Strategic Voting over Strategic Proposals[1]

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Abstract

Prior research on "strategic voting" has reached the conclusion that unanimity rule is uniquely bad: it results in destruction of information, and hence makes voting individuals worse off. We show that this conclusion depends critically on the assumption that the issue being voted on is independent of the voting rule used, i.e., is exogenous. We depart from the existing literature by endogenizing the proposal that is put to a vote, and establish that under many circumstances unanimity rule makes voters better off. Moreover, in some cases unanimity *also* makes the proposing individual better off, and is the Pareto dominant voting rule. Voters prefer unanimity rule because it induces the proposing individual to make a more attractive proposal; the proposing individual may also benefit because the acceptance probabilities for moderately favorable proposals are higher.

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1 Introduction

Many collective decisions are made by holding a vote over an endogenously determined agenda. Examples include debt restructuring negotiations between a troubled company and its bondholders; congressional votes over presidential appointments in the U.S. and elsewhere; shareholder votes on executive compensation; and collective bargaining between a firm and union members. The voting rules used for different decisions differ, and the choice of voting rule has two consequences. First, the voting rule affects whether a given proposal is adopted. Second, the voting rule affects the proposal that is being voted over.

A large and influential recent literature has analyzed voting when individuals have different information.[1] This "strategic voting" literature has dealt exclusively with the first consequence of the voting rule — whether a given proposal is adopted — and reached the conclusion that unanimity rule is inferior to majority rule.[2,3] Specifically, while majority rule aggregates information efficiently when the number of voters is large enough, unanimity rule always results in mistaken decisions. As such, when the issue being voted over is exogenous, unanimity rule is a suboptimal voting rule, and reduces the expected payoff of voting individuals.

Nevertheless, in practice unanimity rule is employed in many settings. For example, under the Trust Indenture Act of 1939 outstanding debt can be restructured only if all creditors agree. Likewise, promotion decisions in a number of professions require unanimous approval, as do the decisions of many international organizations. The

[1] See, for example, Austen-Smith and Banks (1996), Feddersen and Pesendorfer (1996, 1997, 1998), McLennan (1998), Duggan and Martinelli (2001), Doraszelski *et al* (2003), Persico (2004), Yariv (2004), Martinelli (2005), Meirowitz (2005), Gerardi and Yariv (forthcoming).

[2] By majority rule, we mean any threshold voting rule: that is, a proposal is accepted if the fraction voting to accept exceed a pre-specified threshold.

[3] The main exception is Coughlan (2000), who shows that if pre-vote communication is possible and voter preferences are both common knowledge and closely aligned, then both unanimity and majority rules may allow efficient aggregation of information. However, Austen-Smith and Feddersen (2006) show that if voter preferences are not common knowledge then unanimity is again the inferior voting rule from the perspective of information aggregation. Additionally, even Coughlan does not argue that unanamity rule is strictly superior to majority rule in the standard two-alternative voting game.

results of the aforementioned voting literature suggest that a majority vote would be more efficient in such settings.

In this paper we show that the conclusion that majority voting rules are superior depends critically on the assumption that the proposal being voted over is exogenous. We do so by studying the second consequence of the voting rule mentioned above, namely that it affects the proposal being voted upon. We show that under many circumstances unanimity rule increases the expected utility of voting individuals, because it induces the proposing individual to make a more attractive offer.[4] Further, in a subset of such circumstances unanimity rule is Pareto superior, because it also increases the proposing individual's expected utility — even though we model his interests as being diametrically opposed to those of the voting individuals.

Specifically, we consider the following setting. One individual — the *proposer* — makes a take-it-or-leave-it offer to a large group. The group must collectively decide whether to accept or reject the offer, and we assume that it does so by holding a vote. The fraction of votes required to accept the proposer's offer is fixed prior to the offer (either by law, by contract, or by the common consent of group members). As such, when the proposer makes his offer he takes the voting rule used by the group as given. The main assumption we make regarding preferences is that offers can be totally ordered, with voters preferring higher offers and the proposer preferring lower offers. That is, the proposer and voter have opposing interests, as is the case in many (though certainly not all) voting situations. We study the expected equilibrium payoffs when the number of voting individuals is large.

As one would expect, and regardless of the agreement rule, the acceptance probability is increasing in the attractiveness of the offer to the voting individuals. Consequently, the proposer faces a trade-off between a high offer that is accepted more often and a low offer that is accepted less often. Equilibrium offers are determined by this trade-off.

[4]For promotion decisions, the issue being voted upon is effectively the candidate's performance over the evaluation period, which is certainly endogenous.

As in the prior literature, the group (asymptotically) makes the correct decision under majority rule but makes mistakes under unanimity rule. In particular, voters reject low offers more often than they should, and accept high offers more often than they should.

Provided the proposer's payoff under disagreement is not too low, the mistakes that arise under unanimity benefit the group members. In this case, when facing a group using majority rule, the proposer is not willing to make a high offer, but rather prefers a smaller offer accepted less often. Since the group makes mistakes under unanimity by rejecting low offers more often than it should, the proposer needs to make an offer that is higher than he would make under majority rule.

Even though the group receives a better offer from the proposer when the proposer's payoff under disagreement is not too low, it is still not obvious whether the group prefers unanimity rule or majority rule. The reason is that the higher offer is made as a direct consequence of the group's mistakes. However, we establish that a form of the envelope theorem holds in voting environments. As such one can evaluate the effect of the higher offer simply by considering the direct effect, which is of course positive. It follows that the group's expected utility — as well as the equilibrium offer — is higher under unanimity rule.

Moreover, and surprisingly, when the proposer's payoff is neither too high nor too low, the proposer *also* prefers unanimity rule, making it the Pareto dominating voting rule. The key to this result is that against a group using unanimity rule the proposer is able to get a moderate offer accepted with very high probability. In contrast, as described above the proposer's best offer against majority rule is a lower offer that is accepted with a lower probability. In this case group members prefer unanimity because they get a higher offer than they would under majority, and the proposer prefers unanimity because he can secure acceptance more often than he could under majority rule at a cost he is willing to bear.

Overall, our results highlight the importance of the endogeneity of the agenda in the determination of optimal voting rules. While unanimity rule is inferior when the offers are exogenous, it may Pareto dominate all other voting rules once the agenda is endogenous.

Related Literature

As discussed above, our paper develops the strategic voting literature by endogenizing the issue being voted over. This literature studies how differentially informed individuals vote over an exogenously specified agenda by explicitly taking into account that a vote only matters when it is pivotal, and so each voter should condition on the information implied by him being pivotal. In particular, it is not an equilibrium for each voter to vote sincerely, i.e., purely according to his own information (Austen-Smith and Banks 1996). When the number of voters is large, in equilibrium information is nonetheless aggregated efficiently in a majority voting game. In contrast, unanimity rule does not lead to efficient information aggregation, and therefore results in mistakes (see Feddersen and Pesendorfer 1997, 1997, 1998, and also Duggan and Martinelli 2001). Given these results, one might be tempted to conclude that unanimity rule is inefficient, and in particular, hurts the voters. Our results show that neither is true when the agenda being voted on is endogenous.

In addition, our paper is also related to the extensive recent literature on multilateral bargaining, in which more than two agents must agree on the division of a pie.[5] However, in many negotiations a proposal must treat all members of some group equally, either for technological reasons (e.g., the building of a bridge), or for institutional/legal reasons (e.g., wage determination, debt restructuring). The literature analyzing this important class of bargaining problems is much smaller — see Banks and Duggan (2001), Cho and Duggan (2003), Cardona and Ponsati (2005), and Manzini and Mariotti (2005). These papers are deterministic complete information models, and as such, informational issues do not arise. Moreover, since agreement is always reached, there is no risk of breakdown of agreement from having a "tougher" bargaining stance. In contrast, the possibility

[5] The classic paper is Baron and Ferejohn (1989).

of failing to agree to a Pareto improving proposal is central to our analysis and results. Finally, Chae and Moulin (2004) provide a family of solutions to group bargaining from an axiomatic viewpoint. Elbittar *et al* (2004) provide experimental evidence that the choice of voting rule used by a group in bargaining affects outcomes.

In our model, bargaining takes place under two-sided asymmetric information. The literature on bilateral bargaining under asymmetric information is extensive.[6] We add to this literature by considering how the internal organization of one of the parties affects equilibrium outcomes.

PAPER OUTLINE

The paper proceeds as follows. In Section 2 we illustrate the main results and intuition through an example. We formalize and generalize this example in the following sections. Section 3 describes the model. Sections 4 and 5 establish equilibrium existence and characterize basic equilibrium properties. Section 6 bounds the equilibrium outcomes of the bargaining game when the group uses unanimity rule. Section 7 conducts the same exercise when the group adopts majority rule. Section 8 compares outcomes and payoffs from different voting rules. Section 9 concludes. All proofs are in the Appendix.

2 An example

To illustrate the main results of the paper, it is useful to consider the following example. A firm, which is initially wholly owned by a single individual (the *debtor*), seeks to restructure its outstanding debt by offering a group of creditors a share of its future cash flow. If the creditors accept the offer, the debtor continues to run the firm — in which case he receives a utility equivalent to \$120, in addition to his share of firm cash flows. If instead creditors decline the offer, they liquidate the firm and obtain \$100, while the debtor receives nothing, but derives a utility of $\bar{V} < \$120$ from his outside option.

[6]See Kennan and Wilson (1993) for a review. Of most relevance for our paper are Samuelson (1984), Chatterjee and Samuelson (1987), Evans (1989), Vincent (1989), and Schweizer (1989), all of which study common values environments.

The future cash flow of the firm (if not liquidated) is uncertain: it is either \$100 or \$200, with *ex ante* equal probability. Each creditor possesses private information about the relative likelihood of the two valuations. To keep the example as transparent as possible, we assume the debtor has no private information about the future cash flow. We relax this assumption in our formal model below.

Assume that the number of creditors is large, and consider first the case in which the creditors use a majority rule. Since majority rule aggregates information efficiently[7] the debtor's choice boils down to the following: either he can offer creditors 1/2 of the future cash flow, so that they accept whenever the true cash flow of the firm is \$200; or he can offer creditors all the future cash flow, and gain acceptance all the time. Under the former alternative, the debtor's expected payoff is $\frac{1}{2}(100 + 120) + \frac{1}{2}\bar{V}$, while the latter alternative yields a payoff of 120. Consequently, if the creditors are using majority rule, the debtor offers all the firm's cash flows if his outside option is low enough — specifically, if $\bar{V} < 20$ — and half of the cash flows otherwise. His expected utility is 120 in the former case, and $110 + \frac{\bar{V}}{2}$ in the latter case, whereas creditors' expected payoff is $\frac{1}{2}200 + \frac{1}{2}100 = 150$ in the former case, and $\frac{1}{2}100 + \frac{1}{2}100 = 100$ in the latter case.

Consider now the case in which creditors instead use unanimity rule. Suppose also that the most negative signal received by an individual creditor, denoted by $\underline{\sigma}$, is twice as likely to be received when the true state is L than H, and so, the probability of state H conditional on $\underline{\sigma}$ is 1/3. This implies that when the debtor offers 3/4 of the cash flows to the creditors, voting to accept irrespective of one's own signal when everyone else is using this strategy is a best response. This is because when all creditors vote to accept all the time, being pivotal does not convey any information. As such, given our assumption about the information content of the most negative signal, conditional on all other creditors accepting, a creditor who observes the most negative signal is indifferent

[7] See references above.

between accepting and rejecting the offer: if he accepts, the offer is accepted, and he gets

$$\frac{1}{3} \times \frac{3}{4} \times 200 + \frac{2}{3} \times \frac{3}{4} \times 100 = 100,$$

while if he rejects the offer is rejected and creditors get 100 in liquidation.

Given that creditors always accept an offer of 3/4 of the cash flows, facing unanimity the debtor would not offer more than this amount. Moreover, given his outside option $\bar{V} < 120$ he clearly prefers offering 3/4 of the cash flows and gaining certain acceptance to offering nothing and ensuring certain rejection. Numerical simulations show that in this example the debtor's payoff is convex in his offer,[8] and so the debtor's best offer under a unanimity voting rule is 3/4. As a result, the debtor's expected payoff is $120 + \frac{1}{2}50 + \frac{1}{2}25 = 157.5$, whereas the creditors' unconditional expected payoff is $\frac{1}{2}150 + \frac{1}{2}75 = 112.5$ for all $\bar{V}$.

Summarizing, under majority rule, the debtor offers all the cash flows to the creditors if his utility from the outside option is low enough; otherwise, he offers half of the cash flows. When he offers all the cash flows, creditors accept with probability one, while when he offers half of the cash flows creditors accept only when the true cash flow is high. Under unanimity, regardless of his utility from the outside option, he offers a fraction 3/4 of the cash flows to the creditors, and this offer is accepted all the time. It follows that, when the debtor's utility from his outside option is high enough ($\bar{V} > 20$), the creditors prefer unanimity, but when the debtor's utility from his outside option is low ($\bar{V} < 20$) the creditors prefer majority. In contrast, the debtor prefers unanimity when his utility from his outside option is low enough ($\bar{V} < 95$), and he prefers majority when his utility from his outside option is high ($\bar{V} > 95$).

Interestingly, it is possible that both the debtor and the creditors prefer unanimity rule to any majority rule. This happens when the debtor's outside option takes an intermediate value ($20 < \bar{V} < 95$). The creditors prefer the unanimity rule because

[8] More generally, the proposing agent's payoff is convex in the offer whenever the signal quality of each voting agent is sufficiently small — see Lemma B-1 in Appendix B, available on the authors' webpages.

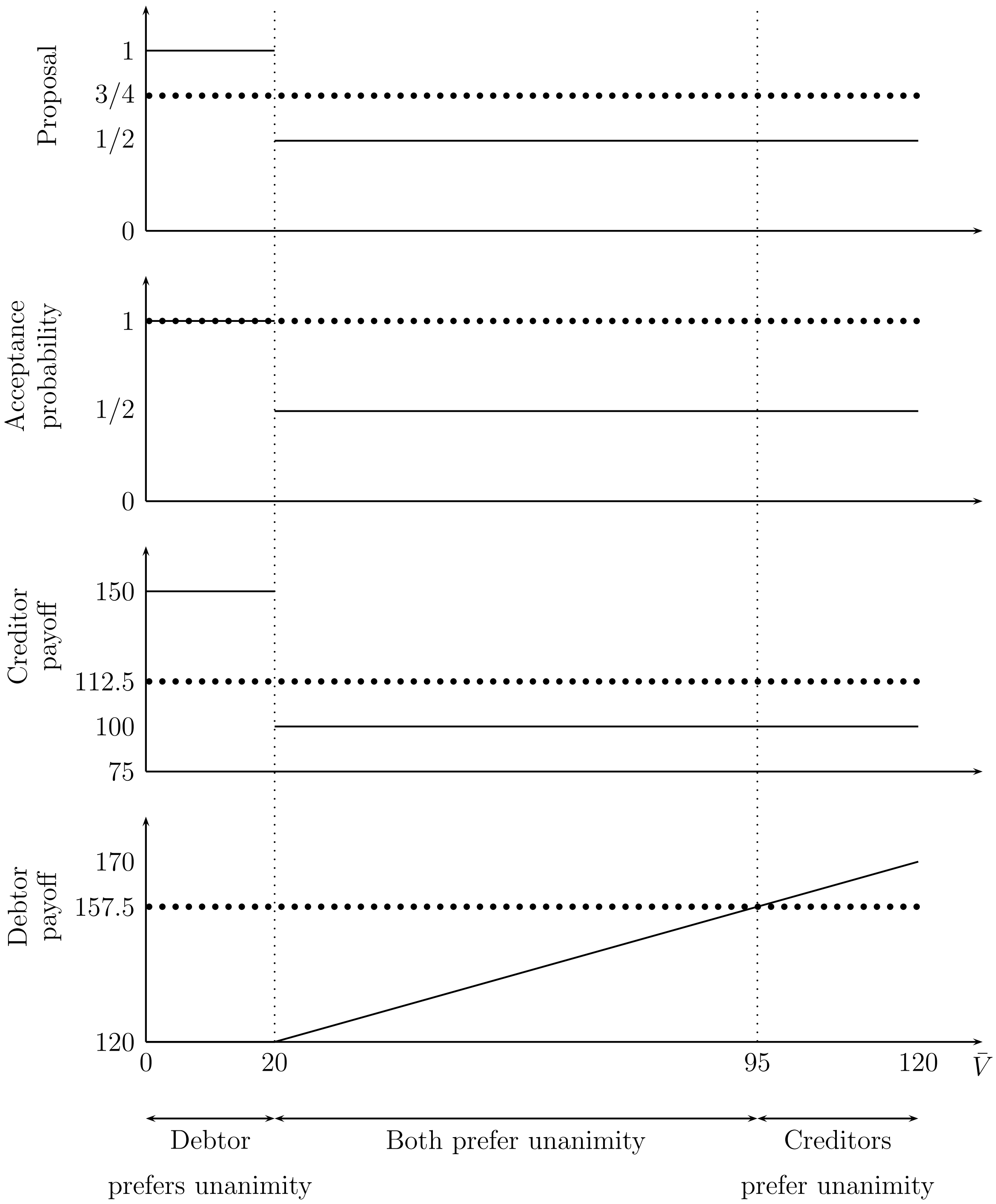

Figure 1: The graphs plot the equilibrium proposal, acceptance probability and expected payoffs. The solid lines correspond to majority rule while the dotted lines correspond to unanimity rule.

it helps to get a higher offer relative to majority rule. The debtor prefers unanimity rule because he can get acceptance all the time at a smaller cost than he could under majority rule — the reason being that under unanimity rule, creditors mistakenly accept moderately high offers too often.

In the remainder of the paper we generalize this example and formally establish the claims made above. In particular, relative to this example, we:

- establish these results formally: A major (and unsatisfactory) shortcut in our presentation of the example is that we considered the offers made in response to the acceptance probabilities in an economy with infinitely many voters. Below, we instead first characterize the equilibrium set for a finite economy, and then take the limit. Doing so requires us to characterize the convergence properties of the acceptance probability function.

- establish these results for a fairly general specification of preferences: One particular property of preferences in the example is that the interests of creditors are completely aligned (pure common values). In our analysis we relax this assumption by working with preferences which allow for both private and common values. We establish all our results for preferences that are sufficiently close to pure common values. In doing so, we show that there is no discontinuity at the common values extreme.

- generalize the results to the case in which the proposing agent has some information: In the example, we assumed that the debtor has no information about the relative likelihood of the two states. In our analysis below we do not make this assumption. This introduces a signalling aspect to the game.

- establish that the envelope theorem holds in the voting game we analyze — even though the voting outcome reflects the decisions of many different voters. This step is necessary in order to show that voters prefer higher offers in spite of the changing incidence of mistakes.

3 Model

There is a single proposer (agent 0), and a group of $n \geq 2$ responders, labelled $i = 1, \ldots, n$. The timing is as follows: (1) Each agent $i \in \{0, 1, \ldots, n\}$ privately observes a random variable $\sigma_i \in [\underline{\sigma}, \bar{\sigma}]$. As we detail below, the realization of σ_i affects agent i's preferences and/or information. (2) The proposer selects a proposal $x \in [0, 1]$. (3) Responders simultaneously cast ballots to accept or reject the proposal. (4) If at least a fraction α of the responders vote to accept,[9] the proposal is implemented, while otherwise the status quo prevails. We take the voting rule α to be exogenously given:[10] in particular, it cannot be changed after the proposer makes his offer. Common examples include the simple majority rule, $\alpha = 1/2$; the supermajority rule, $\alpha = 2/3$; and the unanimity rule, $\alpha = 1$.

Preferences

Agent i's relative preferences over the proposal x and the status quo are determined by σ_i and an unobserved state variable $\omega \in \{L, H\}$. The probability of state ω is p^ω. We write responder i's utility associated with offer x as $U^\omega(x, \sigma_i, \lambda)$, where $\lambda \in [0, 1]$ is a parameter that describes the relative importance of ω and σ_i. We assume that $U^\omega(x, \sigma_i, \lambda)$ is independent of σ_i at $\lambda = 0$, and $U^L(\cdot, \cdot, \lambda) \equiv U^H(\cdot, \cdot, \lambda)$ at $\lambda = 1$. Likewise, we write $\bar{U}^\omega(\sigma_i, \lambda)$ for responder i's utility under the status quo, and make parallel assumptions for $\lambda = 0, 1$. As such, our framework includes pure *common values* ($\lambda = 0$) and pure *private values* ($\lambda = 1$) as special cases. A key object in our

[9] Throughout, we ignore the issue of whether or not $n\alpha$ is an integer. This issue could easily be handled formally by replacing $n\alpha$ with $[n\alpha]$ everywhere, where $[n\alpha]$ denotes the smallest integer weakly greater than $n\alpha$. Since this formality has no impact on our results, we prefer to avoid the extra notation and instead proceed as if $n\alpha$ were an integer.

[10] Since our main results characterize the proposer's and responders preferences over different voting rules, it would be straightforward to endogenize the choice of voting rule by having either the responders or proposer select it at an *ex ante* stage before σ_i is realized.

analysis is the utility of a responder from the proposal above and beyond the status quo. Accordingly, we define

$$\Delta^{\omega}(x, \sigma_i, \lambda) \equiv U^{\omega}(x, \sigma_i, \lambda) - \bar{U}^{\omega}(\sigma_i, \lambda).$$

Similarly, we write the proposer's utility from having his offer accepted as $V^{\omega}(x, \sigma_0)$, and his utility under the status quo as $\bar{V}^{\omega}(\sigma_0)$. Note that we do not require the relative weights of ω and σ_0 in determining the proposer's preferences to match the relative weights (given by λ) of ω and σ_i in determining responder i's preferences.

For all preferences $\lambda < 1$, the realization of σ_i provides responder i with useful (albeit noisy) information about the unobserved state variable ω. We assume that the random variables $\{\sigma_i : i = 0, 1, \ldots, n\}$ are independent conditional on ω, and that except for σ_0 (which is observed by the proposer) are identically distributed. Let $F(\cdot|\omega)$ and $F_0(\cdot|\omega)$ denote the distribution functions for the responders and proposer respectively. We assume that both distributions have associated continuous density functions, which we write $f(\cdot|\omega)$ and $f_0(\cdot|\omega)$. We let $\ell(\sigma)$ denote the likelihood ratio $\frac{f(\sigma|H)}{f(\sigma|L)}$, and $\ell_0(\sigma)$ denote the likelihood ratio $\frac{f_0(\sigma|H)}{f_0(\sigma|L)}$. The realization of σ_i is informative about ω, in the sense that the monotone likelihood ratio property (MLRP) holds strictly;[11] but no realization is perfectly informative, i.e., $\ell(\underline{\sigma}) > 0$ and $\ell(\bar{\sigma}) < \infty$, with similar inequalities for ℓ_0. We denote the probability of state ω conditional on σ_i by $p^{\omega}(\sigma_i)$.

Interpretations

Possible interpretations of the model include the following:

(A) An indebted firm offers n creditors an equity stake x in exchange for the retirement of existing debt claims. If the creditors reject the offer the firm is liquidated. Let $\frac{1}{n}U^{\omega}(x, \sigma_i, \lambda)$ be the value of an x/n share to creditor i, $\frac{1}{n}\bar{U}^{\omega}(\sigma_i, \lambda)$ be the value of receiving $1/n$ of the liquidation value,[12] $V^{\omega}(x, \sigma_0)$ be the debtor's valuation of the remaining $1 - x$ share if his offer is accepted, and $\bar{V}^{\omega}(\sigma_0)$ his payoff in liquidation.

[11] That is, $\ell(\sigma)$ and $\ell_0(\sigma)$ are strictly increasing in σ.

[12] These preferences are isomorphic under any monotone transformation, and in particular, multiplication by n.

(B) An employer is in wage negotiations with n workers. He offers a wage x, which worker i values at $U^{\omega}(x, \sigma_i, \lambda)$. If the offer is rejected, workers strike: $\bar{U}^{\omega}(\sigma_i, \lambda)$ is worker i's expected payoff from the strike. The firm's total profits if the offer is accepted are $nV^{\omega}(x, \sigma_0)$, and its expected total profits if a strike ensues are $n\bar{V}^{\omega}(\sigma_0)$.

(C) A president proposes a policy x.[13] The proposal is adopted only if passed by the legislature. This requires the support of a sufficient fraction of legislators from the opposing party to the president.

Equilibrium

We examine the sequential equilibria of the game just described. The proposer's strategy is a mapping from set of possible signals, $[\underline{\sigma}, \bar{\sigma}]$, to probability distributions over the offer set $[0, 1]$. Conditional on the proposer's offer, and as is standard in the strategic voting literature on which we build, we restrict attention to equilibria in which the *ex ante* identical responders behave symmetrically.[14]

Responders are potentially able to infer information about the proposer's observation of σ_0 from his offer, and thus information about the state variable ω. Since only the latter affects responders' preferences, we focus directly on the beliefs about ω after observing an offer x. Let $\beta_n(x; \lambda, \alpha)$ denote the responders' belief that $\omega = H$ after observing offer x in the game with n responders using voting rule α, and preference parameter λ.

A sequential equilibrium thus consists of an offer strategy for the proposer, a set of responder beliefs $\beta_n(\cdot; \lambda, \alpha)$ and a voting strategy $[\underline{\sigma}, \bar{\sigma}] \rightarrow \{\text{accept,reject}\}$ for each responder such that the proposer's strategy is a best response to the responders' (identical) strategies; and each responder's strategy maximizes his expected payoff given that all other responders use the same strategy, and his beliefs are $\beta_n(\cdot; \lambda, \alpha)$; and the beliefs themselves are consistent. At a minimum, belief consistency requires that having received an offer x, responders are not more (respectively, less) confident that the state is

[13] A judicial nominee, for example.

[14] Duggan and Martinelli (2001) give conditions under which the symmetric voting equilibrium is the unique equilibrium for unanimity rule.

H than the proposer himself is after he sees the most (respectively, least) pro-H signal $\sigma_0 = \bar{\sigma}$ (respectively, $\sigma_0 = \underline{\sigma}$). That is, for all offers x,

$$\frac{\beta_n(x;\lambda,\alpha)}{1-\beta_n(x;\lambda,\alpha)} \in \left[\ell_0(\underline{\sigma})\frac{p^H}{p^L}, \ell_0(\bar{\sigma})\frac{p^H}{p^L}\right]. \tag{1}$$

Consequently consistency implies that $\beta_n(x;\lambda,\alpha) \in \left[\underline{b}, \bar{b}\right]$, for some $0 < \underline{b} < \bar{b} < 1$.

Assumptions

We make the following assumptions:

Assumption 1 *Δ^ω, V and $\bar{V}$ are twice continuously differentiable in their arguments.*

Assumption 2 *$\Delta^H \geq \Delta^L$ and Δ^ω is increasing in σ_i; both relations are strict for $x > 0$.*

Assumption 3 *For all λ, $\Delta^H(0,\bar{\sigma},\lambda) < 0$ and $\Delta^H(1,\bar{\sigma},\lambda) > 0$.*

Assumption 4 *For all x, $V^\omega(x,\sigma_0) - \bar{V}^\omega(\sigma_0) \geq 0$ for $\omega = L, H$ and all σ_0.*

Assumption 5 *Δ^ω is strictly increasing and V is strictly decreasing in x.*

Assumption 1 is standard. For future reference, observe that $|\Delta^\omega|$ is bounded above since Δ^ω is continuous in its arguments and has compact domain.

Assumption 2 says responder i is more pro-acceptance when $\omega = H$ than $\omega = L$, and when the realization of σ_i is higher. Since higher values of σ_i are more likely when $\omega = H$ (by MLRP), the content of Assumption 2 (beyond being a normalization) is that the "private" and "common" components of responder utility act in the same direction.

Assumption 3 says that the responders regard the worst offer ($x = 0$) as worthless, i.e., they prefer the status quo. On the other hand, there are some offers which the responders view as worthwhile under some conditions — in particular, responder i prefers the best offer ($x = 1$) to the status quo when $\omega = H$ and $\sigma_i = \bar{\sigma}$.

Assumption 4 says that the proposer strongly dislikes the status quo relative to the range of possible alternatives: regardless of the state, he would prefer to have any proposal $x \in [0,1]$ implemented.[15]

Finally, Assumption 5 says that the proposer and responders have diametrically opposing preferences: higher x makes the responders more pro-agreement, but reduces the proposer's payoff if his proposal is accepted.

Robustness issues

In our framework, voting is the only means by which group members can share their information. When voters are numerous and dispersed, as is often the case, this is a reasonably realistic assumption. We return to this issue in more detail in the conclusion. Somewhat related, we also take as given the information possessed by group members. Other authors have modelled strategic voting games with costly information acquisition,[16] but have done so under the assumption that the agenda is independent of the voting rule, i.e., is exogenous. We leave the simultaneous integration of costly information acquisition and endogenous agendas into strategic voting for future work.

4 Equilibrium characterization and existence

In this section we establish equilibrium existence, along with a number of characterization results. We first look at voting stage of the game.

The voting stage

Fix a preference parameter λ and a number of responders n. Having observed the proposer's offer x, each responder attaches a subjective probability $b = \beta_n(x; \lambda, \alpha)$ to the state variable ω being H. A central insight of the existing strategic voting literature

[15] In general, one can clearly think of a broader range of proposals $[0, \infty)$, but with the proposer preferring the status quo to offers $x \in (1, \infty)$. The content of Assumption 4 is that $x = 1$ is the highest offer the proposer is prepared to make for *any* pair (ω, σ_0). For instance, in our debt renegotiation example, a debtor (the proposer) would prefer being left with any fraction $1-x$ of the firm to liquidation, if (as is typical) in the latter case he is left with nothing.

[16] See Persico (2004), Martinelli (2005), and Yariv (2004).

is that responder i's voting decision depends on the comparison of his expected utilities from accepting and rejecting, conditional on the event of being pivotal. Taking the strategies of other responders as given, let PIV denote the event that his vote is pivotal. Thus responder i votes to accept offer x after observing σ_i if and only if

$$E_b \left[U^\omega (x, \sigma_i, \lambda) \,|PIV, \sigma_i\right] \geq E_b \left[\bar{U}^\omega (x, \sigma_i, \lambda) \,|PIV, \sigma_i\right], \tag{2}$$

where $\Pr_b$ and E_b denote the subjective probability and expectation given b. Observe that even though responder i does not observe σ_j $(j \neq i)$, and does not know whether or not he is actually pivotal, in casting his vote he considers only the payoffs in events in which he is pivotal, and takes into account any information he can thus infer.

Since the random variables σ_i are independent conditional on ω,

$$\Pr_b (\omega|PIV, \sigma_i) = \frac{\Pr_b (\omega, PIV, \sigma_i)}{\Pr_b (PIV, \sigma_i)} = \frac{\Pr (PIV|\omega) \Pr (\sigma_i|\omega) \Pr_b (\omega)}{\Pr_b (PIV, \sigma_i)}. \tag{3}$$

Substituting (3) into inequality (2), and noting that $\Pr_b (H) = b = 1 - \Pr_b (L)$, responder i votes to accept proposal x after observing σ_i if and only if

$$\Delta^H (x, \sigma_i, \lambda) \Pr (PIV|H) f (\sigma_i|H) b + \Delta^L (x, \sigma_i, \lambda) \Pr (PIV|L) f (\sigma_i|L) (1 - b) \geq 0. \tag{4}$$

By MLRP, it is immediate from (4) that in any equilibrium each responder i follows a cutoff strategy, in the sense of voting to accept if and only if σ_i exceeds some critical level. As noted, throughout we focus on symmetric equilibria in which the *ex ante* identical responders follow the same voting strategy. Let $\sigma_n^*(x, b, \lambda, \alpha) \in [\underline{\sigma}, \bar{\sigma}]$ denote the common cutoff[17] when there are n responders, the offer is x, responders attach a probability b to $\omega = H$, and the preference parameter and voting rule are λ and α respectively. For clarity of exposition, we will suppress the arguments n, x, b, λ and α unless needed, both for σ^* and other variables introduced below.

Evaluating explicitly, the probability that a responder is pivotal is given by

$$\Pr (PIV|\omega) = \binom{n-1}{n\alpha - 1} (1 - F (\sigma^* (x) \,|\omega))^{n\alpha - 1} F (\sigma^* (x) \,|\omega)^{n - n\alpha}. \tag{5}$$

[17] As we show below, there exists a unique cutoff signal.

The acceptance condition (4) then rewrites to:

$$\begin{aligned}
&\Delta^H(x,\sigma_i,\lambda)\left(1-F\left(\sigma^*(x)|H\right)\right)^{n\alpha-1}F\left(\sigma^*(x)|H\right)^{n-n\alpha}f(\sigma_i|H)\,b \\
&+\Delta^L(x,\sigma_i,\lambda)\left(1-F\left(\sigma^*(x)|L\right)\right)^{n\alpha-1}F\left(\sigma^*(x)|L\right)^{n-n\alpha}f(\sigma_i|L)\,(1-b)\geq 0 \qquad (6)
\end{aligned}$$

If there exists a $\sigma^* \in [\underline{\sigma}, \bar{\sigma}]$ such that responder i is indifferent between accepting and rejecting the offer x exactly when he observes the signal $\sigma_i = \sigma^*$, then the equilibrium can be said to be a *responsive equilibrium.* Notationally, we represent a responsive equilibrium by its corresponding cutoff value $\sigma^* \in [\underline{\sigma}, \bar{\sigma}]$.

For a given subjective probability b that $\omega = H$, it is useful to define the function

$$Z(x,\sigma,b,\lambda,\alpha,n) \equiv b\Delta^H(x,\sigma)\,\ell(\sigma)\left(\frac{F(\sigma|H)}{F(\sigma|L)}\right)^{n-n\alpha}\left(\frac{1-F(\sigma|H)}{1-F(\sigma|L)}\right)^{n\alpha-1}+(1-b)\,\Delta^L(x,\sigma).$$

If $Z(x,\sigma)$ is positive (negative), and all but one of the responders use a cutoff strategy σ, then the remaining responder i is better off voting to accept (reject) the proposal x if he observes $\sigma_i = \sigma$. Similarly, if $Z(x,\sigma) = 0$ then there is a responsive equilibrium in which all responders use the cutoff strategy σ.

By the Theorem of the Maximum, $\max_{\sigma\in[\underline{\sigma},\bar{\sigma}]} Z(x,\sigma)$ and $\min_{\sigma\in[\underline{\sigma},\bar{\sigma}]} Z(x,\sigma)$ are both continuous in x. So we can define $\underline{x}_n(b,\lambda,\alpha)$ and $\bar{x}_n(b,\lambda,\alpha)$ that describe the range of offers for which a responsive equilibrium exists:[18]

$$\underline{x}_n(b,\lambda,\alpha) = \begin{cases} \min\{x|\max_\sigma Z(x,\sigma)\geq 0\} & \text{if } \{x|\max_\sigma Z(x,\sigma)\geq 0\}\neq\varnothing \\ 1 & \text{otherwise}\end{cases} \qquad (7)$$

$$\bar{x}_n(b,\lambda,\alpha) = \begin{cases} \max\{x|\min_\sigma Z(x,\sigma)\leq 0\} & \text{if } \{x|\min_\sigma Z(x,\sigma)\leq 0\}\neq\varnothing \\ 0 & \text{otherwise}\end{cases}. \qquad (8)$$

That is, $\underline{x}_n(b,\lambda,\alpha)$ is the lowest offer that is ever accepted in a responsive equilibrium: if $x < \underline{x}_n(b,\lambda,\alpha)$, then $Z(x,\sigma) < 0$ for all σ. Similarly, $\bar{x}_n(b,\lambda,\alpha)$ is the highest offer that is ever rejected in a responsive equilibrium.

[18] Observe that $\underline{x}_n(b,\lambda,\alpha) > 0$ since, by Assumptions 2 and 3, $Z(0,\sigma) < 0$ for all σ.

The following lemma establishes existence and uniqueness of cutoff strategies in the voting stage of the game. Part (1) extends Theorem 1 of Duggan and Martinelli (2001) to our more general preference framework. Parts (2) and (3) establish elementary properties of how the responsive equilibrium is related to the proposer's offer x.

Lemma 1 ***(Existence and uniqueness in the voting stage)*** *Fix beliefs b, a voting rule α and preferences λ. Then:*

(1) For any n, a responsive equilibrium $\sigma^*(x) \in [\underline{\sigma}, \bar{\sigma}]$ exists if and only if $x \in [\underline{x}_n, \bar{x}_n]$. When a responsive equilibrium exists it is the unique symmetric responsive equilibrium.

(2) The equilibrium cutoff $\sigma^*(x)$ is decreasing and continuously differentiable over $(\underline{x}_n, \bar{x}_n)$, with $\sigma^*(\underline{x}_n) = \bar{\sigma}$ and $\sigma^*(\bar{x}_n) = \underline{\sigma}$.

(3) If $\alpha < 1$ and x is such that $\Delta^H(x, \bar{\sigma}) > 0 > \Delta^L(x, \underline{\sigma})$, there exists N such that $x \in (\underline{x}_n, \bar{x}_n)$ for $n \geq N$.

In addition to responsive equilibria, non-responsive equilibria also exist. Specifically, for any $\alpha > \frac{1}{n}$ there is an equilibrium in which each responder votes to reject regardless of his signal, i.e., $\sigma^* = \bar{\sigma}$. Likewise, for any $\alpha < 1 - \frac{1}{n}$ there is an equilibrium in which each responder votes to accept regardless of his signal, i.e., $\sigma^* = \underline{\sigma}$. We follow the literature and assume that if a responsive equilibrium exists, then it is played. From Lemma 1 it follows that as x increases over the interval $(\underline{x}_n, \bar{x}_n)$ the acceptance probability increases continuously from 0 to 1. We thus assume that when $x \leq \underline{x}_n$ the rejection equilibrium is played, while for $x \geq \bar{x}_n$ the acceptance equilibrium is played. In addition to being intuitive and ensuring continuity, this rule selects the unique trembling-hand perfect equilibrium when $x \leq \underline{x}_n$.[19]

[19]Formally, for any beliefs b, preference parameter λ and voting rule $\alpha > \frac{1}{2} + \frac{1}{2n}$, if $x \leq \underline{x}_n$ then the only trembling-hand perfect equilibrium is the non-responsive equilibrium in which each responder always rejects. A proof is available on the authors' webpages.

Moreover, although when $x \geq \bar{x}_n$ both the acceptance and rejection equilibria are trembling-hand perfect, the trembles required to support the rejection equilibrium do not satisfy the cutoff rule property we discussed earlier. Indeed, if tremble strategies were required to satisfy the mild monotonicity restriction that voting to accept is weakly more likely after a higher signal, then the acceptance equilibrium would be the *only* trembling-hand perfect equilibrium when $x \geq \bar{x}_n$.

Equilibrium existence

In our environment, the proposer chooses an offer x from an infinite choice set $[0,1]$. Additionally, the proposer "type" σ_0 is itself drawn from an infinite set $[\underline{\sigma},\bar{\sigma}]$. It is well-known that sequential equilibria may fail to exist in infinite games, even when (as is the case here) payoff functions are continuous.

To establish equilibrium existence, we exploit Manelli's (1996) sufficient conditions for equilibrium existence in a canonical signaling game, in which a single "sender" of unknown type chooses an action, and a single uninformed "receiver" selects a response. To apply his results, we must first show that the aggregate behavior of the n partially informed responders in our model matches that of a single uninformed receiver endowed with suitable preferences. The following result does just this:

Lemma 2 ***(Equivalent sender-receiver game)*** *Fix n, λ, α. Suppose that the proposer makes an offer x and the responders' beliefs about the proposer's observation σ_0 are given by the probability distribution φ on $[\underline{\sigma},\bar{\sigma}]$. Then the equilibrium σ_n^* of the voting stage of the game coincides with the best-response correspondence of a single fictitious player holding the same beliefs and whose payoff depends on the offer x, proposer signal σ_0, and his own action σ' according to*

$$U_n\left(x,\sigma',\sigma_0;\lambda,\alpha\right)\equiv\int_{\underline{\sigma}}^{\sigma'}-Z\left(x,s,b=p^H(\sigma_0),\lambda,\alpha,n\right)ds. \tag{9}$$

Our model is a stylized bargaining model in which an opposing party makes take-it-or-leave-it offers to a group. In practice, there are many instances in which a group of individuals is engaged in collective bargaining. In such instances, it is often tempting to model the group as a single individual. Lemma 2 suggests that to some extent this approach is viable, but that the relation between the true preferences of individuals and those of the "representative" agent may be quite complicated. In particular, the utility function defined by (9) *does not* equal the average expected utility of a group member in the model

From Lemma 2, Manelli's results immediately imply:

Proposition 1 ***(Equilibrium existence)*** *An equilibrium exists.*

5 Equilibrium properties

We next establish general properties of equilibrium strategies and payoffs that we will use in our comparison of voting rules later in the paper. In particular, we show that a higher offer increases the expected utility of responders, even though a higher offer also changes the incidence of voting mistakes.

We start with the following straightforward corollary to Lemma 1:

Corollary 1 ***(Change in offer and responder beliefs)*** *Fix n, α, λ, and suppose that a responsive equilibrium exists given offer x and responder beliefs b. Then the acceptance probability is monotonically increasing in x and responder beliefs b.*

The heart of our analysis concerns the effect of the voting rule on the proposer's offer x, and in turn the effect on responder and proposer payoffs. Notationally, we write $\Pi_n^R(x, b, \lambda, \alpha)$ for a responder's expected payoff from offer x under voting rule α, responder preferences λ, and responder beliefs b; and $\Pi_n^P(x, \sigma_0, b, \lambda, \alpha)$ for the proposer's expected payoff after observing σ_0. Before proceeding, we note a second straightforward corollary of Lemma 1:

Corollary 2 ***(Continuity and differentiability of payoffs)*** *Fix a set of responder beliefs b. Then $\Pi_n^R(x, b, \lambda, \alpha)$ and $\Pi_n^P(x, \sigma_0, b, \lambda, \alpha)$ are continuous functions of the offer x, and are differentiable except at the boundaries of the responsive equilibrium range, $\underline{x}_n(b, \lambda, \alpha)$ and $\bar{x}_n(b, \lambda, \alpha)$.*

One way that voting rules affect payoffs is through their impact on the equilibrium offer. As such, it is important to characterize how the responder payoff Π_n^R depends on x. The main complication in doing so is that as x changes the equilibrium of the voting stage changes, and so the standard envelope theorem does not apply. However, the envelope theorem can be adapted as follows.

Notationally, for an arbitrary profile of responder cutoff voting strategies $\hat{\sigma}_1, \ldots, \hat{\sigma}_n$, define $u_i(x, \hat{\sigma}_1, \ldots, \hat{\sigma}_n, b, \lambda, \alpha)$ as the expected payoff of responder i given offer x. Write $u_i(x, \hat{\sigma}, b, \lambda, \alpha)$ for the special case in which all responders use the same strategy. Evaluating the effect of the offer x on responder payoffs Π_n^R explicitly,

$$\frac{\partial}{\partial x}\Pi_x^R(x, b, \lambda, \alpha) = \frac{\partial}{\partial x}u_i(x, \sigma_n^*(x), b, \lambda, \alpha) + \sum_{j=1}^{n} \frac{\partial \sigma_n^*(x)}{\partial x}\frac{\partial}{\partial \hat{\sigma}_j}u_i(x, \sigma_n^*(x), b, \lambda, \alpha). \quad (10)$$

As in the standard envelope theorem, the fact that $\sigma_n^*(x)$ is an equilibrium strategy implies that

$$\frac{\partial}{\partial \hat{\sigma}_j}u_j(x, \sigma_n^*(x), b, \lambda, \alpha) = 0 \quad (11)$$

for all j, x, b, λ, and α. In the pure common values setting $(\lambda = 0)$ $u_j \equiv u_i$, and so

$$\frac{\partial}{\partial \hat{\sigma}_j}u_i(x, \sigma_n^*(x), b, \lambda, \alpha) = 0$$

for all i, j, x, b, λ, α. So in the pure common values case, in order to evaluate the effect of a change in the offer x on the responder payoffs, it is sufficient to look at its direct effect on the utility evaluated at the best response to this offer — exactly as in the standard envelope theorem. Moreover, when responders use unanimity rule this argument extends to arbitrary preferences:

Proposition 2 ***(Effect of higher offers on responder payoffs)*** *For either $\alpha = 1$ or $\lambda = 0$,*

$$\frac{\partial}{\partial x}\Pi_x^R(x, b, \lambda, \alpha) \geq \frac{\partial}{\partial x}u_i(x, \sigma_n^*(x), b, \lambda, \alpha).$$

6 Unanimity rule

In this section, we characterize equilibrium offers and payoffs under unanimity rule. In order to do so, we first derive the asymptotic acceptance probabilities. We start by introducing some new notation. Define $x_\omega(\lambda)$ as the solution to $\Delta^\omega(x_\omega(\lambda), \underline{\sigma}, \lambda) = 0$, and write $x_\omega(\lambda) = \infty$ if no solution exists. By Assumption 3, $x_H(\lambda = 0) \neq \infty$, and so by continuity there exists $\bar{\lambda} > 0$ such that $x_H(\lambda; \alpha) \neq \infty$ for all $\lambda < \bar{\lambda}$. Economically, $x_\omega(\lambda)$ is the lowest offer that a large coalition of responders would accept under unanimity rule if it were somehow revealed that the true state is ω. Next, define $x_U(b, \lambda)$ as the solution to

$$\Delta^H(x, \underline{\sigma}) \frac{b}{1-b} \ell(\underline{\sigma}) + \Delta^L(x, \underline{\sigma}) = 0. \tag{12}$$

Economically, $x_U(b, \lambda)$ is the lowest offer that the responder coalition accepts with probability 1. For example, we showed in the opening example that $x_U(b, \lambda) = 3/4$. By Assumptions 2, 3 and 5, the lefthand side of (12) is strictly negative at $x = 0$, and is strictly increasing in x. As such, (12) has at most one solution. If the lefthand side is strictly negative at $x = 1$, define $x_U(b, \lambda) = \infty$. Note that if $x_U(b, \lambda) \neq \infty$ then $x_U(b, \lambda) < x_L(\lambda)$. Moreover, $x_U(b, \lambda)$ is decreasing in b.

Define $P_n^\omega(x, b, \lambda, \alpha)$ as the equilibrium acceptance probability in state ω given offer x. The next result gives the limiting behavior of P_n^ω as the number of responders n grows large. The result is an extension of Duggan and Martinelli's (2001) Theorem 4 to cases in which the proposal being voted over is either very unattractive, or very attractive.[20]

Lemma 3 ***(Limit acceptance probability under unanimity)*** *Suppose the unanimity rule is in effect ($\alpha = 1$). Take any $\lambda \in [0, \bar{\lambda})$ and a responder belief $b \in (0, 1)$. If the offer $x \geq x_U(b, \lambda)$ then $P_n^\omega(x, b, \lambda, 1) = 1$ all n, for $\omega = L, H$. Otherwise,*

$$\lim_{n \to \infty} P_n^\omega(x, b, \lambda, 1) = \begin{cases} 0 & \text{if } x \leq x_H(\lambda) \\ \left(-\frac{\Delta^H(x, \underline{\sigma}, \lambda)}{\Delta^L(x, \underline{\sigma}, \lambda)} \frac{b}{1-b} \ell(\underline{\sigma})\right)^{\frac{f(\underline{\sigma}|\omega)}{f(\underline{\sigma}|L) - f(\underline{\sigma}|H)}} & \text{if } x \in (x_H(\lambda), x_U(b, \lambda)) \end{cases}. \tag{13}$$

The limit acceptance probability $\lim_{n\to\infty} P^\omega(x, b, \lambda, 1)$ is continuous and monotone increasing in both x and b.

Lemma 3 reflects the failure of information aggregation under unanimity rule. On the one hand, failure of information aggregation leads offers above x_H to be rejected even

[20] That is, in Duggan and Martinelli's notation, ρ is either non-positive or exceeds $1/L$.

when $\omega = H$. On the other hand, failure of information aggregation leads offers below x_L to be accepted even when $\omega = L$.

Given the limit acceptance probabilities we can characterize the proposer's preferred offers. In order to do so, we must first show that it is legitimate to maximize the proposer's payoff using the limit acceptance probabilities. Mathematically, we must establish that acceptance probabilities converge uniformly:

Lemma 4 ***(Uniform convergence)*** *For any $\varepsilon > 0$, $P_n^\omega(\cdot)$ converges uniformly over $[\varepsilon, 1-\varepsilon] \times [\underline{b}, \bar{b}] \times [0,1]$.*

The proof of Lemma 4 hinges on the monotonicity of the acceptance probability in the offer x. A standard result of real analysis, Helly's Selection Theorem, implies that P_n^ω converges uniformally when treated as a function of the offer only, x. To establish uniform convergence of P_n^ω as a function of x, b, λ, where P_n^ω need not be monotone in λ, we extend Helly's Selection Theorem. Details are in Lemma A-2 which is stated and proved in the appendix.

When the proposer faces a group bound by unanimity, the vote does not aggregate the information of the group efficiently. Because of this, if it were revealed, the proposer's signal has the capacity to affect the acceptance probability, even when the number of responders is large. This observation in turn implies that the proposer may try to signal his own information σ_0 with the offer he makes. As is well known, signalling games often possess multiple equilibria. Nonetheless, our next result bounds the proposer's equilibrium offer *regardless* of the equilibrium played. In doing so, we show that even though information aggregation is imperfect, there is still no equilibrium in which the proposer makes an offer just above the minimum acceptable offer $x_H(\lambda)$, and the responders accept because they think such an offer signals favorable proposer information.

Proposition 3 ***(Equilibrium offer under unanimity)*** *Suppose the unanimity rule is in effect ($\alpha = 1$). Then there exists $\check{\lambda} < \bar{\lambda}$, $\kappa > 0$, and N such that for all σ_0, $\lambda \in [0, \check{\lambda}]$,* *in any equilibrium the proposer's offer always exceeds* $x_H(\lambda) + \kappa$ *when* $n \geq N$*, and is always less than* $x_U(\underline{b}, \lambda)$ *(regardless of* n*).*

Given the uniform convergence of acceptance probabilities established in Lemma 4, it is immediate from Lemma 3 and Proposition 3 that the acceptance probability under unanimity rule is bounded uniformly away from zero when the preferences are close to common values.

Corollary 3 ***(Lower bound on acceptance probability under unanimity)*** *There exists* $\check{\lambda} < \bar{\lambda}$*,* $\kappa > 0$*, and* N *such that for all* $\lambda \in [0, \check{\lambda}]$*, and* $n \geq N$*, in any equilibrium the acceptance probability exceeds* κ*.*

This Corollary in turn combines with Proposition 2 and Proposition 3 to deliver bounds on the responders' equilibrium payoff under unanimity rule:

Corollary 4 ***(Bounds on responder payoffs under unanimity)*** *There exists* $\check{\lambda} < \bar{\lambda}$*,* $\kappa > 0$*, and* N *such that for all* $\lambda \in [0, \check{\lambda}]$*, and* $n \geq N$*, in any equilibrium the payoff of responders exceeds*

$$E_{\sigma_i,\omega}\left[\bar{U}^\omega(\sigma_i, \lambda)\right] + \kappa.$$

Moreover, if $x_U(\underline{b}, \lambda) \neq \infty$*, in any equilibrium the payoff of responders is bounded above by*

$$E_{\sigma_i,\omega}\left[\bar{U}^\omega(\sigma_i, \lambda)\right] + E_{\sigma_i,\omega}\left[\Delta^\omega\left(x_U(\underline{b}, \lambda), \sigma_i, \lambda\right)\right].$$

7 Majority rule

Throughout, we refer to any non-unanimity voting rule α as a majority rule. As we will see, asymptotically (in the number of responders n) all such rules generate the same equilibrium outcomes.

When the responders use majority rule, the limit acceptance probabilities are discontinuous. As such, it is not possible to first analyze the equilibrium of the limit game, and then to show that it is also the limit of the equilibrium of the finite games. Because of this complication, we first extend results from existing strategic voting literature to the case where the proposal being voted over varies with the number of responders.

To state our results, we need to generalize the $x_\omega(\lambda)$ notation we introduced above. For $\omega = L, H$, define $\sigma_\omega(\alpha)$ and $x_\omega(\lambda; \alpha)$ implicitly by

$$1 - F(\sigma_\omega(\alpha) | \omega) = \alpha \quad \text{and} \quad \Delta^\omega(x_\omega(\lambda; \alpha), \sigma_\omega(\alpha), \lambda) = 0.$$

That is, conditional on ω there is a probability α that the realization of σ_i exceeds $\sigma_\omega(\alpha)$; and $x_\omega(\lambda; \alpha)$ is the proposal that gives a responder i the same payoff as the status quo, given ω and $\sigma_i = \sigma_\omega(\alpha)$. As such, if the state ω were public information, then an offer just above $x_\omega(\lambda; \alpha)$ would be accepted with probability converging to 1 as the number of responders n grows large. Note that $x_\omega(\cdot; \alpha = 1) \equiv x_\omega(\cdot)$, so this notation contains the notation of the prior section as a special case. Moreover, under pure common values ($\lambda = 0$) the value $x_\omega(\lambda; \alpha = 1)$ is independent of the voting rule α.

We are now ready to give our result on the limit acceptance probabilities. Because there is no reason to require the proposer's offers to have a well-defined limit, we state our result in terms of the limits infimum and supremum. We show that, as in Feddersen and Pesendorfer (1997) and Duggan and Martinelli (2001), the aggregate response of the voting group to an offer x matches that which would be obtained under full information.

Lemma 5 ***(Acceptance probabilities under majority)*** *Suppose a majority voting rule $\alpha < 1$ is in effect. Take any $\lambda \in [0, 1]$, and consider a sequence of offers x_n. If $\liminf x_n > x_\omega(\lambda; \alpha)$ then $P_n^\omega(x_n) \to 1$ and if $\limsup x_n < x_\omega(\lambda; \alpha)$ then $P_n^\omega(x_n) \to 0$ as $n \to \infty$.*

We next use Lemma 5 to characterize the proposer's equilibrium offer. First, note that the acceptance probabilities are asympototically the same over each of the ranges $[0, x_H]$, $[x_H, x_L]$, and $[x_L, 1]$. Consequently, when facing a large number of responders using majority rule, the proposer will never make an offer that lies far from the lower ends of these ranges, i.e., 0, x_H, x_L. Since the offer 0 is always rejected when the number of responders is large, the proposer's choice approximately boils down to x_H versus x_L. To this end, for any σ_0 define

$$W(\sigma_0; \lambda, \alpha) \equiv p^H(\sigma_0) V^H(x_H, \sigma_0) + p^L(\sigma_0) \bar{V}^L(\sigma_0) - E\left[V^\omega(x_L, \sigma_0) | \sigma_0\right]. \tag{14}$$

The function W has the following interpretation: the first two terms are the proposer's expected payoff from offering x_H if this offer is accepted when $\omega = H$ and rejected when $\omega = L$. The final term is the proposer's expected payoff from offering x_L if this offer is always accepted. In Lemma 5 we established that approximately this acceptance behavior is obtained as the number of responders grows large. As such, a proposer facing a large number of responders will offer x_H whenever $W(\sigma_0; \lambda, \alpha) > 0$; and will offer x_L whenever $W(\sigma_0; \lambda, \alpha) < 0$.

Because under majority voting responders' signals asymptotically reveal the true realization of ω, there is no scope for the proposer's offer to convey useful information. Consequently the signalling aspect of the bargaining game disappears. The equilibrium outcome is then unique,[21] at least in payoff terms.

It follows that when responders hold a majority vote we can precisely characterize the expected equilibrium payoffs of both the proposer and responders. Doing so, however, requires handling one further technical issue. We must show that as the number of responders grows large, equilibrium offers and acceptance probabilities converge uniformly with respect to the proposer's signal σ_0. Our next result does just this; Proposition 4 below then gives the limiting expected utilities.

Lemma 6 ***(Equilibrium offer under majority)*** *Suppose a majority voting rule $\alpha < 1$ is in effect. Then:*

(1) If $x_L(\lambda; \alpha) \neq \infty \neq x_H(\lambda; \alpha)$, then for any $\varepsilon, \delta > 0$ there exists $N(\varepsilon, \delta)$ such that

[21] More accurately, the equilibrium is unique within the class of symmetric responder equilibria, and given our standard equilibrium selection rule that chooses a responsive equilibrium whenever one exists.

(a) If $W(\sigma_0) > \varepsilon$ *and* $n \geq N(\varepsilon, \delta)$ *then for any equilibrium offer* x*,* $|x - x_H(\lambda; \alpha)| < \delta$*;* $P_n^H(x|\sigma_0) > 1 - \delta$*; and* $P_n^L(x|\sigma_0) < \delta$*.*

(b) If $W(\sigma_0) < -\varepsilon$ *and* $n \geq N(\varepsilon, \delta)$ *then for any equilibrium offer* x*,* $|x - x_L(\lambda, \alpha)| < \delta$*;* $P_n^H(x|\sigma_0) > 1 - \delta$*; and* $P_n^L(x|\sigma_0) > 1 - \delta$*.*

(2) If $x_H(\lambda; \alpha) \neq \infty$ *and* $x_L(\lambda; \alpha) = \infty$ *then for any* $\delta > 0$ *there exists* $N(\delta)$ *such that for any equilibrium offer* x*,* $|x - x_H(\lambda; \alpha)| < \delta$ *and* $P_n^H(\cdot|\sigma_0) > 1 - \delta$ *for all* σ_0 *when* $n \geq N(\delta)$*.*

(3) If $x_L(\lambda; \alpha) = x_H(\lambda; \alpha) = \infty$*, for any* $\delta > 0$ *there exists* $N(\delta)$ *such that for any equilibrium offer* x*,* $P_n^\omega(x|\sigma_0) < \delta$ *for all* $\sigma_0, \omega = L, H$ *when* $n \geq N(\delta)$*.*

For use below, we set $W(\sigma_0; \lambda, \alpha) = \infty$ when $x_H(\lambda; \alpha) \neq \infty$ and $x_L(\lambda; \alpha) = \infty$. Lemma 6 says that the proposer will make an offer close to $x_H(\lambda; \alpha)$ (respectively, $x_L(\lambda; \alpha) > x_H(\lambda; \alpha)$) after observing a σ_0 such that $W(\sigma_0)$ is strictly positive (negative). As stated, it does not cover equilibrium behavior when $W(\sigma_0) = 0$. In general, this knife-edge condition will hold only for finitely many realizations of σ_0. In particular, $W(\sigma_0) = 0$ for at most one value of σ_0 if the proposer's payoffs $V^\omega(x, \sigma_0)$ and $\bar{V}^\omega(\sigma_0)$ are independent of σ_0 — or more generally, if the private values component of proposer payoffs is sufficiently small, i.e., $\left|\frac{\partial}{\partial \sigma_0} V^\omega(x, \sigma_0)\right|$ and $\left|\frac{\partial}{\partial \sigma_0} \bar{V}^\omega(\sigma_0)\right|$ are sufficiently small for all x and σ_0. For the remainder of the paper we make the following mild assumption:

Assumption 6 $W(\sigma_0; \lambda, \alpha) = 0$ *for at most finitely many values of* σ_0 *when* $x_L(\lambda; \alpha) \neq \infty \neq x_H(\lambda; \alpha)$*.*

From Lemma 6 it is straightforward to establish the limiting expected payoffs of the proposer and the responders under any majority voting rule. Notationally, we write $\Pi_n^{*P}(\lambda, \alpha)$ and $\Pi_n^{*R}(\lambda, \alpha)$ for the proposer's and responders' expected equilibrium payoffs. Immediate from Lemma 6, we have:

Proposition 4 ***(Equilibrium payoffs under majority)*** *Suppose a majority voting rule $\alpha < 1$ is in effect and $x_H(\lambda;\alpha) \neq \infty$. Then the equilibrium payoffs satisfy:*

$$\begin{aligned}
\Pi_n^{*R}(\lambda,\alpha) &\rightarrow E_{\sigma_i,\omega}\left[\bar{U}^{\omega}(\sigma_i)\right] + \int_{\sigma_0 \text{ s.t. } W(\sigma_0)<0} E_{\sigma_i,\omega}\left[\Delta^{\omega}(x_L,\sigma_i,\lambda)\,|\sigma_0\right] dF_0(\sigma_0) \\
&\quad + \int_{\sigma_0 \text{ s.t. } W(\sigma_0)>0} p^H(\sigma_0) E_{\sigma_i}\left[\Delta^H(x_H,\sigma_i,\lambda)\,|H\right] dF_0(\sigma_0) \\
\Pi_n^{*P}(\lambda,\alpha) &\rightarrow \int_{\sigma_0 \text{ s.t. } W(\sigma_0)<0} E_{\omega}\left[V^{\omega}(x_L,\sigma_0)\,|\sigma_0\right] dF_0(\sigma_0) \\
&\quad + \int_{\sigma_0 \text{ s.t. } W(\sigma_0)>0} \left(p^H(\sigma_0)V^H(x_H,\sigma_0) + p^L(\sigma_0)\bar{V}^L(\sigma_0)\right) dF_0(\sigma_0).
\end{aligned}$$

8 Comparing majority and unanimity voting rules

We are now ready to compare the equilibrium payoffs of the proposer and responders under any majority rule to those under unanimity rule. We focus on cases in which responder preferences are not too far from pure common values (i.e., λ close enough to 0).

Responder preferences

First, suppose $W(\cdot;\lambda,\alpha) > 0$. Directly from Proposition 4, for any majority voting rule $\alpha < 1$,

$$\Pi_n^{*R}(\lambda,\alpha) \rightarrow E_{\sigma_i,\omega}\left[\bar{U}^{\omega}(\sigma_i,\lambda)\right] + p^H E_{\sigma_i}\left[\Delta^H(x_H(\lambda;\alpha),\sigma_i,\lambda)\,|H\right]$$

as the number of responders grows large. The first term is the responders' payoff under the status quo. In general, the second term can be positive or negative. However, by definition, $\Delta^H(x_H(\lambda;\alpha),\sigma_H,\lambda) = 0$, and Δ^H is independent of σ_i when $\lambda = 0$. Consequently $E_{\sigma_i}\left[\Delta^H(x_H(\lambda;\alpha);\sigma_i,\lambda)\,|H\right]$ approaches 0 as $\lambda \rightarrow 0$, and so the responders' payoff approaches their status quo payoff. Put differently, against a majority rule the proposer is able to reduce the responders' payoff all the way to their outside option.

From Corollary 4, the responders' equilibrium payoff when they use unanimity rule is bounded away from their status quo payoff $E_{\sigma_i,\omega}\left[\bar{U}^{\omega}(\sigma_i,\lambda)\right]$. It is then immediate that the responders are better off under unanimity rule when $W(.;\lambda=0,\alpha) > 0$.

Proposition 5 ***(Responders better off under unanimity)*** *Fix a majority rule $\alpha < 1$, and suppose that $W(\cdot; \lambda = 0, \alpha) > 0$. Then there exists $\check{\lambda} > 0$ such that whenever $\lambda < \check{\lambda}$, responders strictly prefer unanimity to the majority rule α (regardless of the equilibrium played).*

To interpret Proposition 5, consider what economic circumstances lead W to be positive, and so to the proposer making the offer x_H against majority rule. First, W is increasing in x_L and decreasing in x_H. As such, W is more likely to be positive if a responder's payoff relative to the status quo in state L is low (i.e., Δ^L high), and/or a responder's payoff relative to the status quo in state H is high (i.e., Δ^H low). Second, turning to the proposer's own preferences, W is more likely to be positive if his status quo payoff in state L (i.e., $\bar{V}^L$) is high and/or the cost of increasing the offer in state H (i.e., $\left|\frac{\partial V^H}{\partial x}\right|$) is high and/or the value of having an offer accepted in state L (i.e., V^L) is low.

Next, suppose instead that $W(\cdot; \lambda, \alpha) < 0$ for all σ_0. In this case, the proposer's offer to responders using majority rule converges to $x_L(\lambda; \alpha)$, the offer which is required to guarantee acceptance in both state L and H. From Proposition 4, as the number of responders grows large their expected utility has the following limit:

$$\Pi_n^{*R}(\lambda, \alpha) \to E_{\sigma_i,\omega}\left[\bar{U}^\omega(\sigma_i, \lambda)\right] + E_{\sigma_i,\omega}\left[\Delta^\omega(x_L(\lambda; \alpha), \sigma_i, \lambda)\right].$$

By definition, $x_L(\lambda; \alpha) \leq 1$ in this case, and so $x_U(\bar{b}, \lambda) < x_L(\lambda; \alpha)$. Consequently, when responders use unanimity rule the proposer is able to gain certain acceptance by offering $x_U(\bar{b}, \lambda)$. As such, the responders are better off under majority rule when $W(.; \lambda = 0, \alpha) < 0$.

Proposition 6 ***(Responders better off under majority)*** *Fix a majority voting rule $\alpha < 1$, and suppose that $W(\cdot; \lambda = 0, \alpha) < 0$. Then there exists $\check{\lambda} > 0$ such that whenever $\lambda \leq \check{\lambda}$, for all n large enough responders strictly prefer the majority rule α to unanimity (regardless of the equilibrium played).*

Proposer preferences

We now turn to proposer preferences. In two significant cases, proposer preferences over the two voting rules are diametrically opposed to the responder preferences.

First, if $W(\cdot;\lambda,\alpha)<0$ the proposer offers $x_L(\lambda;\alpha)$ to responders using majority rule and the offer is always accepted; while if the responders use unanimity rule, the proposer is able to obtain certain acceptance with a lower offer. As such, the proposer prefers to face responders using unanimity rule in this case.

Proposition 7 ***(Proposer better off under unanimity)*** *Fix a majority voting rule $\alpha<1$, and suppose that $W(\cdot;\lambda=0,\alpha)<0$. Then there exists $\check{\lambda}>0$ such that whenever $\lambda\leq\check{\lambda}$, for all n large enough the proposer strictly prefers unanimity rule to the majority rule α to unanimity (regardless of the equilibrium played).*

Second, in many bargaining environments the proposer prefers to face responders using majority rule. In particular, this is often the case when proposer and responder preferences are linear:

Assumption 7 *$U^{\omega}(x,\sigma_i,\lambda)$ and $V^{\omega}(x,\sigma_0)$ are linear in x, and the ratio $U_x^{\omega}(x,\sigma_i,\lambda)/V_x^{\omega}(x,\sigma_0)$ is independent of ω, σ_i and σ_0.*[22]

Assumption 7 says that both the proposer and responders are risk-neutral in x, and the relative value of changes in the offer x for the responders and the proposer is always the same. An immediate consequence is that there exists some constant $C>0$ such that $U^{\omega}(x,\sigma_i,\lambda)+CV^{\omega}(x,\sigma_0)$ is independent of x, for all ω, σ_0, σ_i, and λ. In other words, agreement creates the same total surplus independent of the offer x — which affect only the division of surplus between the bargaining parties. When

$$U^L(\cdot,\sigma_i,\lambda=0)+CV^L(\cdot,\sigma_0)<\bar{U}^L(\sigma_i,\lambda=0)+C\bar{V}^L(\sigma_0)\,, \tag{15}$$

[22]The requirement that the ratio $U_x^{\omega}(x,\sigma_i,\lambda)/V_x^{\omega}(x,\sigma_0)$ be independent of ω and σ_i rules out any benefit from reallocating resources from one state to another. Without this assumption, one voting rule might dominate another because it effectively enables such a reallocation.

agreement destroys surplus in state L whenever the preference parameter λ is close enough 0 (i.e., common values). Note that (15) always holds when $x_L(\lambda = 0; \alpha) = \infty$ and $V^L(x = 1, \sigma_0)$ is sufficiently close to $\bar{V}^L(\sigma_0)$. Note also that by assumptions 3 and 4,

$$U^H(\cdot, \sigma_i, \lambda = 0) + CV^H(\cdot, \sigma_0) > \bar{U}^H(\sigma_i, \lambda = 0) + C\bar{V}^H(\sigma_0),$$

so that agreement always creates surplus in state H.

When $W(\cdot; \lambda, \alpha) > 0$ we know that the proposer offers $x_H(\lambda; \alpha)$ to responders using majority rule, and the responders accept if and only if the state is H. So if (15) holds, the majority decision rule maximizes total surplus. In contrast, if the responders are using unanimity rule, the proposer offers strictly more than $x_H(\lambda; \alpha)$, and his offer is accepted with strictly positive probability in state L. Since total surplus is lower and the responders strictly prefer unanimity to majority, it follows that the proposer has exactly the opposite preferences.

Proposition 8 ***(Proposer prefers majority)*** *Suppose that Assumption 7 and inequality (15) hold. Fix a majority voting rule $\alpha < 1$. If $W(\cdot; \lambda, \alpha) > 0$ then there exists $\check{\lambda} > 0$ such that whenever $\lambda \leq \check{\lambda}$, for all n large enough the proposer strictly prefers the majority rule α to unanimity (regardless of the equilibrium played).*

PARETO DOMINANCE OF UNANIMITY

Propositions 7 and 8 give conditions under which the proposer and responders have opposite preferences over the voting rule used by the responders. However, and as we saw in the opening example, there are also cases in which both sides strictly prefer unanimity to majority. We now establish this result more generally.

We consider first the case in which the proposer's signal is completely uninformative, and responders' preferences are pure common values ($\lambda = 0$). For this case, we establish:

Proposition 9 ***(Pareto dominance of unanimity with uninformed proposer)*** *Suppose $\lambda = 0$ and the proposer's signal is completely uninformative. There exist preferences (i.e., U^ω, $\bar{U}^\omega$, V^ω, $\bar{V}^\omega$) such that for all n sufficiently large both the proposer* *and responders strictly prefer unanimity (regardless of the equilibrium played). In contrast, there do not exist preferences under which the proposer and responders both prefer majority for all* n.

We prove this result in reverse order, and first establish that there are no preferences such that both sides prefer majority rule. When the proposer's signal is uninformative, the function W is a constant independent of σ_0. A necessary condition for the responders to prefer a majority voting rule α to unanimity rule is $W \leq 0$: for if instead $W > 0$, from Proposition 5 the responders strictly prefer unanimity rule. Since $W \leq 0$, the proposer at least weakly prefers having $x_L(0;\alpha)$ accepted always to having $x_H(0;\alpha)$ accepted only in state H. He clearly strictly prefers having $x_U < x_L(0;\alpha)$ accepted always, which is possible under unanimity rule, to having $x_L(0;\alpha)$ accepted always. So for all n large enough, the proposer strictly prefers unanimity rule when $W \leq 0$.

We now establish that for some preferences both sides prefer unanimity. Choose preferences such that $W = 0$ (in the opening example, this is the point $\bar{V} = 20$). Under these preferences, the proposer strictly prefers unanimity to majority for n large enough, by the same argument as above. By continuity, the same is true if $\bar{V}^L$ is increased slightly, so that $W > 0$. Under these perturbed preferences, the responders also prefer unanimity whenever n is sufficiently large.

Proposition 9 establishes that there are conditions under which unanimity Pareto dominates majority voting. As stated, it covers only the case in which the proposer's signal is completely uninformative. Although this assumption greatly simplifies the proof, it is not essential. More generally, we can establish the following:

Proposition 10 ***(Pareto dominance of unanimity with informed proposer)*** *Suppose that Assumption 7 holds, and that responder preferences are such that* $x_L(\lambda = 0) < 1$. *Then provided responder information is sufficiently poor (*$\ell(\underline{\sigma})$ *close enough to* 1*) there exist proposer preferences such that for any majority rule* $\alpha_M < 1$, *there exist* N, $\check{\lambda} > 0$

such that both the proposer and responder strictly prefer unanimity to the majority rule α_M *for* $n \geq N$ *and* $\lambda \leq \check{\lambda}$.

The proof of this last result is omitted for space reasons, and is available from a technical appendix posted on the authors' webpages.

9 Concluding remarks

In this paper we have analyzed a strategic voting game in which the agenda is set endogenously. We have shown that in such an environment, unanimity rule may be the preferred alternative not only of the voting group, but also of the opposing party as well. These results contrast sharply with the results of the existing strategic voting literature that has analyzed voting over exogenous agendas.

Inevitably our analysis has neglected some important issues. We focus almost exclusively on equilibrium payoffs as the group size grows large. The chief reason for this focus is that it allows us to establish our results with fewer assumptions on preferences and the distributional properties of agents' information. Numerical simulations suggest that the group size needed for our asymptotic results to apply is not large — in many cases the equilibrium with ten agents is very close to the limiting equilibrium.

Related to this last point, in our analysis we have focused primarily on common values environments in which the preferences of the voting group are aligned. To ensure that our results to not depend on complete preference alignment, we have established all our main results for the case in which group member preferences are not perfectly aligned, but instead are merely "sufficiently close" to common values. An alternative robustness check would be to move slightly away from homogeneity and allow some individuals to have extreme biases, such as always prefering the status quo, for example. In such circumstances, unanimity would be impossible to obtain asymptotically. However, a version of our results should still hold when the number of voters is not too large. As we discussed above, acceptance probabilities converge relatively quickly to their limiting expressions.

We conclude with a discussion of implications our analysis has for pre-vote communication, i.e., deliberation. In our analysis, the role of voting is to aggregate information, and no communication is permitted. As is well-known, when voters have biases, full information sharing during communication is not always possible (see Coughlan 2000, Meirowitz 2005, Austen-Smith and Feddersen, 2006).[23] In contrast, when there are no biases, as in the pure common values case, voters would share their information truthfully when voting is over an exogenous agenda. The same is true when voting is over an endogenous agenda and the group members are worse off under unanimity rule due to mistakes. Note however that the mistakes sometimes benefit the group members. In this case, group members would actually want to *ex ante* commit not to communicate *ex post*. Of course, *ex post* they still wish to change their minds and communicate, but when the number of voters is large such communication will be hard to achieve without pre-existing arrangements. As such, our analysis complements Austen-Smith and Feddersen's (2006) result that when the agenda is exogenous voters may not communicate truthfully under unanimity rule when their interests are imperfectly aligned. Our analysis implies that even when interests are perfectly aligned, group members may still not communicate truthfully, because by refraining from communication they generate a better (endogenous) offer.

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[23]See also Doraszelski *et al* (2003), and Gerardi and Yariv (forthcoming) for communication in strategic voting games. See also Caillaud and Tirole (2006) which adopts a mechanism design approach to communication, and emphasizes the need to distill information selectively to create persuasion cascades.

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Appendix A

We repeatedly use the following result. The proof is straightforward and available from the authors' webpages.

Lemma A-1 *$F(\sigma|H)/F(\sigma|L)$ is increasing in σ, and is bounded above by 1. Consequently, $F(\sigma|H) \leq F(\sigma|L)$, and is strict if $\sigma \in (\underline{\sigma}, \bar{\sigma})$. Moreover, $(1 - F(\sigma|H))/(1 - F(\sigma|L))$ is increasing in σ, and is bounded above by $\ell(\bar{\sigma}) > 1$.*

Proof of Lemma 1: First note that if $Z(x,\sigma) = 0$, then it must be the case that $\Delta^H(x,\sigma) > 0$ by Assumption 2. This implies that $Z(x,\sigma)$ is strictly increasing in σ whenever $Z(x,\sigma) \geq 0$. In turn, $Z(x,\sigma') < 0$ for all $\sigma' < \sigma$ if $Z(x,\sigma) = 0$.

Part 1: By definition, if $x < \underline{x}_n$ then $Z(x,\cdot) < 0$, while if $x > \bar{x}_n$ then $Z(x,\cdot) > 0$. For $x \in [\underline{x}_n, \bar{x}_n]$ we claim that $Z(x,\sigma) = 0$ for some unique σ, which we write as $\sigma^*(x)$. Existence is immediate, since $\max_\sigma Z(x,\sigma) \geq 0 \geq \min_\sigma Z(x,\sigma)$, and $Z(x,\sigma)$ is continuous in σ. Uniqueness follows from the result we have just shown that $Z(x,\sigma)$ is strictly increasing in σ whenever $Z(x,\sigma) \geq 0$.

Part 2: To see that $\sigma^*(x)$ is decreasing, consider x and $x' > x$ in $(\underline{x}_n, \bar{x}_n)$. Since $Z(x, \sigma^*(x)) = 0$, it follows that $Z(x', \sigma^*(x)) > 0$. Since $Z(x', \sigma)$ is increasing in σ it must be the case that $\sigma^*(x') < \sigma^*(x)$. By the Implicit Function Theorem, $\sigma(x)$ is continuously differentiable over $(\underline{x}_n, \bar{x}_n)$. To see $\sigma^*(\underline{x}_n) = \bar{\sigma}$, suppose to the contrary that $\sigma^*(\underline{x}_n) < \bar{\sigma}$. By definition $Z(\underline{x}_n, \sigma^*(\underline{x}_n)) = 0$, and so $Z(\underline{x}_n, \bar{\sigma}) > 0$. By continuity there exists an $x < \underline{x}_n$ such that $Z(x, \bar{\sigma}) > 0$ as well. This contradicts the definition of $\underline{x}_n$. Likewise, to see $\sigma^*(\bar{x}_n) = \underline{\sigma}$ suppose to the contrary that $\sigma^*(\bar{x}_n) > \underline{\sigma}$. By definition $Z(\bar{x}_n, \sigma^*(\bar{x}_n)) = 0$ which implies that $Z(\bar{x}_n, \underline{\sigma}) < 0$. By continuity there exists an x such that $x > \bar{x}_n$ and $Z(x, \sigma) < 0$, contradicting the definition of $\bar{x}_n$.

Part 3: Immediate from the observation that as $n \to \infty$,

$$\ell(\sigma)\left(\frac{F(\sigma|H)}{F(\sigma|L)}\right)^{n-n\alpha}\left(\frac{1-F(\sigma|H)}{1-F(\sigma|L)}\right)^{n\alpha-1}$$

converges to 0 and ∞ respectively for $\sigma = \underline{\sigma}, \bar{\sigma}$. ∎

Proof of Lemma 2: Define $b_\varphi = \int p^H(\sigma_0)\varphi(d\sigma_0)$. The equilibrium σ_n^* of the voting stage of the game is the unique solution to $Z(x, \sigma, b_\varphi) = 0$, provided a solution exists; is $\underline{\sigma}$ if $Z(x, \sigma, b_\varphi) > 0$ for all $\sigma \in [\underline{\sigma}, \bar{\sigma}]$; and is $\bar{\sigma}$ if $Z(x, \sigma, b_\varphi) < 0$ for all $\sigma \in [\underline{\sigma}, \bar{\sigma}]$.

The fictitious player chooses σ' to maximize

$$\begin{aligned}\int_{\underline{\sigma}}^{\bar{\sigma}} U_n(x, \sigma', \sigma_0)\varphi(d\sigma_0) &= \int_{\underline{\sigma}}^{\bar{\sigma}}\left(\int_{\underline{\sigma}}^{\sigma'} -Z\left(x, s, b = p^H(\sigma_0)\right) ds\right) d\varphi(d\sigma_0) \\ &= \int_{\underline{\sigma}}^{\sigma'}\left(\int_{\underline{\sigma}}^{\bar{\sigma}} -Z\left(x, s, b = p^H(\sigma_0)\right) d\varphi(d\sigma_0)\right) ds \\ &= \int_{\underline{\sigma}}^{\sigma'} -Z(x, s, b_\varphi)\, ds.\end{aligned}$$

(The change of integration order in the second equality follows from standard arguments, while the third equality follows from the linearity of Z in b.)

By prior arguments (see the proof of Lemma 1) we know that if $Z(x, \hat{\sigma}, b_\varphi) = 0$ for some $\hat{\sigma}$, then $Z(x, \sigma, b_\varphi) < 0$ for $\sigma < \hat{\sigma}$ and $Z(x, \sigma, b_\varphi) > 0$ for $\sigma > \hat{\sigma}$. It follows that if $Z(x, \sigma_n^*, b_\varphi) = 0$ then σ_n^* is the unique maximizer of $\int_{\underline{\sigma}}^{\sigma'} -Z(x, s, b_\varphi)\, ds$; if $Z(x, \sigma, b_\varphi) > 0$

for all $\sigma \in [\underline{\sigma}, \bar{\sigma}]$ then the unique maximizer of $\int_{\underline{\sigma}}^{\sigma'} -Z(x, s, b_\varphi)\, ds$ is $\underline{\sigma}$; and finally, if $Z(x, \sigma, b_\varphi) < 0$ for all $\sigma \in [\underline{\sigma}, \bar{\sigma}]$ then the unique maximizer of $\int_{\underline{\sigma}}^{\sigma'} -Z(x, s, b_\varphi)\, ds$ is $\bar{\sigma}$. This completes the proof. ∎

Proof of Proposition 1: As established in Lemma 2, it is possible to replace the n responders with a single uninformed fictitious agent with preferences defined in (9). The proposer strictly prefers more acceptance (lower values of σ_n^*), regardless of his "type" σ_0 and offer x. Moreover, for any beliefs the best-response of the fictitious agent is a pure-strategy. As such, the game is strongly monotonic (see Manelli, page 929), and so possesses a sequential equilibrium (Manelli, Corollary 3). ∎

Proof of Proposition 2: Differentiability follows directly from Lemma 1. For either $x < \underline{x}_n$ or $x > \bar{x}_n$, it is immediate that

$$\frac{\partial}{\partial x} \Pi_x^R(x, b, \lambda, \alpha) = \frac{\partial}{\partial x} u_i(x, \sigma_n^*(x), b, \lambda, \alpha),$$

since $\sigma_n^*(x)$ equals $\bar{\sigma}$ and $\underline{\sigma}$ respectively over these regions. For the intermediate case $x \in (\underline{x}_n, \bar{x}_n)$ we also need to account for the effect changing x has on the equilibrium voting strategies. The main text handles the pure common values case $\lambda = 0$. When $\alpha = 1$ and $\lambda > 0$, note that for any common cutoff strategy $\hat{\sigma}$

$$\begin{aligned}
\frac{\partial}{\partial \hat{\sigma}_i} u_i(x, \hat{\sigma}, b, \lambda, \alpha) &= E_\omega\left[-f(\hat{\sigma}|\omega)(1 - F(\hat{\sigma}|\omega))^{n-1} \Delta^\omega(x, \hat{\sigma})\right] \\
\frac{\partial}{\partial \hat{\sigma}_j} u_i(x, \hat{\sigma}, b, \lambda, \alpha) &= E_\omega\left[-f(\hat{\sigma}|\omega)(1 - F(\hat{\sigma}|\omega))^{n-1} E\left[\Delta^\omega(x, \sigma_i)\,|\sigma_i \geq \hat{\sigma}\right]\right].
\end{aligned}$$

Since $E\left[\Delta^\omega(x, \sigma_i)\,|\sigma_i \geq \hat{\sigma}\right] \geq \Delta^\omega(x, \hat{\sigma})$ by Assumption 2, it follows that

$$\frac{\partial}{\partial \hat{\sigma}_i} u_i(x, \hat{\sigma}, b, \lambda, \alpha) \geq \frac{\partial}{\partial \hat{\sigma}_j} u_i(x, \hat{\sigma}, b, \lambda, \alpha).$$

In equilibrium, (11) holds. Since $\partial \sigma_n^* / \partial x < 0$ (see Lemma 1), the result then follows from (10). ∎

Proof of Lemma 3: First, consider offers $x \geq x_U(b, \lambda)$. We claim that if $x_U(b, \lambda) \neq \infty$ then $\bar{x}_n(b, \lambda, \alpha = 1) = x_U(b, \lambda)$ for all n. To see this, note that when $\alpha = 1$, $Z(x, \underline{\sigma})$

coincides with the lefthand side of (12), regardless of n. As such, $Z(x_U, \underline{\sigma}) = 0$, and so $Z(x_U, \sigma) > 0$ for $\sigma > \underline{\sigma}$. So certainly $\bar{x}_n \geq x_U$. Moreover, it follows that $Z(x, \sigma) > 0$ for all σ if $x > x_U$, so that $\bar{x}_n = x_U$. Given this, the equilibrium for $x \geq x_U$ is the non-responsive acceptance equilibrium.

Second, consider offers $x \in (x_H(\lambda), x_U(b, \lambda))$. By definition, $\Delta^H(x_H(\lambda), \underline{\sigma}) = 0$. It follows that for any $\sigma > 0$, $Z(x_H(\lambda), \sigma) > 0$ for n large enough. So $\underline{x}_n(b, \lambda, \alpha) \leq x_H(\lambda)$ for n large enough. As such, a responsive equilibrium exists whenever n is large enough.

When a responsive equilibrium exists, the equilibrium condition is

$$-\frac{\Delta^H(x, \sigma_n^*, \lambda)}{\Delta^L(x, \sigma_n^*, \lambda)} \frac{b}{1-b} \ell(\sigma_n^*) \left(\frac{1 - F(\sigma_n^*|H)}{1 - F(\sigma_n^*|L)} \right)^{n-1} = 1. \tag{A-1}$$

Since $\frac{1-F(\sigma|H)}{1-F(\sigma|L)} > 1$ for $\sigma > \underline{\sigma}$, it follows that

$$\sigma_n^* \to \underline{\sigma} \text{ as } n \to \infty. \tag{A-2}$$

In the proof of their Theorem 4, Duggan and Martinelli (2001) show that (A-1) and (A-2) together imply that

$$\lim (1 - F(\sigma_n^*|H))^n = \left(-\frac{\Delta^H(x, \underline{\sigma}, \lambda)}{\Delta^L(x, \underline{\sigma}, \lambda)} \frac{b}{1-b} \ell(\underline{\sigma}) \right)^{\frac{f(\underline{\sigma}|H)}{f(\underline{\sigma}|L) - f(\underline{\sigma}|H)}}, \tag{A-3}$$

i.e., equation (13) with $\omega = H$. The limit acceptance probability when $\omega = L$ then follows immediately from (A-1) and (A-2).

Third, consider the offer $x = x_H(\lambda)$, and suppose that contrary to the claimed result $\lim (1 - F(\sigma_n^*|H))^n \neq 0$. As such, there exists a subsequence of (σ_n^*), $(\sigma_{n_m}^*)$ say, such that $\lim_{m\to\infty} (1 - F(\sigma_{n_m}^*|H))^{n_m} > 0$. Since $x_H(\lambda) < x_U(b, \lambda)$, for m large $\sigma_{n_m}^*$ is a responsive equilibrium. As argued above, $\sigma_{n_m}^* \to \underline{\sigma}$. It follows that $\lim_{m\to\infty} (1 - F(\sigma_{n_m}^*|H))^{n_m}$ equals the righthand side of equation (A-3) evaluated at $x = x_H(\lambda)$. However, $-\frac{\Delta^H(x_H(\lambda), \underline{\sigma}, \lambda)}{\Delta^L(x_H(\lambda), \underline{\sigma}, \lambda)} = 0$, and so this contradicts the hypothesis that $\lim_{m\to\infty} (1 - F(\sigma_{n_m}^*|H))^{n_m} > 0$.

Fourth, and finally, if $x < x_H(\lambda)$ then from Corollary 1 $P_n^\omega(x) \leq P_n^\omega(x_H(\lambda))$ for all n. As such, $\lim P_n^\omega(x) = 0$.

Finally, we prove continuity and monotonicity in x and b. For x, observe that as its value increases from $x_H(\lambda)$ to $x_U(b,\lambda) < x_L(\lambda)$, the term $\Delta^H(x,\underline{\sigma},\lambda)$ increases from 0, and $\Delta^L(x,\underline{\sigma},\lambda)$ increases and remains strictly negative. As such, the term inside parentheses in (A-3) increases from 0, and equals 1 at $x = x_U(b,\lambda)$ (see definition (12)). For b, observe that if $x \leq x_H(\lambda)$, the limit acceptance probability is 0 for all b; while if $x \geq \sup_{b\in(0,1)} x_U(b,\lambda)$, the limit acceptance probability is 1 for all b. For the remaining case of $x \in \left(x_H(\lambda), \sup_{b\in(0,1)} x_U(b,\lambda)\right)$, continuity and monotonicity in b are both immediate if $x < x_U(b,\lambda)$ for all $b \in (0,1)$. Otherwise, since $x_U(b,\lambda)$ is continuous and decreasing in b there exists $\hat{b}$ such that $x = x_U\left(\hat{b},\lambda\right)$, $x < x_U(b,\lambda)$ if $b < \hat{b}$, and $x > x_U(b,\lambda)$ if $b > \hat{b}$. Monotonicity is then immediate, while continuity follows from $-\frac{\Delta^H(x,\underline{\sigma},\lambda)}{\Delta^L(x,\underline{\sigma},\lambda)}\frac{b}{1-b}\ell(\underline{\sigma}) = 1$ at $x = x_U(b,\lambda)$. ∎

Before proving Lemma 4, we establish the following technical result which extends Helly's Selection Theorem from the real line to a compact Euclidian space under certain conditions.

Lemma A-2 *Let $C = [a_i, b_i] \times D$, where D is a compact Euclidian set, and let $g_n : C \to \mathbb{R}$ be a sequence of continuous functions converging pointwise to a continuous function $g : C \to \mathbb{R}$. Suppose that g_n is monotone in its first argument for all n, and has the following property:*

There exists a compact Euclidean set S, a sequence of functions $s_n : C \to S$, and a continuous function $h : C \times S \to \mathbb{R}$ that is strictly monotone and continuously differentiable in its first argument such that, for all n, $g_n(z) = g_n(z')$ whenever $h(z, s_n(z)) = h(z', s_n(z))$.

Then for any $\kappa > 0$, g_n converges uniformly to g over $[a_1 + \kappa, b_1 - \kappa] \times D$.

Proof of Lemma A-2: Given $\kappa > 0$, write $\hat{C} = [a_1 + \kappa, b_1 - \kappa] \times D$. Fix $\varepsilon > 0$ and choose $\mu < \kappa$ such that $|g(z) - g(z')| < \frac{\varepsilon}{4}$ whenever $|z - z'| < \mu$ and $z, z' \in C$.

Define $\psi = \min_{(z,s)\in C\times S} \left|\frac{\partial h(z,s)}{\partial z_1}\right|$, and choose $\delta \in (0,\mu)$ such that $\delta < \psi\mu$.

Choose $\gamma \in (0, \mu)$ such that $|h(z,s) - h(z',s')| < \delta$ whenever $|(z,s) - (z',s')| < \gamma$ and $(z,s), (z',s') \in C \times S$.

Select a finite set $D^* \subset D$ such that for all $z_{-1} \in D$, there exists $z'_{-1} \in D^*$ such that $\left|z_{-1} - z'_{-1}\right| < \gamma$.

By Helly's Selection Theorem, for any $z \in D$ the function sequence $g_n(\cdot, z_{-1})$ converges uniformly to $g(\cdot; z_{-1})$. So there exists some N such that $\left|g_n\left(z_1, z'_{-1}\right) - g\left(z_1, z'_{-1}\right)\right| < \frac{\varepsilon}{2}$ for any $\left(z_1, z'_{-1}\right) \in [a_1, b_1] \times D^*$ whenever $n \geq N$.

Choose any $z \in \hat{C}$, and $n \geq N$. Let $z'_{-1} \in D^*$ be such that $\left|z_{-1} - z'_{-1}\right| < \gamma$. Note that

$$h(z, s_n(z)) \in \left(h\left(z_1, z'_{-1}, s_n(z)\right) - \delta, h\left(z_1, z'_{-1}, s_n(z)\right) + \delta\right).$$

Moreover, as $\tilde{z}_1$ ranges over $(z_1 - \mu, z_1 + \mu)$ the value of $h\left(\tilde{z}_1, z'_{-1}, s_n(z)\right)$ ranges continuously over a superset of

$$\left(h\left(z_1, z'_{-1}, s_n(y,z)\right) - \psi\mu, h\left(z_1, z'_{-1}, s_n(y,z)\right) + \psi\mu\right).$$

Since $\delta < \psi\mu$, there exists some $z'_1 \in (z_1 - \mu, z_1 + \mu)$ such that

$$h(z', s_n(z)) = h(z, s_n(z)),$$

where $z' = \left(z'_1, z'_{-1}\right) \in [a_1, b_1] \times D^*$. By assumption it follows that $g_n(z) = g_n(z')$, and so

$$\begin{aligned} |g_n(z) - g(z)| &\leq |g_n(z') - g(z')| + \left|g(z') - g\left(z_1, z'_{-1}\right)\right| + \left|g\left(z_1, z'_{-1}\right) - g(z)\right| \\ &\leq \frac{\varepsilon}{2} + \frac{\varepsilon}{4} + \frac{\varepsilon}{4} = \varepsilon. \end{aligned}$$

∎

Proof of Lemma 4: The proof involves the application of Lemma A-2. To apply this result, define

$$D = \left[\underline{b}, \bar{b}\right] \times [0,1], \quad C = [0,1] \times D, \quad S = [\underline{\sigma}, \bar{\sigma}] \times [0,1],$$

$$h\left(x, b, \lambda, s^1, s^2\right) = \Delta^H\left(x, s^1, \lambda\right) \frac{b}{1-b} + \Delta^L\left(x, s^1, \lambda\right) \frac{1}{\ell(s^1)} s^2,$$

$$s_n(x,b,\lambda) = \left(\sigma_n^*(x,b,\lambda), \left(\frac{1-F(\sigma_n^*(x,b,\lambda)|L)}{1-F(\sigma_n^*(x,b,\lambda)|H)}\right)^{n-1}\right),$$

and

$$g_n(x,b,\lambda) = P_n^\omega(x,b,\lambda).$$

Note that h is strictly monotone and continuously differentiable in x. It remains to show that if $h(z, s_n(z)) = h(z', s_n(z))$, then $g_n(z) = g_n(z')$.

There are three cases to consider. First, suppose $\sigma_n^*(x,b,\lambda) \in (\underline{\sigma}, \bar{\sigma})$. By construction, $h(z, s_n(z)) = 0$. Suppose that $h(z', s_n(z)) = 0$, Then, by the uniqueness of responsive equilibrium, $\sigma_n^*(x',b',\lambda') = \sigma_n^*(x,b,\lambda)$.

For the next two cases, we claim that if $h_n\left(x,b,\lambda,\sigma,\left(\frac{1-F(\sigma|L)}{1-F(\sigma|H)}\right)^{n-1}\right) \geq 0$ at $\sigma = \underline{\sigma}$, the same is true for all $\sigma \in [\underline{\sigma}, \bar{\sigma}]$; and likewise, if $h_n\left(x,b,\lambda,\sigma,\left(\frac{1-F(\sigma|L)}{1-F(\sigma|H)}\right)^{n-1}\right) \leq 0$ at $\sigma = \bar{\sigma}$, the same is true for all $\sigma \in [\underline{\sigma}, \bar{\sigma}]$. Both these statements follow from the observation that $h_n\left(x,b,\lambda,\sigma,\left(\frac{1-F(\sigma|L)}{1-F(\sigma|H)}\right)^{n-1}\right)$ has the same sign as $Z(x,\sigma,b,\lambda,\alpha,n)$, and observation that if $Z(x,\sigma,b,\lambda,\alpha,n) \geq 0$ for some σ, the same is true for all higher σ (see proof of Lemma 1).

Now consider the case $\sigma_n^*(x,b,\lambda) = \underline{\sigma}$. By definition, $h_n\left(x,b,\lambda,\sigma,\left(\frac{1-F(\sigma|L)}{1-F(\sigma|H)}\right)^{n-1}\right) \geq 0$ for all σ, and so in particular $h_n\left(x,b,\lambda,\underline{\sigma},\left(\frac{1-F(\underline{\sigma}|L)}{1-F(\underline{\sigma}|H)}\right)^{n-1}\right) \geq 0$. So if

$$h_n\left(x',b',\lambda',\underline{\sigma},\left(\frac{1-F(\underline{\sigma}|L)}{1-F(\underline{\sigma}|H)}\right)^{n-1}\right) = h_n\left(x,b,\lambda,\underline{\sigma},\left(\frac{1-F(\underline{\sigma}|L)}{1-F(\underline{\sigma}|H)}\right)^{n-1}\right),$$

it follows (by above claim) that $h_n\left(x',b',\lambda',\sigma,\left(\frac{1-F(\sigma|L)}{1-F(\sigma|H)}\right)^{n-1}\right) \geq 0$ for all σ, and hence $\sigma_n^*(x',b',\lambda') = \underline{\sigma}$.

Finally, consider the case $\sigma_n^*(x,b,\lambda) = \bar{\sigma}$. By definition, $h_n\left(x,b,\lambda,\sigma,\left(\frac{1-F(\sigma|L)}{1-F(\sigma|H)}\right)^{n-1}\right) \leq 0$ for all σ, and so in particular $h_n\left(x,b,\lambda,\bar{\sigma},\left(\frac{1-F(\bar{\sigma}|L)}{1-F(\bar{\sigma}|H)}\right)^{n-1}\right) \leq 0$. So if

$$h_n\left(x',b',\lambda',\bar{\sigma},\left(\frac{1-F(\bar{\sigma}|L)}{1-F(\bar{\sigma}|H)}\right)^{n-1}\right) = h_n\left(x,b,\lambda,\bar{\sigma},\left(\frac{1-F(\bar{\sigma}|L)}{1-F(\bar{\sigma}|H)}\right)^{n-1}\right),$$

it follows (by above claim) that $h_n\left(x', b', \lambda', \sigma, \left(\frac{1-F(\sigma|L)}{1-F(\sigma|H)}\right)^{n-1}\right) \leq 0$ for all σ, and hence $\sigma_n^*(x', b', \lambda') = \bar{\sigma}$. ∎

Proof of Proposition 3: For clarity, we suppress α throughout. Let $v_n(x, \sigma_0, b, \lambda)$ denote the proposer's expected payoff from an offer x when he has observed σ_0, the responders attach belief b to offer x, and have preferences λ, i.e.,

$$v_n(x, \sigma_0, b, \lambda) \equiv E_\omega\left[\bar{V}^\omega(\sigma_0) + P_n^\omega(x, b, \lambda)\left(V^\omega(x, \sigma_0) - \bar{V}^\omega(\sigma_0)\right)|\sigma_0\right]. \qquad \text{(A-4)}$$

Define P^ω and v as the pointwise limits of P_n^ω and v respectively.

We first establish that the proposer's offer is less than $x_U(\underline{b}, \lambda)$. If $x_U(\underline{b}, \lambda) = \infty$ this is vacuously true. If instead $x_U(\underline{b}, \lambda) \leq 1$, then $x_U(b, \lambda) < x_U(\underline{b}, \lambda)$ for all $b > \underline{b}$. From Lemma 3, it follows that the offer $x_U(\underline{b}, \lambda)$ is accepted with probability one no matter what beliefs responders attach to it. As such, there is no equilibrium in which the proposer ever offers strictly more than $x_U(\underline{b}, \lambda)$.

Next, we establish the lower bound on the proposer's equilibrium offer. We know that for any beliefs b, $x_H(\lambda) < \min\{x_U(b, \lambda), 1\}$ at $\lambda = 0$. Choose $\check{\lambda} > 0$ and $\check{x}$ such that

$$\max_{\lambda \in [0, \check{\lambda}]} x_H(\lambda) < \check{x} < \min_{\lambda \in [0, \check{\lambda}]} \min\{x_U(b, \lambda), 1\}.$$

Choose $\varepsilon_x > 0$ such that $\min_{\lambda \in [0, \check{\lambda}]} x_H(\lambda) > \varepsilon_x$ and $\check{x} < 1 - \varepsilon_x$.

From Lemma 3, for any $(b, \lambda) \in [0, \check{\lambda}] \times [\underline{b}, \bar{b}]$ the limit acceptance probabilities $P(\check{x}, b, \lambda|\omega)$ are strictly positive for $\omega = L, H$. By continuity and compactness it follows that there exists $\varepsilon > 0$ such that for any $(\sigma_0, b, \lambda) \in [\underline{\sigma}, \bar{\sigma}] \times [0, \check{\lambda}] \times [\underline{b}, \bar{b}]$,

$$v(\check{x}, \sigma_0, b, \lambda) > E_\omega\left[\bar{V}^\omega(\sigma_0)|\sigma_0\right] + 4\varepsilon.$$

From Lemma 4, v_n converges uniformly to v over $[\varepsilon_x, 1 - \varepsilon_x] \times [\underline{\sigma}, \bar{\sigma}] \times [\underline{b}, \bar{b}] \times [0, 1]$. This implies that there exists N_1 such that whenever $n \geq N_1$,

$$v_n(\check{x}, \sigma_0, b, \lambda) > E_\omega\left[\bar{V}^\omega(\sigma_0)|\sigma_0\right] + 3\varepsilon.$$

Let $\varpi = \max_{\omega,\sigma_0,x} V^{\omega}(x,\sigma_0) - \bar{V}^{\omega}(\sigma_0)$. From Lemma 3, there exists $\kappa > 0$ such that $P(x,b,\lambda) \leq \frac{\varepsilon}{\varpi}$ if $x \in [0, x_H(\lambda) + \kappa]$. Uniform convergence of $P_n^{\omega}(\cdot)$ to $P^{\omega}(\cdot)$ (Lemma 4) implies that there exists N_2 such that $P_n^{\omega}(x_H(\lambda) + \kappa, b, \lambda) \leq \frac{2\varepsilon}{\varpi}$ whenever $n \geq N_2$. Combined with monotonicity of the acceptance probability in the offer x, it follows that if $x \leq x_H(\lambda) + \kappa$ and $n \geq N_2$,

$$v_n(x,\sigma_0,b,\lambda) \leq E_{\omega}\left[\bar{V}^{\omega}(\sigma_0)\,|\sigma_0\right] + 2\varepsilon.$$

Hence for $n \geq \max\{N_1, N_2\}$, for any $(\sigma_0,\lambda) \in [\underline{\sigma},\bar{\sigma}] \times \left[0,\check{\lambda}\right]$ it cannot be an equilibrium for the proposer to offer $x \in [0, x_H(\lambda) + \kappa]$: doing so generates at most 2ε over the status quo payoff, while offering $\check{x}$ generates at least 3ε over the status quo payoff, regardless of beliefs. This completes the proof. ∎

Proof of Lemma 5: We prove the lemma in four steps. For clarity, we suppress λ and α and write x_ω in place of $x_\omega(\lambda;\alpha)$ throughout.

Claim 1 *If* $\limsup x_n < x_H$ *then* $\liminf \sigma_n^* > \sigma_H$*.*

Proof: By hypothesis, there exists ε such that $x_n \leq x_H - \varepsilon$ for all n large enough. Suppose now to the contrary that $\liminf \sigma_n^* \leq \sigma_H$. So for any $\delta > 0$, there exists a subsequence of σ_n^* such that $\sigma_n^* \leq \sigma_H + \delta$. By definition $\Delta^H(x_H,\sigma_H,\lambda) = 0$; so for δ small enough, there exists $\hat{\varepsilon}$ such that $\Delta^H(x_n,\sigma_n^*,\lambda) < -\hat{\varepsilon}$. Moreover, $\Delta^L(x_n,\sigma_n^*,\lambda) \leq \Delta^H(x_n,\sigma_n^*,\lambda)$. Consequently $Z(x_n,\sigma_n^*) < 0$. As such, σ_n^* is not a responsive equilibrium; and since $x_n \leq \bar{x}_n$ then σ_n^* is not an acceptance equilibrium either. The only remaining possibility is that σ_n^* is a rejection equilibrium — but then $\sigma_n^* = \bar{\sigma}$, which gives a contradiction when δ is chosen small enough. ∎

Claim 2 *If* $\limsup x_n < x_L$ *then* $\liminf \sigma_n^* > \sigma_L$*.*

Proof: By hypothesis, there exists ε such that $x_n \leq x_L - \varepsilon$ for all n large enough. Suppose now to the contrary that $\liminf \sigma_n^* \leq \sigma_L$. So for any $\delta > 0$, there exists a subsequence of σ_n^* such that $\sigma_n^* \leq \sigma_L + \delta$. By definition $\Delta^L(x_L, \sigma_L, \lambda) = 0$; so for δ small enough, there exists $\hat{\varepsilon}$ such that $\Delta^L(x_n, \sigma_n^*, \lambda) < -\hat{\varepsilon}$. Next, define

$$\phi = \max_{\sigma \in [\underline{\sigma}, \sigma_L + \delta]} \frac{(1 - F(\sigma|H))^\alpha F(\sigma|H)^{1-\alpha}}{(1 - F(\sigma|L))^\alpha F(\sigma|L)^{1-\alpha}}$$

Note that the function $(1-q)^\alpha q^{1-\alpha}$ is increasing for $q \in (0, 1-\alpha)$ and decreasing for $q \in (1-\alpha, 1)$. Recall that by definition $F(\sigma_L|L) = 1 - \alpha$, and by Lemma A-1 $F(\sigma|H) < F(\sigma|L)$ for all $\sigma \in (\underline{\sigma}, \bar{\sigma})$. It follows that $\phi < 1$ for δ chosen small enough, and so

$$\left(\frac{(1 - F(\sigma^*|H))^\alpha F(\sigma^*|H)^{1-\alpha}}{(1 - F(\sigma^*|L))^\alpha F(\sigma^*|L)^{1-\alpha}} \right)^n \leq \phi^n \to 0.$$

Since σ_n^* is bounded away from $\bar{\sigma}$, then $1 - F(\sigma_n^*|H)$ is bounded away from 0. By belief consistency, $\frac{\beta_n(x_n)}{1-\beta_n(x_n)}$ is bounded away from infinity. Consequently $Z(x_n, \sigma_n^*) < 0$ for n sufficiently large. A contradiction then follows as in Claim 1. ∎

Claim 3 *If* $\liminf x_n > x_L$ *then* $\limsup \sigma_n^* < \sigma_L$.

Proof: By hypothesis, there exists ε such that $x_n \geq x_L + \varepsilon$ for all n large enough. Suppose that contrary to the claim $\limsup \sigma_n^* \geq \sigma_L$. So for any δ, there exists a subsequence such that $\sigma_n^* \geq \sigma_L - \delta$. By definition, $\Delta^L(x_L, \sigma_L, \lambda) = 0$; so for δ small enough, there exists $\hat{\varepsilon}$ such that $\Delta^L(x_n, \sigma_n^*, \lambda) > \hat{\varepsilon}$. Moreover, $\Delta^H(x_n, \sigma_n^*, \lambda) \geq \Delta^L(x_n, \sigma_n^*, \lambda)$. Consequently $Z(x_n, \sigma_n^*) > 0$ for n sufficiently large. So σ_n^* cannot be a responsive equilibrium; and since $x_n \geq \underline{x}_n$ it is not a rejection equilibrium either. The only remaining possibility is that σ_n^* is an acceptance equilibrium — but then $\sigma_n^* = \underline{\sigma}$, which gives a contradiction when δ is chosen small enough. ∎

Claim 4 *If* $\liminf x_n > x_H$ *then* $\limsup \sigma_n^* < \sigma_H$.

Proof: By hypothesis, there exists ε such that $x_n \geq x_H + \varepsilon$ for all n large enough. Suppose now to the contrary that $\limsup \sigma_n^* \geq \sigma_H$. So for any $\delta > 0$, there exists a subsequence of σ_n^* such that $\sigma_n^* \geq \sigma_H - \delta$. By definition $\Delta^H(x_H, \sigma_H, \lambda) = 0$; so for δ small enough, there exists $\hat{\varepsilon}$ such that $\Delta^H(x_n, \sigma_n^*, \lambda) > \hat{\varepsilon}$. Next, define

$$\phi = \min_{\sigma \in [\sigma_H - \delta, \bar{\sigma}]} \frac{(1 - F(\sigma|H))^\alpha F(\sigma|H)^{1-\alpha}}{(1 - F(\sigma|L))^\alpha F(\sigma|L)^{1-\alpha}}$$

Recall that by definition $F(\sigma_H|H) = 1 - \alpha$. By arguments similar to those in Claim 2, it follows that $\phi > 1$ for δ chosen small enough, and so

$$\left(\frac{(1 - F(\sigma^*|H))^\alpha F(\sigma^*|H)^{1-\alpha}}{(1 - F(\sigma^*|L))^\alpha F(\sigma^*|L)^{1-\alpha}} \right)^n \geq \phi^n \to \infty.$$

From Lemma A-1, the term $\frac{1-F(\sigma|L)}{1-F(\sigma|H)}$ lies above $\ell(\bar{\sigma})$. By belief consistency, $\frac{\beta_n(x_n)}{1-\beta_n(x_n)}$ is bounded away from zero. Consequently $Z(x_n, \sigma_n^*) > 0$ for n sufficiently large. A contradiction then follows as in Claim 3. ∎

Proof of Lemma 6: We focus on Part 1a. (Part 1b and 2 are proved by similar arguments, which we omit for conciseness. Part 3 is immediate from Lemma 5.) The main idea is straightforward: for any σ_0 such that $W(\sigma_0) > 0$, the proposer prefers offering $x_H(\lambda; \alpha)$ and gaining acceptance if and only if $\omega = H$ to offering $x_L(\lambda, \alpha)$ and gaining acceptance all the time. Given the limiting behavior of responders established in Lemma 5, intuitively it follows that the proposer's offer converges to $x_H(\lambda; \alpha)$ as the number of responders grows large. The main difficulty encountered in the formal proof is establishing uniform convergence: for any $\varepsilon, \delta > 0$, there is some $N(\varepsilon, \delta)$ such that when $n \geq N(\varepsilon, \delta)$, the proposer's offer lies within δ of $x_H(\lambda; \alpha)$ for all σ_0 such that $W(\sigma_0) > \varepsilon$.

Take any $\varepsilon, \delta > 0$. Throughout the proof, we omit all λ and α arguments for readability. We define $\Delta_0^\omega(x, \sigma_0) \equiv V^\omega(x, \sigma_0) - \bar{V}^\omega(\sigma_0)$, the proposer's gain to offer x being accepted conditional on ω.

Preliminaries: The first part of the proof consists of defining bounds which we will use to establish uniform convergence below. Choose μ, δ_1, δ_2, $\delta_3 \in (0, \delta]$ such that

$x_H+\mu<x_L-\mu$, and for all σ_0 for which $W(\sigma_0)>\varepsilon$,

$$p^H(\sigma_0)V^H\left(x_H+\frac{\mu}{2},\sigma_0\right)+p^L(\sigma_0)\bar{V}^L(\sigma_0)\geq E\left[V^\omega(x_L-\mu,\sigma_0)\,|\sigma_0\right]+\frac{\varepsilon}{2}, \tag{A-5}$$

$$\delta_1\Delta_0^H\left(x_H+\frac{\mu}{2},\sigma_0\right)\leq\frac{\varepsilon}{4}, \tag{A-6}$$

$$\delta_2\left(p^H(\sigma_0)\Delta_0^H(0,\sigma_0)+p^L(\sigma_0)\Delta_0^L(0,\sigma_0)\right)<(1-\delta_1)\,p^H(\sigma_0)\Delta_0^H\left(x_H+\frac{\mu}{2},\sigma_0\right), \tag{A-7}$$

$$p^H(\sigma_0)\left((1-\delta_1)\,\Delta_0^H\left(x_H+\frac{\mu}{2},\sigma_0\right)-\Delta_0^H(x_H+\mu,\sigma_0)\right)>p^L(\sigma_0)\delta_3\Delta_0^L(x_H+\mu,\sigma_0), \tag{A-8}$$

$$\begin{aligned} & p^H(\sigma_0)\left((1-\delta_1)\,V^H\left(x_H+\frac{\mu}{2},\sigma_0\right)+\delta_1\bar{V}^H(\sigma_0)\right)+p^L(\sigma_0)\bar{V}^L(\sigma_0) \\ > \; & p^H(\sigma_0)\left((1-\delta)\,V^H(x_H-\mu,\sigma_0)+\delta\bar{V}^H(\sigma_0)\right) \\ + \; & p^L(\sigma_0)\left(\delta_3V^L(x_H-\mu,\sigma_0)+(1-\delta_3)\,\bar{V}^L(\sigma_0)\right). \end{aligned} \tag{A-9}$$

A choice of μ, δ_1, δ_2, δ_3 exists such that (A-5), (A-6), (A-7), (A-8), and (A-9) hold as follows. First, choose μ such that (A-5) holds, along with

$$V^H\left(x_H+\frac{\mu}{2},\sigma_0\right)>(1-\delta)\,V^H(x_H-\mu,\sigma_0)+\delta\bar{V}^H(\sigma_0). \tag{A-10}$$

It is possible to choose $\mu>0$ that satisfies these two inequalities for all σ_0 since $|V_x^\omega|$ is bounded. The same argument applies in choosing $\delta_1,\delta_2,\delta_3$ below. Second, choose δ_1 such that (A-6) holds, along with

$$(1-\delta_1)\,\Delta_0^H\left(x_H+\frac{\mu}{2},\sigma_0\right)-\Delta_0^H(x_H+\mu,\sigma_0)>0, \tag{A-11}$$

$$\left((1-\delta_1)\,V^H\left(x_H+\frac{\mu}{2},\sigma_0\right)+\delta_1\bar{V}^H(\sigma_0)\right)-\left((1-\delta)\,V^H(x_H-\mu,\sigma_0)+\delta\bar{V}^H(\sigma_0)\right)>0, \tag{A-12}$$

where (A-12) is possible by (A-10). Third, choose δ_2 such that (A-7) holds. Finally, choose δ_3 such that (A-8) and (A-9) hold, which is possible by (A-11) and (A-12) respectively.

Let $\underline{b}$ and $\bar{b}$ respectively denote the most pro-L and pro-H beliefs possible. Fix a realization of σ_0 such that $W(\sigma_0)\geq\varepsilon$. Define the following offer sequences, which we use throughout the proof:

$$x_n^{H+} \equiv x_H + \frac{\mu}{2}, \quad x_n^{H-} \equiv x_H - \mu, \quad x_n^{L-} \equiv x_L - \mu.$$

By Lemma 5, $P_n^H\left(x_n^{H+}, \underline{b}\right) \to 1$ and $P_n^L\left(x_n^{H+}, \bar{b}\right) \to 0$; $P_n^\omega\left(x_n^{H-}, \bar{b}\right) \to 0$ for $\omega = L, H$; and $P_n^L\left(x_n^{L-}, \bar{b}\right) \to 0$. Thus there exist N_1, N_2, N_3 such that $P_n^H\left(x_n^{H+}, \underline{b}\right) \geq 1 - \delta_1$ and $P_n^L\left(x_n^{H+}, \bar{b}\right) \leq \delta_1$ for $n \geq N_1$; $P_n^\omega\left(x_n^{H-}, \bar{b}\right) \leq \delta_2$ for $\omega = L, H$ and $n \geq N_2$; and $P_n^L\left(x_n^{L-}, \bar{b}\right) \leq \delta_3$ for $n \geq N_3$. Let $N(\varepsilon, \delta) = \max\{N_1, N_2, N_3\}$. Note that $N(\varepsilon, \delta)$ depends only on ε and δ, and not σ_0.

Given σ_0, choose x_n from the set of equilibrium offers when the number of responders is n.

Part A: If $W(\sigma_0) \geq \varepsilon$ and $n \geq N(\varepsilon, \delta)$, then for any equilibrium offer x_n, $P_n^L(x_n) \leq \delta_3 \leq \delta$.

Proof: If $x_n \leq x_n^{L-}$ then $P_n^L(x_n) \leq P_n^L\left(x^{L-}, \bar{b}\right) \leq \delta_3$ for $n \geq N(\varepsilon, \delta)$. Consequently it suffices to show that $x_n \leq x_n^{L-}$ for all $n \geq N(\varepsilon, \delta)$. If this were not the case, there must exist some $m \geq N(\varepsilon, \delta)$ such that $x_m > x_m^{L-}$. By Assumption 4 the proposer is always better off when his offer is accepted; and so if $x_m > x_m^{L-}$ the proposer's expected payoff is bounded above by $E\left[V^\omega\left(x_m^{L-}, \sigma_0\right)|\sigma_0\right]$. In contrast, since $m \geq N(\varepsilon, \delta)$, the proposer's payoff from the offer x_m^{H+} is bounded below by

$$\begin{aligned}
& p^H(\sigma_0)\left((1-\delta_1)V^H\left(x_m^{H+}, \sigma_0\right) + \delta_1 \bar{V}^H(\sigma_0)\right) + p^L(\sigma_0)\bar{V}^L(\sigma_0) \\
= \; & p^H(\sigma_0)\left(V^H\left(x_m^{H+}, \sigma_0\right) - \delta_1 \Delta_0^H\left(x_m^{H+}, \sigma_0\right)\right) + p^L(\sigma_0)\bar{V}^L(\sigma_0) \\
\geq \; & p^H(\sigma_0)V^H\left(x_m^{H+}, \sigma_0\right) + p^L(\sigma_0)\bar{V}^L(\sigma_0) - \frac{\varepsilon}{4}
\end{aligned}$$

where the inequality follows by (A-6) (and the fact that $p^H(\sigma_0) \leq 1$). By (A-5) this lower bound exceeds $E\left[V^\omega\left(x_m^{L-}, \sigma_0\right)|\sigma_0\right]$, contradicting the optimality of x_n.

Part B: If $W(\sigma_0) \geq \varepsilon$ and $n \geq N(\varepsilon, \delta)$, then for any equilibrium offer x_n, $|x_n - x_H| \leq \mu \leq \delta$.

Proof: First, we claim that $x_n > x_n^{H-}$ whenever $n \geq N(\varepsilon, \delta)$. If this were not the case, there must exist some $m \geq N(\varepsilon, \delta)$ such that $x_m \leq x_m^{H-}$. The acceptance probability of

x_m given ω is consequently less than that of x_m^{H-} under the most pro-acceptance beliefs $\bar{b}$, which is in turn less than δ_2. The acceptance probability of x_m^{H+} given H is at least $1-\delta_1$. It follows from (A-7) that the proposer's payoff is higher under x_m^{H+} than under x_m. But this contradicts the optimality of the proposer's offer x_m. Second, we claim that $x_n \leq x_H + \mu$ whenever $n \geq N(\varepsilon, \delta)$. If not, there exists $m \geq N(\varepsilon, \delta)$ such that $x_m > x_H + \mu$. By Part A, proposer's payoff under x_m is bounded above by

$$p^H(\sigma_0)\left(\Delta_0^H(x_H+\mu, \sigma_0) + \bar{V}^H(\sigma_0)\right) + p^L(\sigma_0)\left(\delta_3\Delta_0^L(x_H+\mu, \sigma_0) + \bar{V}^L(\sigma_0)\right).$$

In contrast, since $m \geq N(\varepsilon, \delta)$, the proposer's payoff from the offer x_m^{H+} is bounded below by

$$p^H(\sigma_0)\left((1-\delta_1)\Delta_0^H\left(x_m^{H+}, \sigma_0\right) + \bar{V}^H(\sigma_0)\right) + p^L(\sigma_0)\bar{V}^L(\sigma_0),$$

which exceeds the payoff from the offer x_m by (A-8), contradicting optimality of x_m.

Part C: If $W(\sigma_0) \geq \varepsilon$ and $n \geq N(\varepsilon, \delta)$, then for any equilibrium offer x_n, $P_n^H(x_n) \geq 1-\delta$.

Proof: Suppose that contrary to the claim, there exists $m \geq N(\varepsilon, \delta)$ such that $P_m^H(x_m) < 1-\delta$. By Part A, $P_n^L\left(x_m^{L-}, \bar{b}\right) \leq \delta_3$, and by Part B, $x_m \geq x_m^{H-}$ and hence proposer's payoff is bounded above by

$$p^H(\sigma_0)\left((1-\delta)V^H\left(x_m^{H-}, \sigma_0\right) + \delta\bar{V}^H(\sigma_0)\right) + p^L(\sigma_0)\left(\delta_3 V^L\left(x_m^{H-}, \sigma_0\right) + (1-\delta_3)\bar{V}^L(\sigma_0)\right).$$

In contrast, under the offers x_m^{H+}, the proposer's payoff is bounded below by

$$p^H(\sigma_0)\left((1-\delta_1)V^H\left(x_m^{H+}, \sigma_0\right) + \delta_1\bar{V}^H(\sigma_0)\right) + p^L(\sigma_0)\bar{V}^L(\sigma_0).$$

By (A-9) the latter is strictly greater, contradicting the optimality of the offers x_n. ∎