Robust Herding with Endogenous Ordering and One-Sided Commitment

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Abstract

When making sequential decisions under imperfect information, one may learn from other people's choices. Herding occurs when people ignore their own information and follow their predecessors. Consequently, their decisions are uninformative to others, which prevents information aggregation. Therefore, the initial realization of signals can have long-term consequences and herd behavior is often error prone. We analyze an endogenous ordering sequential decision model with one-sided commitment in which decision makers are allowed to choose the time of acting or waiting. We characterize herd behavior under endogenous ordering and compare it with herd behavior under exogenous ordering, in which only one decision maker moves at each period in an exogenously given order. We then show that with endogenous ordering, if decision makers are patient enough, at any fixed time, nearly all decision makers wait due to the negligible information disclosed. In this case, if decision makers can be forced to move with an exogenous order, the resulting equilibrium is more efficient because exogenous ordering tends to aggregate more information.

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1 Introduction

How do people make sequential decisions under imperfect information? One may learn from his own experiences or from other people's choices. For instance, individuals currently using a particular software package may also have the choice of upgrading to a new software package. They may have some knowledge about the new software package. But if the new software package is brand new and private information is limited, individuals may be inclined to wait for other people to discourse more information about the newly released software before they take any action. If the information previously aggregated dominates their own private information, individuals ignore their own private information and follow their predecessors – **herding** occurs. Herding prevents the aggregation of information. Therefore, the initial realization of signals can have long-term consequences and herd behavior is often error prone. The decisions of the first few individuals' can have a disproportional effect.

Bikhchandani, Hirshleifer, and Welch (1992), hereafter BHW, and Banerjee (1992) investigate herd behavior under **exogenous ordering**, in which the decision ordering is exogenously given and only one individual moves at each period. The restaurant example in Banerjee (1992) may fit the exogenous ordering setting.[1] But in many other cases, **endogenous ordering** which allows individuals to choose the time of acting or waiting may be more appropriate. For instance, when individuals decide to buy a new car or computer, they have the option to buy immediately or to wait. With endogenous ordering, there exist strategic interactions among decision makers. Due to the free-rider problem, some decision makers may have incentives to delay their decisions and learn from other decision makers, while others make decisions immediately if they feel confident that their decisions will produce desirable results. Furthermore, more than one individual can act or wait during the same period and consequently their decisions can be clustered together. Thus, under the endogenous ordering setting, the insight will be completely different from that under the exogenous ordering setting. Our main question for inquiry is: if we allow decision makers to choose the time of acting or waiting, will herd behavior be more or less error prone?

Continuing with the software upgrading example, there is a new software package A available for upgrading. Individuals are currently using a software package B. It is known that with some prior probability A is better than B. Each individual also gets a private signal indicating whether A is better or not. Upgrading to A is an irreversible choice. Once they upgrade to A, they are committed to their decisions.[2] But there is no commitment to continuing using B. If individuals have not upgraded,

[1] In the restaurant example in Banerjee (1992), there are two restaurants next to each other. Individuals arrive at the restaurants in sequence. Observing the choices made by people before them, they decide on either one of the two restaurants.

[2] There exists extremely high "disruption costs" involved in upgrading. In other words, we could see this upgrade as a perpetual American call option. Individuals are free to exercise the option at any time they want. But once they exercise the option, they cannot reverse their decision.

they continue to have the option of doing so.[3] Thus, the software upgrading example belongs to the setting of **one-sided commitment**.

In contrast, the restaurant example in Banerjee (1992) is a **two-sided commitment** decision problem. Individuals choose between two restaurants. Choosing either one of the two restaurants is irreversible. Once an individual chooses one restaurant, he cannot go to the other any more. For exogenous ordering, one-sided commitment is equivalent to two-sided commitment because once an individual chooses A or B at his turn, he is out of the game and cannot change his decision any more. But for endogenous ordering, individuals in a one-sided commitment decision problem have two choices: A or B. If they choose A, they cannot change. If they choose B, they still have the option of choosing A later. Individuals in a two-sided commitment decision problem have three choices: A, B or wait. If they choose A or B, they cannot change. If they choose to wait, they still have the option of choosing A or B later. In other words, waiting is equivalent to choosing B in a one-sided commitment decision problem with endogenous ordering.

In this paper we concentrate on the one-sided commitment case.[4] We analyze an endogenous ordering sequential decision model in which decision makers are allowed to choose the time of acting (upgrading to the new software package A) or waiting (continuing using the current software package B). To emphasize the information aspect, we focus on pure information externalities: each decision maker's payoff only depends on his own action and the state of nature. Our main results are summarized below.

1. With endogenous ordering, we show the existence of a symmetric equilibrium with the following monotonicity property: at each period there exists a critical type of individuals who upgrade with probability less than one; all types of individuals with private signals indicating a higher value of A upgrade with probability one; all others wait.

2. In this particular equilibrium, there is a strategic phase, followed by a herding phase. In the **strategic phase**, depending on their own private signals, some individuals upgrade, while others wait. In the **herding phase**, all the remaining individuals either upgrade immediately or wait forever regardless of their own private signals. Compared with the exogenous ordering setting, disclosure of public information has a completely different impact on the strategic and herding behavior of individuals. In particular, if the game is at the upgrade herding phase, all the remaining individuals upgrade immediately and the game ends in one period. Further disclosure of public information will not have any effect.

[3]Throughout the paper, we use the software upgrading example to illustrate our model.
[4]Its companion (Zhang 2006) investigates the two-sided commitment case.

3. With endogenous ordering, if individuals are patient enough, at any fixed time, nearly all individuals wait due to the negligible information disclosed. In this case, if individuals can be forced to move with an exogenous order, the resulting equilibrium is more efficient because exogenous ordering tends to aggregate more information.

There are some papers which investigate the decision problem with endogenous ordering. For example, Chamley and Gale (1994) investigate a discrete time investment model which assumes the timing of decisions is endogenous, that is, individuals try to find the best place in the decision-making queue. In their model, there are only two types of individuals: those with investment options and those without. Those individuals without investment options are assumed to be passive. In contrast, in our model we allow for a finite or an infinite number of types of individuals. Given one's own signals, each individual decides whether to upgrade immediately or to wait and learn the true value of the new software package A by observing other individuals' actions.

The rest of the paper is organized as follows. Section 2 begins with an example of two types of individuals in an attempt to capture our main idea. Section 3 provides the setup of a general model and shows the existence of a symmetric equilibrium with the monotonicity property. Then we characterize herd behavior under exogenous ordering and endogenous ordering and discuss our main results. Several extensions and modifications of the general model are presented in section 4 before we offer our conclusion in Section 5.

2 An Example

We begin with an example of two types of individuals who choose to either upgrade to the new software package A or to continue using the current software package B. If an individual continues using the current software package B, he gets a reservation utility V^0, normalized to zero. The benefit from A, denoted by V, is the same for all individuals and is either $1/2$ or $-1/2$, with equal prior probability. Each individual privately observes a conditionally independent signal about the true value of V. Individual i's signal μ_i is either H or L as described in the following table, where $p > 1/2$.

	$Pr(\mu_i = H \vert V)$	$Pr(\mu_i = L \vert V)$
$V = 1/2$	p	$1-p$
$V = -1/2$	$1-p$	p

The common discount factor is δ. Although the discount factor does not play a role in the decision making under the exogenous ordering setting, it does under the endogenous ordering setting.

Before characterizing and comparing the equilibrium results of exogenous and endogenous ordering settings, we describe some benchmark cases for comparison. If there are no interactions among the individuals, each individual makes a **self-decision** using his own private signal and the prior probabilities. The probability for each individual making the correct choice is p, the precision of the private signal. If there is a **social planner** who can gather the private information from all individuals, then based on all private signals and the prior probabilities, we can imagine that the probability for the social planner of making the correct choice is increasing in the number of conditionally independent signals. Certainly, in the **complete information** case, the true value of the new software package A is known and everyone makes the correct choice. In the other extreme case, if individuals make **random decision**, based on only the prior probabilities, then only half of the individuals will make the correct choice.

2.1 Setting I: exogenous ordering

The ordering of individuals is an exogenous sequence and known to all. Individuals differ in their positions in the queue and only one individual moves at each period. Each individual observes the actions of those before him. When it is his turn to make a decision, he decides to upgrade or to reject A according to current public information and his own private signal. With N individuals, the game ends in N periods. Following the tie-breaking rule in BHW, we assume that an individual indifferent between upgrading and rejecting A chooses to upgrade or reject A with equal probability.[5]

Similar to the specific model in BHW, the equilibrium decision rule is described as follows. At period 1, the first individual rejects A if his signal is L and upgrades to A if his signal is H as the signal precision $p > 1/2$. At period 2, the second individual can infer the first individual's signal from his predecessor's decision. Based on his own private signal and the inferred first individual's signal, the second individual makes the following decision: if the first individual rejects A, he rejects A if his signal is L and rejects or upgrades to A with equal probability $1/2$ if his signal is H; if the first individual upgrades to A, he upgrades to A if his signal is H and rejects or upgrades to A with equal probability $1/2$ if his signal is L. At period 3, we have one of the following three situations: (1) if both predecessors reject A, then the rejecting herding phase starts – the following individuals reject A regardless of their own signals; (2) if both predecessors upgrade to A, the upgrade herding phase starts – the following individuals upgrade to A regardless of their own signals; (3) if one predecessor rejects

[5]The tie-breaking rule matters for the efficiency of the exogenous ordering setting. In section 4.1, we discuss the general tie-breaking rule.

A while the other upgrades to A, the third individual and the forth individual are in the same situation as the first individual and the second individual respectively. The following individuals are in the similar situation until the game ends at period N.

2.2 Setting II: endogenous ordering

Individuals are allowed to choose the time of acting (upgrading to A) or waiting (continuing using B). At any period t, each individual decides to wait or to upgrade to A if he has not upgraded to A yet. If he waits, he gets reservation utility $V^0 = 0$ and has the option of upgrading to A later.

The equilibrium decision rule has the following properties (See the Appendix):

> (i) **Period 1:** For the symmetric equilibrium, there exists a $\delta^*(N, p)$, which is decreasing in N and increasing in p. At period 1, L type individuals will wait to see H type's action. H type individuals will upgrade to A for sure if $\delta \leq \delta^*(N, p)$. Otherwise, H type individuals will upgrade to A with some probability $0 < p_{H,1} < 1$, where $p_{H,1}$ is decreasing in δ and N, and increasing in p.
>
> (ii) **Patient Individuals:** If individuals are patient enough, at any fixed time, nearly all individuals wait due to the negligible information disclosed.

Intuitively, at period 1, for L type individuals, the expected benefit from upgrading to A is $(\frac{1}{2} - p) < 0$. The expected benefit from waiting is greater than or equal to the benefit from waiting forever, which equals zero. Therefore, L type individual will wait for sure at period 1. For H type individuals, at period 1, the expected benefit from upgrading to A is $(p - \frac{1}{2}) > 0$. If no one else upgrades to A, the expected benefit from waiting is equal to the benefit from waiting forever, which equals zero. An H type individual will upgrade to A if no one else upgrades. For a symmetric equilibrium, this means $p_{H,1}$ (the probability of H type individuals upgrading to A at period 1) is greater than zero. If discount factor δ is low enough, H type individuals will upgrade to A for sure. As the number of individuals N increases, precision of signals p decreases, and discount factor δ increases, H type individuals have a higher incentive to wait and $p_{H,1}$ decreases.

We say individuals are patient enough if H type individuals have enough incentive to wait at period 1 such that $p_{H,1}$ is approximately equal to 0. At any period $\infty > t > 1$, the game is "almost" the same as the period 1 game. The probability of H type individuals upgrading to A at period t, which is denoted by $p_{H,t}$, is equal to 0 or approximately equal to 0. Thus, at any fixed time, there is a negligible proportion of individuals upgrading to A and so is the information disclosed.

2.3 Expected Number of Correct Choices

Let $X(N)$ represent the expected number of correct choices with N individuals in the game. Subsequently, $X(N)/N$ is the average expected number of correct choices. We present the following results.

Result 1 *(See the Appendix) For the example above, given δ and p, there exists an N^*, such that if $N < N^*$, the equilibrium with endogenous ordering is more efficient in terms of inducing a larger expected number of correct choices; there also exists an N^{**}, such that if $N > N^{**}$, the equilibrium with exogenous ordering is more efficient in terms of inducing a larger expected number of correct choices.*

Figure 1 sketches out the implication of Result 1. We can see that if N is large enough (stage III in the figure), endogenous ordering is worse than self-decision, not to mention exogenous ordering (if individuals can be forced to move with an exogenous order).

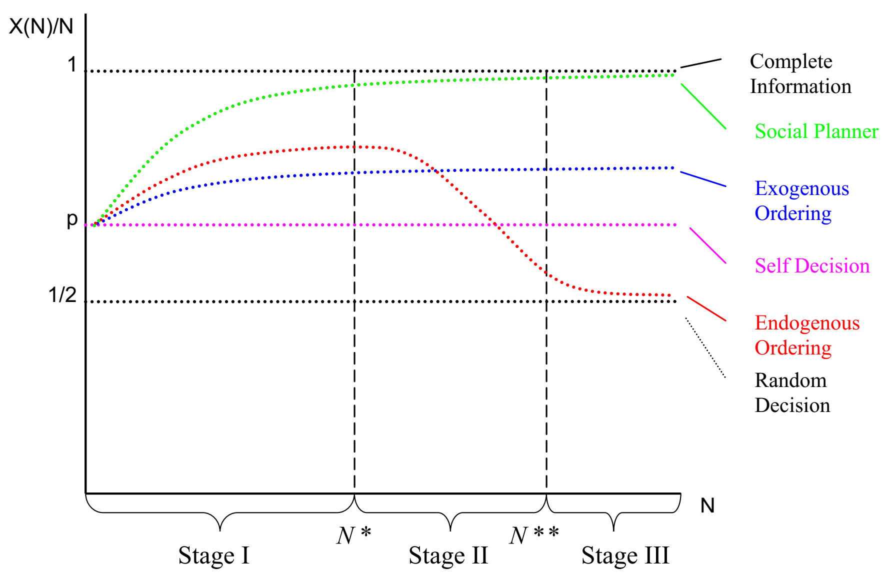

Figure 1: Average Expected Number of Correct Choices

3 The General Model

In this section, we first provide the basic setup of our general model. Then we characterize herd behavior under exogenous ordering and endogenous ordering.

3.1 Basic Setup

There are N individuals. All are rational and risk neutral. There is a new software package A available for upgrading. Individuals currently use software package B. Assume that the true value of A, denoted by V, is chosen by nature at the beginning of the game, and is unknown to the individuals.[6] Individuals only know V follows some prior distribution $F_0(V)$, with density $f_0(V)$. To emphasize the information aspect, we concentrate on pure information externalities: each individual's payoff only depends on his own action and the state of nature.

We focus on the case that upgrading to A is an irreversible binary choice.[7] The indivisibility of the action space is important. As in Banerjee (1992), since the choices made by individuals are not sufficient statistics for the information they have, the error prone herding can occur.[8]

At the beginning of the game, individual i in the market freely observes some conditionally independent private signal $\mu_i \in [\underline{\mu}, \overline{\mu}]$, which follows some distribution $F(\mu_i|V)$, with density $f(\mu_i|V)$. Assume individuals are more likely to get a higher private signal (indicating higher value of A) if the underlying V is higher.

Assumption 1: $F(\mu_i|V)$ satisfies the *Monotone Likelihood Ratio Property* (MLRP)[9] with respect to V, i.e.

$$\frac{f(\mu_i|V_1)}{f(\mu_i|V_2)} \quad \text{increasing in} \quad \mu_i \quad \forall V_1 > V_2$$

If individual i upgrades to A at period t, then at the following periods, everyone

[6] Rosenberg (1976) points out that there exist two types of technological uncertainty. First, when an innovation is introduced, it may have some imperfections: "Innumerable 'bugs' may need to be worked out. The first user often takes considerable risk." In addition, current innovation could be improved further in the future. There are two possible situations for the future possible improvement: expected or unexpected. If it is expected, then it only increases the benefit from waiting by some constant amount. If it is unexpected, it will not affect the strategic interactions of the current game until it happens. Thus, we ignore the second type of technological uncertainty here. When we investigate the switch from one innovation to another, the future improvement, either expected or unexpected, could be incorporated.

[7] There exists extremely high "disruption costs" involved in upgrading. In other words, we could see this upgrade as a perpetual American call option as in Grenadier (1999). In Grenadier (1999), decisions are made in continuous time and there is a state variable, which follows some exogenous continuous time stochastic process. In this paper, we assume discrete time decision and no exogenous state variable.

[8] Banerjee (1992) assumes a continuous action space and gets similar herding results as BHW. This is due to the degenerate payoff function as pointed out by BHW. Park (2001) assumes perfect observability. Therefore, in his model players share the same information and hidden information is not an issue.

[9] Landsberger and Meilijson (1990) point out that this property holds for exponential type families (binomial with the same number of trials, normal with equal variances, etc.) as well as for some non-exponential cases such as uniform with the same left endpoint.

knows individual i upgrades to A at period t. The public information available at the beginning of period t is denoted by h_t, which includes the prior information of V, actions and the equilibrium strategy profile of all individuals before t. If an individual doesn't upgrade to A, he gets reservation utility V^0, normalized to zero. The common discount factor is δ.

3.2 Herd Behavior with Exogenous Ordering

If we assume the ordering of individuals is exogenous, in which only one individual moves at each period in an exogenously given order, then there are no strategic interactions among individuals. When it is one's turn to make a decision, he decides whether to upgrade or to reject A given the current public information and his own private signal.

The equilibrium decision rule is a sequence of critical values

$$\{\mu_t^*(h_t)\}_t$$

such that the individual making the decision at period t upgrades to A if his private signal $\mu_t > \mu_t^*(h_t)$; otherwise, he rejects A.[10] We can see this sequence of critical values is not monotone. If the individual at period t upgrades to A, which indicates $\mu_t > \mu_t^*(h_t)$, this is "good" news for the individual at period $t+1$. Thus, at period $t+1$, $\mu_{t+1}^*(h_{t+1}) \leq \mu_t^*(h_t)$. And vice versa, if the individual at period t rejects A, $\mu_{t+1}^*(h_{t+1}) \geq \mu_t^*(h_t)$.

The game is in the strategic phase when the sequence of critical values $\{\mu_t^*(h_t)\}_t$ fluctuates in between $\underline{\mu}$ and $\overline{\mu}$. In the strategic phase, each individual's decision depends on both the current public information and his own private signal.

Once the sequence of critical values $\{\mu_t^*(h_t)\}_t$ "breaks" either one of the boundaries, herding occurs. The upgrade herding phase starts at period τ if $\mu_\tau^*(h_\tau) = \underline{\mu}$. The individual at period τ will upgrade to A regardless of his own private signal. His decision is, therefore, uninformative to others. Thus, $\mu_t^*(h_t) = \mu_{t-1}^*(h_{t-1}) = \underline{\mu}$ $\forall t > \tau$ (see figure 2). All the following individuals will upgrade to A. Similarly, the rejecting herding phase starts at period τ if $\mu_\tau^*(h_\tau) = \overline{\mu}$. All the following individuals will reject A and $\mu_t^*(h_t) = \mu_{t-1}^*(h_{t-1}) = \overline{\mu}$ $\forall t > \tau$ (see figure 3).

Since public information disclosed only needs to offset the information from the last individual's action before the herding phase starts, both upgrade herding and rejecting herding are not robust to the public disclosure of information. If at a certain period $N \geq t \geq \tau$ there is some public information disclosed such that $\underline{\mu} < \mu_t^*(h_t) < \overline{\mu}$, then the strategic phase starts again.

[10] For notation simplicity, we assume the following tie-breaking rule: if an individual i is indifferent between upgrading and rejecting A, he rejects it whenever $\mu_i \in (\underline{\mu}, \overline{\mu}]$ and upgrades whenever $\mu_i = \underline{\mu}$.

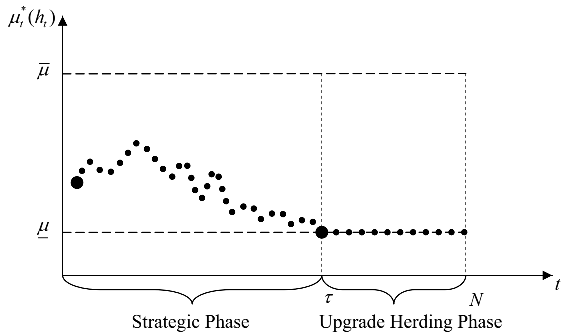

Figure 2: Upgrade herding with exogenous ordering

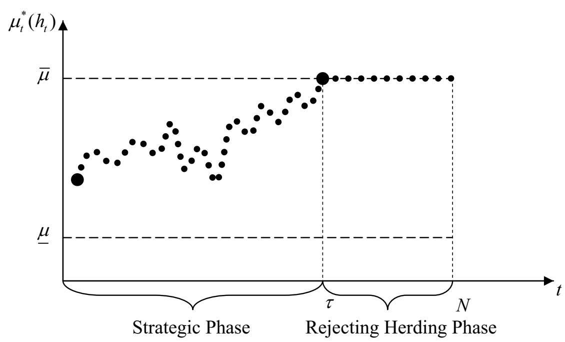

Figure 3: Rejecting herding with exogenous ordering

3.3 Herd Behavior with Endogenous Ordering

If we allow the individuals to choose the time of acting (upgrading to the new software package A) or waiting (continuing using the current software package B), there exist strategic interactions among the individuals.

The timing of endogenous ordering is as follows:

> At period 1, each individual decides whether or not to upgrade to A. If he doesn't upgrade to A at period 1, he gets reservation utility $V^0 = 0$ and has the option of upgrading later.
>
> At period 2, all the remaining individuals decide to upgrade to A or to wait after observing others' actions at period 1.
>
> The subsequent periods are the same as period 2. The game continues until everyone upgrades to A. The time period is denoted by t, $t = 1, 2, 3, ...$

The benefit from waiting is the information revealed about the new software package A by other individuals. The cost of waiting is the difference between the gain from A and the reservation utility.

We first investigate the relationship between the incentive to wait and private information. We prove any possible symmetric equilibrium must be monotone with respect to personal private signals. Then, we show the existence and describe characteristics of a symmetric equilibrium with the monotonicity property by backward induction in two cases: a continuous private signal space and a finite discrete private signal space.[11]

3.3.1 Information and Incentives

The following remark shows that if an individual gets a higher private signal, given the same history, he believes that V will be higher, i.e., the posterior distribution of V satisfies MLRP with respect to private signals.

Remark 1

$$\frac{f(V|\mu_i, h_t)}{f(V|\mu_i', h_t)} \quad \textit{increasing in} \quad V \quad \forall \mu_i > \mu_i'$$

Proof. See the Appendix. ■

The benefit from upgrading to A at period t for individual i is:

$$U^A(\mu_i; h_t) = E_{V|\mu_i;h_t} V \tag{1}$$

The benefit from waiting at period t for individual i is:

$$U^W(\mu_i; h_t; s_{-i,t}) = \delta E_{H_{t+1}(h_t;s_{-i,t})}[max\{U^A(\mu_i; h_{t+1}); U^W(\mu_i; h_{t+1}; s_{-i,t+1})\}] \tag{2}$$

where $s_{-i,t}$ represents the strategy profile of all other individuals except for individual i starting from period t; $H_{t+1}(h_t; s_{-i,t})$ represents the set of histories at the beginning of period $t+1$ given h_t and $s_{-i,t}$. From the above equation, we can solve the benefit from waiting forever $\underline{U}^W = 0$. In this paper, we focus on the symmetric equilibrium.

Lemma 1 *Under the worst news, individuals will never upgrade to A. In our model, the worst news from period t is no one upgrading to A at period t. Under this worst news, the waiting herding phase starts at period $t+1$. Thus, with finite number of N individuals, the game lasts at most N periods before a herding phase starts.*

[11] Since the information disclosed through the backward induction construction process may not be monotone, the equilibrium is not necessarily unique.

Proof. See the Appendix. ■

The next proposition proves that for any possible symmetric equilibrium, it must be monotone with respect to personal private signals. That is, individuals with private signals indicating higher value of A have a higher incentive to upgrade.

Proposition 1

$$U^A(\mu_i; h_t) - U^W(\mu_i; h_t; s_{-i,t}) \quad \textit{increasing in} \quad \mu_i \quad \forall h_t; s_{-i,t}$$

Proof. See the Appendix. ■

3.3.2 Symmetric Equilibrium with the Monotonicity Property

Proposition 2 *There exists a symmetric equilibrium with the following monotonicity property.*

(i) ***Case I: Continuous private signal space***

The equilibrium strategy profile is a sequence of decreasing critical values: $\{\mu_t^*(h_t)\}_t$. *At period* t *with history* h_t, *individuals with* $\mu > \mu_t^*(h_t)$ *upgrade; others wait.*

Case II: Finite discrete private signal space

The equilibrium strategy profile is a sequence of decreasing critical values $\{\mu_t^*(h_t)\}_t$ *and a sequence of probability of this critical type* $\{p_{\mu_t^*(h_t)}\}_t$. *At period* t *with history* h_t, *individuals with* $\mu > \mu_t^*(h_t)$ *upgrade; the critical type individuals upgrade with probability* $p_{\mu_t^*(h_t)}$; *others wait.*[12]

(ii) ***Patient Individuals:*** *If individuals are patient enough, at any fixed time, nearly all individuals wait due to the negligible information disclosed.*

Proof. See the Appendix. ■

Part (i) is from the construction in the above proposition. For part (ii), we say individuals are patient enough if the highest types of individuals have enough incentive to wait at period 1 such that: either $\mu_1^*(h_1) = \overline{\mu}$ or $\mu_1^*(h_1) \approx \overline{\mu}$ (finite discrete private signal space: either $p_{\overline{\mu}} = 0$ or $p_{\overline{\mu}} \approx 0$). At any period $\infty > t > 1$, the game is either the same or "almost" the same as the period 1 game: either $\mu_t^*(h_t) = \mu_{t-1}^*(h_{t-1})$ or $\mu_t^*(h_t) \approx \mu_{t-1}^*(h_{t-1})$ (finite discrete private signal space: either $p_{\overline{\mu}} = 0$ or $p_{\overline{\mu}} \approx 0$).

[12] For simplicity, $p_{\mu_t^*(h_t)} < 1$ is chosen in the construction process so that there is the possibility for the $\mu_t^*(h_t)$ type of individuals to remain in the game at period $t+1$. Moreover, $\mu_t^*(h_t)$ is the highest type at period $t+1$. If a herding phase starts, all the remaining individuals in the game either upgrade ($p_{\mu_t^*(h_t)} = 1$) or wait forever ($p_{\mu_t^*(h_t)} = 0$).

Thus, at any fixed time, there is a negligible proportion of individuals upgrading to A and so is the information disclosed.

From the above proposition, with endogenous ordering the sequence of critical values is monotone. Intuitively, at any period t, all the individuals with $\mu > \mu^*_{t-1}(h_{t-1})$ upgraded before t. At period t, we only need to consider the individuals with private signals between $\underline{\mu}$ and $\mu^*_{t-1}(h_{t-1})$. Thus, $\mu^*_t(h_t) \leq \mu^*_{t-1}(h_{t-1})$.

Upgrade herding occurs at period τ when $\mu^*_\tau(h_\tau) = \underline{\mu}$ (finite discrete signal space: $\mu^*_\tau(h_\tau) = \underline{\mu}; p_{\mu^*_\tau(h_\tau)} = 1$). All the remaining individuals upgrade to A at period τ regardless of their own private signal, and then the game ends (see figure 4).

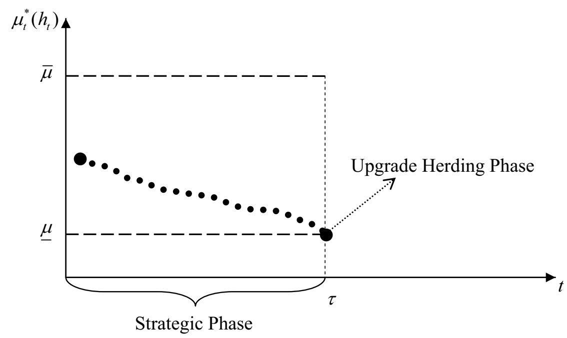

Figure 4: Upgrade herding with endogenous ordering

Waiting herding occurs at period τ when $\mu^*_\tau(h_\tau) = \mu^*_{\tau-1}(h_{\tau-1})$ (finite discrete signal space: $\mu^*_\tau(h_\tau) = \mu^*_{\tau-1}(h_{\tau-1}); p_{\mu^*_\tau(h_\tau)} = 0$). Since no new information is disclosed at the following periods, the game remains the same. $\mu^*_t(h_t) = \mu^*_{t-1}(h_{t-1})\ \forall t > \tau$ (finite discrete signal space: $\mu^*_t(h_t) = \mu^*_{t-1}(h_{t-1}); p_{\mu^*_t(h_t)} = p_{\mu^*_{t-1}(h_{t-1})} = 0\ \forall t > \tau$) (see figure 5).

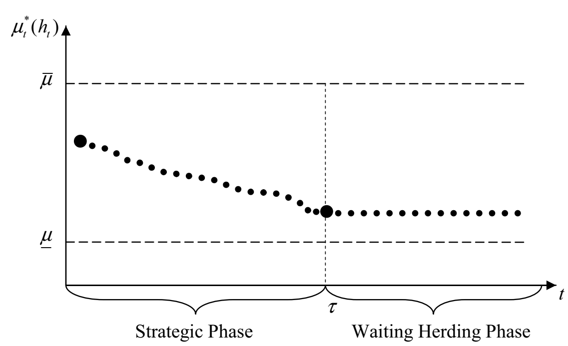

Figure 5: Waiting herding with endogenous ordering

Since the public information disclosed only needs to offset the information from individuals' actions at the last period before the waiting herding phase starts, the waiting herding phase is not robust to the public disclosure of information. If at some period $t > \tau$ there is some public information disclosed such that $\mu_t^*(h_t) < \mu_{t-1}^*(h_{t-1})$ (finite discrete signal space: $\mu_t^*(h_t) \leq \mu_{t-1}^*(h_{t-1})$ with $p_{\mu_t^*(h_t)} > 0$), then the strategic phase starts again. However, if the game falls into the upgrade herding phase, disclosure of public information after τ will not have any effect since the upgrading herding phase only lasts one period.

3.4 Robustness with Respect to the Disclosure of Public Information

We summarize the results of the impact of public information disclosure on herding behavior under exogenous and endogenous ordering settings respectively in the following table.[13]

		exogenous ordering		endogenous ordering	
		upgrade herding	rejecting herding	upgrade herding	waiting herding
time of disclosure	$N \geq t > \tau$	Not Robust	Not Robust	Robust	Not Robust
of public information	$t > N$	Robust	Robust	Robust	Not Robust

By the game construction, under the exogenous ordering setting, the game lasts exactly N periods. Disclosure of public information after period N will not have any effect. Under the endogenous ordering setting, the upgrading herding phase only lasts one period. Disclosure of public information after τ will not have any effect. In contrast, the waiting herding phase under the endogenous ordering setting could last forever. Disclosure of public information after τ or even N may have some effect.

3.5 Expected Number of Correct Choices

Proposition 3 *In the general model, at the case of patient individuals, exogenous ordering is more efficient than endogenous ordering in terms of inducing a larger expected number of correct choices, if individuals can be forced to move with an exogenous order.*

Proof. From proposition 2, with endogenous ordering, if individuals are patient enough, at any fixed time, nearly all individuals wait due to the negligible information disclosed. In contrast, in the self-decision case, each individual still utilizes his own private information and the prior probabilities. Exogenous ordering is even better since it tends to aggregate more information by forcing some of the individuals to

[13] Here, we only talk about the unexpected disclosure of public information. Section 4.3 investigates more variations of public information disclosure.

make decisions at some given periods. Similar to result 1, if individuals are patient enough, as long as the private signals and the prior probabilities are informative, exogenous ordering is more likely to induce a larger expected number of correct choices than endogenous ordering. ■

4 Extensions and Modifications

In this section, we first discuss the general tie-breaking rule for the exogenous ordering setting in the example from section 2 (with two types of individuals). Then we study the asymptotic properties of the equilibrium with endogenous ordering as number of individuals goes to infinity and the effect of the disclosure of public information for the general model.

4.1 General Tie-breaking Rule

In the example presented in section 2.1 with exogenous ordering, we follow the same tie-breaking rule in BHW. An individual indifferent between upgrading and rejecting A chooses to upgrade or reject A with equal probability. Now, we consider the **general tie-breaking rule**: whenever individuals are indifferent between upgrading and rejecting A, H type individuals and L type individuals choose to upgrade to A with probability p_H and p_L respectively. We denote the general tie-breaking rule as $\{p_H; p_L\}$. The tie-breaking rule in BHW is a special case of the general tie-breaking rule with $\{p_H = 1/2; p_L = 1/2\}$. With the general tie-breaking rule $\{p_H; p_L\}$, the equilibrium decision rule is the same as the equilibrium decision rule in section 2.1 except for the tie-breaking cases.

Result 2 *(See the Appendix) In the example with exogenous ordering presented in section 2.1, with the general tie-breaking rule $\{p_H; p_L\}$, the expected number of correct choices is increasing in $p_H - p_L$. In particular, $\{p_H = 1; p_L = 0\}$ is the optimal tie-breaking rule in terms of inducing the maximum expected number of correct choices. Conversely, $\{p_H = 0; p_L = 1\}$ is the worst tie-breaking rule and the equilibrium result of exogenous ordering in this case is the same as the result of self-decision in terms of inducing the same expected number of correct choices.*

Intuitively, when H type individuals are indifferent between upgrading and rejecting A, L type individuals will for sure reject. In this case, for H type individuals, the higher p_H is, the more informative their actions are to the followers in the sense of revealing their own private signals. Conversely, for L type individuals, when they are indifferent between upgrading and rejecting A, the lower p_L is, the more informative their actions are to the followers.

4.2 Large Number of Individuals

For the general model, as the number of individuals goes to infinity, there are three possible cases.

> First, no one upgrades to A. Since no new information is disclosed, the game at the next period will be the same as the current game. Everyone will continue to wait. The benefit from waiting is $\underline{U}^W = 0$ as defined earlier.
>
> Second, there is an infinite number of upgrades. With an infinite number of upgrades, the true value of A will be revealed. In this case, the benefit from waiting achieves its maximum $\overline{U^W}(\mu_i; h_t)$ given μ_i and h_t. Returning to equation 2, we have
>
> $$\overline{U^W}(\mu_i; h_t) = \delta E_{V|\mu_i;h_t}[max\{\frac{1}{1-\delta}V; 0\}]$$
>
> Third, there is a finite number of upgrades. Due to the infinite number of individuals, the proportion of individuals upgrading must be approaching zero.

The following proposition shows the asymptotic properties of the symmetric equilibrium in Proposition 2 in these three different cases.

Proposition 4 *As $N \to \infty$, there are three possible cases for the symmetric equilibrium in Proposition 2:*

(i) Zero amount of information disclosed *No one upgrades. At each of the following periods, everyone will continue to wait.*

(ii) Infinite amount of information disclosed *There is an infinite number of upgrades. At period t, individuals with $\mu > \mu_t^*$ will upgrade to A. Others wait. At period $t+1$, all the individuals remaining in the game will upgrade to A if $\frac{1}{1-\delta}V \geq \underline{U}^W$. Otherwise, they will wait forever. The critical value μ_t^* is the solution to the following equations and inequalities.*

Case I: Continuous private signal space

$$U^A(\mu_t^*; h_t) = \overline{U^W}(\mu_t^*; h_t)$$

Case II: Finite discrete private signal space

$$U^A(\mu_t^*; h_t) \leq \overline{U^W}(\mu_t^*; h_t)$$
$$U^A(\mu; h_t) > \overline{U^W}(\mu; h_t) \quad \forall \mu_{t-1}^* \geq \mu > \mu_t^*$$

(iii) Finite amount of information disclosed *There is a finite number of upgrades. By Poisson approximation, the number of upgrades at period t satisfies Poisson distribution with parameter $\beta(\mu_{t-1}^*; h_t)$, which is the solution of the following equation.*

$$0 = (1-\delta)\{E_{H_{t+1}^A(\mu_{t-1}^*;h_t;\beta(\mu_{t-1}^*;h_t))}U^A(\mu_{t-1}^*; h_{t+1}) + E_{H_{t+1}^{interim}(\mu_{t-1}^*;h_t;\beta(\mu_{t-1}^*;h_t))}U^A(\mu_{t-1}^*; h_{t+1})\} + E_{H_{t+1}^W(\mu_{t-1}^*;h_t;\beta(\mu_{t-1}^*;h_t))}[U^A(\mu_{t-1}^*; h_{t+1}) - \delta \underline{U}^W]$$

where $H_{t+1}^A(\mu_{t-1}^*; h_t; \beta(\mu_{t-1}^*; h_t))$ is the set of histories at period $t+1$ in which individuals with μ_{t-1}^* will upgrade; $H_{t+1}^{interim}(\mu_{t-1}^*; h_t; \beta(\mu_{t-1}^*; h_t))$ is the set of histories at period $t+1$ in which individuals upgrading follows some Poisson distribution; $H_{t+1}^W(\mu_{t-1}^*; h_t; \beta(\mu_{t-1}^*; h_t))$ is the set of histories at period $t+1$ in which individuals with μ_{t-1}^* will wait.

Proof. See the Appendix. ■

4.3 Disclosure of Public Information

Disclosure of public information could have an influence on the strategic and herding behavior of individuals. In BHW, with exogenous ordering, initial public disclosure can make some individuals worse off ex ante. All individuals welcome public information once a herding phase has begun. Herding is delicate with respect to new information. A small amount of public information can shatter a long-lasting herd. With multiple public information disclosures, individuals eventually settle into the correct herd.

In contrast, in our general model with endogenous ordering, we distinguish the unexpected and expected disclosures of public information and announcements of future disclosure of public information at the strategic phase and the waiting herding phase respectively.

Proposition 5 ***(i) Disclosure of public information in the strategic phase***

If there is a disclosure of public information in the strategic phase, either unexpected or expected, all the remaining individuals welcome the new information in the ex ante sense. The announcements of future disclosure of public information will increase the individuals' incentive to wait. However, ex post some individuals may be worse off.

(ii) Disclosure of public information in the waiting herding phase

If there is a disclosure of public information in the waiting herding phase, either unexpected or expected, all the remaining individuals welcome the new information. And the waiting herding phase is delicate with respect to the new information. A small *amount of public information can shatter a waiting herding phase. The announcements of future disclosure of public information don't have any effect on the waiting herding phase until the disclosure of public information actually happens.*

(iii) Multiple disclosures of public information

Multiple disclosures of public information do not always let individuals settle into the correct herd.

Proof. See the Appendix. ■

5 Conclusion

In this paper, we investigate herd behavior of sequential decisions under imperfect information with one-sided commitment. We provide a framework of endogenous ordering to allow decision makers to choose the time of acting or waiting. We show the existence and characteristics of the equilibrium. We find that herd behavior under endogenous ordering is not necessarily less error prone than herd behavior under exogenous ordering due to the free-rider problem. In particular, if individuals are patient enough, under endogenous ordering nearly all individuals are willing to wait and free-ride on others. Consequently, nearly all individuals wait due to the negligible information disclosed. In this case, if decision makers can be forced to move with an exogenous order, the resulting equilibrium is more efficient because exogenous ordering tends to aggregate more information. That is to say, more "freedom" may not be better.

Another feature of the endogenous ordering framework is that in a herding phase, all the remaining individuals either act immediately or wait forever regardless of their own private signals. Thus, there exists an investment surge or collapse when herding starts. Compared with the exogenous ordering setting, disclosure of public information has a completely different impact on the strategic and herding behavior of individuals. In particular, if the game is at the upgrade herding phase, all the remaining individuals upgrade immediately and the game ends in one period. Further disclosure of public information will not have any effect.

Appendix

Proof of the Equilibrium Decision Rule with Endogenous Ordering (The Example from Section 2)

(i) Period 1:

At period 1, for L type individuals, the expected benefit from upgrading to A is $(\frac{1}{2}-p) < 0$. The expected benefit from waiting is greater than or equal to the benefit from waiting forever, which equals zero. Therefore, L type individual will wait for sure at period 1. For H type individuals, the expected benefit from upgrading to A is $(p - \frac{1}{2}) > 0$. If no one else upgrades to A, the expected benefit from waiting is equal to the benefit from waiting forever, which equals zero. Thus, an H type individual will upgrade to A if no one else upgrades.

Let us check the condition that an H type individual i will upgrade to A for sure at period 1 when all other H type individuals upgrade to A for sure at period 1. The expected benefit from upgrading to A is $(p - \frac{1}{2})$. Since all other H type individuals upgrade to A for sure at period 1, all of the possible information is disclosed for individual i at period 2. If the number of individuals upgrading to A at period 1 is greater than or equal to $\lceil \frac{N-1}{2} \rceil$, where $\lceil \frac{N-1}{2} \rceil$ is the smallest integer greater than or equal to $\frac{N-1}{2}$, at period 2 individual i will upgrade to A. Thus, the expected benefit from waiting is

$$\delta \sum_{j=\lceil \frac{N-1}{2} \rceil}^{N-1} Prob(\text{j upgrade at period } 1|\mu_i = H)[Prob(V = 1/2|\text{j+1 H in N}) - 1/2]$$

where

$$Prob(\text{j upgrade at period } 1|\mu_i = H) = \binom{N-1}{j}[p^{j+1}(1-p)^{N-1-j} + (1-p)^{j+1}p^{N-1-j}]$$

$$Prob(V = 1/2|\text{j+1 H in N}) = \frac{p^{2j+2-N}}{p^{2j+2-N} + (1-p)^{2j+2-N}}$$

Let the expected benefit from upgrading to A equal the expected benefit from waiting. We get

$$\delta^*(N,p) = \frac{p - \frac{1}{2}}{\sum_{j=\lceil \frac{N-1}{2} \rceil}^{N-1} \binom{N-1}{j}[p^{j+1}(1-p)^{N-1-j} + (1-p)^{j+1}p^{N-1-j}][\frac{p^{2j+2-N}}{p^{2j+2-N}+(1-p)^{2j+2-N}} - 1/2]}$$

Clearly, $\delta^*(N,p)$ is an increasing function of p and decreasing function of N.

Figure 6 illustrates the locus of $\delta^*(N, p = 0.6)$ decreasing in N. The upper right area is the mixed strategy area of H type individuals, in which H type individuals upgrade to A with some probability $0 < p_{H,1} < 1$. The lower left area is the pure strategy area that H type individuals will for sure upgrade to A at period 1. The intuition is that as the number of individuals increases, everyone has a higher incentive to wait. To induce H type individuals to upgrade to A with probability one, the discount factor should be low.

Figure 7 illustrates the locus of $\delta^*(N,p)$ shifting up as p increases. As the precision of signals p increases, H type individuals have a higher incentive to upgrade to A. Thus, the pure strategy area of H type individuals becomes larger as p increases.

For the symmetric equilibrium, at period 1 L type individuals will wait to see H type's action. H type individuals will upgrade to A for sure if $\delta \leq \delta^*(N,p)$. Otherwise, H type individuals will upgrade to A with some probability $0 < p_{H,1} < 1$.

Similar to the proof of Proposition 1 in Chamley and Gale (1994), as $p_{H,1}$ and N increases, information at period 2 is more informative in the sense of Blackwell. From

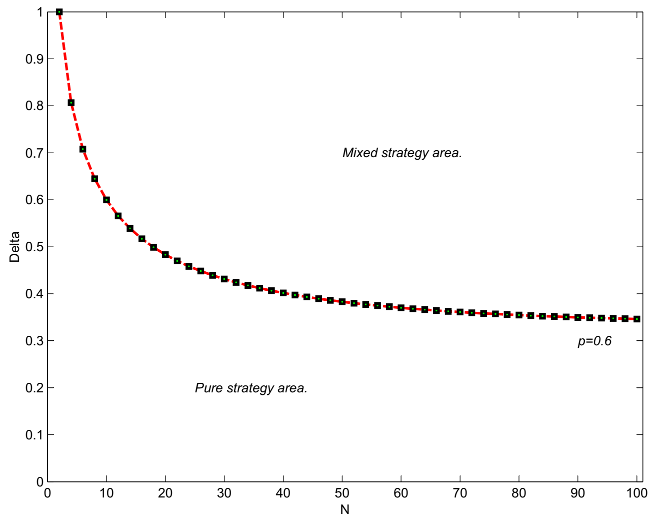

Figure 6: The Example from Section 2 – Endogenous Ordering

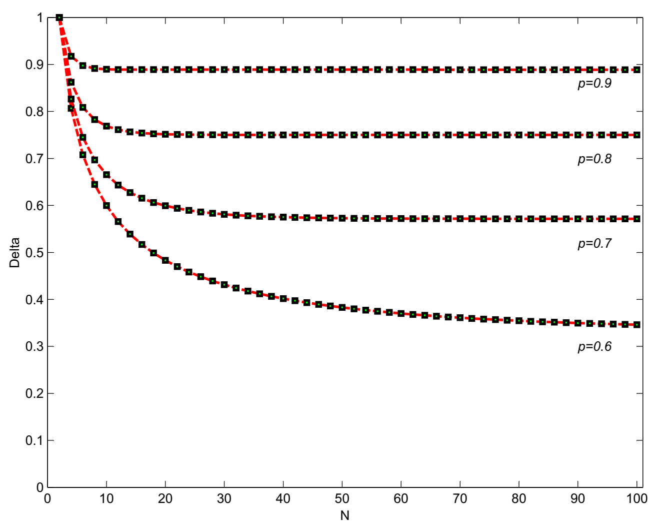

Figure 7: The Example from Section 2 – Endogenous Ordering

Blackwell's theorem, the expected benefit from waiting increases. As δ increases, the expected benefit from waiting increases. As p decreases, the expected benefit from upgrading to A decreases.

To keep H type individuals indifferent between upgrading to A immediately and waiting, we must have $p_{H,1}$ decreasing in δ and N, and increasing in p. Figure 8 illustrates the locus of $p_{H,1}$ when $N = 3$.

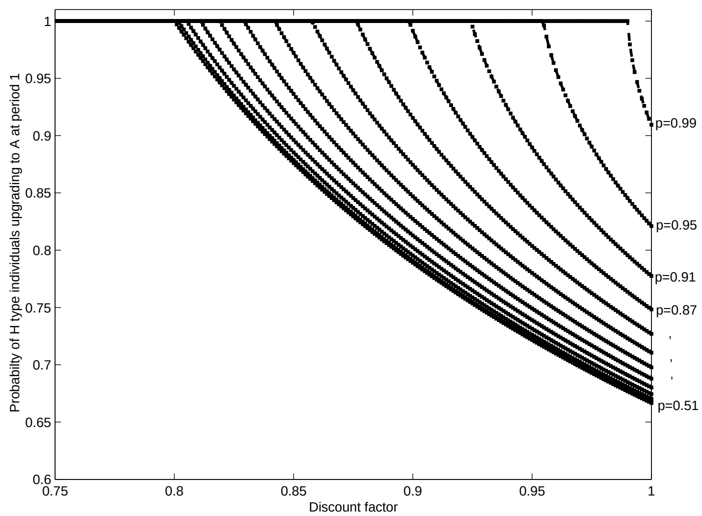

Figure 8: The Example from Section 2 – Endogenous Ordering: N=3

(ii) Patient Individuals:

From the above proof, $p_{H,1}$ is decreasing in δ and N, and increasing in p. If N is large, δ approaches 1, and p approaches 1/2, then $p_{H,1} \approx 0$. At any period $\infty > t > 1$, the game is "almost" the same as the period 1 game. The probability of H type individuals upgrading to A at period t, which is denoted by $p_{H,t}$, is equal to 0 or approximately equal to 0. Otherwise, if there exists some finite period T such that $p_{H,T} \gg 0$, then all individuals will wait till period $T+1$ since they are "patient enough". This means $p_{H,t} = 0\ \forall t \leq T$. That is a contradiction.

Thus, if individuals are patient enough, at any fixed time, nearly all individuals wait due to the negligible information disclosed. ■

Proof of Result 1

(Complete information) With complete information, the true value of A is known. Everyone makes the correct choice, which means $X_{CI}(N)/N = 1$. Certainly, this is the upper bound of $X(N)/N$.

(Social planner) If there is a social planner who can gather the private information from all the individuals, then based on all the private signals and the prior probabilities,

$$X_{SP}(N)/N = \sum_{j>N/2}^{N} \binom{N}{j} p^j (1-p)^{N-j} + \mathbf{1}_{\{\text{N is even}\}} \left[\binom{N}{N/2} p^{N/2} (1-p)^{N/2} \frac{1}{2} \right]$$

where $\mathbf{1}_{\{\text{N is even}\}}$ is the indicator function; if N is even, $\mathbf{1}_{\{\text{N is even}\}} = 1$; otherwise, $\mathbf{1}_{\{\text{N is even}\}} = 0$.

(Self-decision) If there are no interactions among the individuals, each individual makes a self-decision using his own private signal and the prior probabilities. Then based on the precision of private signals, $X_{SD}(N)/N = p$.

(Exogenous ordering) With exogenous ordering, according to the equilibrium decision rule in section 2.1, we have

$$\begin{cases} X_{EX}(1)/1 = p \\ X_{EX}(2)/2 = p \\ X_{EX}(N)/N = p^2 + p(1-p)\left\{\frac{N-2}{N}\left[\frac{1}{N-2}X_{EX}(N-2) - \frac{1}{2}\right] + 1\right\} \ \forall N \geq 3 \end{cases}$$

We can easily check that $X_{EX}(3)/3 > p$ and $X_{EX}(4)/4 > p$. Then plugging back to the above formula and by induction, we have $X_{EX}(N)/N > X_{SD}(N)/N = p$, $\forall N \geq 3$. As $N \to \infty$, $X_{EX}(N)/N \to p\frac{1/2(1+p)}{1-p+p^2}$, where $\frac{1/2(1+p)}{1-p+p^2} > 1$.

(Endogenous ordering) With endogenous ordering, $X_{EN}(1)/1 = p$; $X_{EN}(2)/2 = p$. The results are equivalent to the cases of self-decision and exogenous ordering with 1 or 2 individuals respectively. According to the equilibrium decision rule in section 2.2, given δ and p, there exists an N^* such that $\delta = \delta^*(N^*, p)$.

If $N \leq N^*$, then $\delta \leq \delta^*(N, p)$. At period 1, H type individuals will upgrade to A for sure and L type individuals will wait to see H type's action. At period 2, all the possible information is disclosed. Thus, conditional on $V = 1/2$,

$$X_{EN|V=1/2}(N)/N = \sum_{j>N/2}^{N}\binom{N}{j}p^j(1-p)^{N-j} + \mathbf{1}_{\{\text{N is even}\}}\left[\binom{N}{N/2}p^{N/2}(1-p)^{N/2}3/4\right] + \sum_{j=0}^{j<N/2}\binom{N}{j}p^j(1-p)^{N-j}j/N$$

Conditional on $V = -1/2$,

$$X_{EN|V=-1/2}(N)/N = \mathbf{1}_{\{\text{N is even}\}}\left[\binom{N}{N/2}(1-p)^{N/2}p^{N/2}1/4\right] + \sum_{j=0}^{j<N/2}\binom{N}{j}(1-p)^j p^{N-j}(N-j)/N$$

Unconditional average expected number of corrected choices,

$$X_{EN}(N)/N = \frac{1}{2}X_{EN|V=1/2}(N)/N + \frac{1}{2}X_{EN|V=-1/2}(N)/N$$

Using the exhaustion method (Matlab simulation), we can check that $X_{SP}(N)/N > X_{EN}(N)/N > X_{EX}(N)/N$ for $N \geq 3$ and not too large. For N large but still less than N^*, $X_{EN|V=1/2}(N)/N$ converges to 1 and $X_{EN|V=-1/2}(N)/N$ to p. Then $X_{EN}(N)/N$ converges to $p\frac{1+p}{2p}$. $\frac{1+p}{2p} > \frac{1/2(1+p)}{1-p+p^2}$ implies $X_{EN}(N)/N > X_{EX}(N)/N$ for N large but still less than N^*.

If N is large enough, according to the equilibrium decision rule in section 2.2, for any finite period t, $p_{H,t}$ is either zero or approximately equal to zero. At any fixed time, nearly all individuals wait due to the negligible information disclosed. Thus, there exists an N^{**} such that for all $N > N^{**}$, $X_{EN|V=1/2}(N)/N$ converges to zero and $X_{EN|V=-1/2}(N)/N$ to one. Then $X_{EN}(N)/N$ converges to 1/2, which is less than $X_{SD}(N)/N = p$. ■

Proof of Remark 1

Since it has been assumed that the private signals μ are independent conditional on V, we have

$$f(V|\mu_i, h_t) = \frac{f(\mu_i, h_t|V)f_0(V)}{f(\mu_i, h_t)} = \frac{f(\mu_i|V)f(h_t|V)f_0(V)}{f(\mu_i, h_t)}$$
$$f(V|\mu_i', h_t) = \frac{f(\mu_i', h_t|V)f_0(V)}{f(\mu_i', h_t)} = \frac{f(\mu_i'|V)f(h_t|V)f_0(V)}{f(\mu_i', h_t)}$$

This implies

$$\frac{f(V|\mu_i, h_t)}{f(V|\mu_i', h_t)} = \frac{\frac{f(\mu_i|V)f(h_t|V)f_0(V)}{f(\mu_i,h_t)}}{\frac{f(\mu_i'|V)f(h_t|V)f_0(V)}{f(\mu_i',h_t)}} = \frac{f(\mu_i|V)}{f(\mu_i'|V)}\frac{f(\mu_i', h_t)}{f(\mu_i, h_t)}$$

Then $\forall V_1 > V_2$,

$$\frac{\frac{f(V_1|\mu_i,h_t)}{f(V_1|\mu_i',h_t)}}{\frac{f(V_2|\mu_i,h_t)}{f(V_2|\mu_i',h_t)}} = \frac{\frac{f(\mu_i|V_1)}{f(\mu_i'|V_1)}\frac{f(\mu_i',h_t)}{f(\mu_i,h_t)}}{\frac{f(\mu_i|V_2)}{f(\mu_i'|V_2)}\frac{f(\mu_i',h_t)}{f(\mu_i,h_t)}} = \frac{\frac{f(\mu_i|V_1)}{f(\mu_i|V_2)}}{\frac{f(\mu_i'|V_1)}{f(\mu_i'|V_2)}}$$

By Assumption 1,

$$\frac{f(\mu_i|V_1)}{f(\mu_i|V_2)} \quad \text{increasing in} \quad \mu_i \quad \forall V_1 > V_2$$

So, we have $\forall \mu_i > \mu_i'$

$$\frac{\frac{f(V_1|\mu_i,h_t)}{f(V_1|\mu_i',h_t)}}{\frac{f(V_2|\mu_i,h_t)}{f(V_2|\mu_i',h_t)}} = \frac{\frac{f(\mu_i|V_1)}{f(\mu_i|V_2)}}{\frac{f(\mu_i'|V_1)}{f(\mu_i'|V_2)}} \geq 1$$

which means

$$\frac{f(V|\mu_i, h_t)}{f(V|\mu_i', h_t)} \quad \text{increasing in} \quad V \quad \forall \mu_i > \mu_i'$$

■

Proof of Lemma 1

If at period t individual i chooses to wait, then $U^A(\mu_i; h_t) \leq U^W(\mu_i; h_t; s_{-i,t})$.

By the Martingale property,

$$U^A(\mu_i; h_t) = E_{H_{t+1}(h_t; s_{-i,t})} U^A(\mu_i; h_{t+1})$$

The set of histories $H_{t+1}(h_t; s_{-i,t})$ can be decomposed into two disjoint sets: $H^A_{t+1}(\mu_i; h_t; s_{-i,t})$ and $H^W_{t+1}(\mu_i; h_t; s_{-i,t})$, where $H^A_{t+1}(\mu_i; h_t; s_{-i,t})$ is the set of histories at period $t+1$ in which individual i will upgrade to A according to some strategy s_i of individual i; $H^W_{t+1}(\mu_i; h_t; s_{-i,t})$ is the set of histories at period $t+1$ in which individual i will wait according to some strategy s_i of individual i. Then we have

$$U^A(\mu_i; h_t) = E_{H^A_{t+1}(\mu_i; h_t; s_{-i,t})} U^A(\mu_i; h_{t+1}) + E_{H^W_{t+1}(\mu_i; h_t; s_{-i,t})} U^A(\mu_i; h_{t+1})$$

By equation 2,

$$\begin{aligned} U^W(\mu_i; h_t; s_{-i,t}) &= \delta E_{H_{t+1}(\mu_i; h_t; s_{-i,t})} [max\{U^A(\mu_i; h_{t+1}); U^W(\mu_i; h_{t+1}; s_{-i,t+1})\}] \\ &= \delta [E_{H^A_{t+1}(\mu_i; h_t; s_{-i,t})} U^A(\mu_i; h_{t+1}) + E_{H^W_{t+1}(\mu_i; h_t; s_{-i,t})} U^W(\mu_i; h_{t+1}; s_{-i,t+1})] \end{aligned}$$

Suppose under the worst news from period t individual i still upgrades at period $t+1$. Then he will for sure upgrade at period $t+1$, which means $H^W_{t+1}(\mu_i; h_t; s_{-i,t}) = \emptyset$.

Back to the above equations, we have

$$\begin{aligned} U^A(\mu_i; h_t) &= E_{H^A_{t+1}(\mu_i; h_t; s_{-i,t})} U^A(\mu_i; h_{t+1}) \\ U^W(\mu_i; h_t; s_{-i,t}) &= \delta E_{H^A_{t+1}(\mu_i; h_t; s_{-i,t})} U^A(\mu_i; h_{t+1}) \end{aligned}$$

Since $H^W_{t+1}(\mu_i; h_t; s_{-i,t}) = \emptyset$, $U^W(\mu_i; h_t; s_{-i,t}) > \underline{U}^W = 0$. We have $U^A(\mu_i; h_t) > U^W(\mu_i; h_t; s_{-i,t})$. This is a contradiction.

In our model, the worst news at period t is no one upgrades. Under this worst news, the waiting herding phase starts at period $t+1$. To keep the upgrade going, at least one individual must upgrade to A in each period. Thus, with finite number of N individuals, the game lasts at most N periods before a herding phase starts. ■

Proof of Proposition 1

By Remark 1, $f(V|\mu, h_t)$ satisfies MLRP with respect to μ. According to Landsberger and Meilijson (1990), $F(V|\mu_i, h_t)$ first order stochastic dominates (FOSD) $F(V|\mu_i', h_t)$ for any $\mu_i > \mu_i'$. So, $U^A(\mu_i; h_t) \geq U^A(\mu_i'; h_t)$ for any h_t.

Similar to the proof of Lemma 1, by the Martingale property,

$$U^A(\mu_i; h_t) = E_{H^A_{t+1}(\mu_i; h_t; s_{-i,t})} U^A(\mu_i; h_{t+1}) + E_{H^W_{t+1}(\mu_i; h_t; s_{-i,t})} U^A(\mu_i; h_{t+1})$$

$$\begin{aligned} U^W(\mu_i; h_t; s_{-i,t}) &= \delta E_{H_{t+1}(\mu_i; h_t; s_{-i,t})} [max\{U^A(\mu_i; h_{t+1}); U^W(\mu_i; h_{t+1}; s_{-i,t+1})\}] \\ &= \delta [E_{H^A_{t+1}(\mu_i; h_t; s_{-i,t})} U^A(\mu_i; h_{t+1}) + E_{H^W_{t+1}(h_t; s_{-i,t})} U^W(\mu_i; h_{t+1}; s_{-i,t+1})] \end{aligned}$$

Thus, for any non-negative integer j

$$\begin{aligned}U^A(\mu_i;h_t)-\delta^j U^W(\mu_i;h_t;s_{-i,t}) &= (1-\delta^{j+1})E_{H^A_{t+1}(\mu_i;h_t;s_{-i,t})}U^A(\mu_i;h_{t+1}) \\ &+ E_{H^W_{t+1}(\mu_i;h_t;s_{-i,t})}[U^A(\mu_i;h_{t+1})-\delta^{j+1}U^W(\mu_i;h_{t+1};s_{-i,t+1})]\end{aligned} \tag{3}$$

Let us check the incentives of waiting and upgrading for individual i who has a lower private signal $\mu_i' < \mu_i$. Similarly, we have

$$\begin{aligned}U^A(\mu_i';h_t)-\delta^j U^W(\mu_i';h_t;s_{-i,t}) &= (1-\delta^{j+1})E_{H^A_{t+1}(\mu_i';h_t;s_{-i,t})}U^A(\mu_i';h_{t+1}) \\ &+ E_{H^W_{t+1}(\mu_i';h_t;s_{-i,t})}[U^A(\mu_i';h_{t+1})-\delta^{j+1}U^W(\mu_i';h_{t+1};s_{-i,t+1})]\end{aligned} \tag{4}$$

By Lemma 1, the game lasts at most N periods before a herding phase starts. Suppose either an upgrading or a waiting herding phase starts at period $T \le N$, which means no one will upgrade to A after period T given history h_T and strategy profile $(s_{i,T}, s_{-i,T})$. With a herding phase starting at period T, no more new information is disclosed at period $T+1$, which means $U^W(\mu_i;h_T;s_{-i,T}) = U^W(\mu_i';h_T;s_{-i,T}) = 0$. Thus, at period T,

$$U^A(\mu_i;h_T)-\delta^j U^W(\mu_i;h_T;s_{-i,T}) \ge U^A(\mu_i';h_T)-\delta^j U^W(\mu_i';h_T;s_{-i,T})$$

Back to period $T-1$, since herding starts at period T, $H^A_T(\mu_i;h_{T-1};s_{-i,T-1}) = H^A_T(\mu_i';h_{T-1};s_{-i,T-1})$ and $H^W_T(\mu_i;h_{T-1};s_{-i,T-1}) = H^W_T(\mu_i';h_{T-1};s_{-i,T-1})$. By equation 3 and 4,

$$U^A(\mu_i;h_{T-1})-\delta^j U^W(\mu_i;h_{T-1};s_{-i,T-1}) \ge U^A(\mu_i';h_{T-1})-\delta^j U^W(\mu_i';h_{T-1};s_{-i,T-1})$$

When $j=0$, the above formula implies that individuals with private signals indicating higher value of the new software package A have a higher incentive to upgrade given the same public history at period $T-1$. That is to say,

$$\begin{aligned}H^A_{T-1}(\mu_i;h_{T-2};s_{-i,T-2}) &\supseteq H^A_{T-1}(\mu_i';h_{T-2};s_{-i,T-2}) \\ H^W_{T-1}(\mu_i;h_{T-2};s_{-i,T-2}) &\subseteq H^W_{T-1}(\mu_i';h_{T-2};s_{-i,T-2})\end{aligned}$$

Back to period $T-2$, by equation 3 and 4,

$$\begin{aligned}U^A(\mu_i;h_{T-2})-\delta^j U^W(\mu_i;h_{T-2};s_{-i,T-2}) &= (1-\delta^{j+1})E_{H^A_{T-1}(\mu_i';h_{T-2};s_{-i,T-2})}U^A(\mu_i;h_{T-1}) \\ &+(1-\delta^{j+1})E_{H^A_{T-1}(\mu_i;h_{T-2};s_{-i,T-2})\cap H^W_{T-1}(\mu_i';h_{T-2};s_{-i,T-2})}U^A(\mu_i;h_{T-1}) \\ &+E_{H^W_{T-1}(\mu_i;h_{T-2};s_{-i,T-2})}[U^A(\mu_i;h_{T-1})-\delta^{j+1}U^W(\mu_i;h_{T-1};s_{-i,T-1})]\end{aligned}$$

$$\begin{aligned}U^A(\mu_i';h_{T-2})-\delta^j U^W(\mu_i';h_{T-2};s_{-i,T-2}) &= (1-\delta^{j+1})E_{H^A_{T-1}(\mu_i';h_{T-2};s_{-i,T-2})}U^A(\mu_i';h_{T-1}) \\ &+E_{H^A_{T-1}(\mu_i;h_{T-2};s_{-i,T-2})\cap H^W_{T-1}(\mu_i';h_{T-2};s_{-i,T-2})}[U^A(\mu_i';h_{T-1})-\delta^{j+1}U^W(\mu_i';h_{T-1};s_{-i,T-1})] \\ &+E_{H^W_{T-1}(\mu_i;h_{T-2};s_{-i,T-2})}[U^A(\mu_i';h_{T-1})-\delta^{j+1}U^W(\mu_i';h_{T-1};s_{-i,T-1})]\end{aligned}$$

For $h_{T-1} \in [H^A_{T-1}(\mu_i;h_{T-2};s_{-i,T-2}) \cap H^W_{T-1}(\mu_i';h_{T-2};s_{-i,T-2})]$, $U^W(\mu_i';h_{T-1};s_{-i,T-1}) \ge U^A(\mu_i';h_{T-1})$, which implies $U^A(\mu_i';h_{T-1})-\delta^{j+1}U^W(\mu_i';h_{T-1};s_{-i,T-1}) \le (1-\delta^{j+1})U^A(\mu_i';h_{T-1}) \le (1-\delta^{j+1})U^A(\mu_i;h_{T-1})$. Thus,

$$U^A(\mu_i;h_{T-2})-\delta^j U^W(\mu_i;h_{T-2};s_{-i,T-2}) \ge U^A(\mu_i';h_{T-2})-\delta^j U^W(\mu_i';h_{T-2};s_{-i,T-2})$$

And so on, for any $t \leq T$

$$U^A(\mu_i; h_t) - \delta^j U^W(\mu_i; h_t; s_{-i,t}) \geq U^A(\mu_i'; h_t) - \delta^j U^W(\mu_i'; h_t; s_{-i,t})$$

Let $j = 0$. We are done. ■

Proof of Proposition 2

(i) Case I: Continuous private signal space

Let $\mathcal{G}_n(h_t)$ represent the subgame starting from period t with history h_t, where n is the number of individuals remaining in this subgame.[14] Use backward induction.

Step 1 Start from the subgame with only one individual, $\mathcal{G}_1(h_t)$. We can find a critical value $\mu_t^*(h_t)$ which is the solution of $U^A(\mu_t^*(h_t); h_t) = 0$. An individual with $\mu > \mu_t^*(h_t)$ upgrades; otherwise, he waits forever.

Step 2 Now consider the subgame with two individuals $\mathcal{G}_2(h_t)$, by Lemma 1, this subgame lasts at most 2 periods. At period $t+1$, there are three possible cases: (1) there are still two individuals remaining in the game (waiting herding starts); (2) there is only one individual remaining in the game ($\mathcal{G}_1(h_{t+1})$); (3) there is no one remaining in the game (game ends). By Proposition 1, for any symmetric equilibrium, individuals with private signals indicating higher value of A have a higher incentive to upgrade. We can find a critical value $\mu_t^*(h_t)$ which is a function of $\mu_{t+1}^*(h_{t+1})$ in the subsequent subgame $\mathcal{G}_1(h_{t+1})$. Individuals with $\mu > \mu_t^*(h_t)$ upgrade; otherwise, they wait.

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Step N Continue to the subgame with N individuals $\mathcal{G}_N(h_t)$, by Lemma 1, this subgame lasts at most N periods. Similarly, there are $N+1$ possible cases: (1) there are still N individuals remaining in the game (waiting herding starts); (2) there are $N-1$ individuals remaining in the game ($\mathcal{G}_{N-1}(h_{t+1})$);. . .; ($N$) there is only 1 individual remaining in the game ($\mathcal{G}_1(h_{t+1})$); ($N+1$) there is no one remaining in the game (game ends). We can find a critical value $\mu_t^*(h_t)$ which is a function of $\mu_{t+1}^*(h_{t+1})$ in the subsequent subgames $\mathcal{G}_{N-1}(h_{t+1}), \ldots, \mathcal{G}_1(h_{t+1})$. Individuals with $\mu > \mu_t^*(h_t)$ upgrade; otherwise, they wait.

We can see in the final Step N if we replace h_t with h_1 then $\mathcal{G}_N(h_t)$ is the original game.

Case II: Finite discrete private signal space

Denote the private signal space by $\{\mu_1, \mu_2, \ldots, \mu_K\}$, where $\mu_1 < \mu_2 < \ldots < \mu_K$. The strategy profile starting from period t, $s_t = \{P_\tau\}_{\tau=t}^{\infty}$, where $P_\tau = \{p_{\mu_k,\tau}\}_{k=1}^{K}$ and $p_{\mu_k,\tau}$ represents the probability of type μ_k upgrading to A at period τ. For μ_i, $U^A(\mu_i; h_t) - U^W(\mu_i; h_t; \{P_\tau\}_{\tau=t}^{\infty})$ is continuous in $p_{\mu_k,\tau}$ $\forall \mu_k, \tau$. Let $\mathcal{G}_M(h_t)$ represent the subgame starting

[14] n must be compatible with h_t.

from period t with history h_t, where M is the set of possible types remaining in this subgame.[15]

By Proposition 1, for any symmetric equilibrium, individuals with private signals indicating higher value of A have a higher incentive to upgrade. Thus, with a finite number of individuals and a finite number of individual types, backward induction can be used to construct the symmetric equilibrium through the following steps.

Step 1 Start from the subgame $\mathcal{G}_{\{\mu_1\}}(h_t)$ with only the lowest type of individuals μ_1. That is, $M = \{\mu_1\}$. Since all the information is disclosed, $U^W(\mu_1; h_t; \{p_{\mu_1,t}\}) = \underline{U}^W = 0$. There are three possible cases:

1.1 If $U^A(\mu_1; h_t) < 0$, $\{p_{\mu_1,t} = 0\}$ is the equilibrium strategy profile at period t. The continuation game at the following periods is the same as the period t game since $h_\tau = h_t \ \forall \tau > t$.

1.2 If $U^A(\mu_1; h_t) = 0$, $\{0 \leq p_{\mu_1,t} \leq 1\}$ will be the equilibrium strategy profile at period t. If the game does not end at period t, the continuation game at period $t+1$ is the same as the period t game since $h_{t+1} = h_t$.

1.3 If $U^A(\mu_1; h_t) > 0$, $\{p_{\mu_1,t} = 1\}$ is the equilibrium and game ends.

Step 2 Now consider the subgame $\mathcal{G}_{\{\mu_1,\mu_2\}}(h_t)$ with two possible types of individuals μ_1, μ_2. That is, $M = \{\mu_1, \mu_2\}$. There are three possible cases:

2.1 If $0 \geq U^A(\mu_2; h_t) \geq U^A(\mu_1; h_t)$, then $\{p_{\mu_1,t} = 0, p_{\mu_2,t} = 0\}$ is an equilibrium strategy profile at period t. The continuation game at the following periods is the same as the period t game since $h_\tau = h_t \ \forall \tau > t$.

2.2 If $U^A(\mu_2; h_t) > 0 > U^A(\mu_1; h_t)$, then μ_1 type will for sure wait at period t. Consider the strategy profile at period t: $\{p_{\mu_1,t} = 0, p_{\mu_2,t} = 1\}$. There are two possible cases:

2.2.1 If $U^A(\mu_2; h_t) - U^W(\mu_2; h_t; \{p_{\mu_1,t} = 0, p_{\mu_2,t} = 1\}) \geq 0 > U^A(\mu_1; h_t) - U^W(\mu_1; h_t; \{p_{\mu_1,t} = 0, p_{\mu_2,t} = 1\})$, then $\{p_{\mu_1,t} = 0, p_{\mu_2,t} = 1\}$ is an equilibrium strategy profile at period t. The continuation game at period $t+1$ is $\mathcal{G}_{\{\mu_1\}}(h_{t+1})$ if the game does not end at period t.

2.2.2 If $0 > U^A(\mu_2; h_t) - U^W(\mu_2; h_t; \{p_{\mu_1,t} = 0, p_{\mu_2,t} = 1\}) \geq U^A(\mu_1; h_t) - U^W(\mu_1; h_t; \{p_{\mu_1,t} = 0, p_{\mu_2,t} = 1\})$, then decreasing $p_{\mu_2,t}$ till $0 = U^A(\mu_2; h_t) - U^W(\mu_2; h_t; \{p_{\mu_1,t} = 0, p_{\mu_2,t}\}) \geq U^A(\mu_1; h_t) - U^W(\mu_1; h_t; \{p_{\mu_1,t} = 0, p_{\mu_2,t}\})$ by continuity. The solution $\{p_{\mu_1,t} = 0, p_{\mu_2,t}\}$ to the above formula is an equilibrium strategy profile at period t. The continuation game at period $t+1$ is $\mathcal{G}_{\{\mu_1,\mu_2\}}(h_{t+1})$ if the game does not end at period t.

2.3 If $U^A(\mu_2; h_t) \geq U^A(\mu_1; h_t) \geq 0$, then $\{p_{\mu_1,t} = 1, p_{\mu_2,t} = 1\}$ is an equilibrium and game ends.

[15] M must be compatible with h_t.

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Step K Continue to the subgame $\mathcal{G}_{\{\mu_1,\mu_2,\dots,\mu_K\}}(h_t)$ with all the possible types of individuals $\mu_1, \mu_2, \dots, \mu_K$. That is, $M = \{\mu_1; \mu_2, \dots, \mu_K\}$. There are $K+1$ possible cases:

K.1 If $0 \geq U^A(\mu_K; h_t) \geq U^A(\mu_{K-1}; h_t) \geq \dots \geq U^A(\mu_1; h_t)$, then $\{p_{\mu_1,t} = 0, p_{\mu_2,t} = 0, \dots, p_{\mu_K,t} = 0\}$ is an equilibrium. The continuation game at the following periods is the same as the period t game since $h_\tau = h_t \; \forall \tau > t$.

K.2 If $U^A(\mu_K; h_t) > 0 \geq U^A(\mu_{K-1}; h_t) \geq \dots \geq U^A(\mu_1; h_t)$, then $\mu_1, \mu_2, \dots, \mu_{K-1}$ types will for sure wait at period t. Consider the strategy profile at period t: $\{p_{\mu_1,t} = 0, p_{\mu_2,t} = 0, \dots, p_{\mu_{K-1},t} = 0, p_{\mu_K,t} = 1\}$. There are two possible cases:

K.2.1 If $U^A(\mu_K; h_t) - U^W(\mu_K; h_t; \{p_{\mu_1,t} = 0, p_{\mu_2,t} = 0, \dots, p_{\mu_{K-1},t} = 0, p_{\mu_K,t} = 1\}) \geq 0 > U^A(\mu_{K-1}; h_t) - U^W(\mu_{K-1}; h_t; \{p_{\mu_1,t} = 0, p_{\mu_2,t} = 0, \dots, p_{\mu_{K-1},t} = 0, p_{\mu_K,t} = 1\})$, then $\{p_{\mu_1,t} = 0, p_{\mu_2,t} = 0, \dots, p_{\mu_{K-1},t} = 0, p_{\mu_K,t} = 1\}$ is an equilibrium strategy profile at period t. The continuation game at period $t+1$ is $\mathcal{G}_{\{\mu_1,\mu_2,\dots,\mu_{K-1}\}}(h_{t+1})$ if the game does not end at period t.

K.2.2 If $0 > U^A(\mu_K; h_t) - U^W(\mu_K; h_t; \{p_{\mu_1,t} = 0, p_{\mu_2,t} = 0, \dots, p_{\mu_{K-1},t} = 0, p_{\mu_K,t} = 1\})$, then decreasing $p_{\mu_K,t}$ till $0 = U^A(\mu_K; h_t) - U^W(\mu_K; h_t; \{p_{\mu_1,t} = 0, p_{\mu_2,t} = 0, \dots, p_{\mu_{K-1},t} = 0, p_{\mu_K,t} = 1\})$ by continuity. The solution $\{p_{\mu_1,t} = 0, p_{\mu_2,t} = 0, \dots, p_{\mu_{K-1},t} = 0, p_{\mu_K,t}\}$ to the above formula is an equilibrium strategy profile at period t. The continuation game at period $t+1$ is $\mathcal{G}_{\{\mu_1,\mu_2,\dots,\mu_K\}}(h_{t+1})$ if the game does not end at period t.

⋮

⋮

K.K If $U^A(\mu_K; h_t) \geq U^A(\mu_{K-1}; h_t) \geq \dots \geq U^A(\mu_2; h_t) > 0 > U^A(\mu_1; h_t)$, then μ_1 type will for sure wait at period t. Consider the strategy profile at period t: $\{p_{\mu_1,t} = 0, p_{\mu_2,t} = 1, \dots, p_{\mu_K,t} = 1\}$. There are K possible cases. Check if this strategy profile is an equilibrium strategy profile at period t. If not, decrease $p_{\mu_2,t}$ from 1 to 0. Then decrease $p_{\mu_3,t}$ from 1 to 0. And so on, decrease $p_{\mu_K,t}$ from 1 to 0. Eventually, by continuity, we will find a symmetric equilibrium for subgame $\mathcal{G}_{\{\mu_1,\mu_2,\dots,\mu_K\},N}(h_t)$.

K.K+1 If $U^A(\mu_K; h_t) \geq U^A(\mu_{K-1}; h_t) \geq \dots \geq U^A(\mu_1; h_t) \geq 0$, then $\{p_{\mu_1,t} = 1, p_{\mu_2,t} = 1, \dots, p_{\mu_K,t} = 1\}$ is an equilibrium and game ends.

We can see at the final Step K if we replace h_t with h_1 then $\mathcal{G}_{\{\mu_1,\mu_2,\dots,\mu_K\}}(h_1)$ is the original game.

(ii) Patient Individuals:

If individuals are patient enough, at period 1 the highest types of individuals have enough incentive to wait such that: either $\mu_1^*(h_1) = \overline{\mu}$ or $\mu_1^*(h_1) \approx \overline{\mu}$ (finite discrete private signal space: either $p_{\overline{\mu}} = 0$ or $p_{\overline{\mu}} \approx 0$). At any period $\infty > t > 1$, the game is either the same or "almost" the same as the period 1 game: either $\mu_t^*(h_t) = \mu_{t-1}^*(h_{t-1})$ or $\mu_t^*(h_t) \approx \mu_{t-1}^*(h_{t-1})$ (finite discrete private signal space: either $p_{\overline{\mu}} = 0$ or $p_{\overline{\mu}} \approx 0$). Otherwise, if there exists some finite period T such that $\mu_t^*(h_t) \ll \mu_{t-1}^*(h_{t-1})$ (finite discrete private signal space: $p_{\overline{\mu}} \gg 0$), then all individuals will wait until period $T+1$ since they are "patient enough". This means the probability for all types of individuals upgrading to A is equal to zero $\forall t \leq T$. That is a contradiction.

Thus, if individuals are patient enough, at any fixed time, there is a negligible proportion of individuals upgrading to A and so is the information disclosed. ■

Proof of Result 2

According to the equilibrium decision rule in section 2.1, with the general tie-breaking rule $\{p_H; p_L\}$, we have $X_{EX}(1)/1 = X_{EX}(2)/2 = p$ and $\forall N \geq 3$

$$X_{EX}(N)/N = p^2 + p(1-p)\left\{\frac{N-2}{N}(1 + p_H - p_L)\left[\frac{1}{N-2}X_{EX}(N-2) - \frac{1}{2}\right] + 1\right\}$$

We can easily check that $X_{EX}(3)/3 \geq p$ and $X_{EX}(4)/4 \geq p$. Then plugging back to the above formula and by induction, we have $X_{EX}(N)/N \geq p$, $\forall N$. As $N \to \infty$, $X_{EX}(N)/N \to \frac{p^2+1/2p(1-p)(1-(p_H-p_L))}{1-p(1-p)(1+(p_H-p_L))}$.

Since $\forall N$, $X_{EX}(N)/N \geq p > \frac{1}{2}$, from the above formula, we can see $X_{EX}(N)/N$ increasing in $p_H - p_L$. In particular, $X_{EX}(N)/N$ achieves its maximum when $\{p_H = 1; p_L = 0\}$. $X_{EX}(N)/N$ achieves its minimum p when $\{p_H = 0; p_L = 1\}$. In other words, with the tie-breaking rule $\{p_H = 0; p_L = 1\}$, the equilibrium result of exogenous ordering is the same as the result of self-decision in terms of inducing the same expected number of correct choices. ■

Proof of Proposition 4

Given h_t and that μ_{t-1}^* is the highest type remaining in the game at period t, we can calculate $U^A(\mu_{t-1}^*; h_t)$, $\underline{U}^W$, and $\overline{U^W}(\mu_{t-1}^*; h_t)$. Because we know $\overline{U^W}(\mu_{t-1}^*; h_t) \geq \underline{U}^W$, there are only three possible cases:

$$\begin{cases} 1, & U^A(\mu_{t-1}^*; h_t) \leq \underline{U}^W; \\ 2, & \overline{U^W}(\mu_{t-1}^*; h_t) < U^A(\mu_{t-1}^*; h_t); \\ 3, & \underline{U}^W < U^A(\mu_{t-1}^*; h_t) \leq \overline{U^W}(\mu_{t-1}^*; h_t). \end{cases}$$

(i) Zero amount of information disclosed

For the first case, since the highest type μ_{t-1}^* will wait, others will also wait. No new information is disclosed. The next period game will be the same as the current period game. Everyone will continue to wait. The benefit from waiting is $\underline{U}^W$.

(ii) Infinite amount of information disclosed

For the second case, the highest type μ_{t-1}^* will upgrade at period t even under the case that at period $t+1$ the true value of the new software package A will be revealed. With an infinite number of individuals, there will be an infinite number of upgrades at period t and the true value of the new software package A will be revealed.

At period t, individuals with $\mu > \mu_t^*$ will upgrade. Others wait. At period $t+1$, all the individuals remaining in the game will upgrade if $\frac{1}{1-\delta}V \geq \underline{U}^W$. Otherwise, they will wait forever. The critical type μ_t^* is the solution of the following equations and inequalities.

Case I: Continuous private signal space

$$U^A(\mu_t^*; h_t) = \overline{U^W}(\mu_t^*; h_t)$$

Case II: Finite discrete private signal space

$$U^A(\mu_t^*; h_t) \leq \overline{U^W}(\mu_t^*; h_t)$$
$$U^A(\mu; h_t) > \overline{U^W}(\mu; h_t) \quad \forall \mu_{t-1}^* \geq \mu > \mu_t^*$$

(iii) Finite amount of information disclosed

For the third case, the benefit from upgrading to A lies in between $\underline{U}^W$ and $\overline{U^W}(\mu_{t-1}^*; h_t)$. If all others wait at period t, the highest type μ_{t-1}^* will upgrade. But if there is an infinite number of upgrades and the true value of the new software package A will be revealed at period $t+1$, then the highest type μ_{t-1}^* will wait.

Case I: Continuous private signal space

Define $\varphi(\mu_i; h_t) = Prob(\mu_t^* < \mu \leq \mu_{t-1}^* | \mu_i; h_t)$. As $N \to \infty$, $\varphi(\mu_i; h_t) \to 0$, $N\varphi(\mu_i; h_t)$ should be bounded from 0 and infinity to keep a finite number of upgrades. Suppose $N\varphi(\mu_i; h_t) \to \beta(\mu_i; h_t)$. By Poisson approximation, individuals upgrading at period t follows the Poisson distribution with parameter $\beta(\mu_{t-1}^*; h_t)$. The probability of m upgrades at period t is

$$b(m; \beta(\mu_i; h_t)) = e^{-\beta(\mu_i; h_t)} \frac{(\beta(\mu_i; h_t))^m}{m!}$$

Case II: Finite discrete private signal space

Define $\phi(\mu_i; h_t) = Prob(\mu = \mu_{t-1}^* | \mu_i; h_t)$. As $N \to \infty$, $p_{\mu_{t-1}^*} \to 0$, $N\phi(\mu_i; h_t)p_{\mu_{t-1}^*}$ should be bounded from 0 and infinity to keep a finite number of upgrades. Suppose $N\phi(\mu_i; h_t)p_{\mu_{t-1}^*} \to \beta(\mu_i; h_t)$. By Poisson approximation, individuals upgrading at period t follows the Poisson distribution with parameter $\beta(\mu_{t-1}^*; h_t)$.

For the highest type μ_{t-1}^*, we have $0 = U^A(\mu_{t-1}^*; h_t) - U^W(\mu_{t-1}^*; h_t; \beta(\mu_{t-1}^*; h_t))$. Decompose the set of histories at period $t+1$ $H_{t+1}(h_t; \beta(\mu_{t-1}^*; h_t))$ into three disjoint sets: $H_{t+1}^A(\mu_{t-1}^*; h_t; \beta(\mu_{t-1}^*; h_t))$, $H_{t+1}^{interim}(\mu_{t-1}^*; h_t; \beta(\mu_{t-1}^*; h_t))$, and $H_{t+1}^W(\mu_{t-1}^*; h_t; \beta(\mu_{t-1}^*; h_t))$, where $H_{t+1}^A(\mu_{t-1}^*; h_t; \beta(\mu_{t-1}^*; h_t))$ is the set of histories at period $t+1$ in which individuals with μ_{t-1}^* will upgrade; $H_{t+1}^{interim}(\mu_{t-1}^*; h_t; \beta(\mu_{t-1}^*; h_t))$ is the set of histories at period $t+1$ in which individuals upgrading follows some Poisson distribution; $H_{t+1}^W(\mu_{t-1}^*; h_t; \beta(\mu_{t-1}^*; h_t))$ is the set of histories at period $t+1$ in which individuals with μ_{t-1}^* will wait.

Following the same logic as presented in the proof of Proposition 1, we have

$$
\begin{aligned}
0 =& U^A(\mu^*_{t-1}; h_t) - U^W(\mu^*_{t-1}; h_t; \beta(\mu^*_{t-1}; h_t)) \\
=& (1-\delta)\{E_{H^A_{t+1}(\mu^*_{t-1}; h_t; \beta(\mu^*_{t-1}; h_t))} U^A(\mu^*_{t-1}; h_{t+1}) + E_{H^{interim}_{t+1}(\mu^*_{t-1}; h_t; \beta(\mu^*_{t-1}; h_t))} U^A(\mu^*_{t-1}; h_{t+1})\} \\
&+ E_{H^W_{t+1}(\mu^*_{t-1}; h_t; \beta(\mu^*_{t-1}; h_t))}[U^A(\mu^*_{t-1}; h_{t+1}) - \delta \underline{U}^W]
\end{aligned}
$$

Note, if at period $t+1$ $h_{t+1} \in H^{interim}_{t+1}(\mu^*_{t-1}; h_t; \beta(\mu^*_{t-1}; h_t))$, then $U^A(\mu^*_{t-1}; h_{t+1}) = U^W(\mu^*_{t-1}; h_{t+1}; \beta(\mu^*_{t-1}; h_{t+1}))$.

Since $\underline{U}^W \leq U^W(\mu^*_{t-1}; h_t; \beta(\mu^*_{t-1}; h_t)) \leq \overline{U^W}(\mu^*_{t-1}; h_t)$ and $U^W(\mu^*_{t-1}; h_t; \beta(\mu^*_{t-1}; h_t))$ is increasing in $\beta(\mu^*_{t-1}; h_t)$, there exists some $\beta(\mu^*_{t-1}; h_t)$ solving the above equation. ■

Proof of Proposition 5

(i) Disclosure of public information in the strategic phase

Suppose there is a disclosure of public information at the beginning of period t which belongs to the strategic phase, either unexpected or expected. In the ex ante sense, without the new information, the expected payoff for individual i is

$$max\{U^A(\mu_i; h_t); U^W(\mu_i; h_t; s_{-i,t})\}$$

With the new information, the expected payoff for individual i is

$$E_{\tilde{H}^A_t(\mu_i; h_t; s_{-i,t})} U^A(\mu_i; \tilde{h}_t) + E_{\tilde{H}^W_t(\mu_i; h_t; s_{-i,t})} U^W(\mu_i; \tilde{h}_t)$$

where $\tilde{H}^A_t(\mu_i; h_t; s_{-i,t})$ is the set of histories at period t in which with the new information individual i will upgrade according to some strategy s_i. $\tilde{H}^W_t(\mu_i; h_t; s_{-i,t})$ is the set of histories at period t in which with the new information individual i will wait according to some strategy s_i.

Following the same logic as presented in the proof of Proposition 1, by the Martingale property,

$$U^A(\mu_i; h_t) = E_{\tilde{H}_t(h_t; s_{-i,t})} U^A(\mu_i; \tilde{h}_t) = E_{\tilde{H}^A_t(\mu_i; h_t; s_{-i,t})} U^A(\mu_i; \tilde{h}_t) + E_{\tilde{H}^W_t(\mu_i; h_t; s_{-i,t})} U^A(\mu_i; \tilde{h}_t)$$

$$U^W(\mu_i; h_t) = E_{\tilde{H}_t(h_t; s_{-i,t})} U^W(\mu_i; \tilde{h}_t) = E_{\tilde{H}^A_t(\mu_i; h_t; s_{-i,t})} U^W(\mu_i; \tilde{h}_t) + E_{\tilde{H}^W_t(\mu_i; h_t; s_{-i,t})} U^W(\mu_i; \tilde{h}_t)$$

Since

$$
\begin{aligned}
\forall \tilde{h}_t \in \tilde{H}^A_t(\mu_i; h_t; s_{-i,t}), U^A(\mu_i; \tilde{h}_t) \geq U^W(\mu_i; \tilde{h}_t) \\
\forall \tilde{h}_t \in \tilde{H}^W_t(\mu_i; h_t; s_{-i,t}), U^W(\mu_i; \tilde{h}_t) \geq U^A(\mu_i; \tilde{h}_t)
\end{aligned}
$$

we have

$$E_{\tilde{H}^A_t(\mu_i; h_t; s_{-i,t})} U^A(\mu_i; \tilde{h}_t) + E_{\tilde{H}^W_t(\mu_i; h_t; s_{-i,t})} U^W(\mu_i; \tilde{h}_t) \geq max\{U^A(\mu_i; h_t); U^W(\mu_i; h_t; s_{-i,t})\}$$

All the remaining individuals prefer to wait for the new information and make the appropriate decision. Thus, they welcome the new information in the ex ante sense.

The announcements of future disclosure of public information will increase the individuals' incentive to wait until its disclosure. In this case, some individuals may be worse off. For example, for the continuous private signal space, given the original equilibrium strategy profile $\{\mu_t^*(h_t)\}_t$, now at the beginning of period τ there is an announcement saying that at period $\tau + T$ there will be a disclosure of public information. With this announcement the equilibrium strategy profile changes to $\{\tilde{\mu}_t^*(h_t)\}_t$. Then at any history h_t, we have $\tilde{\mu}_t^*(h_t) \geq \mu_t^*(h_t)\ \forall \tau \leq t < \tau + T$. There exists the possibility that $\tilde{\mu}_{t+1}^*(h_{t+1}) < \mu_i \leq \mu_t^*(h_t)$. For these individuals, they will upgrade at period $t+1$ in both equilibria. But with less information in the new equilibrium, they are worse off.

(ii) Disclosure of public information in the waiting herding phase

Similar to the disclosure of public information in the strategic phase, if there is a disclosure of public information in the waiting herding phase, either unexpected or expected, all the remaining individuals welcome the new information. They prefer to wait for the new information and make the appropriate decision. The waiting herding phase is indeed not robust as the disclosure of public information only needs to induce the most optimistic individuals among the remainders to upgrade. A small amount of positive information about the new software package A can shatter a waiting herding phase. Then a new strategic phase starts. Everyone is better off in the ex ante sense.

Since at the waiting herding phase everyone has already waited, the announcements of future disclosure of public information even more greatly increase the incentive to wait. The waiting herding continues until the disclosure of public information happens.

(iii) Multiple disclosures of public information

Unlike BHW, even though multiple disclosures of public information can eventually shatter a waiting herding phase, they cannot always rule out the possibility that individuals settle into the wrong upgrade herding. Suppose the multiple disclosures of public information eventually reveal that waiting is the better choice. But as long as the individuals are optimistic enough, they will not wait for the possible future multiple disclosures of public information. Furthermore, the upgrade herding phase could start before the true value of the new software package A is revealed. The game may end with the wrong upgrade herd conclusion. ■

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