^i(x, C_A) = \frac{\alpha x}{v(0) - v\left(C_A/\theta^i\right)} \quad \text{for } x, C_A > 0,\ \ i = H, M, L \tag{4}$$

Those workers with γ lower than $\tilde{\gamma}^i(x, C_A)$ for each ability type attend college A and the others attend college B. Since college A admits one-and-a-half unit measures of workers, the cut-off performance C_A for college A is uniquely determined by the following condition:

$$\sum_{i=H,M,L} F\left(\tilde{\gamma}^i(x, C_A)\right) = 1.5 \text{ for } x > 0. \tag{5}$$

The above condition implicitly defines C_A as a function of x and thus I can write $\tilde{\gamma}^i(x, C_A)$ as $\tilde{\gamma}^i(x)$. Since $\tilde{\gamma}^i(x)$ ($i = H, M, L$) pins down the unique ability distribution across the colleges, the cross-college ability difference $\psi(x)$ is uniquely determined (thus single valued) for $x > 0$.

$$\begin{aligned} \psi(x) &\equiv E(\theta|A, x) - E(\theta|B, x) \\ &= \frac{1}{1.5} \sum_{i=H,M,L} \theta^i F\left(\tilde{\gamma}^i(x)\right) - \frac{1}{1.5} \sum_{i=H,M,L} \theta^i \left\{1 - F\left(\tilde{\gamma}^i(x)\right)\right\} \text{ for } x > 0. \end{aligned} \tag{6}$$

So far I have assumed that α is fixed when characterizing $\tilde{\gamma}^i(x)$ and $\psi(x)$. Since I want to relate α to the existence of US equilibrium, I slightly modify the notations in order to follow the effect of a change in α on $\tilde{\gamma}^i(x)$ and $\psi(x)$. Let $\tilde{\gamma}^i(x; \alpha')$ and $\psi(x; \alpha')$ denote $\tilde{\gamma}^i(x)$ and $\psi(x)$ with $\alpha = \alpha'$ respectively. The following lemma is crucial for analyzing the conditions determining the existence of US equilibrium.

Lemma 1 *Let $\underline{\gamma}$, $\bar{\gamma}$, θ^H, θ^M, θ^L and F satisfy Assumption 1.*

(i) There exists $\gamma_0^i \in \left(\underline{\gamma}, \bar{\gamma}\right)$ such that $\lim_{\alpha \searrow 0} \tilde{\gamma}^i(x; \alpha) = \gamma_0^i$ for all $x > 0$ $(i = H, M, L)$.

(ii) For all $\alpha \geq 0$, $\psi(x; \alpha)$ is continuous and weakly increasing in $x > 0$.

(iii) There exists $y_0 \in \left(0, (2/3)\left(\theta^H - \theta^L\right)\right)$ such that (a) $\lim_{x \searrow 0} \psi(x; \alpha) = y_0$ for all $\alpha \geq 0$ and (b) $\lim_{\alpha \searrow 0} \psi\left((2/3)\left(\theta^H - \theta^L\right); \alpha\right) = y_0$.

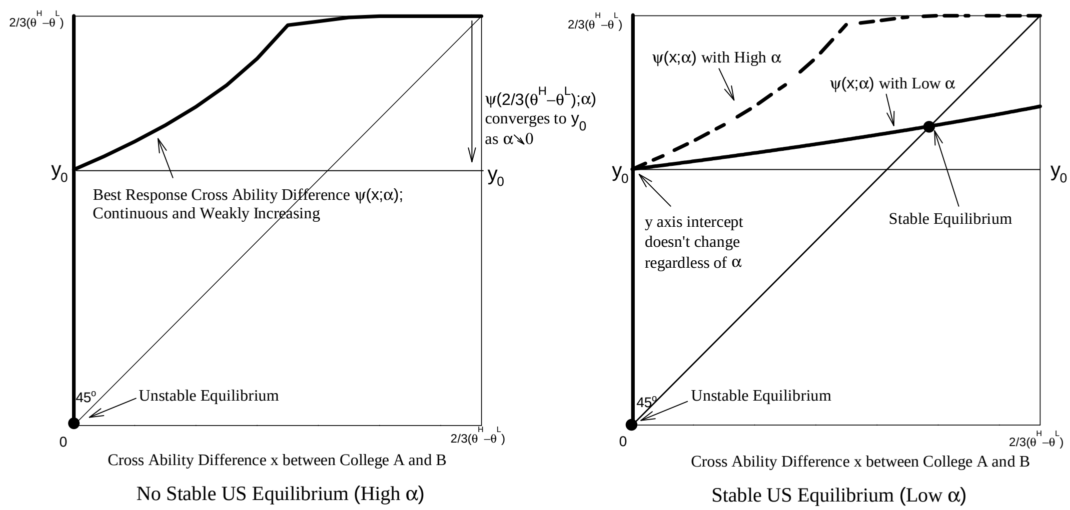

Figure 1: The Existence of Stable US Equilibrium and Networking Effect Coefficient α

Proof. See Appendix. ■

Part (i) of Lemma 1 says that each college has all three ability types (note that $\underline{\gamma} < \gamma_0^i < \bar{\gamma}$) for sufficiently small networking coefficient $\alpha > 0$. Parts (ii) and (iii) of Lemma 1, illustrated by Figure 1, characterize the graphical properties of $\psi(x;\alpha)$. Part (iii) of Lemma 1 says that (a) y axis intercept y_0 of $\psi(x;\alpha)$ does not depend on α and is strictly between 0 and $(2/3)\left(\theta^H - \theta^L\right)$, and that (b) $\psi(x;\alpha)$ uniformly converges down to a flat line crossing the y axis intercept (note that $\psi(x;\alpha)$ is weakly increasing in x due to part (ii)).

In US equilibrium, the initial networking effect has to coincide with the new networking effect resulting from workers' best-response college decisions. Therefore, the following condition has to hold:

$$x^* \in \psi(x^*). \tag{7}$$

Suppose $x^* = 0$. Condition (7) is satisfied because $0 \in \Pi = \psi(0)$, and there exists a trivial US equilibrium where all workers are indifferent between both colleges and each worker flips a fair coin between them. In this trivial US equilibrium, each college has the same share of all three ability types. However, this trivial US equilibrium is not stable as defined in the following sense.

Definition 3 *Denote* $x_{n+1} \equiv \psi(x_n)$ *for all* $n \in \mathbb{N}$*. A US equilibrium with the cross-ability difference* x^* *is (locally) stable if there exists* $\delta > 0$ *such that* $\lim_{n\to\infty} x_n = x^*$ *for all* $x_0 \in (x^* - \delta, x^* + \delta) \cap \Pi$.

Definition 3 is a usual definition of stability, that the system returns to the original equilibrium after small disturbances. The trivial US equilibrium at $x^* = 0$ is not stable, because y axis intercept y_0 of $\psi(x;\alpha)$ is strictly greater than 0 and $\psi(x;\alpha)$ is continuous in x due to parts (ii) and (iii) of Lemma 1.

Suppose $x^* > 0$. The intermediate value theorem implies that there exists $x^* \in \left(0, (2/3)\left(\theta^H - \theta^L\right)\right)$ satisfying condition (7) for sufficiently small $\alpha > 0$, due to parts (ii) and (iii) of Lemma 1. Moreover, each college has all three ability types for sufficiently small $\alpha > 0$, due to part (i) of Lemma 1. Therefore, US equilibrium exists for sufficiently small $\alpha > 0$. The stability condition is satisfied for at least one US equilibrium because there exists at least one x^* where ψ intersects the 45° line from above and ψ is weakly increasing in x.

Proposition 4 *Let* $\underline{\gamma}$, $\bar{\gamma}$, θ^H, θ^M, θ^L *and* F *satisfy Assumption 1. There exists a stable US equilibrium for* $\alpha > 0$ *sufficiently close to* 0.

The intuition behind Proposition 4 is the opposite to that of Proposition 3. Assumption 1 guarantees that low ability workers with $\underline{\gamma}$ outperform the high ability workers with $\bar{\gamma}$ when the benefit of attending college A becomes sufficiently small.

4.3 Asian High School vs. US High School

Now I compare Asian and US high school outcomes. The benefit of attending college A is greater in Asian equilibrium than in US equilibrium (Proposition 2) and thus workers in Asian equilibrium are willing to study more in high school to enter college A. This makes college A cut-off performance higher in Asian equilibrium and average high school performance better there because high school students perform only as much as the cut-off performance of the colleges they attend.

Lemma 2 *Whenever both Asian and US equilibria coexist with the same parameters, the average performance of high school students is strictly better in Asian equilibrium.*

Now I compare high school performance across US and Asian equilibria when networking effect coefficients α are different but the other parameters are the same. Proposition 5 directly follows from Lemma 2 because the benefit of attending college A becomes even larger as the networking effect coefficient α increases.

Proposition 5 *Suppose that the networking coefficient α of Asian equilibrium is weakly greater than that of US equilibrium (the other parameters being the same). The Asian equilibrium has strictly better high school performance than the US equilibrium.*

These predictions are fairly weak because I expect that Asian equilibrium usually has a better high school performance even when Asian equilibrium has a lower α than US equilibrium. The extreme ability distribution across colleges in Asian equilibrium, which strengthens the networking effect, and the sorting effect found only in Asian equilibrium are usually more than enough to compensate for the loss of the benefit of attending college A resulting from low α.

Rather surprisingly, it is not always true that workers in high school study more in Asian equilibrium even when they have better high school performance. In this model, only those attending college A study in high school. Since workers attending college A in Asian equilibrium have better average abilities than their counterparts in US equilibrium, they need less study time to achieve the same level of performance. It is thus possible that high school students work less in Asian equilibrium even when their performance is better. However, in most cases I expect high school students to work more in Asian equilibrium, as will be shown later in the simulation section.

4.4 High School vs. College

Now I compare study hours between high school and college within each equilibrium. In Asian equilibrium, the benefit of attending college A increases to infinity as the networking effect coefficient α increases to infinity. Thus, students work harder in high school than in college for sufficiently large

α. In US equilibrium, the benefit of attending college A converges down to 0 as the networking coefficient α converges down to 0. Thus, students work harder in college than in high school for sufficiently small $\alpha > 0$.

Proposition 6 *(i) In Asian equilibrium, students study more in high school than in college for sufficiently large α.*

(ii) In US equilibrium, students study more in college than in high school for sufficiently small $\alpha > 0$.

5 Simulation

In this section I run a simulation varying the networking coefficient α. The main purpose of this simulation is to show that there exist a range of αs where both Asian and US equilibria coexist with the same parameters. In addition, the simulation also provides a concrete example illustrating how big or small the coefficient should be for each theoretical result to hold. In the simulation, the networking effect coefficient α varies from 0 to 0.9, and other parameters used in the simulation are the following:

$$\theta^H = 3,\ \theta^M = 2,\ \theta^L = 1, v(n) = \log(1-n), \gamma \sim Uniform[0.2, 1.8].$$

Figure 2 summarizes the simulation result. First, different equilibria exist with different networking coefficients α. Only Asian equilibrium exists for large αs and only US equilibrium exists for small αs. Both equilibria coexist for medium-size αs. Second, Asian students study harder in high school than in college while US students study harder in college than in high school. Moreover, high school students study harder in Asian equilibrium while college students study harder in US equilibrium. These are all consistent with the stylized facts.

Figure 2 suggests that there are two different kinds of explanations, both consistent with the theory, for why US equilibrium occurs in the US and why Asian equilibrium occurs in East Asia. The first explanation is a multiple equilibria argument. Figure 2 shows that both US and Asian

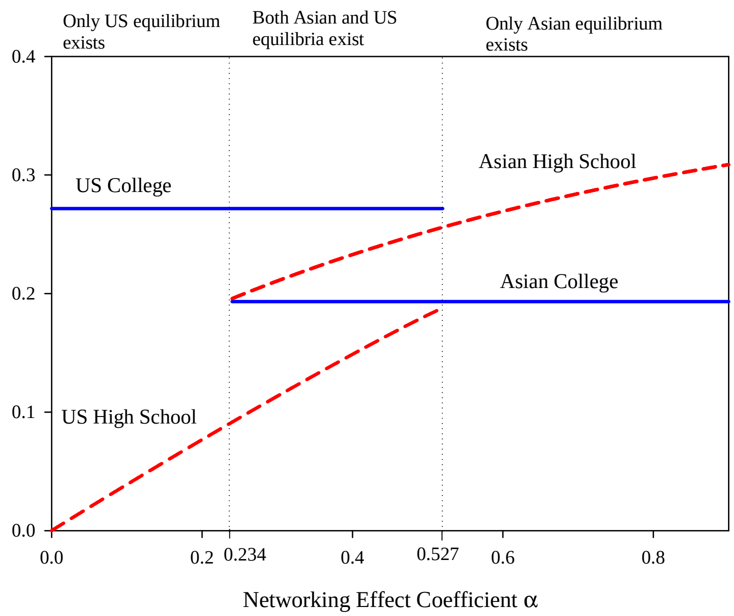

Figure 2: Networking Effect, Existence of Equilibrium, and Study Time

equilibria coexist for medium size α. In this case both the US and East Asia are interpreted as having the same parameters and the equilibrium is selected based only on the society's self-fulfilling belief. In East Asia, workers study hard in high school because firms believe that college names are better signals of abilities; college names actually become better signals of abilities just because workers compete hard in high school to enter better colleges. In the US, workers study hard in college because firms believe that college GPAs are better signals of abilities; college GPAs actually become better signals just because workers compete hard in college to get better GPAs.

The second explanation is that East Asia and the US actually have different fundamental parameters, especially the networking effect coefficient α. East Asia seems to have a higher α. Networking has been regarded as one of the most important factors to be successful in East Asia. For example, in major firms in Japan and Korea there are college alumni associations which promote the success of their members. Alumni connections are considered so important that they have their own names: "Gakubatsu" in Japan and "Hakbul" in Korea.

6 Empirical Evidence

This section provides empirical evidence. A testable implication of the theory is that high ability workers in East Asia should be relatively more concentrated among a few top colleges than their counterparts in the US. I examine this implication using college distribution data for the largest firms' CEOs in the US and Korea.

The theory claims that college names are better signals of workers' abilities in East Asia than in the US. Its immediate implication is that students within an average US college should be more heterogeneous than students within an average East Asian college: US equilibrium has all three ability types within each college while Asian equilibrium has only two ability types. Although this conjecture seems quite plausible, this implication is hard to test due to data availability.

An equivalent testable implication is that high ability workers should be more concentrated in a few colleges in East Asia than in the US: Asian equilibrium has high ability workers only in college A while US equilibrium has high ability workers in both colleges. I examine this implication by looking at college distribution of top CEOs for the largest firms in the US and Korea. Clearly these CEOs are high ability workers and I find that Korean CEOs are substantially more concentrated among a few top colleges as compared with their US counterparts.

Data come from several sources. I look at top CEOs of 2004 Standard and Poors 500 firms in the US and 2003 Hankyung business ranking top 81 firms in Korea. Some CEOs did not graduate from colleges and some firms have co-CEOs. I excluded those who did not graduate from colleges and included both co-CEOs in the sample. This leaves me with 494 US CEOs for the US and 82 Korean CEOs. I use 6 times as many US CEOs as Korean CEOs because the US is 6 times bigger in population. I also control for differences in alumni size across colleges, using freshmen enrollment size as its proxy. I use 2003 freshmen enrollment data for each college from the National Center for Education Statistics (NCES) and the Korea National Center for Education Statistics and Information (KNCESI).

Table 4 shows top US and Korean universities in terms of the number of CEOs among their

Ranking	University	Number of CEOs	Cumulative CEO Percentage
1	Harvard Univ.	15	3%
2	Univ. of Wisconsin – Madison	14	6%
3	Princeton Univ.	10	8%
	Stanford Univ.	10	10%
5	Univ. of Texas – Austin	9	12%
6	Yale Univ.	8	13%
7	US Naval Academy	7	15%
	Univ. of Pennsylvania	7	16%
	Univ. of Washington – Seattle	7	17%

(a) US

Ranking	University	Number of CEOs	Cumulative CEO Percentage
1	Seoul National Univ.	39	48%
2	Korea Univ.	12	62%
3	Yonsei Univ.	10	74%

(b) Korea

Table 4: University Ranking in Number of Top CEOs

graduates. It is striking that Seoul National University alone accounts for 48 percent of the Korean CEOs. The top three colleges in Korea account for as much as 74 percent of the CEOs. On the other hand, the top ten US colleges all together account for only 17 percent of the CEOs.

However, Table 4 can be misleading for the following two reasons. First, Korea has a smaller number of college graduates, and thus universities of the same size would account for a bigger share of CEOs in Korea. Second, Korean universities may tend to be bigger in alumni size than US universities and thus each university may account for a bigger share of CEOs. In order to control for these differences, I derive something similar to the Lorenz curve, weighted by school size. First, I calculate per capita number of CEOs for each school by dividing the number of CEOs by year 2003 freshmen enrollment, which I use as proxy for alumni size. Second, I rerank schools by the per capita number of CEOs and calculate cumulative percentages of CEOs and alumni size (freshmen enrollment).

Figure 3 shows the result. The figure clearly shows that Korean CEOs are substantially more

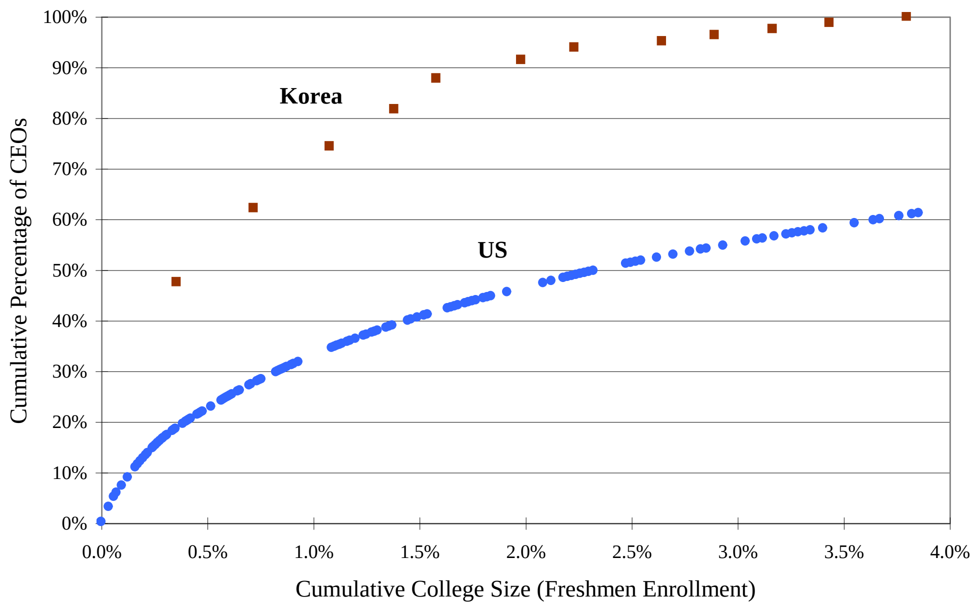

Figure 3: CEO Concentration among Colleges

concentrated among a few top colleges as compared with US CEOs. The curve for Korea is placed uniformly higher than the curve for the US. For example, the top university in Korea (Seoul National University), which accounts for only 0.4 percent of all college graduates, accounts for 48 percent of the CEOs. The top three Korean colleges, which account for 1 percent of college graduates, account for 74 percent of the CEOs. On the other hand, the top 0.4 percent of US universities account for only 19 percent and the top 1 percent of US colleges account for less than 40 percent.

7 Discussion

There may be other explanations for the puzzle of why American students work harder in college than in high school while East Asian students work harder in high school than in college. This section discusses these alternative explanations and explains how they are related to my theory.

7.1 Cultural difference

The difference in study effort pattern between the US and East Asia may be due to cultural difference between the two regions: East Asian parents may be more interested in their children's education than US parents, and this makes East Asian students work harder in high school than in college because parents have more control over their children in high school than in college. This argument is plausible, but leads to another question of how the cultural difference appeared and could have been sustained as an equilibrium.

This paper provides a microeconomic foundation for this cultural difference: East Asian parents are obsessive about their children's high school performance because they know that college names matter tremendously in their children's life. In this respect, this paper is related to the literature on the microfoundations of cultural effects (Cole et al. (1992), Cozzi (1998), Fang (2000)).

7.2 Institutional difference

The difference in study effort pattern may be due to institutional difference across countries regarding college admission. First, the US may have a relatively bigger number of seats in colleges, and this may makes college admission less competitive so students work less hard in high school. It is well known that the US has one of the highest college enrollment rates in the world: A survey by UNESCO (2005) shows that the US college enrollment rate was 81 percent in the years 2002-2003. However, this is also true for some East Asian countries. The same survey shows that the South Korean college enrollment rate was 85 percent in the same period.[6]

Second, US colleges may use more diverse admission criteria besides academic performance as compared with East Asian colleges, and this makes US high school students spend less time studying than their East Asian peers. This argument may explain why East Asian students work harder in high school than US students. However, the admission criteria argument alone does not explain why American students start working harder in college. Moreover, the argument does not explain the higher concentration of high ability workers (top CEOs) among a few colleges in East

[6] Japan has a substantially lower college enrollment rate at 49 percent.

Asia.

8 Conclusion

This paper proposes a signaling explanation for the puzzle of why American students study more in college than in high school while East Asian students study more in high school than in college. Signaling occurs over time both in high school and in college, and the timing of signaling may differ across countries. Students work hard in the signaling stage determined by the society as a whole. The model also shows why a signaling stage may differ across countries. The signaling stage is likely to be high school if networking is important for job performance. A testable implication of the theory is that East Asia has a greater concentration of high ability workers among a few top colleges, as compared with the US. I confirm this implication using college education data for the largest firms' top CEOs.

A Appendix: Proof of Lemma 1

A.1 Preliminary Results

Lemma A 1 *(i)* $C_A(x;\alpha)$ *is continuous and increasing in* x *for all* $\alpha > 0$*, and* $C_A(x;\alpha)$ *is continuous and increasing in* α *for all* $x > 0$*.*
(ii) $\lim_{x \searrow 0} C_A(x;\alpha) = 0$ *for all* $\alpha > 0$*, and* $\lim_{\alpha \searrow 0} C_A(x;\alpha) = 0$ *for all* $x > 0$*.*

Proof. *(i)* C_A is uniquely determined by conditions (4) and (5). $C_A(x;\alpha)$ is continuous in x because F and $\gamma^i(x, C_A)$ are continuous. It follows from condition (4) that $\gamma^i(x, C_A)$ is increasing in x and decreasing in C_A. In order to satisfy the condition (5), C_A has to increase when x increases. Therefore, C_A is increasing in x. Similar arguments show that for all $x > 0$ $C_A(x;\alpha)$ is continuous and increasing in α.

(ii) Suppose that $\lim_{x \searrow 0} C_A(x;\alpha) \neq 0$ for some $\alpha > 0$. It follows from condition (4) that $\lim_{x \searrow 0} \tilde{\gamma}^i(x, C_A) = 0$ for $i = H, M, L$, and therefore $\lim_{x \searrow 0} \sum_{i=H,M,L} F\left(\tilde{\gamma}^i(x, C_A)\right) = 0$. This is a

contradiction to condition (5). Similar arguments show that $\lim_{\alpha \searrow 0} C_A(x;\alpha) = 0$ for all $x > 0$. ■

Lemma A 2 *For all $\alpha > 0$, both $\tilde{\gamma}^H(x;\alpha)/\tilde{\gamma}^M(x;\alpha)$ and $\tilde{\gamma}^M(x;\alpha)/\tilde{\gamma}^L(x;\alpha)$ are increasing in x.*

Proof. It follows from condition (4) that

$$\frac{\tilde{\gamma}^H(x;\alpha)}{\tilde{\gamma}^M(x;\alpha)} = \frac{v(0) - v\left(C_A/\theta^M\right)}{v(0) - v\left(C_A/\theta^H\right)}.$$

In order to show that $\tilde{\gamma}^H(x;\alpha)/\tilde{\gamma}^M(x;\alpha)$ is increasing in x, it suffices to show that $\left\{v(0) - v\left(C_A/\theta^M\right)\right\} / \left\{v(0) - v\right.$
is increasing in C_A because C_A is an increasing function in x due to part (i) of Lemma A1.

$$\begin{aligned}
\frac{\partial}{\partial C_A}\frac{v(0) - v\left(C_A/\theta^M\right)}{v(0) - v\left(C_A/\theta^H\right)} &= \left\{\begin{array}{c} v(0) - v\left(C_A/\theta^M\right) \cdot v'\left(C_A/\theta^H\right) \cdot \frac{1}{\theta^H} \\ -v(0) - v\left(C_A/\theta^H\right) \cdot v'\left(C_A/\theta^M\right) \cdot \frac{1}{\theta^M} \end{array}\right\} / v(0) - v\left(C_A/\theta^H\right)^2 \\
&> 0
\end{aligned}$$

The last line comes because $v(0) - v\left(C_A/\theta^H\right) > v(0) - v\left(C_A/\theta^M\right) > 0$, $v'\left(C_A/\theta^M\right) < v'\left(C_A/\theta^H\right) < 0$ ($\because$ v is concave) and $1/\theta^M > 1/\theta^H > 0$. Similar arguments show that $\tilde{\gamma}^M(x;\alpha)/\tilde{\gamma}^L(x;\alpha)$ are increasing in x. ■

Lemma A 3 (i) *For all $\alpha > 0$, $\lim_{x \searrow 0} \tilde{\gamma}^H(x;\alpha)/\tilde{\gamma}^M(x;\alpha) = \theta^H/\theta^M$ and $\lim_{x \searrow 0} \tilde{\gamma}^M(x;\alpha)/\tilde{\gamma}^L(x;\alpha) = \frac{\theta^M}{\theta^L}$.*
(ii) *For all $x > 0$, $\lim_{\alpha \searrow 0} \tilde{\gamma}^H(x;\alpha)/\tilde{\gamma}^M(x;\alpha) = \theta^H/\theta^M$ and $\lim_{\alpha \searrow 0} \tilde{\gamma}^M(x;\alpha)/\tilde{\gamma}^L(x;\alpha) = \theta^M/\theta^L$.*

Proof. (i)

$$\lim_{x \searrow 0} \frac{\tilde{\gamma}^H}{\tilde{\gamma}^M} = \lim_{C_A \searrow 0} \frac{v(0) - v\left(C_A/\theta^M\right)}{v(0) - v\left(C_A/\theta^H\right)} = \frac{\theta^H}{\theta^M}$$

The first equation comes from condition (4) and Lemma A1. The second equation comes from l'Hôpital's rule. Similar arguments show $\lim_{x \searrow 0} \tilde{\gamma}^M/\tilde{\gamma}^L = \theta^M/\theta^L$. Part (ii) can be proved in the same way. ■

Lemma A 4 *For all $\alpha > 0$, $\lim_{x \searrow 0} \tilde{\gamma}^L(x;\alpha) > \underline{\gamma}$ and $\lim_{x \searrow 0} \tilde{\gamma}^H(x;\alpha) < \bar{\gamma}$.*

Proof. Suppose that $\lim_{x \searrow 0} \tilde{\gamma}^L(x) \leq \underline{\gamma}$. It implies that no low ability workers exist in college A for x near 0. Since the capacity of college A is 1.5, college A has to have at least a half-unit measure of medium ability workers. Thus, $\lim_{x\searrow 0}\tilde{\gamma}^{M}$ has to be weakly greater than γ_m where $F(\gamma_m) = 0.5$, and I obtain

$$\lim_{x\searrow 0}\frac{\tilde{\gamma}^{M}(x;\alpha)}{\tilde{\gamma}^{L}(x;\alpha)} \geq \frac{\gamma_m}{\underline{\gamma}}.$$

However, using part (i) of Lemma A3 and Assumption 1 I obtain the following contradicting result.

$$\lim_{x\searrow 0}\frac{\tilde{\gamma}^{M}(x;\alpha)}{\tilde{\gamma}^{L}(x;\alpha)} = \frac{\theta^M}{\theta^L} < \frac{\gamma_m}{\underline{\gamma}}.$$

Similar arguments show that $\lim_{x\searrow 0}\tilde{\gamma}^{H}(x) < \bar{\gamma}$. ■

Lemma A 5 *There exists* $\gamma_0^i \in (\underline{\gamma}, \bar{\gamma})$ *such that* (a) $\lim_{x\searrow 0}\tilde{\gamma}^{i}(x;\alpha) = \gamma_0^i$ *for all* $\alpha \geq 0$ (b) $\lim_{\alpha\searrow 0}\tilde{\gamma}^{i}(x;\alpha) = \gamma_0^i$ *for all* $x > 0$ $(i = H, M, L.)$

Proof. Part (i) of Lemma A3 implies that $\lim_{x\searrow 0}\tilde{\gamma}^{L}(x;\alpha) < \lim_{x\searrow 0}\tilde{\gamma}^{M}(x;\alpha) < \lim_{x\searrow 0}\tilde{\gamma}^{H}(x;\alpha)$ for $\alpha > 0$. Therefore, Lemma A4 implies that $\underline{\gamma} < \gamma_0^i \equiv \lim_{x\searrow 0}\tilde{\gamma}^{i}(x;\alpha) < \bar{\gamma}$ for $\alpha > 0$ $(i = H, M, L.)$ Lemma A3 also implies that the limiting ability distribution is identical whether x converges to 0 with α fixed or α converges to 0 with x fixed. Therefore, $\lim_{x\searrow 0}\tilde{\gamma}^{i}(x;\alpha)$ with $\alpha > 0$ fixed is equal to $\lim_{\alpha\searrow 0}\tilde{\gamma}^{i}(x;\alpha)$ with $x > 0$ fixed $(i = H, M, L.)$ ■

A.2 Proof for part (i) of Lemma 1

Proof. Lemma A5 encompasses part (i) of Lemma 1. ■

A.3 Proof for part (ii) of Lemma 1

Proof. C_A is continuous in x according to Lemma A1. I thus obtain from condition (4) that $\tilde{\gamma}^{i}(x;\alpha)$ is continuous in x $(i = H, M, L.)$ It follows from condition (6) that ψ is continuous in x. Lemma A2 implies that there will be weakly more higher ability workers in college A relative to lower ability workers as x increases. Therefore, ψ is weakly increasing in x. ■

A.4 Proof for part (iii) of Lemma 1

Proof. Part (iii) of Lemma 1 directly follows from Lemma A5. ■

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